Matrix Addition and Multiplication

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Scalar Multiplication/Division, Addition/Subtraction

If we multiply a matrix by a number, we multiply every element of that matrix by that number. Addition, subtraction, and division of a matrix with a sclar value work the same way

```
c = 10
c = 10
matA = rand(3,2)
matA = 3 \times 2
    0.3111
               0.1848
    0.9234
               0.9049
    0.4302
               0.9797
c*matA
ans = 3 \times 2
    3.1110
               1.8482
               9.0488
    9.2338
               9.7975
    4.3021
matA/c
ans = 3 \times 2
    0.0311
               0.0185
    0.0923
               0.0905
    0.0430
               0.0980
matA - c
ans = 3 \times 2
   -9.6889
              -9.8152
              -9.0951
   -9.0766
   -9.5698
              -9.0203
matA + c
ans = 3 \times 2
              10.1848
   10.3111
              10.9049
   10.9234
              10.9797
   10.4302
```

Addition and Subtraction

You can add/subtract together two matrixes of the same size. We can add up the two 3 by 1 vectors from above, and the two 2 by 3 matrixes from above.

```
colVecA = rand(3,1)

colVecA = 3×1
    0.4389
    0.1111
```

```
colVecB = rand(3,1)
colVecB = 3 \times 1
    0.4087
    0.5949
    0.2622
matA = rand(3,2)
matA = 3 \times 2
    0.6028
              0.1174
    0.7112
              0.2967
    0.2217
              0.3188
matB = rand(3,2)
matB = 3 \times 2
    0.4242
              0.2625
    0.5079
              0.8010
    0.0855
              0.0292
colVecA + colVecB
ans = 3 \times 1
    0.8476
    0.7060
    0.5203
matA - matB
ans = 3 \times 2
    0.1787
              -0.1451
    0.2034
             -0.5043
    0.1362
              0.2896
```

When using matlab, even if you add up to a single column or single row with a matrix that has multiple rows and columns, if the column count or row count matches up, matlab will **broadcast** rules, and addition will still be legal. In the example below, matA is 3 by 2, and colVecA is 3 by 1, matlab duplicate colVecA and add it to each column of matA (*Broadcast rules are important for efficient storage and computation*):

```
matA + colVecA

ans = 3×2
    1.0417    0.5563
    0.8223    0.4078
    0.4798    0.5768
```

Matrix Multiplication

When we try to multiply two matrixes together: $A \cdot B$ for example, the *number of columns* of matrix A and the *number of rows* of matrix B have to match up.

If the matrix A is has L rows and M columns, and the matrix B has M rows and N columns, then the resulting matrix of $C = A \cdot B$ has to have L rows and N columns.

Each of the (l,n) cell in the product matrix $C=A\cdot B$, is equal to:

$$C_{l,n} = \sum_{m=1}^{M} A_{l,m} \cdot B_{m,n}$$

Note that we are summing over M: row l in matrix A, and column n in matrix B both have M elements. We multiply each m of the M element from the row in A and column in B together one by one, and then sum them up to end up with the value for the lth row and nth column in matrix C.

```
% (3 by 4) times (4 by 2) end up with (3 by 2)
L = 3;
M = 4;
N = 2;
matA = rand(L, M)
matA = 3 \times 4
                        0.9631
                                  0.2316
   0.9289
              0.5785
   0.7303
              0.2373
                        0.5468
                                  0.4889
   0.4886
              0.4588
                        0.5211
                                  0.6241
matB = rand(M, N)
matB = 4 \times 2
   0.6791
              0.0377
   0.3955
              0.8852
   0.3674
              0.9133
   0.9880
              0.7962
matC = matA*matB
matC = 3 \times 2
    1.4423
              1.6111
    1.2738
              1.1262
   1.3214
              1.3974
% (2 by 10) times (10 by 1) end up with (2 by 1)
L = 2;
M = 10;
N = 1;
matA = rand(L, M)
matA = 2 \times 10
   0.0987
              0.3354
                                  0.1068
                                             0.4942
                                                       0.7150
                                                                 0.8909
                                                                            0.6987 ...
                        0.1366
   0.2619
              0.6797
                        0.7212
                                  0.6538
                                             0.7791
                                                       0.9037
                                                                 0.3342
                                                                            0.1978
matB = rand(M, N)
matB = 10 \times 1
   0.9047
   0.6099
   0.6177
   0.8594
   0.8055
   0.5767
   0.1829
   0.2399
```

0.8865

0.0287

matC = matA*matB

 $matC = 2 \times 1$

1.6524

3.5895