Positive and Negative Exponents when x is Below or Above 1

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2021-05-23

Contents

1 Positive and Negative Exponents

Go to the **RMD**, **R**, **PDF**, or **HTML** version of this file. Go back to fan's REconTools Package, R Code Examples Repository (bookdown site), or Intro Stats with R Repository (bookdown site).

1.1 Positive Exponents

Define exponents to consider and x-values to consider.

```
# positive value exponents
ar_exponents_posv <- c(0.05, 0.5, 1, 1.5)
# positive and negative values of the base
ar_baseval_pos <- seq(1e-10, 1.5, length.out=1000)
# base to power
mt_x2a_val <- matrix(data=NA, nrow=length(ar_exponents_posv), ncol=length(ar_baseval_pos))
# Generate values
it_row_ctr <- 0
for (fl_exponents_posv in ar_exponents_posv) {
  it_row_ctr <- it_row_ctr + 1
  mt_x2a_val[it_row_ctr,] <- ar_baseval_pos^fl_exponents_posv
}
```

Note that the smaller exponents functions are higher when $x < 1$, but lower when $x > 1$.

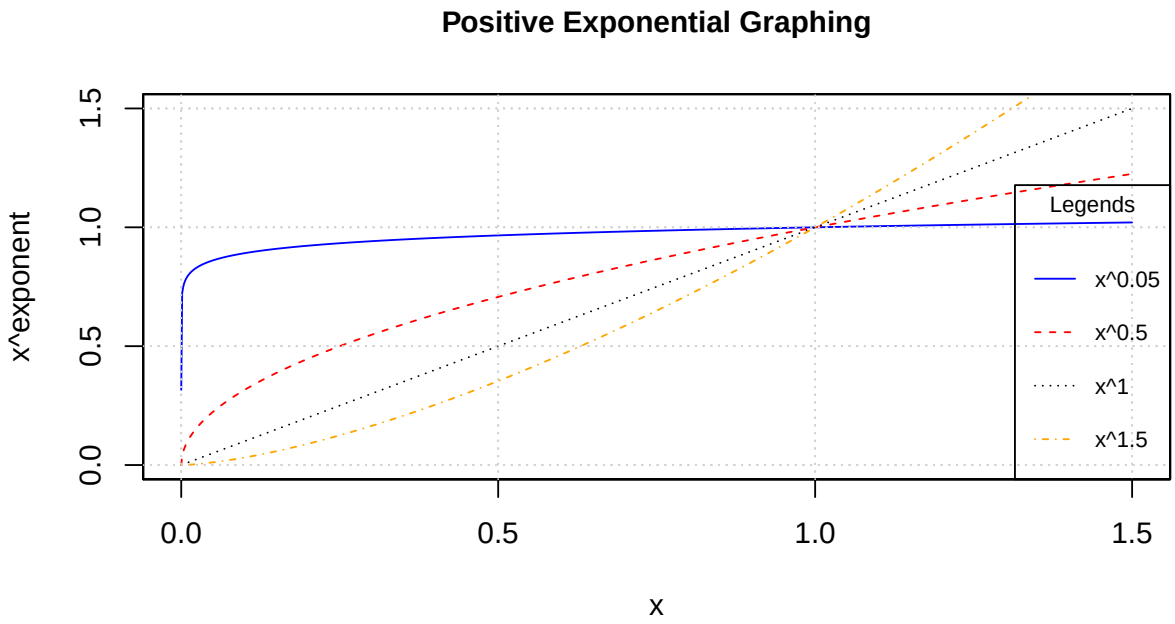
$$\text{if } b > a > 0, \text{ then, } \left(x^a - x^b\right) > 0, \text{ for all } 1 > x > 0$$

$$\text{if } b > a > 0, \text{ then, } \left(x^a - x^b\right) < 0, \text{ for all } x > 1$$

