

# Equilibrium Interest Rate

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We derived demand and supply for credit here: [Demand and Supply Derivation and Graphs](#).

We rewrite here the supply curve for credit which is a function of interest rate  $r$ :

- $$\text{Supply}(R) = Q_s = a - \frac{b}{(1+r)}$$

We can also rewrite the demand curve for credit which is a function of interest rate  $r$ :

- $$\text{Demand}(r) = Q_d = \frac{h}{r^k}$$

At equilibrium, demand equals to supply, shown graphically as the intersection point in [Demand and Supply Derivation and Graphs](#).

We can solve for equilibrium by trying out a vector of interest rate points, or using nonlinear solution methods.

Alternatively, although this is not a system of linear equations, we can approximate these equations using first order taylor approximation, then they become a system of linear equations. We can then using *linsolve* to find approximate equilibrium  $Q$  and  $r$ .

## First Order Taylor Approximation

Here, we discussed the formula for First Order Taylor Approximation: [Definition of Differentials](#). Using the formula we have from there:

- $$f(x) \approx f(a) + f'(a) \cdot (x - a)$$

We approximate the demand and Supply curves. Now  $x$  is the interest rate,  $f(x)$  is the demand or supply at interest rate  $x$  we are interested in.  $a$  is the interest rate level where we solve for actual demand or supply. We approximate the  $f(x)$  by using information from  $f(a)$ .

For the problem here, let us approximate around  $a = r_0 = 1$ , this is 100 percent interest rate.

Note the demand and supply curves are monotonic, and they are somewhat linear for segments of  $r$  values. If they are not monotonically increasing or decreasing, we should not use taylor approximation.

## Approximate the Supply

The Supply equation comes from [Optimal Savings Choice in a 2 period Model with initial Wealth](#), applying the formula above with  $a = r_0 = 1$ :

```
clear all
syms a b r
% Supply equation
```

```
S = a - b/(1+r);
% For Approximation, need to get the derivative with respect to R
SDiffR = diff(S, r)
```

```
SDiffR =
```

$$\frac{b}{(r+1)^2}$$

```
% Now evaluate S at r = 1 and evaluate S'(r) also at r = 1
SatRis1 = subs(S, r, 1)
```

```
SatRis1 =
```

$$a - \frac{b}{2}$$

```
SDiffRris1 = subs(SDiffR, r, 1)
```

```
SDiffRris1 =
```

$$\frac{b}{4}$$

```
% We now have an equation that approximates supply
SupplyApproximate = SatRis1 + SDiffRris1*(r-1)
```

```
SupplyApproximate =
```

$$a - \frac{b}{2} + \frac{b(r-1)}{4}$$

## Approximate the Demand

The Demand equation comes from [Optimal Borrowing Choice Firm Maximization](#), Applying the formula above with  $a = r_0 = 1$ :

```
clear all
syms h k r
% Supply equation
D = h/r^k;
% For Approximation, need to get the derivative with respect to R
DDiffR = diff(D, r)
```

```
DDiffR =
```

$$-\frac{hk}{r^{k+1}}$$

```
% Now evaluate D at r = 1 and evaluate D'(r) also at r = 1
DatRis1 = subs(D, r, 1)
```

```
DatRis1 = h
```

```
DDiffRris1 = subs(DDiffR, r, 1)
```

```
DDiffRris1 = -hk
```

% We now have an equation that approximates supply  
DemandApproximate = DatRis1 + DDiffRis1\*(r-1)

$$\text{DemandApproximate} = h - h k (r - 1)$$

## Solve approximate Demand and Supply using a System of Linear Equations

Now we have two linear equations with two unknowns, we can rearrange the terms. Note that only  $r$  and  $Q = Q_d = Q_s$  are unknowns, the other letters are parameters.

Starting with the equations from above:

- $S(r) \approx (a - \frac{b}{2}) + \frac{b}{4}(r - 1)$
- $D(r) \approx h - k \cdot h(r - 1)$

we end up with this system of two equations and two unknowns ([Solving for Two Equations and Two Unknowns](#)):

$$\bullet \begin{bmatrix} 1 & -\frac{b}{4} \\ 1 & k \cdot h \end{bmatrix} \cdot \begin{bmatrix} Q \\ r \end{bmatrix} = \begin{bmatrix} a - \frac{3}{4}b \\ h + k \cdot h \end{bmatrix}$$

We can plug this into matlab and solve for it

```
syms a b h k r
COEFMAT = [1, -b/4; 1, k*h];
OUTVEC = [a-(3*b)/4; h + k*h];
approximateSolution = linsolve(COEFMAT, OUTVEC);
QEquiApproximate = approximateSolution(1)
```

$$\text{QEquiApproximate} = \frac{b h + 4 a h k - 2 b h k}{b + 4 h k}$$

```
REquiApproximate = approximateSolution(2)
```

$$\text{REquiApproximate} = \frac{3 b - 4 a + 4 h + 4 h k}{b + 4 h k}$$

Now we have approximate analytical equations for demand and supply. If our  $a = r_0 = 1$  was close to true equilibrium rate, we would have a good approximation of how parameters of the model, the  $a, b, h, k$  constants, impact the equilibrium interest rate and quantity demanded and supplied.

See this page for how this is applied to the credit demand and supply example: [First Order Taylor Approximation of Demand and Supply for Capital](#)