

Panel Data and Optimization with R

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Preface

This is a work-in-progress [website](#) consisting of R panel data and optimization examples for Statistics/Econometrics/Economic Analysis. Materials gathered from various [projects](#) in which R code is used. Files are from [Fan's R4Econ](#) repository. This is not a R package, but a list of examples in PDF/HTML/Rmd formats. [REconTools](#) is a package that can be installed with tools used in [projects](#) involving R.

Bullet points show which [base R](#), [tidyverse](#) or other functions/commands are used to achieve various objectives. An effort is made to use only [base R](#) ([R Core Team, 2019](#)) and [tidyverse](#) ([Wickham, 2019](#)) packages whenever possible to reduce dependencies. The goal of this repository is to make it easier to find/re-use codes produced for various projects. Some functions also rely on or correspond to functions from [REconTools](#) ([Wang, 2020](#)).

From [Fan's](#) other repositories: For dynamic borrowing and savings problems, see [Dynamic Asset Repository](#); For code examples, see also [Matlab Example Code](#) and [Stata Example Code](#); For intro econ with Matlab, see [Intro Mathematics for Economists](#), and for intro stat with R, see [Intro Statistics for Undergraduates](#). See [here](#) for all of [Fan's](#) public repositories.

The site is built using [Bookdown](#) ([Xie, 2020](#)).

Please contact [FanWangEcon](#) for issues or problems.

Chapter 1

Array, Matrix, Dataframe

1.1 List

1.1.1 Multiple Dimensional List

Go back to [fan's REconTools](#) Package, [R4Econ](#) Repository ([bookdown site](#)), or [Intro Stats with R](#) Repository.

- r list tutorial
- r vector vs list
- r initialize empty multiple element list
- r name rows and columns of 2 dimensional list
- r row and colum names of list
- list dimnames

1.1.1.1 One Dimensional Named List

1. define list
2. slice list

```
# Define Lists
ls_num <- list(1,2,3)
ls_str <- list('1','2','3')
ls_num_str <- list(1,2,'3')

# Named Lists
ar_st_names <- c('e1','e2','e3')
ls_num_str_named <- ls_num_str
names(ls_num_str_named) <- ar_st_names

# Add Element to Named List
ls_num_str_named$e4 <- 'this is added'

# display
print(paste0('ls_num:', str(ls_num)))

## List of 3
## $ : num 1
## $ : num 2
## $ : num 3
## [1] "ls_num:"

print(paste0('ls_num[2:3]:', str(ls_num[2:3])))
```

```
## List of 2
```

```
## $ : num 2
## $ : num 3
## [1] "ls_num[2:3]:"
```

```
print(paste0('ls_str:', str(ls_str)))
```

```
## List of 3
## $ : chr "1"
## $ : chr "2"
## $ : chr "3"
## [1] "ls_str:"
```

```
print(paste0('ls_str[2:3]:', str(ls_str[2:3])))
```

```
## List of 2
## $ : chr "2"
## $ : chr "3"
## [1] "ls_str[2:3]:"
```

```
print(paste0('ls_num_str:', str(ls_num_str)))
```

```
## List of 3
## $ : num 1
## $ : num 2
## $ : chr "3"
## [1] "ls_num_str:"
```

```
print(paste0('ls_num_str[2:4]:', str(ls_num_str[2:4])))
```

```
## List of 3
## $ : num 2
## $ : chr "3"
## $ : NULL
## [1] "ls_num_str[2:4]:"
```

```
print(paste0('ls_num_str_named:', str(ls_num_str_named)))
```

```
## List of 4
## $ e1: num 1
## $ e2: num 2
## $ e3: chr "3"
## $ e4: chr "this is added"
## [1] "ls_num_str_named:"
```

```
print(paste0('ls_num_str_named[c(\"e2\", \"e3\", \"e4\")]', str(ls_num_str_named[c('e2', 'e3', 'e4')])))
```

```
## List of 3
## $ e2: num 2
## $ e3: chr "3"
## $ e4: chr "this is added"
## [1] "ls_num_str_named[c('e2', 'e3', 'e4')]"
```

1.1.1.2 Two Dimensional Unnamed List

Generate a multiple dimensional list:

1. Initiate with an N element empty list
2. Reshape list to M by Q
3. Fill list elements
4. Get list element by row and column number

List allows for different data types to be stored together.

Note that element specific names in named list are not preserved when the list is reshaped to be two dimensional. Two dimensional list, however, could have row and column names.

```
# Dimensions
it_M <- 2
it_Q <- 3
it_N <- it_M*it_Q

# Initiate an Empty MxQ=N element list
ls_2d_flat <- vector(mode = "list", length = it_N)
ls_2d <- ls_2d_flat

# Named flat
ls_2d_flat_named <- ls_2d_flat
names(ls_2d_flat_named) <- paste0('e',seq(1,it_N))
ls_2d_named <- ls_2d_flat_named

# Reshape
dim(ls_2d) <- c(it_M, it_Q)
# named 2d list can not carry 1d name after reshape
dim(ls_2d_named) <- c(it_M, it_Q)
```

Print Various objects generated above:

```
# display
print('ls_2d_flat')

## [1] "ls_2d_flat"

print(ls_2d_flat)

## [[1]]
## NULL
##
## [[2]]
## NULL
##
## [[3]]
## NULL
##
## [[4]]
## NULL
##
## [[5]]
## NULL
##
## [[6]]
## NULL

print('ls_2d_flat_named')

## [1] "ls_2d_flat_named"

print(ls_2d_flat_named)

## $e1
## NULL
##
## $e2
## NULL
##
## $e3
```

```
## NULL
##
## $e4
## NULL
##
## $e5
## NULL
##
## $e6
## NULL
```

```
print('ls_2d')
```

```
## [1] "ls_2d"
```

```
print(ls_2d)
```

```
##      [,1] [,2] [,3]
## [1,] NULL NULL NULL
## [2,] NULL NULL NULL
```

```
print('ls_2d_named')
```

```
## [1] "ls_2d_named"
```

```
print(ls_2d_named)
```

```
##      [,1] [,2] [,3]
## [1,] NULL NULL NULL
## [2,] NULL NULL NULL
```

Select element from list:

```
# Select Values, double bracket to select from 2dim list
print('ls_2d[[1,2]]')
```

```
## [1] "ls_2d[[1,2]]"
```

```
print(ls_2d[[1,2]])
```

```
## NULL
```

1.1.1.3 Define Two Dimensional Named Llist

For naming two dimensional lists, *rowname* and *colname* does not work. Rather, we need to use *dimnames*. Note that in addition to *dimnames*, we can continue to have element specific names. Both can co-exist. But note that the element specific names are not preserved after dimension transform, so need to be redefined afterwards.

How to select an element of a two dimensional list:

1. row and column names: *dimnames*, *ls_2d_flat_named*[[*'row2'*,*'col2'*]]
2. named elements: *names*, *ls_2d_flat_named*[[*'e5'*]]
3. select by index: *index*, *ls_2d_flat_named*[[5]]
4. converted two dimensional named list to tibble/matrix

Neither *dimnames* nor *names* are required, but both can be used to select elements.

```
# Dimensions
it_M <- 3
it_Q <- 4
it_N <- it_M*it_Q
```

```
# Initiate an Empty MxQ=N element list
ls_2d_flat_named <- vector(mode = "list", length = it_N)
```

```

dim(ls_2d_flat_named) <- c(it_M, it_Q)

# Fill with values
for (it_Q_ctr in seq(1,it_Q)) {
  for (it_M_ctr in seq(1,it_M)) {
    # linear index
    ls_2d_flat_named[[it_M_ctr, it_Q_ctr]] <- (it_Q_ctr-1)*it_M+it_M_ctr
  }
}

# Replace row names, note rownames does not work
dimnames(ls_2d_flat_named)[[1]] <- paste0('row',seq(1,it_M))
dimnames(ls_2d_flat_named)[[2]] <- paste0('col',seq(1,it_Q))

# Element Specific Names
names(ls_2d_flat_named) <- paste0('e',seq(1,it_N))

# Convert to Matrix
tb_2d_flat_named <- as_tibble(ls_2d_flat_named) %>% unnest()
mt_2d_flat_named <- as.matrix(tb_2d_flat_named)

```

Print various objects generated above:

```

# These are not element names, can still name each element
# display
print('ls_2d_flat_named')

## [1] "ls_2d_flat_named"

print(ls_2d_flat_named)

##      col1 col2 col3 col4
## row1  1    4    7    10
## row2  2    5    8    11
## row3  3    6    9    12
## attr(,"names")
## [1] "e1" "e2" "e3" "e4" "e5" "e6" "e7" "e8" "e9" "e10" "e11" "e12"

print('str(ls_2d_flat_named)')

## [1] "str(ls_2d_flat_named)"

print(str(ls_2d_flat_named))

## List of 12
## $ e1 : num 1
## $ e2 : num 2
## $ e3 : num 3
## $ e4 : num 4
## $ e5 : num 5
## $ e6 : num 6
## $ e7 : num 7
## $ e8 : num 8
## $ e9 : num 9
## $ e10: num 10
## $ e11: num 11
## $ e12: num 12
## - attr(*, "dim")= int [1:2] 3 4
## - attr(*, "dimnames")=List of 2
## ..$ : chr [1:3] "row1" "row2" "row3"
## ..$ : chr [1:4] "col1" "col2" "col3" "col4"

```

```
## NULL
print('tb_2d_flat_named')

## [1] "tb_2d_flat_named"
print(tb_2d_flat_named)
print('mt_2d_flat_named')

## [1] "mt_2d_flat_named"
print(mt_2d_flat_named)

##      col1 col2 col3 col4
## [1,]    1    4    7   10
## [2,]    2    5    8   11
## [3,]    3    6    9   12
```

Select elements from list:

```
# Select elements with with dimnames
print('ls_2d_flat_named[["row2","col2"]]' )

## [1] "ls_2d_flat_named[["row2","col2"]]"
print(ls_2d_flat_named[["row2","col2"]])

## [1] 5
# Select elements with element names
print('ls_2d_flat_named[["e5"]]' )

## [1] "ls_2d_flat_named[["e5"]]"
print(ls_2d_flat_named[["e5"]])

## [1] 5
# Select elements with index
print('ls_2d_flat_named[[5]]' )

## [1] "ls_2d_flat_named[[5]]"
print(ls_2d_flat_named[[5]])

## [1] 5
```

1.2 Array

1.2.1 Array Basics

Go back to [fan's REconTools](#) Package, [R4Econ](#) Repository ([bookdown site](#)), or [Intro Stats with R](#) Repository.

1.2.1.1 Multidimensional Arrays

```
# Multidimensional Array
# 1 is r1c1t1, 1.5 in r2c1t1, 0 in r1c2t1, etc.
# Three dimensions, row first, column second, and tensor third
x <- array(c(1, 1.5, 0, 2, 0, 4, 0, 3), dim=c(2, 2, 2))
dim(x)
```

1.2.1.1.1 Generate 2 Dimensional Array

```
## [1] 2 2 2
```

```
print(x)

## , , 1
##
##      [,1] [,2]
## [1,]  1.0   0
## [2,]  1.5   2
##
## , , 2
##
##      [,1] [,2]
## [1,]    0   0
## [2,]    4   3
```

1.2.1.2 Array Slicing

1.2.1.2.1 Remove Elements of Array Select elements with direct indexing, or with head and tail functions. Get the first two elements of three elements array.

```
# Remove last element of array
vars.group.bydf <- c('23', 'dfa', 'wer')
vars.group.bydf[-length(vars.group.bydf)]
```

```
## [1] "23" "dfa"
```

```
# Use the head function to remove last element
head(vars.group.bydf, -1)
```

```
## [1] "23" "dfa"
```

```
head(vars.group.bydf, 2)
```

```
## [1] "23" "dfa"
```

Get last two elements of array.

```
# Remove first element of array
vars.group.bydf <- c('23', 'dfa', 'wer')
vars.group.bydf[2:length(vars.group.bydf)]
```

```
## [1] "dfa" "wer"
```

```
# Use Tail function
tail(vars.group.bydf, -1)
```

```
## [1] "dfa" "wer"
```

```
tail(vars.group.bydf, 2)
```

```
## [1] "dfa" "wer"
```

1.2.1.3 NA in Array

```
# Convert Inf and -Inf to NA
x <- c(1, -1, Inf, 10, -Inf)
na_if(na_if(x, -Inf), Inf)
```

1.2.1.3.1 Check if NA is in Array

```
## [1] 1 -1 NA 10 NA
```

1.2.2 Generate Arrays

Go back to [fan's REconTools Package](#), [R4Econ Repository \(bookdown site\)](#), or [Intro Stats with R Repository](#).

1.2.2.1 Generate Special Arrays

1.2.2.1.1 Log Space Arrays Often need to generate arrays on log rather than linear scale, below is log 10 scaled grid.

```
# Parameters
it.lower.bd.inc.cnt <- 3
fl.log.lower <- -10
fl.log.higher <- -9
fl.min.rescale <- 0.01
it.log.count <- 4

# Generate
ar.fl.log.rescaled <- exp(log(10)*seq(log10(fl.min.rescale),
                                     log10(fl.min.rescale +
                                             (fl.log.higher-fl.log.lower)),
                                     length.out=it.log.count))
ar.fl.log <- ar.fl.log.rescaled + fl.log.lower - fl.min.rescale

# Print
ar.fl.log

## [1] -10.000000 -9.963430 -9.793123 -9.000000
```

1.2.3 String Arrays

Go back to [fan's REconTools Package](#), [R4Econ Repository \(bookdown site\)](#), or [Intro Stats with R Repository](#).

1.2.3.1 String Replace

```
# String replacement
gsub(x = paste0(unique(df.slds.stats.perc$it.inner.counter), ':',
                    unique(df.slds.stats.perc$z_n_a_n), collapse = ';'),
     pattern = "\\n",
     replacement = "")
gsub(x = var, pattern = "\\n", replacement = "")
gsub(x = var.input, pattern = "\\.", replacement = "_")
```

1.2.3.1.1 Search If and Which String Contains

- r if string contains
- r if string contains either or grepl
- [Use grepl to search either of multiple substrings in a text](#)

Search for a single substring in a single string:

```
st_example_a <- 'C:/Users/fan/R4Econ/amto/tibble/fs_tib_basics.Rmd'
st_example_b <- 'C:/Users/fan/R4Econ/amto/tibble/_main.html'
grepl('_main', st_example_a)

## [1] FALSE

grepl('_main', st_example_b)

## [1] TRUE
```

Search for if one of a set of substring exists in a set of strings. In particular which one of the elements of *ls_spn* contains at least one of the elements of *ls_str_if_contains*. In the example below, only the first

path does not contain either the word *aggregate* or *index* in the path. This can be used after all paths have been found recursively in some folder to select only desired paths from the full set of possibilities:

```
ls_spn <- c("C:/Users/fan/R4Econ//panel/basic/fs_genpanel.Rmd",
           "C:/Users/fan/R4Econ//summarize/aggregate/_main.Rmd",
           "C:/Users/fan/R4Econ//summarize/index/fs_index_populate.Rmd")
ls_str_if_contains <- c("aggregate", "index")
str_if_contains <- paste(ls_str_if_contains, collapse = "|")
grepl(str_if_contains, ls_spn)
```

```
## [1] FALSE TRUE TRUE
```

1.2.3.2 String Concatenate

```
# Simple Collapse
vars.group.by <- c('abc', 'efg')
paste0(vars.group.by, collapse='|')
```

```
## [1] "abc|efg"
```

1.2.3.3 String Add Leading Zero

```
# Add Leading zero for integer values to allow for sorting when
# integers are combined into strings
it_z_n <- 1
it_a_n <- 192
print(sprintf("%02d", it_z_n))
```

```
## [1] "01"
```

```
print(sprintf("%04d", it_a_n))
```

```
## [1] "0192"
```

1.2.3.4 Substring and File Name

From path, get file name without suffix.

- `r` string split
- `r` list last element
- `r` get file name from path
- `r` get file path no name

```
st_example <- 'C:/Users/fan/R4Econ/amto/tibble/fs_tib_basics.Rmd'
st_file_wth_suffix <- tail(strsplit(st_example, "/")[[1]], n=1)
st_file_wno_suffix <- sub('\\.Rmd$', '', basename(st_example))
st_fullpath_nosufx <- sub('\\.Rmd$', '', st_example)
st_lastpath_noname <- (dirname(st_example))
st_fullpath_noname <- dirname(st_example)

print(strsplit(st_example, "/"))
```

```
## [[1]]
```

```
## [1] "C:" "Users" "fan" "R4Econ" "amto"
```

```
print(st_file_wth_suffix)
```

```
## [1] "fs_tib_basics.Rmd"
```

```
print(st_file_wno_suffix)
```

```
## [1] "fs_tib_basics"
```

```
print(st_fullpath_nosufx)

## [1] "C:/Users/fan/R4Econ/amto/tibble/fs_tib_basics"

print(st_lastpath_noname)

## [1] "C:/Users/fan/R4Econ/amto/tibble"

print(st_fullpath_noname)

## [1] "C:/Users/fan/R4Econ/amto/tibble"
```

1.2.4 Mesh Matrices, Arrays and Scalars

Go back to [fan's REconTools Package](#), [R4Econ Repository](#) ([bookdown site](#)), or [Intro Stats with R Repository](#).

- `r.expand.grid` meshed array to matrix
- `r.meshgrid`
- `r.array` to matrix
- `r.reshape` array to matrix
- `dplyr` permutations rows of matrix and element of array
- `tidyr.expand_grid` mesh matrix and vector

1.2.4.1 Mesh Two or More Vectors with `expand_grid`

In the example below, we have a matrix that is 2 by 2 (endogenous states), a vector that is 3 by 1 (choices), and another matrix that is 4 by 3 (exogenous states shocks).

We want to generate a tibble dataset that meshes the matrix and the vector, so that all combinations show up. Additionally, we want to add some additional values that are common across all rows to the meshed dataframe.

Note `expand_grid` is a from `tidyr` 1.0.0.

```
# A. Generate the 5 by 2 Matrix (ENDO STATES)
# it_child_count = N, the number of children
it_N_child_cnt = 2
# P fixed parameters, nN is N dimensional, nP is P dimensional
ar_nN_A = seq(-2, 2, length.out = it_N_child_cnt)
ar_nN_alpha = seq(0.1, 0.9, length.out = it_N_child_cnt)
fl_rho = 0.1
fl_lambda = 1.1
mt_nP_A_alpha = cbind(ar_nN_A, ar_nN_alpha, fl_rho, fl_lambda)
ar_st_varnames <- c('s_A', 's_alpha', 'p_rho', 'p_lambda')
tb_states_endo <- as_tibble(mt_nP_A_alpha) %>%
  rename_all(~c(ar_st_varnames)) %>%
  rowid_to_column(var = "state_id")

# B. Choice Grid
it_N_choice_cnt = 3
fl_max = 10
fl_min = 0
ar_nN_d = seq(fl_min, fl_max, length.out = it_N_choice_cnt)
ar_st_varnames <- c('c_food')
tb_choices <- as_tibble(ar_nN_d) %>%
  rename_all(~c(ar_st_varnames)) %>%
  rowid_to_column(var = "choice_id")

# C. Shock Grid
set.seed(123)
it_N_shock_cnt = 4
```



```

ar_nQ_shocks = exp(rnorm(it_N_shock_cnt, mean=0, sd=1))
ar_st_varnames <- c('s_eps')
tb_states_exo <- as_tibble(ar_nQ_shocks) %>%
  rename_all(~c(ar_st_varnames)) %>%
  rowid_to_column(var = "shock_id")

# dataframe expand with other non expanded variables
ar_st_varnames <-
tb_states_shk_choices <- tb_states_endo %>%
  expand_grid(tb_choices) %>%
  expand_grid(tb_states_exo) %>%
  select(state_id, choice_id, shock_id,
         s_A, s_alpha, s_eps, c_food,
         p_rho, p_lambda)

# display
kable(tb_states_shk_choices) %>% kable_styling_fc()

```

state_id	choice_id	shock_id	s_A	s_alpha	s_eps	c_food	p_rho	p_lambda
1	1	1	-2	0.1	0.5709374	0	0.1	1.1
1	1	2	-2	0.1	0.7943926	0	0.1	1.1
1	1	3	-2	0.1	4.7526783	0	0.1	1.1
1	1	4	-2	0.1	1.0730536	0	0.1	1.1
1	2	1	-2	0.1	0.5709374	5	0.1	1.1
1	2	2	-2	0.1	0.7943926	5	0.1	1.1
1	2	3	-2	0.1	4.7526783	5	0.1	1.1
1	2	4	-2	0.1	1.0730536	5	0.1	1.1
1	3	1	-2	0.1	0.5709374	10	0.1	1.1
1	3	2	-2	0.1	0.7943926	10	0.1	1.1
1	3	3	-2	0.1	4.7526783	10	0.1	1.1
1	3	4	-2	0.1	1.0730536	10	0.1	1.1
2	1	1	2	0.9	0.5709374	0	0.1	1.1
2	1	2	2	0.9	0.7943926	0	0.1	1.1
2	1	3	2	0.9	4.7526783	0	0.1	1.1
2	1	4	2	0.9	1.0730536	0	0.1	1.1
2	2	1	2	0.9	0.5709374	5	0.1	1.1
2	2	2	2	0.9	0.7943926	5	0.1	1.1
2	2	3	2	0.9	4.7526783	5	0.1	1.1
2	2	4	2	0.9	1.0730536	5	0.1	1.1
2	3	1	2	0.9	0.5709374	10	0.1	1.1
2	3	2	2	0.9	0.7943926	10	0.1	1.1
2	3	3	2	0.9	4.7526783	10	0.1	1.1
2	3	4	2	0.9	1.0730536	10	0.1	1.1

Using `expand_grid` directly over arrays

```

# expand grid with dplyr
expand_grid(x = 1:3, y = 1:2, z = -3:-1)

```

1.2.4.2 Mesh Arrays with `expand.grid`

Given two arrays, mesh the two arrays together.

```

# use expand.grid to generate all combinations of two arrays

```

```

it_ar_A = 5
it_ar_alpha = 10

```

```

ar_A = seq(-2, 2, length.out=it_ar_A)
ar_alpha = seq(0.1, 0.9, length.out=it_ar_alpha)

mt_A_alpha = expand.grid(A = ar_A, alpha = ar_alpha)

mt_A_meshed = mt_A_alpha[,1]
dim(mt_A_meshed) = c(it_ar_A, it_ar_alpha)

mt_alpha_meshed = mt_A_alpha[,2]
dim(mt_alpha_meshed) = c(it_ar_A, it_ar_alpha)

# display
kable(mt_A_meshed) %>%
  kable_styling_fc()

```

-2	-2	-2	-2	-2	-2	-2	-2	-2	-2
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2

```

kable(mt_alpha_meshed) %>%
  kable_styling_fc_wide()

```

0.1	0.1888889	0.2777778	0.3666667	0.4555556	0.5444444	0.6333333	0.7222222	0.8111111	0.9
0.1	0.1888889	0.2777778	0.3666667	0.4555556	0.5444444	0.6333333	0.7222222	0.8111111	0.9
0.1	0.1888889	0.2777778	0.3666667	0.4555556	0.5444444	0.6333333	0.7222222	0.8111111	0.9
0.1	0.1888889	0.2777778	0.3666667	0.4555556	0.5444444	0.6333333	0.7222222	0.8111111	0.9
0.1	0.1888889	0.2777778	0.3666667	0.4555556	0.5444444	0.6333333	0.7222222	0.8111111	0.9

Two Identical Arrays, individual attributes, each column is an individual for a matrix, and each row is also an individual.

```

# use expand.grid to generate all combinations of two arrays

it_ar_A = 5

ar_A = seq(-2, 2, length.out=it_ar_A)
mt_A_A = expand.grid(Arow = ar_A, Arow = ar_A)
mt_Arow = mt_A_A[,1]
dim(mt_Arow) = c(it_ar_A, it_ar_A)
mt_Acol = mt_A_A[,2]
dim(mt_Acol) = c(it_ar_A, it_ar_A)

# display
kable(mt_Arow) %>%
  kable_styling_fc()

```

-2	-2	-2	-2	-2
-1	-1	-1	-1	-1
0	0	0	0	0
1	1	1	1	1
2	2	2	2	2

```

kable(mt_Acol) %>%
  kable_styling_fc()

```

-2	-1	0	1	2
-2	-1	0	1	2
-2	-1	0	1	2
-2	-1	0	1	2
-2	-1	0	1	2

1.3 Matrix

1.3.1 Generate Matrixes

Go back to [fan's REconTools Package](#), [R4Econ Repository \(bookdown site\)](#), or [Intro Stats with R Repository](#).

1.3.1.1 Create a N by 2 Matrix from 3 arrays

Names of each array become row names automatically.

```
ar_row_one <- c(-1,+1)
ar_row_two <- c(-3,-2)
ar_row_three <- c(0.35,0.75)

mt_n_by_2 <- rbind(ar_row_one, ar_row_two, ar_row_three)
kable(mt_n_by_2) %>%
  kable_styling_fc()
```

ar_row_one	-1.00	1.00
ar_row_two	-3.00	-2.00
ar_row_three	0.35	0.75

1.3.1.2 Generate Random Matrixes

Random draw from the normal distribution, random draw from the uniform distribution, and combine resulting matrixes.

```
# Generate 15 random normal, put in 5 rows, and 3 columns
mt_rnorm <- matrix(rnorm(15,mean=0,sd=1), nrow=5, ncol=3)

# Generate 15 random normal, put in 5 rows, and 3 columns
mt_runif <- matrix(runif(15,min=0,max=1), nrow=5, ncol=5)

# Combine
mt_rnorm_runif <- cbind(mt_rnorm, mt_runif)

# Display
kable(mt_rnorm_runif) %>%
  kable_styling_fc_wide()
```

0.1292877	-0.4456620	-0.5558411	0.3181810	0.3688455	0.2659726	0.3181810	0.3688455
1.7150650	1.2240818	1.7869131	0.2316258	0.1524447	0.8578277	0.2316258	0.1524447
0.4609162	0.3598138	0.4978505	0.1428000	0.1388061	0.0458312	0.1428000	0.1388061
-1.2650612	0.4007715	-1.9666172	0.4145463	0.2330341	0.4422001	0.4145463	0.2330341
-0.6868529	0.1106827	0.7013559	0.4137243	0.4659625	0.7989248	0.4137243	0.4659625

1.3.2 Linear Algebra

Go back to [fan's REconTools Package](#), [R4Econ Repository \(bookdown site\)](#), or [Intro Stats with R Repository](#).

1.3.2.1 Matrix Multiplication

Multiply Together a 3 by 2 matrix and a 2 by 1 vector

```
ar_row_one <- c(-1,+1)
ar_row_two <- c(-3,-2)
ar_row_three <- c(0.35,0.75)
mt_n_by_2 <- rbind(ar_row_one, ar_row_two, ar_row_three)
```

```
ar_row_four <- c(3,4)
```

```
# Matrix Multiplication
```

```
mt_out <- mt_n_by_2 %*% ar_row_four
print(mt_n_by_2)
```

```
##           [,1] [,2]
## ar_row_one -1.00  1.00
## ar_row_two -3.00 -2.00
## ar_row_three  0.35  0.75
```

```
print(ar_row_four)
```

```
## [1]  3  4
```

```
print(mt_out)
```

```
##           [,1]
## ar_row_one  1.00
## ar_row_two -17.00
## ar_row_three  4.05
```

1.4 Variables in Dataframes

1.4.1 Generate Dataframe

Go back to [fan's REconTools](#) Package, [R4Econ](#) Repository ([bookdown site](#)), or [Intro Stats with R](#) Repository.

1.4.1.1 Generate Tibble given Matrixes and Arrays

Given Arrays and Matrixes, Generate Tibble and Name Variables/Columns

- naming tibble columns
- tibble variable names
- dplyr rename tibble
- dplyr rename tibble all variables
- dplyr rename all columns by index
- dplyr tibble add index column
- see also: [SO-51205520](#)

```
# Base Inputs
```

```
ar_col <- c(-1,+1)
mt_rnorm_a <- matrix(rnorm(4,mean=0,sd=1), nrow=2, ncol=2)
mt_rnorm_b <- matrix(rnorm(4,mean=0,sd=1), nrow=2, ncol=4)
```

```
# Combine Matrix
```

```
mt_combine <- cbind(ar_col, mt_rnorm_a, mt_rnorm_b)
colnames(mt_combine) <- c('ar_col',
                          paste0('matcolvar_grpa_', seq(1,dim(mt_rnorm_a)[2])),
                          paste0('matcolvar_grpb_', seq(1,dim(mt_rnorm_b)[2])))
```

```
# Variable Names
```

```

ar_st_varnames <- c('var_one',
                    paste0('tibcolvar_ga_', c(1,2)),
                    paste0('tibcolvar_gb_', c(1,2,3,4)))

# Combine to tibble, add name col1, col2, etc.
tb_combine <- as_tibble(mt_combine) %>% rename_all(~c(ar_st_varnames))

# Add an index column to the dataframe, ID column
tb_combine <- tb_combine %>% rowid_to_column(var = "ID")

# Change all gb variable names
tb_combine <- tb_combine %>%
  rename_at(vars(starts_with("tibcolvar_gb_")),
            funs(str_replace(., "_gb_", "gbrenamed_")))

# Tibble back to matrix
mt_tb_combine_back <- data.matrix(tb_combine)

# Display
kable(mt_combine) %>% kable_styling_fc_wide()

```

ar_col	matcolvar_grpa_1	matcolvar_grpa_2	matcolvar_grpb_1	matcolvar_grpb_2	matcolvar_grpb_3	matcolvar_grpb_4
-1	-1.1655448	0.6849361	-1.3115224	-0.1294107	-1.3115224	-0.1294107
1	-0.8185157	-0.3200564	-0.5996083	0.8867361	-0.5996083	0.8867361

```
kable(tb_combine) %>% kable_styling_fc_wide()
```

ID	var_one	tibcolvar_ga_1	tibcolvar_ga_2	tibcolvar_gbrenamed_1	tibcolvar_gbrenamed_2	tibcolvar_gbrenamed_3	tibcolvar_gbrenamed_4
1	-1	-1.1655448	0.6849361	-1.3115224	-0.1294107	-1.3115224	-0.1294107
2	1	-0.8185157	-0.3200564	-0.5996083	0.8867361	-0.5996083	0.8867361

```
kable(mt_tb_combine_back) %>% kable_styling_fc_wide()
```

ID	var_one	tibcolvar_ga_1	tibcolvar_ga_2	tibcolvar_gbrenamed_1	tibcolvar_gbrenamed_2	tibcolvar_gbrenamed_3	tibcolvar_gbrenamed_4
1	-1	-1.1655448	0.6849361	-1.3115224	-0.1294107	-1.3115224	-0.1294107
2	1	-0.8185157	-0.3200564	-0.5996083	0.8867361	-0.5996083	0.8867361

1.4.1.2 Rename Tibble with Numeric Column Names

After reshaping, often could end up with variable names that are all numeric, integers for example, how to rename these variables to add a common prefix for example.

```

# Base Inputs
ar_col <- c(-1,+1)
mt_rnorm_c <- matrix(rnorm(4,mean=0,sd=1), nrow=5, ncol=10)
mt_combine <- cbind(ar_col, mt_rnorm_c)

# Variable Names
ar_it_cols_ctr <- seq(1, dim(mt_rnorm_c)[2])
ar_st_varnames <- c('var_one', ar_it_cols_ctr)

# Combine to tibble, add name col1, col2, etc.
tb_combine <- as_tibble(mt_combine) %>% rename_all(~c(ar_st_varnames))

# Add an index column to the dataframe, ID column
tb_combine_ori <- tb_combine %>% rowid_to_column(var = "ID")

# Change all gb variable names
tb_combine <- tb_combine_ori %>%
  rename_at(

```

```
vars(num_range(' ', ar_it_cols_ctr)),
funs(paste0("rho", . , 'var'))
)
```

Display

```
kable(tb_combine_ori) %>% kable_styling_fc_wide()
```

ID	var_one	1	2	3	4	5	6	7	8	9	10
1	-1	-0.1513960	0.3297912	-3.2273228	-0.7717918	-0.1513960	0.3297912	-3.2273228	-0.7717918	-0.1513960	0.3297912
2	1	0.3297912	-3.2273228	-0.7717918	-0.1513960	0.3297912	-3.2273228	-0.7717918	-0.1513960	0.3297912	-3.2273228
3	-1	-3.2273228	-0.7717918	-0.1513960	0.3297912	-3.2273228	-0.7717918	-0.1513960	0.3297912	-3.2273228	-0.7717918
4	1	-0.7717918	-0.1513960	0.3297912	-3.2273228	-0.7717918	-0.1513960	0.3297912	-3.2273228	-0.7717918	-0.1513960
5	-1	-0.1513960	0.3297912	-3.2273228	-0.7717918	-0.1513960	0.3297912	-3.2273228	-0.7717918	-0.1513960	0.3297912

```
kable(tb_combine) %>% kable_styling_fc_wide()
```

ID	var_one	rho1var	rho2var	rho3var	rho4var	rho5var	rho6var	rho7var	rho8var	rho9var	rho10var
1	-1	-0.1513960	0.3297912	-3.2273228	-0.7717918	-0.1513960	0.3297912	-3.2273228	-0.7717918	-0.1513960	0.3297912
2	1	0.3297912	-3.2273228	-0.7717918	-0.1513960	0.3297912	-3.2273228	-0.7717918	-0.1513960	0.3297912	-3.2273228
3	-1	-3.2273228	-0.7717918	-0.1513960	0.3297912	-3.2273228	-0.7717918	-0.1513960	0.3297912	-3.2273228	-0.7717918
4	1	-0.7717918	-0.1513960	0.3297912	-3.2273228	-0.7717918	-0.1513960	0.3297912	-3.2273228	-0.7717918	-0.1513960
5	-1	-0.1513960	0.3297912	-3.2273228	-0.7717918	-0.1513960	0.3297912	-3.2273228	-0.7717918	-0.1513960	0.3297912

1.4.1.3 Tibble Row and Column and Summarize

Show what is in the table: 1, column and row names; 2, contents inside table.

```
tb_iris <- as_tibble(iris)
print(rownames(tb_iris))
```

```
## [1] "1" "2" "3" "4" "5" "6" "7" "8" "9" "10" "11" "12" "13" "14" "15" "
## [28] "28" "29" "30" "31" "32" "33" "34" "35" "36" "37" "38" "39" "40" "41" "42" "
## [55] "55" "56" "57" "58" "59" "60" "61" "62" "63" "64" "65" "66" "67" "68" "69" "
## [82] "82" "83" "84" "85" "86" "87" "88" "89" "90" "91" "92" "93" "94" "95" "96" "
## [109] "109" "110" "111" "112" "113" "114" "115" "116" "117" "118" "119" "120" "121" "122" "123" "
## [136] "136" "137" "138" "139" "140" "141" "142" "143" "144" "145" "146" "147" "148" "149" "150"
```

```
colnames(tb_iris)
```

```
## [1] "Sepal.Length" "Sepal.Width" "Petal.Length" "Petal.Width" "Species"
```

```
colnames(tb_iris)
```

```
## [1] "Sepal.Length" "Sepal.Width" "Petal.Length" "Petal.Width" "Species"
```

```
summary(tb_iris)
```

```
## Sepal.Length Sepal.Width Petal.Length Petal.Width Species
## Min. :4.300 Min. :2.000 Min. :1.000 Min. :0.100 setosa :50
## 1st Qu.:5.100 1st Qu.:2.800 1st Qu.:1.600 1st Qu.:0.300 versicolor:50
## Median :5.800 Median :3.000 Median :4.350 Median :1.300 virginica :50
## Mean :5.843 Mean :3.057 Mean :3.758 Mean :1.199
## 3rd Qu.:6.400 3rd Qu.:3.300 3rd Qu.:5.100 3rd Qu.:1.800
## Max. :7.900 Max. :4.400 Max. :6.900 Max. :2.500
```

1.4.1.4 Tibble Sorting

- dplyr arrange desc reverse
- dplyr sort

Sort in Ascending Order

```
tb_iris %>% select(Species, Sepal.Length, everything()) %>%
  arrange(Species, Sepal.Length) %>% head(10) %>%
  kable() %>% kable_styling_fc()
```

Species	Sepal.Length	Sepal.Width	Petal.Length	Petal.Width
setosa	4.3	3.0	1.1	0.1
setosa	4.4	2.9	1.4	0.2
setosa	4.4	3.0	1.3	0.2
setosa	4.4	3.2	1.3	0.2
setosa	4.5	2.3	1.3	0.3
setosa	4.6	3.1	1.5	0.2
setosa	4.6	3.4	1.4	0.3
setosa	4.6	3.6	1.0	0.2
setosa	4.6	3.2	1.4	0.2
setosa	4.7	3.2	1.3	0.2

Sort in Descending Order

```
tb_iris %>% select(Species, Sepal.Length, everything()) %>%
  arrange(desc(Species), desc(Sepal.Length)) %>% head(10) %>%
  kable() %>% kable_styling_fc()
```

Species	Sepal.Length	Sepal.Width	Petal.Length	Petal.Width
virginica	7.9	3.8	6.4	2.0
virginica	7.7	3.8	6.7	2.2
virginica	7.7	2.6	6.9	2.3
virginica	7.7	2.8	6.7	2.0
virginica	7.7	3.0	6.1	2.3
virginica	7.6	3.0	6.6	2.1
virginica	7.4	2.8	6.1	1.9
virginica	7.3	2.9	6.3	1.8
virginica	7.2	3.6	6.1	2.5
virginica	7.2	3.2	6.0	1.8

1.4.1.5 REconTools Summarize over Tible

Use R4Econ's summary tool.

```
df_summ_stats <- ff_summ_percentiles(tb_iris)
kable(t(df_summ_stats)) %>% kable_styling_fc_wide()
```

stats	n	NAobs	ZEROobs	mean	sd	cv	min	p01	p05	p10	p25	p50	p75	p90	p95	p99	max
Petal.Length	150	0	0	3.758000	1.7652982	0.4697441	1.0	1.149	1.300	1.4	1.6	4.35	5.1	5.80	6.100	6.700	6.9
Petal.Width	150	0	0	1.199333	0.7622377	0.6355511	0.1	0.100	0.200	0.2	0.3	1.30	1.8	2.20	2.300	2.500	2.5
Sepal.Length	150	0	0	5.843333	0.8280661	0.1417113	4.3	4.400	4.600	4.8	5.1	5.80	6.4	6.90	7.255	7.700	7.9
Sepal.Width	150	0	0	3.057333	0.4358663	0.1425642	2.0	2.200	2.345	2.5	2.8	3.00	3.3	3.61	3.800	4.151	4.4

1.4.2 Factor Label and Combine

Go back to [fan's REconTools Package](#), [R4Econ Repository \(bookdown site\)](#), or [Intro Stats with R Repository](#).

