Laws of Matrix Algebra

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6 Old Rules, 5 Still Apply

We had associative, commutative and distributive laws for scalar algebra, we can think of them as the six bullet points below. Only the multiplicative-commutative law no longer works for matrix, the other rules work for matrix as well as scalar algebra.

Associative laws work as in scalar algebra for matrix

- $(A+B)+C = A+(B+C)$
- $(A \cdot B) \cdot C = A \cdot (B \cdot C)$

Commutative Law works as well for addition

- $A+B = B+A$
- with scalars, we know $3 \cdot 4 = 4 \cdot 3$, but commutative law for matrix multiplication does not work, Matrix $A \cdot B \neq B \cdot A$, because the matrix dimensions no longer match up for multiplication.

And Distributive Law still applies to matrix

- $A \cdot (B+C) = A \cdot B + A \cdot C$
- $(B+C) \cdot A = B \cdot A + C \cdot A$

Example for $A \cdot B \neq B \cdot A$

```
% Non-Square
A = rand(2,3)
```

```
A = 2×3
    0.6787    0.7431    0.6555
    0.7577    0.3922    0.1712
```

```
B = rand(3,4)
```

```
B = 3×4
    0.7060    0.0462    0.6948    0.0344
    0.0318    0.0971    0.3171    0.4387
    0.2769    0.8235    0.9502    0.3816
```

```
% This is OK
disp(A*B)
```

```
    0.6844    0.6433    1.3301    0.5995
    0.5949    0.2140    0.8135    0.2635
```

```
% This does not work
try
    B*A
catch ME
```

```
    disp('does not work! Dimension mismatch')
end
```

```
does not work! Dimension mismatch
```

```
% Square
A = rand(3,3)
```

```
A = 3×3
    0.7655    0.4898    0.7094
    0.7952    0.4456    0.7547
    0.1869    0.6463    0.2760
```

```
B = rand(3,3)
```

```
B = 3×3
    0.6797    0.1190    0.3404
    0.6551    0.4984    0.5853
    0.1626    0.9597    0.2238
```

```
% This is OK
A*B
```

```
ans = 3×3
    0.9565    1.0160    0.7060
    0.9551    1.0410    0.7004
    0.5953    0.6092    0.5037
```

```
% This works, but result differs from A*B
B*A
```

```
ans = 3×3
    0.6786    0.6059    0.6659
    1.0072    0.9212    1.0024
    0.9295    0.6519    0.9014
```

4 New Rules for Transpose

In scalar algebra, transpose does not make sense. Given matrix *A*, A^T is the transpose matrix of *A* where each row of *A* becomes columns in A^T. If *A* is *M* by *N*, then A^T is *N* by *M*.

Given matrix *A* and scalar value *r*:

- **1**: $(r \cdot A)^T = r \cdot A^T$
- **2**: $(A^T)^T = A$
- **3:** $(A + B)^T = A^T + B^T$
- **4**: $(A \cdot B)^T = B^T \cdot A^T$

For the 4th rule, suppose matrix *A* is has *L* rows and *M* columns, and the matrix *B* has *M* rows and *N*columns. $(A \cdot B)$ is a *L* by *N* matrix, $(A \cdot B)^T$ is a *N* by *L* matrix. This is equal to $B^T \cdot A^T$, where we have a *N* by *M* matrix B^T multiplied by a *M* by *L* matrix A^T, and the resulting matrix is *N* by *L*.

```
A = rand(2,3)
```

```
A = 2×3
    0.7513    0.5060    0.8909
    0.2551    0.6991    0.9593
```

```
A_transpose = (A')
```

```
A_transpose = 3×2
    0.7513    0.2551
    0.5060    0.6991
    0.8909    0.9593
```