Laws of Matrix Algebra

back to Fan's Intro Math for Econon, Matlab Examples, or MEconTools Repositories

6 Old Rules, 5 Still Apply

We had associative, commutative and distributive laws for scalar algebra, we can think of them as the six bullet points below. Only the multiplicative-commutative law no longer works for matrix, the other rules work for matrix as well as scalar algebra.

Associative laws work as in scalar algebra for matrix

- $(A+B)+C=A+(B+C)$
- $(A \cdot B) \cdot C=A \cdot (B \cdot C)$

Commutative Law works as well for addition

- $A+B=B+A$
- with scalars, we know $3 \cdot 4=4 \cdot 3$, but commutative law for matrix multiplication does not work, Matrix $A \cdot B \neq B \cdot A$. The matrix dimensions might not even match up for multiplication. (see below for examples)

And Distributive Law still applies to matrix

- $A \cdot (B+C)=A \cdot B+A \cdot C$
- $(B+C) \cdot A=B \cdot A+C \cdot A$

Example for $A \cdot B \neq B \cdot A$

```
% Non-Square
A = rand(2,3)
```

```
A = 2×3
    0.6959    0.6385    0.0688
    0.6999    0.0336    0.3196
```

```
B = rand(3,4)
```

```
B = 3×4
    0.5309    0.8200    0.5313    0.6110
    0.6544    0.7184    0.3251    0.7788
    0.4076    0.9686    0.1056    0.4235
```

```
% This is OK
disp(A*B)
```

```
    0.8154    1.0960    0.5847    0.9516
    0.5238    0.9076    0.4166    0.5891
```

```
% This does not work
try
```

```
    B*A
catch ME
    disp('does not work! Dimension mismatch')
end
```

```
does not work! Dimension mismatch
```

```
% Square
A = rand(3,3)
```

```
A = 3×3
    0.0908    0.2810    0.4574
    0.2665    0.4401    0.8754
    0.1537    0.5271    0.5181
```

```
B = rand(3,3)
```

```
B = 3×3
    0.9436    0.2407    0.6718
    0.6377    0.6761    0.6951
    0.9577    0.2891    0.0680
```

```
% This is OK
A*B
```

```
ans = 3×3
    0.7030    0.3441    0.2875
    1.3704    0.6147    0.5445
    0.9773    0.5431    0.5049
```

```
% This works, but result differs from A*B
B*A
```

```
ans = 3×3
    0.2531    0.7252    0.9904
    0.3449    0.8432    1.2437
    0.1745    0.4322    0.7263
```

4 New Rules for Transpose

In scalar algebra, transpose does not make sense. Given matrix A, A^T is the transpose matrix of A where each row of A becomes columns in A^T. If A is M by N, then A^T is N by M.

Given matrix A and scalar value r:

- **1**: $(r \cdot A)^T = r \cdot A^T$
- **2**: $(A^T)^T = A$
- **3:** $(A + B)^T = A^T + B^T$
- **4**: $(A \cdot B)^T = B^T \cdot A^T$

For the 4th rule, suppose matrix A is has L rows and M columns, and the matrix B has M rows and Ncolumns. $(A \cdot B)$ is a L by N matrix, $(A \cdot B)^T$ is a N by L matrix. This is equal to $B^T \cdot A^T$, where we have a N by M matrix B^T multiplied by a M by L matrix A^T, and the resulting matrix is N by L.

```
A = rand(2,3)
```

```
A = 2×3
    0.2548    0.6678    0.3445
    0.2240    0.8444    0.7805
```

```
Atranspose = (A')
```

```
Atranspose = 3×2
    0.2548    0.2240
    0.6678    0.8444
    0.3445    0.7805
```