BFW Equilibrium Gender LFP and Wage Code Companion

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Contents

Preface

This is a work-in-progress Matlab package consisting of functions that solve the equilibrium gender labor force participation and wage model in Bhalotra, Fernández and Wang (2022). Tested with Matlab 2021b (The MathWorks Inc, 2021).

All functions are parts of a matlab toolbox that can be installed:

> Download and install the Matlab toolbox: PrjLabEquiBFW.mltbx

The Code Companion can also be accessed via the bookdown site and PDF linked below:

> **bookdown pdf**, **MathWorks File Exchange**

This bookdown file is a collection of mlx based vignettes for functions that are available from PrjLabEquiBFW. Each Vignette file contains various examples for invoking each function.

The package relies on MEconTools, which needs to be installed first. The package does not include allocation functions, only simulation code to generate the value of each stimulus check increments for households.

The site is built using Bookdown (Xie, 2020).

Please contact FanWangEcon for issues or problems.

Chapter 1

Introduction

1.1 Bhalotra, Fernández, and Wang (2022)

In Bhalotra, Fernández, and Wang (2022).

Chapter 2

Core Functions

2.1 CES Demand Core Functions

This is the example vignette for function: **bfw_mp_func_demand** from the **PrjLabEquiBFW Package.** This function generates a container map with key CES demand-side equation for a particular sub-nest.

2.1.1 Default Test

Default test

```
bl_verbose = true;
mp_func_demand = bfw_mp_func_demand(bl_verbose);
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_func Functions
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                                  i      idx
                                 ----    ----    -----------------------------------------------------

    fc_OMEGA                     "1"     "1"     "@(p1,p2,rho,beta_1,beta_2)p1.*fc_d1(p1,p2,1,1,rho,be
    fc_d1                        "2"     "2"     "@(p1,p2,Y,Z,rho,beta_1,beta_2)(Y/Z).*(beta_1+beta_2.
    fc_d2                        "3"     "3"     "@(p1,p2,Y,Z,rho,beta_1,beta_2)(Y/Z).*(beta_1.*((p2./
    fc_lagrange_x1               "4"     "4"     "@(p1,rho,beta_1,beta_2,x_1,x_2)p1/(((beta_1*x_1^(rho
    fc_lagrange_x2               "5"     "5"     "@(p2,rho,beta_1,beta_2,x_1,x_2)p2/(((beta_1*x_1^(rho
    fc_output_nest               "6"     "6"     "@(q1,q2,rho,beta_1,beta_2)((beta_1)*q1^(rho)+beta_2*
    fc_p1_foc                    "7"     "7"     "@(lagrangem,rho,beta_1,beta_2,x_1,x_2)lagrangem*(((b
    fc_p2_foc                    "8"     "8"     "@(lagrangem,rho,beta_1,beta_2,x_1,x_2)lagrangem*(((b
    fc_share_given_elas_foc      "9"     "9"     "@(rho,p1,p2,x1,x2)fc_share_given_elas_foc_Q(rho,p1,p
    fc_w1dw2                     "10"    "10"    "@(x_1,x_2,rho,beta_1,beta_2)(x_2/x_1)^(1-rho)*(beta_
    fc_yz_ratio                  "11"    "11"    "@(p1,p2,q1,q2,rho,beta_1,beta_2)fc_revenue(p1,p2,q1,
```

Chapter 3

Demand

3.1 Solve Nested CES Optimal Demand (CRS)

Testing the **bfw_crs_nested_ces** function from the **PrjLabEquiBFW Package.** This function solves optimal choices given CES production function under cost minimization. Works with Constant Elasticity of Substitution problems with constant returns, up to four nest layers, and two inputs in each sub-nest. Takes as inputs share and elasticity parameters across layers of sub-nests, as well as input unit costs at the bottom-most layer. Works with Constant Elasticity of Substitution problems with constant returns, up to four nest layers, and two inputs in each sub-nest. Allows for uneven branches, so that some branches go up to four layers, but others have less layers, works with BFW (2022) nested labor input problem.

3.1.1 Key Inputs and Outputs for bfw_mp_func_demand

Here are the key inputs for the CES demand solver function:

- **FL_YZ** float output divided by productivity, aggregate single term
- **CL_MN_PRHO** cell array of rho (elasticity) parameter between negative infinity and 1. For example, suppose there are four nest layers, and there are two branches at each layer, then we have 1, 2, 4, and 8 ρ parameter values at the 1st, 2nd, 3rd, and 4th nest layers: size(CL_MN_PRHO{1})= $[1, 1]$, size(CL_MN_PRHO{2}) = $[1, 2]$, size(CL_MN_PRHO{3}) = $[2, 2]$, size(CL_MN_PRHO{4}) = $[2, 2, 2]$. Note that if the model has 4 nest layers, not all cells need to be specified, some branches could be deeper than others.
- **CL_MN_PSHARE** cell array of share (between 0 and 1) for the first input of the two inputs for each nest. The structure for this is similar to CL_MN_PRHO.
- **CL_MN_PRICE** cell array of wages for both wages for the first and second nest, the last index in each element of the cell array indicates first (1) or second (2) wage. For example, suppose we have four layers, with 2 branches at each layer, as in the example for CL_MN_PRHO, then we have 2, 4, 8, and 16 wage values at the 1st, 2nd, 3rd, and 4th nest layers: size(CL_MN_PRICE{1}) = $[1, 2]$, size(CL_MN_PRICE{2}) = $[2, 2]$, size(CL_MN_PRICE{3}) = $[2, 2, 2]$, size(CL_MN_PRICE{4}) = $[2, 2, 2, 2]$. Note that only the last layer of wage needs to be specified, in this case, the 16 wages at the 4th layer. Given optimal solutions, we solve for the 2, 4, and 8 aggregate wages at the higher nest layers. If some branches are deeper than other branches, then can specific NA for non-reached layers along some branches.
- **BL_BFW_MODEL** boolean true by default if true then will output outcomes specific to the BFW 2022 problem.

Here are the key outputs for the CES demand solver function:

- **CL_MN_YZ_CHOICES** has the same dimension as CL_MN_PRICE, suppose there are four layers, the CL_MN_PRICE{4} results at the lowest layer includes quantity choices that might be observed in the data. CL_MN_PRICE cell values at non-bottom layers include aggregate quantity outcomes.

- **CL_MN_PRICE** includes at the lowest layer observed wages, however, also includes higher layer aggregate solved waves. CL_MN_PRHO and CL_MN_PSHARE are identical to inputs.

3.1.2 Single Nest Layer Two Inputs CES Problem (Demand)

In this first example, we solve a constant returns to scale problem with a single nest, meaning just two inputs and a single output.

```
clc;
close all;
clear all;

% Output requirement
fl_yz = 1;
% rho = 0.5, 1/(1-0.5)=2, elasticity of substitution of 2
cl_mn_prho = {[0.1]};
% equal share, similar "productivity"
cl_mn_pshare = {[0.5]};
% wages for the two inputs, identical wage
cl_mn_price = {[1.5, 0.75]};
% print option
bl_verbose = true;
mp_func = bfw_mp_func_demand();
bl_bfw_model = false;
[cl_mn_yz_choices, cl_mn_price] = ...
    bfw_crs_nested_ces(fl_yz, cl_mn_prho, cl_mn_pshare, cl_mn_price, ...
    mp_func, bl_verbose, bl_bfw_model);
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                i    idx    ndim    numel    rowN    colN     sum       mean       std      coefvari
                -    ---    ----    -----    ----    ----    ------    ------    -------    --------

    price_c1    1     2      2        2       1       2        2.25     1.125    0.53033     0.4714
    yz_c1       2     4      2        2       1       2      2.1343    1.0671    0.55403    0.51918

xxx TABLE:price_c1 xxxxxxxxxxxxxxxxxx
          c1      c2
          ---    ----

    r1    1.5    0.75

xxx TABLE:yz_c1 xxxxxxxxxxxxxxxxxx
            c1         c2
          -------    ------

    r1    0.67537    1.4589

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                 i    idx    value
                 -    ---    -----
```

```
    prho_c1      1     1      0.1
    pshare_c1    2     3      0.5
```

