Solve Nested CES Optimal Demand (CRS)

Testing the **bfw_crs_nested_ces** function from the **PrjLabEquiBFW Package.** This function solves optimal choices given CES production function under cost minimization. Works with Constant Elasticity of Substitution problems with constant returns, up to four nest layers, and two inputs in each sub-nest. Takes as inputs share and elasticity parameters across layers of sub-nests, as well as input unit costs at the bottom-most layer. Works with Constant Elasticity of Substitution problems with constant returns, up to four nest layers, and two inputs in each sub-nest. Allows for uneven branches, so that some branches go up to four layers, but others have less layers, works with BFW (2022) nested labor input problem.

Key Inputs and Outputs for bfw_mp_func_demand

Here are the key inputs for the CES demand solver function:

- **FL_YZ** float output divided by productivity, aggregate single term
- **CL_MN_PRHO** cell array of rho (elasticity) parameter between negative infinity and 1. For example, suppose there are four nest layers, and there are two branches at each layer, then we have 1, 2, 4, and 8 ρ parameter values at the 1st, 2nd, 3rd, and 4th nest layers: size(CL_MN_PRHO{1})= $[1,1]$, size(CL_MN_PRHO{2}) = $[1,2]$, size(CL_MN_PRHO{3}) = $[2,2]$, size(CL_MN_PRHO{4}) = $[2,2,2]$. Note that if the model has 4 nest layers, not all cells need to be specified, some branches could be deeper than others.
- **CL_MN_PSHARE** cell array of share (between 0 and 1) for the first input of the two inputs for each nest. The structure for this is similar to CL_MN_PRHO.
- **CL_MN_PRICE** cell array of wages for both wages for the first and second nest, the last index in each element of the cell array indicates first (1) or second (2) wage. For example, suppose we have four layers, with 2 branches at each layer, as in the example for CL_MN_PRHO, then we have 2, 4, 8, and 16 wage values at the 1st, 2nd, 3rd, and 4th nest layers: size(CL_MN_PRICE{1}) = $[1,2]$, size(CL_MN_PRICE{2}) = $[2,2]$, size(CL_MN_PRICE{3}) = $[2,2,2]$, size(CL_MN_PRICE{4}) = $[2,2,2,2]$. Note that only the last layer of wage needs to be specified, in this case, the 16 wages at the 4th layer. Given optimal solutions, we solve for the 2, 4, and 8 aggregate wages at the higher nest layers. If some branches are deeper than other branches, then can specific NA for non-reached layers along some branches.
- **BL_BFW_MODEL** boolean true by default if true then will output outcomes specific to the BFW 2022 problem.

Here are the key outputs for the CES demand solver function:

- **CL_MN_YZ_CHOICES** has the same dimension as CL_MN_PRICE, suppose there are four layers, the CL_MN_PRICE{4} results at the lowest layer includes quantity choices that might be observed in the data. CL_MN_PRICE cell values at non-bottom layers include aggregate quantity outcomes.
- **CL_MN_PRICE** includes at the lowest layer observed wages, however, also includes higher layer aggregate solved waves. CL_MN_PRHO and CL_MN_PSHARE are identical to inputs.

Single Nest Layer Two Inputs CES Problem (Demand)

In this first example, we solve a constant returns to scale problem with a single nest, meaning just two inputs and a single output.

```
clc;
close all;
clear all;

% Output requirement
fl_yz = 1;
% rho = 0.5, 1/(1-0.5)=2, elasticity of substitution of 2
cl_mn_prho = {[0.1]};
% equal share, similar "productivity"
cl_mn_pshare = {[0.5]};
% wages for the two inputs, identical wage
cl_mn_price = {[1.5, 0.75]};
% print option
bl_verbose = true;
mp_func = bfw_mp_func_demand();
bl_bfw_model = false;
[cl_mn_yz_choices, cl_mn_price] = ...
    bfw_crs_nested_ces(fl_yz, cl_mn_prho, cl_mn_pshare, cl_mn_price, ...
    mp_func, bl_verbose, bl_bfw_model);
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                i    idx    ndim    numel    rowN    colN     sum       mean       std      coefvari      min
                _    ___    ____    _____    ____    ____    ______    ______    _______    ________    _______

    price_c1    1     2      2        2       1       2        2.25     1.125    0.53033     0.4714        0.75
    yz_c1       2     4      2        2       1       2      2.1343    1.0671    0.55403    0.51918    0.67537

xxx TABLE:price_c1 xxxxxxxxxxxxxxxxxx
          c1      c2 
          ___    ____

    r1    1.5    0.75

xxx TABLE:yz_c1 xxxxxxxxxxxxxxxxxx
            c1          c2  
          _______    ______

    r1    0.67537    1.4589

----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                 i    idx    value
                 _    ___    _____

    prho_c1      1     1      0.1 
    pshare_c1    2     3      0.5 
```

