R Integrate Over Normal Guassian Process Shock

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Contents

Integrate Over Normal Guassian Process Shock

> Go to the **RMD**, **R**, **PDF**, or **HTML** version of this file. Go back to fan's REconTools Package, R4Econ Repository (bookdown site), or Intro Stats with R Repository.

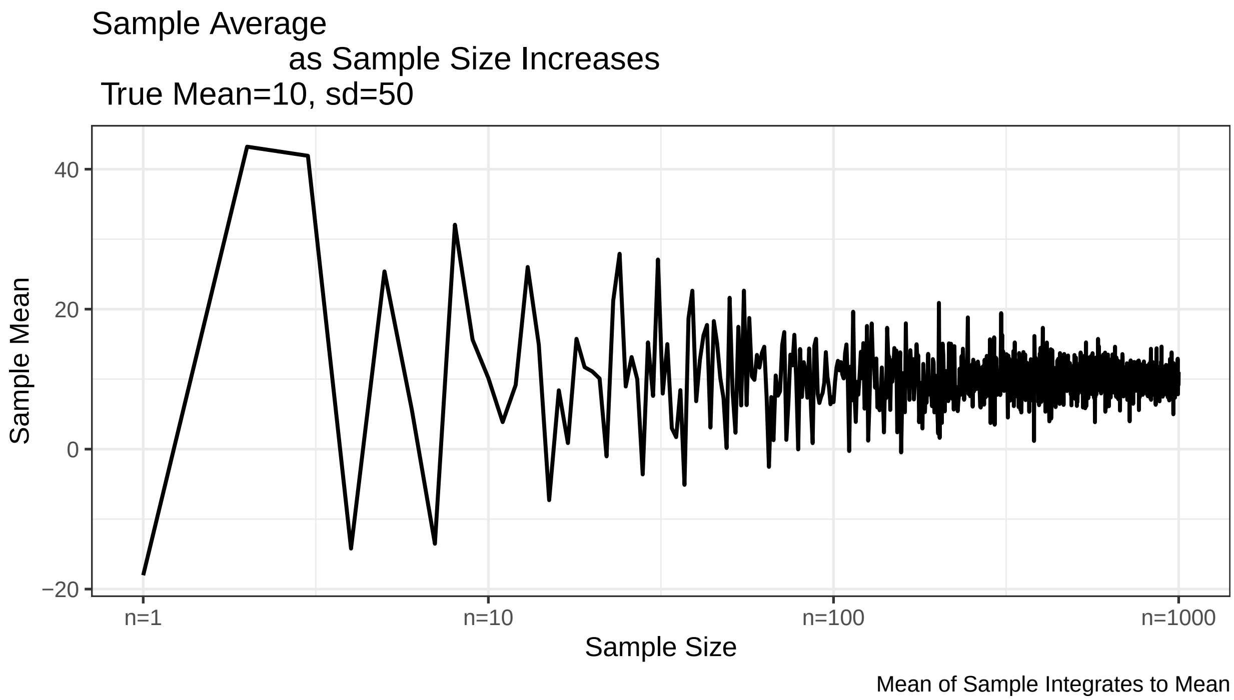
Some Common parameters

```
fl_eps_mean = 10
fl_eps_sd = 50
fl_cdf_min = 0.000001
fl_cdf_max = 0.999999
ar_it_draws <- seq(1, 1000)
```

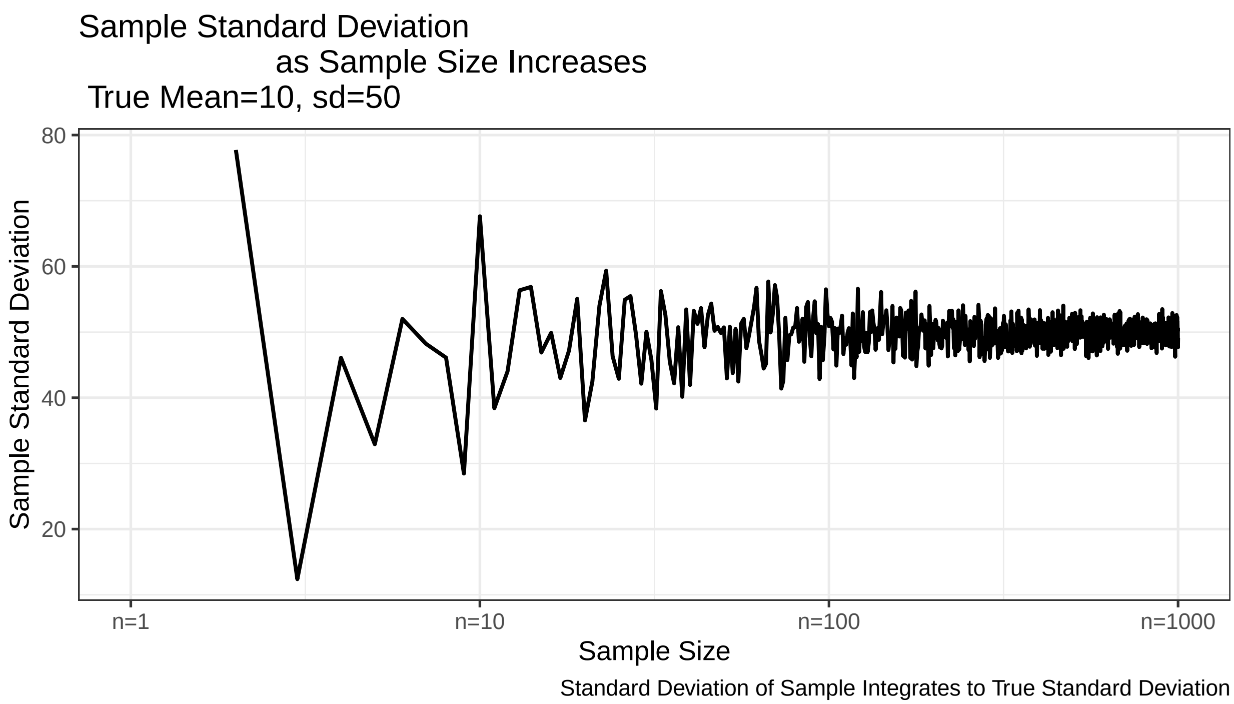
Randomly Sample and Integrate (Monte Carlo integration) Compare randomly drawn normal shock mean and known mean. How does simulated mean change with draws. Actual integral equals to 10, as sample size increases, the sample mean approaches the integration results, but this is expensive, even with ten thousand draws, not very exact.

```
# Simulate Draws
set.seed(123)
ar_fl_means <-
  sapply(ar_it_draws, function(x)