1.4.2.1 Factor, Label, Cross and Graph

Generate a Scatter plot with different colors representing different categories. There are multiple underlying factor/categorical variables, for example two binary variables. Generate scatter plot with colors for the combinations of these two binary variables.

We combine here the *vs* and *am* variables from the *mtcars* dataset. *vs* is engine shape, *am* is auto or manual shift. We will generate a scatter plot of *mpg* and *qsec* over four categories with different colors.

- *am*: Transmission (0 = automatic, 1 = manual)
- *vs*: Engine (0 = V-shaped, 1 = straight)
- *mpg*: miles per gallon

- *qsec*: 1/4 mile time

```
# First make sure these are factors
tb_mtcars <- as_tibble(mtcars) %>%
  mutate(vs = as_factor(vs), am = as_factor(am))

# Second Label the Factors
am_levels <- c(auto_shift = "0", manual_shift = "1")
vs_levels <- c(vshaped_engine = "0", straight_engine = "1")
tb_mtcars <- tb_mtcars %>%
  mutate(vs = fct_recode(vs, !!!vs_levels),
         am = fct_recode(am, !!!am_levels))

# Third Combine Factors
tb_mtcars_selected <- tb_mtcars %>%
  mutate(vs_am = fct_cross(vs, am, sep='_', keep_empty = FALSE)) %>%
  select(mpg, qsec, vs_am)
print(tb_mtcars_selected)
```

Now we generate scatter plot based on the combined factors

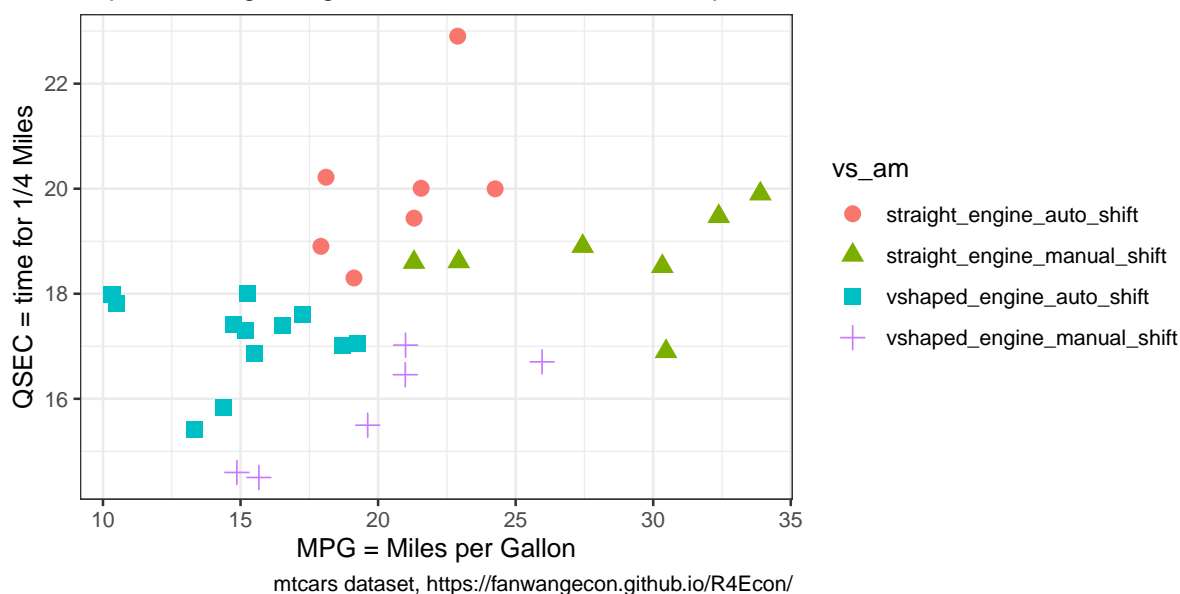
```
# Labeling
st_title <- paste0('Distribution of MPG and QSEC from mtcars')
st_subtitle <- paste0('https://fanwangecon.github.io/',
                     'R4Econ/amto/tibble/htmlpdf/fr/fs_tib_factors.html')
st_caption <- paste0('mtcars dataset, ',
                    'https://fanwangecon.github.io/R4Econ/')
st_x_label <- 'MPG = Miles per Gallon'
st_y_label <- 'QSEC = time for 1/4 Miles'

# Graphing
plt_mtcars_scatter <-
  ggplot(tb_mtcars_selected,
        aes(x=mpg, y=qsec, colour=vs_am, shape=vs_am)) +
  geom_jitter(size=3, width = 0.15) +
  labs(title = st_title, subtitle = st_subtitle,
       x = st_x_label, y = st_y_label, caption = st_caption) +
  theme_bw()

# show
print(plt_mtcars_scatter)
```


Distribution of MPG and QSEC from mtcars

https://fanwangecon.github.io/R4Econ/amto/tibble/htmlpdf/fs_tib_factors.html



1.4.3 Drawly Random Rows

Go back to [fan's REconTools Package](#), [R4Econ Repository](#) ([bookdown site](#)), or [Intro Stats with R Repository](#).

1.4.3.1 Draw Random Subset of Sample

- `r` random discrete

We have a sample of N individuals in some dataframe. Draw without replacement a subset $M < N$ of rows.

```
# parameters, it_M < it_N
it_N <- 10
it_M <- 5

# Draw it_m from indexed list of it_N
set.seed(123)
ar_it_rand_idx <- sample(it_N, it_M, replace=FALSE)

# dataframe
df_full <- as_tibble(matrix(rnorm(4,mean=0,sd=1), nrow=it_N, ncol=4)) %>% rowid_to_column(var = "ID")

# random Subset
df_rand_sub_a <- df_full[ar_it_rand_idx,]

# Random subset also
df_rand_sub_b <- df_full[sample(dim(df_full)[1], it_M, replace=FALSE),]

# Print
# Display
kable(df_full) %>% kable_styling_fc()

kable(df_rand_sub_a) %>% kable_styling_fc()

kable(df_rand_sub_b) %>% kable_styling_fc()
```

ID	V1	V2	V3	V4
1	0.1292877	0.4609162	0.1292877	0.4609162
2	1.7150650	-1.2650612	1.7150650	-1.2650612
3	0.4609162	0.1292877	0.4609162	0.1292877
4	-1.2650612	1.7150650	-1.2650612	1.7150650
5	0.1292877	0.4609162	0.1292877	0.4609162
6	1.7150650	-1.2650612	1.7150650	-1.2650612
7	0.4609162	0.1292877	0.4609162	0.1292877
8	-1.2650612	1.7150650	-1.2650612	1.7150650
9	0.1292877	0.4609162	0.1292877	0.4609162
10	1.7150650	-1.2650612	1.7150650	-1.2650612

ID	V1	V2	V3	V4
3	0.4609162	0.1292877	0.4609162	0.1292877
10	1.7150650	-1.2650612	1.7150650	-1.2650612
2	1.7150650	-1.2650612	1.7150650	-1.2650612
8	-1.2650612	1.7150650	-1.2650612	1.7150650
6	1.7150650	-1.2650612	1.7150650	-1.2650612

ID	V1	V2	V3	V4
5	0.1292877	0.4609162	0.1292877	0.4609162
3	0.4609162	0.1292877	0.4609162	0.1292877
9	0.1292877	0.4609162	0.1292877	0.4609162
1	0.1292877	0.4609162	0.1292877	0.4609162
4	-1.2650612	1.7150650	-1.2650612	1.7150650

1.4.3.2 Random Subset of Panel

There are N individuals, each could be observed M times, but then select a subset of rows only, so each person is randomly observed only a subset of times. Specifically, there are 3 unique students with student ids, and the second variable shows the random dates in which the student showed up in class, out of the 10 classes available.

```
# Define
it_N <- 3
it_M <- 10
svr_id <- 'student_id'

# dataframe
set.seed(123)
df_panel_rand <- as_tibble(matrix(it_M, nrow=it_N, ncol=1)) %>%
  rowid_to_column(var = svr_id) %>%
  uncount(V1) %>%
  group_by(!!sym(svr_id)) %>% mutate(date = row_number()) %>%
  ungroup() %>% mutate(in_class = case_when(rnorm(n(), mean=0, sd=1) < 0 ~ 1, TRUE ~ 0)) %>%
  filter(in_class == 1) %>% select(!!sym(svr_id), date) %>%
  rename(date_in_class = date)

# Print
kable(df_panel_rand) %>% kable_styling_fc()
```

1.4.4 Variable NA Values

Go back to [fan's REconTools Package](#), [R4Econ Repository \(bookdown site\)](#), or [Intro Stats with R Repository](#).

student_id	date_in_class
1	1
1	2
1	8
1	9
1	10
2	5
2	8
2	10
3	1
3	2
3	3
3	4
3	5
3	6
3	9

1.4.4.1 Find and Replace

Find and Replace in Dataframe.

```
# For dataframe
df.reg <- df.reg %>% na_if(-Inf) %>% na_if(Inf)
# For a specific variable in dataframe
df.reg.use %>% mutate(!!(var.input) := na_if(!sym(var.input), 0))

# Setting to NA
df.reg.use <- df.reg.guat %>% filter(!sym(var.mth) != 0)
df.reg.use.log <- df.reg.use
df.reg.use.log[which(is.nan(df.reg.use$prot.imputed.log)),] = NA
df.reg.use.log[which(df.reg.use$prot.imputed.log==Inf),] = NA
df.reg.use.log[which(df.reg.use$prot.imputed.log==--Inf),] = NA
df.reg.use.log <- df.reg.use.log %>% drop_na(prot.imputed.log)
# df.reg.use.log$prot.imputed.log
```

1.4.5 String Values

Go back to [fan's REconTools Package](#), [R4Econ Repository \(bookdown site\)](#), or [Intro Stats with R Repository](#).

1.4.5.1 Find and Replace

Find and Replace in Dataframe.

```
# if string value is contained in variable
("bridex.B" %in% (df.reg.out.all$vars_var.y))
# if string value is not contained in variable:
# 1. type is variable name
# 2. Toyota/Mazda are strings to be excluded
filter(mtcars, !grepl('Toyota|Mazda', type))

# filter does not contain string
rs_hgt_prot_log_tidy %>% filter(!str_detect(term, 'prot'))
```


Chapter 2

Summarize Data

2.1 Counting Observation

2.1.1 Uncount

Go back to [fan's REconTools Package](#), [R4Econ Repository \(bookdown site\)](#), or [Intro Stats with R Repository](#).

In some panel, there are N individuals, each observed for Y_i years. Given a dataset with two variables, the individual index, and the Y_i variable, expand the dataframe so that there is a row for each individual index's each unique year in the survey.

Search:

- [r duplicate row by variable](#)

Links:

- see: [Create duplicate rows based on a variable](#)

Algorithm:

1. generate testing frame, the individual attribute dataset with invariant information over panel
2. uncount, duplicate rows by years in survey
3. group and generate sorted index
4. add individual specific stat year to index

```
# 1. Array of Years in the Survey
ar_years_in_survey <- c(2,3,1,10,2,5)
ar_start_yaer <- c(1,2,3,1,1,1)
ar_end_year <- c(2,4,3,10,2,5)
mt_combine <- cbind(ar_years_in_survey, ar_start_yaer, ar_end_year)

# This is the individual attribute dataset, attributes that are invariant acrosss years
tb_indi_attributes <- as_tibble(mt_combine) %>% rowid_to_column(var = "ID")

# 2. Sort and generate variable equal to sorted index
tb_indi_panel <- tb_indi_attributes %>% uncount(ar_years_in_survey)

# 3. Panel now construct exactly which year in survey, note that all needed is sort index
# Note sorting not needed, all rows identical now
tb_indi_panel <- tb_indi_panel %>%
  group_by(ID) %>%
  mutate(yr_in_survey = row_number())

tb_indi_panel <- tb_indi_panel %>%
  mutate(calendar_year = yr_in_survey + ar_start_yaer - 1)
```

```
# Show results Head 10
tb_indi_panel %>% head(10) %>%
  kable() %>%
  kable_styling_fc()
```

ID	ar_start_yaer	ar_end_year	yr_in_survey	calendar_year
1	1	2	1	1
1	1	2	2	2
2	2	4	1	2
2	2	4	2	3
2	2	4	3	4
3	3	3	1	3
4	1	10	1	1
4	1	10	2	2
4	1	10	3	3
4	1	10	4	4

2.2 Sorting, Indexing, Slicing

2.2.1 Sorting

Go back to [fan's REconTools Package](#), [R4Econ Repository \(bookdown site\)](#), or [Intro Stats with R Repository](#).

2.2.1.1 Generate Sorted Index within Group with Repeating Values

There is a variable, sort by this variable, then generate index from 1 to N representing sorted values of this index. If there are repeating values, still assign index, different index each value.

- r generate index sort
- dplyr mutate equals index

```
# Sort and generate variable equal to sorted index
df_iris <- iris %>% arrange(Sepal.Length) %>%
  mutate(Sepal.Len.Index = row_number()) %>%
  select(Sepal.Length, Sepal.Len.Index, everything())

# Show results Head 10
df_iris %>% head(10) %>%
  kable() %>%
  kable_styling_fc_wide()
```

Sepal.Length	Sepal.Len.Index	Sepal.Width	Petal.Length	Petal.Width	Species
4.3	1	3.0	1.1	0.1	setosa
4.4	2	2.9	1.4	0.2	setosa
4.4	3	3.0	1.3	0.2	setosa
4.4	4	3.2	1.3	0.2	setosa
4.5	5	2.3	1.3	0.3	setosa
4.6	6	3.1	1.5	0.2	setosa
4.6	7	3.4	1.4	0.3	setosa
4.6	8	3.6	1.0	0.2	setosa
4.6	9	3.2	1.4	0.2	setosa
4.7	10	3.2	1.3	0.2	setosa

2.2.1.2 Populate Value from Lowest Index to All other Rows

We would like to calculate for example the ratio of each individual's highest to the the person with the lowest height in a dataset. We first need to generated sorted index from lowest to highest, and then populate the lowest height to all rows, and then divide.

Search Terms:

- r spread value to all rows from one row
- r other rows equal to the value of one row
- Conditional assignment of one variable to the value of one of two other variables
- dplyr mutate conditional
- dplyr value from one row to all rows
- dplyr mutate equal to value in another cell

Links:

- see: dplyr [rank](#)
- see: dplyr [case_when](#)

2.2.1.2.1 Short Method: mutate and min We just want the lowest value to be in its own column, so that we can compute various statistics using the lowest value variable and the original variable.

```
# 1. Sort
df_iris_m1 <- iris %>% mutate(Sepal.Len.Lowest.all = min(Sepal.Length)) %>%
  select(Sepal.Length, Sepal.Len.Lowest.all, everything())

# Show results Head 10
df_iris_m1 %>% head(10) %>%
  kable() %>%
  kable_styling_fc_wide()
```

Sepal.Length	Sepal.Len.Lowest.all	Sepal.Width	Petal.Length	Petal.Width	Species
5.1	4.3	3.5	1.4	0.2	setosa
4.9	4.3	3.0	1.4	0.2	setosa
4.7	4.3	3.2	1.3	0.2	setosa
4.6	4.3	3.1	1.5	0.2	setosa
5.0	4.3	3.6	1.4	0.2	setosa
5.4	4.3	3.9	1.7	0.4	setosa
4.6	4.3	3.4	1.4	0.3	setosa
5.0	4.3	3.4	1.5	0.2	setosa
4.4	4.3	2.9	1.4	0.2	setosa
4.9	4.3	3.1	1.5	0.1	setosa

2.2.1.2.2 Long Method: row_number and case_when This is the long method, using row_number, and case_when. The benefit of this method is that it generates several intermediate variables that might be useful. And the key final step is to set a new variable (A=Sepal.Len.Lowest.all) equal to another variable's (B=Sepal.Length's) value at the index that satisfies condition based a third variable (C=Sepal.Len.Index).

```
# 1. Sort
# 2. generate index
# 3. value at lowest index (case_when)
# 4. spread value from lowest index to other rows
# Note step 4 does not require step 3
df_iris_m2 <- iris %>% arrange(Sepal.Length) %>%
  mutate(Sepal.Len.Index = row_number()) %>%
  mutate(Sepal.Len.Lowest.one =
```

```

      case_when(row_number()==1 ~ Sepal.Length)) %>%
mutate(Sepal.Len.Lowest.all =
      Sepal.Length[Sepal.Len.Index==1]) %>%
select(Sepal.Length, Sepal.Len.Index,
      Sepal.Len.Lowest.one, Sepal.Len.Lowest.all)

# Show results Head 10
df_iris_m2 %>% head(10) %>%
  kable() %>%
  kable_styling_fc_wide()

```

Sepal.Length	Sepal.Len.Index	Sepal.Len.Lowest.one	Sepal.Len.Lowest.all
4.3	1	4.3	4.3
4.4	2	NA	4.3
4.4	3	NA	4.3
4.4	4	NA	4.3
4.5	5	NA	4.3
4.6	6	NA	4.3
4.6	7	NA	4.3
4.6	8	NA	4.3
4.6	9	NA	4.3
4.7	10	NA	4.3

2.2.1.3 Generate Sorted Index based on Deviations

Generate Positive and Negative Index based on Ordered Deviation from some Number.

There is a variable that is continuous, subtract a number from this variable, and generate index based on deviations. Think of the index as generating intervals indicating where the value lies. 0th index indicates the largest value in sequence that is smaller than or equal to number x , 1st index indicates the smallest value in sequence that is larger than number x .

The solution below is a little bit convoluted and long, there is likely a much quicker way. The process below shows various intermediary outputs that help arrive at deviation index *Sepal.Len.Devi.Index* from initial sorted index *Sepal.Len.Index*.

search:

- dplyr arrange ignore na
- dplyr index deviation from order number sequence
- dplyr index below above
- dplyr index order below above value

```

# 1. Sort and generate variable equal to sorted index
# 2. Plus or minus deviations from some value
# 3. Find the zero, which means, the number closests to zero including zero from the negative side
# 4. Find the index at the highest zero and below deviation point
# 5. Difference of zero index and original sorted index
sc_val_x <- 4.65
df_iris_deviate <- iris %>% arrange(Sepal.Length) %>%
  mutate(Sepal.Len.Index = row_number()) %>%
  mutate(Sepal.Len.Devi = (Sepal.Length - sc_val_x)) %>%
  mutate(Sepal.Len.Devi.Neg =
    case_when(Sepal.Len.Devi <= 0 ~ (-1)*(Sepal.Len.Devi))) %>%
  arrange((Sepal.Len.Devi.Neg), desc(Sepal.Len.Index)) %>%

```



```

mutate(Sepal.Len.Index.Zero =
  case_when(row_number() == 1 ~ Sepal.Len.Index)) %>%
mutate(Sepal.Len.Devi.Index =
  Sepal.Len.Index - Sepal.Len.Index.Zero[row_number() == 1]) %>%
arrange(Sepal.Len.Index) %>%
select(Sepal.Length, Sepal.Len.Index, Sepal.Len.Devi,
  Sepal.Len.Devi.Neg, Sepal.Len.Index.Zero, Sepal.Len.Devi.Index)

# Show results Head 10
df_iris_deviate %>% head(20) %>%
  kable() %>%
  kable_styling_fc_wide()

```

Sepal.Length	Sepal.Len.Index	Sepal.Len.Devi	Sepal.Len.Devi.Neg	Sepal.Len.Index.Zero	Sepal.Len.Devi.Index
4.3	1	-0.35	0.35	NA	-8
4.4	2	-0.25	0.25	NA	-7
4.4	3	-0.25	0.25	NA	-6
4.4	4	-0.25	0.25	NA	-5
4.5	5	-0.15	0.15	NA	-4
4.6	6	-0.05	0.05	NA	-3
4.6	7	-0.05	0.05	NA	-2
4.6	8	-0.05	0.05	NA	-1
4.6	9	-0.05	0.05	9	0
4.7	10	0.05	NA	NA	1
4.7	11	0.05	NA	NA	2
4.8	12	0.15	NA	NA	3
4.8	13	0.15	NA	NA	4
4.8	14	0.15	NA	NA	5
4.8	15	0.15	NA	NA	6
4.8	16	0.15	NA	NA	7
4.9	17	0.25	NA	NA	8
4.9	18	0.25	NA	NA	9
4.9	19	0.25	NA	NA	10
4.9	20	0.25	NA	NA	11

2.3 Group Statistics

2.3.1 Groups Statistics

Go back to [fan's REconTools Package](#), [R4Econ Repository \(bookdown site\)](#), or [Intro Stats with R Repository](#).

2.3.1.1 Aggrgate Groups only Unique Group and Count

There are two variables that are numeric, we want to find all the unique groups of these two variables in a dataset and count how many times each unique group occurs

- r unique occurrence of numeric groups
- How to add count of unique values by group to R data.frame

```

# Numeric value combinations unique Groups
vars.group <- c('hgt0', 'wgt0')

# dataset subsetting
df_use <- df_hgt_wgt %>% select(!!!syms(c(vars.group))) %>%
  mutate(hgt0 = round(hgt0/5)*5, wgt0 = round(wgt0/2000)*2000) %>%
  drop_na()

# Group, count and generate means for each numeric variables
# mutate_at(vars.group, funs(as.factor(.))) %>%

```

```
df.group.count <- df_use %>% group_by(!!!syms(vars.group)) %>%
  arrange(!!!syms(vars.group)) %>%
  summarise(n_obs_group=n())

# Show results Head 10
df.group.count %>% kable() %>% kable_styling_fc()
```

hgt0	wgt0	n_obs_group
40	2000	122
45	2000	4586
45	4000	470
50	2000	9691
50	4000	13106
55	2000	126
55	4000	1900
60	6000	18

2.3.1.2 Aggrgate Groups only Unique Group Show up With Means

Several variables that are grouping identifiers. Several variables that are values which mean be unique for each group members. For example, a Panel of income for N households over T years with also household education information that is invariant over time. Want to generate a dataset where the unit of observation are households, rather than household years. Take average of all numeric variables that are household and year specific.

A complicating factor potentially is that the number of observations differ within group, for example, income might be observed for all years for some households but not for other households.

- r dplyr aggregate group average
- Aggregating and analyzing data with dplyr
- column can't be modified because it is a grouping variable
- see also: [Aggregating and analyzing data with dplyr](#)

```
# In the df_hgt_wgt from R4Econ, there is a country id, village id,
# and individual id, and various other statistics
vars.group <- c('S.country', 'vil.id', 'indi.id')
vars.values <- c('hgt', 'momEdu')
```

```
# dataset subsetting
df_use <- df_hgt_wgt %>% select(!!!syms(c(vars.group, vars.values)))
```

```
# Group, count and generate means for each numeric variables
df.group <- df_use %>% group_by(!!!syms(vars.group)) %>%
  arrange(!!!syms(vars.group)) %>%
  summarise_if(is.numeric,
    funs(mean = mean(., na.rm = TRUE),
          sd = sd(., na.rm = TRUE),
          n = sum(is.na(.)==0)))
```

```
# Show results Head 10
df.group %>% head(10) %>%
  kable() %>%
  kable_styling_fc_wide()
```

```
# Show results Head 10
df.group %>% tail(10) %>%
  kable() %>%
  kable_styling_fc_wide()
```

S.country	vil.id	indi.id	hgt_mean	momEdu_mean	hgt_sd	momEdu_sd	hgt_n	momEdu_n
Cebu	1	1	61.80000	5.3	9.520504	0	7	18
Cebu	1	2	68.86154	7.1	9.058931	0	13	18
Cebu	1	3	80.45882	9.4	29.894231	0	17	18
Cebu	1	4	88.10000	13.9	35.533166	0	18	18
Cebu	1	5	97.70556	11.3	41.090366	0	18	18
Cebu	1	6	87.49444	7.3	35.586439	0	18	18
Cebu	1	7	90.79412	10.4	38.722385	0	17	18
Cebu	1	8	68.45385	13.5	10.011961	0	13	18
Cebu	1	9	86.21111	10.4	35.126057	0	18	18
Cebu	1	10	87.67222	10.5	36.508127	0	18	18

S.country	vil.id	indi.id	hgt_mean	momEdu_mean	hgt_sd	momEdu_sd	hgt_n	momEdu_n
Guatemala	14	2014	66.97000	NaN	8.967974	NaN	10	0
Guatemala	14	2015	71.71818	NaN	11.399984	NaN	11	0
Guatemala	14	2016	66.33000	NaN	9.490352	NaN	10	0
Guatemala	14	2017	76.40769	NaN	14.827871	NaN	13	0
Guatemala	14	2018	74.55385	NaN	12.707846	NaN	13	0
Guatemala	14	2019	70.47500	NaN	11.797390	NaN	12	0
Guatemala	14	2020	60.28750	NaN	7.060036	NaN	8	0
Guatemala	14	2021	84.96000	NaN	15.446193	NaN	10	0
Guatemala	14	2022	79.38667	NaN	15.824749	NaN	15	0
Guatemala	14	2023	66.50000	NaN	8.613113	NaN	8	0

2.3.2 One Variable Group Summary

Go back to [fan's REconTools Package](#), [R4Econ Repository \(bookdown site\)](#), or [Intro Stats with R Repository](#).

There is a categorical variable (based on one or the interaction of multiple variables), there is a continuous variable, obtain statistics for the continuous variable conditional on the categorical variable, but also unconditionally.

Store results in a matrix, but also flatten results wide to row with appropriate keys/variable-names for all group statistics.

Pick which statistics to be included in final wide row

2.3.2.1 Build Program

```
# Single Variable Group Statistics (also generate overall statistics)
ff_summ_by_group_summ_one <- function(
  df, vars.group, var.numeric, str.stats.group = 'main',
  str.stats.specify = NULL, boo.overall.stats = TRUE){

  # List of statistics
  # https://rdr.io/cran/dplyr/man/summarise.html
  str.center <- c('mean', 'median')
  str.spread <- c('sd', 'IQR', 'mad')
  str.range <- c('min', 'max')
  str.pos <- c('first', 'last')
  str.count <- c('n_distinct')

  # Grouping of Statistics
  if (missing(str.stats.specify)) {
    if (str.stats.group == 'main') {
      str.all <- c('mean', 'min', 'max', 'sd')
    }
    if (str.stats.group == 'all') {
      str.all <- c(str.center, str.spread, str.range, str.pos, str.count)
    }
  }
}
```

```

} else {
  str.all <- str.stats.specify
}

# Start Transform
df <- df %>% drop_na() %>%
  mutate(!!(var.numeric) := as.numeric(!!sym(var.numeric)))

# Overall Statistics
if (boo.overall.stats) {
  df.overall.stats <- df %>%
    summarize_at(vars(var.numeric), funs(!!!str.all))
  if (length(str.all) == 1) {
    # give it a name, otherwise if only one stat, name of stat not saved
    df.overall.stats <- df.overall.stats %>%
      rename(!!str.all := !!sym(var.numeric))
  }
  names(df.overall.stats) <-
    paste0(var.numeric, '.', names(df.overall.stats))
}

# Group Sort
df.select <- df %>%
  group_by(!!!syms(vars.group)) %>%
  arrange(!!!syms(c(vars.group, var.numeric)))

# Table of Statistics
df.table.grp.stats <- df.select %>%
  summarize_at(vars(var.numeric), funs(!!!str.all))

# Add Stat Name
if (length(str.all) == 1) {
  # give it a name, otherwise if only one stat, name of stat not saved
  df.table.grp.stats <- df.table.grp.stats %>%
    rename(!!str.all := !!sym(var.numeric))
}

# Row of Statistics
str.vars.group.combine <- paste0(vars.group, collapse='_')
if (length(vars.group) == 1) {
  df.row.grp.stats <- df.table.grp.stats %>%
    mutate(!!(str.vars.group.combine) :=
      paste0(var.numeric, '.',
              vars.group, '.g',
              (!!!syms(vars.group)))) %>%
    gather(variable, value, -one_of(vars.group)) %>%
    unite(str.vars.group.combine, c(str.vars.group.combine, 'variable')) %>%
    spread(str.vars.group.combine, value)
} else {
  df.row.grp.stats <- df.table.grp.stats %>%
    mutate(vars.groups.combine :=
      paste0(paste0(vars.group, collapse='.'),
              !(str.vars.group.combine) :=
                paste0(interaction(!!!syms(vars.group)))) %>%
    mutate(!!(str.vars.group.combine) :=
      paste0(var.numeric, '.', vars.groups.combine, '.'),

```

```

    (!!sym(str.vars.group.combine))) %>%
  ungroup() %>%
  select(-vars.groups.combine, -one_of(vars.group)) %>%
  gather(variable, value, -one_of(str.vars.group.combine)) %>%
  unite(str.vars.group.combine, c(str.vars.group.combine, 'variable')) %>%
  spread(str.vars.group.combine, value)
}

# Clean up name strings
names(df.table.grp.stats) <-
  gsub(x = names(df.table.grp.stats), pattern = "_", replacement = "\\.")
names(df.row.grp.stats) <-
  gsub(x = names(df.row.grp.stats), pattern = "_", replacement = "\\.")

# Return
list.return <-
  list(df_table_grp_stats = df.table.grp.stats,
       df_row_grp_stats = df.row.grp.stats)

# Overall Statistics, without grouping
if (boo.overall.stats) {
  df.row.stats.all <- c(df.row.grp.stats, df.overall.stats)
  list.return <- append(list.return,
                       list(df_overall_stats = df.overall.stats,
                            df_row_stats_all = df.row.stats.all))
}

# Return
return(list.return)
}

```

2.3.2.2 Test

Load data and test

```

# Library
library(tidyverse)

# Load Sample Data
setwd('C:/Users/fan/R4Econ/_data/')
df <- read_csv('height_weight.csv')

```

2.3.2.2.1 Function Testing By Gender Groups Need two variables, a group variable that is a factor, and a numeric

```

vars.group <- 'sex'
var.numeric <- 'hgt'

```

```
df.select <- df %>% select(one_of(vars.group, var.numeric)) %>% drop_na()
```

Main Statistics:

```

# Single Variable Group Statistics
ff_summ_by_group_summ_one(
  df.select, vars.group = vars.group, var.numeric = var.numeric,
  str.stats.group = 'main')$df_table_grp_stats

```

Specify Two Specific Statistics:

```
ff_summ_by_group_summ_one(
  df.select, vars.group = vars.group, var.numeric = var.numeric,
  str.stats.specify = c('mean', 'sd'))$df_table_grp_stats
```

Specify One Specific Statistics:

```
ff_summ_by_group_summ_one(
  df.select, vars.group = vars.group, var.numeric = var.numeric,
  str.stats.specify = c('mean'))$df_table_grp_stats
```

2.3.2.2.2 Function Testing By Country and Gender Groups Need two variables, a group variable that is a factor, and a numeric. Now joint grouping variables.

```
vars.group <- c('S.country', 'sex')
var.numeric <- 'hgt'
```

```
df.select <- df %>% select(one_of(vars.group, var.numeric)) %>% drop_na()
```

Main Statistics:

```
ff_summ_by_group_summ_one(
  df.select, vars.group = vars.group, var.numeric = var.numeric,
  str.stats.group = 'main')$df_table_grp_stats
```

Specify Two Specific Statistics:

```
ff_summ_by_group_summ_one(
  df.select, vars.group = vars.group, var.numeric = var.numeric,
  str.stats.specify = c('mean', 'sd'))$df_table_grp_stats
```

Specify One Specific Statistics:

```
ff_summ_by_group_summ_one(
  df.select, vars.group = vars.group, var.numeric = var.numeric,
  str.stats.specify = c('mean'))$df_table_grp_stats
```

2.3.3 Nested within Group Stats

Go back to [fan's REconTools Package](#), [R4Econ Repository](#) ([bookdown site](#)), or [Intro Stats with R Repository](#).

By Multiple within Individual Groups Variables, Averages for All Numeric Variables within All Groups of All Group Variables (Long to very Wide). Suppose you have an individual level final outcome. The individual is observed for N periods, where each period the inputs differ. What inputs impacted the final outcome?

Suppose we can divide N periods in which the individual is in the data into a number of years, a number of semi-years, a number of quarters, or uneven-staggered lengths. We might want to generate averages across individuals and within each of these different possible groups averages of inputs.

Then we want to version of the data where each row is an individual, one of the variables is the final outcome, and the other variables are these different averages: averages for the 1st, 2nd, 3rd year in which individual is in data, averages for 1st, ..., final quarter in which individual is in data.

2.3.3.1 Build Function

This function takes as inputs:

1. **vars.not.groups2avg**: a list of variables that are not the within-individual or across-individual grouping variables, but the variables we want to average over. Within individual grouping averages will be calculated for these variables using the not-listed variables as within individual groups (excluding vars.indi.grp groups).

2. **vars.indi.grp**: a list of individual variables, and also perhaps villages, province, etc id variables that are higher than individual ID. Note the groups are across individual higher level group variables.
3. the remaining variables are all within individual grouping variables.

the function output is a dataframe:

1. each row is an individual
2. initial variables individual ID and across individual groups from *vars.indi.grp*.
3. other variables are all averages for the variables in *vars.not.groups2avg*
 - if there are 2 within individual group variables, and the first has 3 groups (years), the second has 6 groups (semi-years), then there would be 9 average variables.
 - each average variable has the original variable name from *vars.not.groups2avg* plus the name of the within individual grouping variable, and at the end 'c_x', where x is an integer representing the category within the group (if 3 years, x=1, 2, 3)

```
# Data Function
# https://fanwangecon.github.io/R4Econ/summarize/summ/ByGroupsSummWide.html
f.by.groups.summ.wide <- function(df.groups.to.average,
                                vars.not.groups2avg,
                                vars.indi.grp = c('S.country', 'ID'),
                                display=TRUE) {

  # 1. generate categoricals for full year (m.12), half year (m.6), quarter year (m.4)
  # 2. generate categoricals also for uneven years (m12t14) using
  # stagger (+2 rather than -1)
  # 3. reshape wide to long, so that all categorical date groups appear in var=value,
  # and categories in var=variable
  # 4. calculate mean for all numeric variables for all date groups
  # 5. combine date categorical variable and value, single var:
  # m.12.c1= first year average from m.12 averaging

  #####
  # Step 1
  #####
  # 1. generate categoricals for full year (m.12), half year (m.6), quarter year (m.4)
  # 2. generate categoricals also for uneven years (m12t14) using stagger
  # (+2 rather than -1)

  #####
  # S2: reshape wide to long, so that all categorical date groups appear in var=value,
  # and categories in var=variable; calculate mean for all
  # numeric variables for all date groups
  #####
  df.avg.long <- df.groups.to.average %>%
    gather(variable, value, -one_of(c(vars.indi.grp,
                                     vars.not.groups2avg))) %>%
    group_by(!!!syms(vars.indi.grp), variable, value) %>%
    summarise_if(is.numeric, funs(mean(., na.rm = TRUE)))

  if (display){
    dim(df.avg.long)
    options(repr.matrix.max.rows=10, repr.matrix.max.cols=20)
    print(df.avg.long)
  }

  #####
  # S3 combine date categorical variable and value, single var:
  # m.12.c1= first year average from m.12 averaging; to do this make
  # data even longer first
```

```
#####

# We already have the averages, but we want them to show up as variables,
# mean for each group of each variable.
df.avg.allvars.wide <- df.avg.long %>%
  ungroup() %>%
  mutate(all_m_cate = paste0(variable, '_c', value)) %>%
  select(all_m_cate, everything(), -variable, -value) %>%
  gather(variable, value, -one_of(vars.indi.grp), -all_m_cate) %>%
  unite('var_mcate', variable, all_m_cate) %>%
  spread(var_mcate, value)

if (display){
  dim(df.avg.allvars.wide)
  options(repr.matrix.max.rows=10, repr.matrix.max.cols=10)
  print(df.avg.allvars.wide)
}

return(df.avg.allvars.wide)
}
```

2.3.3.2 Test Program

In our sample dataset, the number of nutrition/height/income etc information observed within each country and month of age group are different. We have a panel dataset for children observed over different months of age.