3.1.3 Single Nest Layer Two Inputs CES Problem, Vary Share and Elasticity (Demand)

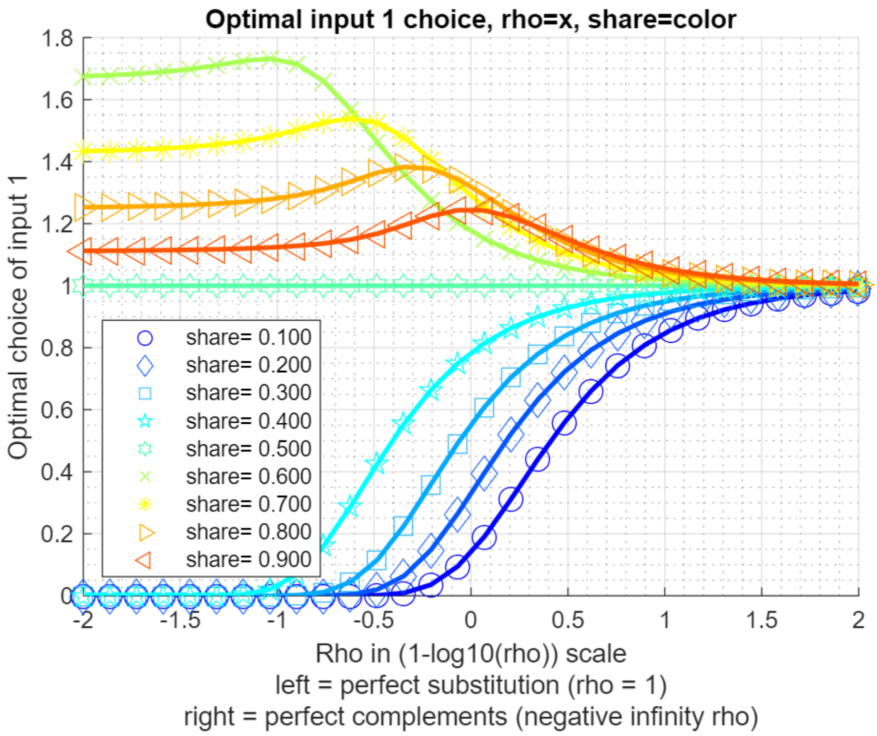
In this second example, we test over different rho values, explore optimal relative choices, as share and elasticity change. In this exercise, we also check, at every combination of rho and share parameter, whether the FOC condition is satisfied by the optimal choices. Also check if at the optimal choices, the minimization output requirement is met.

```
% Approximately close function
rel_tol=1e-09;
abs_tol=0.0;
if_is_close = @(a,b) (abs(a-b) <= max(rel_tol * max(abs(a), abs(b)), abs_tol));

% Define share and rho arrays
fl_yz = 1;
ar_pshare = linspace(0.1, 0.9, 9);
ar_prho = 1 - 10.^(linspace(-2, 2, 30));
% Loop over share and rho values
mt_rela_opti = NaN([length(ar_pshare), length(ar_prho)]);
mt_x1_opti = NaN([length(ar_pshare), length(ar_prho)]);
for it_pshare_ctr = 1:length(ar_pshare)
    for it_prho_ctr = 1:length(ar_prho)

        % A. Parameters
        % rho
        fl_prho = ar_prho(it_prho_ctr);
        cl_mn_prho = {[fl_prho]};
        % share
        fl_pshare = ar_pshare(it_pshare_ctr);
        cl_mn_pshare = {[fl_pshare]};
        % wages for the two inputs, identical wage
        cl_mn_price = {[1, 1]};
        % print option
        bl_verbose = false;

        % B. Call function
        [cl_mn_yz_choices, cl_mn_price, cl_mn_prho, cl_mn_pshare] = ...
            bfw_crs_nested_ces(fl_yz, cl_mn_prho, cl_mn_pshare, cl_mn_price, ...
            mp_func, bl_verbose, bl_bfw_model);
        % Store results for optimal choice
        fl_opti_x1 = cl_mn_yz_choices{1}(1);
        fl_opti_x2 = cl_mn_yz_choices{1}(2);
        mt_x1_opti(it_pshare_ctr, it_prho_ctr) = fl_opti_x1;

        % C. Check if relative optimality FOC condition is met
        fl_rela_opti = fl_opti_x1/fl_opti_x2;
        % From FOC give wages = 1 both
        % Using What is above Equation A.20 in draft.
        fl_rela_opti_foc = (((fl_pshare/(1-fl_pshare)))*(1))^(1/(1-ar_prho(it_prho_ctr)));
        if (~if_is_close(fl_rela_opti_foc, fl_rela_opti))
            error('There is an error, optimal relative not equal to expected foc ratio')
        end

        % D. Check if output quantity requirement is met
        fl_output = ((fl_pshare)*fl_opti_x1^(fl_prho) + (1-fl_pshare)*fl_opti_x2^(fl_prho))^(1/fl_pr
```

```
        if (~if_is_close(fl_output, fl_yz))
            error('There is an error, output is not equal to required expenditure minimizing output'
        end

    end
end
```

Key results: (1) As share parameter of input 1 goes to zero, optimal choice goes to zero when inputs are elastic; (2) When inputs are inelasticty, even very low share input 1 asymptote to equal input 2; (3) When input 1 is more productive (higher share), actually hire less as productivity (share) increases, becasue less of it is needed to achieve production for high rho, elastictic production function; (4) For inelastic production, monotonic relationship between input and shares.

```
% Visualize
% Generate some Data
rng(456);
ar_row_grid = ar_pshare;
ar_col_grid = log(1-ar_prho)/log(10);
rng(123);
mt_value = mt_x1_opti;
% container map settings
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'Optimal input 1 choice, rho=x, share=color'};
mp_support_graph('cl_st_ytitle') = {'Optimal choice of input 1'};
mp_support_graph('cl_st_xtitle') = {'Rho in (1-log10(rho)) scale', ...
    'left = perfect substitution (rho = 1)', ...
    'right = perfect complements (negative infinity rho)'};
mp_support_graph('st_legend_loc') = 'southwest';
mp_support_graph('bl_graph_logy') = false; % do not log
mp_support_graph('st_rowvar_name') = 'share=';
mp_support_graph('it_legend_select') = 5; % how many shock legends to show
mp_support_graph('st_rounding') = '6.3f'; % format shock legend
mp_support_graph('cl_colors') = 'jet'; % any predefined matlab colormap
% Call function
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```



3.1.4 Doubly Nest Layer Two Inputs Each Sub-nest CES Problem (Demand)

In this third example, solve for optimal choices for a doubly nested problem. Below, we first solve for the optimal choices, then we do a number of checks, to make sure that the solutions are correct, as expected.

```
% output requirement
fl_yz = 2.1;
% upper nest 0.1, lower nests 0.35 and -1 separately for rho values
cl_mn_prho = {[0.1], [0.35, -1]};
% unequal shares of share values
cl_mn_pshare = {[0.4], [0.3, 0.88]};
% differential wages
% in lower-left nest, not productive and very expensive, not very elastic
% last index for left or right,
cl_mn_price = {[nan, nan], [10, 1;3, 4]};
% print option
bl_verbose = true;
[cl_mn_yz_choices, cl_mn_price, cl_mn_prho, cl_mn_pshare] = ...
    bfw_crs_nested_ces(fl_yz, cl_mn_prho, cl_mn_pshare, cl_mn_price, ...
    mp_func, bl_verbose, bl_bfw_model);
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                 i    idx    ndim    numel    rowN    colN     sum       mean       std       coefvar
                 -    ---    ----    -----    ----    ----    ------    ------    -------    -------

    prho_c2      1     2      2        2       1       2       -0.65    -0.325    0.95459    -2.9372
    price_c1     2     3      2        2       1       2      7.7788    3.8894     2.0959    0.53886
    price_c2     3     4      2        4       2       2          18       4.5      3.873    0.86066
    pshare_c2    4     6      2        2       1       2        1.18      0.59    0.41012    0.69512
    yz_c1        5     7      2        2       1       2      4.4862    2.2431    0.68863      0.307
    yz_c2        6     8      2        4       2       2      9.0506    2.2626     2.7086     1.1971

xxx TABLE:prho_c2 xxxxxxxxxxxxxxxxxx
           c1     c2
          ----    --

    r1    0.35    -1

xxx TABLE:price_c1 xxxxxxxxxxxxxxxxxx
            c1        c2
          ------    ------