    return(mean(rnorm(x[1], mean=fl_eps_mean, sd=fl_eps_sd))))
ar_fl_sd <-
  sapply(ar_it_draws, function(x)
    return(sd(rnorm(x[1], mean=fl_eps_mean, sd=fl_eps_sd))))

mt_sample_means <- cbind(ar_it_draws, ar_fl_means, ar_fl_sd)
colnames(mt_sample_means) <- c('draw_count', 'mean', 'sd')
tb_sample_means <- as_tibble(mt_sample_means)

# Graph
# x-labels
x.labels <- c('n=1', 'n=10', 'n=100', 'n=1000')
x.breaks <- c(1, 10, 100, 1000)

# Graph Results--Draw
plt_mean <- tb_sample_means %>%
  ggplot(aes(x=draw_count, y=mean)) +
  geom_line(size=0.75) +
```

```r
  labs(title = paste0('Sample Average
                      as Sample Size Increases\n True Mean=',
                      fl_eps_mean,', sd=',fl_eps_sd),
       x = 'Sample Size',
       y = 'Sample Mean',
       caption = 'Mean of Sample Integrates to Mean') +
  scale_x_continuous(trans='log10', labels = x.labels, breaks = x.breaks) +
  theme_bw()
print(plt_mean)
```




```r
plt_sd <- tb_sample_means %>%
  ggplot(aes(x=draw_count, y=sd)) +
  geom_line(size=0.75) +
