Multinomial-Logit and CRRA

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1 Multinomial Logit and CRRA

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1.1 Multinomial Logit Expected Value

The log-sum term. U_{tj} denotes pecuniary related utility at time t for choice alternative j.

$$\log\left(\exp\left(U_{11}\right)+\exp\left(U_{12}\right)\right)+\log\left(\exp\left(U_{21}\right)+\exp\left(U_{22}\right)\right)$$

1.2 Two Regimes with Utility (Does not Work)

With numerical values, utility under regime A.

```
UA11 = -0.5
UA12 = -1
UA21 = -0.6
UA22 = -1.1
fl_EV_A = log(exp(UA11) + exp(UA12)) + log(exp(UA21) + exp(UA22))
print(fl_EV_A)
```

```
## [1] -0.151846
```

With numerical values, utility under regime B.

```
UB11 = -0.1
UB12 = -0.65
UB21 = -0.21
UB22 = -0.2
fl_EV_B = log(exp(UB11) + exp(UB12)) + log(exp(UB21) + exp(UB22))
print(fl_EV_B)
```

```
## [1] 0.8436522
```

How much must U under A increase by to match overall U under B? Note that the structure below does not make sense, but U is already negative, it can not be about how chaning Shares of U impact things.

```
ar_increase = seq(0, 0.5, length.out=10)
fl_EV_A = log(exp(UA11*(1+ar_increase)) + exp(UA12*(1+ar_increase))) + log(exp(UA21*(1+ar_increase)) + e
print(fl_EV_A)
```

```
##  [1] -0.1518460 -0.2337507 -0.3152953 -0.3964823 -0.4773146 -0.5577950 -0.6379265 -0.7177120 -0.7971
```

1.3 Two Regimes with Consumption

Preference CRRA

```
gamma = 1.3
# Define utility Function
ffi_crra <- function(fl_c){
  fl_U = (fl_c^(1-gamma))/(1-gamma)
  return(fl_U)
}
```

```
fl_increment = 1.5
CA11 = -0.5+fl_increment
CA12 = -1+fl_increment
CA21 = -0.6+fl_increment
CA22 = -1.1+fl_increment
fl_EV_A = log(exp(ffi_crra(CA11)) + exp(ffi_crra(CA12))) + log(exp(ffi_crra(CA21)) + exp(ffi_crra(CA22))
print(fl_EV_A)
```

```
## [1] -6.065728
```

With numerical values, utility under regime B.

```
fl_increment = 3
CB11 = -0.1+fl_increment
CB12 = -0.65+fl_increment
CB21 = -0.21+fl_increment
CB22 = -0.2+fl_increment
fl_EV_B = log(exp(ffi_crra(CB11)) + exp(ffi_crra(CB12))) + log(exp(ffi_crra(CB21)) + exp(ffi_crra(CB22))
print(fl_EV_B)
```

```
## [1] -3.560238
```

How much must C under A increase by to match overall U under B? This structure works, negative or positive values for overall EV does not matter, work in either case.

```
ar_increase = seq(0, 3, length.out=10)
# Method 1
fl_EV_A_with_c_increments =
  log(exp(ffi_crra(CA11*(1+ar_increase))) +
        exp(ffi_crra(CA12*(1+ar_increase)))) +
  log(exp(ffi_crra(CA21*(1+ar_increase))) +
        exp(ffi_crra(CA22*(1+ar_increase))))
print(fl_EV_A_with_c_increments)
```

```
##  [1] -6.065728 -5.462530 -5.027880 -4.692944 -4.423210 -4.199089 -4.008473 -3.843392 -3.698348 -3.56
```

```
# Method 2
fl_EV_A_with_c_increments_m2 =
  log(exp((1+ar_increase)^(1-gamma)*ffi_crra(CA11)) +
```

```
        exp((1+ar_increase)^(1-gamma)*ffi_crra(CA12))) +
  log(exp((1+ar_increase)^(1-gamma)*ffi_crra(CA21)) +
        exp((1+ar_increase)^(1-gamma)*ffi_crra(CA22)))
print(fl_EV_A_with_c_increments_m2)
```

```
##  [1] -6.065728 -5.462530 -5.027880 -4.692944 -4.423210 -4.199089 -4.008473 -3.843392 -3.698348 -3.56
```

```
# Method 3
fl_EV_A_with_c_increments_m3 =
  log(exp((1+ar_increase)^(1-gamma))*exp(ffi_crra(CA11)) +
        exp((1+ar_increase)^(1-gamma))*exp(ffi_crra(CA12))) +
  log(exp((1+ar_increase)^(1-gamma))*exp(ffi_crra(CA21)) +
        exp((1+ar_increase)^(1-gamma))*exp(ffi_crra(CA22)))
print(fl_EV_A_with_c_increments_m3)
```

```
##  [1] -4.065728 -4.231098 -4.349893 -4.441223 -4.514638 -4.575546 -4.627282 -4.672037 -4.711323 -4.74
```

```
# Method 3
fl_EV_A_with_c_increments_m3 =
  (1+ar_increase)^(1-gamma) +
  log(exp(ffi_crra(CA11)) + exp(ffi_crra(CA12))) +
  (1+ar_increase)^(1-gamma) +
  log(exp(ffi_crra(CA21)) + exp(ffi_crra(CA22)))
print(fl_EV_A_with_c_increments_m3)
```

```
##  [1] -4.065728 -4.231098 -4.349893 -4.441223 -4.514638 -4.575546 -4.627282 -4.672037 -4.711323 -4.74
```

1.4 Analytically Show

What is analytically the CEV with CRRA + Mlogit?

$$\log\left(\exp\left(U\left(C_{11}\right)\right)+\exp\left(U\left(C_{12}\right)\right)\right)+\log\left(\exp\left(U\left(C_{21}\right)\right)+\exp\left(U\left(C_{22}\right)\right)\right)$$

Given CRRA Utility

$$U(C;\psi)=\frac{\left(c\cdot\left(1+\psi\right)\right)^{1-\gamma}}{1-\gamma}U(C;\psi)=\frac{\left(1+\psi\right)^{1-\gamma}\left(c\right)^{1-\gamma}}{1-\gamma}=\left(1+\psi\right)^{1-\gamma}\cdot U(C)$$

Plugging ψ into the equation

$$\log\left(\exp\left(\left(1+\psi\right)^{1-\gamma}\cdot U\left(C_{11}\right)\right)+\exp\left(\left(1+\psi\right)^{1-\gamma}\cdot U\left(C_{12}\right)\right)\right)+\log\left(\exp\left(\left(1+\psi\right)^{1-\gamma}\cdot U\left(C_{21}\right)\right)+\exp\left(\left(1+\psi\right)^{1-\gamma}\cdot U\left(C_{22}\right)\right)\right)$$

This is as far as we can easily go, note:

$$\exp(a\cdot b)\neq\exp(a)\cdot\exp(b)$$