We have two key grouping variables: 1. country: data are observed for guatemala and cebu 2. month-age (survey month round=svymthRound): different months of age at which each individual child is observed

A child could be observed for many months, or just a few months. A child's height information could be observed for more months-of-age than nutritional intake information. We eventually want to run regressions where the outcome is height/weight and the input is nutrition. The regressions will be at the month-of-age level. We need to know how many times different variables are observed at the month-of-age level.

```
# Library
library(tidyverse)

# Load Sample Data
setwd('C:/Users/fan/R4Econ/_data/')
df <- read_csv('height_weight.csv')
```

2.3.3.2.1 Generate Within Individual Groups In the data, children are observed for different number of months since birth. We want to calculate quarterly, semi-year, annual, etc average nutritional intakes. First generate these within-individual grouping variables. We can also generate uneven-staggered calendar groups as shown below.

```
mth.var <- 'svymthRound'
df.groups.to.average <- df %>%
  filter(!sym(mth.var) >= 0 & !sym(mth.var) <= 24) %>%
  mutate(m12t24=(floor((!sym(mth.var) - 12) %/% 14) + 1),
         m8t24=(floor((!sym(mth.var) - 8) %/% 18) + 1),
         m12 = pmax((floor((!sym(mth.var)-1) %/% 12) + 1), 1),
         m6 = pmax((floor((!sym(mth.var)-1) %/% 6) + 1), 1),
         m3 = pmax((floor((!sym(mth.var)-1) %/% 3) + 1), 1))

# Show Results
options(repr.matrix.max.rows=30, repr.matrix.max.cols=20)
vars.arrange <- c('S.country', 'indi.id', 'svymthRound')
```



```

ar_test_scores_ec3 <- c(107.72,101.28,105.92,109.31,104.27,110.27,91.92846154,81.8,109.0071429,103.0
ar_test_scores_ec1 <- c(101.72,101.28,99.92,103.31,100.27,104.27,90.23615385,77.8,103.4357143,97.07,
mt_test_scores <- cbind(ar_test_scores_ec1, ar_test_scores_ec3)
ar_st_varnames <- c('course_total_ec1p','course_total_ec3p')
tb_final_twovar <- as_tibble(mt_test_scores) %>% rename_all(~c(ar_st_varnames))
summary(tb_final_twovar)

```

2.4.1.1.1 A Dataset with only Two Continuous Variable

```

## course_total_ec1p course_total_ec3p
## Min. : 40.48 Min. : 44.23
## 1st Qu.: 76.46 1st Qu.: 79.91
## Median : 86.35 Median : 89.28
## Mean : 83.88 Mean : 87.90
## 3rd Qu.: 95.89 3rd Qu.:100.75
## Max. :104.27 Max. :112.22

ff_summ_percentiles(df = tb_final_twovar, bl_statsasrows = TRUE, col2varname = FALSE)

```

```

ar_final_scores <- c(94.28442509,95.68817475,97.25219512,77.89268293,95.08795497,93.27380863,92.3,84
mt_test_scores <- cbind(seq(1,length(ar_final_scores)), ar_final_scores)
ar_st_varnames <- c('index', 'course_final')
tb_onevar <- as_tibble(mt_test_scores) %>% rename_all(~c(ar_st_varnames))
summary(tb_onevar)

```

2.4.1.1.2 A Dataset with one Continuous Variable and Histogram

```

## index course_final
## Min. : 1.0 Min. : 2.293
## 1st Qu.:12.5 1st Qu.: 76.372
## Median :24.0 Median : 86.959
## Mean :24.0 Mean : 82.415
## 3rd Qu.:35.5 3rd Qu.: 94.686
## Max. :47.0 Max. :100.898

ff_summ_percentiles(df = tb_onevar, bl_statsasrows = TRUE, col2varname = FALSE)

```

```

#load in data empirically by hand
txt_test_data <- "init_prof, later_prof, class_id, exam_score
'SW', 'SW', 1, 102
'SW', 'SW', 1, 102
'SW', 'SW', 1, 101
'SW', 'SW', 1, 100
'SW', 'SW', 1, 100
'SW', 'SW', 1, 99
'SW', 'SW', 1, 98.5
'SW', 'SW', 1, 98.5
'SW', 'SW', 1, 97
'SW', 'SW', 1, 95
'SW', 'SW', 1, 94
'SW', 'SW', 1, 91
'SW', 'SW', 1, 91
'SW', 'SW', 1, 90
'SW', 'SW', 1, 89
'SW', 'SW', 1, 88.5
'SW', 'SW', 1, 88

```

```
'SW', 'SW', 1, 87
'SW', 'SW', 1, 87
'SW', 'SW', 1, 87
'SW', 'SW', 1, 86
'SW', 'SW', 1, 86
'SW', 'SW', 1, 84
'SW', 'SW', 1, 82
'SW', 'SW', 1, 78.5
'SW', 'SW', 1, 76
'SW', 'SW', 1, 72
'SW', 'SW', 1, 70.5
'SW', 'SW', 1, 67.5
'SW', 'SW', 1, 67.5
'SW', 'SW', 1, 67
'SW', 'SW', 1, 63.5
'SW', 'SW', 1, 60
'SW', 'SW', 1, 59
'SW', 'SW', 1, 44.5
'SW', 'SW', 1, 44
'SW', 'SW', 1, 42.5
'SW', 'SW', 1, 40.5
'SW', 'SW', 1, 40.5
'SW', 'SW', 1, 36.5
'SW', 'SW', 1, 35.5
'SW', 'SW', 1, 21.5
'SW', 'SW', 1, 4
'MP', 'MP', 2, 105
'MP', 'MP', 2, 103
'MP', 'MP', 2, 102
'MP', 'MP', 2, 101
'MP', 'MP', 2, 101
'MP', 'MP', 2, 100.5
'MP', 'MP', 2, 100
'MP', 'MP', 2, 99
'MP', 'MP', 2, 97
'MP', 'MP', 2, 97
'MP', 'MP', 2, 97
'MP', 'MP', 2, 97
'MP', 'MP', 2, 96
'MP', 'MP', 2, 95
'MP', 'MP', 2, 91
'MP', 'MP', 2, 89
'MP', 'MP', 2, 85
'MP', 'MP', 2, 84
'MP', 'MP', 2, 84
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'MP', 'MP', 2, 83.5
'MP', 'MP', 2, 82.5
'MP', 'MP', 2, 81.5
'MP', 'MP', 2, 80.5
'MP', 'MP', 2, 80
'MP', 'MP', 2, 77
'MP', 'MP', 2, 77
'MP', 'MP', 2, 75
'MP', 'MP', 2, 75
'MP', 'MP', 2, 71
'MP', 'MP', 2, 70
'MP', 'MP', 2, 68
```

```
'MP', 'MP', 2, 63
'MP', 'MP', 2, 56
'MP', 'MP', 2, 56
'MP', 'MP', 2, 55.5
'MP', 'MP', 2, 49.5
'MP', 'MP', 2, 48.5
'MP', 'MP', 2, 47.5
'MP', 'MP', 2, 44.5
'MP', 'MP', 2, 34.5
'MP', 'MP', 2, 29.5
'CA', 'MP', 3, 103
'CA', 'MP', 3, 103
'CA', 'MP', 3, 101
'CA', 'MP', 3, 96.5
'CA', 'MP', 3, 93.5
'CA', 'MP', 3, 93
'CA', 'MP', 3, 93
'CA', 'MP', 3, 92
'CA', 'MP', 3, 90
'CA', 'MP', 3, 90
'CA', 'MP', 3, 89
'CA', 'MP', 3, 86.5
'CA', 'MP', 3, 84.5
'CA', 'MP', 3, 83
'CA', 'MP', 3, 83
'CA', 'MP', 3, 82
'CA', 'MP', 3, 78
'CA', 'MP', 3, 75
'CA', 'MP', 3, 74.5
'CA', 'MP', 3, 70
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'CA', 'MP', 3, 52
'CA', 'MP', 3, 50
'CA', 'MP', 3, 42
'CA', 'MP', 3, 36.5
'CA', 'MP', 3, 28
'CA', 'MP', 3, 26
'CA', 'MP', 3, 11
'CA', 'SN', 4, 103
'CA', 'SN', 4, 103
'CA', 'SN', 4, 102
'CA', 'SN', 4, 102
'CA', 'SN', 4, 101
'CA', 'SN', 4, 100
'CA', 'SN', 4, 98
'CA', 'SN', 4, 98
'CA', 'SN', 4, 98
'CA', 'SN', 4, 95
'CA', 'SN', 4, 95
'CA', 'SN', 4, 92.5
'CA', 'SN', 4, 92
'CA', 'SN', 4, 91
'CA', 'SN', 4, 90
'CA', 'SN', 4, 85.5
'CA', 'SN', 4, 84
'CA', 'SN', 4, 82.5
'CA', 'SN', 4, 81
'CA', 'SN', 4, 77.5
```

```

'CA', 'SN', 4, 77
'CA', 'SN', 4, 72
'CA', 'SN', 4, 71.5
'CA', 'SN', 4, 69
'CA', 'SN', 4, 68.5
'CA', 'SN', 4, 68
'CA', 'SN', 4, 67
'CA', 'SN', 4, 65.5
'CA', 'SN', 4, 62.5
'CA', 'SN', 4, 62
'CA', 'SN', 4, 61.5
'CA', 'SN', 4, 61
'CA', 'SN', 4, 57.5
'CA', 'SN', 4, 54
'CA', 'SN', 4, 52.5
'CA', 'SN', 4, 51
'CA', 'SN', 4, 50.5
'CA', 'SN', 4, 50
'CA', 'SN', 4, 49
'CA', 'SN', 4, 43
'CA', 'SN', 4, 39.5
'CA', 'SN', 4, 32.5
'CA', 'SN', 4, 25.5
'CA', 'SN', 4, 18"

csv_test_data = read.csv(text=txt_test_data, header=TRUE)
ar_st_varnames <- c('first_half_professor',
                    'second_half_professor',
                    'course_id', 'exam_score')
tb_test_data <- as_tibble(csv_test_data) %>%
  rename_all(~c(ar_st_varnames))
summary(tb_test_data)

```

2.4.1.1.3 A Dataset with Multiple Variables

##	first_half_professor	second_half_professor	course_id	exam_score
##	'CA':72	'MP':70	Min. :1.000	Min. : 4.00
##	'MP':42	'SN':44	1st Qu.:1.000	1st Qu.: 60.00
##	'SW':43	'SW':43	Median :2.000	Median : 82.00
##			Mean :2.465	Mean : 75.08
##			3rd Qu.:4.000	3rd Qu.: 94.00
##			Max. :4.000	Max. :105.00

2.4.1.2 Test Score Distributions

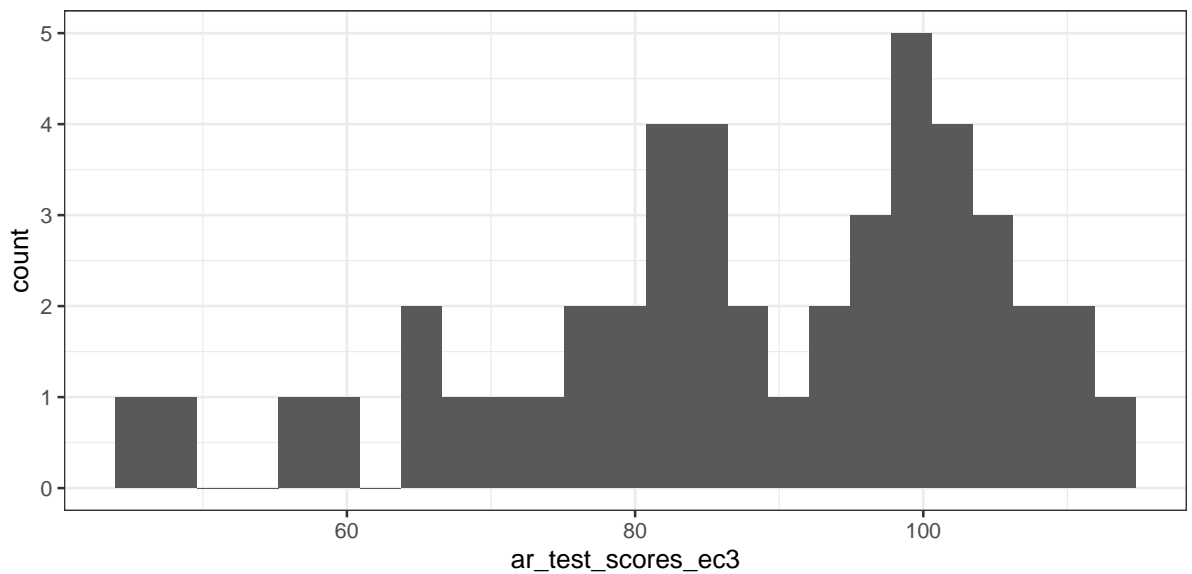
```

ggplot(tb_final_twovar, aes(x=ar_test_scores_ec3)) +
  geom_histogram(bins=25) +
  labs(title = paste0('Sandbox: Final Distribution (Econ 2370, FW)'),
       caption = paste0('FW Section, formula:',
                        '0.3*exam1Perc + 0.3*exam2Perc + ',
                        '0.42*HWtotalPerc + 0.03*AttendancePerc \n',
                        '+ perfect attendance + 0.03 per Extra Credit')) +
  theme_bw()

```

2.4.1.2.1 Histogram

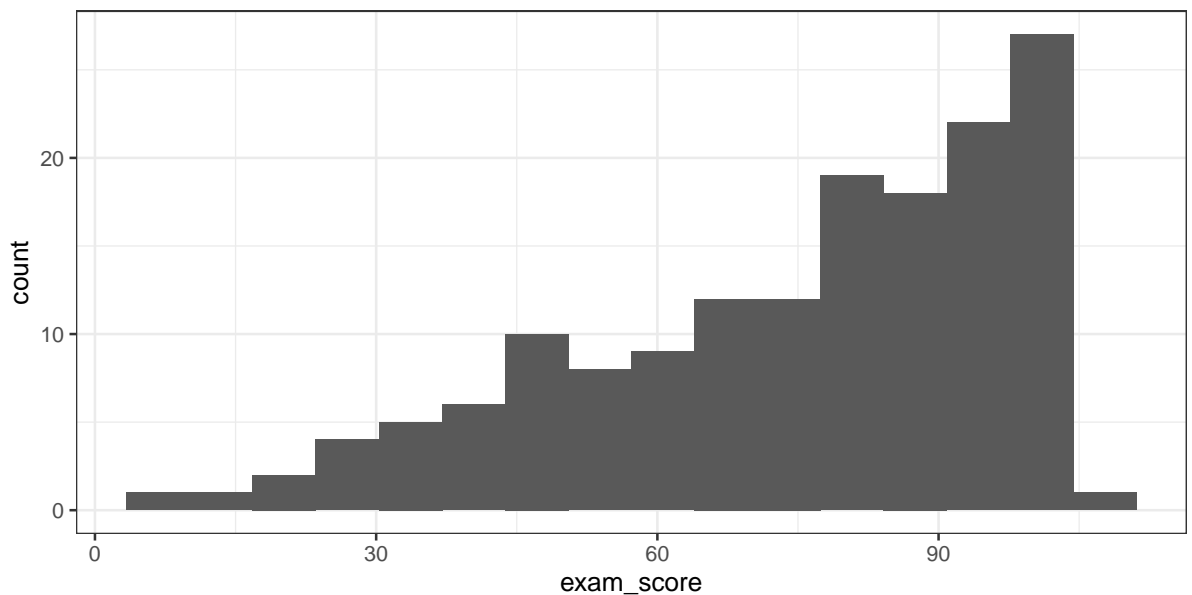
Sandbox: Final Distribution (Econ 2370, FW)



FW Section, formula: $0.3 \times \text{exam1Perc} + 0.3 \times \text{exam2Perc} + 0.42 \times \text{HWtotalPerc} + 0.03 \times \text{AttendancePerc}$
 + perfect attendance + 0.03 per Extra Credit

```
ggplot(tb_test_data, aes(x=exam_score)) +
  geom_histogram(bins=16) +
  labs(title = paste0('Exam Distribution'),
       caption = 'All Sections') +
  theme_bw()
```

Exam Distribution



All Sections

2.5 Summarize Multiple Variables

2.5.1 Generate Replace Variables

Go back to [fan's REconTools Package](#), [R4Econ Repository \(bookdown site\)](#), or [Intro Stats with R Repository](#).

2.5.1.1 Replace NA for Multiple Variables

Replace some variables NA by some values, and other variables' NAs by other values.

```
# Define
it_N <- 3
it_M <- 5
svr_id <- 'date'

# NA dataframe
df_NA <- as_tibble(matrix(NA, nrow=it_N, ncol=it_M)) %>%
  rowid_to_column(var = svr_id) %>%
  rename_at(vars(starts_with("V")),
            funs(str_replace(., "V", "var")))
kable(df_NA) %>%
  kable_styling_fc()
```

date	var1	var2	var3	var4	var5
1	NA	NA	NA	NA	NA
2	NA	NA	NA	NA	NA
3	NA	NA	NA	NA	NA

```
# Replace NA
df_NA_replace <- df_NA %>%
  mutate_at(vars(one_of(c('var1', 'var2'))), list(~replace_na(., 0))) %>%
  mutate_at(vars(one_of(c('var3', 'var5'))), list(~replace_na(., 99)))
kable(df_NA_replace) %>%
  kable_styling_fc()
```

date	var1	var2	var3	var4	var5
1	0	0	99	NA	99
2	0	0	99	NA	99
3	0	0	99	NA	99

2.5.1.2 Cumulative Sum Multiple Variables

Each row is a different date, each column is the profit a firms earns on a date, we want to compute cumulatively how much a person is earning. Also renames variable names below jointly.

```
# Define
it_N <- 3
it_M <- 5
svr_id <- 'date'

# random dataframe, daily profit of firms
# dp_fx: daily profit firm ID something
set.seed(123)
df_daily_profit <- as_tibble(matrix(rnorm(it_N*it_M), nrow=it_N, ncol=it_M)) %>%
  rowid_to_column(var = svr_id) %>%
  rename_at(vars(starts_with("V")),
            funs(str_replace(., "V", "dp_f")))
kable(df_daily_profit) %>%
  kable_styling_fc()
```

date	dp_f1	dp_f2	dp_f3	dp_f4	dp_f5
1	-0.5604756	0.0705084	0.4609162	-0.4456620	0.4007715
2	-0.2301775	0.1292877	-1.2650612	1.2240818	0.1106827
3	1.5587083	1.7150650	-0.6868529	0.3598138	-0.5558411

```
# cumulative sum with suffix
```

```
df_cummu_profit_suffix <- df_daily_profit %>%
  mutate_at(vars(contains('dp_f')), .funs = list(cumu = ~cumsum(.)))
kable(df_cummu_profit_suffix) %>%
  kable_styling_fc_wide()
```

date	dp_f1	dp_f2	dp_f3	dp_f4	dp_f5	dp_f1_cumu	dp_f2_cumu	dp_f3_cumu	dp_f4_cumu	dp_f5_cumu
1	-0.5604756	0.0705084	0.4609162	-0.4456620	0.4007715	-0.5604756	0.0705084	0.4609162	-0.4456620	0.4007715
2	-0.2301775	0.1292877	-1.2650612	1.2240818	0.1106827	-0.7906531	0.1997961	-0.8041450	0.7784198	0.5114542
3	1.5587083	1.7150650	-0.6868529	0.3598138	-0.5558411	0.7680552	1.9148611	-1.4909979	1.1382337	-0.0443870

```
# cumulative sum variables naming to prefix
```

```
df_cummu_profit <- df_cummu_profit_suffix %>%
  rename_at(vars(contains( "_cumu" )), list(~paste("cp_f", gsub("_cumu", "", .), sep = ""))) %>%
  rename_at(vars(contains( "cp_f" )), list(~gsub("dp_f", "", .)))
kable(df_cummu_profit) %>%
  kable_styling_fc_wide()
```

date	dp_f1	dp_f2	dp_f3	dp_f4	dp_f5	cp_f1	cp_f2	cp_f3	cp_f4	cp_f5
1	-0.5604756	0.0705084	0.4609162	-0.4456620	0.4007715	-0.5604756	0.0705084	0.4609162	-0.4456620	0.4007715
2	-0.2301775	0.1292877	-1.2650612	1.2240818	0.1106827	-0.7906531	0.1997961	-0.8041450	0.7784198	0.5114542
3	1.5587083	1.7150650	-0.6868529	0.3598138	-0.5558411	0.7680552	1.9148611	-1.4909979	1.1382337	-0.0443870

Chapter 3

Functions

3.1 Dataframe Mutate

3.1.1 Row Input Functions

Go back to [fan's REconTools Package](#), [R4Econ Repository \(bookdown site\)](#), or [Intro Stats with R Repository](#).

We want evaluate nonlinear function $f(Q_i, y_i, ar_x, ar_y, c, d)$, where c and d are constants, and ar_x and ar_y are arrays, both fixed. x_i and y_i vary over each row of matrix. We would like to evaluate this nonlinear function concurrently across N individuals. The eventual goal is to find the i specific Q that solves the nonlinear equations.

This is a continuation of [R use Apply, Sapply and dplyr Mutate to Evaluate one Function Across Rows of a Matrix](#)

3.1.1.1 Set up Input Arrays

There is a function that takes $M = Q + P$ inputs, we want to evaluate this function N times. Each time, there are M inputs, where all but Q of the M inputs, meaning P of the M inputs, are the same. In particular, $P = Q * N$.

$$M = Q + P = Q + Q * N$$

```
# it_child_count = N, the number of children
it_N_child_cnt = 5
# it_heter_param = Q, number of parameters that are heterogeneous across children
it_Q_hetpa_cnt = 2

# P fixed parameters, nN is N dimensional, nP is P dimensional
ar_nN_A = seq(-2, 2, length.out = it_N_child_cnt)
ar_nN_alpha = seq(0.1, 0.9, length.out = it_N_child_cnt)
ar_nP_A_alpha = c(ar_nN_A, ar_nN_alpha)
ar_nN_N_choice = seq(1, it_N_child_cnt) / sum(seq(1, it_N_child_cnt))

# N by Q varying parameters
mt_nN_by_nQ_A_alpha = cbind(ar_nN_A, ar_nN_alpha, ar_nN_N_choice)
# Show
kable(mt_nN_by_nQ_A_alpha) %>%
  kable_styling_fc()
```

3.1.1.2 Testing Function

Test non-linear Equation.

ar_nN_A	ar_nN_alpha	ar_nN_N_choice
-2	0.1	0.0666667
-1	0.3	0.1333333
0	0.5	0.2000000
1	0.7	0.2666667
2	0.9	0.3333333

```
# Test Parameters
```

```
fl_N_agg = 100
```

```
fl_rho = -1
```

```
fl_N_q = ar_nN_N_choice[4]*fl_N_agg
```

```
ar_A_alpha = mt_nN_by_nQ_A_alpha[4,]
```

```
# Apply Function
```

```
ar_p1_s1 = exp((ar_A_alpha[1] - ar_nN_A)*fl_rho)
```

```
ar_p1_s2 = (ar_A_alpha[2]/ar_nN_alpha)
```

```
ar_p1_s3 = (1/(ar_nN_alpha*fl_rho - 1))
```

```
ar_p1 = (ar_p1_s1*ar_p1_s2)^ar_p1_s3
```

```
ar_p2 = fl_N_q^((ar_A_alpha[2]*fl_rho-1)/(ar_nN_alpha*fl_rho-1))
```

```
ar_overall = ar_p1*ar_p2
```

```
fl_overall = fl_N_agg - sum(ar_overall)
```

```
print(fl_overall)
```

```
## [1] -598.2559
```

Implement the non-linear problem's evaluation using apply over all N individuals.

```
# Define Implicit Function
```

```
ffi_nonlin_dplyrdo <- function(fl_A, fl_alpha, fl_N, ar_A, ar_alpha, fl_N_agg, fl_rho){
```

```
  # ar_A_alpha[1] is A
```

```
  # ar_A_alpha[2] is alpha
```

```
  ## Test Parameters
```

```
  # fl_N = 100
```

```
  # fl_rho = -1
```

```
  # fl_N_q = 10
```

```
  # Apply Function
```

```
  ar_p1_s1 = exp((fl_A - ar_A)*fl_rho)
```

```
  ar_p1_s2 = (fl_alpha/ar_alpha)
```

```
  ar_p1_s3 = (1/(ar_alpha*fl_rho - 1))
```

```
  ar_p1 = (ar_p1_s1*ar_p1_s2)^ar_p1_s3
```

```
  ar_p2 = fl_N^((fl_alpha*fl_rho-1)/(ar_alpha*fl_rho-1))
```

```
  ar_overall = ar_p1*ar_p2
```

```
  fl_overall = fl_N_agg - sum(ar_overall)
```

```
  return(fl_overall)
```

```
}
```

```
# Parameters
```

```
fl_rho = -1
```

```
# Evaluate Function
```

```
print(ffi_nonlin_dplyrdo(mt_nN_by_nQ_A_alpha[1,1],
                        mt_nN_by_nQ_A_alpha[1,2],
                        mt_nN_by_nQ_A_alpha[1,3]*fl_N_agg,
                        ar_nN_A, ar_nN_alpha, fl_N_agg, fl_rho))
```

```
## [1] 81.86645
```

```

for (i in seq(1,dim(mt_nN_by_nQ_A_alpha)[1])){
  fl_eval = ffi_nonlin_dplyrdo(mt_nN_by_nQ_A_alpha[i,1],
                              mt_nN_by_nQ_A_alpha[i,2],
                              mt_nN_by_nQ_A_alpha[i,3]*fl_N_agg,
                              ar_nN_A, ar_nN_alpha, fl_N_agg, fl_rho)

  print(fl_eval)
}

```

```

## [1] 81.86645
## [1] 54.48885
## [1] -65.5619
## [1] -598.2559
## [1] -3154.072

```

3.1.1.3 Evaluate Nonlinear Function using dplyr mutate

```

# Convert Matrix to Tibble
ar_st_col_names = c('fl_A', 'fl_alpha', 'fl_N')
tb_nN_by_nQ_A_alpha <- as_tibble(mt_nN_by_nQ_A_alpha) %>% rename_all(~c(ar_st_col_names))

# Define Implicit Function
ffi_nonlin_dplyrdo <- function(fl_A, fl_alpha, fl_N, ar_A, ar_alpha, fl_N_agg, fl_rho){

  # Test Parameters
  # ar_A = ar_nN_A
  # ar_alpha = ar_nN_alpha
  # fl_N = 100
  # fl_rho = -1
  # fl_N_q = 10

  # Apply Function
  ar_p1_s1 = exp((fl_A - ar_A)*fl_rho)
  ar_p1_s2 = (fl_alpha/ar_alpha)
  ar_p1_s3 = (1/(ar_alpha*fl_rho - 1))
  ar_p1 = (ar_p1_s1*ar_p1_s2)^ar_p1_s3
  ar_p2 = (fl_N*fl_N_agg)^((fl_alpha*fl_rho-1)/(ar_alpha*fl_rho-1))
  ar_overall = ar_p1*ar_p2
  fl_overall = fl_N_agg - sum(ar_overall)

  return(fl_overall)
}

# fl_A, fl_alpha are from columns of tb_nN_by_nQ_A_alpha
tb_nN_by_nQ_A_alpha = tb_nN_by_nQ_A_alpha %>% rowwise() %>%
  mutate(dplyr_eval = ffi_nonlin_dplyrdo(fl_A, fl_alpha, fl_N,
                                          ar_nN_A, ar_nN_alpha,
                                          fl_N_agg, fl_rho))

# Show
kable(tb_nN_by_nQ_A_alpha) %>%
  kable_styling_fc()

```

fl_A	fl_alpha	fl_N	dplyr_eval
-2	0.1	0.0666667	81.86645
-1	0.3	0.1333333	54.48885
0	0.5	0.2000000	-65.56190
1	0.7	0.2666667	-598.25595
2	0.9	0.3333333	-3154.07226

3.1.2 Evaluate Choices Across States

Go back to [fan's REconTools Package](#), [R4Econ Repository \(bookdown site\)](#), or [Intro Stats with R Repository](#).

See the `ff_opti_bisect_pmap_multi` function from [Fan's REconTools Package](#), which provides a reusable function based on the algorithm worked out here.

We want evaluate linear function $0 = f(z_{ij}, x_i, y_i, \mathbf{X}, \mathbf{Y}, c, d)$. There are i functions that have i specific x and y . For each i function, we evaluate along a grid of feasible values for z , over $j \in J$ grid points, potentially looking for the j that is closest to the root. \mathbf{X} and \mathbf{Y} are arrays common across the i equations, and c and d are constants.

The evaluation strategy is the following, given min and max for z that are specific for each j , and given common number of grid points, generate a matrix of z_{ij} . Suppose there the number of i is I , and the number of grid points for j is J .

1. Generate a $J \cdot I$ by 3 matrix where the columns are z, x, y as tibble
2. Follow [this](#) Mutate to evaluate the $f(\cdot)$ function.
3. Add two categorical columns for grid levels and wich i, i and j index. Plot Mutate output evaluated column categorized by i as color and j as x-axis.

3.1.2.1 Set up Input Arrays

There is a function that takes $M = Q + P$ inputs, we want to evaluate this function N times. Each time, there are M inputs, where all but Q of the M inputs, meaning P of the M inputs, are the same. In particular, $P = Q * N$.

$$M = Q + P = Q + Q * N$$

Now we need to expand this by the number of choice grid. Each row, representing one equation, is expanded by the number of choice grids. We are graphically searching, or rather brute force searching, which means if we have 100 individuals, we want to plot out the nonlinear equation for each of these lines, and show graphically where each line crosses zero. We achieve this, by evaluating the equation for each of the 100 individuals along a grid of feasible choices.

In this problem here, the feasible choices are shared across individuals.

```
# Parameters
fl_rho = 0.20
svr_id_var = 'INDI_ID'

# it_child_count = N, the number of children
it_N_child_cnt = 4
# it_heter_param = Q, number of parameters that are heterogeneous across children
it_Q_hetpa_cnt = 2

# P fixed parameters, nN is N dimensional, nP is P dimensional
ar_nN_A = seq(-2, 2, length.out = it_N_child_cnt)
ar_nN_alpha = seq(0.1, 0.9, length.out = it_N_child_cnt)
ar_nP_A_alpha = c(ar_nN_A, ar_nN_alpha)

# N by Q varying parameters
mt_nN_by_nQ_A_alpha = cbind(ar_nN_A, ar_nN_alpha)

# Choice Grid for nutritional feasible choices for each
fl_N_agg = 100
fl_N_min = 0
it_N_choice_cnt_ttest = 3
it_N_choice_cnt_dense = 100
ar_N_choices_ttest = seq(fl_N_min, fl_N_agg, length.out = it_N_choice_cnt_ttest)
```

```

ar_N_choices_dense = seq(fl_N_min, fl_N_agg, length.out = it_N_choice_cnt_dense)

# Mesh Expand
tb_states_choices <- as_tibble(mt_nN_by_nQ_A_alpha) %>% rowid_to_column(var=svr_id_var)
tb_states_choices_ttest <- tb_states_choices %>% expand_grid(choices = ar_N_choices_ttest)
tb_states_choices_dense <- tb_states_choices %>% expand_grid(choices = ar_N_choices_dense)

# display
summary(tb_states_choices_dense)

##      INDI_ID      ar_nN_A      ar_nN_alpha      choices
##  Min.   :1.00   Min.   :-2    Min.   :0.1   Min.   : 0
##  1st Qu.:1.75   1st Qu.: -1   1st Qu.:0.3   1st Qu.: 25
##  Median :2.50   Median : 0    Median :0.5   Median : 50
##  Mean   :2.50   Mean    : 0    Mean   :0.5   Mean   : 50
##  3rd Qu.:3.25   3rd Qu.: 1    3rd Qu.:0.7   3rd Qu.: 75
##  Max.   :4.00   Max.    : 2    Max.   :0.9   Max.   :100

kable(tb_states_choices_ttest) %>%
  kable_styling_fc()

```

INDI_ID	ar_nN_A	ar_nN_alpha	choices
1	-2.0000000	0.1000000	0
1	-2.0000000	0.1000000	50
1	-2.0000000	0.1000000	100
2	-0.6666667	0.3666667	0
2	-0.6666667	0.3666667	50
2	-0.6666667	0.3666667	100
3	0.6666667	0.6333333	0
3	0.6666667	0.6333333	50
3	0.6666667	0.6333333	100
4	2.0000000	0.9000000	0
4	2.0000000	0.9000000	50
4	2.0000000	0.9000000	100

3.1.2.2 Apply Same Function all Rows, Some Inputs Row-specific, other Shared

There are two types of inputs, row-specific inputs, and inputs that should be applied for each row. The Function just requires all of these inputs, it does not know what is row-specific and what is common for all row. Dplyr recognizes which parameter inputs already existing in the piped dataframe/tibble, given rowwise, those will be row-specific inputs. Additional function parameters that do not exist in dataframe as variable names, but that are pre-defined scalars or arrays will be applied to all rows.

- ? string variable name of input where functions are evaluated, these are already contained in the dataframe, existing variable names, row specific, rowwise computation over these, each rowwise calculation using different rows: *fl_A*, *fl_alpha*, *fl_N*
- ? scalar and array values that are applied to every rowwise calculation, all rowwise calculations using the same scalars and arrays: *ar_A*, *ar_alpha*, *fl_N_agg*, *fl_rho*
- ? string output variable name

The function looks within group, finds min/max etc that are relevant.

```

# Convert Matrix to Tibble
ar_st_col_names = c(svr_id_var, 'fl_A', 'fl_alpha')
tb_states_choices <- tb_states_choices %>% rename_all(~c(ar_st_col_names))
ar_st_col_names = c(svr_id_var, 'fl_A', 'fl_alpha', 'fl_N')
tb_states_choices_ttest <- tb_states_choices_ttest %>% rename_all(~c(ar_st_col_names))

```

```

tb_states_choices_dense <- tb_states_choices_dense %>% rename_all(~c(ar_st_col_names))

# Define Implicit Function
ffi_nonlin_dplyrdo <- function(fl_A, fl_alpha, fl_N, ar_A, ar_alpha, fl_N_agg, fl_rho){
  # scalar value that are row-specific, in dataframe already: *fl_A*, *fl_alpha*, *fl_N*
  # array and scalars not in dataframe, common all rows: *ar_A*, *ar_alpha*, *fl_N_agg*, *fl_rho*

  # Test Parameters
  # ar_A = ar_nN_A
  # ar_alpha = ar_nN_alpha
  # fl_N = 100
  # fl_rho = -1
  # fl_N_q = 10

  # Apply Function
  ar_p1_s1 = exp((fl_A - ar_A)*fl_rho)
  ar_p1_s2 = (fl_alpha/ar_alpha)
  ar_p1_s3 = (1/(ar_alpha*fl_rho - 1))
  ar_p1 = (ar_p1_s1*ar_p1_s2)^ar_p1_s3
  ar_p2 = fl_N^((fl_alpha*fl_rho-1)/(ar_alpha*fl_rho-1))
  ar_overall = ar_p1*ar_p2
  fl_overall = fl_N_agg - sum(ar_overall)

  return(fl_overall)
}

```

3.1.2.2.1 3 Points and Denser Dataframes and Define Function

3.1.2.2.2 Evaluate at Three Choice Points and Show Table In the example below, just show results evaluating over three choice points and show table.

```

# fl_A, fl_alpha are from columns of tb_nN_by_nQ_A_alpha
tb_states_choices_ttest_eval = tb_states_choices_ttest %>% rowwise() %>%
  mutate(dplyr_eval = ffi_nonlin_dplyrdo(fl_A, fl_alpha, fl_N,
                                          ar_nN_A, ar_nN_alpha,
                                          fl_N_agg, fl_rho))

# Show
kable(tb_states_choices_ttest_eval) %>%
  kable_styling_fc()

```

INDI_ID	fl_A	fl_alpha	fl_N	dplyr_eval
1	-2.0000000	0.1000000	0	100.00000
1	-2.0000000	0.1000000	50	-5666.95576
1	-2.0000000	0.1000000	100	-12880.28392
2	-0.6666667	0.3666667	0	100.00000
2	-0.6666667	0.3666667	50	-595.73454
2	-0.6666667	0.3666667	100	-1394.70698
3	0.6666667	0.6333333	0	100.00000
3	0.6666667	0.6333333	50	-106.51058
3	0.6666667	0.6333333	100	-323.94216
4	2.0000000	0.9000000	0	100.00000
4	2.0000000	0.9000000	50	22.55577
4	2.0000000	0.9000000	100	-51.97161

3.1.2.2.3 Evaluate at Many Choice Points and Show Graphically Same as above, but now we evaluate the function over the individuals at many choice points so that we can graph things out.

```
# fl_A, fl_alpha are from columns of tb_nN_by_nQ_A_alpha
tb_states_choices_dense_eval = tb_states_choices_dense %>% rowwise() %>%
  mutate(dplyr_eval = ffi_nonlin_dpdyrdo(fl_A, fl_alpha, fl_N,
                                         ar_nN_A, ar_nN_alpha,
                                         fl_N_agg, fl_rho))

# Labeling
st_title <- paste0('Evaluate Non-Linear Functions to Search for Roots')
st_subtitle <- paste0('https://fanwangecon.github.io/',
                     'R4Econ/function/mutatef/htmlpdf/fs_func_choice_states.html')
st_caption <- paste0('Evaluating the function, ',
                    'https://fanwangecon.github.io/R4Econ/')
st_x_label <- 'x values'
st_y_label <- 'f(x)'

# Show
dim(tb_states_choices_dense_eval)
```

```
## [1] 400 5
```

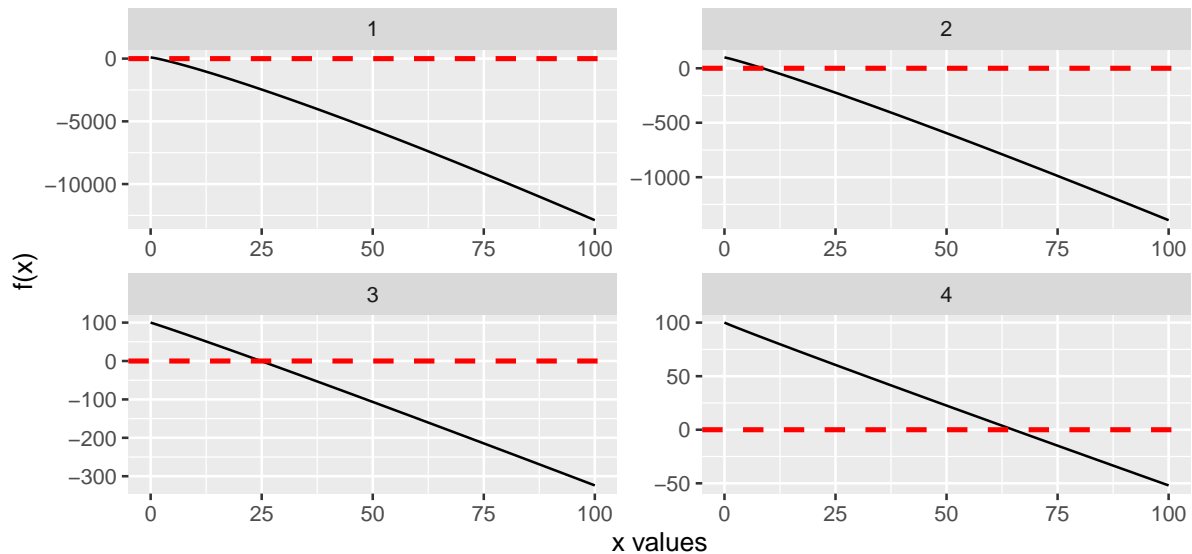
```
summary(tb_states_choices_dense_eval)
```

```
##      INDI_ID      fl_A      fl_alpha      fl_N      dplyr_eval
## Min.   :1.00  Min.   :-2    Min.   :0.1    Min.   : 0    Min.   : -12880.28
## 1st Qu.:1.75  1st Qu.: -1    1st Qu.:0.3    1st Qu.: 25    1st Qu.: -1167.29
## Median :2.50  Median : 0    Median :0.5    Median : 50    Median : -202.42
## Mean   :2.50  Mean   : 0    Mean   :0.5    Mean   : 50    Mean   : -1645.65
## 3rd Qu.:3.25  3rd Qu.: 1    3rd Qu.:0.7    3rd Qu.: 75    3rd Qu.: 0.96
## Max.   :4.00  Max.   : 2    Max.   :0.9    Max.   :100    Max.   : 100.00
```

```
lineplot <- tb_states_choices_dense_eval %>%
  ggplot(aes(x=fl_N, y=dplyr_eval)) +
    geom_line() +
    facet_wrap( . ~ INDI_ID, scales = "free") +
    geom_hline(yintercept=0, linetype="dashed",
              color = "red", size=1) +
    labs(title = st_title,
         subtitle = st_subtitle,
         x = st_x_label,
         y = st_y_label,
         caption = st_caption)
print(lineplot)
```

Evaluate Non-Linear Functions to Search for Roots

https://fanwangecon.github.io/R4Econ/function/mutatef/htmlpdf/fs_func_choice_states.html



Evaluating the function, <https://fanwangecon.github.io/R4Econ/>

3.2 Dataframe Do Anything

3.2.1 MxQ to MxP Rows

Go back to [fan's REconTools Package](#), [R4Econ Repository](#) ([bookdown site](#)), or [Intro Stats with R Repository](#).

