Elasticity and Derivative

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Demand and Supply

At price p_0, the current price level, the demand and supply of good *x* (*x* could be capital, labor, apples ect) could be written as:

- $x_{\text{demand}} = \text{D}(p_0)$
- $x_{\text{supply}} = \text{S}(p_0)$

Note that we solve for the maximization problem of the demander of good *x* and the supplier of good *x* at price p_0 to find the quantity demanded and quantity supplied at this particular price. We derive the demand and supply curves by solving for quantity demanded and supplied at many prices points and connecting the resulting pairs of price and quantity demanded and supplied in a graph together.

How does demand (or supply) respond to a change in price?

What happens to demand and supply if p_0 increases to $p_0 + h$?

- $x_{\text{d}} = \text{D}(p_0 + h)$
- $x_{\text{s}} = \text{S}(p_0 + h)$

With normal goods, we expect that demand for *x* decreases when price increases, and supply for *x* increases when price increases.

How sensitive are demands to price changes?

If when movie ticket doubles in price, the number of theater goers goes down just a little bit, perhaps theater chains could make a lot more money by raising price. In this case, price has a hard time shifting demand, hence demand is fairly inelastic with respect to price. If orange juice buyers find apple juice to be largely substitutable, then if the price of orange juice goes up, demand for orange juice might decrease a lot as consumers switch to apple juice. In this case, price has an easy time shifting demand, hence demand is fairly elastic with repsect to price.

To avoid thinking about the unit of price and unit of goods, we think of percentage changes: what is the **percent** change in quantity of goods demanded given a **percent** change in the price of that good?

- $\frac{\text{Percent change in demand given } h \text{ change in price}}{\text{Percent change in price when price increase by } h}$

The price elasticity of demand at price p_0 given *h* increase in price is:

- $$\frac{\left(\frac{D(p_0+h)-D(p_0)}{D(p_0)}\right)}{\left(\frac{p_0+h-p_0}{p_0}\right)} = \left(\frac{D(p_0+h)-D(p_0)}{D(p_0)}\right) \cdot \left(\frac{p_0}{h}\right)$$

If we know how to solve for the optimal demand, we can calculate this at every point x_0 for small *h*.

Point Elasticity and Derivative

If you solve for the elasticity formula above, you will find that as *h* decreases, the price elasticity of demand at p_0 converges to a number. The number that the elasticity formula converges to is the **point price elasticity of demand:**

- $$\text{DemandElasticity}(p_0) = \lim_{h\to 0}\left(\left(\frac{D(p_0+h)-D(p_0)}{D(p_0)}\right) \cdot \left(\frac{p_0}{h}\right)\right)$$

Some of the terms in the fomrula do not include *h*, we can move them outside of the $\lim$ symbol

- $$\text{DemandElasticity}(p_0) = \left(\lim_{h\to 0}\left(\frac{D(p_0+h)-D(p_0)}{h}\right)\right) \cdot \frac{p_0}{D(p_0)}$$

This should look very familiary, it is exactly the formula for derivative of the demand funtion at p_0.

- $$\text{DemandElasticity}(p_0) = D'(p_0) \cdot \frac{p_0}{D(p_0)}$$

This formula applies to all price *p*

- $$\text{DemandElasticity}(p_0) = D'(p) \cdot \frac{p}{D(p)}$$

If we can derive the demand function, and it is differentiable over the domain of *p*, then we can solve analytically for demand elasticity as a function of *p*.

Inelastic, elastic and unit elastic

If the elasticity is 0, that means demand is fixed and does not change with price. If the demand elasticity obtained above is between 0 and -1, the good is inelastic with respect to price. If the price elasticity is less than -1, the good is elastic with respect to price. At 1, the good is unit elastic:

- $\begin{cases} -1 < \mathrm{D}'(p) \cdot \dfrac{p}{\mathrm{D}(p)} < 0, \text{ inelastic} \\ \mathrm{D}'(p) \cdot \dfrac{p}{\mathrm{D}(p)} = -1 \text{ , unit elastic} \\ \mathrm{D}'(p) \cdot \dfrac{p}{\mathrm{D}(p)} < -1 \text{ , elastic} \end{cases}$

Theorem 3.6: If a good is inelastic, an increasein price leads to an increase in total expenditure, for an elastic good, an increase in price leads to a decrease in total expenditures.