Solving A Constrained Maximization Problem with Fixed Cost and Minimum Choice Bound

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Discrete and Continuous

Go back to fan's REconTools Package, R4Econ Repository, or Intro Stats with R Repository.

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library(tidyverse)
library(tidyr)
library(knitr)
library(kableExtra)
```

Study the relationship between fixed cost ϕ and minimum choice bounds a^{min}. This is a general problem that appears in a lot of context.

There is a cost to choosing U over B. And when choosing B, there is a minimum choice associated with choosing B category. So the question is, should we choose U or B, and if we choose B, how much of a should we choose.

Individuals are defined by a single state variable z, which determines current wealth, and also helps to determine future wealth. We are interested in solving for the proportion of people choosing U and B, and then within those choosing B, the proportion of individuals choosing the a^{min}

There are three problems to solve:

1. The discrete choice problem over U and B
2. The bounding threshold problem
3. The unbounded continuous choice problem

The problem presented here is a simple version of Udupa and Wang (2020)'s savings friction paper.

Unconstrained Savings and Borrowing There are endowments today and tomorrow. The endowment tomorrow is a function of the endowment today, higher endowment today increases endowment tomorrow. Households are defined by a single state variable z. In particular, there are three possible future, same endowment as today, endowment increases by ϵ, or endowment decreases by ϵ

$$V^{unc}(z) = \max_{a} \left(\log\left(z - \phi - a'\right) + \beta \left(\begin{array}{c} P_h \cdot \log\left(\exp(z+\epsilon) + a' \cdot R\right) + \\ P_m \cdot \log\left(\exp(z) + a' \cdot R\right) + \\ P_l \cdot \log\left(\exp(z-\epsilon) + a' \cdot R\right) \end{array} \right) \right)$$

It is straight forward to solve the above problem numerically. we can solve the problem given a dense grid of a and z points, and find approximately exactly at each z point.

We will solve the problem twice, first using grid method, then using the R bisection method from Fan's REconTools. We should see that the discretized solution is almost identical to the bisection solution which should be more exact.

Clearly some of the optimal savings or borrowing choices $a^{\prime,*}$ will be negative, when the household wants to borrow at some z, and it will be positive at other z points where households want to save.

The Asset Choice Constraint Now we add in a constraint, the constraint could be a borrowing or savings constraint.

Compare Utility What would be the optimal choice if households