Quadratic and other Rescaling of Parameters with Fixed Min and Max

Fan Wang

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1 Rescale a Parameter with Fixed Min and Max

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1.1 Using A Quadratic Function to Fit Three Points Uniquely

Given e < x < f, use f(x) to rescale x, such that f(e)=e, f(f)=f, but f(z)=$\alpha \cdot z$ for one particular z between e and f, where $\alpha > 1$. And in this case, assume that $\alpha \cdot z < f$. We can fit these three points using the Quadratic function uniquely. In another word, there is a unique quadratic function that crosses these three points. Note the quadratic function is either concave or convex through the entire domain.

Suppose that $e = 0$, $f = 10$, $z = 2$, and $\alpha = 1.5$. Using a quadratic to fit:

$$y(x) = a \cdot x^2 + b \cdot x + c$$

We have three equations:

$$0 = a \cdot 0 + b \cdot 0 + c2 \cdot 1.5 = a \cdot 2^2 + b \cdot 2 + c10 = a \cdot 10^2 + b \cdot 10 + c$$

Given these, we have, $c = 0$, and subsequently, 2 equations and 2 unknowns:

$$3 = a \cdot 4 + b \cdot 210 = a \cdot 100 + b \cdot 10$$

Hence:

$$a = \frac{3 - 2b}{4}10 = \frac{3 - 2b}{4} \cdot 100 + b \cdot 1010 = 75 - 50b + 10b$$

And finally:

$$a = \frac{3 - 2 * 1.625}{4} = -0.0625b = \frac{65}{40} = 1.625c = 0$$

Generate the a, b and c points above for the quadratic function:

```r
# set values
e <- 0
f <- 10
z <- 2
```

```
alpha <- 1.5
# apply formulas from above
a <- -0.0625
b <- 1.625
c <- 0
# grid of values beween a and b, 11 points covering z = 2
ar_x <- seq(e, f, length.out = 11)
# rescale
ar_grid_quad <- a*ar_x^2 + b*ar_x + c
# show values
kable(print(as_tibble(cbind(ar_x, ar_grid_quad))),
      caption = paste0("Quadratic Fit of Three Equations and Three Unknowns\n",
                       "Satisfies: f(0)=0, f(10)=10, f(2)=3")) %>%
  kable_styling_fc()
```

Quadratic Fit of Three Equations and Three Unknowns Satisfies: f(0)=0, f(10)=10, f(2)=3

ar_x	ar_grid_quad
0	0.0000
1	1.5625
2	3.0000
3	4.3125
4	5.5000
5	6.5625
6	7.5000
7	8.3125
8	9.0000
9	9.5625
10	10.0000

We have three equations:

$$0 = a \cdot 0 + b \cdot 0 + c 0.75 = a \cdot 0.5^2 + b \cdot 0.5 + c 3.5 = a \cdot 3.5^2 + b \cdot 3.5 + c$$

Given these, we have, $c = 0$, and subsequently, 2 equations and 2 unknowns:

$$0.75 = a \cdot 0.25 + b \cdot 0.5 3.5 = a \cdot 12.25 + b \cdot 3.5$$

Hence:

$$a = \frac{0.75 - 0.5b}{0.25} 3.5 = \frac{0.75 - 0.5b}{0.25} \cdot 12.25 + b \cdot 3.5 3.5 = 36.75 - 24.5b + 3.5b$$

And finally:

$$a = \frac{0.75 - 0.5 * 1.58333}{0.25} = -0.1666 b = \frac{36.75 - 3.5}{24.5 - 3.5} = 1.58333 c = 0$$

Generate the a, b and c points above for the quadratic function:

```
# set values
e <- 0
f <- 3.5
z <- 0.5
alpha <- 1.5
# apply formulas from above
a <- -0.16666666
b <- 1.583333333
```

```r
c <- 0
# grid of values beween a and b, 11 points covering z = 2
ar_x <- seq(e, f, length.out = 100000)
# rescale
ar_grid_quad <- a*ar_x^2 + b*ar_x + c
# show values
# cbind(ar_x, ar_grid_quad)
ar_x[which.min(abs(ar_grid_quad - 0.75))]
```

```
## [1] 0.500015
```