  labs(title = paste0('Sample Standard Deviation
                      as Sample Size Increases\n True Mean=',
                      fl_eps_mean,', sd=',fl_eps_sd),
       x = 'Sample Size',
       y = 'Sample Standard Deviation',
       caption = 'Standard Deviation of Sample Integrates to True Standard Deviation') +
  scale_x_continuous(trans='log10', labels = x.labels, breaks = x.breaks) +
  theme_bw()
print(plt_sd)
```

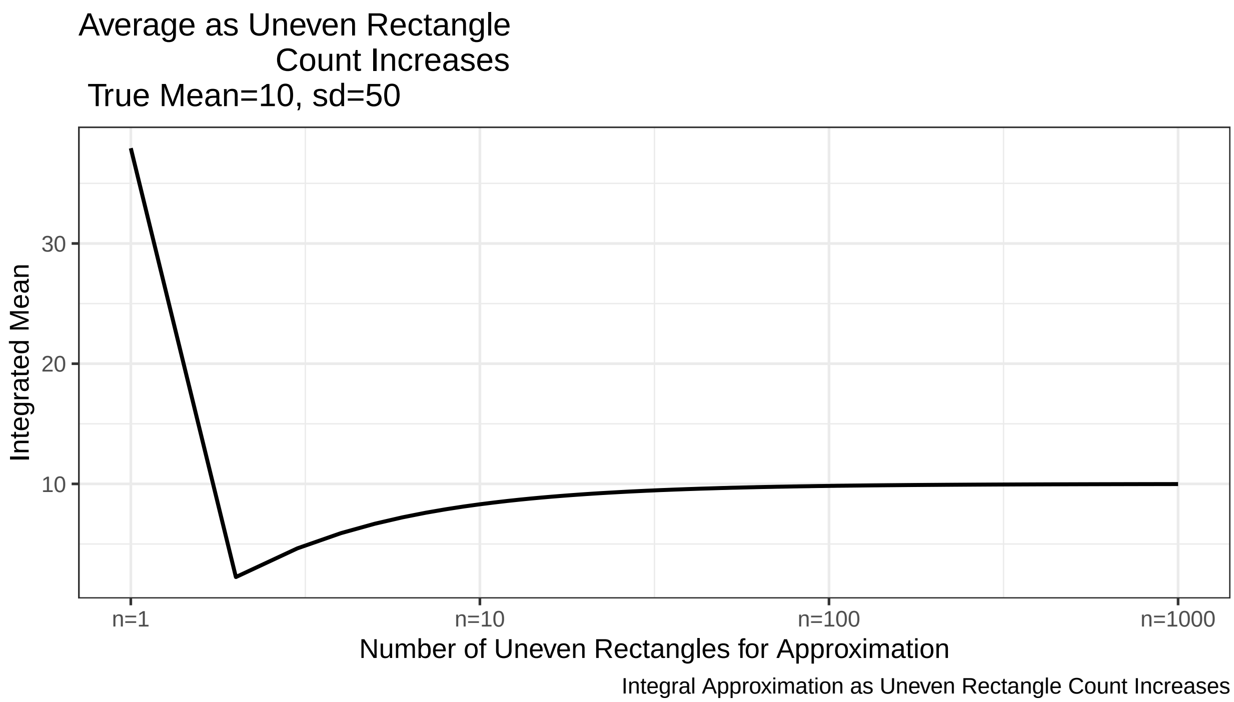


Integration By Symmetric Uneven Rectangle Draw on grid from probability space, and then find use norm inverse to find corresponding x point. Under this approach, each rectangle is suppose to approximate the same same. So even area, but uneven width.

Resulting integration is rectangle based, but rectangle width differ. Th rectangle have wider width as they move away from the mean, and thin width close to the mean. This is much more stable than the random draw method, and approximates the true answer more accurately.