3.2.1.1 MxQ to Mx1 Rows: Within Group Gini

There is a Panel with M individuals and each individual has Q records/rows. A function generate an individual specific outcome given the Q individual specific inputs, along with shared parameters and arrays across the M individuals.

For example, suppose we have a dataframe of individual wage information from different countries, each row is an individual from one country. We want to generate country specific gini based on the individual data for each country in the dataframe. But additionally, perhaps the gini formula requires not just individual income but some additional parameters or shared dataframes as inputs.

Given the within m income observations, we can compute gini statistics that are individual specific based on the observed distribution of incomes. For this, we will use the [ff_dist_gini_vector_pos.html](#) function from [REconTools](#).

To make this more interesting, we will generate large dataframe with more M and more Q each m .

3.2.1.1.1 Large Dataframe There are up to ten thousand income observation per person. And there are ten people.

```
# Parameter Setups
it_M <- 10
it_Q_max <- 10000
fl_rnorm_mu <- 1
ar_rnorm_sd <- seq(0.01, 0.2, length.out=it_M)
ar_it_q <- sample.int(it_Q_max, it_M, replace=TRUE)

# N by Q varying parameters
mt_data = cbind(ar_it_q, ar_rnorm_sd)
tb_M <- as_tibble(mt_data) %>% rowid_to_column(var = "ID") %>%
```



```
rename(sd = ar_rnorm_sd, Q = ar_it_q) %>%
mutate(mean = fl_rnorm_mu)
```

3.2.1.1.2 Compute Group specific gini, NORMAL There is only one input for the gini function `ar_pos`. Note that the gini are not very large even with large SD, because these are normal distributions. By Construction, most people are in the middle. So with almost zero standard deviation, we have perfect equality, as standard deviation increases, inequality increases, but still pretty equal overall, there is no fat upper tail.

Note that there are three ways of referring to variable names with dot, which are all shown below:

1. We can explicitly refer to names
2. We can use the [dollar dot structure](#) to use string variable names in do anything.
3. We can use dot bracket, this is the only option that works with string variable names

```
# A. Normal Draw Expansion, Explicitly Name
set.seed('123')
tb_income_norm_dot_dollar <- tb_M %>% group_by(ID) %>%
  do(income = rnorm(.$Q,
                    mean=.$mean,
                    sd=.$sd)) %>%
  unnest(c(income)) %>%
  left_join(tb_M, by="ID")

# Normal Draw Expansion again, dot dollar differently with string variable name
set.seed('123')
tb_income_norm_dollar_dot <- tb_M %>% group_by(ID) %>%
  do(income = rnorm(`$`(. , 'Q'),
                    mean = `$`(. , 'mean'),
                    sd = `$`(. , 'sd')) %>%
  unnest(c(income)) %>%
  left_join(tb_M, by="ID")

# Normal Draw Expansion again, dot double bracket
set.seed('123')
svr_mean <- 'mean'
svr_sd <- 'sd'
svr_Q <- 'Q'
tb_income_norm_dot_bracket_db <- tb_M %>% group_by(ID) %>%
  do(income = rnorm(.[[svr_Q]],
                    mean = .[[svr_mean]],
                    sd = .[[svr_sd]])) %>%
  unnest(c(income)) %>%
  left_join(tb_M, by="ID")

# display
sum(sum(tb_income_norm_dollar_dot - tb_income_norm_dot_dollar - tb_income_norm_dot_bracket_db))

## [1] -463785175

# display
head(tb_income_norm_dot_dollar, 20)

# Gini by Group
tb_gini_norm <- tb_income_norm_dollar_dot %>% group_by(ID) %>%
  do(inc_gini_norm = ff_dist_gini_vector_pos(.$income)) %>%
  unnest(c(inc_gini_norm)) %>%
  left_join(tb_M, by="ID")

# display
```

```
kable(tb_gini_norm) %>%
  kable_styling_fc()
```

ID	inc_gini_norm	Q	sd	mean
1	0.0056337	9982	0.0100000	1
2	0.0175280	2980	0.0311111	1
3	0.0293986	1614	0.0522222	1
4	0.0422304	555	0.0733333	1
5	0.0535146	4469	0.0944444	1
6	0.0653938	9359	0.1155556	1
7	0.0769135	7789	0.1366667	1
8	0.0894165	9991	0.1577778	1
9	0.1010982	9097	0.1788889	1
10	0.1124019	1047	0.2000000	1

3.2.2 Mx1 to MxQ Rows

Go back to [fan's REconTools Package](#), [R4Econ Repository \(bookdown site\)](#), or [Intro Stats with R Repository](#).

Case One: There is a dataframe with M rows, based on these m specific information, generate dataframes for each m . Stack these individual dataframes together and merge original m specific information in as well. The number of rows for each m is Q_m , each m could have different number of expansion rows.

Generate a panel with M individuals, each individual is observed for different spans of times (*uncount*). Before expanding, generate individual specific normal distribution standard deviation. All individuals share the same mean, but have increasing standard deviations.

3.2.2.1 Generate Dataframe with M Rows.

This is the first step, generate M rows of data, to be expanded. Each row contains the number of normal draws to make and the mean and the standard deviation for normal daraws that are m specific.

```
# Parameter Setups
it_M <- 3
it_Q_max <- 5
fl_rnorm_mu <- 1000
ar_rnorm_sd <- seq(0.01, 200, length.out=it_M)
ar_it_q <- sample.int(it_Q_max, it_M, replace=TRUE)

# N by Q varying parameters
mt_data = cbind(ar_it_q, ar_rnorm_sd)
tb_M <- as_tibble(mt_data) %>% rowid_to_column(var = "ID") %>%
  rename(sd = ar_rnorm_sd, Q = ar_it_q) %>%
  mutate(mean = fl_rnorm_mu)

# display
kable(tb_M) %>%
  kable_styling_fc()
```

ID	Q	sd	mean
1	3	0.010	1000
2	3	100.005	1000
3	1	200.000	1000

3.2.2.2 Random Normal Draw Expansion

The steps are:

1. `do anything`
2. use “.\$” sign to refer to variable names, or `[[‘name’]]`
3. `unnest`
4. `left_join` expanded and original

Note these all give the same results

Use dot dollar to get variables

```
# Generate $Q_m$ individual specific incomes, expanded different number of times for each m
tb_income <- tb_M %>% group_by(ID) %>%
  do(income = rnorm(.$Q, mean=.$mean, sd=.$sd)) %>%
  unnest(c(income))

# Merge back with tb_M
tb_income_full_dd <- tb_income %>%
  left_join(tb_M)

# display
kable(tb_income) %>%
  kable_styling_fc()
```

ID	income
1	1000.0183
1	999.9943
1	999.9822
2	1033.7465
2	1093.1374
2	862.1896
3	988.7742

```
kable(tb_income_full_dd) %>%
  kable_styling_fc()
```

ID	income	Q	sd	mean
1	1000.0183	3	0.010	1000
1	999.9943	3	0.010	1000
1	999.9822	3	0.010	1000
2	1033.7465	3	100.005	1000
2	1093.1374	3	100.005	1000
2	862.1896	3	100.005	1000
3	988.7742	1	200.000	1000

3.3 Apply and pmap

3.3.1 Apply, Sapply, Mutate

Go back to [fan's REconTools Package](#), [R4Econ Repository](#) ([bookdown site](#)), or [Intro Stats with R Repository](#).

- `r` apply matrix to function row by row
- `r` evaluate function on grid
- [Apply a function to every row of a matrix or a data frame](#)
- `r` apply
- `r` sapply
- `sapply` over matrix row by row
- `apply` dplyr vectorize
- function as parameters using formulas

- do

We want evaluate linear function $f(x_i, y_i, ar_x, ar_y, c, d)$, where c and d are constants, and ar_x and ar_y are arrays, both fixed. x_i and y_i vary over each row of matrix. More specifically, we have a functions, this function takes inputs that are individual specific. We would like to evaluate this function concurrently across N individuals.

The function is such that across the N individuals, some of the function parameter inputs are the same, but others are different. If we are looking at demand for a particular product, the prices of all products enter the demand equation for each product, but the product's own price enters also in a different way.

The objective is either to just evaluate this function across N individuals, or this is a part of a nonlinear solution system.

What is the relationship between apply, lapply and vectorization? see [Is the “*apply” family really not vectorized?](#).

3.3.1.1 Set up Input Arrays

There is a function that takes $M = Q + P$ inputs, we want to evaluate this function N times. Each time, there are M inputs, where all but Q of the M inputs, meaning P of the M inputs, are the same. In particular, $P = Q * N$.

$$M = Q + P = Q + Q * N$$

```
# it_child_count = N, the number of children
it_N_child_cnt = 5
# it_heter_param = Q, number of parameters that are
# heterogeneous across children
it_Q_hetpa_cnt = 2

# P fixed parameters, nN is N dimensional, nP is P dimensional
ar_nN_A = seq(-2, 2, length.out = it_N_child_cnt)
ar_nN_alpha = seq(0.1, 0.9, length.out = it_N_child_cnt)
ar_nP_A_alpha = c(ar_nN_A, ar_nN_alpha)

# N by Q varying parameters
mt_nN_by_nQ_A_alpha = cbind(ar_nN_A, ar_nN_alpha)

# display
kable(mt_nN_by_nQ_A_alpha) %>%
  kable_styling_fc()
```

ar_nN_A	ar_nN_alpha
-2	0.1
-1	0.3
0	0.5
1	0.7
2	0.9

3.3.1.2 Using apply

3.3.1.2.1 Apply with Named Function First we use the apply function, we have to hard-code the arrays that are fixed for each of the N individuals. Then apply allows us to loop over the matrix that is N by Q , each row one at a time, from 1 to N .

```
# Define Implicit Function
ffi_linear_hardcode <- function(ar_A_alpha){
  # ar_A_alpha[1] is A
  # ar_A_alpha[2] is alpha
```

```

fl_out = sum(ar_A_alpha[1]*ar_nN_A +
             1/(ar_A_alpha[2] + 1/ar_nN_alpha))

return(fl_out)
}

# Evaluate function row by row
ar_func_apply = apply(mt_nN_by_nQ_A_alpha, 1, ffi_linear_hardcode)

```

3.3.1.2.2 Apply using Anonymous Function

- apply over matrix

Apply with anonymous function generating a list of arrays of different lengths. In the example below, we want to draw N sets of random uniform numbers, but for each set the number of draws we want to have is Q_i . Furthermore, we want to rescale the random uniform draws so that they all become proportions that sum up to one for each i , but then we multiply each row's values by the row specific aggregates.

The anonymous function has hard coded parameters. Using an anonymous function here allows for parameters to be provided inside the function that are shared across each looped evaluation. This is perhaps more convenient than `sapply` with additional parameters.

```

set.seed(1039)

# Define the number of draws each row and total amount
it_N <- 4
fl_unif_min <- 1
fl_unif_max <- 2
mt_draw_define <- cbind(sample(it_N, it_N, replace=TRUE),
                        runif(it_N, min=1, max=10))
tb_draw_define <- as_tibble(mt_draw_define) %>%
  rowid_to_column(var = "draw_group")
print(tb_draw_define)

# apply row by row, anonymous function has hard
# coded min and max
ls_ar_draws_shares_lvls =
  apply(tb_draw_define,
        1,
        function(row) {
          it_draw <- row[2]
          fl_sum <- row[3]
          ar_unif <- runif(it_draw,
                          min=fl_unif_min,
                          max=fl_unif_max)
          ar_share <- ar_unif/sum(ar_unif)
          ar_levels <- ar_share*fl_sum
          return(list(ar_share=ar_share,
                     ar_levels=ar_levels))
        })

# Show Results
print(ls_ar_draws_shares_lvls)

## [[1]]
## [[1]]$ar_share
## [1] 0.2783638 0.2224140 0.2797840 0.2194381
##
## [[1]]$ar_levels

```

```
## [1] 1.492414 1.192446 1.500028 1.176491
##
##
## [[2]]
## [[2]]$ar_share
## [1] 0.5052919 0.4947081
##
## [[2]]$ar_levels
## [1] 3.866528 3.785541
##
##
## [[3]]
## [[3]]$ar_share
## [1] 1
##
## [[3]]$ar_levels
##      V2
## 9.572211
##
##
## [[4]]
## [[4]]$ar_share
## [1] 0.4211426 0.2909812 0.2878762
##
## [[4]]$ar_levels
## [1] 4.051971 2.799640 2.769765
```

We will try to do the same thing as above, but now the output will be a stacked dataframe. Note that within each element of the apply row by row loop, we are generating two variables *ar_share* and *ar_levels*. We will not generate a dataframe with multiple columns, storing *ar_share*, *ar_levels* as well as information on *min*, *max*, number of draws and rescale total sum.

```
set.seed(1039)
# apply row by row, anonymous function has hard coded min and max
ls_mt_draws_shares_lvls =
  apply(tb_draw_define, 1, function(row) {

    it_draw_group <- row[1]
    it_draw <- row[2]
    fl_sum <- row[3]

    ar_unif <- runif(it_draw,
                     min=fl_unif_min,
                     max=fl_unif_max)
    ar_share <- ar_unif/sum(ar_unif)
    ar_levels <- ar_share*fl_sum

    mt_all_res <- cbind(it_draw_group, it_draw, fl_sum,
                       ar_unif, ar_share, ar_levels)
    colnames(mt_all_res) <-
      c('draw_group', 'draw_count', 'sum',
        'unif_draw', 'share', 'rescale')
    rownames(mt_all_res) <- NULL

    return(mt_all_res)
  })
mt_draws_shares_lvls_all <- do.call(rbind, ls_mt_draws_shares_lvls)
# Show Results
kable(mt_draws_shares_lvls_all) %>% kable_styling_fc()
```

draw_group	draw_count	sum	unif_draw	share	rescale
1	4	5.361378	1.125668	0.1988606	1.066167
1	4	5.361378	1.668536	0.2947638	1.580340
1	4	5.361378	1.419382	0.2507483	1.344356
1	4	5.361378	1.447001	0.2556274	1.370515
2	2	7.652069	1.484598	0.4605236	3.523959
2	2	7.652069	1.739119	0.5394764	4.128110
3	1	9.572211	1.952468	1.0000000	9.572211
4	3	9.621375	1.957931	0.3609352	3.472693
4	3	9.621375	1.926995	0.3552324	3.417824
4	3	9.621375	1.539678	0.2838324	2.730858

3.3.1.3 Using sapply

3.3.1.3.1 sapply with named function

- r convert matrix to list
- Convert a matrix to a list of vectors in R

Sapply allows us to not have to hard code in the A and alpha arrays. But Sapply works over List or Vector, not Matrix. So we have to convert the N by Q matrix to a N element list. Now update the function with sapply.

```
ls_ar_nN_by_nQ_A_alpha = as.list(data.frame(t(mt_nN_by_nQ_A_alpha)))

# Define Implicit Function
ffi_linear_sapply <- function(ar_A_alpha, ar_A, ar_alpha){
  # ar_A_alpha[1] is A
  # ar_A_alpha[2] is alpha

  fl_out = sum(ar_A_alpha[1]*ar_nN_A +
              1/(ar_A_alpha[2] + 1/ar_nN_alpha))

  return(fl_out)
}

# Evaluate function row by row
ar_func_sapply = sapply(ls_ar_nN_by_nQ_A_alpha, ffi_linear_sapply,
                        ar_A=ar_nN_A, ar_alpha=ar_nN_alpha)
```

3.3.1.3.2 sapply using anonymous function

- sapply anonymous function
- r anonymous function multiple lines

Sapply with anonymous function generating a list of arrays of different lengths. In the example below, we want to draw N sets of random uniform numbers, but for each set the number of draws we want to have is Q_i . Furthermore, we want to rescale the random uniform draws so that they all become proportions that sum up to one for each i .

```
it_N <- 4
fl_unif_min <- 1
fl_unif_max <- 2

# Generate using runif without anonymous function
set.seed(1039)
ls_ar_draws = sapply(seq(it_N),
                     runif,
                     min=fl_unif_min, max=fl_unif_max)

print(ls_ar_draws)
```

```
## [[1]]
## [1] 1.125668
##
## [[2]]
## [1] 1.668536 1.419382
##
## [[3]]
## [1] 1.447001 1.484598 1.739119
##
## [[4]]
## [1] 1.952468 1.957931 1.926995 1.539678

# Generate Using Anonymous Function
set.seed(1039)
ls_ar_draws_shares = sapply(seq(it_N),
                             function(n, min, max) {
                               ar_unif <- runif(n,min,max)
                               ar_share <- ar_unif/sum(ar_unif)
                               return(ar_share)
                             },
                             min=fl_unif_min, max=fl_unif_max)

# Print Share
print(ls_ar_draws_shares)

## [[1]]
## [1] 1
##
## [[2]]
## [1] 0.5403432 0.4596568
##
## [[3]]
## [1] 0.3098027 0.3178522 0.3723451
##
## [[4]]
## [1] 0.2646671 0.2654076 0.2612141 0.2087113

# Sapply with anonymous function to check sums
sapply(seq(it_N), function(x) {sum(ls_ar_draws[[x]])})

## [1] 1.125668 3.087918 4.670717 7.377071

sapply(seq(it_N), function(x) {sum(ls_ar_draws_shares[[x]])})

## [1] 1 1 1 1
```

3.3.1.4 Using dplyr mutate rowwise

- dplyr mutate own function
- dplyr all row function
- dplyr do function
- apply function each row dplyr
- applying a function to every row of a table using dplyr
- dplyr rowwise

```
# Convert Matrix to Tibble
ar_st_col_names = c('fl_A', 'fl_alpha')
tb_nN_by_nQ_A_alpha <- as_tibble(mt_nN_by_nQ_A_alpha) %>%
  rename_all(~c(ar_st_col_names))
# Show
kable(tb_nN_by_nQ_A_alpha) %>%
  kable_styling_fc()
```


fl_A	fl_alpha
-2	0.1
-1	0.3
0	0.5
1	0.7
2	0.9

```
# Define Implicit Function
ffi_linear_dplyrdo <- function(fl_A, fl_alpha, ar_nN_A, ar_nN_alpha){
  # ar_A_alpha[1] is A
  # ar_A_alpha[2] is alpha

  print(paste0('cur row, fl_A=', fl_A, ', fl_alpha=', fl_alpha))
  fl_out = sum(fl_A*ar_nN_A + 1/(fl_alpha + 1/ar_nN_alpha))

  return(fl_out)
}

# Evaluate function row by row of tibble
# fl_A, fl_alpha are from columns of tb_nN_by_nQ_A_alpha
tb_nN_by_nQ_A_alpha_show <- tb_nN_by_nQ_A_alpha %>%
  rowwise() %>%
  mutate(dplyr_eval =
    ffi_linear_dplyrdo(fl_A, fl_alpha, ar_nN_A, ar_nN_alpha))

## [1] "cur row, fl_A=-2, fl_alpha=0.1"
## [1] "cur row, fl_A=-1, fl_alpha=0.3"
## [1] "cur row, fl_A=0, fl_alpha=0.5"
## [1] "cur row, fl_A=1, fl_alpha=0.7"
## [1] "cur row, fl_A=2, fl_alpha=0.9"

# Show
kable(tb_nN_by_nQ_A_alpha_show) %>%
  kable_styling_fc()
```

fl_A	fl_alpha	dplyr_eval
-2	0.1	2.346356
-1	0.3	2.094273
0	0.5	1.895316
1	0.7	1.733708
2	0.9	1.599477

same as before, still rowwise, but hard code some inputs:

```
# Define function, fixed inputs are not parameters, but
# defined earlier as a part of the function
# ar_nN_A, ar_nN_alpha are fixed, not parameters
ffi_linear_dplyrdo_func <- function(fl_A, fl_alpha){
  fl_out <- sum(fl_A*ar_nN_A + 1/(fl_alpha + 1/ar_nN_alpha))
  return(fl_out)
}

# Evaluate function row by row of tibble
tbfunc_A_nN_by_nQ_A_alpha_rowwise = tb_nN_by_nQ_A_alpha %>% rowwise() %>%
  mutate(dplyr_eval = ffi_linear_dplyrdo_func(fl_A, fl_alpha))

# Show
kable(tbfunc_A_nN_by_nQ_A_alpha_rowwise) %>%
  kable_styling_fc()
```

fl_A	fl_alpha	dplyr_eval
-2	0.1	2.346356
-1	0.3	2.094273
0	0.5	1.895316
1	0.7	1.733708
2	0.9	1.599477

3.3.1.5 Using Dplyr Mutate with Pmap

Apparently `rowwise()` is not a good idea, and `pmap` should be used, below is the `pmap` solution to the problem. Which does seem nicer. Crucially, don't have to define input parameter names, automatically I think they are matching up to the names in the function

- dplyr mutate pass function
- r function quosure string multiple
- r function multiple parameters as one string
- dplyr mutate anonymous function
- quosure style lambda
- pmap tibble rows
- dplyr pwalk

```
# Define function, fixed inputs are not parameters, but defined
# earlier as a part of the function Rorate fl_alpha and fl_A name
# compared to before to make sure pmap tracks by names
ffi_linear_dplyrdo_func <- function(fl_alpha, fl_A){
  fl_out <- sum(fl_A*ar_nN_A + 1/(fl_alpha + 1/ar_nN_alpha))
  return(fl_out)
}

# Evaluate a function row by row of dataframe, generate list,
# then to vector
tb_nN_by_nQ_A_alpha %>% pmap(ffi_linear_dplyrdo_func) %>% unlist()

## [1] 2.346356 2.094273 1.895316 1.733708 1.599477

# Same as above, but in line line and save output as new column
# in dataframe note this ONLY works if the tibble only has variables
# that are inputs for the function if tibble contains additional
# variables, those should be dropped, or only the ones needed selected,
# inside the pmap call below.
tbfunc_A_nN_by_nQ_A_alpha_pmap <- tb_nN_by_nQ_A_alpha %>%
  mutate(dplyr_eval_pmap =
    unlist(
      pmap(tb_nN_by_nQ_A_alpha, ffi_linear_dplyrdo_func)
    )
  )

# Show
kable(tbfunc_A_nN_by_nQ_A_alpha_pmap) %>%
  kable_styling_fc()
```

fl_A	fl_alpha	dplyr_eval_pmap
-2	0.1	2.346356
-1	0.3	2.094273
0	0.5	1.895316
1	0.7	1.733708
2	0.9	1.599477

3.3.1.6 DPLYR Three Types of Inputs ROWWISE

Now, we have three types of parameters, for something like a bisection type calculation. We will supply the program with a function with some hard-coded value inside, and as parameters, we will have one parameter which is a row in the current matrix, and another parameter which is a scalar values. The three types of parameters are dealt with separately:

1. parameters that are fixed for all bisection iterations, but differ for each row
 - these are hard-coded into the function
2. parameters that are fixed for all bisection iterations, but are shared across rows
 - these are the first parameter of the function, a list
3. parameters that differ for each iteration, but differ across iterations
 - second scalar value parameter for the function
 - dplyr mutate function apply to each row dot notation
 - note `rowwise` might be bad according to Hadley, should use `pmap`?

```
ffi_linear_dplyrdo_fdot <- function(ls_row, fl_param){
  # Type 1 Param = ar_nN_A, ar_nN_alpha
  # Type 2 Param = ls_row$fl_A, ls_row$fl_alpha
  # Type 3 Param = fl_param

  fl_out <- (sum(ls_row$fl_A*ar_nN_A +
                1/(ls_row$fl_alpha + 1/ar_nN_alpha))) + fl_param
  return(fl_out)
}

cur_func <- ffi_linear_dplyrdo_fdot
fl_param <- 0
dplyr_eval_flex <- tb_nN_by_nQ_A_alpha %>% rowwise() %>%
  do(dplyr_eval_flex = cur_func(., fl_param)) %>%
  unnest(dplyr_eval_flex)
tbfunc_B_nN_by_nQ_A_alpha <- tb_nN_by_nQ_A_alpha %>% add_column(dplyr_eval_flex)
# Show
kable(tbfunc_B_nN_by_nQ_A_alpha) %>%
  kable_styling_fc()
```

fl_A	fl_alpha	dplyr_eval_flex
-2	0.1	2.346356
-1	0.3	2.094273
0	0.5	1.895316
1	0.7	1.733708
2	0.9	1.599477

3.3.1.7 Compare Apply and Mutate Results

```
# Show overall Results
mt_results <- cbind(ar_func_apply, ar_func_sapply,
  tb_nN_by_nQ_A_alpha_show['dplyr_eval'],
  tbfunc_A_nN_by_nQ_A_alpha_rowwise['dplyr_eval'],
  tbfunc_A_nN_by_nQ_A_alpha_pmap['dplyr_eval_pmap'],
  tbfunc_B_nN_by_nQ_A_alpha['dplyr_eval_flex'],
  mt_nN_by_nQ_A_alpha)
colnames(mt_results) <- c('eval_lin_apply', 'eval_lin_sapply',
  'eval_dplyr_mutate',
  'eval_dplyr_mutate_hcode',
```

[illegible]

Chapter 4

Panel

4.1 Generate and Join

4.1.1 Generate Panel Structure

Go back to [fan's REconTools Package](#), [R4Econ Repository \(bookdown site\)](#), or [Intro Stats with R Repository](#).

4.1.1.1 Balanced Panel Skeleton

There are N individuals, each could be observed M times. In the example below, there are 3 students, each observed over 4 dates. This just uses the `uncount` function from *tidyr*.

```
# Define
it_N <- 3
it_M <- 5
svr_id <- 'student_id'
svr_date <- 'class_day'

# dataframe
df_panel_skeleton <- as_tibble(matrix(it_M, nrow=it_N, ncol=1)) %>%
  rowid_to_column(var = svr_id) %>%
  uncount(V1) %>%
  group_by(!!sym(svr_id)) %>% mutate(!!sym(svr_date) := row_number()) %>%
  ungroup()

# Print
kable(df_panel_skeleton) %>%
  kable_styling_fc()
```

4.1.2 Join Datasets

Go back to [fan's REconTools Package](#), [R4Econ Repository \(bookdown site\)](#), or [Intro Stats with R Repository](#).

4.1.2.1 Join Panel with Multiple Keys

We have two datasets, one for student enrollment, panel over time, but some students do not show up on some dates. The other is a skeleton panel with all student ID and all dates. Often we need to join dataframes together, and we need to join by the student ID and the panel time Key at the same time. When students show up, there is a quiz score for that day, so the joined panel should have as data column quiz score

student_id	class_day
1	1
1	2
1	3
1	4
1	5
2	1
2	2
2	3
2	4
2	5
3	1
3	2
3	3
3	4
3	5

Student count is N , total dates are M . First we generate two panels below, then we join by both keys using `left_join`. First, define dataframes:

```
# Define
it_N <- 4
it_M <- 3
svr_id <- 'sid'
svr_date <- 'classday'
svr_attend <- 'date_in_class'

# Panel Skeleton
df_panel_balanced_skeleton <- as_tibble(matrix(it_M, nrow=it_N, ncol=1)) %>%
  rowid_to_column(var = svr_id) %>%
  uncount(V1) %>%
  group_by(!!sym(svr_id)) %>% mutate(!!sym(svr_date) := row_number()) %>%
  ungroup()

# Print
kable(df_panel_balanced_skeleton) %>%
  kable_styling_fc()
```

sid	classday
1	1
1	2
1	3
2	1
2	2
2	3
3	1
3	2
3	3
4	1
4	2
4	3

```
# Smaller Panel of Random Days in School
set.seed(456)
df_panel_attend <- as_tibble(matrix(it_M, nrow=it_N, ncol=1)) %>%
  rowid_to_column(var = svr_id) %>%
  uncount(V1) %>%
  group_by(!!sym(svr_id)) %>% mutate(!!sym(svr_date) := row_number()) %>%
```

```

ungroup() %>% mutate(in_class = case_when(rnorm(n(),mean=0,sd=1) < 0 ~ 1, TRUE ~ 0)) %>%
filter(in_class == 1) %>% select(!!sym(svr_id), !!sym(svr_date)) %>%
rename(!!sym(svr_attend) := !!sym(svr_date)) %>%
mutate(dayquizscore = rnorm(n(),mean=80,sd=10))
# Print
kable(df_panel_attend) %>%
  kable_styling_fc()

```

sid	date_in_class	dayquizscore
1	1	89.88726
2	1	96.53929
2	2	65.59195
2	3	99.47356
4	2	97.36936

Second, now join dataframes:

```

# Join with explicit names
df_quiz_joined_multikey <- df_panel_balanced_skeleton %>%
  left_join(df_panel_attend,
    by=(c('sid'='sid', 'classday'='date_in_class')))

# Join with setname strings
df_quiz_joined_multikey_setnames <- df_panel_balanced_skeleton %>%
  left_join(df_panel_attend, by=setNames(c('sid', 'date_in_class'), c('sid', 'classday')))

# Print
kable(df_quiz_joined_multikey) %>%
  kable_styling_fc()

```

sid	classday	dayquizscore
1	1	89.88726
1	2	NA
1	3	NA
2	1	96.53929
2	2	65.59195
2	3	99.47356
3	1	NA
3	2	NA
3	3	NA
4	1	NA
4	2	97.36936
4	3	NA

```

kable(df_quiz_joined_multikey_setnames) %>%
  kable_styling_fc()

```

4.1.2.2 Stack Panel Frames Together

There are multiple panel dataframe, each for different subsets of dates. All variable names and units of observations are identical. Use DPLYR `bind_rows`.

```

# Define
it_N <- 2 # Number of individuals
it_M <- 3 # Number of Months
svr_id <- 'sid'
svr_date <- 'date'

```

sid	classday	dayquizscore
1	1	89.88726
1	2	NA
1	3	NA
2	1	96.53929
2	2	65.59195
2	3	99.47356
3	1	NA
3	2	NA
3	3	NA
4	1	NA
4	2	97.36936
4	3	NA

```

# Panel First Half of Year
df_panel_m1tom3 <- as_tibble(matrix(it_M, nrow=it_N, ncol=1)) %>%
  rowid_to_column(var = svr_id) %>%
  uncount(V1) %>%
  group_by(!!sym(svr_id)) %>% mutate(!!sym(svr_date) := row_number()) %>%
  ungroup()

# Panel Second Half of Year
df_panel_m4tom6 <- as_tibble(matrix(it_M, nrow=it_N, ncol=1)) %>%
  rowid_to_column(var = svr_id) %>%
  uncount(V1) %>%
  group_by(!!sym(svr_id)) %>% mutate(!!sym(svr_date) := row_number() + 3) %>%
  ungroup()

# Bind Rows
df_panel_m1tm6 <- bind_rows(df_panel_m1tom3, df_panel_m4tom6) %>% arrange(!!!syms(c(svr_id, svr_date)))

# Print
kable(df_panel_m1tom3) %>%
  kable_styling_fc()

```

sid	date
1	1
1	2
1	3
2	1
2	2
2	3

```

kable(df_panel_m4tom6) %>%
  kable_styling_fc()

```

sid	date
1	4
1	5
1	6
2	4
2	5
2	6


```
kable(df_panel_m1tm6) %>%
  kable_styling_fc()
```

sid	date
1	1
1	2
1	3
1	4
1	5
1	6
2	1
2	2
2	3
2	4
2	5
2	6

4.2 Wide and Long

4.2.1 Long to Wide

Go back to [fan's REconTools Package](#), [R4Econ Repository \(bookdown site\)](#), or [Intro Stats with R Repository](#).

Using the `pivot_wider` function in `tidyr` to reshape panel or other data structures

4.2.1.1 Panel Long Attendance Roster to Wide

There are N students in class, but only a subset of them attend class each day. If student id_i is in class on day Q , the teacher records on a sheet the date and the student ID. So if the student has been in class 10 times, the teacher has ten rows of recorded data for the student with two columns: column one is the student ID, and column two is the date on which the student was in class. Suppose there were 50 students, who on average attended exactly 10 classes each during the semester, this means we have $10 \cdot 50$ rows of data, with differing numbers of rows for each student. This is shown as `df_panel_attend_date` generated below.

Now we want to generate a new dataframe, where each row is a date, and each column is a student. The values in the new dataframe shows, at the Q^{th} day, how many classes student i has attended so far. The following results is also in a REconTools Function. This is shown as `df_attend_cumu_by_day` generated below.

First, generate the raw data structure, `df_panel_attend_date`:

```
# Define
it_N <- 3
it_M <- 5
svr_id <- 'student_id'

# from : support/rand/fs_rand_draws.Rmd
set.seed(222)
df_panel_attend_date <- as_tibble(matrix(it_M, nrow=it_N, ncol=1)) %>%
  rowid_to_column(var = svr_id) %>%
  uncount(V1) %>%
  group_by(!sym(svr_id)) %>% mutate(date = row_number()) %>%
  ungroup() %>% mutate(in_class = case_when(rnorm(n(), mean=0, sd=1) < 0 ~ 1, TRUE ~ 0)) %>%
  filter(in_class == 1) %>% select(!sym(svr_id), date) %>%
  rename(date_in_class = date)
```

```
# Print
kable(df_panel_attend_date) %>%
  kable_styling_fc()
```

student_id	date_in_class
1	2
1	4
2	1
2	2
2	5
3	2
3	3
3	5

Second, generate wider data structure, *df_attend_cumu_by_day*:

```
# Define
svr_id <- 'student_id'
svr_date <- 'date_in_class'
st_idcol_prefix <- 'sid_'

# Generate cumulative enrollment counts by date
df_panel_attend_date_addone <- df_panel_attend_date %>% mutate(attended = 1)
kable(df_panel_attend_date_addone) %>%
  kable_styling_fc()
```

student_id	date_in_class	attended
1	2	1
1	4	1
2	1	1
2	2	1
2	5	1
3	2	1
3	3	1
3	5	1

```
# Pivot Wide
df_panel_attend_date_wider <- df_panel_attend_date_addone %>%
  pivot_wider(names_from = svr_id,
              values_from = attended)
kable(df_panel_attend_date_wider) %>%
  kable_styling_fc()
```

date_in_class	1	2	3
2	1	1	1
4	1	NA	NA
1	NA	1	NA
5	NA	1	1
3	NA	NA	1

```
# Sort and rename
# rename see: https://fanwangecon.github.io/R4Econ/amto/tibble/fs\_tib\_basics.html
ar_unique_ids <- sort(unique(df_panel_attend_date %>% pull(!sym(svr_id))))
df_panel_attend_date_wider_sort <- df_panel_attend_date_wider %>%
  arrange(!sym(svr_date)) %>%
  rename_at(vars(num_range(' ', ar_unique_ids))
            , list(~paste0(st_idcol_prefix, . , ' ')))
```

```
)
kable(df_panel_attend_date_wider_sort) %>%
  kable_styling_fc()
```

date_in_class	sid_1	sid_2	sid_3
1	NA	1	NA
2	1	1	1
3	NA	NA	1
4	1	NA	NA
5	NA	1	1

```
# replace NA and cumusum again
# see: R4Econ/support/function/fs_func_multivar for renaming and replacing
df_attend_cumu_by_day <- df_panel_attend_date_wider_sort %>%
  mutate_at(vars(contains(st_idcol_prefix)), list(~replace_na(., 0))) %>%
  mutate_at(vars(contains(st_idcol_prefix)), list(~cumsum(.)))

kable(df_attend_cumu_by_day) %>%
  kable_styling_fc()
```

date_in_class	sid_1	sid_2	sid_3
1	0	1	0
2	1	2	1
3	1	2	2
4	2	2	2
5	2	3	3

The structure above is also a function in Fan's [REconTools](#) Package, here the function is tested:

```
# Parameters
df <- df_panel_attend_date
svr_id_i <- 'student_id'
svr_id_t <- 'date_in_class'
st_idcol_prefix <- 'sid_'

# Invoke Function
ls_df_rosterwide <- ff_panel_expand_longrosterwide(df, svr_id_t, svr_id_i, st_idcol_prefix)
df_rosterwide_func <- ls_df_rosterwide$df_rosterwide
df_rosterwide_cumu_func <- ls_df_rosterwide$df_rosterwide_cumu

# Print
print(df_rosterwide_func)
print(df_rosterwide_cumu_func)
```


Chapter 5

Linear Regression

5.1 OLS and IV

Back to [fan's R4Econ Homepage](#) [Table of Content](#)

5.1.1 OLS and IV Regression

Go back to [fan's REconTools](#) Package, [R4Econ](#) Repository ([bookdown site](#)), or [Intro Stats with R](#) Repository.

