R Compute Gini Coefficient for Discrete Samples

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Gini Discrete Sample

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This works out how the ff_dist_gini_vector_pos function works from Fan's *REconTools* Package.

Gini Formula for Discrete Sample There is an vector values (all positive). This could be height information for N individuals. It could also be income information for N individuals. Calculate the GINI coefficient treating the given vector as population. This is not an estimation exercise where we want to estimate population gini based on a sample. The given array is the population. The population is discrete, and only has these N individuals in the length n vector.

Note that when the sample size is small, there is a limit to inequality using the formula defined below given each N. So for small N, can not really compare inequality across arrays with different N, can only compare arrays with the same N. In another word, if 1 of N individual holds all resource, as N increases, GINI will asymptote to 1, but it is very far away from 1 for low N.

The GINI formula used here is:

$$GINI = 1 - \frac{2}{N+1} \cdot \left(\sum_{i=1}^{N} \sum_{j=1}^{i} x_j \right) \cdot \left(\sum_{i=1}^{N} x_i \right)^{-1}$$

Derive the formula in the steps below.

Step 1 Area Formula

$$\Gamma = \sum_{i=1}^{N} \frac{1}{N} \cdot \left(\sum_{j=1}^{i} \left(\frac{x_j}{\sum_{\widehat{j}=1}^{N} x_{\widehat{j}}} \right) \right)$$

Step 2 Total Area Given Perfect equality

With perfect equality $x_i = a$ for all i, so need to divide by that.

$$\Gamma^{\text{equal}} = \sum_{i=1}^{N} \frac{1}{N} \cdot \left(\sum_{j=1}^{i} \left(\frac{a}{\sum_{\widehat{j}=1}^{N} a} \right) \right) = \frac{N+1}{N} \cdot \frac{1}{2}$$

As the number of elements of the vecotr increases:

$$\lim_{N \to \infty} \Gamma^{\text{equal}} = \lim_{N \to \infty} \frac{N+1}{N} \cdot \frac{1}{2} = \frac{1}{2}$$

Step 3 Arriving at Finite Vector Gini Formula

Given what we have from above, we obtain the gini formula, divide by total area below 45 degree line.

$$GINI = 1 - \left(\sum_{i=1}^{N} \sum_{j=1}^{i} x_j \right) \cdot \left(N \cdot \sum_{i=1}^{N} x_i \right)^{-1} \cdot \left(\frac{N+1}{N} \cdot \frac{1}{2} \right)^{-1} = 1 - \frac{2}{N+1} \cdot \left(\sum_{i=1}^{N} \sum_{j=1}^{i} x_j \right) \cdot \left(\sum_{i=1}^{N} x_i \right)^{-1}$$

Step 4 Maximum Inequality given N

Suppose $x_i = 0$ for all $i < N$, then:

$$GINI^{x_i=0 \text{ except } i=N} = 1 - \frac{2}{N+1} \cdot X_N \cdot (X_N)^{-1} = 1 - \frac{2}{N+1}$$

$$\lim_{N \to \infty} GINI^{x_i=0 \text{ except } i=N} = 1 - \lim_{N \to \infty} \frac{2}{N+1} = 1$$

Note that for small N, for example if $N = 10$, even when one person holds all income, all others have 0 income, the formula will not produce gini is zero, but that gini is equal to $\frac{2}{11} \approx 0.1818$. If $N = 2$, inequality is at most, $\frac{2}{3} \approx 0.667$.

$$MostUnequalGINI(N) = 1 - \frac{2}{N+1} = \frac{N-1}{N+1}$$

Implement GINI Formula The **GINI** formula just derived is trivial to compute.

1. scalar: $\frac{2}{N+1}$
2. cumsum: $\sum_{j=1}^{i} x_j$
3. sum of cumsum: $\left(\sum_{i=1}^{N} \sum_{j=1}^{i} x_j \right)$
4. sum: $\sum_{i=1}^{N} X_i$

There are no package dependencies. Define the formula here:

```r
# Formula, directly implement the GINI formula Following Step 4 above
fv_dist_gini_vector_pos_test <- function(ar_pos) {
  # Check length and given warning
  it_n <- length(ar_pos)
  if (it_n <= 100)  warning('Data vector has n=',it_n,', max-inequality/max-gini=',(it_n-1)/(it_n + 1))
  # Sort
  ar_pos <- sort(ar_pos)
  # formula implement
  fl_gini <- 1 - ((2/(it_n+1)) * sum(cumsum(ar_pos))*(sum(ar_pos))^(-1))
  return(fl_gini)
}
```

Generate a number of examples Arrays for testing

```r
# Example Arrays of data
ar_equal_n1 = c(1)
ar_ineql_n1 = c(100)

ar_equal_n2 = c(1,1)
ar_ineql_alittle_n2 = c(1,2)
ar_ineql_somewht_n2 = c(1,2^3)
ar_ineql_alotine_n2 = c(1,2^5)
ar_ineql_veryvry_n2 = c(1,2^8)
ar_ineql_mostmst_n2 = c(1,2^13)

ar_equal_n10 = c(2,2,2,2,2,2, 2, 2, 2, 2)
ar_ineql_some_n10 = c(1,2,3,5,8,13,21,34,55,89)
ar_ineql_very_n10 = c(1,2^2,3^2,5^2,8^2,13^2,21^2,34^2,55^2,89^2)
ar_ineql_extr_n10 = c(1,2^2,3^3,5^4,8^5,13^6,21^7,34^8,55^9,89^10)
```

Now test the example arrays above using the function based no our formula:

```
##
## Small N=1 Hard-Code
## ar_equal_n1: 0
## ar_ineql_n1: 0
##
## Small N=2 Hard-Code, converge to 1/3, see formula above
## ar_ineql_alittle_n2: 0.1111111
## ar_ineql_somewht_n2: 0.2592593
## ar_ineql_alotine_n2: 0.3131313
## ar_ineql_veryvry_n2: 0.3307393
##
## Small N=10 Hard-Code, convege to 9/11=0.8181, see formula above
## ar_equal_n10: 0
## ar_ineql_some_n10: 0.5395514
## ar_ineql_very_n10: 0.7059554
## ar_ineql_extr_n10: 0.8181549
```