Single Nest Layer Two Inputs CES Problem, Vary Share and Elasticity (Demand)

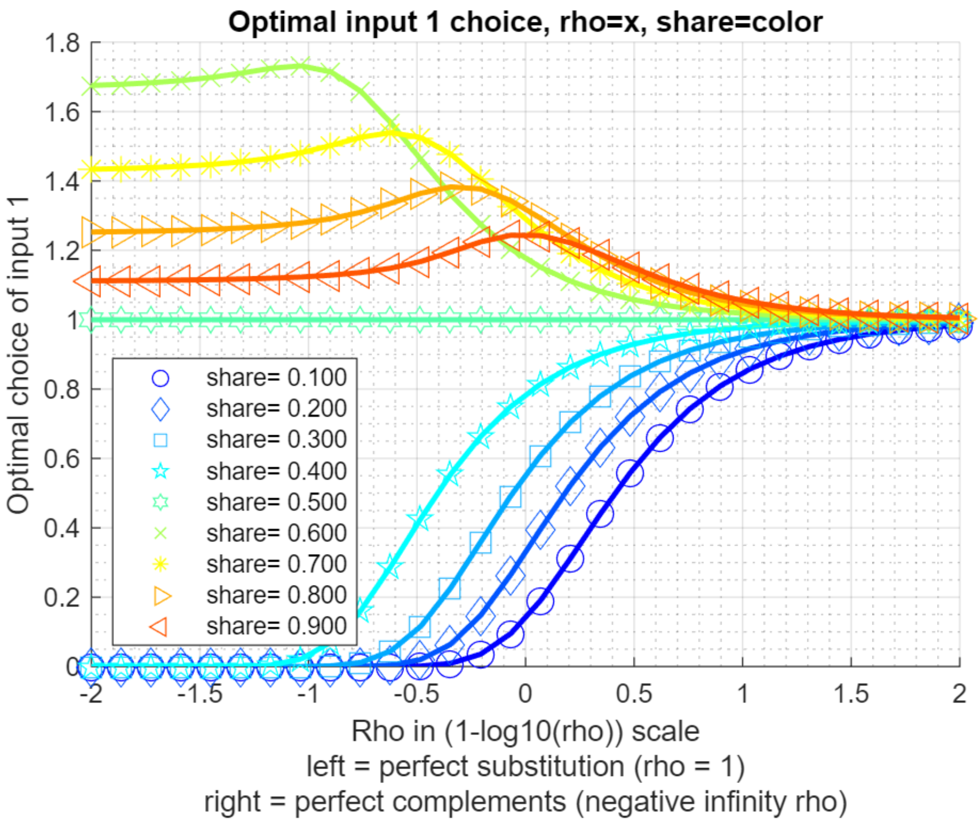
In this second example, we test over different rho values, explore optimal relative choices, as share and elasticity change. In this exercise, we also check, at every combination of rho and share parameter, whether the FOC condition is satisfied by the optimal choices. Also check if at the optimal choices, the minimization output requirement is met.

```
% Approximately close function
rel_tol=1e-09;
abs_tol=0.0;
if_is_close = @(a,b) (abs(a-b) <= max(rel_tol * max(abs(a), abs(b)), abs_tol));

% Define share and rho arrays
fl_yz = 1;
ar_pshare = linspace(0.1, 0.9, 9);
ar_prho = 1 - 10.^(linspace(-2, 2, 30));
% Loop over share and rho values
mt_rela_opti = NaN([length(ar_pshare), length(ar_prho)]);
mt_x1_opti = NaN([length(ar_pshare), length(ar_prho)]);
for it_pshare_ctr = 1:length(ar_pshare)
    for it_prho_ctr = 1:length(ar_prho)

        % A. Parameters
        % rho
        fl_prho = ar_prho(it_prho_ctr);
        cl_mn_prho = {[fl_prho]};
        % share
        fl_pshare = ar_pshare(it_pshare_ctr);
        cl_mn_pshare = {[fl_pshare]};
        % wages for the two inputs, identical wage
        cl_mn_price = {[1, 1]};
        % print option
        bl_verbose = false;

        % B. Call function
        [cl_mn_yz_choices, cl_mn_price, cl_mn_prho, cl_mn_pshare] = ...
            bfw_crs_nested_ces(fl_yz, cl_mn_prho, cl_mn_pshare, cl_mn_price, ...
            mp_func, bl_verbose, bl_bfw_model);
        % Store results for optimal choice
        fl_opti_x1 = cl_mn_yz_choices{1}(1);
        fl_opti_x2 = cl_mn_yz_choices{1}(2);
        mt_x1_opti(it_pshare_ctr, it_prho_ctr) = fl_opti_x1;

        % C. Check if relative optimality FOC condition is met
        fl_rela_opti = fl_opti_x1/fl_opti_x2;
        % From FOC give wages = 1 both
        % Using What is above Equation A.20 in draft.
        fl_rela_opti_foc = (((fl_pshare/(1-fl_pshare)))*(1))^(1/(1-ar_prho(it_prho_ctr)));
        if (~if_is_close(fl_rela_opti_foc, fl_rela_opti))
            error('There is an error, optimal relative not equal to expected foc ratio')
        end

        % D. Check if output quantity requirement is met
        fl_output = ((fl_pshare)*fl_opti_x1^(fl_prho) + (1-fl_pshare)*fl_opti_x2^(fl_prho))^(1/
        if (~if_is_close(fl_output, fl_yz))
```

```
            error('There is an error, output is not equal to required expenditure minimizing ou
        end

    end
end
```

