

Approximating Credit Demand and Supply Curves--First Order Taylor Linear Approximation

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We derived demand and supply for credit here: [Demand and Supply Derivation and Graphs](#).

We rewrite here the supply curve for credit which is a function of interest rate r .

- $$\text{Supply}(R) = Q_s = a - \frac{b}{(1+r)}$$

We can also rewrite the demand curve for credit which is a function of interest rate r .

- $$\text{Demand}(r) = Q_d = \frac{h}{r^k}$$

At equilibrium, demand equals to supply, shown graphically as the intersection point in [Demand and Supply Derivation and Graphs](#).

We can solve for equilibrium by trying out a vector of interest rate points, or using nonlinear solution methods.

Alternatively, although this is not a system of linear equations, we can approximate these equations using first order taylor approximation, then they become a system of linear equations. We can then using *linsolve* to find approximate equilibrium Q and r .

First Order Taylor Approximation

Here, we discussed the formula for First Order Taylor Approximation: [Definition of Differentials](#). Using the formula we have from there:

- $$f(x) \approx f(a) + f'(a) \cdot (x - a)$$

We approximate the demand and Supply curves. Now x is the interest rate, $f(x)$ is the demand or supply at interest rate x we are interested in. a is the interest rate level where we solve for actual demand or supply. We approximate the $f(x)$ by using information from $f(a)$.

For the problem here, let us approximate around $a = r_0 = 1$, this is 100 percent interest rate.

Note the demand and supply curves are monotonic, and they are somewhat linear for segments of r values. If they are not monotonically increasing or decreasing, we should not use taylor approximation.

Approximate the Supply

The Supply equation comes from [Optimal Savings Choice in a 2 period Model with initial Wealth](#), applying the formula above with $a = r_0 = 1$:

```
clear all
syms a b r
```

```
% Supply equation
```

```
S = a - b/(1+r);
```

```
% For Approximation, need to get the derivative with respect to R
```

```
S_diff_r = diff(S, r)
```

```
S_diff_r =
```

$$\frac{b}{(r+1)^2}$$

```
% Now evaluate S at r = 1 and evaluate S'(r) also at r = 1
```

```
S_at_ris1 = subs(S, r, 1)
```

```
S_at_ris1 =
```

$$a - \frac{b}{2}$$

```
S_diff_r_ris1 = subs(S_diff_r, r, 1)
```

```
S_diff_r_ris1 =
```

$$\frac{b}{4}$$

```
% We now have an equation that approximates supply
```

```
Supply_Approximate = S_at_ris1 + S_diff_r_ris1*(r-1)
```

```
Supply_Approximate =
```

$$a - \frac{b}{2} + \frac{b(r-1)}{4}$$

Approximate the Demand

The Demand equation comes from [Optimal Borrowing Choice Firm Maximization](#), Applying the formula above with $a = r_0 = 1$:

```
clear all
```

```
syms h k r
```

```
% Supply equation
```

```
D = h/r^k;
```

```
% For Approximation, need to get the derivative with respect to R
```

```
D_diff_r = diff(D, r)
```

```
D_diff_r =
```

$$-\frac{hk}{r^{k+1}}$$

```
% Now evaluate D at r = 1 and evaluate D'(r) also at r = 1
```

```
D_at_ris1 = subs(D, r, 1)
```

```
D_at_ris1 = h
```

```
D_diff_r_ris1 = subs(D_diff_r, r, 1)
```

$$D_{diff_r_ris1} = -h k$$

% We now have an equation that approximates supply
 $Demand_Approximate = D_{at_ris1} + D_{diff_r_ris1} \cdot (r-1)$

$$Demand_Approximate = h - h k (r - 1)$$

Solve approximate Demand and Supply using a System of Linear Equations

Now we have two linear equations with two unknowns, we can rearrange the terms. Note that only r and $Q = Q_d = Q_s$ are unknowns, the other letters are parameters.

Starting with the equations from above:

- $S(r) \approx (a - \frac{b}{2}) + \frac{b}{4}(r - 1)$
- $D(r) \approx h - k \cdot h(r - 1)$

we end up with this system of two equations and two unknowns ([Solving for Two Equations and Two Unknowns](#)):

$$\begin{bmatrix} 1 & -\frac{b}{4} \\ 1 & k \cdot h \end{bmatrix} \cdot \begin{bmatrix} Q \\ r \end{bmatrix} = \begin{bmatrix} a - \frac{3}{4}b \\ h + k \cdot h \end{bmatrix}$$

We can plug this into matlab and solve for it

```
syms a b h k r
COEF_MAT = [1, -b/4; 1, k*h];
OUT_VEC = [a-(3*b)/4; h + k*h];
approximate_solution = linsolve(COEF_MAT, OUT_VEC);
Q_equi_approximate = approximate_solution(1)
```

$Q_{equi_approximate} =$

$$\frac{b h + 4 a h k - 2 b h k}{b + 4 h k}$$

$R_{equi_approximate} =$

$R_{equi_approximate} =$

$$\frac{3 b - 4 a + 4 h + 4 h k}{b + 4 h k}$$

Now we have approximate analytical equations for demand and supply. If our $a = r_0 = 1$ was close to true equilibrium rate, we would have a good approximation of how parameters of the model, the a, b, h, k constants, impact the equilibrium interest rate and quantity demanded and supplied.

See this page for how this is applied to the credit demand and supply example: [First Order Taylor Approximation of Demand and Supply for Capital](#)