Higher Order Derivatives--Cobb Douglas

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We have the following general form for the Cobb-Douglas Production Function

$$Y(K,L) = K^{\alpha} \cdot L^{\beta}$$

The first order condition is

$$\frac{dY(K,L)}{dL} = (\beta) \cdot K^{\alpha} \cdot L^{\beta-1}$$

The derivative we have obtained is just another function. We can take additional derivatives with respect to this function.

$$\frac{\mathrm{d}^2 Y(K,L)}{dL^2} = (\beta) \cdot (\beta-1) \cdot K^{\alpha} \cdot L^{\beta-2}$$

Matlab symbolic toolbox gives us the same answer:

```
syms L K0 alpha beta
f(L, K0, alpha) = K0^(alpha)*L^(beta);
first_deri = diff(f, L)
```

first_deri(L, K0, alpha) = $K_0^{\alpha} L^{\beta-1} \beta$

```
second_deri = diff(diff(f, L),L)
```

second_deri(L, K0, alpha) = $K_0^{\alpha} L^{\beta-2} \beta (\beta-1)$

You can specify an additional parameter for the matlab *diff* function, if we want to take multiple derivatives:

```
syms L K0 alpha beta
f(L, K0, alpha) = K0^(alpha)*L^(beta);
% 5 for 5th derivative
tenth_deri = diff(f, L, 5)
```

tenth_deri(L, K0, alpha) = $K_0^{\alpha} L^{\beta-5} \beta (\beta-1) (\beta-2) (\beta-3) (\beta-4)$

Curvature and Second Derivative, Concave Function

Let's graph out the second derivative when $\beta = 0.5$. The production function is concave (concave down). For a function that is twice continuously differentiable, the function is convex if and only if its second derivative is non-positive (never accelerating).

```
alpha = 0.5;
beta = 0.5;
K0 = 1;
% Note that we have 1 symbolic variable now, the others are numbers
syms L
f(L) = K0^(alpha)*L^(beta);
```

```
% note f_diff1_L >= 0 always
f_diff1_L = diff(f, L)
```

```
f_diff1_L(L) =
```

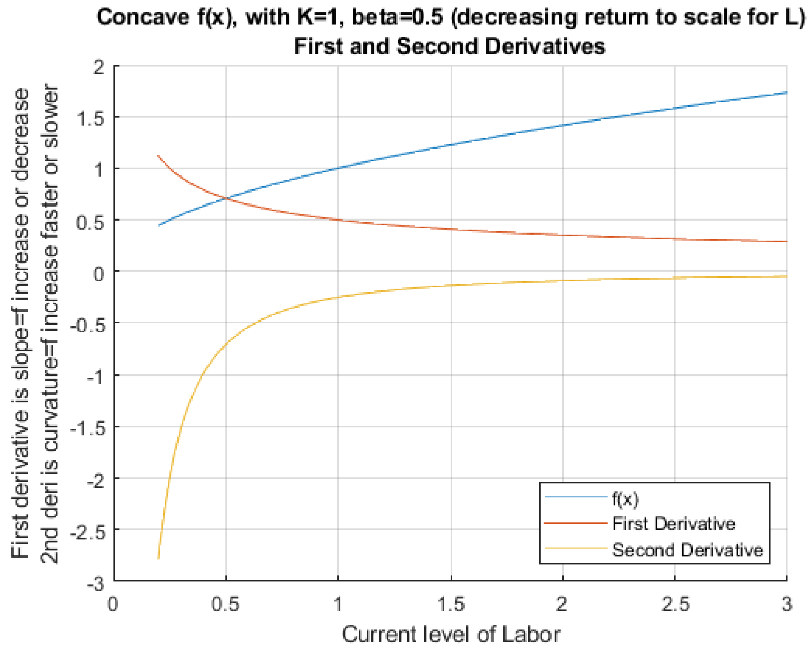
$$\frac{1}{2\sqrt{L}}$$

```
% note f_diff2_l <= 0 always
f_diff2_L = diff(f, L, 2)
```

```
f_diff2_L(L) =
```

$$-\frac{1}{4\,L^{3/2}}$$

```
% Start figure
figure();
hold on;
% fplot plots a function with one symbolic variable
fplot(f, [0.2, 3])
fplot(f_diff1_L, [0.2, 3])
fplot(f_diff2_L, [0.2, 3])
title({'Concave f(x), with K=1, beta=0.5 (decreasing return to scale for L)' 'First and Second
ylabel({'First derivative is slope=f increase or decrease' '2nd deri is curvature=f increase fa
xlabel('Current level of Labor')
legend(['f(x)'], ['First Derivative'], ['Second Derivative'], 'Location','SE');
grid on
```



Curvature and Second Derivative, Convex Function

Let's graph out the second derivative when $\beta = 1.2$. The production function is convex (concave up). For a function that is twice continuously differentiable, the function is convex if and only if its second derivative is non-negative (never decelerating).

```
alpha = 0.5;
beta = 1.2;
K0 = 1;
% Note that we have 1 symbolic variable now, the others are numbers
syms L
f(L) = K0^(alpha)*L^(beta);
% Note here f_diff1_L >= 0
f_diff1_L = diff(f, L)
```

f_diff1_L(L) =

$$\frac{6\,L^{1/5}}{5}$$

```
% Note here f_diff2_L >= 0
f_diff2_L = diff(f, L, 2)
```

f_diff2_L(L) =

$$\frac{6}{25\,L^{4/5}}$$

```
% Start figure
figure();
hold on;
% fplot plots a function with one symbolic variable
fplot(f, [0.1, 3])
fplot(f_diff1_L, [0.1, 3])
fplot(f_diff2_L, [0.1, 3])
title({'Convex f(x), with K=1, beta=1.2 (increasing return to scale for L)' 'First and Second D
ylabel({'First derivative is slope=f increase or decrease' '2nd deri is curvature=f increase fa
xlabel('Current level of Labor')
legend(['f(x)'], ['First Derivative'], ['Second Derivative'], 'Location','NW');
grid on
```