Derivative Definition and Rules

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Definition

(SB) Let $(x_0, f(x_0))$ be a point on te graph of $y = f(x)$.

The **derivative** of *f* at x_0 is the slope of the tangent line to the graph of *f* at $(x_0, f(x_0))$.

There are some common ways of denoting derivative of funtion *f* at x_0:

- $f'(x_0)$
- $\frac{df}{dx}(x_0)$
- $\frac{dy}{dx}(x_0)$
- $f_x(x_0)$: this is popular in economics

We write this analyticaly as:

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

If this limit exists, then the function *f* is **differentiable** at x_0.

We will use this formula to derive first order taylor approximation. And this will also appear when we derive the formula for point elasticity.

Derivative Rules--Constant Rule

given constant *k*,:

- $f(x) = a \cdot x$
- $f'(x_0) = a$

```
syms x a
f(x, a) = a*x
```

f(x, a) = $a\,x$

```
diff_f_k = diff(f,x)
```

diff_f_k(x, a) = a

Derivative Rules--Power Rule (Polynomial Rule)

(SB) For any positive integer *k* (or real number *k*), the derivative of $f(x) = x^k$ at x_0 is:

- $f(x) = x^k$
- $f'(x_0) = k \cdot x_0^{k-1}$

```
syms x a k
f(x, a, k) = a*x^k
```

f(x, a, k) = $a\,x^k$

```
diff_f_k = diff(f,x)
```

diff_f_k(x, a, k) = $a\,k\,x^{k-1}$

Derivative Rules--Chain Rule

- $f(x) = p(q(x))$
- $f'(x_0) = p'(q(x_0)) \cdot q'(x_0)$

```
syms x a k
f(x, a, k) = (a*x)^k
```

f(x, a, k) = $(a\,x)^k$

```
diff_f_k = diff(f,x)
```

diff_f_k(x, a, k) = $a\,k\,(a\,x)^{k-1}$

Derivative Rules--Sum (and difference) Rule

Given functions *p* and *q* that are differentiable at *x*, then:

- $f(x) = p(x) + q(x)$
- $f'(x) = p'(x) + q'(x)$

```
syms x a b c d
f(x, a, b, c, d) = a*x^b + c*x^d
```

f(x, a, b, c, d) = $a\,x^b + c\,x^d$

```
diff_f_k = diff(f,x)
```

diff_f_k(x, a, b, c, d) = $a\,b\,x^{b-1} + c\,d\,x^{d-1}$

Derivative Rules--Product Rule

Given functions *p* and *q* that are differentiable at *x*, then:

- $f(x) = p(x) \cdot q(x)$
- $f'(x) = p'(x) \cdot q(x) + p(x) \cdot q'(x)$

```
syms x a b c d
f(x, a, b, c) = (a*x^b)*(c*x^d)
```

f(x, a, b, c) = $a\,c\,x^b\,x^d$

```
diff_f_k = diff(f,x)
```

diff_f_k(x, a, b, c) = $a\,b\,c\,x^d\,x^{b-1} + a\,c\,d\,x^b\,x^{d-1}$

Derivative Rules--Quotient Rule

Given functions *p* and *q* that are differentiable at *x*, then:

- $f(x) = \frac{p(x)}{q(x)}$
- $f'(x) = \frac{p'(x) \cdot q(x) - p(x) \cdot q'(x)}{(q(x))^2}$

Note that the quotient rule is based on the product rule, because:

- $f(x) = \frac{p(x)}{q(x)} = p(x) \cdot \frac{1}{q(x)}$

So you can derive the quotient rule formula based on the product rule where the first term is $p(x)$ and the second term is $\frac{1}{q(x)}$.

```
syms x a b c d
f(x, a, b, c) = (a*x^b)/(c*x^d)
```

f(x, a, b, c) =

$\frac{a\,x^b}{c\,x^d}$

```
diff_f_k = diff(f,x)
```

diff_f_k(x, a, b, c) =

$\frac{a\,b\,x^{b-1}}{c\,x^d} - \frac{a\,d\,x^b}{c\,x^{d+1}}$

Derivative Rules--Exponential

We use exponential functions in econmonics a lot:

- $f(x) = \exp(a \cdot x)$

- $f'(x) = a \cdot \exp(a \cdot x)$

```
syms x a
f(x, a) = exp(a*x)
```

f(x, a) = $e^{a\,x}$

```
diff_f_k = diff(f,x)
```

diff_f_k(x, a) = $a\,e^{a\,x}$

This is a special case of any power function

- $f(x) = c^{a \cdot x}$
- $f'(x) = a \cdot (\log c) \cdot c^{a \cdot x}$

note that $log(exp(c)) = c$

```
syms x a c
f(x, a, c) = c^(a*x)
```

f(x, a, c) = $c^{a\,x}$

```
diff_f_k = diff(f,x)
```

diff_f_k(x, a, c) = $a\,c^{a\,x}\log(c)$

Derivative Rules--Log

We use Log functions in economnics a lot:

- $f(x) = \log(a \cdot x)$
- $f'(x) = \frac{1}{x}$

note that the c cancels out.

```
syms x a
f(x, a) = log(a*x)
```

f(x, a) = $\log(a\,x)$

```
diff_f_k = diff(f,x)
```

diff_f_k(x, a) =

$$\frac{1}{x}$$