Matrix Addition and Multiplication

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Scalar Multiplication/Division, Addition/Subtraction

If we multiply a matrix by a number, we multiply every element of that matrix by that number. Addition, subtraction, and division of a matrix with a sclar value work the same way

```
c = 10
```

```
c = 10
```

```
matA = rand(3,2)
```

```
matA = 3×2
    0.3111    0.1848
    0.9234    0.9049
    0.4302    0.9797
```

```
c*matA
```

```
ans = 3×2
    3.1110    1.8482
    9.2338    9.0488
    4.3021    9.7975
```

```
matA/c
```

```
ans = 3×2
    0.0311    0.0185
    0.0923    0.0905
    0.0430    0.0980
```

```
matA - c
```

```
ans = 3×2
   -9.6889   -9.8152
   -9.0766   -9.0951
   -9.5698   -9.0203
```

```
matA + c
```

```
ans = 3×2
   10.3111   10.1848
   10.9234   10.9049
   10.4302   10.9797
```

Addition and Subtraction

You can add/subtract together two matrixes of the same size. We can add up the two 3 by 1 vectors from above, and the two 2 by 3 matrixes from above.

```
colVecA = rand(3,1)
```

```
colVecA = 3×1
    0.4389
    0.1111
```

```
    0.2581
```

```
colVecB = rand(3,1)
```

```
colVecB = 3×1
    0.4087
    0.5949
    0.2622
```

```
matA = rand(3,2)
```

```
matA = 3×2
    0.6028    0.1174
    0.7112    0.2967
    0.2217    0.3188
```

```
matB = rand(3,2)
```

```
matB = 3×2
    0.4242    0.2625
    0.5079    0.8010
    0.0855    0.0292
```

```
colVecA + colVecB
```

```
ans = 3×1
    0.8476
    0.7060
    0.5203
```

```
matA - matB
```

```
ans = 3×2
    0.1787   -0.1451
    0.2034   -0.5043
    0.1362    0.2896
```

When using matlab, even if you add up to a single column or single row with a matrix that has multiple rows and columns, if the column count or row count matches up, matlab will ***broadcast*** rules, and addition will still be legal. In the example below, matA is 3 by 2, and colVecA is 3 by 1, matlab duplicate colVecA and add it to each column of matA (*Broadcast rules are important for efficient storage and computation*):

```
matA + colVecA
```

```
ans = 3×2
    1.0417    0.5563
    0.8223    0.4078
    0.4798    0.5768
```

Matrix Multiplication

When we try to multiply two matrixes together: $A \cdot B$ for example, the ***number of columns*** of matrix *A* and the ***number of rows*** of matrix *B* have to match up.

If the matrix *A* is has *L* rows and *M* columns, and the matrix *B* has *M* rows and *N*columns, then the resulting matrix of $C = A \cdot B$ has to have *L* rows and *N* columns.

Each of the (l, n) cell in the product matrix $C = A \cdot B$, is equal to:

- $$C_{l,n} = \sum_{m=1}^{M} A_{l,m} \cdot B_{m,n}$$

Note that we are summing over *M*: row *l* in matrix *A*, and column *n* in matrix *B* both have *M* elements. We multiply each *m* of the *M* element from the row in *A* and column in *B* together one by one, and then sum them up to end up with the value for the *l*th row and *n*th column in matrix *C*.

```
% (3 by 4) times (4 by 2) end up with (3 by 2)
L = 3;
M = 4;
N = 2;
matA = rand(L, M)
```

```
matA = 3×4
    0.9289    0.5785    0.9631    0.2316
    0.7303    0.2373    0.5468    0.4889
    0.4886    0.4588    0.5211    0.6241
```

```
matB = rand(M, N)
```

```
matB = 4×2
    0.6791    0.0377
    0.3955    0.8852
    0.3674    0.9133
    0.9880    0.7962
```

```
matC = matA*matB
```

```
matC = 3×2
    1.4423    1.6111
    1.2738    1.1262
    1.3214    1.3974
```

```

% (2 by 10) times (10 by 1) end up with (2 by 1)
L = 2;
M = 10;
N = 1;
matA = rand(L, M)
```

```
matA = 2×10
    0.0987    0.3354    0.1366    0.1068    0.4942    0.7150    0.8909    0.6987 ···
    0.2619    0.6797    0.7212    0.6538    0.7791    0.9037    0.3342    0.1978
```

```
matB = rand(M, N)
```

```
matB = 10×1
    0.9047
    0.6099
    0.6177
    0.8594
    0.8055
    0.5767
    0.1829
    0.2399
```

```
    0.8865
    0.0287
```

```
matC = matA*matB
```

```
matC = 2×1
    1.6524
    3.5895
```