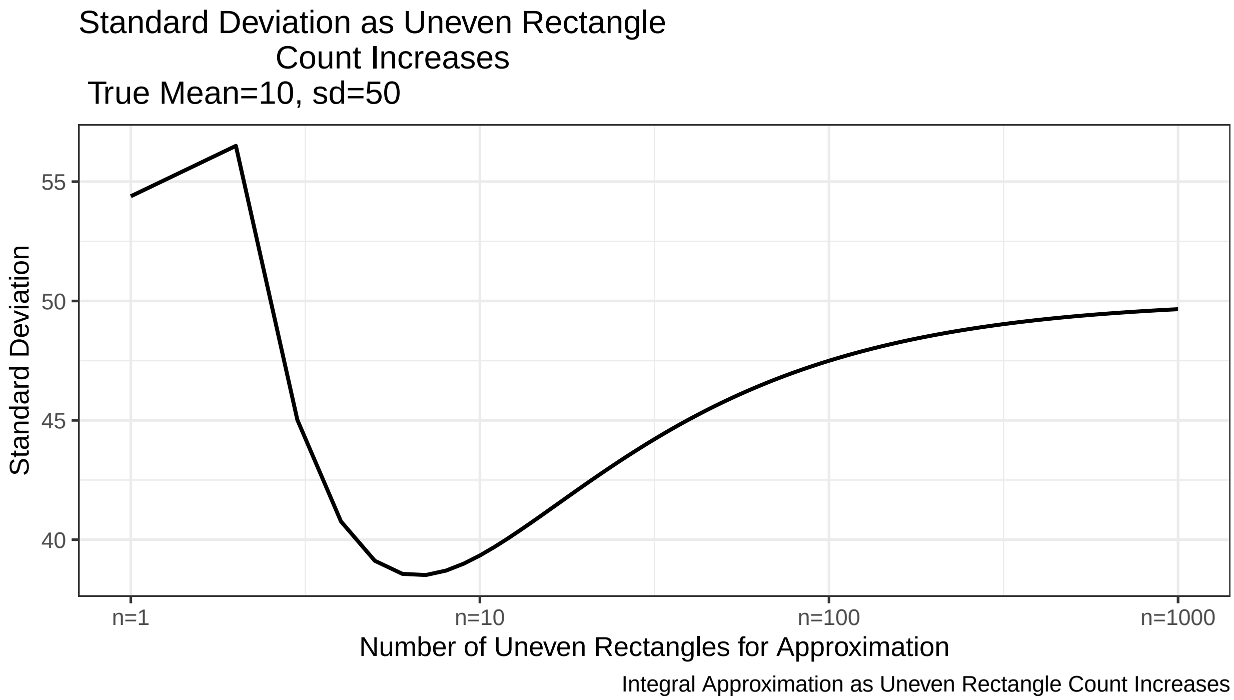
```
mt_fl_means <-
  sapply(ar_it_draws, function(x) {

    fl_prob_break = (fl_cdf_max - fl_cdf_min)/(x[1])
    ar_eps_bounds <- qnorm(seq(fl_cdf_min, fl_cdf_max,
                               by=(fl_cdf_max - fl_cdf_min)/(x[1])),
                           mean = fl_eps_mean, sd = fl_eps_sd)
    ar_eps_val <- (tail(ar_eps_bounds, -1) + head(ar_eps_bounds, -1))/2
    ar_eps_prb <- rep(fl_prob_break/(fl_cdf_max - fl_cdf_min), x[1])
    ar_eps_fev <- dnorm(ar_eps_val,
                        mean = fl_eps_mean, sd = fl_eps_sd)

    fl_cdf_total_approx <- sum(ar_eps_fev*diff(ar_eps_bounds))
    fl_mean_approx <- sum(ar_eps_val*(ar_eps_fev*diff(ar_eps_bounds)))
    fl_sd_approx <- sqrt(sum((ar_eps_val-fl_mean_approx)^2*(ar_eps_fev*diff(ar_eps_bounds))))

    return(list(cdf=fl_cdf_total_approx, mean=fl_mean_approx, sd=fl_sd_approx))
  })

mt_sample_means <- cbind(ar_it_draws, as_tibble(t(mt_fl_means)) %>% unnest())
colnames(mt_sample_means) <- c('draw_count', 'cdf', 'mean', 'sd')
tb_sample_means <- as_tibble(mt_sample_means)
```

```r
# Graph
# x-labels
x.labels <- c('n=1', 'n=10', 'n=100', 'n=1000')
x.breaks <- c(1, 10, 100, 1000)

