

# Exponentiation and Infinitely Compounding Interest Rate

back to [Fan's Intro Math for Econ](#), [Matlab Examples](#), or [Dynamic Asset Repositories](#)

See also: [Exponential Function and Log Function](#).

## Exponential Function

- **Exponential Function:** Functions where the variable  $x$  appears as an *exponent*:  $a^x$
- $a$  is the base of Exponential function.

Remember that

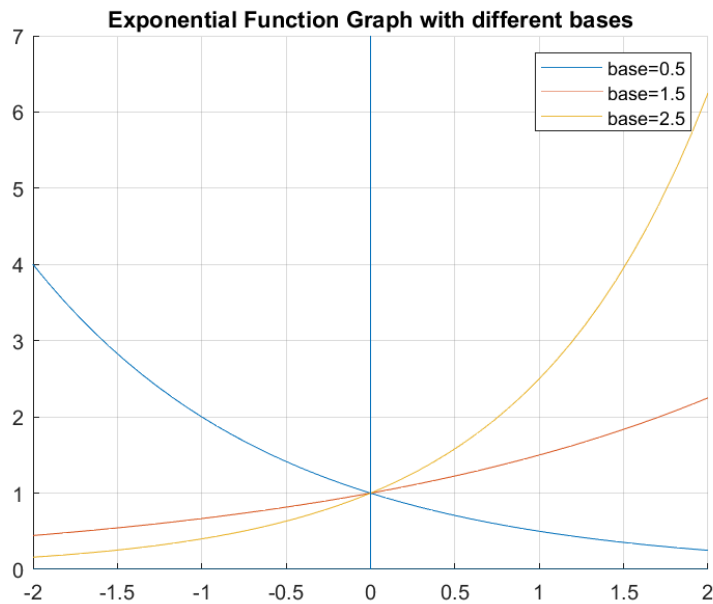
- $a^0 = 1$
- $a^{\frac{1}{2}} = \sqrt{a}$
- if  $a^b = c$ , we can also write,  $a = c^{\frac{1}{b}}$ , for example,  $2^3 = 8$ , and  $2 = 8^{\frac{1}{3}}$
- $a^{-b} = \frac{1}{a^b}$
- $x^a \cdot x^b = x^{a+b}$
- $x^{a \cdot b} = (x^a)^b$

## Exponential Function Graphs?

- Note that the domain of exponential function includes positive and negative  $x$ , and the exponential function will always be positive.
- If base is below 1, then the curve is monotonically downward sloping
- If base is above 1, then the curve is monotonically upwards sloping
- If base is above 1, higher base leads to steeper curvature.

```
syms x
a1 = 0.5;
f_a1 = a1^(x);
a2 = 1.5;
f_a2 = a2^(x);
a3 = 2.5;
f_a3 = a3^(x);
figure();
hold on;
fplot(f_a1, [-2, 2]);
fplot(f_a2, [-2, 2]);
fplot(f_a3, [-2, 2]);
line([0,0],ylim);
line(xlim, [0,0]);
title('Exponential Function Graph with different bases')
legend(['base=',num2str(a1)], ['base=',num2str(a2)], ['base=',num2str(a3)]);
```

grid on;



## Infinitely Compounding Interest rate

with 100 percent interest rate (APR), if we compound  $N$  times within a year, interest we pay at the end of the year is

- $(1 + \frac{1}{N})^N - 1$

Suppose  $N = 5$  (You can also think of this as a loan with interest rate of 20% for every 73 days), then we pay 159% interest rate by the end of the year.

```
r = 1.05;  
N = 5;  
(1 + r/N)^N - 1
```

```
ans = 1.5937
```

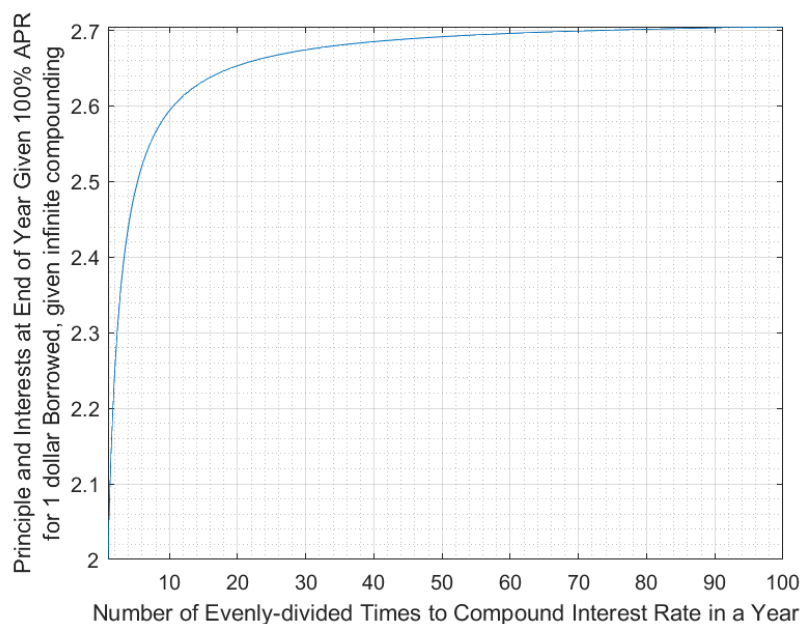
What if we do more and more compounding, if we say interest rate compounds 10, 50, 100 times over the year, what happens? With APR at 100%, the total interest rate you pay at the end of the year does not go to infinity, rather, it converges to this special number  $e$ , the Exponential number, 2.7182818, it is a magical number like  $\pi$ . This means if every second the interest rate is compounding, with an APR of 100%, you end up paying 272% of what you borrowed by the end of the year, which is 172% interest rate.

- $\lim_{N \rightarrow \infty} (1 + \frac{1}{N})^N = e \approx 2.7182818$

We can visualize this limit below

```
r = 1;
```

```
syms N
f_compoundR = (1 + r/N)^N;
figure();
fplot(f_compoundR, [1,100])
ylabel({'Principle and Interests at End of Year Given 100% APR' 'for 1 dollar Borrowed, given infinite compounding'})
xlabel('Number of Evenly-divided Times to Compound Interest Rate in a Year')
grid on;
grid minor
```



```
double(subs(f_compoundR,[1,2,3,4,5,6,7,8,9,10]))
```

```
ans = 1×10
2.0000    2.2500    2.3704    2.4414    2.4883    2.5216    2.5465    2.5658 ...
```

## Infinitely compounding Interest rate, different $r$ (APR $r$ )

Given:

- $\lim_{N \rightarrow \infty} \left(1 + \frac{1}{N}\right)^N = e \approx 2.7182818$

What is

- $\lim_{N \rightarrow \infty} \left(1 + \frac{r}{N}\right)^N?$

We can replace  $N$  by  $N = r \cdot M$

- $\lim_{N \rightarrow \infty} \left(1 + \frac{r}{N}\right)^N = \lim_{M \rightarrow \infty} \left(1 + \frac{r}{r \cdot M}\right)^{r \cdot M} = \left(\lim_{M \rightarrow \infty} \left(1 + \frac{1}{M}\right)^M\right)^r = e^r$

This gives the base  $e$  exponential function a financial interpretation.

```

syms x
f_e = exp(x);
figure();
hold on;
fplot(f_e, [-3, 3]);
line([0,0],ylim);
line(xlim, [0,0]);
title('Exponential Function Graph with base e')
xlabel('r = interest rate');
ylabel({'Principle and Interests at End of Year Given 100% APR' 'for 1 dollar Borrowed, given infinite compounding'});
grid on;

```