IV regression using AER package. Option to store all results in dataframe row for combining results from other estimations together. Produce Row Statistics.

5.1.1.1 Construct Program

```
# IV regression function
# The code below uses the AER library's regresison function
# All results are stored in a single row as data_frame
# This functoin could work with dplyr do
# var.y is single outcome, vars.x, vars.c and vars.z are vectors of endogenous variables, controls a
regf.iv <- function(var.y, vars.x,
                    vars.c, vars.z, df, transpose=TRUE) {

  # A. Set-Up Equation
  str.vars.x <- paste(vars.x, collapse='+')
  str.vars.c <- paste(vars.c, collapse='+')

  df <- df %>%
    select(one_of(var.y, vars.x, vars.c, vars.z)) %>%
    drop_na() %>% filter_all(all_vars(!is.infinite(.)))

  if (length(vars.z) >= 1) {
    # library(AER)
    str.vars.z <- paste(vars.z, collapse='+')
    equa.iv <- paste(var.y,
                     paste(paste(str.vars.x, str.vars.c, sep='+'),
                           paste(str.vars.z, str.vars.c, sep='+'),
                           sep='|'),
                     sep='~')

    # print(equa.iv)

    # B. IV Regression
    ivreg.summ <- summary(ivreg(as.formula(equa.iv), data=df),
```

```

vcov = sandwich, df = Inf, diagnostics = TRUE)

# C. Statistics from IV Regression
#   ivreg.summ$coef
#   ivreg.summ$diagnostics

# D. Combine Regression Results into a Matrix
df.results <- suppressWarnings(suppressMessages(
  as_tibble(ivreg.summ$coef, rownames='rownames') %>%
    full_join(as_tibble(ivreg.summ$diagnostics, rownames='rownames')) %>%
    full_join(tibble(rownames=c('vars'),
                      var.y=var.y,
                      vars.x=str.vars.x,
                      vars.z=str.vars.z,
                      vars.c=str.vars.c))))
} else {

  # OLS regression
  equa.ols <- paste(var.y,
                   paste(paste(vars.x, collapse='+'),
                         paste(vars.c, collapse='+'), sep='+'),
                   sep='~')

  lmreg.summ <- summary(lm(as.formula(equa.ols), data=df))

  lm.diagnostics <- as_tibble(
    list(df1=lmreg.summ$df[[1]],
         df2=lmreg.summ$df[[2]],
         df3=lmreg.summ$df[[3]],
         sigma=lmreg.summ$sigma,
         r.squared=lmreg.summ$r.squared,
         adj.r.squared=lmreg.summ$adj.r.squared)) %>%
    gather(variable, value) %>%
    rename(rownames = variable) %>%
    rename(v = value)

  df.results <- suppressWarnings(suppressMessages(
    as_tibble(lmreg.summ$coef, rownames='rownames') %>%
      full_join(lm.diagnostics) %>%
      full_join(tibble(rownames=c('vars'),
                        var.y=var.y,
                        vars.x=str.vars.x,
                        vars.c=str.vars.c))))
}

# E. Flatten Matrix, All IV results as a single tibble
# row to be combined with other IV results
df.row.results <- df.results %>%
  gather(variable, value, -rownames) %>%
  drop_na() %>%
  unite(esti.val, rownames, variable) %>%
  mutate(esti.val = gsub(' ', '', esti.val))

if (transpose) {
  df.row.results <- df.row.results %>% spread(esti.val, value)
}

```

```
# F. Return
return(data.frame(df.row.results))
}
```

5.1.1.2 Program Testing

Load Data

```
# Library
library(tidyverse)
library(AER)

# Load Sample Data
setwd('C:/Users/fan/R4Econ/_data/')
df <- read_csv('height_weight.csv')

# One Instrucments
var.y <- c('hgt')
vars.x <- c('prot')
vars.z <- NULL
vars.c <- c('sex', 'hgt0', 'wgt0')
# Regression
regf.iv(var.y, vars.x, vars.c, vars.z, df, transpose=FALSE) %>%
  kable() %>%
  kable_styling_fc()
```

esti.val	value
(Intercept)_Estimate	52.1186286658651
prot_Estimate	0.374472386357917
sexMale_Estimate	0.611043720578292
hgt0_Estimate	0.148513781160842
wgt0_Estimate	0.00150560230505631
(Intercept)_Std.Error	1.57770483608693
prot_Std.Error	0.00418121191133815
sexMale_Std.Error	0.118396259120659
hgt0_Std.Error	0.0393807494783186
wgt0_Std.Error	0.000187123663624397
(Intercept)_tvalue	33.0344608660332
prot_tvalue	89.5607288744356
sexMale_tvalue	5.16100529794248
hgt0_tvalue	3.77122790013449
wgt0_tvalue	8.04602836377991
(Intercept)_Pr(> t)	9.92126150975783e-233
prot_Pr(> t)	0
sexMale_Pr(> t)	2.48105505495642e-07
hgt0_Pr(> t)	0.000162939618371183
wgt0_Pr(> t)	9.05257561534111e-16
df1_v	5
df2_v	18958
df3_v	5
sigma_v	8.06197784622979
r.squared_v	0.319078711001325
adj.r.squared_v	0.318935041565942
vars_var.y	hgt
vars_vars.x	prot
vars_vars.c	sex+hgt0+wgt0

5.1.1.2.1 Example No Instrument, OLS

```
# One Instrumcments
var.y <- c('hgt')
vars.x <- c('prot')
vars.z <- c('momEdu')
vars.c <- c('sex', 'hgt0', 'wgt0')
# Regression
regf.iv(var.y, vars.x, vars.c, vars.z, df, transpose=FALSE) %>%
  kable() %>%
  kable_styling_fc()
```

esti.val	value
(Intercept)_Estimate	43.4301969117558
prot_Estimate	0.130833343849446
sexMale_Estimate	0.868121847262411
hgt0_Estimate	0.412093881817148
wgt0_Estimate	0.000858630042617921
(Intercept)_Std.Error	1.82489550971182
prot_Std.Error	0.0192036220809189
sexMale_Std.Error	0.13373016700542
hgt0_Std.Error	0.0459431912927002
wgt0_Std.Error	0.00022691057702563
(Intercept)_zvalue	23.798730766023
prot_zvalue	6.81295139521853
sexMale_zvalue	6.49159323361366
hgt0_zvalue	8.96963990141069
wgt0_zvalue	3.7840018472164
(Intercept)_Pr(> z)	3.4423766196876e-125
prot_Pr(> z)	9.56164541643828e-12
sexMale_Pr(> z)	8.49333228172763e-11
hgt0_Pr(> z)	2.97485394526792e-19
wgt0_Pr(> z)	0.000154326676608523
Weakinstruments_df1	1
Wu-Hausman_df1	1
Sargan_df1	0
Weakinstruments_df2	16394
Wu-Hausman_df2	16393
Weakinstruments_statistic	935.817456612075
Wu-Hausman_statistic	123.595856606729
Weakinstruments_p-value	6.39714929178024e-200
Wu-Hausman_p-value	1.30703637796748e-28
vars_var.y	hgt
vars_vars.x	prot
vars_vars.z	momEdu
vars_vars.c	sex+hgt0+wgt0

5.1.1.2.2 Example 1 Instrument

```
# Multiple Instrumcments
var.y <- c('hgt')
vars.x <- c('prot')
vars.z <- c('momEdu', 'wealthIdx', 'p.A.prot', 'p.A.nProt')
vars.c <- c('sex', 'hgt0', 'wgt0')
# Regression
```



```
regf.iv(var.y, vars.x, vars.c, vars.z, df, transpose=FALSE) %>%
  kable() %>%
  kable_styling_fc()
```

esti.val	value
(Intercept)_Estimate	42.2437613555242
prot_Estimate	0.26699945194704
sexMale_Estimate	0.695548488812932
hgt0_Estimate	0.424954881263031
wgt0_Estimate	0.000486951420329484
(Intercept)_Std.Error	1.85356686789642
prot_Std.Error	0.0154939347964083
sexMale_Std.Error	0.133157977814374
hgt0_Std.Error	0.0463195803786233
wgt0_Std.Error	0.000224867994873235
(Intercept)_zvalue	22.7905246296649
prot_zvalue	17.2325142357597
sexMale_zvalue	5.22348341593581
hgt0_zvalue	9.17441129192849
wgt0_zvalue	2.16549901022595
(Intercept)_Pr(> z)	5.69294074735747e-115
prot_Pr(> z)	1.51424021931607e-66
sexMale_Pr(> z)	1.75588197502565e-07
hgt0_Pr(> z)	4.54048595587756e-20
wgt0_Pr(> z)	0.030349491114332
Weakinstruments_df1	4
Wu-Hausman_df1	1
Sargan_df1	3
Weakinstruments_df2	14914
Wu-Hausman_df2	14916
Weakinstruments_statistic	274.147084958343
Wu-Hausman_statistic	17.7562545747101
Sargan_statistic	463.729664547249
Weakinstruments_p-value	8.61731956233366e-228
Wu-Hausman_p-value	2.52567249124181e-05
Sargan_p-value	3.45452874915475e-100
vars_var.y	hgt
vars_vars.x	prot
vars_vars.z	momEdu+wealthIdx+p.A.prot+p.A.nProt
vars_vars.c	sex+hgt0+wgt0

5.1.1.2.3 Example Multiple Instrucments

```
# Multiple Instrucments
var.y <- c('hgt')
vars.x <- c('prot', 'cal')
vars.z <- c('momEdu', 'wealthIdx', 'p.A.prot', 'p.A.nProt')
vars.c <- c('sex', 'hgt0', 'wgt0')
# Regression
regf.iv(var.y, vars.x, vars.c, vars.z, df, transpose=FALSE) %>%
  kable() %>%
  kable_styling_fc()
```

5.1.1.2.4 Example Multiple Endogenous Variables

esti.val	value
(Intercept)_Estimate	44.0243196254297
prot_Estimate	-1.4025623247106
cal_Estimate	0.065104895750151
sexMale_Estimate	0.120832787571818
hgt0_Estimate	0.286525437984517
wgt0_Estimate	0.000850481389651033
(Intercept)_Std.Error	2.75354847244082
prot_Std.Error	0.198640060273635
cal_Std.Error	0.00758881298880996
sexMale_Std.Error	0.209984580636303
hgt0_Std.Error	0.0707828182888255
wgt0_Std.Error	0.00033711210444429
(Intercept)_zvalue	15.9882130516502
prot_zvalue	-7.06082309267581
cal_zvalue	8.57906181719737
sexMale_zvalue	0.575436478267434
hgt0_zvalue	4.04795181812859
wgt0_zvalue	2.52284441418383
(Intercept)_Pr(> z)	1.54396598126854e-57
prot_Pr(> z)	1.65519210848649e-12
cal_Pr(> z)	9.56500648203187e-18
sexMale_Pr(> z)	0.564996139463599
hgt0_Pr(> z)	5.16677787108928e-05
wgt0_Pr(> z)	0.0116409892837831
Weakinstruments(prot)_df1	4
Weakinstruments(cal)_df1	4
Wu-Hausman_df1	2
Sargan_df1	2
Weakinstruments(prot)_df2	14914
Weakinstruments(cal)_df2	14914
Wu-Hausman_df2	14914
Weakinstruments(prot)_statistic	274.147084958343
Weakinstruments(cal)_statistic	315.036848606231
Wu-Hausman_statistic	94.7020085425169
Sargan_statistic	122.081979628898
Weakinstruments(prot)_p-value	8.61731956233366e-228
Weakinstruments(cal)_p-value	1.18918641220866e-260
Wu-Hausman_p-value	1.35024050408262e-41
Sargan_p-value	3.09196773720398e-27
vars_var.y	hgt
vars_vars.x	prot+cal
vars_vars.z	momEdu+wealthIdx+p.A.prot+p.A.nProt
vars_vars.c	sex+hgt0+wgt0

5.1.1.2.5 Examples Line by Line The examples are just to test the code with different types of variables.

```
# Selecting Variables
var.y <- c('hgt')
vars.x <- c('prot', 'cal')
vars.z <- c('momEdu', 'wealthIdx', 'p.A.prot', 'p.A.nProt')
vars.c <- c('sex', 'hgt0', 'wgt0')
```

```
# A. create Equation
str.vars.x <- paste(vars.x, collapse='+')
str.vars.c <- paste(vars.c, collapse='+')
```

```

str.vars.z <- paste(vars.z, collapse='+')
print(str.vars.x)

## [1] "prot+cal"
print(str.vars.c)

## [1] "sex+hgt0+wgt0"
print(str.vars.z)

## [1] "momEdu+wealthIdx+p.A.prot+p.A.nProt"
equa.iv <- paste(var.y,
                 paste(paste(str.vars.x, str.vars.c, sep='+'),
                       paste(str.vars.z, str.vars.c, sep='+'),
                       sep='|'),
                 sep='~')
print(equa.iv)

## [1] "hgt~prot+cal+sex+hgt0+wgt0|momEdu+wealthIdx+p.A.prot+p.A.nProt+sex+hgt0+wgt0"
# B. regression
res.ivreg <- ivreg(as.formula(equa.iv), data=df)
coef(res.ivreg)

##      (Intercept)      prot      cal      sexMale      hgt0      wgt0
## 44.0243196254 -1.4025623247  0.0651048958  0.1208327876  0.2865254380  0.0008504814
# C. Regression Summary
ivreg.summ <- summary(res.ivreg, vcov = sandwich, df = Inf, diagnostics = TRUE)

ivreg.summ$coef

##              Estimate   Std. Error   z value   Pr(>|z|)
## (Intercept) 44.0243196254 2.7535484724 15.9882131 1.543966e-57
## prot        -1.4025623247 0.1986400603 -7.0608231 1.655192e-12
## cal          0.0651048958 0.0075888130  8.5790618 9.565006e-18
## sexMale      0.1208327876 0.2099845806  0.5754365 5.649961e-01
## hgt0          0.2865254380 0.0707828183  4.0479518 5.166778e-05
## wgt0          0.0008504814 0.0003371121  2.5228444 1.164099e-02
## attr(,"df")
## [1] 0

ivreg.summ$diagnostics

##              df1   df2 statistic      p-value
## Weak instruments (prot)  4 14914 274.14708 8.617320e-228
## Weak instruments (cal)  4 14914 315.03685 1.189186e-260
## Wu-Hausman              2 14914  94.70201  1.350241e-41
## Sargan                  2    NA 122.08198  3.091968e-27
# D. Combine Regression Results into a Matrix
df.results <- suppressMessages(as_tibble(ivreg.summ$coef, rownames='rownames') %>%
  full_join(as_tibble(ivreg.summ$diagnostics, rownames='rownames')) %>%
  full_join(tibble(rownames=c('vars'),
                    var.y=var.y,
                    vars.x=str.vars.x,
                    vars.z=str.vars.z,
                    vars.c=str.vars.c)))
# E. Flatten Matrix, All IV results as a single tibble row to be combined with other IV results
df.row.results <- df.results %>%
  gather(variable, value, -rownames) %>%

```

```

drop_na() %>%
unite(esti.val, rownames, variable) %>%
mutate(esti.val = gsub(' ', '', esti.val))

# F. Results as Single Column
# df.row.results

# G. Results as Single Row
# df.row.results

# t(df.row.results %>% spread(esti.val, value)) %>%
# kable() %>%
# kable_styling_fc_wide()

```

5.1.2 IV Loop over RHS

Go back to [fan's REconTools](#) Package, [R4Econ](#) Repository ([bookdown site](#)), or [Intro Stats with R](#) Repository.

Regression with a Variety of Outcome Variables and Right Hand Side Variables. There are M outcome variables, and there are N alternative right hand side variables. Regress each M outcome variable and each N alternative right hand side variable, with some common sets of controls and perhaps shared instruments. The output file is a M by N matrix of coefficients, with proper variable names and row names. The matrix stores coefficients for this key endogenous variable.

- Dependency: *R4Econ/linreg/ivreg/ivregdfrow.R*

5.1.2.1 Construct Program

The program relies on double lapply. lapply is used for convenience, not speed.

```

ff_reg_mbyn <- function(list.vars.y, list.vars.x,
                        vars.c, vars.z, df,
                        return_all = FALSE,
                        stats_ends = 'value', time = FALSE) {

  # regf.iv() function is from C:\Users\fan\R4Econ\linreg\ivreg\ivregdfrow.R
  if (time) {
    start_time <- Sys.time()
  }

  if (return_all) {
    df.reg.out.all <-
      bind_rows(lapply(list.vars.x,
                        function(x) (
                          bind_rows(
                            lapply(list.vars.y, regf.iv,
                                  vars.x=x, vars.c=vars.c, vars.z=vars.z, df=df))
                          )))
  } else {
    df.reg.out.all <-
      (lapply(list.vars.x,
               function(x) (
                 bind_rows(
                   lapply(list.vars.y, regf.iv,
                         vars.x=x, vars.c=vars.c, vars.z=vars.z, df=df)) %>%
                   select(vars_var.y, starts_with(x)) %>%
                   select(vars_var.y, ends_with(stats_ends))
                 ))) %>% reduce(full_join)
  }
}

```

```

}

if (time) {
  end_time <- Sys.time()
  print(paste0('Estimation for all ys and xs took (seconds):',
              end_time - start_time))
}

return(df.reg.out.all)
}

```

5.1.2.2 Prepare Data

```

# Library
library(tidyverse)
library(AER)

# Load Sample Data
setwd('C:/Users/fan/R4Econ/_data/')
df <- read_csv('height_weight.csv')

# Source Dependency
source('C:/Users/fan/R4Econ/linreg/ivreg/ivregdfrow.R')

# Setting
options(repr.matrix.max.rows=50, repr.matrix.max.cols=50)

```

Parameters.

```

var.y1 <- c('hgt')
var.y2 <- c('wgt')
var.y3 <- c('vil.id')
list.vars.y <- c(var.y1, var.y2, var.y3)

var.x1 <- c('prot')
var.x2 <- c('cal')
var.x3 <- c('wealthIdx')
var.x4 <- c('p.A.prot')
var.x5 <- c('p.A.nProt')
list.vars.x <- c(var.x1, var.x2, var.x3, var.x4, var.x5)

vars.z <- c('indi.id')
vars.c <- c('sex', 'wgt0', 'hgt0', 'svymthRound')

```

5.1.2.3 Program Testing

```

vars.z <- NULL
suppressWarnings(suppressMessages(
  ff_reg_mbyn(list.vars.y, list.vars.x,
              vars.c, vars.z, df,
              return_all = FALSE,
              stats_ends = 'value')))) %>%
  kable() %>%
  kable_styling_fc_wide()

```

5.1.2.3.1 Test Program OLS Z-Stat

vars_var.y	prot_tvalue	cal_tvalue	wealthIdx_tvalue	p.A.prot_tvalue	p.A.nProt_tvalue
hgt	18.8756010031786	23.4421863484661	13.508899618216	3.83682180045518	32.5448257554855
wgt	16.3591125056062	17.3686031309332	14.1390521528113	1.36958319982295	12.0961557911467
vil.id	-14.9385580468907	-19.6150110809452	34.0972558327347	8.45943342783186	17.7801422421419

```
vars.z <- c('indi.id')
suppressWarnings(suppressMessages(
  ff_reg_mbyn(list.vars.y, list.vars.x,
    vars.c, vars.z, df,
    return_all = FALSE,
    stats_ends = 'value')))) %>%
kable() %>%
kable_styling_fc_wide()
```

vars_var.y	prot_zvalue	cal_zvalue	wealthIdx_zvalue	p.A.prot_zvalue	p.A.nProt_zvalue
hgt	8.87674929300964	12.0739764947235	4.62589553677969	26.6373587567312	32.1162192385744
wgt	5.60385871756365	6.1225187008946	5.17869536991717	11.9295584469998	12.3509307017263
vil.id	-9.22106223347162	-13.0586007975839	-51.5866689219593	-29.9627476577329	-38.3528894620707

5.1.2.3.2 Test Program IV T-stat

```
vars.z <- NULL
suppressWarnings(suppressMessages(
  ff_reg_mbyn(list.vars.y, list.vars.x,
    vars.c, vars.z, df,
    return_all = FALSE,
    stats_ends = 'Estimate')))) %>%
kable() %>%
kable_styling_fc_wide()
```

vars_var.y	prot_Estimate	cal_Estimate	wealthIdx_Estimate	p.A.prot_Estimate	p.A.nProt_Estimate
hgt	0.049431093806755	0.00243408846205622	0.21045655488185	3.86952250259526e-05	0.00542428867316449
wgt	16.5557424523585	0.699072500364623	106.678721085969	0.00521731297924587	0.779514232050632
vil.id	-0.0758835879205584	-0.00395676177098486	0.451733304543324	0.000149388430455142	0.00526237555581024

5.1.2.3.3 Test Program OLS Coefficient

```
vars.z <- c('indi.id')
suppressWarnings(suppressMessages(
  ff_reg_mbyn(list.vars.y, list.vars.x,
    vars.c, vars.z, df,
    return_all = FALSE,
    stats_ends = 'Estimate')))) %>%
kable() %>%
kable_styling_fc_wide()
```

vars_var.y	prot_Estimate	cal_Estimate	wealthIdx_Estimate	p.A.prot_Estimate	p.A.nProt_Estimate
hgt	0.859205733632614	0.0238724384575419	0.144503490136948	0.00148073028434642	0.0141317656200726
wgt	98.9428234201406	2.71948246216953	69.1816142883022	0.221916473012486	2.11856940494335
vil.id	-6.02451379136132	-0.168054407187466	-1.91414470908345	-0.00520794333267238	-0.0494468877742109

5.1.2.3.4 Test Program IV coefficient

```
vars.z <- NULL
t(suppressWarnings(suppressMessages(
```

```
ff_reg_mbyn(list.vars.y, list.vars.x,
            vars.c, vars.z, df,
            return_all = TRUE,
            stats_ends = 'Estimate')))) %>%
kable() %>%
kable_styling_fc_wide()
```

```
lapply(list.vars.y, function(y) (mean(df[[var.x1]], na.rm=TRUE) +
                                mean(df[[y]], na.rm=TRUE)))
```

5.1.2.4.2 Nested Lapply Test

```
## [[1]]
## [1] 98.3272
##
## [[2]]
## [1] 13626.51
##
## [[3]]
## [1] 26.11226

lapplytwice <- lapply(
  list.vars.x, function(x) (
    lapply(list.vars.y, function(y) (mean(df[[x]], na.rm=TRUE) +
                                    mean(df[[y]], na.rm=TRUE))))
# lapplytwice
```

```
df.reg.out.all <- bind_rows(
  lapply(list.vars.x,
    function(x) (
      bind_rows(
        lapply(list.vars.y, regf.iv,
          vars.x=x, vars.c=vars.c, vars.z=vars.z, df=df))
      )))
```

```
# df.reg.out.all %>%
# kable() %>%
# kable_styling_fc_wide()
```

5.1.2.4.3 Nested Lapply All

```
df.reg.out.all <-
  (lapply(list.vars.x,
    function(x) (
      bind_rows(lapply(list.vars.y, regf.iv,
        vars.x=x, vars.c=vars.c, vars.z=vars.z, df=df)) %>%
        select(vars_var.y, starts_with(x)) %>%
        select(vars_var.y, ends_with('value'))
      ))) %>% reduce(full_join)
```

```
df.reg.out.all %>%
  kable() %>%
  kable_styling_fc_wide()
```

vars_var.y	prot_tvalue	cal_tvalue	wealthIdx_tvalue	p.A.prot_tvalue	p.A.nProt_tvalue
hgt	18.8756010031786	23.4421863484661	13.508899618216	3.83682180045518	32.5448257554855
wgt	16.3591125056062	17.3686031309332	14.1390521528113	1.36958319982295	12.0961557911467
vil.id	-14.9385580468907	-19.6150110809452	34.0972558327347	8.45943342783186	17.7801422421419

5.1.2.4.4 Nested Lapply Select

5.2 Decomposition

5.2.1 Decompose RHS

Go back to [fan's REconTools](#) Package, [R4Econ](#) Repository ([bookdown site](#)), or [Intro Stats with R](#) Repository.

One runs a number of regressions. With different outcomes, and various right hand side variables.

What is the remaining variation in the left hand side variable if right hand side variable one by one is set to the average of the observed values.

- Dependency: *R4Econ/linreg/ivreg/ivregdfrow.R*

The code below does not work with categorical variables (except for dummies). Dummy variable inputs need to be converted to zero/one first. The examples are just to test the code with different types of variables.

```
# Library
library(tidyverse)
library(AER)

# Load Sample Data
setwd('C:/Users/fan/R4Econ/_data/')
df <- read_csv('height_weight.csv')

# Source Dependency
source('C:/Users/fan/R4Econ/linreg/ivreg/ivregdfrow.R')
```

Data Cleaning.

```
# Convert Variable for Sex which is categorical to Numeric
df <- df
df$male <- (as.numeric(factor(df$sex)) - 1)
summary(factor(df$sex))
```

```
## Female    Male
##  16446    18619
```

```
summary(df$male)
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    0.000   0.000   1.000   0.531   1.000   1.000
```

```
df.use <- df %>% filter(S.country == 'Guatemala') %>%
  filter(svymthRound %in% c(12, 18, 24))
dim(df.use)
```

```
## [1] 2022    16
```

Setting Up Parameters.

```
# Define Left Hand Side Variables
var.y1 <- c('hgt')
var.y2 <- c('wgt')
vars.y <- c(var.y1, var.y2)
# Define Right Hand Side Variables
vars.x <- c('prot')
vars.c <- c('male', 'wgt0', 'hgt0', 'svymthRound')
# vars.z <- c('p.A.prot')
vars.z <- c('vil.id')
# vars.z <- NULL
vars.xc <- c(vars.x, vars.c)

# Other variables to keep
```

```
vars.other.keep <- c('S.country', 'vil.id', 'indi.id', 'svymthRound')

# Decompose sequence
vars.tomean.first <- c('male', 'hgt0')
var.tomean.first.name.suffix <- '_mh02m'
vars.tomean.second <- c(vars.tomean.first, 'hgt0', 'wgt0')
var.tomean.second.name.suffix <- '_mh0me2m'
vars.tomean.third <- c(vars.tomean.second, 'prot')
var.tomean.third.name.suffix <- '_mh0mep2m'
vars.tomean.fourth <- c(vars.tomean.third, 'svymthRound')
var.tomean.fourth.name.suffix <- '_mh0mepm2m'
list.vars.tomean = list(
  # vars.tomean.first,
  vars.tomean.second,
  vars.tomean.third,
  vars.tomean.fourth
)
list.vars.tomean.name.suffix <- list(
  # var.tomean.first.name.suffix,
  var.tomean.second.name.suffix,
  var.tomean.third.name.suffix,
  var.tomean.fourth.name.suffix
)
```

5.2.1.1 Obtain Regression Coefficients from somewhere

```
# Regressions
# regf.iv from C:\Users\fan\R4Econ\linreg\ivreg\ivregdfrow.R
df.reg.out <- as_tibble(
  bind_rows(lapply(vars.y, regf.iv,
    vars.x=vars.x, vars.c=vars.c, vars.z=vars.z, df=df)))

# Regressions
# reg1 <- regf.iv(var.y = var.y1, vars.x, vars.c, vars.z, df.use)
# reg2 <- regf.iv(var.y = var.y2, vars.x, vars.c, vars.z, df.use)
# df.reg.out <- as_tibble(bind_rows(reg1, reg2))

# df.reg.out
```

```
# Select Variables
str.esti.suffix <- '_Estimate'
arr.esti.name <- paste0(vars.xc, str.esti.suffix)
str.outcome.name <- 'vars_var.y'
arr.columns2select <- c(arr.esti.name, str.outcome.name)
arr.columns2select
```

```
## [1] "prot_Estimate"          "male_Estimate"          "wgt0_Estimate"          "hgt0_Estimate"          "
# Generate dataframe for coefficients
df.coef <- df.reg.out[,c(arr.columns2select)] %>%
  mutate_at(vars(arr.esti.name), as.numeric) %>% column_to_rownames(str.outcome.name)
df.coef %>%
  kable() %>%
  kable_styling_fc()
```

	prot_Estimate	male_Estimate	wgt0_Estimate	hgt0_Estimate	svymthRound_Estimate
hgt	-0.2714772	1.244735	0.0004430	0.6834853	1.133919
wgt	-59.0727542	489.852902	0.7696158	75.4867897	250.778883

```
str(df.coef)
```

```
## 'data.frame':  2 obs. of  5 variables:
## $ prot_Estimate      : num  -0.271 -59.073
## $ male_Estimate      : num   1.24 489.85
## $ wgt0_Estimate      : num  0.000443 0.769616
## $ hgt0_Estimate      : num   0.683 75.487
## $ svymthRound_Estimate: num   1.13 250.78
```

5.2.1.2 Decomposition Step 1

```
# Decomposition Step 1: gather
df.decompose_step1 <- df.use %>%
  filter(svymthRound %in% c(12, 18, 24)) %>%
  select(one_of(c(vars.other.keep, vars.xc, vars.y))) %>%
  drop_na() %>%
  gather(variable, value, -one_of(c(vars.other.keep, vars.xc)))
options(repr.matrix.max.rows=20, repr.matrix.max.cols=20)
dim(df.decompose_step1)
```

```
## [1] 1382  10
```

```
head(df.decompose_step1, 10) %>%
  kable() %>%
  kable_styling_fc()
```

S.country	vil.id	indi.id	svymthRound	prot	male	wgt0	hgt0	variable	value
Guatemala	3	1352	18	13.3	1	2545.2	47.4	hgt	70.2
Guatemala	3	1352	24	46.3	1	2545.2	47.4	hgt	75.8
Guatemala	3	1354	12	1.0	1	3634.3	51.2	hgt	66.3
Guatemala	3	1354	18	9.8	1	3634.3	51.2	hgt	69.2
Guatemala	3	1354	24	15.4	1	3634.3	51.2	hgt	75.3
Guatemala	3	1356	12	8.6	1	3911.8	51.9	hgt	68.1
Guatemala	3	1356	18	17.8	1	3911.8	51.9	hgt	74.1
Guatemala	3	1356	24	30.5	1	3911.8	51.9	hgt	77.1
Guatemala	3	1357	12	1.0	1	3791.4	52.6	hgt	71.5
Guatemala	3	1357	18	12.7	1	3791.4	52.6	hgt	77.8

5.2.1.3 Decomposition Step 2

```
# Decomposition Step 2: mutate_at(vars, funs(mean = mean(.)))
# the xc averaging could have taken place earlier, no difference in mean across variables
df.decompose_step2 <- df.decompose_step1 %>%
  group_by(variable) %>%
  mutate_at(vars(c(vars.xc, 'value')), funs(mean = mean(.))) %>%
  ungroup()

options(repr.matrix.max.rows=20, repr.matrix.max.cols=20)
dim(df.decompose_step2)
```

```
## [1] 1382  16
```

```
head(df.decompose_step2, 10) %>%
  kable() %>%
  kable_styling_fc_wide()
```

S.country	vil.id	indi.id	svynthRound	prot	male	wgt0	hgt0	variable	value	prot_mean	male_mean	wgt0_mean	hgt0_mean	svynthRound_mean	value_mean
Guatemala	3	1352	18	13.3	1	2545.2	47.4	hgt	70.2	20.64819	0.5499276	3312.297	49.75137	18.42547	73.41216
Guatemala	3	1352	24	46.3	1	2545.2	47.4	hgt	75.8	20.64819	0.5499276	3312.297	49.75137	18.42547	73.41216
Guatemala	3	1354	12	1.0	1	3634.3	51.2	hgt	66.3	20.64819	0.5499276	3312.297	49.75137	18.42547	73.41216
Guatemala	3	1354	18	9.8	1	3634.3	51.2	hgt	69.2	20.64819	0.5499276	3312.297	49.75137	18.42547	73.41216
Guatemala	3	1354	24	15.4	1	3634.3	51.2	hgt	75.3	20.64819	0.5499276	3312.297	49.75137	18.42547	73.41216
Guatemala	3	1356	12	8.6	1	3911.8	51.9	hgt	68.1	20.64819	0.5499276	3312.297	49.75137	18.42547	73.41216
Guatemala	3	1356	18	17.8	1	3911.8	51.9	hgt	74.1	20.64819	0.5499276	3312.297	49.75137	18.42547	73.41216
Guatemala	3	1356	24	30.5	1	3911.8	51.9	hgt	77.1	20.64819	0.5499276	3312.297	49.75137	18.42547	73.41216
Guatemala	3	1357	12	1.0	1	3791.4	52.6	hgt	71.5	20.64819	0.5499276	3312.297	49.75137	18.42547	73.41216
Guatemala	3	1357	18	12.7	1	3791.4	52.6	hgt	77.8	20.64819	0.5499276	3312.297	49.75137	18.42547	73.41216

5.2.1.4 Decomposition Step 3 Non-Loop

```
ff_lr_decompose_valadj <- function(df, df.coef, vars.tomean, str.esti.suffix) {
  new_value <- (df$value +
    rowSums((df[paste0(vars.tomean, '_mean')] - df[vars.tomean])
      *df.coef[df$variable, paste0(vars.tomean, str.esti.suffix)]))
  return(new_value)
}
```

5.2.1.5 Decomposition Step 3 With Loop

```
df.decompose_step3 <- df.decompose_step2
for (i in 1:length(list.vars.tomean)) {
  var.decomp.cur <- (paste0('value', list.vars.tomean.name.suffix[[i]]))
  vars.tomean <- list.vars.tomean[[i]]
  var.decomp.cur
  df.decompose_step3 <- df.decompose_step3 %>%
    mutate(!var.decomp.cur) :=
      ff_lr_decompose_valadj(., df.coef, vars.tomean, str.esti.suffix))
}

dim(df.decompose_step3)
```

```
## [1] 1382 19
```

```
head(df.decompose_step3, 10) %>%
  kable() %>%
  kable_styling_fc_wide()
```

S.country	vil.id	indi.id	svynthRound	prot	male	wgt0	hgt0	variable	value	prot_mean	male_mean	wgt0_mean	hgt0_mean	svynthRound_mean	value_mean	value_mh0mc2m	value_mh0mep2m	value_mh0mepm2m
Guatemala	3	1352	18	13.3	1	2545.2	47.4	hgt	70.2	20.64819	0.5499276	3312.297	49.75137	18.42547	73.41216	73.19390	71.19903	71.68148
Guatemala	3	1352	24	46.3	1	2545.2	47.4	hgt	75.8	20.64819	0.5499276	3312.297	49.75137	18.42547	73.41216	78.79390	85.75778	79.43671
Guatemala	3	1354	12	1.0	1	3634.3	51.2	hgt	66.3	20.64819	0.5499276	3312.297	49.75137	18.42547	73.41216	63.61689	58.28285	65.56882
Guatemala	3	1354	18	9.8	1	3634.3	51.2	hgt	69.2	20.64819	0.5499276	3312.297	49.75137	18.42547	73.41216	66.51689	63.57185	64.05430
Guatemala	3	1354	24	15.4	1	3634.3	51.2	hgt	75.3	20.64819	0.5499276	3312.297	49.75137	18.42547	73.41216	72.61689	71.19213	64.87106
Guatemala	3	1356	12	8.6	1	3911.8	51.9	hgt	68.1	20.64819	0.5499276	3312.297	49.75137	18.42547	73.41216	64.33707	61.06626	68.35222
Guatemala	3	1356	18	17.8	1	3911.8	51.9	hgt	74.1	20.64819	0.5499276	3312.297	49.75137	18.42547	73.41216	70.33707	69.56385	70.04630
Guatemala	3	1356	24	30.5	1	3911.8	51.9	hgt	77.1	20.64819	0.5499276	3312.297	49.75137	18.42547	73.41216	73.33707	76.01161	69.69055
Guatemala	3	1357	12	1.0	1	3791.4	52.6	hgt	71.5	20.64819	0.5499276	3312.297	49.75137	18.42547	73.41216	66.83353	61.49949	68.78545
Guatemala	3	1357	18	12.7	1	3791.4	52.6	hgt	77.8	20.64819	0.5499276	3312.297	49.75137	18.42547	73.41216	73.13353	70.97578	71.45823

