# Laws of Matrix Algebra

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## 6 Old Rules, 5 Still Apply

We had associative, commutative and distributive laws for scalar algebra, we can think of them as the six bullet points below. Only the multiplicative-commutative law no longer works for matrix, the other rules work for matrix as well as scalar algebra.

Associative laws work as in scalar algebra for matrix

- $(A+B)+C=A+(B+C)$
- $(A\cdot B)\cdot C=A\cdot(B\cdot C)$

Commutative Law works as well for addition

- $A+B=B+A$
- with scalars, we know $3\cdot 4=4\cdot 3$, but commutative law for matrix multiplication does not work, Matrix $A\cdot B\neq B\cdot A$. The matrix dimensions might not even match up for multiplication. (see below for examples)

And Distributive Law still applies to matrix

- $A\cdot(B+C)=A\cdot B+A\cdot C$
- $(B+C)\cdot A=B\cdot A+C\cdot A$

## Example for $A\cdot B\neq B\cdot A$

```
% Non-Square
A = rand(2,3)
```

```
A = 2×3
    0.7655    0.1869    0.4456
    0.7952    0.4898    0.6463
```

```
B = rand(3,4)
```

```
B = 3×4
    0.7094    0.6797    0.1190    0.3404
    0.7547    0.6551    0.4984    0.5853
    0.2760    0.1626    0.9597    0.2238
```

```
% This is OK
disp(A*B)
```

```
    0.8071    0.7152    0.6119    0.4697
    1.1121    0.9664    0.9590    0.7020
```

```
% This does not work
try
    B*A
```

```
catch ME
    disp('does not work! Dimension mismatch')
end
```

```
does not work! Dimension mismatch
```

```
% Square
A = rand(3,3)
```

```
A = 3×3
    0.7513    0.6991    0.5472
    0.2551    0.8909    0.1386
    0.5060    0.9593    0.1493
```

```
B = rand(3,3)
```

```
B = 3×3
    0.2575    0.8143    0.3500
    0.8407    0.2435    0.1966
    0.2543    0.9293    0.2511
```

```
% This is OK
A*B
```

```
ans = 3×3
    0.9203    1.2905    0.5378
    0.8499    0.5535    0.2992
    0.9747    0.7843    0.4032
```

```
% This works, but result differs from A*B
B*A
```

```
ans = 3×3
    0.5783    1.2412    0.3060
    0.7932    0.9933    0.5232
    0.5551    1.2465    0.3055
```

## 4 New Rules for Transpose

In scalar algebra, transpose does not make sense. Given matrix *A*, $A^T$ is the transpose matrix of *A* where each row of *A* becomes columns in $A^T$. If *A* is *M* by *N*, then $A^T$ is *N* by *M*.

Given matrix *A* and scalar value *r*:

- **1**: $(r \cdot A)^T = r \cdot A^T$
- **2**: $(A^T)^T = A$
- **3:** $(A + B)^T = A^T + B^T$
- **4**: $(A \cdot B)^T = B^T \cdot A^T$

For the 4th rule, suppose matrix *A* is has *L* rows and *M* columns, and the matrix *B* has *M* rows and *N*columns. $(A \cdot B)$ is a *L* by *N* matrix, $(A \cdot B)^T$ is a *N* by *L* matrix. This is equal to $B^T \cdot A^T$, where we have a *N* by *M* matrix $B^T$ multiplied by a *M* by *L* matrix $A^T$, and the resulting matrix is *N* by *L*.

```
A = rand(2,3)
```

```
A = 2×3
    0.6160    0.3517    0.5853
    0.4733    0.8308    0.5497
```

```
A_transpose = (A')
```

```
A_transpose = 3×2
    0.6160    0.4733
    0.3517    0.8308
    0.5853    0.5497
```