    r1    2.4074    5.3714

xxx TABLE:price_c2 xxxxxxxxxxxxxxxxxx
          c1    c2
          --    --

    r1    10    1
    r2     3    4

xxx TABLE:pshare_c2 xxxxxxxxxxxxxxxxxx
          c1      c2
          ---    ----

    r1    0.3    0.88
```

```
xxx TABLE:yz_c1 xxxxxxxxxxxxxxxxxx
           c1       c2   
          ----    ------

    r1    2.73    1.7561

xxx TABLE:yz_c2 xxxxxxxxxxxxxxxxxx
             c1         c2  
          --------    -------

    r1    0.047893     6.0934
    r2      2.2044    0.70496

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                 i    idx    value
                 -    ---    -----

    prho_c1      1     1      0.1 
    pshare_c1    2     5      0.4 

% there are four optimal choices, they are
fl_opti_x11 = cl_mn_yz_choices{2}(1,1);
fl_opti_x12 = cl_mn_yz_choices{2}(1,2);
fl_opti_x21 = cl_mn_yz_choices{2}(2,1);
fl_opti_x22 = cl_mn_yz_choices{2}(2,2);
% display
st_print = strjoin(...
    ["completed double nest test:", ...
    ['nest 1 input 1, fl_opti_x11=' num2str(fl_opti_x11)], ...
    ['nest 1 input 2, fl_opti_x12=' num2str(fl_opti_x12)], ...
    ['nest 2 input 1, fl_opti_x21=' num2str(fl_opti_x21)], ...
    ['nest 2 input 2, fl_opti_x22=' num2str(fl_opti_x22)], ...
    ], ";");
st_out = st_print;
ar_ch_out = char(strsplit(st_print,";")');
disp(ar_ch_out);

completed double nest test:
nest 1 input 1, fl_opti_x11=0.047893
nest 1 input 2, fl_opti_x12=6.0934
nest 2 input 1, fl_opti_x21=2.2044
nest 2 input 2, fl_opti_x22=0.70496
```

3.1.5 Doubly Nest Layer Two Inputs Each Sub-nest CES Problem–Solution Check (Demand)

Checking output equality, if there are problems, would output an error.

```
% A. Check output Equality
fl_pshare_0 = cl_mn_pshare{1}(1);
fl_pshare_1 = cl_mn_pshare{2}(1);
fl_pshare_2 = cl_mn_pshare{2}(2);
fl_prho_0 = cl_mn_prho{1}(1);
fl_prho_1 = cl_mn_prho{2}(1);
fl_prho_2 = cl_mn_prho{2}(2);
```

```
fl_output_1 = ((fl_pshare_1)*fl_opti_x11^(fl_prho_1) + (1-fl_pshare_1)*fl_opti_x12^(fl_prho_1))^(1/f
fl_output_2 = ((fl_pshare_2)*fl_opti_x21^(fl_prho_2) + (1-fl_pshare_2)*fl_opti_x22^(fl_prho_2))^(1/f
fl_output_0 = ((fl_pshare_0)*fl_output_1^(fl_prho_0) + (1-fl_pshare_0)*fl_output_2^(fl_prho_0))^(1/f
if (~if_is_close(fl_output_0, fl_yz))
    error('There is an error, output is not equal to required expenditure minimizing output')
end
```

Checking FOC within-nest optimality, if there are problems, would output an error.

```
% B. Check FOC Optimality inner nest
fl_wage_x11 = cl_mn_price{2}(1,1);
fl_wage_x12 = cl_mn_price{2}(1,2);
fl_wage_x21 = cl_mn_price{2}(2,1);
fl_wage_x22 = cl_mn_price{2}(2,2);

% B1. Checking via Method 1
fl_rela_opti_foc_1 = (((fl_pshare_1/(1-fl_pshare_1)))*(fl_wage_x12/fl_wage_x11))^(1/(1-fl_prho_1));
fl_rela_opti_foc_2 = (((fl_pshare_2/(1-fl_pshare_2)))*(fl_wage_x22/fl_wage_x21))^(1/(1-fl_prho_2));
if (~if_is_close(fl_rela_opti_foc_1, fl_opti_x11/fl_opti_x12))
    error('B1. There is an error, optimal relative not equal to expected foc ratio, nest 1')
end
if (~if_is_close(fl_rela_opti_foc_2, fl_opti_x21/fl_opti_x22))
    error('B1. There is an error, optimal relative not equal to expected foc ratio, nest 2')
end

% B2. Equation left to right, right to left, checking via method 2
% Check FOC Optimality cross nests (actually within) T1
fl_dy_dx11 = fl_pshare_1*(fl_opti_x11^(fl_prho_1-1));
fl_dy_dx12 = (1-fl_pshare_1)*(fl_opti_x12^(fl_prho_1-1));
fl_rwage_x11dx12 = fl_dy_dx11/fl_dy_dx12;
if (~if_is_close(fl_rwage_x11dx12, fl_wage_x11/fl_wage_x12))
    error('B2. There is an error, relative price x11 and x12 does not satisfy within optimality acro
end
```

Generate aggregate prices, if there are problems, would output an error.

```
% C. Aggregate prices and optimality within higher tier
% Is optimality satisfied given aggregate prices?
fl_rela_wage_share_11 = ...
    ((fl_wage_x11/fl_wage_x12)*((1-fl_pshare_1)/(fl_pshare_1)))^(fl_prho_1/(1-fl_prho_1));
fl_rela_wage_share_12 = ...
    ((fl_wage_x12/fl_wage_x11)*((fl_pshare_1)/(1-fl_pshare_1)))^(fl_prho_1/(1-fl_prho_1));
fl_agg_prc_1 = ...
    fl_wage_x11*(fl_pshare_1 + (1-fl_pshare_1)*(fl_rela_wage_share_11))^(-1/fl_prho_1) + ...
    fl_wage_x12*(fl_pshare_1*(fl_rela_wage_share_12) + (1-fl_pshare_1))^(-1/fl_prho_1);

fl_rela_wage_share_21 = ...
    ((fl_wage_x21/fl_wage_x22)*((1-fl_pshare_2)/(fl_pshare_2)))^(fl_prho_2/(1-fl_prho_2));
fl_rela_wage_share_22 = ...
    ((fl_wage_x22/fl_wage_x21)*((fl_pshare_2)/(1-fl_pshare_2)))^(fl_prho_2/(1-fl_prho_2));
fl_agg_prc_2 = ...
    fl_wage_x21*(fl_pshare_2 + (1-fl_pshare_2)*(fl_rela_wage_share_21))^(-1/fl_prho_2) + ...
    fl_wage_x22*(fl_pshare_2*(fl_rela_wage_share_22) + (1-fl_pshare_2))^(-1/fl_prho_2);

% What is returned by the omega function that is suppose to have aggregate prices?
mp_func = bfw_mp_func_demand();
params_group = values(mp_func, {'fc_OMEGA', 'fc_d1', 'fc_d2'});
[fc_OMEGA, fc_d1, fc_d2] = params_group{:};

% Aggregate price
```

```
fl_aggregate_price_1 = fc_OMEGA(...
    fl_wage_x11, fl_wage_x12, ...
    fl_prho_1, ...
    fl_pshare_1, 1 - fl_pshare_1);

fl_aggregate_price_2 = fc_OMEGA(...
    fl_wage_x21, fl_wage_x22, ...
    fl_prho_2, ...
    fl_pshare_2, 1 - fl_pshare_2);
```

Check relative price within nest and across nests, if there are problems, would output an error.