5.2.1.6 Decomposition Step 4 Variance

```
df.decompose_step3 %>%
  select(variable, contains('value')) %>%
  group_by(variable) %>%
  summarize_all(funs(mean = mean, var = var)) %>%
  select(matches('value')) %>% select(ends_with("_var")) %>%
  mutate_if(is.numeric, funs( frac = (./value_var))) %>%
  mutate_if(is.numeric, round, 3) %>%
  kable() %>%
  kable_styling_fc_wide()
```

value_var	value_mean_var	value_mh0mc2m_var	value_mh0mep2m_var	value_mh0mepm2m_var	value_var_frac	value_mean_var_frac	value_mh0mc2m_var_frac	value_mh0mep2m_var_frac	value_mh0mepm2m_var_frac
21.864	NA	25.35	49.047	23.06	1	NA	1.159	2.243	1.055
2965693.245	NA	2949187.64	4192769.518	3147506.60	1	NA	0.994	1.414	1.061

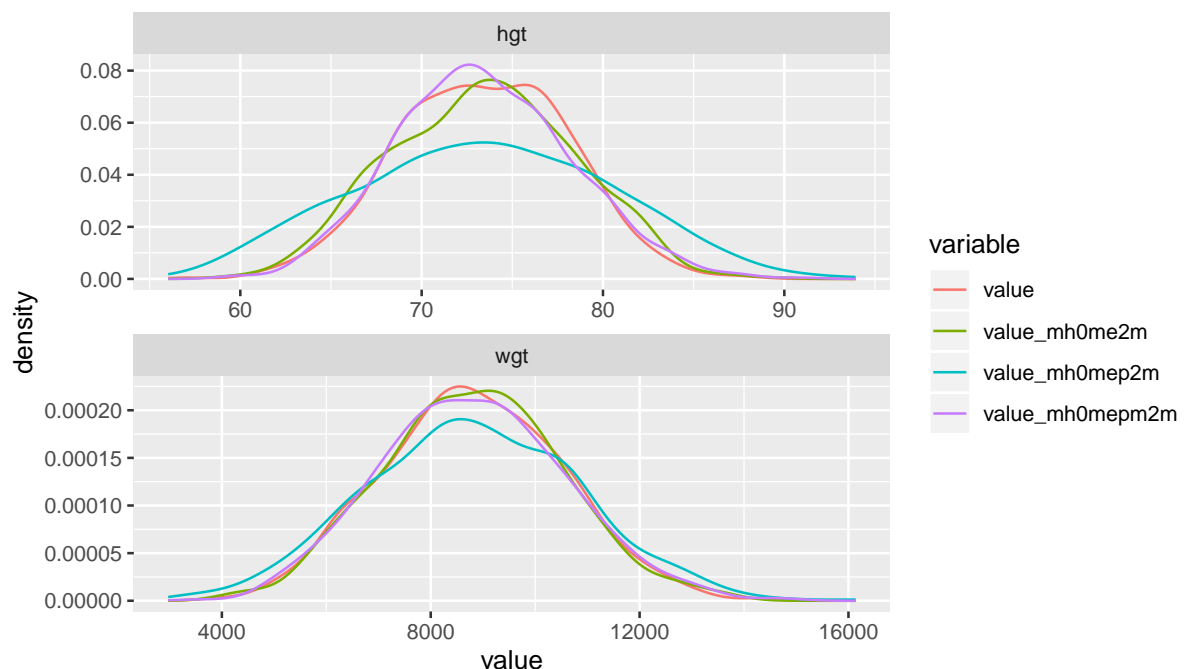
5.2.1.7 Graphical Results

Graphically, difficult to pick up exact differences in variance, a 50 percent reduction in variance visually does not look like 50 percent. Intuitively, we are kind of seeing standard deviation, not variance on the graph if we think about the x-scale.

```
head(df.decompose_step3 %>%
  select(variable, contains('value'), -value_mean), 10) %>%
  kable() %>%
  kable_styling_fc()
```

variable	value	value_mh0me2m	value_mh0mep2m	value_mh0mepm2m
hgt	70.2	73.19390	71.19903	71.68148
hgt	75.8	78.79390	85.75778	79.43671
hgt	66.3	63.61689	58.28285	65.56882
hgt	69.2	66.51689	63.57185	64.05430
hgt	75.3	72.61689	71.19213	64.87106
hgt	68.1	64.33707	61.06626	68.35222
hgt	74.1	70.33707	69.56385	70.04630
hgt	77.1	73.33707	76.01161	69.69055
hgt	71.5	66.83353	61.49949	68.78545
hgt	77.8	73.13353	70.97578	71.45823

```
df.decompose_step3 %>%
  select(variable, contains('value'), -value_mean) %>%
  rename(outcome = variable) %>%
  gather(variable, value, -outcome) %>%
  ggplot(aes(x=value, color = variable, fill = variable)) +
    geom_line(stat = "density") +
    facet_wrap(~ outcome, scales='free', nrow=2)
```



5.2.1.8 Additional Decomposition Testings

```
head(df.decompose_step2[vars.tomean.first], 3)
head(df.decompose_step2[paste0(vars.tomean.first, '_mean')], 3)
```

```

head(df.coef[df.decompose_step2$variable,
           paste0(vars.tomean.first, str.esti.suffix)], 3)
df.decompose.tomean.first <- df.decompose_step2 %>%
  mutate(pred_new = df.decompose_step2$value +
         rowSums((df.decompose_step2[paste0(vars.tomean.first, '_mean')]
                  - df.decompose_step2[vars.tomean.first])
                 *df.coef[df.decompose_step2$variable,
                           paste0(vars.tomean.first, str.esti.suffix)])) %>%
  select(variable, value, pred_new)
head(df.decompose.tomean.first, 10)
df.decompose.tomean.first %>%
  group_by(variable) %>%
  summarize_all(funs(mean = mean, sd = sd)) %>%
  kable() %>%
  kable_styling_fc()

```

variable	value_mean	pred_new_mean	value_sd	pred_new_sd
hgt	73.41216	73.41216	4.675867	4.534947
wgt	8807.87656	8807.87656	1722.118824	1695.221845

Note the r-square from regression above matches up with the 1 - ratio below. This is the proper decomposition method that is equivalent to r^2 .

```

df.decompose_step2 %>%
  mutate(pred_new = df.decompose_step2$value +
         rowSums((df.decompose_step2[paste0(vars.tomean.second, '_mean')]
                  - df.decompose_step2[vars.tomean.second])
                 *df.coef[df.decompose_step2$variable,
                           paste0(vars.tomean.second, str.esti.suffix)])) %>%
  select(variable, value, pred_new) %>%
  group_by(variable) %>%
  summarize_all(funs(mean = mean, var = var)) %>%
  mutate(ratio = (pred_new_var/value_var)) %>%
  kable() %>%
  kable_styling_fc()

```

variable	value_mean	pred_new_mean	value_var	pred_new_var	ratio
hgt	73.41216	73.41216	2.186374e+01	25.3504	1.1594724
wgt	8807.87656	8807.87656	2.965693e+06	2949187.6357	0.9944345

Chapter 6

Nonlinear Regression

6.1 Logit Regression

6.1.1 Binary Logit

Go back to [fan's REconTools Package](#), [R4Econ Repository \(bookdown site\)](#), or [Intro Stats with R Repository](#).

Data Preparation

```
df_mtcars <- mtcars

# X-variables to use on RHS
ls_st_xs <- c('mpg', 'qsec')
ls_st_xs <- c('mpg')
ls_st_xs <- c('qsec')
ls_st_xs <- c('wt')
ls_st_xs <- c('mpg', 'wt', 'vs')

svr_binary <- 'hpLowHigh'
svr_binary_lb0 <- 'LowHP'
svr_binary_lb1 <- 'HighHP'
svr_outcome <- 'am'
sdt_name <- 'mtcars'

# Discretize hp
df_mtcars <- df_mtcars %>%
  mutate(!!sym(svr_binary) := cut(hp,
                                breaks=c(-Inf, 210, Inf),
                                labels=c(svr_binary_lb0, svr_binary_lb1)))
```

6.1.1.1 Logit Regression and Prediction

logit regression with glm, and predict using estimation data. Prediction and estimation with one variable.

- [LOGIT REGRESSION R DATA ANALYSIS EXAMPLES](#)
- [Generalized Linear Models](#)

```
# Regress
rs_logit <- glm(as.formula(paste(svr_outcome, "~", paste(ls_st_xs, collapse="+"))),
               ,data = df_mtcars, family = "binomial")
summary(rs_logit)
```

```
##
## Call:
```

```
## glm(formula = as.formula(paste(svr_outcome, "~", paste(ls_st_xs,
##   collapse = "+"))), family = "binomial", data = df_mtcars)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -1.73603  -0.25477  -0.04891   0.13402   1.90321
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)  22.69008   13.95112   1.626   0.1039
## mpg         -0.01786    0.33957  -0.053   0.9581
## wt          -6.73804    3.01400  -2.236   0.0254 *
## vs          -4.44046    2.84247  -1.562   0.1182
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##    Null deviance: 43.230  on 31  degrees of freedom
## Residual deviance: 13.092  on 28  degrees of freedom
## AIC: 21.092
##
## Number of Fisher Scoring iterations: 7
# Predict Using Regression Data
df_mtcars$p_mpg <- predict(rs_logit, newdata = df_mtcars, type = "response")
```

6.1.1.1.1 Prediction with Observed Binary Input Logit regression with a continuous variable and a binary variable. Predict outcome with observed continuous variable as well as observed binary input variable.

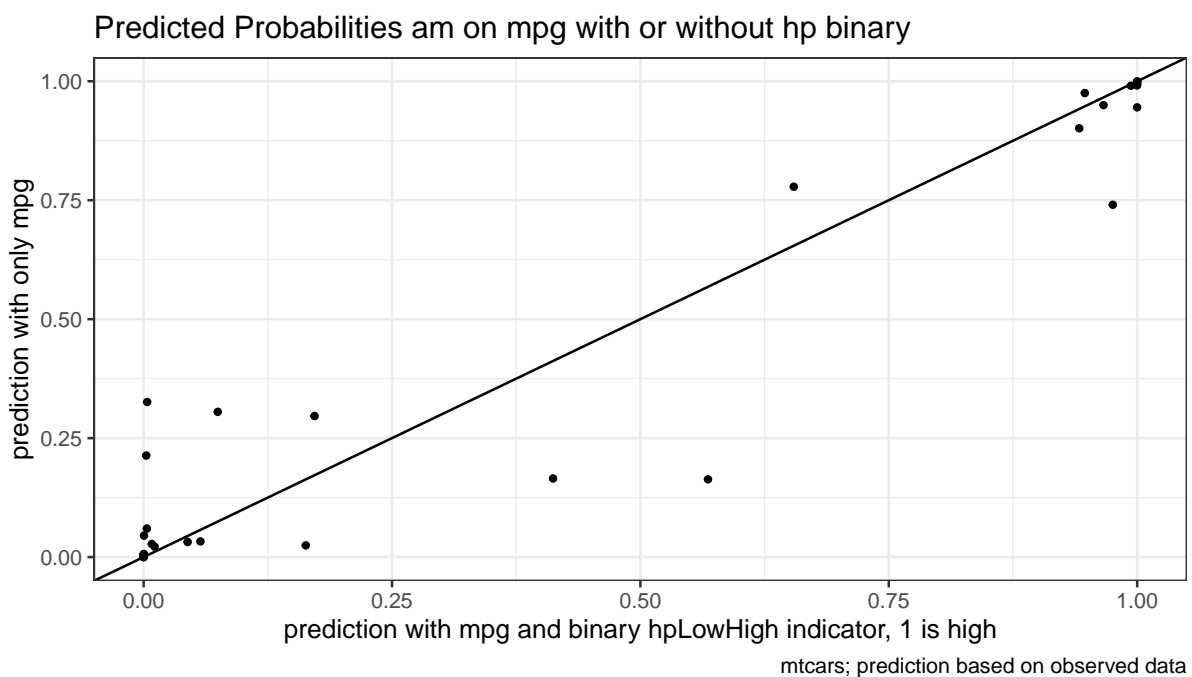
```
# Regress
rs_logit_bi <- glm(as.formula(paste(svr_outcome,
                                   "~ factor(", svr_binary,") + ",
                                   paste(ls_st_xs, collapse="+"))),
                  , data = df_mtcars, family = "binomial")
summary(rs_logit_bi)

##
## Call:
## glm(formula = as.formula(paste(svr_outcome, "~ factor(", svr_binary,
##   ") + ", paste(ls_st_xs, collapse = "+"))), family = "binomial",
##   data = df_mtcars)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -1.45771  -0.09563  -0.00875   0.00555   1.87612
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)      3.8285   18.0390   0.212   0.8319
## factor(hpLowHigh)HighHP  6.9907   5.5176   1.267   0.2052
## mpg              0.8985    0.8906   1.009   0.3131
## wt              -6.7291    3.3166  -2.029   0.0425 *
## vs              -5.9206    4.1908  -1.413   0.1577
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
```



```
##
## Null deviance: 43.2297 on 31 degrees of freedom
## Residual deviance: 8.9777 on 27 degrees of freedom
## AIC: 18.978
##
## Number of Fisher Scoring iterations: 9
# Predict Using Regression Data
df_mtcars$p_mpg_hp <- predict(rs_logit_bi, newdata = df_mtcars, type = "response")

# Predicted Probabilities am on mpg with or without hp binary
scatter <- ggplot(df_mtcars, aes(x=p_mpg_hp, y=p_mpg)) +
  geom_point(size=1) +
  # geom_smooth(method=lm) + # Trend line
  geom_abline(intercept = 0, slope = 1) + # 45 degree line
  labs(title = paste0('Predicted Probabilities ', svr_outcome, ' on ', ls_st_xs, ' with or without ', svr_binary, ' indicator, 1 is high'),
        x = paste0('prediction with ', ls_st_xs, ' and binary ', svr_binary, ' indicator, 1 is high'),
        y = paste0('prediction with only ', ls_st_xs),
        caption = 'mtcars; prediction based on observed data') +
  theme_bw()
print(scatter)
```



6.1.1.1.2 Prediction with Binary set to 0 and 1 Now generate two predictions. One set where binary input is equal to 0, and another where the binary inputs are equal to 1. Ignore whether in data binary input is equal to 0 or 1. Use the same regression results as what was just derived.

Note that given the example here, the probability changes a lot when we

```
# Previous regression results
summary(rs_logit_bi)

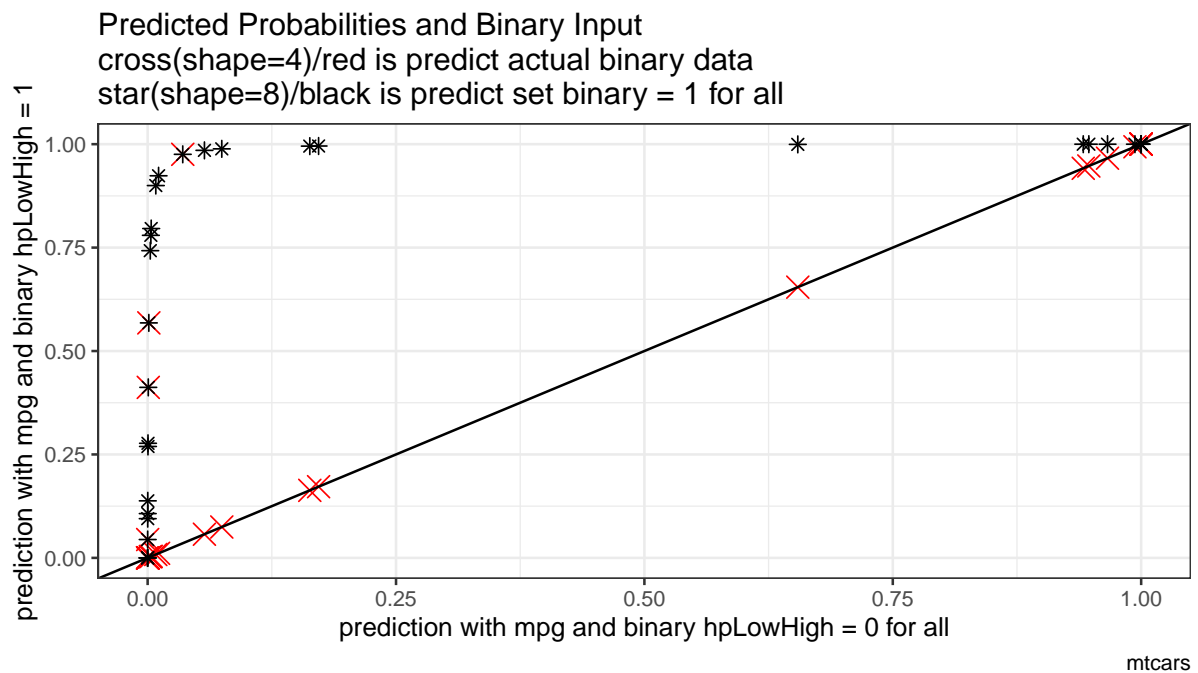
##
## Call:
## glm(formula = as.formula(paste(svr_outcome, "~ factor(", svr_binary,
## " ) + ", paste(ls_st_xs, collapse = "+"))), family = "binomial",
## data = df_mtcars)
```

```
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -1.45771  -0.09563  -0.00875   0.00555   1.87612
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)      3.8285     18.0390   0.212   0.8319
## factor(hpLowHigh)HighHP  6.9907     5.5176   1.267   0.2052
## mpg              0.8985     0.8906   1.009   0.3131
## wt              -6.7291     3.3166  -2.029   0.0425 *
## vs              -5.9206     4.1908  -1.413   0.1577
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##      Null deviance: 43.2297  on 31  degrees of freedom
## Residual deviance:  8.9777  on 27  degrees of freedom
## AIC: 18.978
##
## Number of Fisher Scoring iterations: 9

# Two different dataframes, mutate the binary regressor
df_mtcars_bi0 <- df_mtcars %>% mutate(!sym(svr_binary) := svr_binary_lb0)
df_mtcars_bi1 <- df_mtcars %>% mutate(!sym(svr_binary) := svr_binary_lb1)

# Predict Using Regression Data
df_mtcars$p_mpg_hp_bi0 <- predict(rs_logit_bi, newdata = df_mtcars_bi0, type = "response")
df_mtcars$p_mpg_hp_bi1 <- predict(rs_logit_bi, newdata = df_mtcars_bi1, type = "response")

# Predicted Probabilities and Binary Input
scatter <- ggplot(df_mtcars, aes(x=p_mpg_hp_bi0)) +
  geom_point(aes(y=p_mpg_hp), size=4, shape=4, color="red") +
  geom_point(aes(y=p_mpg_hp_bi1), size=2, shape=8) +
  # geom_smooth(method=lm) + # Trend line
  geom_abline(intercept = 0, slope = 1) + # 45 degree line
  labs(title = paste0('Predicted Probabilities and Binary Input',
    '\ncross(shape=4)/red is predict actual binary data',
    '\nstar(shape=8)/black is predict set binary = 1 for all'),
    x = paste0('prediction with ', ls_st_xs, ' and binary ', svr_binary, ' = 0 for all'),
    y = paste0('prediction with ', ls_st_xs, ' and binary ', svr_binary, ' = 1'),
    caption = paste0(sdt_name)) +
  theme_bw()
print(scatter)
```

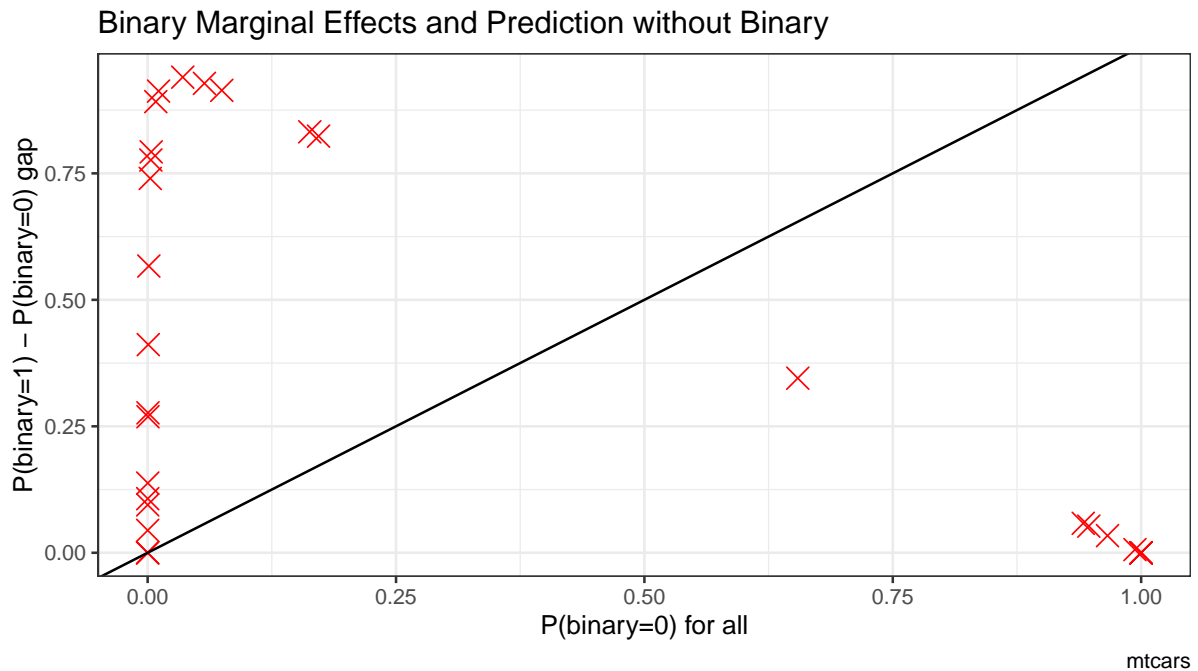


6.1.1.1.3 Prediction with Binary set to 0 and 1 Difference What is the difference in probability between binary = 0 vs binary = 1. How does that relate to the probability of outcome of interest when binary = 0 for all.

In the binary logit case, the relationship will be hump-shaped by construction between A_i and α_i . In the exponential wage cases, the relationship is convex upwards.

```
# Generate Gap Variable
df_mtcars <- df_mtcars %>% mutate(alpha_i = p_mpg_hp_bi1 - p_mpg_hp_bi0) %>%
  mutate(A_i = p_mpg_hp_bi0)

# Binary Marginal Effects and Prediction without Binary
scatter <- ggplot(df_mtcars, aes(x=A_i)) +
  geom_point(aes(y=alpha_i), size=4, shape=4, color="red") +
  geom_abline(intercept = 0, slope = 1) + # 45 degree line
  labs(title = paste0('Binary Marginal Effects and Prediction without Binary'),
       x = 'P(binary=0) for all',
       y = 'P(binary=1) - P(binary=0) gap',
       caption = paste0(sdt_name)) +
  theme_bw()
print(scatter)
```



6.1.1.1.4 X variables and A and alpha Given the x-variables included in the logit regression, how do they relate to A_i and α_i

```
# Generate Gap Variable
df_mtcars <- df_mtcars %>% mutate(alpha_i = p_mpg_hp_bi1 - p_mpg_hp_bi0) %>%
  mutate(A_i = p_mpg_hp_bi0)

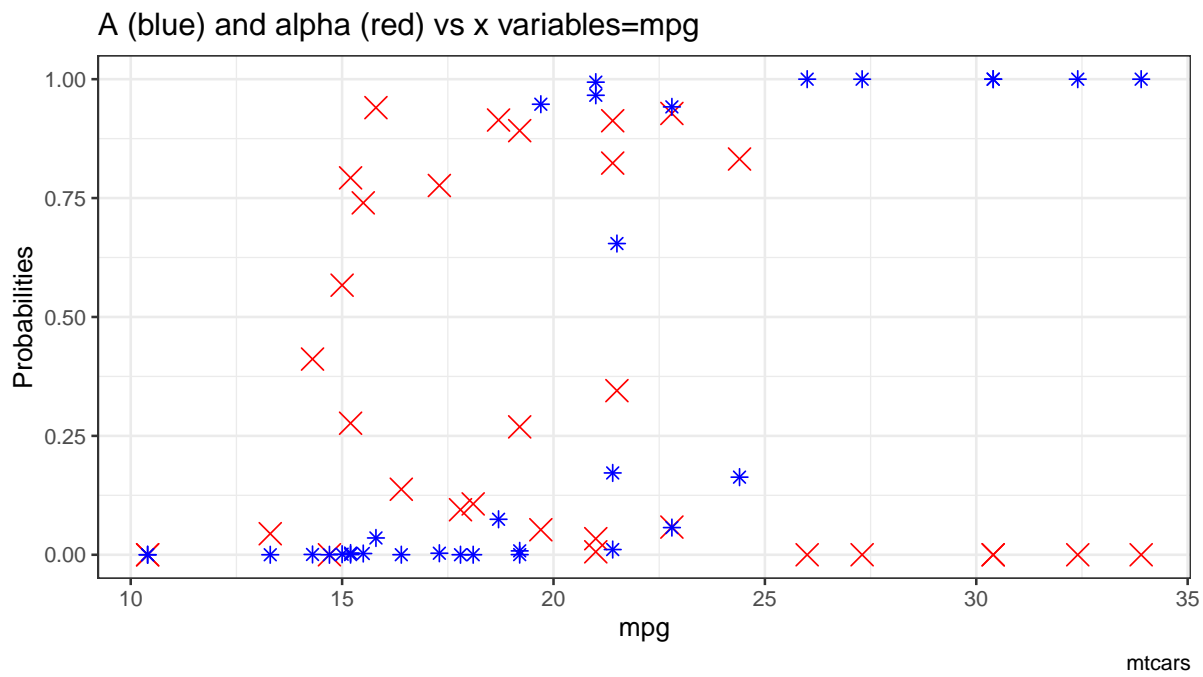
# Binary Marginal Effects and Prediction without Binary
ggplot.A.alpha.x <- function(svr_x, df,
                             svr_alpha = 'alpha_i', svr_A = "A_i"){

  scatter <- ggplot(df, aes(x=!!sym(svr_x))) +
    geom_point(aes(y=alpha_i), size=4, shape=4, color="red") +
    geom_point(aes(y=A_i), size=2, shape=8, color="blue") +
    geom_abline(intercept = 0, slope = 1) + # 45 degree line
    labs(title = paste0('A (blue) and alpha (red) vs x variables=', svr_x),
         x = svr_x,
         y = 'Probabilities',
         caption = paste0(sdt_name)) +
    theme_bw()

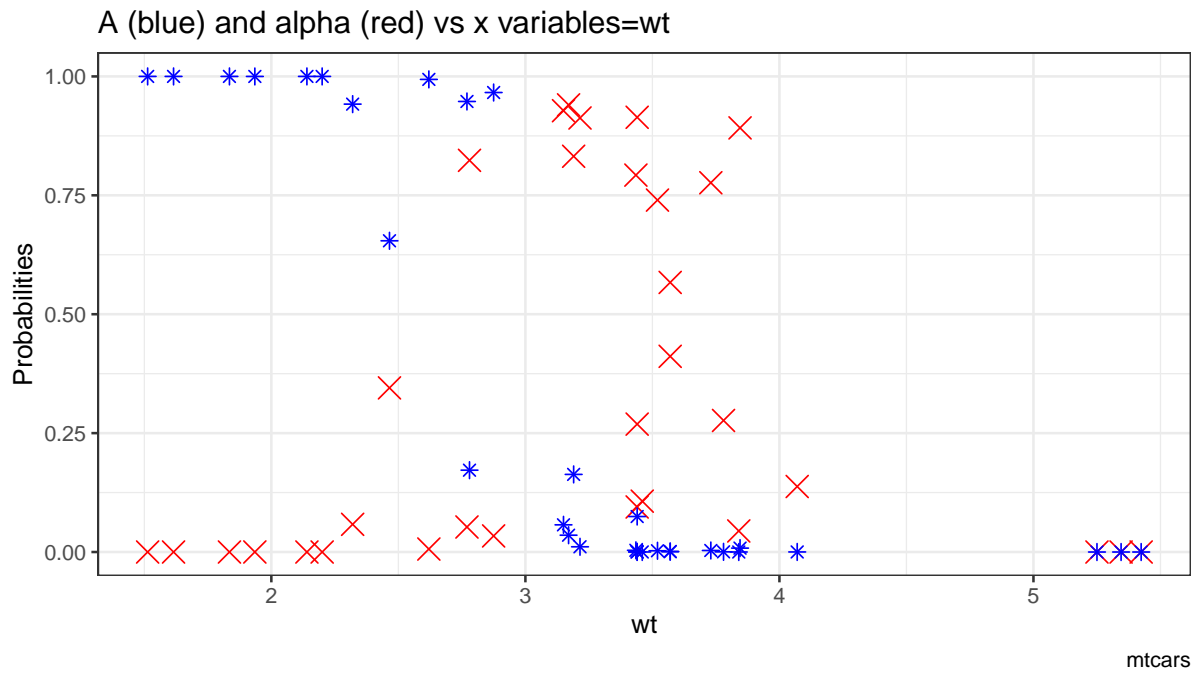
  return(scatter)
}

# Plot over multiple
lapply(ls_st_xs,
       ggplot.A.alpha.x,
       df = df_mtcars)
```

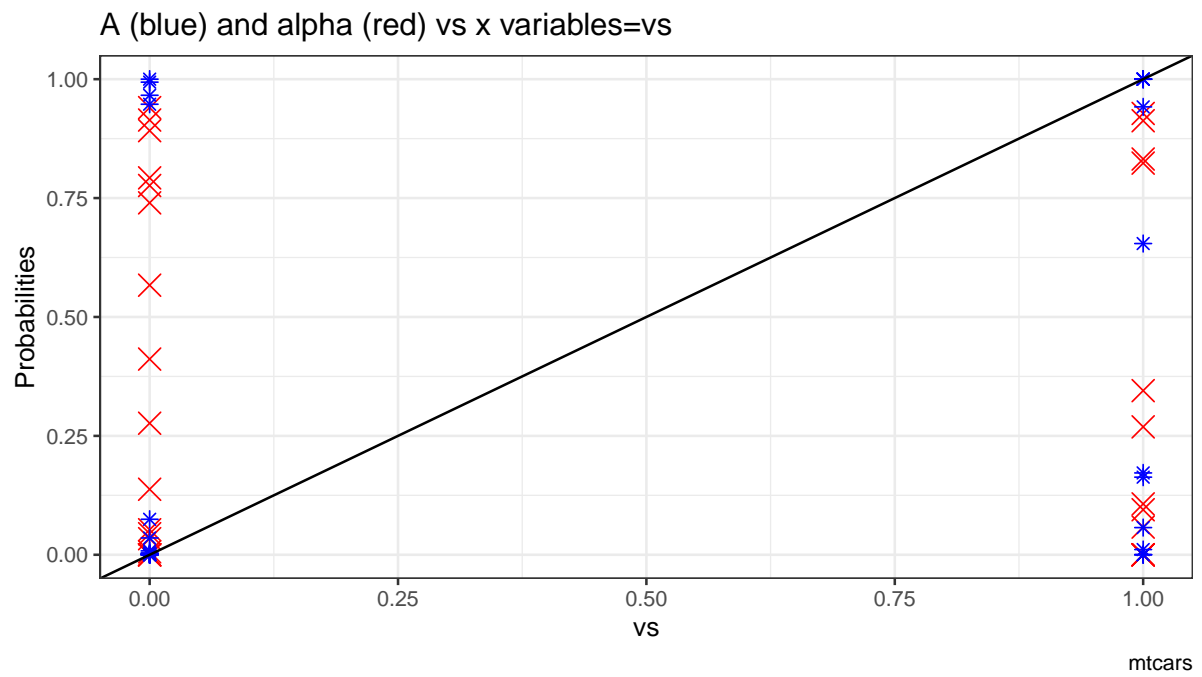
```
## [[1]]
```



```
##  
## [[2]]
```



```
##  
## [[3]]
```



Chapter 7

Optimization

7.1 Bisection

7.1.1 Bisection

Go back to [fan's REconTools Package](#), [R4Econ Repository \(bookdown site\)](#), or [Intro Stats with R Repository](#).

See the `ff_opti_bisect_pmap_multi` function from [Fan's REconTools Package](#), which provides a reusable function based on the algorithm worked out here.

The bisection specific code does not need to do much.

- list variables in file for grouping, each group is an individual for whom we want to calculate optimal choice for using bisection.
- string variable name of input where functions are evaluated, these are already contained in the dataframe, existing variable names, row specific, rowwise computation over these, each rowwise calculation using different rows.
- scalar and array values that are applied to every rowwise calculation, all rowwise calculations using the same scalars and arrays.
- string output variable name

This is how I implement the bisection algorithm, when we know the bounding minimum and maximum to be below and above zero already.

1. Evaluate $f_a^0 = f(a^0)$ and $f_b^0 = f(b^0)$, min and max points.
2. Evaluate at $f_p^0 = f(p^0)$, where $p_0 = \frac{a^0 + b^0}{2}$.
3. if $f_a^i \cdot f_p^i < 0$, then $b_{i+1} = p_i$, else, $a_{i+1} = p_i$ and $f_a^{i+1} = p_i$.
4. iterate until convergence.

Generate New columns of a and b as we iterate, do not need to store p, p is temporary. Evaluate the function below which we have already tested, but now, in the dataframe before generating all permutations, `tb_states_choices`, now the `fl_N` element will be changing with each iteration, it will be row specific. `fl_N` are first min and max, then each subsequent ps.

7.1.1.1 Initialize Matrix

Prepare Input Data:

```
# Parameters
fl_rho = 0.20
svr_id_var = 'INDI_ID'

# P fixed parameters, nN is N dimensional, nP is P dimensional
ar_nN_A = seq(-2, 2, length.out = 4)
ar_nN_alpha = seq(0.1, 0.9, length.out = 4)
```

```

# Choice Grid for nutritional feasible choices for each
fl_N_agg = 100
fl_N_min = 0

# Mesh Expand
tb_states_choices <- as_tibble(cbind(ar_nN_A, ar_nN_alpha)) %>%
  rowid_to_column(var=svr_id_var)

# Convert Matrix to Tibble
ar_st_col_names = c(svr_id_var, 'fl_A', 'fl_alpha')
tb_states_choices <- tb_states_choices %>% rename_all(~c(ar_st_col_names))

```

Prepare Function:

```

# Define Implicit Function
ffi_nonlin_dplyrdo <- function(fl_A, fl_alpha, fl_N, ar_A, ar_alpha, fl_N_agg, fl_rho){

  ar_p1_s1 = exp((fl_A - ar_A)*fl_rho)
  ar_p1_s2 = (fl_alpha/ar_alpha)
  ar_p1_s3 = (1/(ar_alpha*fl_rho - 1))
  ar_p1 = (ar_p1_s1*ar_p1_s2)^ar_p1_s3
  ar_p2 = fl_N^(((fl_alpha*fl_rho-1)/(ar_alpha*fl_rho-1))
  ar_overall = ar_p1*ar_p2
  fl_overall = fl_N_agg - sum(ar_overall)

  return(fl_overall)
}

```

Initialize the matrix with a_0 and b_0 , the initial min and max points:

```

# common prefix to make reshaping easier
st_bisec_prefix <- 'bisec_'
svr_a_lst <- paste0(st_bisec_prefix, 'a_0')
svr_b_lst <- paste0(st_bisec_prefix, 'b_0')
svr_fa_lst <- paste0(st_bisec_prefix, 'fa_0')
svr_fb_lst <- paste0(st_bisec_prefix, 'fb_0')

# Add initial a and b
tb_states_choices_bisec <- tb_states_choices %>%
  mutate(!sym(svr_a_lst) := fl_N_min, !sym(svr_b_lst) := fl_N_agg)

# Evaluate function f(a_0) and f(b_0)
tb_states_choices_bisec <- tb_states_choices_bisec %>%
  rowwise() %>%
  mutate(!sym(svr_fa_lst) := ffi_nonlin_dplyrdo(fl_A, fl_alpha, !sym(svr_a_lst),
                                                ar_nN_A, ar_nN_alpha,
                                                fl_N_agg, fl_rho),
        !sym(svr_fb_lst) := ffi_nonlin_dplyrdo(fl_A, fl_alpha, !sym(svr_b_lst),
                                                ar_nN_A, ar_nN_alpha,
                                                fl_N_agg, fl_rho))

# Summarize
dim(tb_states_choices_bisec)

## [1] 4 7

# summary(tb_states_choices_bisec)

```

7.1.1.2 Iterate and Solve for $f(p)$, update $f(a)$ and $f(b)$

Implement the DPLYR based Concurrent bisection algorithm.


```

# fl_tol = float tolerance criteria
# it_tol = number of iterations to allow at most
fl_tol <- 10^-2
it_tol <- 100

# fl_p_dist2zr = distance to zero to initialize
fl_p_dist2zr <- 1000
it_cur <- 0
while (it_cur <= it_tol && fl_p_dist2zr >= fl_tol ) {

  it_cur <- it_cur + 1

  # New Variables
  svr_a_cur <- paste0(st_bisec_prefix, 'a_', it_cur)
  svr_b_cur <- paste0(st_bisec_prefix, 'b_', it_cur)
  svr_fa_cur <- paste0(st_bisec_prefix, 'fa_', it_cur)
  svr_fb_cur <- paste0(st_bisec_prefix, 'fb_', it_cur)

  # Evaluate function f(a_0) and f(b_0)
  # 1. generate p
  # 2. generate f_p
  # 3. generate f_p*f_a
  tb_states_choices_bisec <- tb_states_choices_bisec %>%
    rowwise() %>%
    mutate(p = ((!!sym(svr_a_lst) + !!sym(svr_b_lst))/2)) %>%
    mutate(f_p = ffi_nonlin_dplyrdo(fl_A, fl_alpha, p,
                                   ar_nN_A, ar_nN_alpha,
                                   fl_N_agg, fl_rho)) %>%
    mutate(f_p_t_f_a = f_p*!!sym(svr_fa_lst))
  # fl_p_dist2zr = sum(abs(p))
  fl_p_dist2zr <- mean(abs(tb_states_choices_bisec %>% pull(f_p)))

  # Update a and b
  tb_states_choices_bisec <- tb_states_choices_bisec %>%
    mutate(!!sym(svr_a_cur) :=
           case_when(f_p_t_f_a < 0 ~ !!sym(svr_a_lst),
                     TRUE ~ p)) %>%
    mutate(!!sym(svr_b_cur) :=
           case_when(f_p_t_f_a < 0 ~ p,
                     TRUE ~ !!sym(svr_b_lst)))
  # Update f(a) and f(b)
  tb_states_choices_bisec <- tb_states_choices_bisec %>%
    mutate(!!sym(svr_fa_cur) :=
           case_when(f_p_t_f_a < 0 ~ !!sym(svr_fa_lst),
                     TRUE ~ f_p)) %>%
    mutate(!!sym(svr_fb_cur) :=
           case_when(f_p_t_f_a < 0 ~ f_p,
                     TRUE ~ !!sym(svr_fb_lst)))

  # Save from last
  svr_a_lst <- svr_a_cur
  svr_b_lst <- svr_b_cur
  svr_fa_lst <- svr_fa_cur
  svr_fb_lst <- svr_fb_cur

  # Summar current round
  print(paste0('it_cur:', it_cur, ', fl_p_dist2zr:', fl_p_dist2zr))
  summary(tb_states_choices_bisec %>%
          select(one_of(svr_a_cur, svr_b_cur, svr_fa_cur, svr_fb_cur)))
}

```

```

}

## [1] "it_cur:1, fl_p_dist2zr:1597.93916362849"
## [1] "it_cur:2, fl_p_dist2zr:676.06602535902"
## [1] "it_cur:3, fl_p_dist2zr:286.850590132782"
## [1] "it_cur:4, fl_p_dist2zr:117.225493866655"
## [1] "it_cur:5, fl_p_dist2zr:37.570593471664"
## [1] "it_cur:6, fl_p_dist2zr:4.60826664896022"
## [1] "it_cur:7, fl_p_dist2zr:14.4217689135683"
## [1] "it_cur:8, fl_p_dist2zr:8.38950830086659"
## [1] "it_cur:9, fl_p_dist2zr:3.93347761455868"
## [1] "it_cur:10, fl_p_dist2zr:1.88261338941038"
## [1] "it_cur:11, fl_p_dist2zr:0.744478952222305"
## [1] "it_cur:12, fl_p_dist2zr:0.187061801237917"
## [1] "it_cur:13, fl_p_dist2zr:0.117844913432613"
## [1] "it_cur:14, fl_p_dist2zr:0.0275365951418891"
## [1] "it_cur:15, fl_p_dist2zr:0.0515488156908255"
## [1] "it_cur:16, fl_p_dist2zr:0.0191152349149135"
## [1] "it_cur:17, fl_p_dist2zr:0.00385372194545752"

```

7.1.1.3 Reshape Wide to long to Wide

To view results easily, how iterations improved to help us find the roots, convert table from wide to long. Pivot twice. This allows us to easily graph out how bisection is working out iteration by iteration.