Key results: (1) As share parameter of input 1 goes to zero, optimal choice goes to zero when inputs are elastic; (2) When inputs are inelasticty, even very low share input 1 asymptote to equal input 2; (3) When input 1 is more productive (higher share), actually hire less as productivity (share) increases, becasue less of it is needed to achieve production for high rho, elastictic production function; (4) For inelastic production, monotonic relationship between input and shares.

```
% Visualize
% Generate some Data
rng(456);
ar_row_grid = ar_pshare;
ar_col_grid = log(1-ar_prho)/log(10);
rng(123);
mt_value = mt_x1_opti;
% container map settings
mp_support_graph = containers.Map('KeyType', 'char', 'ValueType', 'any');
mp_support_graph('cl_st_graph_title') = {'Optimal input 1 choice, rho=x, share=color'};
mp_support_graph('cl_st_ytitle') = {'Optimal choice of input 1'};
mp_support_graph('cl_st_xtitle') = {'Rho in (1-log10(rho)) scale', ...
    'left = perfect substitution (rho = 1)', ...
    'right = perfect complements (negative infinity rho)'};
mp_support_graph('st_legend_loc') = 'southwest';
mp_support_graph('bl_graph_logy') = false; % do not log
mp_support_graph('st_rowvar_name') = 'share=';
mp_support_graph('it_legend_select') = 5; % how many shock legends to show
mp_support_graph('st_rounding') = '6.3f'; % format shock legend
mp_support_graph('cl_colors') = 'jet'; % any predefined matlab colormap
% Call function
ff_graph_grid(mt_value, ar_row_grid, ar_col_grid, mp_support_graph);
```



Doubly Nest Layer Two Inputs Each Sub-nest CES Problem (Demand)

In this third example, solve for optimal choices for a doubly nested problem. Below, we first solve for the optimal choices, then we do a number of checks, to make sure that the solutions are correct, as expected.

```
% output requirement
fl_yz = 2.1;
% upper nest 0.1, lower nests 0.35 and -1 separately for rho values
cl_mn_prho = {[0.1], [0.35, -1]};
% unequal shares of share values
cl_mn_pshare = {[0.4], [0.3, 0.88]};
% differential wages
% in lower-left nest, not productive and very expensive, not very elastic
% last index for left or right,
cl_mn_price = {[nan, nan], [10, 1;3, 4]};
% print option
bl_verbose = true;
[cl_mn_yz_choices, cl_mn_price, cl_mn_prho, cl_mn_pshare] = ...
    bfw_crs_nested_ces(fl_yz, cl_mn_prho, cl_mn_pshare, cl_mn_price, ...
    mp_func, bl_verbose, bl_bfw_model);
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                 i    idx    ndim    numel    rowN    colN     sum      mean      std     coefvari     min
                 _    ___    ____    _____    ____    ____    ______    ______    _______    ________    ________
```