```
% D. Check FOC Optimality cross nests

% D1a. Two within-nest relative wages and four cross-nest relative wages
% within
fl_rwage_x11dx12 = fl_wage_x11/fl_wage_x12;
fl_rwage_x21dx22 = fl_wage_x21/fl_wage_x22;
% across
fl_rwage_x11dx21 = fl_wage_x11/fl_wage_x21;
fl_rwage_x11dx22 = fl_wage_x11/fl_wage_x22;
fl_rwage_x12dx21 = fl_wage_x12/fl_wage_x21;
fl_rwage_x12dx22 = fl_wage_x12/fl_wage_x22;

% D1b. Generate relative wages within nest and across nests own equations
fl_dy_dx1_shared = (fl_pshare_0*(fl_output_1)^(fl_prho_0-1))*((fl_output_1)^(1-fl_prho_1));
fl_dy_dx11 = fl_dy_dx1_shared*(fl_pshare_1*fl_opti_x11^(fl_prho_1-1));
fl_dy_dx12 = fl_dy_dx1_shared*((1-fl_pshare_1)*fl_opti_x12^(fl_prho_1-1));

fl_dy_dx2_shared = ((1-fl_pshare_0)*(fl_output_2)^(fl_prho_0-1))*((fl_output_2)^(1-fl_prho_2));
fl_dy_dx21 = fl_dy_dx2_shared*(fl_pshare_2*fl_opti_x21^(fl_prho_2-1));
fl_dy_dx22 = fl_dy_dx2_shared*((1-fl_pshare_2)*fl_opti_x22^(fl_prho_2-1));

% within
fl_rwage_x11dx12_foc = fl_dy_dx11/fl_dy_dx12;
fl_rwage_x21dx22_foc = fl_dy_dx21/fl_dy_dx22;
% across
fl_rwage_x11dx21_foc = fl_dy_dx11/fl_dy_dx21;
fl_rwage_x11dx22_foc = fl_dy_dx11/fl_dy_dx22;
fl_rwage_x12dx21_foc = fl_dy_dx12/fl_dy_dx21;
fl_rwage_x12dx22_foc = fl_dy_dx12/fl_dy_dx22;

if (~if_is_close(fl_rwage_x11dx21_foc, fl_wage_x11/fl_wage_x21))
    error('There is an error, relative price x11 and x21 does not satisfy cross optimality across ne
end
if (~if_is_close(fl_rwage_x12dx22_foc, fl_wage_x12/fl_wage_x22))
    error('There is an error, relative price x12 and x22 does not satisfy cross optimality across ne
end

% D2. Check FOC Optimality cross nests, simplified equation
fl_rela_wage_x11_x21 = log((fl_pshare_0/(1-fl_pshare_0))* ...
    ((fl_pshare_1*fl_opti_x11^(fl_prho_1-1)*fl_output_2^(fl_prho_2))/(fl_pshare_2*fl_opti_x21^(fl_pr
    fl_prho_0*log(fl_output_1/fl_output_2);
if (~if_is_close(fl_rela_wage_x11_x21, log(fl_wage_x11/fl_wage_x21)))
    error('There is an error, relative price x11 and x21 does not satisfy cross optimality across ne
end
```

3.1.6 BFW (2022) Nested Three Branch (Four Layer) Problem (Demand)

The model BFW 2022 has three branches and four layers. one of the branches go down only three layers, the other two branches go down four layers.

First, we prepare the various inputs:

```
% Controls
bl_verbose = true;
bl_bfw_model = true;

% Given rho and beta, solve for equilibrium quantities
bl_log_wage = false;
mp_func = bfw_mp_func_demand(bl_log_wage);

% Following instructions in: PrjFLFPMexicoBFW\solvedemand\README.md

% Nests/layers
it_nests = 4;

% Input cell of mn matrixes
it_prho_cl = 1;
it_pshare_cl = 2;
it_price_cl = 3;
for it_cl_ctr = [1,2,3]

    cl_mn_cur = cell(it_nests,1);

    % Fill each cell element with NaN mn array
    for it_cl_mn = 1:it_nests

        bl_price = (it_cl_ctr == it_price_cl);

        if (~bl_price && it_cl_mn == 1)
            mn_nan = NaN;
        elseif (~bl_price && it_cl_mn == 2) || (bl_price && it_cl_mn == 1)
            mn_nan = [NaN, NaN];
        elseif (~bl_price && it_cl_mn == 3) || (bl_price && it_cl_mn == 2)
            mn_nan = NaN(2,2);
        elseif (~bl_price && it_cl_mn == 4) || (bl_price && it_cl_mn == 3)
            mn_nan = NaN(2,2,2);
        elseif (~bl_price && it_cl_mn == 5) || (bl_price && it_cl_mn == 4)
            mn_nan = NaN(2,2,2,2);
        elseif (~bl_price && it_cl_mn == 6) || (bl_price && it_cl_mn == 5)
            mn_nan = NaN(2,2,2,2,2);
        end
        cl_mn_cur{it_cl_mn} = mn_nan;
    end

    % Name cell arrays
    if (it_cl_ctr == it_prho_cl)
        cl_mn_prho = cl_mn_cur;
    elseif (it_cl_ctr == it_pshare_cl)
        cl_mn_pshare = cl_mn_cur;
    elseif (it_cl_ctr == it_price_cl)
        cl_mn_price = cl_mn_cur;
    end
end

% Initialize share matrix
```

```
rng(123);
for it_cl_mn = 1:it_nests
    mn_pshare = cl_mn_pshare{it_cl_mn};
    if it_cl_mn == 4
        mn_pshare(2,:,:) = rand(2,2);
    else
        mn_pshare = rand(size(mn_pshare));
    end
    cl_mn_pshare{it_cl_mn} = mn_pshare;
end

% Initialize rho matrix
rng(456);
for it_cl_mn = 1:it_nests
    mn_prho = cl_mn_prho{it_cl_mn};
    if it_cl_mn == 4
        mn_prho(2,:,:) = rand(2,2);
    else
        mn_prho = rand(size(mn_prho));
    end
    % Scalling rho between 0.7500 and -3.0000
    % 1 - 2.^(linspace(-2,2,5))
    mn_prho = 1 - 2.^(mn_prho*(4) - 2);
    cl_mn_prho{it_cl_mn} = mn_prho;
end

% Initialize wage matrix
rng(789);
for it_cl_mn = 1:it_nests
    mn_price = cl_mn_price{it_cl_mn};
    if it_cl_mn == 3
        mn_price(1,:,:) = rand(2,2);
    elseif it_cl_mn == 4
        mn_price(2,:,:,:) = rand(2,2,2);
    end
    % Scalling rho between 3 amd 5
    mn_price = mn_price*(2) + 3;
    cl_mn_price{it_cl_mn} = mn_price;
end

% Initialize yz matrix
rng(101112);
fl_yz = rand();
```

Second, display created inputs:

```
disp(['fl_yz=' num2str(fl_yz)]);
```

```
fl_yz=0.89726
```

```
celldisp(cl_mn_prho);
```

```
cl_mn_prho

cl_mn_prho{1} =

    0.5017
```

```
cl_mn_prho{2} =

    0.6071   -1.1955



cl_mn_prho{3} =

   -1.3523   -0.3346
   -0.4167   -1.9136



cl_mn_prho{4} =


(:,:,1) =

       NaN       NaN
   -1.0512    0.5869


(:,:,2) =

       NaN       NaN
    0.6209    0.1633



celldisp(cl_mn_pshare);

cl_mn_pshare

cl_mn_pshare{1} =

    0.6965



cl_mn_pshare{2} =

    0.2861    0.2269



cl_mn_pshare{3} =

    0.5513    0.4231
    0.7195    0.9808



cl_mn_pshare{4} =


(:,:,1) =
```

```
       NaN       NaN
    0.6848    0.4809


(:,:,2) =

       NaN       NaN
    0.3921    0.3432



celldisp(cl_mn_price);

cl_mn_price

cl_mn_price{1} =

   NaN   NaN



cl_mn_price{2} =

   NaN   NaN
   NaN   NaN



cl_mn_price{3} =


(:,:,1) =

    3.6467    3.4605
       NaN       NaN


(:,:,2) =

    4.5876    4.2488
       NaN       NaN



cl_mn_price{4} =


(:,:,1,1) =

       NaN       NaN
    4.9508    4.5178


(:,:,2,1) =

       NaN       NaN
    3.0212    3.0495
```

```
(:,:,1,2) =

      NaN       NaN
   3.2221    4.0763


(:,:,2,2) =

      NaN       NaN
   3.0909    4.1031
```

Third, call function and solve for optimal demand:

```
% Call function
[cl_mn_yz_choices, cl_mn_price, cl_mn_prho, cl_mn_pshare] = ...
    bfw_crs_nested_ces(fl_yz, cl_mn_prho, cl_mn_pshare, cl_mn_price, ...
    mp_func, bl_verbose, bl_bfw_model);
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                            i     idx    ndim    numel    rowN    colN      sum         mean
                            --    ---    ----    -----    ----    ----    --------    --------    --

    mt_fl_labor_demanded     1      1      2       12       4       3       5.4455     0.45379    0
    prho_c2                  2      3      2        2       1       2     -0.58844    -0.29422
    prho_c3                  3      4      2        4       2       2      -4.0173     -1.0043    0
    prho_c4                  4      5      3        8       2       4          NaN         NaN
    price_c1                 5      6      2        2       1       2       35.345      17.673
    price_c2                 6      7      2        4       2       2       40.906      10.226
    price_c3                 7      8      3        8       2       4       45.403      5.6754
    price_c4                 8      9      4       16       2       8          NaN         NaN
    pshare_c2                9     11      2        2       1       2      0.51299      0.2565    0.
    pshare_c3               10     12      2        4       2       2       2.6747     0.66866    0
    pshare_c4               11     13      3        8       2       4          NaN         NaN
    yz_c1                   12     14      2        2       1       2       1.6003     0.80016
    yz_c2                   13     15      2        4       2       2        2.645     0.66124
    yz_c3                   14     16      3        8       2       4       5.1962     0.64953
    yz_c4                   15     17      4       16       2       8          NaN         NaN

xxx TABLE:mt_fl_labor_demanded xxxxxxxxxxxxxxxxxx
              c1          c2          c3
           --------    --------    -------

    r1     0.020122    0.024929     2.1857
    r2     0.060227    0.037985     2.3642
    r3     0.069088    0.093774    0.21107
    r4     0.058349     0.14469    0.17539

xxx TABLE:prho_c2 xxxxxxxxxxxxxxxxxx
             c1         c2
           -------    -------

    r1     0.60709    -1.1955

xxx TABLE:prho_c3 xxxxxxxxxxxxxxxxxx
              c1          c2
```

```
          --------    --------

    r1     -1.3523    -0.33464
    r2    -0.41668     -1.9136

xxx TABLE:prho_c4 xxxxxxxxxxxxxxxxxx
            c1         c2         c3         c4
          -------    -------    -------    -------

    r1        NaN        NaN        NaN        NaN
    r2    -1.0512    0.58694    0.62089    0.16334

xxx TABLE:price_c1 xxxxxxxxxxxxxxxxxx
            c1       c2
          ------    -----

    r1    12.695    22.65

xxx TABLE:price_c2 xxxxxxxxxxxxxxxxxx
            c1        c2
          ------    ------

    r1    8.1518    7.7015
    r2    13.522     11.53

xxx TABLE:price_c3 xxxxxxxxxxxxxxxxxx
            c1        c2        c3        c4
          ------    ------    ------    ------

    r1    3.6467    3.4605    4.5876    4.2488
    r2    8.1184    8.5114    5.7986    7.0309

xxx TABLE:price_c4 xxxxxxxxxxxxxxxxxx
            c1        c2        c3        c4        c5        c6        c7        c8
          ------    ------    ------    ------    ------    ------    ------    ------

    r1       NaN       NaN       NaN       NaN       NaN       NaN       NaN       NaN
    r2    4.9508    4.5178    3.0212    3.0495    3.2221    4.0763    3.0909    4.1031

xxx TABLE:pshare_c2 xxxxxxxxxxxxxxxxxx
            c1         c2
          -------    -------

    r1    0.28614    0.22685

xxx TABLE:pshare_c3 xxxxxxxxxxxxxxxxxx
            c1         c2
          -------    -------

    r1    0.55131    0.42311
    r2    0.71947    0.98076

xxx TABLE:pshare_c4 xxxxxxxxxxxxxxxxxx
            c1         c2         c3         c4
          -------    -------    -------    -------

    r1        NaN        NaN        NaN        NaN
    r2    0.68483    0.48093    0.39212    0.34318
```

```
xxx TABLE:yz_c1 xxxxxxxxxxxxxxxxxx
           c1         c2
          -----    --------

    r1    1.511    0.089284

xxx TABLE:yz_c2 xxxxxxxxxxxxxxxxxx
             c1         c2
          --------    ------

    r1     0.19312    2.2864
    r2    0.057461     0.108

xxx TABLE:yz_c3 xxxxxxxxxxxxxxxxxx
            c1         c2          c3          c4
          -------    -------    --------    -------

    r1    0.21107     2.1857     0.17539     2.3642
    r2    0.06529    0.11907    0.042587    0.03298

xxx TABLE:yz_c4 xxxxxxxxxxxxxxxxxx
             c1          c2          c3          c4          c5          c6         c7          c8
          --------    --------    --------    --------    --------    -------    --------    -------

    r1         NaN         NaN         NaN         NaN         NaN        NaN         NaN         Na
    r2    0.069088    0.093774    0.020122    0.024929    0.058349    0.14469    0.060227    0.03798

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                 i    idx     value
                 -    ---    -------

    prho_c1      1     2     0.50172
    pshare_c1    2    10     0.69647
```

3.2 Compute Nested CES MPL Given Demand (CRS)

Testing the **bfw_crs_nested_ces_mpl** function from the **PrjLabEquiBFW Package.** Given labor quantity demanded, using first-order relative optimality conditions, find the marginal product of labor given CES production function. Results match up with correct relative wages, but not wage levels. Takes as inputs share and elasticity parameters across layers of sub-nests, as well as quantity demanded at each bottom-most CES nest layer. Works with Constant Elasticity of Substitution problems with constant returns, up to four nest layers, and two inputs in each sub-nest. Allows for uneven branches, so that some branches go up to four layers, but others have less layers, works with BFW (2022) nested labor input problem.

3.2.1 Key Inputs and Outputs for bfw_crs_nested_ces_mpl

Here are the key inputs for the CES demand solver function:

- **CL_MN_PRHO** cell array of rho (elasticity) parameter between negative infinity and 1. For example, suppose there are four nest layers, and there are two branches at each layer, then we have 1, 2, 4, and 8 ρ parameter values at the 1st, 2nd, 3rd, and 4th nest layers: size(CL_MN_PRHO{1})= $[1, 1]$, size(CL_MN_PRHO{2}) = $[1, 2]$, size(CL_MN_PRHO{3}) = $[2, 2]$, size(CL_MN_PRHO{4}) = $[2, 2, 2]$. Note that if the model has 4 nest layers, not all cells need to be specified, some branches could be deeper than others.

- **CL_MN_PSHARE** cell array of share (between 0 and 1) for the first input of the two inputs for each nest. The structure for this is similar to CL_MN_PRHO.
- **CL_MN_YZ_CHOICES** cell array of quantity demanded for the first and second inputs of the bottom-most layer of sub-nests. The last index in each element of the cell array indicates first (1) or second (2) quantities. For example, suppose we have four layers, with 2 branches at each layer, as in the example for CL_MN_PRHO, then we have 2, 4, 8, and 16 quantity values at the 1st, 2nd, 3rd, and 4th nest layers: size(CL_MN_YZ_CHOICES{1})= $[1, 2]$, size(CL_MN_YZ_CHOICES{2})= $[2, 2]$, size(CL_MN_YZ_CHOICES{3})= $[2, 2, 2]$, size(CL_MN_YZ_CHOICES{4})= $[2, 2, 2, 2]$. Note that only the last layer of quantities needs to be specified, in this case, the 16 quantities at the 4th layer. Given first order conditions, we solve for the 2, 4, and 8 aggregate quantities at the higher nest layers. If some branches are deeper than other branches, then can specific NA for non-reached layers along some branches.
- **BL_BFW_MODEL** boolean true by default if true then will output outcomes specific to the BFW 2022 problem.

Here are the key outputs for the CES demand solver function:

- **CL_MN_MPL_PRICE** has the same dimension as CL_MN_YZ_CHOICES, suppose there are four layers, the CL_MN_MPL_PRICE{4} results at the lowest layer includes wages that might be observed in the data. CL_MN_MPL_PRICE cell values at non-bottom layers include aggregate wages.
- **CL_MN_YZ_CHOICES** includes at the lowest layer observed wages, however, also includes higher layer aggregate solved quantities. CL_MN_PRHO and CL_MN_PSHARE are identical to inputs.

3.2.2 Single Nest Layer Two Inputs CES Problem (MPL)

In this first example, we solve a constant returns to scale problem with a single nest, meaning just two inputs and a single output.

