

# Laws of Matrix Algebra

back to [Fan's Intro Math for Econ](#), [Matlab Examples](#), or [Dynamic Asset Repositories](#)

## 6 Old Rules, 5 Still Apply

We had associative, commutative and distributive laws for scalar algebra, we can think of them as the six bullet points below. Only the multiplicative-commutative law no longer works for matrix, the other rules work for matrix as well as scalar algebra.

Associative laws work as in scalar algebra for matrix

- $(A + B) + C = A + (B + C)$
- $(A \cdot B) \cdot C = A \cdot (B \cdot C)$

Commutative Law works as well for addition

- $A + B = B + A$
- with scalars, we know  $3 \cdot 4 = 4 \cdot 3$ , but commutative law for matrix multiplication does not work, Matrix  $A \cdot B \neq B \cdot A$ . The matrix dimensions might not even match up for multiplication. (see below for examples)

And Distributive Law still applies to matrix

- $A \cdot (B + C) = A \cdot B + A \cdot C$
- $(B + C) \cdot A = B \cdot A + C \cdot A$

## Example for $A \cdot B \neq B \cdot A$

```
% Non-Square  
A = rand(2,3)
```

```
A = 2x3  
    0.6959    0.6385    0.0688  
    0.6999    0.0336    0.3196
```

```
B = rand(3,4)
```

```
B = 3x4  
    0.5309    0.8200    0.5313    0.6110  
    0.6544    0.7184    0.3251    0.7788  
    0.4076    0.9686    0.1056    0.4235
```

```
% This is OK  
disp(A*B)
```

```
    0.8154    1.0960    0.5847    0.9516  
    0.5238    0.9076    0.4166    0.5891
```

```
% This does not work  
try  
    B*A
```

```
catch ME
    disp('does not work! Dimension mismatch')
end
```

does not work! Dimension mismatch

```
% Square
A = rand(3,3)
```

```
A = 3x3
    0.0908    0.2810    0.4574
    0.2665    0.4401    0.8754
    0.1537    0.5271    0.5181
```

```
B = rand(3,3)
```

```
B = 3x3
    0.9436    0.2407    0.6718
    0.6377    0.6761    0.6951
    0.9577    0.2891    0.0680
```

```
% This is OK
A*B
```

```
ans = 3x3
    0.7030    0.3441    0.2875
    1.3704    0.6147    0.5445
    0.9773    0.5431    0.5049
```

```
% This works, but result differs from A*B
B*A
```

```
ans = 3x3
    0.2531    0.7252    0.9904
    0.3449    0.8432    1.2437
    0.1745    0.4322    0.7263
```

## 4 New Rules for Transpose

In scalar algebra, transpose does not make sense. Given matrix  $A$ ,  $A^T$  is the transpose matrix of  $A$  where each row of  $A$  becomes columns in  $A^T$ . If  $A$  is  $M$  by  $N$ , then  $A^T$  is  $N$  by  $M$ .

Given matrix  $A$  and scalar value  $r$ :

- **1:**  $(r \cdot A)^T = r \cdot A^T$
- **2:**  $(A^T)^T = A$
- **3:**  $(A + B)^T = A^T + B^T$
- **4:**  $(A \cdot B)^T = B^T \cdot A^T$

For the 4th rule, suppose matrix  $A$  has  $L$  rows and  $M$  columns, and the matrix  $B$  has  $M$  rows and  $N$  columns.  $(A \cdot B)$  is a  $L$  by  $N$  matrix,  $(A \cdot B)^T$  is a  $N$  by  $L$  matrix. This is equal to  $B^T \cdot A^T$ , where we have a  $N$  by  $M$  matrix  $B^T$  multiplied by a  $M$  by  $L$  matrix  $A^T$ , and the resulting matrix is  $N$  by  $L$ .

```
A = rand(2,3)
```

```
A = 2×3  
    0.2548    0.6678    0.3445  
    0.2240    0.8444    0.7805
```

```
Atranspose = (A')
```

```
Atranspose = 3×2  
    0.2548    0.2240  
    0.6678    0.8444  
    0.3445    0.7805
```