Anazlying Atkinson Family Utility (CES)

Go back to fan's REconTools Package, R4Econ Repository, or Intro Stats with R Repository.

```
rm(list = ls(all.names = TRUE))
options(knitr.duplicate.label = 'allow')
```

```
library(tidyverse)
library(knitr)
library(kableExtra)
library(REconTools)
# file name
st_file_name = 'fs_atkinson_ces'
# Generate R File
purl(paste0(st_file_name, ".Rmd"), output=paste0(st_file_name, ".R"), documentation = 2)
# Generate PDF and HTML
# rmarkdown::render("C:/Users/fan/R4Econ/math/function/fs_atkinson_ces", "pdf_document")
# rmarkdown::render("C:/Users/fan/R4Econ/math/function/fs_atkinson_ces", "html_document")
```

Atkinson Family Utility

How does the Aktinson Family utility function work? THe Atkinson Family Utility has the following functional form.

$$V^{\text{social}} = \left(\alpha \cdot A^{\lambda} + \beta \cdot B^{\lambda}\right)^{\frac{1}{\lambda}}$$

Several key issues here:

1. V^{social} is the utility of some social planner
2. A and B are allocations for Alex and Ben.
3. α and β are biases that a social planner has for Alex and Ben: $\alpha + \beta = 1$, $\alpha > 0$, and $\beta > 0$
4. $-\infty < \lambda \leq 1$ is a measure of inequality aversion
    - $\lambda = 1$ is when the planner cares about weighted total allocations (efficient, Utilitarian)
    - $\lambda = -\infty$ is when the planner cares about only the minimum between A and B allocations (equality, Rawlsian)

What if only care about Alex?

Clearly, if the planner only cares about Ben, $\beta = 1$, then:

$$V^{\text{social}} = \left(B^{\lambda}\right)^{\frac{1}{\lambda}} = B$$

Clearly, regardless of the value of λ, as B increases V increases.

What Happens to V when A or B increases?

What is the derivative of V with respect to A or B?

$$\frac{\partial V}{\partial A} = \frac{1}{\lambda}\left(\alpha A^{\lambda} + \beta B^{\lambda}\right)^{\frac{1}{\lambda}-1} \cdot \lambda \alpha A^{\lambda-1}$$

With just a little bit of simplification, we have:

$$\frac{\partial V}{\partial A} = \left(\alpha A^{\lambda} + \beta B^{\lambda}\right)^{\frac{1-\lambda}{\lambda}} \cdot \alpha A^{\lambda-1} > 0$$

It is important to note that $\frac{\partial V}{\partial A} > 0$. When $\lambda < 0$, $Z^{\lambda} > 0$. For example $10^{-2} = \frac{1}{100}$. And For example $0.1^{\frac{3}{-2}} = \frac{1}{0.1^{1.5}}$. Still Positive.

This might be surprising, because when $\lambda < 0$:

$$\text{if } \lambda < 0 \text{ then } \frac{d\left(\alpha A^{\lambda} + \beta B^{\lambda}\right)}{dA} = \alpha \lambda A^{\lambda-1} < 0$$

so if we did not have the outter $\frac{1}{\lambda}$ power, negative λ would lead to decreasing weighted sum. But:

$$\text{if } \lambda < 0 \text{ then } \frac{dG^{\frac{1}{\lambda}}}{dG} = \frac{1}{\lambda} \cdot G^{\frac{1-\lambda}{\lambda}} < 0$$

so when G is increasing and $\lambda < 0$, V would decrease. But when $G(A, B)$ is decreasing, due to increasing A when $\lambda < 0$, V will actually increase.