```
clc;
close all;
clear all;

% rho = 0.5, 1/(1-0.5)=2, elasticity of substitution of 2
cl_mn_prho = {[0.1]};
% equal share, similar "productivity"
cl_mn_pshare = {[0.5]};
% levels of the two inputs, Values picked from demand problem parallel
% example.
cl_mn_yz_choices = {[0.67537, 1.4589]};
% print option
bl_verbose = true;
mp_func = bfw_mp_func_demand();
bl_bfw_model = false;
[cl_mn_yz_choices, cl_mn_mpl_price] = ...
    bfw_crs_nested_ces_mpl(cl_mn_prho, cl_mn_pshare, cl_mn_yz_choices, ...
    mp_func, bl_verbose, bl_bfw_model);
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum       mean        std      coe
                    -    ---    ----    -----    ----    ----    ------    -------    -------    ---

    mpl_price_c1    1     1      2        2       1       2     1.0678    0.53388    0.25168    0.4
```

```
    yz_c1           2     4      2        2       1       2      2.1343     1.0671    0.55404    0.5

xxx TABLE:mpl_price_c1 xxxxxxxxxxxxxxxxxx
            c1         c2
          -------    -------

    r1    0.71184    0.35592

xxx TABLE:yz_c1 xxxxxxxxxxxxxxxxxx
            c1         c2
          -------    ------

    r1    0.67537    1.4589

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                 i    idx    value
                 -    ---    -----

    prho_c1      1     2      0.1
    pshare_c1    2     3      0.5
```

3.2.3 Single Nest Layer Two Inputs CES Problem, Vary Share and Elasticity (MPL)

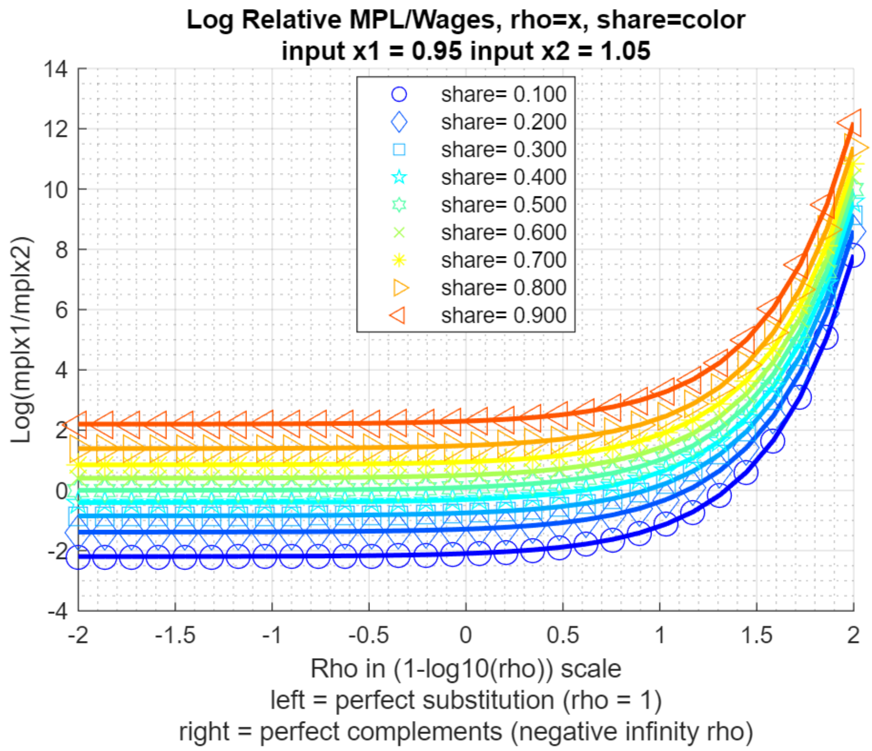
In this second example, we test over different rho values, explore optimal relative choices, as share and elasticity change. In this exercise, we also check, at every combination of rho and share parameter, whether the FOC condition is satisfied by the optimal choices. Also check if at the optimal choices, the minimization output requirement is met.

```
% Approximately close function
rel_tol=1e-09;
abs_tol=0.0;
if_is_close = @(a,b) (abs(a-b) <= max(rel_tol * max(abs(a), abs(b)), abs_tol));

% Input 1 and 2 fixed
fl_x_1 = 0.95;
fl_x_2 = 1.05;

% Define share and rho arrays
ar_pshare = linspace(0.1, 0.9, 9);
ar_prho = 1 - 10.^(linspace(-2, 2, 30));
% Loop over share and rho values
mt_rela_opti = NaN([length(ar_pshare), length(ar_prho)]);
mt_rela_wage = NaN([length(ar_pshare), length(ar_prho)]);
for it_pshare_ctr = 1:length(ar_pshare)
    for it_prho_ctr = 1:length(ar_prho)

        % A. Parameters
        % rho
        fl_prho = ar_prho(it_prho_ctr);
        cl_mn_prho = {[fl_prho]};
        % share
        fl_pshare = ar_pshare(it_pshare_ctr);
        cl_mn_pshare = {[fl_pshare]};
        % wages for the two inputs, identical wage
        % Note that if chosee {[1,1]} below, log(1/1) = log(1) = 0,
```

```
        % elasticity does not matter.
        cl_mn_yz_choices = {[fl_x_1, fl_x_2]};
        % print option
        bl_verbose = false;

        % B. Call function
        [cl_mn_yz_choices, cl_mn_mpl_price] = ...
            bfw_crs_nested_ces_mpl(cl_mn_prho, cl_mn_pshare, cl_mn_yz_choices, ...
            mp_func, bl_verbose, bl_bfw_model);
        % Store results for mpl given input choices
        fl_mpl_x1 = cl_mn_mpl_price{1}(1);
        fl_mpl_x2 = cl_mn_mpl_price{1}(2);
        mt_rela_wage(it_pshare_ctr, it_prho_ctr) = log(fl_mpl_x1/fl_mpl_x2);
    end
end
```

Key results: (1) As share parameter of input 1 goes to zero, input 1 is less productive, and the log(mplx1/mplx2) ratio is lower. (2) Becaus x2 input in this example is larger than x1 input, so as two inputs become more inelastic (more leontief), relative MPL for the lower level input is now larger. At the Leontief extreme, the MPL of the input provided at lower level is infinity.

```
% Visualize
% Generate some Data
rng(456);
ar_row_grid = ar_pshare;
ar_col_grid = log(1-ar_prho)/log(10);
rng(123);
mt_value = mt_rela_wage;
% container map settings
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {...
    ['Log Relative MPL/Wages, rho=x, share=color'] ...
    ['input x1 = ' num2str(fl_x_1) ' input x2 = ' num2str(fl_x_2)]
    };
mp_support_graph('cl_st_ytitle') = {'Log(mplx1/mplx2)'};
mp_support_graph('cl_st_xtitle') = {'Rho in (1-log10(rho)) scale', ...
    'left = perfect substitution (rho = 1)', ...
    'right = perfect complements (negative infinity rho)'};
mp_support_graph('st_legend_loc') = 'best';
mp_support_graph('bl_graph_logy') = false; % do not log
mp_support_graph('st_rowvar_name') = 'share=';
mp_support_graph('it_legend_select') = 5; % how many shock legends to show
mp_support_graph('st_rounding') = '6.3f'; % format shock legend
mp_support_graph('cl_colors') = 'jet'; % any predefined matlab colormap
% Call function
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```