# Graph Results--Draw
plt_mean <- tb_sample_means %>%
  ggplot(aes(x=draw_count, y=mean)) +
  geom_line(size=0.75) +
  labs(title = paste0('Average as Uneven Rectangle
                      Count Increases\n True Mean=',
                      fl_eps_mean,', sd=',fl_eps_sd),
       x = 'Number of Uneven Rectangles for Approximation',
       y = 'Integrated Mean',
       caption = 'Integral Approximation as Uneven Rectangle Count Increases') +
  scale_x_continuous(trans='log10', labels = x.labels, breaks = x.breaks) +
  theme_bw()
print(plt_mean)
```

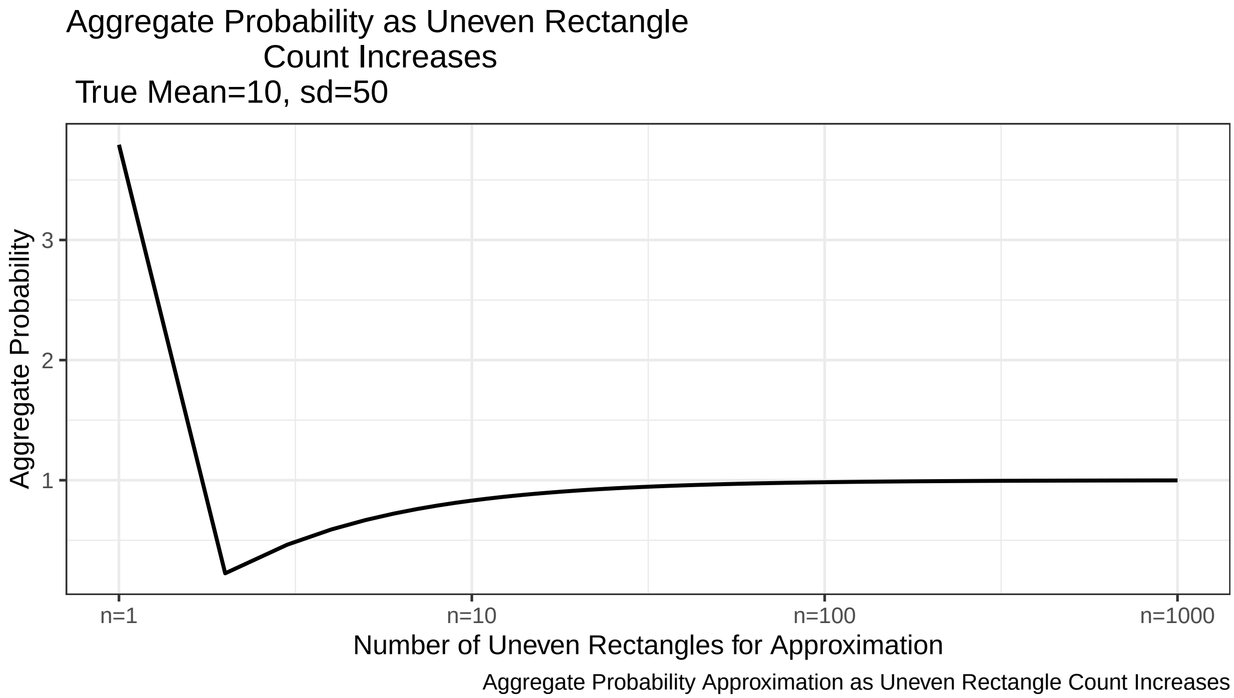



```r
plt_sd <- tb_sample_means %>%
  ggplot(aes(x=draw_count, y=sd)) +
  geom_line(size=0.75) +
  labs(title = paste0('Standard Deviation as Uneven Rectangle
                      Count Increases\n True Mean=',
                      fl_eps_mean,', sd=',fl_eps_sd),
       x = 'Number of Uneven Rectangles for Approximation',
       y = 'Standard Deviation',
       caption = 'Integral Approximation as Uneven Rectangle Count Increases') +
  scale_x_continuous(trans='log10', labels = x.labels, breaks = x.breaks) +
  theme_bw()
```

```r
print(plt_sd)
```



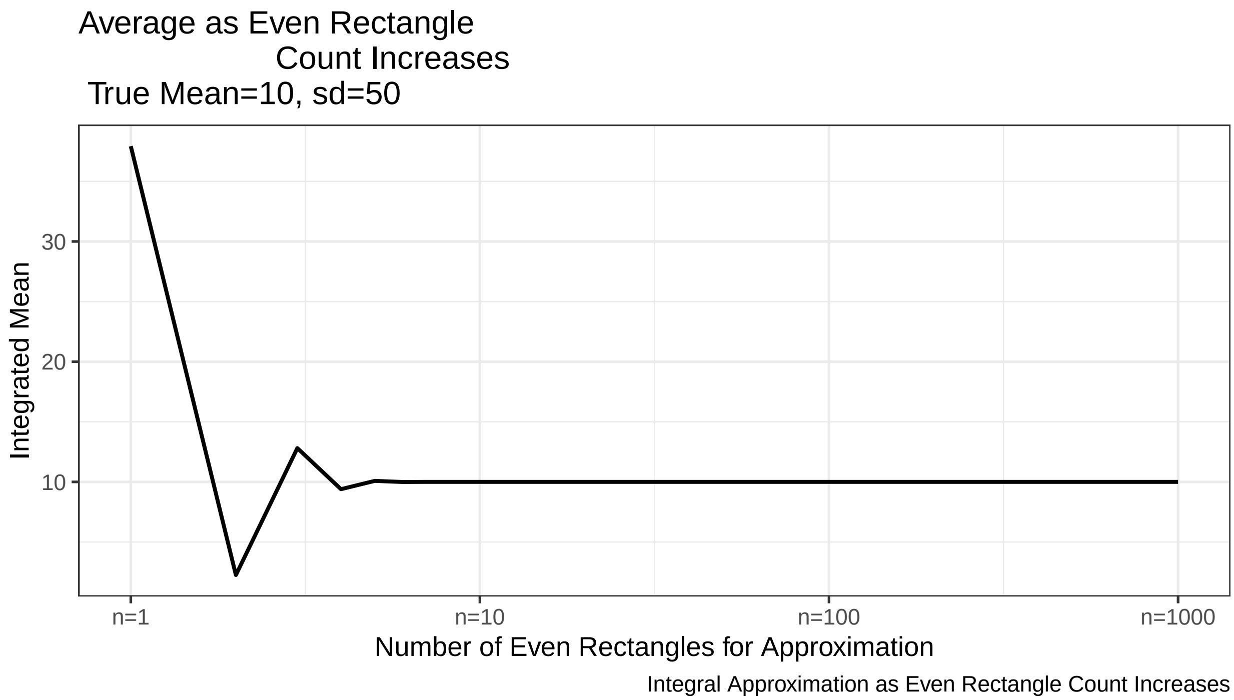

```r
plt_cdf <- tb_sample_means %>%