prho_c2	1	2	2	2	1	2	-0.65	-0.325	0.95459	-2.9372	-1
price_c1	2	3	2	2	1	2	7.7788	3.8894	2.0959	0.53886	2.4074
price_c2	3	4	2	4	2	2	18	4.5	3.873	0.86066	1
pshare_c2	4	6	2	2	1	2	1.18	0.59	0.41012	0.69512	0.3
yz_c1	5	7	2	2	1	2	4.4862	2.2431	0.68863	0.307	1.7561
yz_c2	6	8	2	4	2	2	9.0506	2.2626	2.7086	1.1971	0.047893

xxx TABLE:prho_c2 xxxxxxxxxxxxxxxxxx

	c1	c2
r1	0.35	-1

xxx TABLE:price_c1 xxxxxxxxxxxxxxxxxx

	c1	c2
r1	2.4074	5.3714

xxx TABLE:price_c2 xxxxxxxxxxxxxxxxxx

	c1	c2
r1	10	1
r2	3	4

xxx TABLE:pshare_c2 xxxxxxxxxxxxxxxxxx

	c1	c2
r1	0.3	0.88

xxx TABLE:yz_c1 xxxxxxxxxxxxxxxxxx

	c1	c2
r1	2.73	1.7561

xxx TABLE:yz_c2 xxxxxxxxxxxxxxxxxx

	c1	c2
r1	0.047893	6.0934
r2	2.2044	0.70496

----
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

	i	idx	value
prho_c1	1	1	0.1
pshare_c1	2	5	0.4

```
% there are four optimal choices, they are
fl_opti_x11 = cl_mn_yz_choices{2}(1,1);
fl_opti_x12 = cl_mn_yz_choices{2}(1,2);
fl_opti_x21 = cl_mn_yz_choices{2}(2,1);
fl_opti_x22 = cl_mn_yz_choices{2}(2,2);
% display
st_print = strjoin(...
    ["completed double nest test:", ...
```

```
    ['nest 1 input 1, fl_opti_x11=' num2str(fl_opti_x11)], ...
    ['nest 1 input 2, fl_opti_x12=' num2str(fl_opti_x12)], ...
    ['nest 2 input 1, fl_opti_x21=' num2str(fl_opti_x21)], ...
    ['nest 2 input 2, fl_opti_x22=' num2str(fl_opti_x22)], ...
    ], ";");
st_out = st_print;
ar_ch_out = char(strsplit(st_print,";")');
disp(ar_ch_out);
```

```
completed double nest test:
nest 1 input 1, fl_opti_x11=0.047893
nest 1 input 2, fl_opti_x12=6.0934
nest 2 input 1, fl_opti_x21=2.2044
nest 2 input 2, fl_opti_x22=0.70496
```

Doubly Nest Layer Two Inputs Each Sub-nest CES Problem--Solution Check (Demand)

Checking output equality, if there are problems, would output an error.

```
% A. Check output Equality
fl_pshare_0 = cl_mn_pshare{1}(1);
fl_pshare_1 = cl_mn_pshare{2}(1);
fl_pshare_2 = cl_mn_pshare{2}(2);
fl_prho_0 = cl_mn_prho{1}(1);
fl_prho_1 = cl_mn_prho{2}(1);
fl_prho_2 = cl_mn_prho{2}(2);
fl_output_1 = ((fl_pshare_1)*fl_opti_x11^(fl_prho_1) + (1-fl_pshare_1)*fl_opti_x12^(fl_prho_1)
fl_output_2 = ((fl_pshare_2)*fl_opti_x21^(fl_prho_2) + (1-fl_pshare_2)*fl_opti_x22^(fl_prho_2)
fl_output_0 = ((fl_pshare_0)*fl_output_1^(fl_prho_0) + (1-fl_pshare_0)*fl_output_2^(fl_prho_0)
if (~if_is_close(fl_output_0, fl_yz))
    error('There is an error, output is not equal to required expenditure minimizing output')
end
```