3.2.4 Doubly Nest Layer Two Inputs Each Sub-nest CES Problem

In this third example, solve for optimal choices for a doubly nested problem. Below, we first solve for the optimal choices, then we do a number of checks, to make sure that the solutions are correct, as expected.

```
% output requirement
fl_yz = 2.1;
% upper nest 0.1, lower nests 0.35 and -1 separately for rho values
cl_mn_prho = {[0.1], [0.35, -1]};
% unequal shares of share values
cl_mn_pshare = {[0.4], [0.3, 0.88]};
% differential wages
% in lower-left nest, not productive and very expensive, not very elastic
% last index for left or right. Values picked from demand problem parallel
% example.
cl_mn_yz_choices = {[nan, nan], [0.04789, 6.0934; 2.2044, 0.70496]};
% print option
bl_verbose = true;
[cl_mn_yz_choices, cl_mn_mpl_price] = ...
    bfw_crs_nested_ces_mpl(cl_mn_prho, cl_mn_pshare, cl_mn_yz_choices, ...
    mp_func, bl_verbose, bl_bfw_model);
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i    idx    ndim    numel    rowN    colN     sum      mean       std      coe
                    -    ---    ----    -----    ----    ----    ------    -------    -------    ---

    mpl_price_c1    1     1      2        2        1       2     1.0206    0.51032    0.27499    0.5
    mpl_price_c2    2     2      2        4        2       2     2.3618    0.59045     0.5082    0.8
    prho_c2         3     4      2        2        1       2      -0.65     -0.325    0.95459    -2.
    pshare_c2       4     6      2        2        1       2       1.18       0.59    0.41012    0.6
    yz_c1           5     7      2        2        1       2     4.4862     2.2431    0.68863      0
```

```
    yz_c2           6     8      2        4       2       2      9.0507      2.2627      2.7086      1.

xxx TABLE:mpl_price_c1 xxxxxxxxxxxxxxxxxx
           c1         c2
          -------    -------

    r1    0.31587    0.70476

xxx TABLE:mpl_price_c2 xxxxxxxxxxxxxxxxxx
           c1         c2
          -------    -------

    r1     1.3121    0.13121
    r2    0.39362    0.52484

xxx TABLE:prho_c2 xxxxxxxxxxxxxxxxxx
           c1     c2
          ----    --

    r1    0.35    -1

xxx TABLE:pshare_c2 xxxxxxxxxxxxxxxxxx
          c1      c2
          ---    ----

    r1    0.3    0.88

xxx TABLE:yz_c1 xxxxxxxxxxxxxxxxxx
           c1       c2
          ----    ------

    r1    2.73    1.7562

xxx TABLE:yz_c2 xxxxxxxxxxxxxxxxxx
           c1         c2
          -------    -------

    r1    0.04789     6.0934
    r2     2.2044    0.70496

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                 i    idx    value
                 -    ---    -----

    prho_c1      1     3      0.1
    pshare_c1    2     5      0.4

% there are four optimal choices, they are
fl_mpl_x11 = cl_mn_mpl_price{2}(1,1);
fl_mpl_x12 = cl_mn_mpl_price{2}(1,2);
fl_mpl_x21 = cl_mn_mpl_price{2}(2,1);
fl_mpl_x22 = cl_mn_mpl_price{2}(2,2);
% display
st_print = strjoin(...
    ["completed double nest test:", ...
```

```
    ['nest 1 input 1, fl_mpl_x11=' num2str(fl_mpl_x11)], ...
    ['nest 1 input 2, fl_mpl_x12=' num2str(fl_mpl_x12)], ...
    ['nest 2 input 1, fl_mpl_x21=' num2str(fl_mpl_x21)], ...
    ['nest 2 input 2, fl_mpl_x22=' num2str(fl_mpl_x22)], ...
    ['nest 1 input 1, fl_mpl_x11/fl_mpl_x11=' num2str(fl_mpl_x11/fl_mpl_x11)], ...
    ['nest 1 input 2, fl_mpl_x12/fl_mpl_x11=' num2str(fl_mpl_x12/fl_mpl_x11)], ...
    ['nest 2 input 1, fl_mpl_x21/fl_mpl_x11=' num2str(fl_mpl_x21/fl_mpl_x11)], ...
    ['nest 2 input 2, fl_mpl_x22/fl_mpl_x11=' num2str(fl_mpl_x22/fl_mpl_x11)], ...
    ], ";");
st_out = st_print;
ar_ch_out = char(strsplit(st_print,";")');
disp(ar_ch_out);
```

```
completed double nest test:
nest 1 input 1, fl_mpl_x11=1.3121
nest 1 input 2, fl_mpl_x12=0.13121
nest 2 input 1, fl_mpl_x21=0.39362
nest 2 input 2, fl_mpl_x22=0.52484
nest 1 input 1, fl_mpl_x11/fl_mpl_x11=1
nest 1 input 2, fl_mpl_x12/fl_mpl_x11=0.099995
nest 2 input 1, fl_mpl_x21/fl_mpl_x11=0.29998
nest 2 input 2, fl_mpl_x22/fl_mpl_x11=0.39999
```

3.2.5 BFW (2022) Nested Three Branch (Four Layer) Problem (MPL)

The model BFW 2022 has three branches and four layers. one of the branches go down only three layers, the other two branches go down four layers.

First, we prepare the various inputs:

```
% Controls
bl_verbose = true;
bl_bfw_model = true;

% Given rho and beta, solve for equilibrium quantities
mp_func = bfw_mp_func_demand();

% Following instructions in: PrjFLFPMexicoBFW\solvedemand\README.md

% Nests/layers
it_nests = 4;

% Input cell of mn matrixes
it_prho_cl = 1;
it_pshare_cl = 2;
it_yz_share_cl = 3;
for it_cl_ctr = [1,2,3]

    cl_mn_cur = cell(it_nests,1);

    % Fill each cell element with NaN mn array
    for it_cl_mn = 1:it_nests

        bl_yz_share = (it_cl_ctr == it_yz_share_cl);

        if (~bl_yz_share && it_cl_mn == 1)
            mn_nan = NaN;
        elseif (~bl_yz_share && it_cl_mn == 2) || (bl_yz_share && it_cl_mn == 1)
            mn_nan = [NaN, NaN];
```

```
        elseif (~bl_yz_share && it_cl_mn == 3) || (bl_yz_share && it_cl_mn == 2)
            mn_nan = NaN(2,2);
        elseif (~bl_yz_share && it_cl_mn == 4) || (bl_yz_share && it_cl_mn == 3)
            mn_nan = NaN(2,2,2);
        elseif (~bl_yz_share && it_cl_mn == 5) || (bl_yz_share && it_cl_mn == 4)
            mn_nan = NaN(2,2,2,2);
        elseif (~bl_yz_share && it_cl_mn == 6) || (bl_yz_share && it_cl_mn == 5)
            mn_nan = NaN(2,2,2,2,2);
        end
        cl_mn_cur{it_cl_mn} = mn_nan;
    end

    % Name cell arrays
    if (it_cl_ctr == it_prho_cl)
        cl_mn_prho = cl_mn_cur;
    elseif (it_cl_ctr == it_pshare_cl)
        cl_mn_pshare = cl_mn_cur;
    elseif (it_cl_ctr == it_yz_share_cl)
        cl_mn_yz_choices = cl_mn_cur;
    end
end

% Initialize share matrix
rng(123);
for it_cl_mn = 1:it_nests
    mn_pshare = cl_mn_pshare{it_cl_mn};
    if it_cl_mn == 4
        mn_pshare(2,:,:) = rand(2,2);
    else
        mn_pshare = rand(size(mn_pshare));
    end
    cl_mn_pshare{it_cl_mn} = mn_pshare;
end

% Initialize rho matrix
rng(456);
for it_cl_mn = 1:it_nests
    mn_prho = cl_mn_prho{it_cl_mn};
    if it_cl_mn == 4
        mn_prho(2,:,:) = rand(2,2);
    else
        mn_prho = rand(size(mn_prho));
    end
    % Scalling rho between 0.7500 and -3.0000
    % 1 - 2.^(linspace(-2,2,5))
    mn_prho = 1 - 2.^(mn_prho*(4) - 2);
    cl_mn_prho{it_cl_mn} = mn_prho;
end

% Initialize quantities matrix
rng(789);
for it_cl_mn = 1:it_nests
    mn_yz_choices = cl_mn_yz_choices{it_cl_mn};
    if it_cl_mn == 3
        mn_yz_choices(1,:,:) = rand(2,2);
    elseif it_cl_mn == 4
        mn_yz_choices(2,:,:,:) = rand(2,2,2);
    end
```

```
    % Scalling quantities between 3 amd 5
    mn_yz_choices = mn_yz_choices*(2) + 3;
    cl_mn_yz_choices{it_cl_mn} = mn_yz_choices;
end

% Initialize yz matrix
rng(101112);
```