Note we also have: $\lim_{a \to 0} x^a = 1$ and $\lim_{a \to 1} x^a = x$ bounds. When $a > 1$, function becomes convex.

```
# x and bounds
ar_xlim = c(min(ar_baseval_pos), max(ar_baseval_pos))
ar_ylim = c(0, 1.5)
# function line
st_line_1_y_legend <- paste0('x^', ar_exponents_posv[1])
st_line_2_y_legend <- paste0('x^', ar_exponents_posv[2])
st_line_3_y_legend <- paste0('x^', ar_exponents_posv[3])
```

```r
st_line_4_y_legend <- paste0('x^', ar_exponents_posv[4])
# Color and line
st_point_1_pch <- 10
st_point_1_cex <- 2
ar_colors <- c('blue', 'red', 'black', 'orange')
ar_ltys <- c('solid', 'dashed', 'dotted', 'dotdash')
# Graph and combine
for (it_graph in c(1,2,3,4)) {
  if (it_graph != 1) {
    par(new=T)
  }
  ar_y_current <- mt_x2a_val[it_graph, ]
  plot(ar_baseval_pos, ar_y_current, type="l",
       col = ar_colors[it_graph], lty = ar_ltys[it_graph],
       pch = 10, cex = 2, xlim = ar_xlim, ylim = ar_ylim, panel.first = grid(),
       ylab = '', xlab = '', yaxt='n', xaxt='n', ann=FALSE)
  plot_line <- recordPlot()
}
# CEX sizing Contorl Titling and Legend Sizes
fl_ces_fig_reg = 1
fl_ces_fig_small = 0.75
# R Legend
st_title <- paste0('Positive Exponential Graphing')
st_subtitle <- paste0('https://fanwangecon.github.io/',
                      'R4Econ/math/solutions/htmlpdfr/fs_inequality.html')
st_x_label <- 'x'
st_y_label <- 'x^exponent'
title(main = st_title, sub = st_subtitle, xlab = st_x_label, ylab = st_y_label,
      cex.lab=fl_ces_fig_reg,
      cex.main=fl_ces_fig_reg,
      cex.sub=fl_ces_fig_small)
axis(1, cex.axis=fl_ces_fig_reg)
axis(2, cex.axis=fl_ces_fig_reg)
grid()
# Legend sizing CEX
legend("bottomright",
       inset=c(0,0),
       xpd=TRUE,
       c(st_line_1_y_legend, st_line_2_y_legend, st_line_3_y_legend, st_line_4_y_legend),
       col = c(ar_colors[1], ar_colors[2], ar_colors[3], ar_colors[4]),
       cex = fl_ces_fig_small,
       lty = c(ar_ltys[1], ar_ltys[2], ar_ltys[3], ar_ltys[4]),
       title = 'Legends',
       y.intersp=2)
```



https://fanwangecon.github.io/R4Econ/math/solutions/htmlpdfr/fs_inequality.html

1.2 Negative Exponents

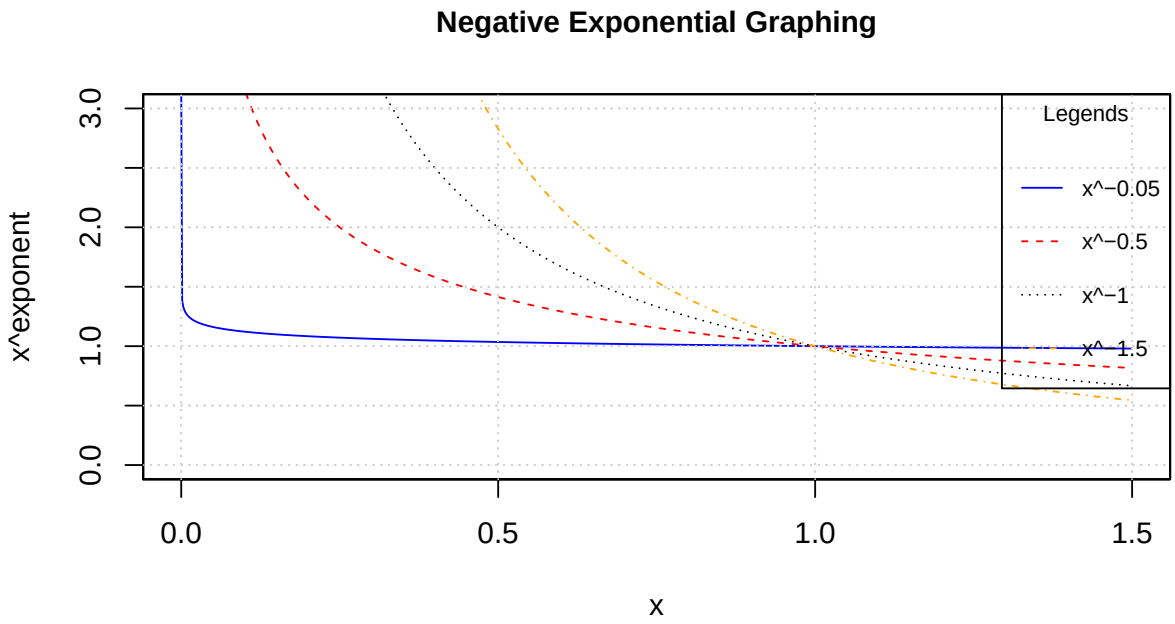
Similar to above, but now with negative exonents.

```
# positive value exponents
ar_exponents_posv <- -c(0.05, 0.5, 1, 1.5)
# positive and negative values of the base
ar_baseval_pos <- seq(1e-10, 1.5, length.out=1000)
# base to power
mt_x2a_val <- matrix(data=NA, nrow=length(ar_exponents_posv), ncol=length(ar_baseval_pos))
# Generate values
it_row_ctr <- 0
for (fl_exponents_posv in ar_exponents_posv) {
  it_row_ctr <- it_row_ctr + 1
  mt_x2a_val[it_row_ctr,] <- ar_baseval_pos^fl_exponents_posv
}
```