  ggplot(aes(x=draw_count, y=cdf)) +
  geom_line(size=0.75) +
  labs(title = paste0('Aggregate Probability as Uneven Rectangle
                      Count Increases\n True Mean=',
                      fl_eps_mean,', sd=',fl_eps_sd),
       x = 'Number of Uneven Rectangles for Approximation',
       y = 'Aggregate Probability',
       caption = 'Aggregate Probability Approximation as Uneven Rectangle Count Increases') +
  scale_x_continuous(trans='log10', labels = x.labels, breaks = x.breaks) +
  theme_bw()
print(plt_cdf)
```



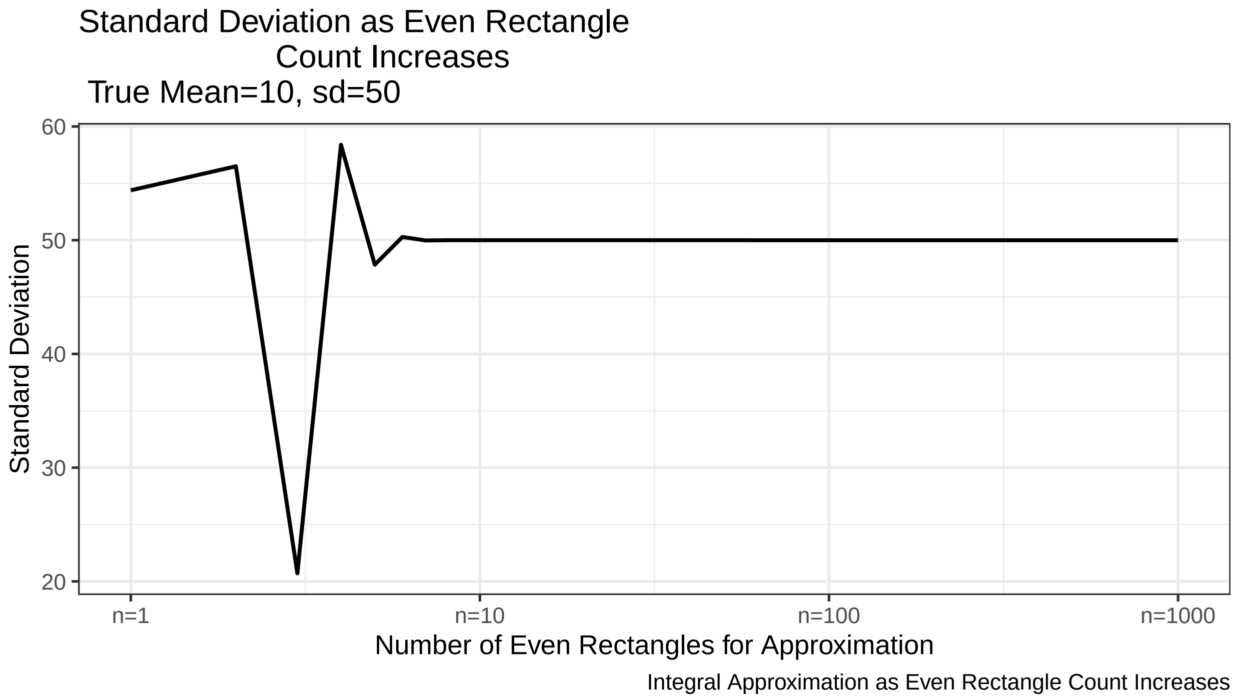
Integration By Constant Width Rectangle (Trapezoidal rule) This is implementing even width recentagle, even along x-axis take points, and measure $f(x)$. Rectangle width are the same. This is even width, but uneven area. Note that this method approximates the true answer much better and more quickly.

```
mt_fl_means <-
  sapply(ar_it_draws, function(x) {

    fl_eps_min <- qnorm(fl_cdf_min, mean = fl_eps_mean, sd = fl_eps_sd)
    fl_eps_max <- qnorm(fl_cdf_max, mean = fl_eps_mean, sd = fl_eps_sd)
    fl_gap <- (fl_eps_max-fl_eps_min)/(x[1])
    ar_eps_bounds <- seq(fl_eps_min, fl_eps_max, by=fl_gap)
    ar_eps_val <- (tail(ar_eps_bounds, -1) + head(ar_eps_bounds, -1))/2