Second, display created inputs:

```
celldisp(cl_mn_prho);


cl_mn_prho{1} =

    0.5017



cl_mn_prho{2} =

    0.6071   -1.1955



cl_mn_prho{3} =

   -1.3523   -0.3346
   -0.4167   -1.9136



cl_mn_prho{4} =


(:,:,1) =

       NaN       NaN
   -1.0512    0.5869


(:,:,2) =

       NaN       NaN
    0.6209    0.1633



celldisp(cl_mn_pshare);


cl_mn_pshare{1} =

    0.6965



cl_mn_pshare{2} =
```

```
    0.2861    0.2269



cl_mn_pshare{3} =

    0.5513    0.4231
    0.7195    0.9808



cl_mn_pshare{4} =


(:,:,1) =

       NaN       NaN
    0.6848    0.4809


(:,:,2) =

       NaN       NaN
    0.3921    0.3432



celldisp(cl_mn_yz_choices);


cl_mn_yz_choices{1} =

   NaN   NaN



cl_mn_yz_choices{2} =

   NaN   NaN
   NaN   NaN



cl_mn_yz_choices{3} =


(:,:,1) =

    3.6467    3.4605
       NaN       NaN


(:,:,2) =

    4.5876    4.2488
       NaN       NaN
```

```
cl_mn_yz_choices{4} =


(:,:,1,1) =

      NaN       NaN
   4.9508    4.5178


(:,:,2,1) =

      NaN       NaN
   3.0212    3.0495


(:,:,1,2) =

      NaN       NaN
   3.2221    4.0763


(:,:,2,2) =

      NaN       NaN
   3.0909    4.1031
```

Third, call function and solve for optimal demand:

```
% Call function
[cl_mn_yz_choices, cl_mn_mpl_price] = ...
    bfw_crs_nested_ces_mpl(cl_mn_prho, cl_mn_pshare, cl_mn_yz_choices, ...
    mp_func, bl_verbose, bl_bfw_model);
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                    i     idx    ndim    numel    rowN    colN      sum         mean        std
                    --    ---    ----    -----    ----    ----    --------    --------    --------

    mpl_price_c1     1      1      2        2       1       2        1.0002      0.5001     0.28686
    mpl_price_c2     2      2      2        4       2       2        1.0009     0.25022     0.17949
    mpl_price_c3     3      3      3        8       2       4        1.0088      0.1261     0.10191
    mpl_price_c4     4      4      4       16       2       8           NaN         NaN         NaN
    prho_c2          5      6      2        2       1       2      -0.58844    -0.29422      1.2746
    prho_c3          6      7      2        4       2       2       -4.0173     -1.0043     0.76195
    prho_c4          7      8      3        8       2       4           NaN         NaN         NaN
    pshare_c2        8     10      2        2       1       2       0.51299      0.2565    0.041923
    pshare_c3        9     11      2        4       2       2        2.6747     0.66866     0.24087
    pshare_c4       10     12      3        8       2       4           NaN         NaN         NaN
    yz_c1           11     13      2        2       1       2        8.0897      4.0448       0.173
    yz_c2           12     14      2        4       2       2        16.015      4.0039     0.19166
    yz_c3           13     15      3        8       2       4        31.235      3.9044     0.51337
    yz_c4           14     16      4       16       2       8           NaN         NaN         NaN

xxx TABLE:mpl_price_c1 xxxxxxxxxxxxxxxxxx
           c1         c2
          -------    -------
```

```
    r1    0.70294    0.29725

xxx TABLE:mpl_price_c2 xxxxxxxxxxxxxxxxxx
             c1         c2
          --------    -------

    r1     0.19946    0.50351
    r2    0.080381    0.21754

xxx TABLE:mpl_price_c3 xxxxxxxxxxxxxxxxxx
             c1         c2          c3          c4
          --------    -------    --------    ---------

    r1     0.13727    0.24893    0.065108      0.25809
    r2    0.050551    0.21139    0.031132    0.0063057

xxx TABLE:mpl_price_c4 xxxxxxxxxxxxxxxxxx
            c1          c2          c3          c4           c5          c6          c7          c8
          -------    --------    --------    ---------    --------    -------    --------    -------

    r1        NaN         NaN         NaN          NaN         NaN        NaN         NaN          N
    r2    0.02507    0.099481    0.012272    0.0025507    0.027845    0.11203    0.018861    0.00380

xxx TABLE:prho_c2 xxxxxxxxxxxxxxxxxx
            c1         c2
          -------    -------

    r1    0.60709    -1.1955

xxx TABLE:prho_c3 xxxxxxxxxxxxxxxxxx
             c1          c2
          --------    --------

    r1     -1.3523    -0.33464
    r2    -0.41668     -1.9136

xxx TABLE:prho_c4 xxxxxxxxxxxxxxxxxx
            c1         c2         c3         c4
          -------    -------    -------    -------

    r1        NaN        NaN        NaN        NaN
    r2    -1.0512    0.58694    0.62089    0.16334

xxx TABLE:pshare_c2 xxxxxxxxxxxxxxxxxx
            c1         c2
          -------    -------

    r1    0.28614    0.22685

xxx TABLE:pshare_c3 xxxxxxxxxxxxxxxxxx
            c1         c2
          -------    -------

    r1    0.55131    0.42311
    r2    0.71947    0.98076

xxx TABLE:pshare_c4 xxxxxxxxxxxxxxxxxx
```

```
          c1         c2         c3         c4
        -------    -------    -------    -------

    r1      NaN        NaN        NaN        NaN
    r2  0.68483    0.48093    0.39212    0.34318

xxx TABLE:yz_c1 xxxxxxxxxxxxxxxxxx
          c1        c2
        ------    ------

    r1  3.9225    4.1672

xxx TABLE:yz_c2 xxxxxxxxxxxxxxxxxx
          c1        c2
        ------    ------

    r1  4.0073    3.8887
    r2  3.8468    4.2727

xxx TABLE:yz_c3 xxxxxxxxxxxxxxxxxx
          c1        c2        c3        c4
        ------    ------    ------    ------

    r1  3.6467    3.4605    4.5876    4.2488
    r2    4.23    4.2863    3.0635    3.7118

xxx TABLE:yz_c4 xxxxxxxxxxxxxxxxxx
          c1        c2        c3        c4        c5        c6        c7        c8
        ------    ------    ------    ------    ------    ------    ------    ------

    r1     NaN       NaN       NaN       NaN       NaN       NaN       NaN       NaN
    r2  4.9508    4.5178    3.0212    3.0495    3.2221    4.0763    3.0909    4.1031

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                 i    idx     value
                 -    ---    -------

    prho_c1      1     5     0.50172
    pshare_c1    2     9     0.69647
```

Appendix A

Index and Code Links

A.1 Introduction links

1. The Labor Demand and Supply Problem: **mlx** | **m** | **pdf** | **html**
    - The Labor Demand and Supply Problem

A.2 Core Functions links

1. CES Demand Core Functions: **mlx** | **m** | **pdf** | **html**
    - This function generates a container map with key CES demand-side equation for a particular sub-nest.
    - **PrjLabEquiBFW**: *bfw_mp_func_demand()*

A.3 Demand links

1. Solve Nested CES Optimal Demand (CRS): **mlx** | **m** | **pdf** | **html**
    - This function solves optimal choices given CES production function under cost minimization.
    - Works with Constant Elasticity of Substitution problems with constant returns, up to four nest layers, and two inputs in each sub-nest.
    - Takes as inputs share and elasticity parameters across layers of sub-nests, as well as input unit costs at the bottom-most layer.
    - Works with Constant Elasticity of Substitution problems with constant returns, up to four nest layers, and two inputs in each sub-nest.
    - **PrjLabEquiBFW**: *bfw_crs_nested_ces()*
2. Compute Nested CES MPL Given Demand (CRS): **mlx** | **m** | **pdf** | **html**
    - Given labor quantity demanded, using first-order relative optimality conditions, find the marginal product of labor given CES production function.
    - Takes as inputs share and elasticity parameters across layers of sub-nests, as well as quantity demanded at each bottom-most CES nest layer.
    - Works with Constant Elasticity of Substitution problems with constant returns, up to four nest layers, and two inputs in each sub-nest.
    - Allows for uneven branches, so that some branches go up to four layers, but others have less layers, works with BFW (2022) nested labor input problem.
    - **PrjLabEquiBFW**: *bfw_crs_nested_ces_mpl()*

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