Checking FOC within-nest optimality, if there are problems, would output an error.

```
% B. Check FOC Optimality inner nest
fl_wage_x11 = cl_mn_price{2}(1,1);
fl_wage_x12 = cl_mn_price{2}(1,2);
fl_wage_x21 = cl_mn_price{2}(2,1);
fl_wage_x22 = cl_mn_price{2}(2,2);

% B1. Checking via Method 1
fl_rela_opti_foc_1 = (((fl_pshare_1/(1-fl_pshare_1)))*(fl_wage_x12/fl_wage_x11))^(1/(1-fl_prho_
fl_rela_opti_foc_2 = (((fl_pshare_2/(1-fl_pshare_2)))*(fl_wage_x22/fl_wage_x21))^(1/(1-fl_prho_
if (~if_is_close(fl_rela_opti_foc_1, fl_opti_x11/fl_opti_x12))
    error('B1. There is an error, optimal relative not equal to expected foc ratio, nest 1')
end
if (~if_is_close(fl_rela_opti_foc_2, fl_opti_x21/fl_opti_x22))
    error('B1. There is an error, optimal relative not equal to expected foc ratio, nest 2')
end

% B2. Equation left to right, right to left, checking via method 2
% Check FOC Optimality cross nests (actually within) T1
```

```
fl_dy_dx11 = fl_pshare_1*(fl_opti_x11^(fl_prho_1-1));
fl_dy_dx12 = (1-fl_pshare_1)*(fl_opti_x12^(fl_prho_1-1));
fl_rwage_x11dx12 = fl_dy_dx11/fl_dy_dx12;
if (~if_is_close(fl_rwage_x11dx12, fl_wage_x11/fl_wage_x12))
    error('B2. There is an error, relative price x11 and x12 does not satisfy within optimality
end
```

Generate aggregate prices, if there are problems, would output an error.

```
% C. Aggregate prices and optimality within higher tier
% Is optimality satisfied given aggregate prices?
fl_rela_wage_share_11 = ...
    ((fl_wage_x11/fl_wage_x12)*((1-fl_pshare_1)/(fl_pshare_1)))^(fl_prho_1/(1-fl_prho_1));
fl_rela_wage_share_12 = ...
    ((fl_wage_x12/fl_wage_x11)*((fl_pshare_1)/(1-fl_pshare_1)))^(fl_prho_1/(1-fl_prho_1));
fl_agg_prc_1 = ...
    fl_wage_x11*(fl_pshare_1 + (1-fl_pshare_1)*(fl_rela_wage_share_11))^(-1/fl_prho_1) + ...
    fl_wage_x12*(fl_pshare_1*(fl_rela_wage_share_12) + (1-fl_pshare_1))^(-1/fl_prho_1);

fl_rela_wage_share_21 = ...
    ((fl_wage_x21/fl_wage_x22)*((1-fl_pshare_2)/(fl_pshare_2)))^(fl_prho_2/(1-fl_prho_2));
fl_rela_wage_share_22 = ...
    ((fl_wage_x22/fl_wage_x21)*((fl_pshare_2)/(1-fl_pshare_2)))^(fl_prho_2/(1-fl_prho_2));
fl_agg_prc_2 = ...
    fl_wage_x21*(fl_pshare_2 + (1-fl_pshare_2)*(fl_rela_wage_share_21))^(-1/fl_prho_2) + ...
    fl_wage_x22*(fl_pshare_2*(fl_rela_wage_share_22) + (1-fl_pshare_2))^(-1/fl_prho_2);

% What is returned by the omega function that is suppose to have aggregate prices?
mp_func = bfw_mp_func_demand();
params_group = values(mp_func, {'fc_OMEGA', 'fc_d1', 'fc_d2'});
[fc_OMEGA, fc_d1, fc_d2] = params_group{:};

% Aggregate price
fl_aggregate_price_1 = fc_OMEGA(...
    fl_wage_x11, fl_wage_x12, ...
    fl_prho_1, ...
    fl_pshare_1, 1 - fl_pshare_1);

fl_aggregate_price_2 = fc_OMEGA(...
    fl_wage_x21, fl_wage_x22, ...
    fl_prho_2, ...
    fl_pshare_2, 1 - fl_pshare_2);
```