    ar_eps_prb <- dnorm(ar_eps_val, mean = fl_eps_mean, sd = fl_eps_sd)*fl_gap

    fl_cdf_total_approx <- sum(ar_eps_prb)
    fl_mean_approx <- sum(ar_eps_val*ar_eps_prb)
    fl_sd_approx <- sqrt(sum((ar_eps_val-fl_mean_approx)^2*ar_eps_prb))

    return(list(cdf=fl_cdf_total_approx, mean=fl_mean_approx, sd=fl_sd_approx))
  })

mt_sample_means <- cbind(ar_it_draws, as_tibble(t(mt_fl_means)) %>% unnest())
colnames(mt_sample_means) <- c('draw_count', 'cdf', 'mean', 'sd')
tb_sample_means <- as_tibble(mt_sample_means)

# Graph
# x-labels
x.labels <- c('n=1', 'n=10', 'n=100', 'n=1000')
x.breaks <- c(1, 10, 100, 1000)

# Graph Results--Draw
```

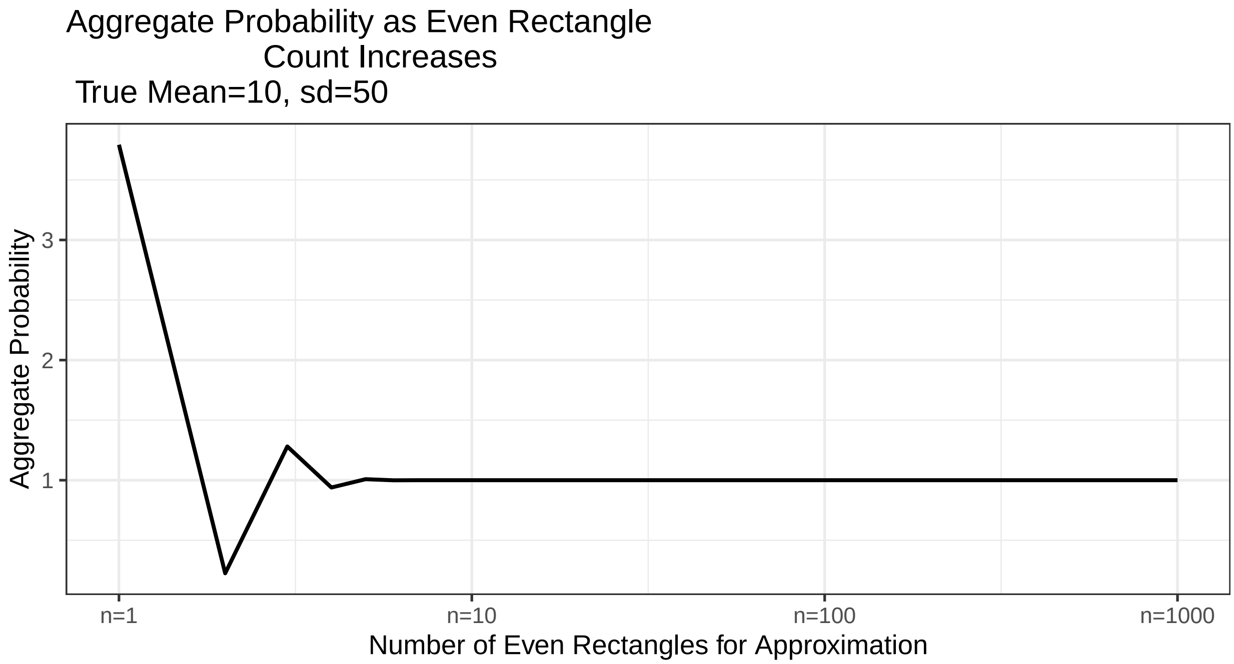
```r
plt_mean <- tb_sample_means %>%
  ggplot(aes(x=draw_count, y=mean)) +
  geom_line(size=0.75) +
  labs(title = paste0('Average as Even Rectangle
                      Count Increases\n True Mean=',
                      fl_eps_mean,', sd=',fl_eps_sd),
       x = 'Number of Even Rectangles for Approximation',
       y = 'Integrated Mean',
       caption = 'Integral Approximation as Even Rectangle Count Increases') +
  scale_x_continuous(trans='log10', labels = x.labels, breaks = x.breaks) +