For positive exponents, when $x < 1$, $x^a < 1$, when $x > 1$, $x^a > 1$. For negative exponents, when $x < 1$, $x^a > 1$, and when $x > 1$, $x^a < 1$. Large positive exponents generate small values when $x < 1$, and large negative exponents generate very large values when $x < 1$.

```
# x and bounds
ar_xlim = c(min(ar_baseval_pos), max(ar_baseval_pos))
ar_ylim = c(0, 3)
# function line
st_line_1_y_legend <- paste0('x^', ar_exponents_posv[1])
st_line_2_y_legend <- paste0('x^', ar_exponents_posv[2])
st_line_3_y_legend <- paste0('x^', ar_exponents_posv[3])
st_line_4_y_legend <- paste0('x^', ar_exponents_posv[4])
# Color and line
st_point_1_pch <- 10
st_point_1_cex <- 2
```

```
ar_colors <- c('blue', 'red', 'black', 'orange')
ar_ltys <- c('solid', 'dashed', 'dotted', 'dotdash')
# Graph and combine
for (it_graph in c(1,2,3,4)) {
  if (it_graph != 1) {
    par(new=T)
  }
  ar_y_current <- mt_x2a_val[it_graph, ]
  plot(ar_baseval_pos, ar_y_current, type="l",
       col = ar_colors[it_graph], lty = ar_ltys[it_graph],
       pch = 10, cex = 2, xlim = ar_xlim, ylim = ar_ylim, panel.first = grid(),
       ylab = '', xlab = '', yaxt='n', xaxt='n', ann=FALSE)
  plot_line <- recordPlot()
}
# CEX sizing Contorl Titling and Legend Sizes
fl_ces_fig_reg = 1
fl_ces_fig_small = 0.75
# R Legend
st_title <- paste0('Negative Exponential Graphing')
st_subtitle <- paste0('https://fanwangecon.github.io/',
                      'R4Econ/math/solutions/htmlpdfr/fs_inequality.html')
st_x_label <- 'x'
st_y_label <- 'x^exponent'
title(main = st_title, sub = st_subtitle, xlab = st_x_label, ylab = st_y_label,
      cex.lab=fl_ces_fig_reg,
      cex.main=fl_ces_fig_reg,
      cex.sub=fl_ces_fig_small)
axis(1, cex.axis=fl_ces_fig_reg)
axis(2, cex.axis=fl_ces_fig_reg)
grid()
# Legend sizing CEX
legend("topright",
       inset=c(0,0),
       xpd=TRUE,
       c(st_line_1_y_legend, st_line_2_y_legend, st_line_3_y_legend, st_line_4_y_legend),
       col = c(ar_colors[1], ar_colors[2], ar_colors[3], ar_colors[4]),
       cex = fl_ces_fig_small,
       lty = c(ar_ltys[1], ar_ltys[2], ar_ltys[3], ar_ltys[4]),
       title = 'Legends',
       y.intersp=2)
```



https://fanwangecon.github.io/R4Econ/math/solutions/htmlpdfr/fs_inequality.html

1.3 Inequality and Exponents

Suppose we have the inequality $0 < a < b$, if we apply positive exponents to them, the direction of the inequality will stay the same: If $0 < a < b$, then $0 < a^{|\alpha|} < b^{|\alpha|}$ if $\alpha < 0$. Think about the graphs above, think of a and b as points along the x-axis, note that positive exponents are strictly increasing (although some concavely and some convexly) along the x-axis. Comparing x^{α} at $0 < b < a$ anywhere along the x-axis has still has $b^{\alpha} < a^{\alpha}$.

In contrast, if $0 < a < b$, then $a^{-|\alpha|} > b^{-|\alpha|} > 0$ if $\alpha < 0$. Sign flips. Visually from above, the sign-flipping happens because negative exponential is strictly decreasing along $x > 0$.