Check relative price within nest and across nests, if there are problems, would output an error.

```
% D. Check FOC Optimality cross nests

% D1a. Two within-nest relative wages and four cross-nest relative wages
% within
fl_rwage_x11dx12 = fl_wage_x11/fl_wage_x12;
fl_rwage_x21dx22 = fl_wage_x21/fl_wage_x22;
% across
fl_rwage_x11dx21 = fl_wage_x11/fl_wage_x21;
fl_rwage_x11dx22 = fl_wage_x11/fl_wage_x22;
```

```
fl_rwage_x12dx21 = fl_wage_x12/fl_wage_x21;
fl_rwage_x12dx22 = fl_wage_x12/fl_wage_x22;

% D1b. Generate relative wages within nest and across nests own equations
fl_dy_dx1_shared = (fl_pshare_0*(fl_output_1)^(fl_prho_0-1))*((fl_output_1)^(1-fl_prho_1));
fl_dy_dx11 = fl_dy_dx1_shared*(fl_pshare_1*fl_opti_x11^(fl_prho_1-1));
fl_dy_dx12 = fl_dy_dx1_shared*((1-fl_pshare_1)*fl_opti_x12^(fl_prho_1-1));

fl_dy_dx2_shared = ((1-fl_pshare_0)*(fl_output_2)^(fl_prho_0-1))*((fl_output_2)^(1-fl_prho_2));
fl_dy_dx21 = fl_dy_dx2_shared*(fl_pshare_2*fl_opti_x21^(fl_prho_2-1));
fl_dy_dx22 = fl_dy_dx2_shared*((1-fl_pshare_2)*fl_opti_x22^(fl_prho_2-1));

% within
fl_rwage_x11dx12_foc = fl_dy_dx11/fl_dy_dx12;
fl_rwage_x21dx22_foc = fl_dy_dx21/fl_dy_dx22;
% across
fl_rwage_x11dx21_foc = fl_dy_dx11/fl_dy_dx21;
fl_rwage_x11dx22_foc = fl_dy_dx11/fl_dy_dx22;
fl_rwage_x12dx21_foc = fl_dy_dx12/fl_dy_dx21;
fl_rwage_x12dx22_foc = fl_dy_dx12/fl_dy_dx22;

if (~if_is_close(fl_rwage_x11dx21_foc, fl_wage_x11/fl_wage_x21))
    error('There is an error, relative price x11 and x21 does not satisfy cross optimality acro
end
if (~if_is_close(fl_rwage_x12dx22_foc, fl_wage_x12/fl_wage_x22))
    error('There is an error, relative price x12 and x22 does not satisfy cross optimality acro
end

% D2. Check FOC Optimality cross nests, simplified equation
fl_rela_wage_x11_x21 = log((fl_pshare_0/(1-fl_pshare_0))* ...
    ((fl_pshare_1*fl_opti_x11^(fl_prho_1-1)*fl_output_2^(fl_prho_2))/(fl_pshare_2*fl_opti_x21^(
    fl_prho_0*log(fl_output_1/fl_output_2);
if (~if_is_close(fl_rela_wage_x11_x21, log(fl_wage_x11/fl_wage_x21)))
    error('There is an error, relative price x11 and x21 does not satisfy cross optimality acro
end
```

BFW (2022) Nested Three Branch (Four Layer) Problem (Demand)

The model BFW 2022 has three branches and four layers. one of the branches go down only three layers, the other two branches go down four layers.