  theme_bw()
print(plt_mean)
```




```r
plt_sd <- tb_sample_means %>%
  ggplot(aes(x=draw_count, y=sd)) +
  geom_line(size=0.75) +
  labs(title = paste0('Standard Deviation as Even Rectangle
                      Count Increases\n True Mean=',
                      fl_eps_mean,', sd=',fl_eps_sd),
       x = 'Number of Even Rectangles for Approximation',
       y = 'Standard Deviation',
       caption = 'Integral Approximation as Even Rectangle Count Increases') +
  scale_x_continuous(trans='log10', labels = x.labels, breaks = x.breaks) +
  theme_bw()
print(plt_sd)
```




```
plt_cdf <- tb_sample_means %>%
  ggplot(aes(x=draw_count, y=cdf)) +
  geom_line(size=0.75) +
  labs(title = paste0('Aggregate Probability as Even Rectangle
                      Count Increases\n True Mean=',
                      fl_eps_mean,', sd=',fl_eps_sd),
       x = 'Number of Even Rectangles for Approximation',
       y = 'Aggregate Probability',
       caption = 'Aggregate Probability Approximation as Even Rectangle Count Increases') +
  scale_x_continuous(trans='log10', labels = x.labels, breaks = x.breaks) +
  theme_bw()
print(plt_cdf)
```



Aggregate Probability Approximation as Even Rectangle Count Increases