

System of Linear Equations

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Linear Equation

If we have an equation: $a \cdot x_1 + b \cdot x_2 + c \cdot x_3 = o$, we can write this in matrix form:

- $$\begin{bmatrix} a & b & c \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = [a \cdot x_1 + b \cdot x_2 + c \cdot x_3] = [o]$$

This is a linear equation, where we have a sequence of variables multiplied by coefficients, more generally, this is a linear equation with n unknown variables, and $n + 1$ known coefficients, note the a at the beginning:

- $$a + \beta_1 \cdot x_1 + \beta_2 \cdot x_2 + \dots + \beta_{n-1} \cdot x_{n-1} + \beta_n \cdot x_n = 0$$

In 2 dimension (with two unknowns), this is a line; in 3 dimension, this is a surface.

System of Linear Equations

We have a system of linear equations, 3 equations and 3 unknowns:

- $$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} a \cdot x_1 + b \cdot x_2 + c \cdot x_3 \\ d \cdot x_1 + e \cdot x_2 + f \cdot x_3 \\ g \cdot x_1 + h \cdot x_2 + i \cdot x_3 \end{bmatrix} = \begin{bmatrix} o \\ p \\ q \end{bmatrix}$$

We can define these:

- $$W = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$
- $$\mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$
- $$v = \begin{bmatrix} o \\ p \\ q \end{bmatrix}$$

Augmented Form

We can write W and v together like this, this is the augmented matrix of the system of linear equations:

- Augmented Matrix:**
$$\left[\begin{array}{ccc|c} a & b & c & o \\ d & e & f & p \\ g & h & i & q \end{array} \right]$$