First, we prepare the various inputs:

```
% Controls
bl_verbose = true;
bl_bfw_model = true;

% Given rho and beta, solve for equilibrium quantities
bl_log_wage = false;
mp_func = bfw_mp_func_demand(bl_log_wage);

% Following instructions in: PrjFLFPMexicoBFW\solvedemand\README.md

% Nests/layers
```

```
it_nests = 4;

% Input cell of mn matrixes
it_prho_cl = 1;
it_pshare_cl = 2;
it_price_cl = 3;
for it_cl_ctr = [1,2,3]

    cl_mn_cur = cell(it_nests,1);

    % Fill each cell element with NaN mn array
    for it_cl_mn = 1:it_nests

        bl_price = (it_cl_ctr == it_price_cl);

        if (~bl_price && it_cl_mn == 1)
            mn_nan = NaN;
        elseif (~bl_price && it_cl_mn == 2) || (bl_price && it_cl_mn == 1)
            mn_nan = [NaN, NaN];
        elseif (~bl_price && it_cl_mn == 3) || (bl_price && it_cl_mn == 2)
            mn_nan = NaN(2,2);
        elseif (~bl_price && it_cl_mn == 4) || (bl_price && it_cl_mn == 3)
            mn_nan = NaN(2,2,2);
        elseif (~bl_price && it_cl_mn == 5) || (bl_price && it_cl_mn == 4)
            mn_nan = NaN(2,2,2,2);
        elseif (~bl_price && it_cl_mn == 6) || (bl_price && it_cl_mn == 5)
            mn_nan = NaN(2,2,2,2,2);
        end
        cl_mn_cur{it_cl_mn} = mn_nan;
    end

    % Name cell arrays
    if (it_cl_ctr == it_prho_cl)
        cl_mn_prho = cl_mn_cur;
    elseif (it_cl_ctr == it_pshare_cl)
        cl_mn_pshare = cl_mn_cur;
    elseif (it_cl_ctr == it_price_cl)
        cl_mn_price = cl_mn_cur;
    end
end

% Initialize share matrix
rng(123);
for it_cl_mn = 1:it_nests
    mn_pshare = cl_mn_pshare{it_cl_mn};
    if it_cl_mn == 4
        mn_pshare(2,:,:) = rand(2,2);
    else
        mn_pshare = rand(size(mn_pshare));
    end
    cl_mn_pshare{it_cl_mn} = mn_pshare;
end

% Initialize rho matrix
```

```
rng(456);
for it_cl_mn = 1:it_nests
    mn_prho = cl_mn_prho{it_cl_mn};
    if it_cl_mn == 4
        mn_prho(2,:,:) = rand(2,2);
    else
        mn_prho = rand(size(mn_prho));
    end
    % Scalling rho between 0.7500 and -3.0000
    % 1 - 2.^(linspace(-2,2,5))
    mn_prho = 1 - 2.^(mn_prho*(4) - 2);
    cl_mn_prho{it_cl_mn} = mn_prho;
end

% Initialize wage matrix
rng(789);
for it_cl_mn = 1:it_nests
    mn_price = cl_mn_price{it_cl_mn};
    if it_cl_mn == 3
        mn_price(1,:,:) = rand(2,2);
    elseif it_cl_mn == 4
        mn_price(2,:,:,:) = rand(2,2,2);
    end
    % Scalling rho between 3 amd 5
    mn_price = mn_price*(2) + 3;
    cl_mn_price{it_cl_mn} = mn_price;
end

% Initialize yz matrix
rng(101112);
fl_yz = rand();
```

Second, display created inputs:

```
disp(['fl_yz=' num2str(fl_yz)]);
```

```
fl_yz=0.89726
```

```
celldisp(cl_mn_prho);
```

```
cl_mn_prho

cl_mn_prho{1} =

    0.5017



cl_mn_prho{2} =

    0.6071   -1.1955



cl_mn_prho{3} =

   -1.3523   -0.3346
```

```
   -0.4167   -1.9136


cl_mn_prho{4} =


(:,:,1) =

       NaN       NaN
   -1.0512    0.5869


(:,:,2) =

       NaN       NaN
    0.6209    0.1633
```

```
celldisp(cl_mn_pshare);
```

```
cl_mn_pshare

cl_mn_pshare{1} =

    0.6965



cl_mn_pshare{2} =

    0.2861    0.2269



cl_mn_pshare{3} =

    0.5513    0.4231
    0.7195    0.9808



cl_mn_pshare{4} =


(:,:,1) =

       NaN       NaN
    0.6848    0.4809


(:,:,2) =

       NaN       NaN
    0.3921    0.3432
```

```
celldisp(cl_mn_price);
```

```
cl_mn_price
```

```
cl_mn_price{1} =

   NaN   NaN


cl_mn_price{2} =

   NaN   NaN
   NaN   NaN


cl_mn_price{3} =


(:,:,1) =

    3.6467    3.4605
       NaN       NaN


(:,:,2) =

    4.5876    4.2488
       NaN       NaN


cl_mn_price{4} =


(:,:,1,1) =

       NaN       NaN
    4.9508    4.5178


(:,:,2,1) =

       NaN       NaN
    3.0212    3.0495


(:,:,1,2) =

       NaN       NaN
    3.2221    4.0763


(:,:,2,2) =

       NaN       NaN
    3.0909    4.1031
```

Third, call function and solve for optimal demand:

```
% Call function
[cl_mn_yz_choices, cl_mn_price, cl_mn_prho, cl_mn_pshare] = ...
    bfw_crs_nested_ces(fl_yz, cl_mn_prho, cl_mn_pshare, cl_mn_price, ...
```

```
    mp_func, bl_verbose, bl_bfw_model);
```