Here, we will first show what the raw table looks like, the wide only table, and then show the long version, and finally the version that is medium wide.

7.1.1.3.1 Table One—Very Wide Show what the `tb_states_choices_bisec` looks like.

Variables are formatted like: `bisec_xx_yy`, where `yy` is the iteration indicator, and `xx` is either a, b, fa, or fb.

```

kable(head(t(tb_states_choices_bisec), 25)) %>%
  kable_styling_fc()

```

```

# str(tb_states_choices_bisec)

```

7.1.1.3.2 Table Two—Very Wide to Very Long We want to treat the iteration count information that is the suffix of variable names as a variable by itself. Additionally, we want to treat the a,b,fa,fb as a variable. Structuring the data very long like this allows for easy graphing and other types of analysis. Rather than dealing with many many variables, we have only 3 core variables that store bisection iteration information.

Here we use the very nice `pivot_longer` function. Note that to achieve this, we put a common prefix in front of the variables we wanted to convert to long. This is helpful, because we can easily identify which variables need to be reshaped.

```

# New variables
svr_bisect_iter <- 'biseciter'
svr_abfafb_long_name <- 'varname'
svr_number_col <- 'value'
svr_id_bisect_iter <- paste0(svr_id_var, '_bisect_ier')

# Pivot wide to very long
tb_states_choices_bisec_long <- tb_states_choices_bisec %>%
  pivot_longer(
    cols = starts_with(st_bisec_prefix),
    names_to = c(svr_abfafb_long_name, svr_bisect_iter),
    names_pattern = paste0(st_bisec_prefix, "(.*)_(.*)"),

```

INDI_ID	1.000000e+00	2.0000000	3.0000000	4.0000000
fl_A	-2.000000e+00	-0.6666667	0.6666667	2.0000000
fl_alpha	1.000000e-01	0.3666667	0.6333333	0.9000000
bisec_a_0	0.000000e+00	0.0000000	0.0000000	0.0000000
bisec_b_0	1.000000e+02	100.0000000	100.0000000	100.0000000
bisec_fa_0	1.000000e+02	100.0000000	100.0000000	100.0000000
bisec_fb_0	-1.288028e+04	-1394.7069782	-323.9421599	-51.9716069
p	1.544952e+00	8.5838318	24.8359680	65.0367737
f_p	-7.637200e-03	-0.0052211	-0.0016162	-0.0009405
f_p_t_f_a	-3.800000e-04	-0.0000237	-0.0000025	-0.0000002
bisec_a_1	0.000000e+00	0.0000000	0.0000000	50.0000000
bisec_b_1	5.000000e+01	50.0000000	50.0000000	100.0000000
bisec_fa_1	1.000000e+02	100.0000000	100.0000000	22.5557704
bisec_fb_1	-5.666956e+03	-595.7345364	-106.5105843	-51.9716069
bisec_a_2	0.000000e+00	0.0000000	0.0000000	50.0000000
bisec_b_2	2.500000e+01	25.0000000	25.0000000	75.0000000
bisec_fa_2	1.000000e+02	100.0000000	100.0000000	22.5557704
bisec_fb_2	-2.464562e+03	-224.1460032	-0.6857375	-14.8701831
bisec_a_3	0.000000e+00	0.0000000	12.5000000	62.5000000
bisec_b_3	1.250000e+01	12.5000000	25.0000000	75.0000000
bisec_fa_3	1.000000e+02	100.0000000	50.8640414	3.7940196
bisec_fb_3	-1.041574e+03	-51.1700464	-0.6857375	-14.8701831
bisec_a_4	0.000000e+00	6.2500000	18.7500000	62.5000000
bisec_b_4	6.250000e+00	12.5000000	25.0000000	68.7500000
bisec_fa_4	1.000000e+02	29.4271641	25.2510409	3.7940196

```

values_to = svr_number_col
)

# Print
# summary(tb_states_choices_bisec_long)
kable(head(tb_states_choices_bisec_long %>%
  select(-one_of('p', 'f_p', 'f_p_t_f_a')), 15)) %>%
  kable_styling_fc()

```

INDI_ID	fl_A	fl_alpha	varname	biseciter	value
1	-2	0.1	a	0	0.000
1	-2	0.1	b	0	100.000
1	-2	0.1	fa	0	100.000
1	-2	0.1	fb	0	-12880.284
1	-2	0.1	a	1	0.000
1	-2	0.1	b	1	50.000
1	-2	0.1	fa	1	100.000
1	-2	0.1	fb	1	-5666.956
1	-2	0.1	a	2	0.000
1	-2	0.1	b	2	25.000
1	-2	0.1	fa	2	100.000
1	-2	0.1	fb	2	-2464.562
1	-2	0.1	a	3	0.000
1	-2	0.1	b	3	12.500
1	-2	0.1	fa	3	100.000

```

kable(tail(tb_states_choices_bisec_long %>%
  select(-one_of('p', 'f_p', 'f_p_t_f_a')), 15)) %>%
  kable_styling_fc()

```

INDI_ID	fl_A	fl_alpha	varname	biseciter	value
4	2	0.9	b	14	65.0390625
4	2	0.9	fa	14	0.0047633
4	2	0.9	fb	14	-0.0043628
4	2	0.9	a	15	65.0360107
4	2	0.9	b	15	65.0390625
4	2	0.9	fa	15	0.0002003
4	2	0.9	fb	15	-0.0043628
4	2	0.9	a	16	65.0360107
4	2	0.9	b	16	65.0375366
4	2	0.9	fa	16	0.0002003
4	2	0.9	fb	16	-0.0020812
4	2	0.9	a	17	65.0360107
4	2	0.9	b	17	65.0367737
4	2	0.9	fa	17	0.0002003
4	2	0.9	fb	17	-0.0009405

7.1.1.3.3 Table Two—Very Very Long to Wider Again But the previous results are too long, with the a, b, fa, and fb all in one column as different categories, they are really not different categories, they are in fact different types of variables. So we want to spread those four categories of this variable into four columns, each one representing the a, b, fa, and fb values. The rows would then be uniquely identified by the iteration counter and individual ID.

```
# Pivot wide to very long to a little wide
tb_states_choices_bisec_wider <- tb_states_choices_bisec_long %>%
  pivot_wider(
    names_from = !!sym(svr_abfafb_long_name),
    values_from = svr_number_col
  )

# Print
# summary(tb_states_choices_bisec_wider)
kable(head(tb_states_choices_bisec_wider %>%
  select(-one_of('p', 'f_p', 'f_p_t_f_a')), 10)) %>%
  kable_styling_fc_wide()
```

INDI_ID	fl_A	fl_alpha	biseciter	a	b	fa	fb
1	-2	0.1	0	0.000000	100.0000	100.00000	-12880.283918
1	-2	0.1	1	0.000000	50.0000	100.00000	-5666.955763
1	-2	0.1	2	0.000000	25.0000	100.00000	-2464.562178
1	-2	0.1	3	0.000000	12.5000	100.00000	-1041.574253
1	-2	0.1	4	0.000000	6.2500	100.00000	-408.674764
1	-2	0.1	5	0.000000	3.1250	100.00000	-126.904283
1	-2	0.1	6	0.000000	1.5625	100.00000	-1.328965
1	-2	0.1	7	0.781250	1.5625	54.69612	-1.328965
1	-2	0.1	8	1.171875	1.5625	27.46061	-1.328965
1	-2	0.1	9	1.367188	1.5625	13.23495	-1.328965

```
kable(head(tb_states_choices_bisec_wider %>%
  select(-one_of('p', 'f_p', 'f_p_t_f_a')), 10)) %>%
  kable_styling_fc_wide()
```

7.1.1.4 Graph Bisection Iteration Results

Actually we want to graph based on the long results, not the wider. Wider easier to view in table.

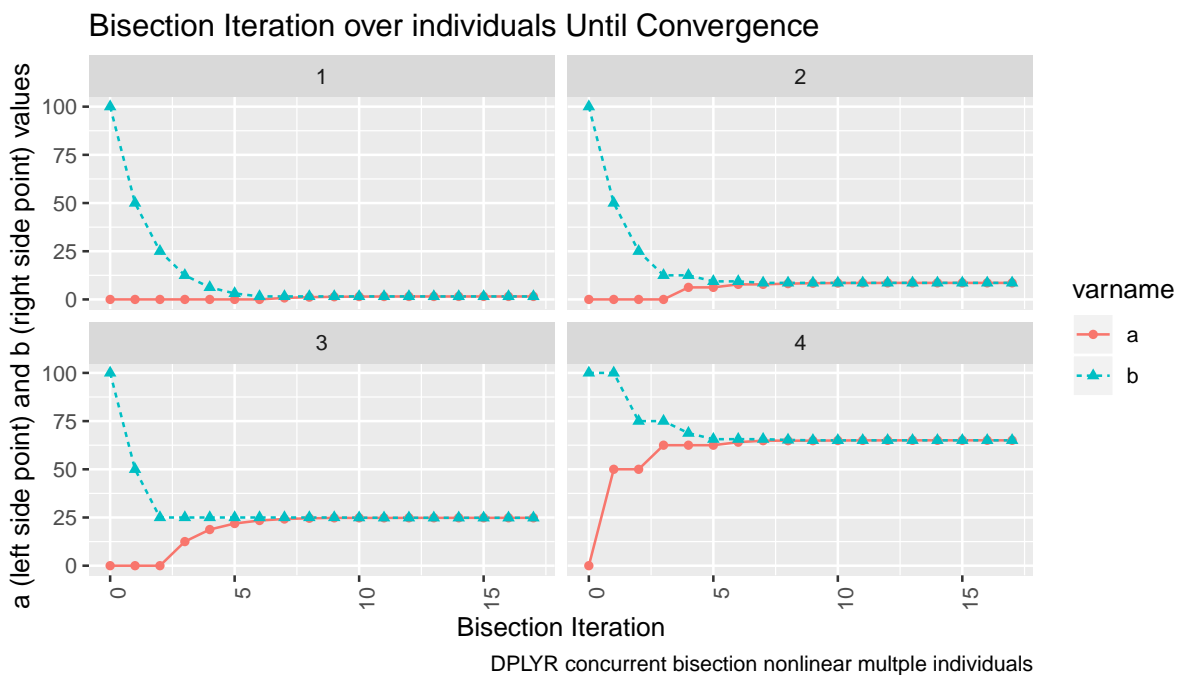
INDI_ID	fl_A	fl_alpha	biseciter	a	b	fa	fb
1	-2	0.1	0	0.000000	100.0000	100.00000	-12880.283918
1	-2	0.1	1	0.000000	50.0000	100.00000	-5666.955763
1	-2	0.1	2	0.000000	25.0000	100.00000	-2464.562178
1	-2	0.1	3	0.000000	12.5000	100.00000	-1041.574253
1	-2	0.1	4	0.000000	6.2500	100.00000	-408.674764
1	-2	0.1	5	0.000000	3.1250	100.00000	-126.904283
1	-2	0.1	6	0.000000	1.5625	100.00000	-1.328965
1	-2	0.1	7	0.781250	1.5625	54.69612	-1.328965
1	-2	0.1	8	1.171875	1.5625	27.46061	-1.328965
1	-2	0.1	9	1.367188	1.5625	13.23495	-1.328965

Graph results

```

lineplot <- tb_states_choices_bisec_long %>%
  mutate(!sym(svr_bisect_iter) := as.numeric(!sym(svr_bisect_iter))) %>%
  filter(!sym(svr_abfafb_long_name) %in% c('a', 'b')) %>%
  ggplot(aes(x=!sym(svr_bisect_iter), y=!sym(svr_number_col),
             colour=!sym(svr_abfafb_long_name),
             linetype=!sym(svr_abfafb_long_name),
             shape=!sym(svr_abfafb_long_name))) +
  facet_wrap( ~ INDI_ID) +
  geom_line() +
  geom_point() +
  labs(title = 'Bisection Iteration over individuals Until Convergence',
       x = 'Bisection Iteration',
       y = 'a (left side point) and b (right side point) values',
       caption = 'DPLYR concurrent bisection nonlinear multiple individuals') +
  theme(axis.text.x = element_text(angle = 90, hjust = 1))
print(lineplot)

```



Chapter 8

Mathematics and Statistics

8.1 Distributions

8.1.1 Integrate Over Normal Guassian Process Shock

Go back to [fan's REconTools Package](#), [R4Econ Repository](#) ([bookdown site](#)), or [Intro Stats with R Repository](#).

Some Common parameters

```
fl_eps_mean = 10
fl_eps_sd = 50
fl_cdf_min = 0.000001
fl_cdf_max = 0.999999
ar_it_draws <- seq(1, 1000)
```

8.1.1.1 Randomly Sample and Integrate (Monte Carlo integration)

Compare randomly drawn normal shock mean and known mean. How does simulated mean change with draws. Actual integral equals to 10, as sample size increases, the sample mean approaches the integration results, but this is expensive, even with ten thousand draws, not very exact.

```
# Simulate Draws
set.seed(123)
ar_fl_means <-
  sapply(ar_it_draws, function(x)
    return(mean(rnorm(x[1], mean=fl_eps_mean, sd=fl_eps_sd))))
ar_fl_sd <-
  sapply(ar_it_draws, function(x)
    return(sd(rnorm(x[1], mean=fl_eps_mean, sd=fl_eps_sd))))

mt_sample_means <- cbind(ar_it_draws, ar_fl_means, ar_fl_sd)
colnames(mt_sample_means) <- c('draw_count', 'mean', 'sd')
tb_sample_means <- as_tibble(mt_sample_means)

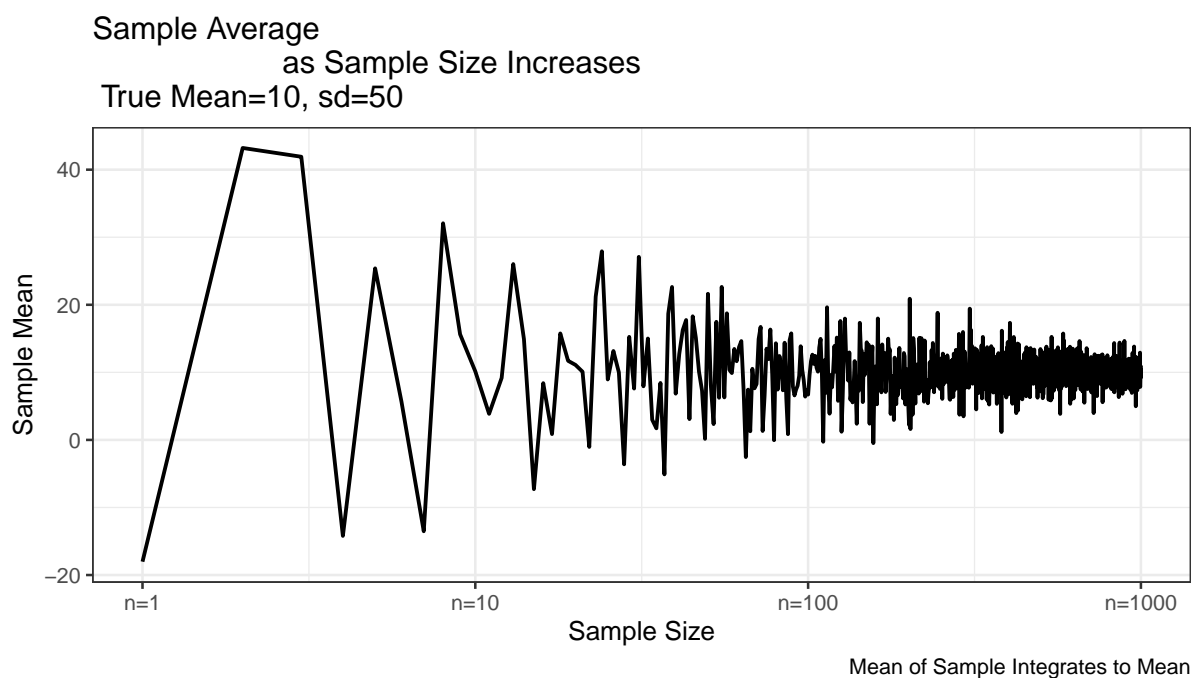
# Graph
# x-labels
x.labels <- c('n=1', 'n=10', 'n=100', 'n=1000')
x.breaks <- c(1, 10, 100, 1000)

# Graph Results--Draw
plt_mean <- tb_sample_means %>%
  ggplot(aes(x=draw_count, y=mean)) +
  geom_line(size=0.75) +
```

```

labs(title = paste0('Sample Average
                    as Sample Size Increases\n True Mean=',
                    fl_eps_mean, ', sd=', fl_eps_sd),
     x = 'Sample Size',
     y = 'Sample Mean',
     caption = 'Mean of Sample Integrates to Mean') +
scale_x_continuous(trans='log10', labels = x.labels, breaks = x.breaks) +
theme_bw()
print(plt_mean)

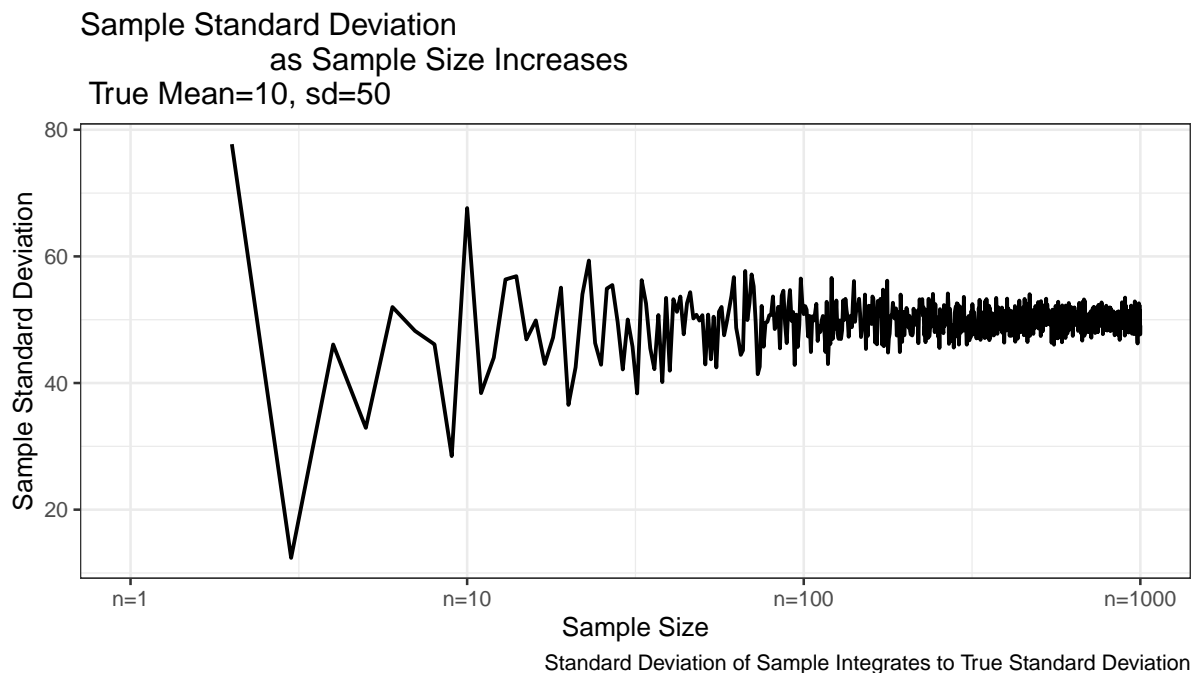
```



```

plt_sd <- tb_sample_means %>%
  ggplot(aes(x=draw_count, y=sd)) +
  geom_line(size=0.75) +
  labs(title = paste0('Sample Standard Deviation
                    as Sample Size Increases\n True Mean=',
                    fl_eps_mean, ', sd=', fl_eps_sd),
     x = 'Sample Size',
     y = 'Sample Standard Deviation',
     caption = 'Standard Deviation of Sample Integrates to True Standard Deviation') +
scale_x_continuous(trans='log10', labels = x.labels, breaks = x.breaks) +
theme_bw()
print(plt_sd)

```

8.1.1.2 Integration By Symmetric Uneven Rectangle

Draw on grid from probability space, and then find use norm inverse to find corresponding x point. Under this approach, each rectangle is suppose to approximate the same same. So even area, but uneven width.

Resulting integration is rectangle based, but rectangle width differ. Th rectangle have wider width as they move away from the mean, and thin width close to the mean. This is much more stable than the random draw method, and approximates the true answer more accurately.

```
mt_fl_means <-
  supply(ar_it_draws, function(x) {

    fl_prob_break = (fl_cdf_max - fl_cdf_min)/(x[1])
    ar_eps_bounds <- qnorm(seq(fl_cdf_min, fl_cdf_max,
                              by=(fl_cdf_max - fl_cdf_min)/(x[1])),
                          mean = fl_eps_mean, sd = fl_eps_sd)
    ar_eps_val <- (tail(ar_eps_bounds, -1) + head(ar_eps_bounds, -1))/2
    ar_eps_prb <- rep(fl_prob_break/(fl_cdf_max - fl_cdf_min), x[1])
    ar_eps_fev <- dnorm(ar_eps_val,
                        mean = fl_eps_mean, sd = fl_eps_sd)

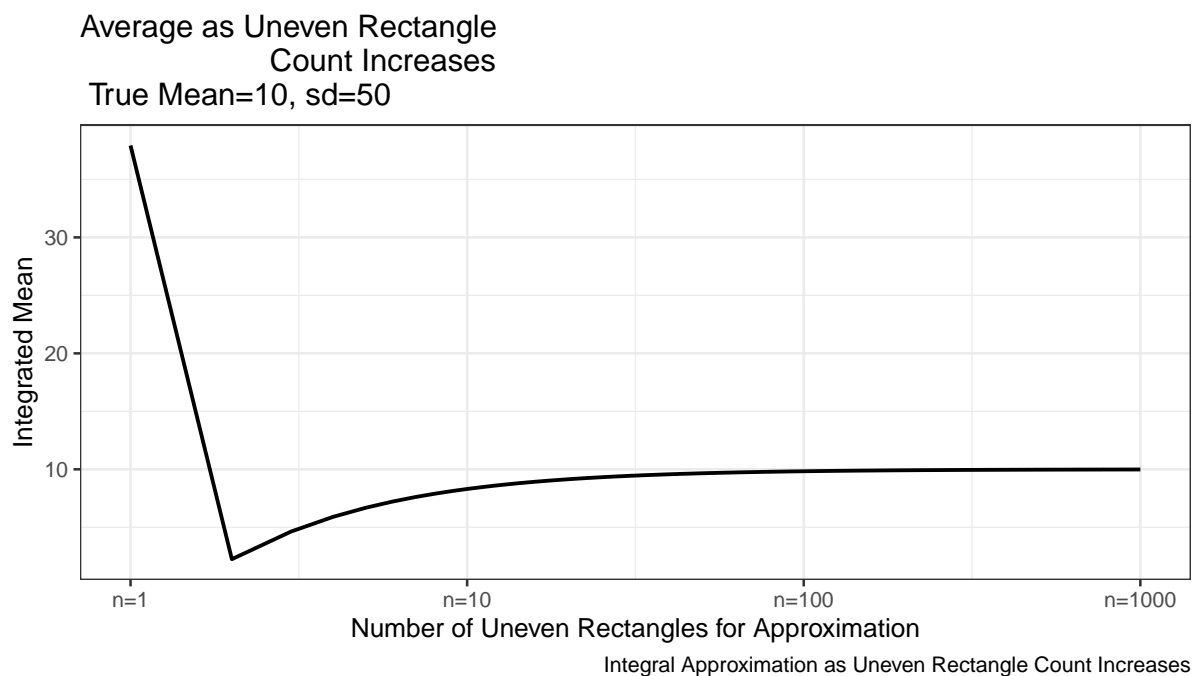
    fl_cdf_total_approx <- sum(ar_eps_fev*diff(ar_eps_bounds))
    fl_mean_approx <- sum(ar_eps_val*(ar_eps_fev*diff(ar_eps_bounds)))
    fl_sd_approx <- sqrt(sum((ar_eps_val-fl_mean_approx)^2*(ar_eps_fev*diff(ar_eps_bounds))))

    return(list(cdf=fl_cdf_total_approx, mean=fl_mean_approx, sd=fl_sd_approx))
  })

mt_sample_means <- cbind(ar_it_draws, as_tibble(t(mt_fl_means)) %>% unnest())
colnames(mt_sample_means) <- c('draw_count', 'cdf', 'mean', 'sd')
tb_sample_means <- as_tibble(mt_sample_means)

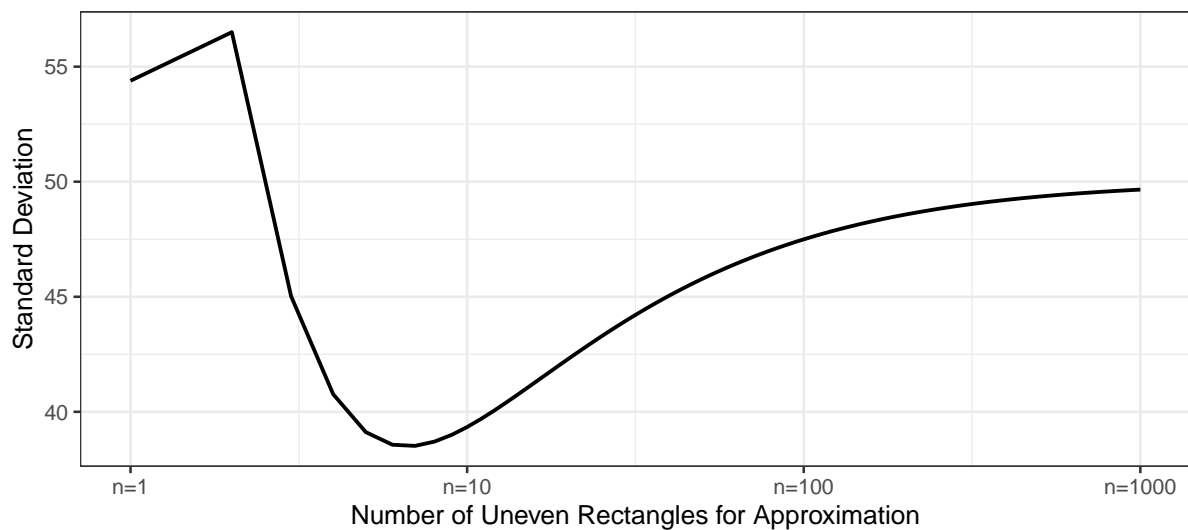
# Graph
# x-labels
x.labels <- c('n=1', 'n=10', 'n=100', 'n=1000')
x.breaks <- c(1, 10, 100, 1000)
```

```
# Graph Results--Draw
plt_mean <- tb_sample_means %>%
  ggplot(aes(x=draw_count, y=mean)) +
  geom_line(size=0.75) +
  labs(title = paste0('Average as Uneven Rectangle
                      Count Increases\n True Mean=',
                      fl_eps_mean, ', sd=', fl_eps_sd),
       x = 'Number of Uneven Rectangles for Approximation',
       y = 'Integrated Mean',
       caption = 'Integral Approximation as Uneven Rectangle Count Increases') +
  scale_x_continuous(trans='log10', labels = x.labels, breaks = x.breaks) +
  theme_bw()
print(plt_mean)
```



```
plt_sd <- tb_sample_means %>%
  ggplot(aes(x=draw_count, y=sd)) +
  geom_line(size=0.75) +
  labs(title = paste0('Standard Deviation as Uneven Rectangle
                      Count Increases\n True Mean=',
                      fl_eps_mean, ', sd=', fl_eps_sd),
       x = 'Number of Uneven Rectangles for Approximation',
       y = 'Standard Deviation',
       caption = 'Integral Approximation as Uneven Rectangle Count Increases') +
  scale_x_continuous(trans='log10', labels = x.labels, breaks = x.breaks) +
  theme_bw()
print(plt_sd)
```

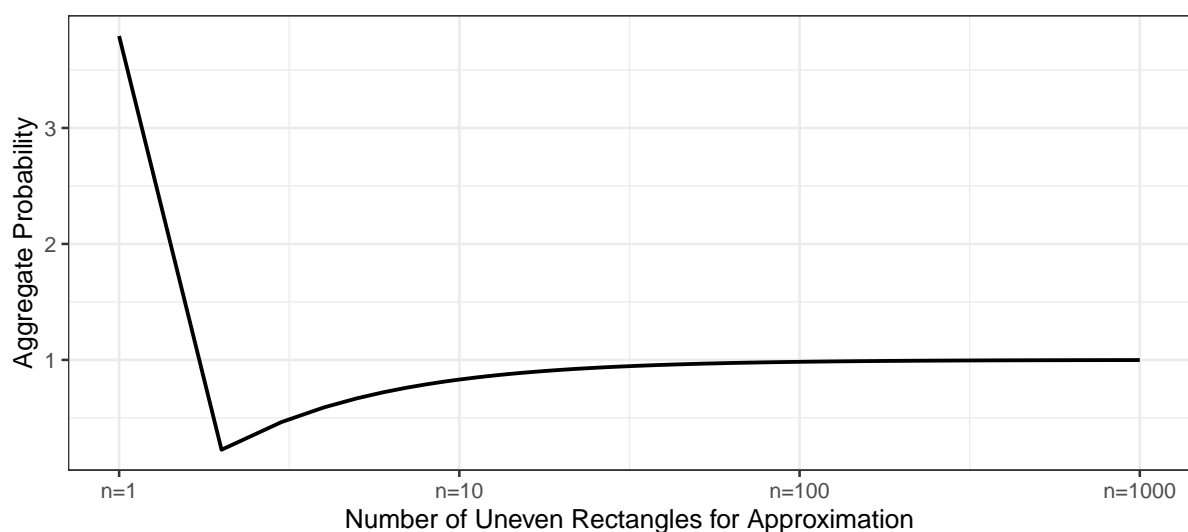
Standard Deviation as Uneven Rectangle
Count Increases
True Mean=10, sd=50



Integral Approximation as Uneven Rectangle Count Increases

```
plt_cdf <- tb_sample_means %>%
  ggplot(aes(x=draw_count, y=cdf)) +
  geom_line(size=0.75) +
  labs(title = paste0('Aggregate Probability as Uneven Rectangle
                        Count Increases\n True Mean=',
                        fl_eps_mean, ', sd=', fl_eps_sd),
        x = 'Number of Uneven Rectangles for Approximation',
        y = 'Aggregate Probability',
        caption = 'Aggregate Probability Approximation as Uneven Rectangle Count Increases') +
  scale_x_continuous(trans='log10', labels = x.labels, breaks = x.breaks) +
  theme_bw()
print(plt_cdf)
```

Aggregate Probability as Uneven Rectangle
Count Increases
True Mean=10, sd=50



Aggregate Probability Approximation as Uneven Rectangle Count Increases

8.1.1.3 Integration By Constant Width Rectangle (Trapezoidal rule)

This is implementing even width rectangle, even along x-axis take points, and measure $f(x)$. Rectangle width are the same. This is even width, but uneven area. Note that this method approximates the true answer much better and more quickly.

```
mt_fl_means <-
  sapply(ar_it_draws, function(x) {

    fl_eps_min <- qnorm(fl_cdf_min, mean = fl_eps_mean, sd = fl_eps_sd)
    fl_eps_max <- qnorm(fl_cdf_max, mean = fl_eps_mean, sd = fl_eps_sd)
    fl_gap <- (fl_eps_max - fl_eps_min) / (x[1])
    ar_eps_bounds <- seq(fl_eps_min, fl_eps_max, by = fl_gap)
    ar_eps_val <- (tail(ar_eps_bounds, -1) + head(ar_eps_bounds, -1)) / 2
    ar_eps_prb <- dnorm(ar_eps_val, mean = fl_eps_mean, sd = fl_eps_sd) * fl_gap

    fl_cdf_total_approx <- sum(ar_eps_prb)
    fl_mean_approx <- sum(ar_eps_val * ar_eps_prb)
    fl_sd_approx <- sqrt(sum((ar_eps_val - fl_mean_approx)^2 * ar_eps_prb))

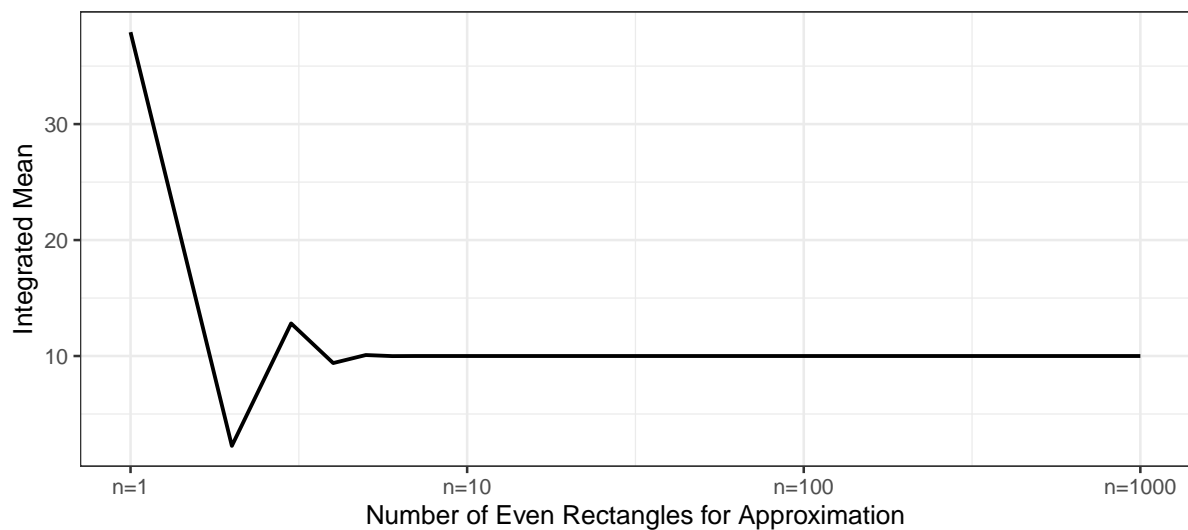
    return(list(cdf = fl_cdf_total_approx, mean = fl_mean_approx, sd = fl_sd_approx))
  })

mt_sample_means <- cbind(ar_it_draws, as_tibble(t(mt_fl_means))) %>% unnest()
colnames(mt_sample_means) <- c('draw_count', 'cdf', 'mean', 'sd')
tb_sample_means <- as_tibble(mt_sample_means)

# Graph
# x-labels
x.labels <- c('n=1', 'n=10', 'n=100', 'n=1000')
x.breaks <- c(1, 10, 100, 1000)

# Graph Results--Draw
plt_mean <- tb_sample_means %>%
  ggplot(aes(x = draw_count, y = mean)) +
  geom_line(size = 0.75) +
  labs(title = paste0('Average as Even Rectangle
                        Count Increases\n True Mean = ',
                        fl_eps_mean, ', sd = ', fl_eps_sd),
       x = 'Number of Even Rectangles for Approximation',
       y = 'Integrated Mean',
       caption = 'Integral Approximation as Even Rectangle Count Increases') +
  scale_x_continuous(trans = 'log10', labels = x.labels, breaks = x.breaks) +
  theme_bw()
print(plt_mean)
```