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map ND Array (Matrix etc)
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
                               i     idx    ndim    numel    rowN    colN      sum         mean        std        coefva
                               __    ___    ____    _____    ____    ____    ________    ________    ________    ______

    mt_fl_labor_demanded        1     1      2       12       4       3        5.4455     0.45379     0.85359       1.8
    prho_c2                     2     3      2        2       1       2      -0.58844    -0.29422      1.2746     -4.33
    prho_c3                     3     4      2        4       2       2       -4.0173     -1.0043     0.76195    -0.758
    prho_c4                     4     5      3        8       2       4           NaN         NaN         NaN         N
    price_c1                    5     6      2        2       1       2        35.345      17.673      7.0394     0.398
    price_c2                    6     7      2        4       2       2        40.906      10.226      2.7834     0.272
    price_c3                    7     8      3        8       2       4        45.403      5.6754      2.0037     0.353
    price_c4                    8     9      4       16       2       8           NaN         NaN         NaN         N
    pshare_c2                   9    11      2        2       1       2       0.51299      0.2565    0.041923     0.163
    pshare_c3                  10    12      2        4       2       2        2.6747     0.66866     0.24087     0.360
    pshare_c4                  11    13      3        8       2       4           NaN         NaN         NaN         N
    yz_c1                      12    14      2        2       1       2        1.6003     0.80016      1.0053      1.25
    yz_c2                      13    15      2        4       2       2         2.645     0.66124      1.0849      1.64
    yz_c3                      14    16      3        8       2       4        5.1962     0.64953      1.0063      1.54
    yz_c4                      15    17      4       16       2       8           NaN         NaN         NaN         N

xxx TABLE:mt_fl_labor_demanded xxxxxxxxxxxxxxxxxx
             c1          c2          c3
          ________    ________    _______

    r1    0.020122    0.024929     2.1857
    r2    0.060227    0.037985     2.3642
    r3    0.069088    0.093774    0.21107
    r4    0.058349     0.14469    0.17539

xxx TABLE:prho_c2 xxxxxxxxxxxxxxxxxx
            c1         c2
          _______    _______

    r1    0.60709    -1.1955

xxx TABLE:prho_c3 xxxxxxxxxxxxxxxxxx
             c1          c2
          ________    ________

    r1     -1.3523    -0.33464
    r2    -0.41668     -1.9136

xxx TABLE:prho_c4 xxxxxxxxxxxxxxxxxx
            c1         c2         c3         c4
          _______    _______    _______    _______

    r1        NaN        NaN        NaN        NaN
    r2    -1.0512    0.58694    0.62089    0.16334

xxx TABLE:price_c1 xxxxxxxxxxxxxxxxxx
            c1       c2
          ______    _____

    r1    12.695    22.65

xxx TABLE:price_c2 xxxxxxxxxxxxxxxxxx
            c1        c2
          ______    ______
```

r1	8.1518	7.7015
r2	13.522	11.53

xxx TABLE:price_c3 xxxxxxxxxxxxxxxxxx

	c1	c2	c3	c4
r1	3.6467	3.4605	4.5876	4.2488
r2	8.1184	8.5114	5.7986	7.0309

xxx TABLE:price_c4 xxxxxxxxxxxxxxxxxx

	c1	c2	c3	c4	c5	c6	c7	c8
r1	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
r2	4.9508	4.5178	3.0212	3.0495	3.2221	4.0763	3.0909	4.1031

xxx TABLE:pshare_c2 xxxxxxxxxxxxxxxxxx

	c1	c2
r1	0.28614	0.22685

xxx TABLE:pshare_c3 xxxxxxxxxxxxxxxxxx

	c1	c2
r1	0.55131	0.42311
r2	0.71947	0.98076

xxx TABLE:pshare_c4 xxxxxxxxxxxxxxxxxx

	c1	c2	c3	c4
r1	NaN	NaN	NaN	NaN
r2	0.68483	0.48093	0.39212	0.34318

xxx TABLE:yz_c1 xxxxxxxxxxxxxxxxxx

	c1	c2
r1	1.511	0.089284

xxx TABLE:yz_c2 xxxxxxxxxxxxxxxxxx

	c1	c2
r1	0.19312	2.2864
r2	0.057461	0.108

xxx TABLE:yz_c3 xxxxxxxxxxxxxxxxxx

	c1	c2	c3	c4
r1	0.21107	2.1857	0.17539	2.3642
r2	0.06529	0.11907	0.042587	0.03298

xxx TABLE:yz_c4 xxxxxxxxxxxxxxxxxx

	c1	c2	c3	c4	c5	c6	c7	c8
r1	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
r2	0.069088	0.093774	0.020122	0.024929	0.058349	0.14469	0.060227	0.037985

```
----------------------------------------
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
CONTAINER NAME: mp_container_map Scalars
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
```

	i	idx	value
prho_c1	1	2	0.50172
pshare_c1	2	10	0.69647