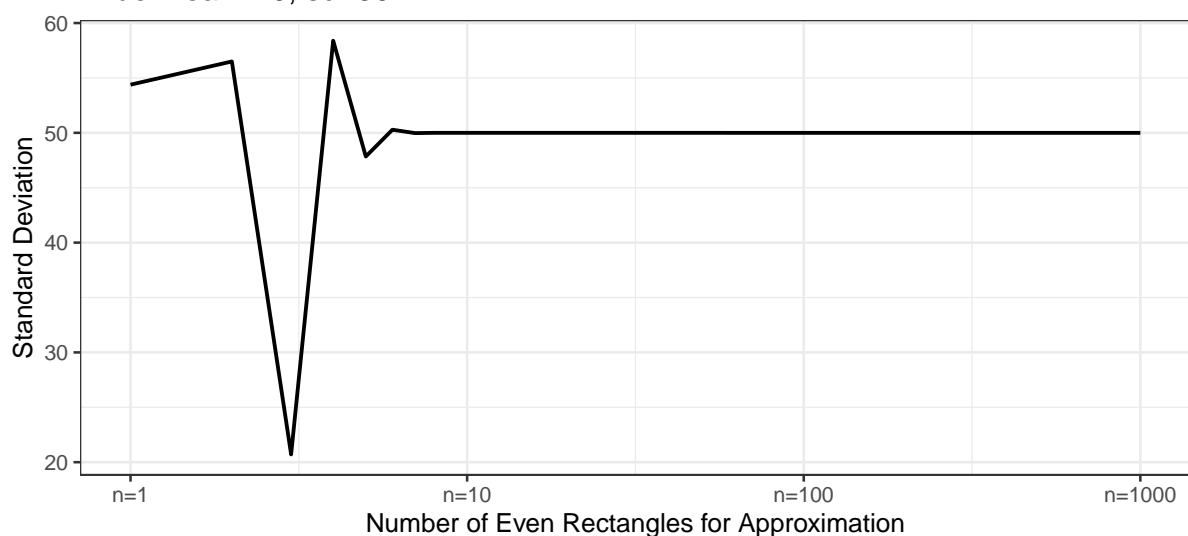
Average as Even Rectangle
Count Increases
True Mean=10, sd=50



Integral Approximation as Even Rectangle Count Increases

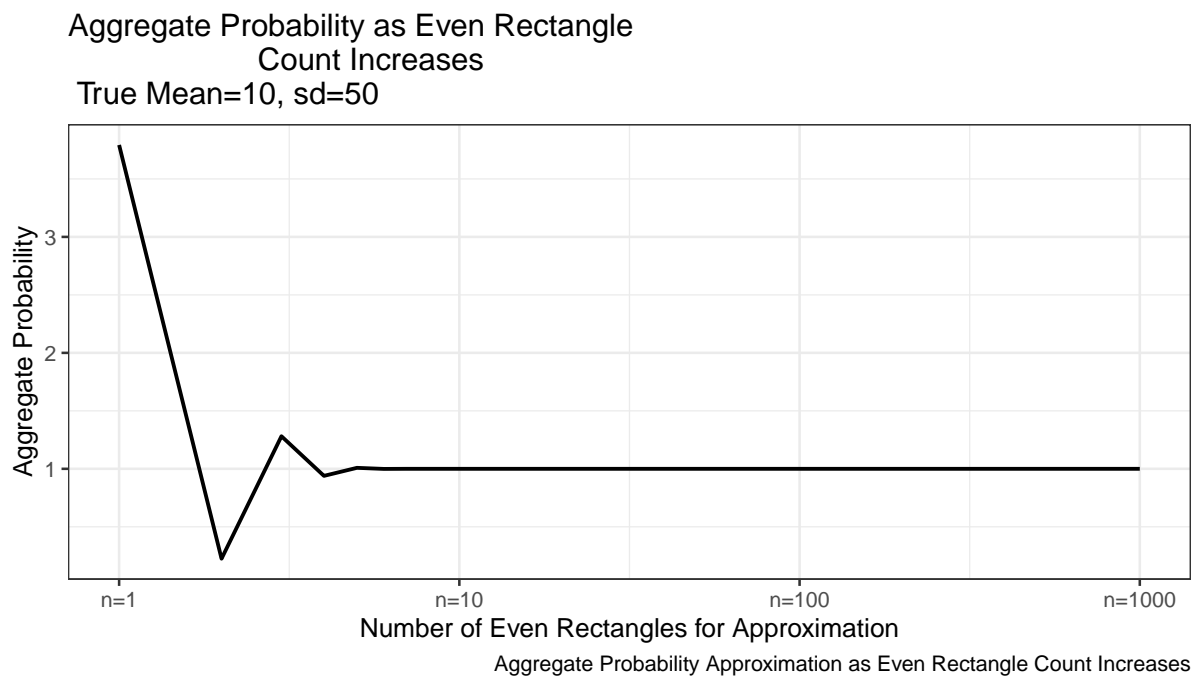
```
plt_sd <- tb_sample_means %>%
  ggplot(aes(x=draw_count, y=sd)) +
  geom_line(size=0.75) +
  labs(title = paste0('Standard Deviation as Even Rectangle
                        Count Increases\n True Mean=',
                        fl_eps_mean, ', sd=', fl_eps_sd),
        x = 'Number of Even Rectangles for Approximation',
        y = 'Standard Deviation',
        caption = 'Integral Approximation as Even Rectangle Count Increases') +
  scale_x_continuous(trans='log10', labels = x.labels, breaks = x.breaks) +
  theme_bw()
print(plt_sd)
```

Standard Deviation as Even Rectangle
Count Increases
True Mean=10, sd=50



Integral Approximation as Even Rectangle Count Increases

```
plt_cdf <- tb_sample_means %>%
  ggplot(aes(x=draw_count, y=cdf)) +
  geom_line(size=0.75) +
  labs(title = paste0('Aggregate Probability as Even Rectangle
                        Count Increases\n True Mean=',
                        fl_eps_mean,', sd=',fl_eps_sd),
        x = 'Number of Even Rectangles for Approximation',
        y = 'Aggregate Probability',
        caption = 'Aggregate Probability Approximation as Even Rectangle Count Increases') +
  scale_x_continuous(trans='log10', labels = x.labels, breaks = x.breaks) +
  theme_bw()
print(plt_cdf)
```



8.2 Analytical Solutions

8.2.1 Linear Scalar $f(x)=0$ Solutions

Go back to [fan's REconTools](#) Package, [R4Econ](#) Repository ([bookdown site](#)), or [Intro Stats with R](#) Repository.

8.2.1.1 Ratio

Here are some common ratios.

8.2.1.1.1 Unif Draw Min and Max Ratio We want to draw numbers such that we have some mean b , and that the possible maximum and minimum value drawn are at most a times apart. Given b and a , solve for x .

$$f(x) = \frac{b+x}{b-x} - a = 0$$

$$b \cdot a - x \cdot a = b + x \cdot a - b = x + x \cdot ab(a-1) = x(a+1) \Rightarrow x = \frac{b(a-1)}{a+1}$$

Uniformly draw

```
b <- 100
a <- 2
x <- (b*(a-1))/(a+1)
ar_unif_draws <- runif(100, min=b-x, max=b+x)
fl_max_min_ratio <- max(ar_unif_draws)/min(ar_unif_draws)
cat('fl_max_min_ratio =', fl_max_min_ratio, 'is close to a =', a, '\n')

## fl_max_min_ratio = 1.965882 is close to a = 2
```

8.3 Inequality Models

8.3.1 Gini Discrete Sample

Go back to [fan's REconTools Package](#), [R4Econ Repository \(bookdown site\)](#), or [Intro Stats with R Repository](#).

This works out how the `ff_dist_gini_vector_pos` function works from [Fan's REconTools Package](#).

8.3.1.1 Gini Formula for Discrete Sample

There is an vector values (all positive). This could be height information for N individuals. It could also be income information for N individuals. Calculate the **GINI** coefficient treating the given vector as population. This is not an estimation exercise where we want to estimate population gini based on a sample. The given array is the population. The population is discrete, and only has these N individuals in the length n vector.

Note that when the sample size is small, there is a limit to inequality using the formula defined below given each N . So for small N , can not really compare inequality across arrays with different N , can only compare arrays with the same N . In another word, if 1 of N individual holds all resource, as N increases, GINI will asymptote to 1, but it is very far away from 1 for low N .

The GINI formula used here is:

$$GINI = 1 - \frac{2}{N+1} \cdot \left(\sum_{i=1}^N \sum_{j=1}^i x_j \right) \cdot \left(\sum_{i=1}^N x_i \right)^{-1}$$

Derive the formula in the steps below.

Step 1 Area Formula

$$\Gamma = \sum_{i=1}^N \frac{1}{N} \cdot \left(\sum_{j=1}^i \left(\frac{x_j}{\sum_{\hat{j}=1}^N x_{\hat{j}}} \right) \right)$$

Step 2 Total Area Given Perfect equality

With perfect equality $x_i = a$ for all i , so need to divide by that.

$$\Gamma^{\text{equal}} = \sum_{i=1}^N \frac{1}{N} \cdot \left(\sum_{j=1}^i \left(\frac{a}{\sum_{\hat{j}=1}^N a} \right) \right) = \frac{N+1}{N} \cdot \frac{1}{2}$$

As the number of elements of the vecotr increases:

$$\lim_{N \rightarrow \infty} \Gamma^{\text{equal}} = \lim_{N \rightarrow \infty} \frac{N+1}{N} \cdot \frac{1}{2} = \frac{1}{2}$$

Step 3 Arriving at Finite Vector Gini Formula

Given what we have from above, we obtain the gini formula, divide by total area below 45 degree line.

$$GINI = 1 - \left(\sum_{i=1}^N \sum_{j=1}^i x_j \right) \cdot \left(N \cdot \sum_{i=1}^N x_i \right)^{-1} \cdot \left(\frac{N+1}{N} \cdot \frac{1}{2} \right)^{-1} = 1 - \frac{2}{N+1} \cdot \left(\sum_{i=1}^N \sum_{j=1}^i x_j \right) \cdot \left(\sum_{i=1}^N x_i \right)^{-1}$$

Step 4 Maximum Inequality given N

Suppose $x_i = 0$ for all $i < N$, then:

$$GINI_{x_i=0 \text{ except } i=N} = 1 - \frac{2}{N+1} \cdot X_N \cdot (X_N)^{-1} = 1 - \frac{2}{N+1}$$

$$\lim_{N \rightarrow \infty} GINI_{x_i=0 \text{ except } i=N} = 1 - \lim_{N \rightarrow \infty} \frac{2}{N+1} = 1$$

Note that for small N , for example if $N = 10$, even when one person holds all income, all others have 0 income, the formula will not produce gini is zero, but that gini is equal to $\frac{2}{11} \approx 0.1818$. If $N = 2$, inequality is at most, $\frac{2}{3} \approx 0.667$.

$$MostUnequalGINI(N) = 1 - \frac{2}{N+1} = \frac{N-1}{N+1}$$

8.3.1.2 Implement GINI Formula

The **GINI** formula just derived is trivial to compute.

1. scalar: $\frac{2}{N+1}$
2. cumsum: $\sum_{j=1}^i x_j$
3. sum of cumsum: $\left(\sum_{i=1}^N \sum_{j=1}^i x_j \right)$
4. sum: $\sum_{i=1}^N X_i$

There are no package dependencies. Define the formula here:

```
# Formula, directly implement the GINI formula Following Step 4 above
fv_dist_gini_vector_pos_test <- function(ar_pos) {
  # Check length and given warning
  it_n <- length(ar_pos)
  if (it_n <= 100) warning('Data vector has n=', it_n, ', max-inequality/max-gini=', (it_n-1)/(it_n +
  # Sort
  ar_pos <- sort(ar_pos)
  # formula implement
  fl_gini <- 1 - ((2/(it_n+1)) * sum(cumsum(ar_pos)) * (sum(ar_pos))^(-1))
  return(fl_gini)
}
```

Generate a number of examples Arrays for testing

```
# Example Arrays of data
ar_equal_n1 = c(1)
ar_ineql_n1 = c(100)

ar_equal_n2 = c(1,1)
ar_ineql_alittle_n2 = c(1,2)
ar_ineql_somewht_n2 = c(1,2^3)
ar_ineql_alotine_n2 = c(1,2^5)
ar_ineql_veryvry_n2 = c(1,2^8)
ar_ineql_mostmst_n2 = c(1,2^13)

ar_equal_n10 = c(2,2,2,2,2,2, 2, 2, 2, 2)
```



```

ar_ineql_some_n10 = c(1,2,3,5,8,13,21,34,55,89)
ar_ineql_very_n10 = c(1,2^2,3^2,5^2,8^2,13^2,21^2,34^2,55^2,89^2)
ar_ineql_extr_n10 = c(1,2^2,3^3,5^4,8^5,13^6,21^7,34^8,55^9,89^10)

```

Now test the example arrays above using the function based on our formula:

```

##
## Small N=1 Hard-Code
## ar_equal_n1: 0
## ar_ineql_n1: 0

##
## Small N=2 Hard-Code, converge to 1/3, see formula above
## ar_ineql_alittle_n2: 0.1111111
## ar_ineql_somewht_n2: 0.2592593
## ar_ineql_alotine_n2: 0.3131313
## ar_ineql_veryvry_n2: 0.3307393

##
## Small N=10 Hard-Code, converge to 9/11=0.8181, see formula above
## ar_equal_n10: 0
## ar_ineql_some_n10: 0.5395514
## ar_ineql_very_n10: 0.7059554
## ar_ineql_extr_n10: 0.8181549

```

8.3.2 Atkinson Family Utility

Go back to [fan's REconTools](#) Package, [R4Econ](#) Repository ([bookdown site](#)), or [Intro Stats with R](#) Repository.

8.3.2.1 Individual Outcomes and Preference

How does the Atkinson Family utility function work? The Atkinson Family Utility has the following functional form.

$$V^{\text{social}} = (\alpha \cdot A^\lambda + \beta \cdot B^\lambda)^{\frac{1}{\lambda}}$$

Several key issues here:

1. V^{social} is the utility of some social planner
2. A and B are allocations for Alex and Ben.
3. α and β are biases that a social planner has for Alex and Ben: $\alpha + \beta = 1$, $\alpha > 0$, and $\beta > 0$
4. $-\infty < \lambda \leq 1$ is a measure of inequality aversion
 - $\lambda = 1$ is when the planner cares about weighted total allocations (efficient, Utilitarian)
 - $\lambda = -\infty$ is when the planner cares about only the minimum between A and B allocations (equality, Rawlsian)

What if only care about Alex? Clearly, if the planner only cares about Ben, $\beta = 1$, then:

$$V^{\text{social}} = (B^\lambda)^{\frac{1}{\lambda}} = B$$

Clearly, regardless of the value of λ , as B increases V increases. What Happens to V when A or B increases? What is the derivative of V with respect to A or B ?

$$\frac{\partial V}{\partial A} = \frac{1}{\lambda} (\alpha A^\lambda + \beta B^\lambda)^{\frac{1}{\lambda}-1} \cdot \lambda \alpha A^{\lambda-1}$$

$$\frac{\partial V}{\partial A} = (\alpha A^\lambda + \beta B^\lambda)^{\frac{1-\lambda}{\lambda}} \cdot \alpha A^{\lambda-1} > 0$$

Note that $\frac{\partial V}{\partial A} > 0$. When $\lambda < 0$, $Z^\lambda > 0$. For example $10^{-2} = \frac{1}{100}$. And For example $0.1^{\frac{3}{-2}} = \frac{1}{0.1^{1.5}}$. Still Positive.

While the overall V increases with increasing A , but if we did not have the outer power term, the situation is different. In particular, when $\lambda < 0$:

$$\text{if } \lambda < 0 \text{ then } \frac{d(\alpha A^\lambda + \beta B^\lambda)}{dA} = \alpha \lambda A^{\lambda-1} < 0$$

Without the outer $\frac{1}{\lambda}$ power, negative λ would lead to decreasing weighted sum. But:

$$\text{if } \lambda < 0 \text{ then } \frac{dG^{\frac{1}{\lambda}}}{dG} = \frac{1}{\lambda} \cdot G^{\frac{1-\lambda}{\lambda}} < 0$$

so when G is increasing and $\lambda < 0$, V would decrease. But when $G(A, B)$ is decreasing, as is the case with increasing A when $\lambda < 0$, V will actually increase. This confirms that $\frac{\partial V}{\partial A} > 0$ for $\lambda < 0$. The result is symmetric for $\lambda > 0$.

8.3.2.2 Indifference Curve Graph

Given V^* , we can show the combinations of A and B points that provide the same utility. We want to be able to potentially draw multiple indifference curves at the same time. Note that indifference curves are defined by α, λ only. Each indifference curve is a set of A and B coordinates. So to generate multiple indifference curves means to generate many sets of A, B associated with different planner preferences, and then these could be graphed out.

```
# A as x-axis, need bounds on A
fl_A_min = 0.01
fl_A_max = 3
it_A_grid = 10000

# Define parameters
# ar_lambda <- 1 - (10^(c(seq(-2,2, length.out=3))))
ar_lambda <- c(1, 0.6, 0.06, -6)
ar_beta <- seq(0.25, 0.75, length.out = 3)
ar_beta <- c(0.3, 0.5, 0.7)
ar_v_star <- seq(1, 2, length.out = 1)
tb_pref <- as_tibble(cbind(ar_lambda)) %>%
  expand_grid(ar_beta) %>% expand_grid(ar_v_star) %>%
  rename_all(~c('lambda', 'beta', 'vstar')) %>%
  rowid_to_column(var = "indiff_id")

# Generate indifference points with apply and anonymous function
# tb_pref, whatever is selected from it, must be all numeric
# if there are strings, would cause conversion error.
ls_df_indiff <- apply(tb_pref, 1, function(x){
  indiff_id <- x[1]
  lambda <- x[2]
  beta <- x[3]
  vstar <- x[4]
  ar_fl_A_indiff <- seq(fl_A_min, fl_A_max, length.out=it_A_grid)
  ar_fl_B_indiff <- (((vstar^lambda) -
    (beta*ar_fl_A_indiff^(lambda)))/(1-beta))^(1/lambda)
  mt_A_B_indiff <- cbind(indiff_id, lambda, beta, vstar,
```

```

      ar_fl_A_indiff, ar_fl_B_indiff)
colnames(mt_A_B_indiff) <- c('indiff_id', 'lambda', 'beta', 'vstar',
                             'indiff_A', 'indiff_B')
tb_A_B_indiff <- as_tibble(mt_A_B_indiff) %>%
  rowid_to_column(var = "A_grid_id") %>%
  filter(indiff_B >= 0 & indiff_B <= max(ar_fl_A_indiff))
return(tb_A_B_indiff)
})
df_indiff <- do.call(rbind, ls_df_indiff) %>% drop_na()

```

Note that many more A grid points are needed to fully plot out the leontief line.

```

# Labeling
st_title <- paste0('Indifference Curves Aktinson Atkinson Utility (CES)')
st_subtitle <- paste0('Each Panel Different beta=A\'s Weight lambda=inequality aversion\n',
                      'https://fanwangecon.github.io/',
                      'R4Econ/math/func_ineq/htmlpdf/fs_atkinson_ces.html')
st_caption <- paste0('Indifference Curve 2 Individuals, ',
                     'https://fanwangecon.github.io/R4Econ/')

st_x_label <- 'A'
st_y_label <- 'B'

# Graphing
plt_indiff <-
  df_indiff %>% mutate(lambda = as_factor(lambda),
                      beta = as_factor(beta),
                      vstar = as_factor(vstar)) %>%
  ggplot(aes(x=indiff_A, y=indiff_B,
             colour=lambda)) +
  facet_wrap( ~ beta) +
  geom_line(size=1) +
  labs(title = st_title, subtitle = st_subtitle,
       x = st_x_label, y = st_y_label, caption = st_caption) +
  theme_bw()

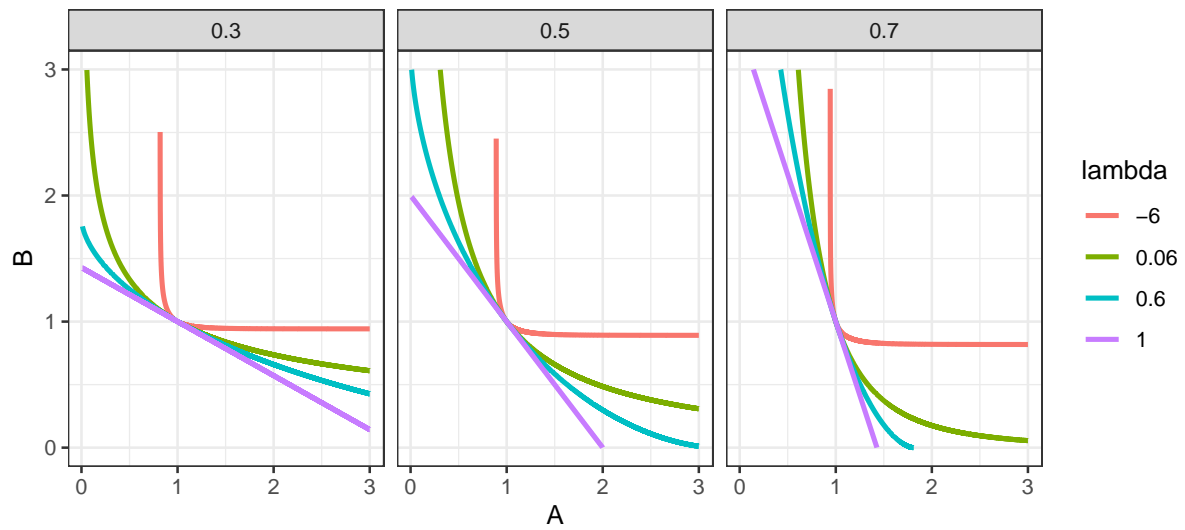
# show
print(plt_indiff)

```

Indifference Curves Aktinson Atkinson Utility (CES)

Each Panel Different beta=A's Weight lambda=inequality aversion

https://fanwangecon.github.io/R4Econ/math/func_ineq/htmlpdf/fs_atkinson_ces.html



Indifference Curve 2 Individuals, <https://fanwangecon.github.io/R4Econ/>

Appendix A

Index and Code Links

A.1 Array, Matrix, Dataframe links

A.1.1 Section 1.1 List links

1. Multi-dimensional Named Lists: [rmd](#) | [r](#) | [pdf](#) | [html](#)
 - Initiate Empty List. Named one and two dimensional lists.
 - **r**: `vector(mode = "list", length = it_N) + names(list) <- paste0('e',seq()) + dimnames(ls2d)[[1]] <- paste0('r',seq()) + dimnames(ls2d)[[2]] <- paste0('c',seq())`
 - **tidyr**: `unnest()`

A.1.2 Section 1.2 Array links

1. Arrays Operations in R: [rmd](#) | [r](#) | [pdf](#) | [html](#)
 - Basic array operations in R.
 - **r**: `head() + tail() + na_if()`
2. Generate Special Arrays: [rmd](#) | [r](#) | [pdf](#) | [html](#)
 - Generate special arrays: log spaced array
 - **r**: `seq()`
3. String Operations: [rmd](#) | [r](#) | [pdf](#) | [html](#)
 - Split, concatenate, subset strings
 - **r**: `paste0() + sub() + gsub() + grepl() + sprintf() + tail() + strsplit() + basename() + dirname()`
4. Meshgrid Matrices, Arrays and Scalars: [rmd](#) | [r](#) | [pdf](#) | [html](#)
 - Meshgrid Matrices, Arrays and Scalars to form all combination dataframe.
 - **tidyr**: `expand_grid() + expand.grid()`

A.1.3 Section 1.3 Matrix links

1. Matrix Basics: [rmd](#) | [r](#) | [pdf](#) | [html](#)
 - Generate and combine fixed and random matrixes
 - **R**: `rbind() + matrix`
2. Linear Algebra Operations: [rmd](#) | [r](#) | [pdf](#) | [html](#)

A.1.4 Section 1.4 Variables in Dataframes links

1. Tibble Basics: [rmd](#) | [r](#) | [pdf](#) | [html](#)
 - generate tibbles, rename tibble variables, tibble row and column names
 - rename numeric sequential columns with string prefix and suffix
 - **dplyr**: `as_tibble(mt) + rename_all(~c(ar_names)) + rename_at(vars(starts_with("xx")), funs(str_replace(., "yy", "yyy"))) + rename_at(vars(num_range("ar_it")), funs(paste0(st,.))) + rowid_to_column() + colnames + rownames`
2. Label and Combine Factor Variables: [rmd](#) | [r](#) | [pdf](#) | [html](#)

- Convert numeric variables to factor variables, generate joint factors, and label factors.
- Graph MPG and 1/4 Miles Time (qsec) from the mtcars dataset over joint shift-type (am) and engine-type (vs) categories.
- **forcats**: `as_factor()` + `fct_recode()` + `fct_cross()`
- 3. **Randomly Draw Subsets of Rows from Matrix**: [rmd](#) | [r](#) | [pdf](#) | [html](#)
 - Given matrix, randomly sample rows, or select if random value is below threshold.
 - **r**: `rnorm()` + `sample()` + `df[sample(dim(df)[1], it_M, replace=FALSE),]`
 - **dplyr**: `case_when()` + `mutate(var = case_when(rnorm(n(), mean=0, sd=1) < 0 ~ 1, TRUE ~ 0))` %>% `filter(var == 1)`
- 4. **R Tibble Dataframe NA Values**: [rmd](#) | [r](#) | [pdf](#) | [html](#)
- 5. **R Tibble Dataframe String Manipulations**: [rmd](#) | [r](#) | [pdf](#) | [html](#)

A.2 Summarize Data links

A.2.1 Section 2.1 Counting Observation links

1. **Counting Basics**: [rmd](#) | [r](#) | [pdf](#) | [html](#)
 - uncount to generate panel skeleton from years in survey
 - **dplyr**: `uncount(yr_n)` + `group_by()` + `mutate(yr = row_number() + start_yr)`

A.2.2 Section 2.2 Sorting, Indexing, Slicing links

1. **Sorted Index, Interval Index and Expand Value from One Row**: [rmd](#) | [r](#) | [pdf](#) | [html](#)
 - Sort and generate index for rows
 - Generate negative and positive index based on deviations
 - Populate Values from one row to other rows
 - **dplyr**: `arrange()` + `row_number()` + `mutate(lowest = min(Sepal.Length))` + `case_when(row_number() == x ~ Sepal.Length)` + `mutate(Sepal.New = Sepal.Length[Sepal.Index == 1])`

A.2.3 Section 2.3 Group Statistics links

1. **Count Unique Groups and Mean within Groups**: [rmd](#) | [r](#) | [pdf](#) | [html](#)
 - Unique groups defined by multiple values and count obs within group.
 - Mean, sd, observation count for non-NA within unique groups.
 - **dplyr**: `group_by()` + `summarise(n())` + `summarise_if(is.numeric, funs(mean = mean(., na.rm = TRUE), n = sum(is.na(.)==0)))`
2. **By Groups, One Variable All Statistics**: [rmd](#) | [r](#) | [pdf](#) | [html](#)
 - Pick stats, overall, and by multiple groups, stats as matrix or wide row with name=(ctsvar + catevar + catelabel).
 - **tidyr**: `group_by()` + `summarize_at(, funs())` + `rename(!var := !!sym(var))` + `mutate(!var := paste0(var, 'str', !!syms(vars)))` + `gather()` + `unite()` + `spread(varcates, value)`
3. **By within Individual Groups Variables, Averages**: [rmd](#) | [r](#) | [pdf](#) | [html](#)
 - By Multiple within Individual Groups Variables.
 - Averages for all numeric variables within all groups of all group variables. Long to Wide to very Wide.
 - **tidyr**: `*gather()` + `group_by()` + `summarise_if(is.numeric, funs(mean(., na.rm = TRUE)))` + `mutate(all_m_cate = paste0(variable, '_c', value))` + `unite()` + `spread()*`

A.2.4 Section 2.4 Distributional Statistics links

1. **Tibble Basics**: [rmd](#) | [r](#) | [pdf](#) | [html](#)
 - input multiple variables with comma separated text strings
 - quantitative/continuous and categorical/discrete variables
 - histogram and summary statistics
 - **tibble**: `ar_one <- c(107.72, 101.28)` + `ar_two <- c(101.72, 101.28)` + `mt_data <- cbind(ar_one, ar_two)` + `as_tibble(mt_data)`

A.2.5 Section 2.5 Summarize Multiple Variables links

1. [Apply the Same Function over Columns of Matrix: rmd | r | pdf | html](#)
 - Replace NA values in selected columns by alternative values.
 - Cumulative sum over multiple variables.
 - Rename various various with common prefix and suffix appended.
 - **r**: `cumsum()` + `gsub()` + `mutate_at(vars(contains("V")), .funs = list(cumu = ~cumsum(.)))` + `rename_at(vars(contains("V")), list(~gsub("M", "", .)))`
 - **dplyr**: `rename_at()` + `mutate_at()` + `rename_at(vars(starts_with("V")), funs(str_replace(., "V", "var")))` + `mutate_at(vars(one_of(c("var1", "var2"))), list(~replace_na(., 99)))`

A.3 Functions links

A.3.1 Section 3.1 Dataframe Mutate links

1. [Nonlinear Function over Rows: rmd | r | pdf | html](#)
 - Evaluate nonlinear function $f(x_i, y_i, ar_x, ar_y, c, d)$, where c and d are constants, and ar_x and ar_y are arrays, both fixed. x_i and y_i vary over each row of matrix.
 - **dplyr**: `rowwise()` + `mutate(out = funct(inputs))`
2. [Evaluate Functions over Rows of Meshes Matrices: rmd | r | pdf | html](#)
 - Mesh states and choices together and rowwise evaluate many matrixes.
 - Cumulative sum over multiple variables.
 - Rename various various with common prefix and suffix appended.
 - **r**: `ffi <- function(fl_A, ar_B)`
 - **tidyr**: `expand_grid()` + `rowwise()` + `df %>% rowwise() %>% mutate(var = ffi(fl_A, ar_B))`
 - **ggplot2**: `geom_line()` + `facet_wrap()` + `geom_hline()` + `facet_wrap(. ~ var_id, scales = 'free')` + `geom_hline(yintercept=0, linetype="dashed", color="red", size=1)` +

A.3.2 Section 3.2 Dataframe Do Anything links

1. [Evaluate Function Do Anything Group Stack Results: rmd | r | pdf | html](#)
 - Group dataframe by categories, compute category specific output scalar or arrays based on within category variable information.
 - **dplyr**: `group_by(ID)` + `do(inc = rnorm(.N, mean = .mn, sd = .sd))` + `unnest(c(inc))` + `left_join(df, by="ID")`
2. [Expand Each Dataframe Row into More Rows: rmd | r | pdf | html](#)
 - Generate row value specific arrays of varying Length, and stack expanded dataframe.
 - **dplyr**: `do()` + `unnest()` + `left_join()` + `df %>% group_by(ID) %>% do(inc = rnorm(.Q, mean = .mean, sd = .sd)) %>% unnest(c(inc))`

A.3.3 Section 3.3 Apply and pmap links

1. [Apply and Mutate over Rows: rmd | r | pdf | html](#)
 - Evaluate function $f(x_i, y_i, c)$, where c is a constant and x and y vary over each row of a matrix, with index i indicating rows.
 - Get same results using `apply`, `sapply`, and `dplyr mutate`.
 - **r**: `do.call()` + `apply(mt, 1, func)` + `sapply(ls_ar, func, ar1, ar2)`
 - **purrr**: `rowwise()` + `unnest(out)` + `pmap(func)` + `unlist()`

A.4 Panel links

A.4.1 Section 4.1 Generate and Join links

1. [TIDYVERSE Generate Panel Data Structures: rmd | r | pdf | html](#)
 - Build skeleton panel frame with N observations and T periods.
 - **tidyr**: `rowid_to_column()` + `uncount()` + `group_by()` + `row_number()` + `ungroup()`
2. [R DPLYR Join Multiple Dataframes Together: rmd | r | pdf | html](#)
 - Join dataframes together with one or multiple keys. Stack dataframes together.

- **dplyr:** `filter() + rename(!sym(vsta) := !sym(vstb)) + mutate(var = rnorm(n())) + left_join(df, by=c('id'='id', 'vt'='vt')) + left_join(df, by=setNames(c('id', 'vt'), c('id', 'vt')))) + bind_rows()`

A.4.2 Section 4.2 Wide and Long links

1. **TIDYR Pivot Wider and Pivot Longer Examples:** [rmd](#) | [r](#) | [pdf](#) | [html](#)
 - Long roster to wide roster and cumulative sum attendance by date.
 - **dplyr:** `mutate(var = case_when(rnorm(n()) < 0 ~ 1, TRUE ~ 0)) + rename_at(vars(num_range("ar_it")), list(~paste0(st_prefix, ., "))) + mutate_at(vars(contains(str)), list(~replace_na(, 0))) + mutate_at(vars(contains(str)), list(~cumsum(.)))`

A.5 Linear Regression links

A.5.1 Section 5.1 OLS and IV links

1. **IV/OLS Regression:** [rmd](#) | [r](#) | [pdf](#) | [html](#)
 - R Instrumental Variables and Ordinary Least Square Regression store all Coefficients and Diagnostics as Dataframe Row.
 - **aer:** `library(aer) + ivreg(as.formula, diagnostics = TRUE)`
2. **M Outcomes and N RHS Alternatives:** [rmd](#) | [r](#) | [pdf](#) | [html](#)
 - There are M outcome variables and N alternative explanatory variables. Regress all M outcome variables on N endogenous/independent right hand side variables one by one, with controls and/or IVs, collect coefficients.
 - **dplyr:** `bind_rows(lapply(listx, function(x)(bind_rows(lapply(listy, regf.iv)))) + starts_with() + ends_with() + reduce(full_join)`

A.5.2 Section 5.2 Decomposition links

1. **Regression Decomposition:** [rmd](#) | [r](#) | [pdf](#) | [html](#)
 - Post multiple regressions, fraction of outcome variables' variances explained by multiple subsets of right hand side variables.
 - **dplyr:** `gather() + group_by(var) + mutate_at(vars, funs(mean = mean(.))) + rowSums(matmat) + mutate_if(is.numeric, funs(frac = (./value_var)))`*

A.6 Nonlinear Regression links

A.6.1 Section 6.1 Logit Regression links

1. **Logit Regression:** [rmd](#) | [r](#) | [pdf](#) | [html](#)
 - Logit regression testing and prediction.
 - **stats:** `glm(as.formula(), data, family='binomial') + predict(rs, newdata, type = "response")`

A.7 Optimization links

A.7.1 Section 7.1 Bisection links

1. **Concurrent Bisection over Dataframe Rows:** [rmd](#) | [r](#) | [pdf](#) | [html](#)
 - Post multiple regressions, fraction of outcome variables' variances explained by multiple subsets of right hand side variables.
 - **tidyr:** `*pivot_longer(cols = starts_with('abc'), names_to = c('a', 'b'), names_pattern = paste0('prefix', "(.)_(.)"), values_to = val) + pivot_wider(names_from = !sym(name), values_from = val) + mutate(!sym(abc) := case_when(efg < 0 ~ !sym(opq), TRUE ~ iso))`*
 - **ggplot2:** `geom_line() + facet_wrap() + geom_hline()`

A.8 Mathematics and Statistics links

A.8.1 Section 8.1 Distributions links

1. [Integrate Normal Shocks](#): [rmd](#) | [r](#) | [pdf](#) | [html](#)
 - Random Sampling (Monte Carlo) integrate shocks.
 - Trapezoidal rule (symmetric rectangles) integrate normal shock.

A.8.2 Section 8.2 Analytical Solutions links

1. [linear solve x with \$f\(x\) = 0\$](#) : [rmd](#) | [r](#) | [pdf](#) | [html](#)
 - Evaluate and solve statistically relevant problems with one equation and one unknown that permit analytical solutions.

A.8.3 Section 8.3 Inequality Models links

1. [Gini for Discrete Samples](#): [rmd](#) | [r](#) | [pdf](#) | [html](#)
 - Given sample of data points that are discrete, compute the approximate gini coefficient.
 - **r**: `sort() + cumsum() + sum()`
2. [CES abd Atkinson Utility](#): [rmd](#) | [r](#) | [pdf](#) | [html](#)
 - Analyze how changing individual outcomes shift utility given inequality preference parameters.
 - Draw Cobb-Douglas, Utilitarian and Leontief indifference curve
 - **r**: `apply(mt, 1, funct(x){}) + do.call(rbind, ls_mt)`
 - **tidyr**: `expand_grid()`
 - **ggplot2**: `geom_line() + facet_wrap()`
3. [README_appendix](#): [rmd](#) | [r](#) | [pdf](#) | [html](#)

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