R Integrate Over Normal Guassian Process Shock

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Integrate Over Normal Guassian Process Shock

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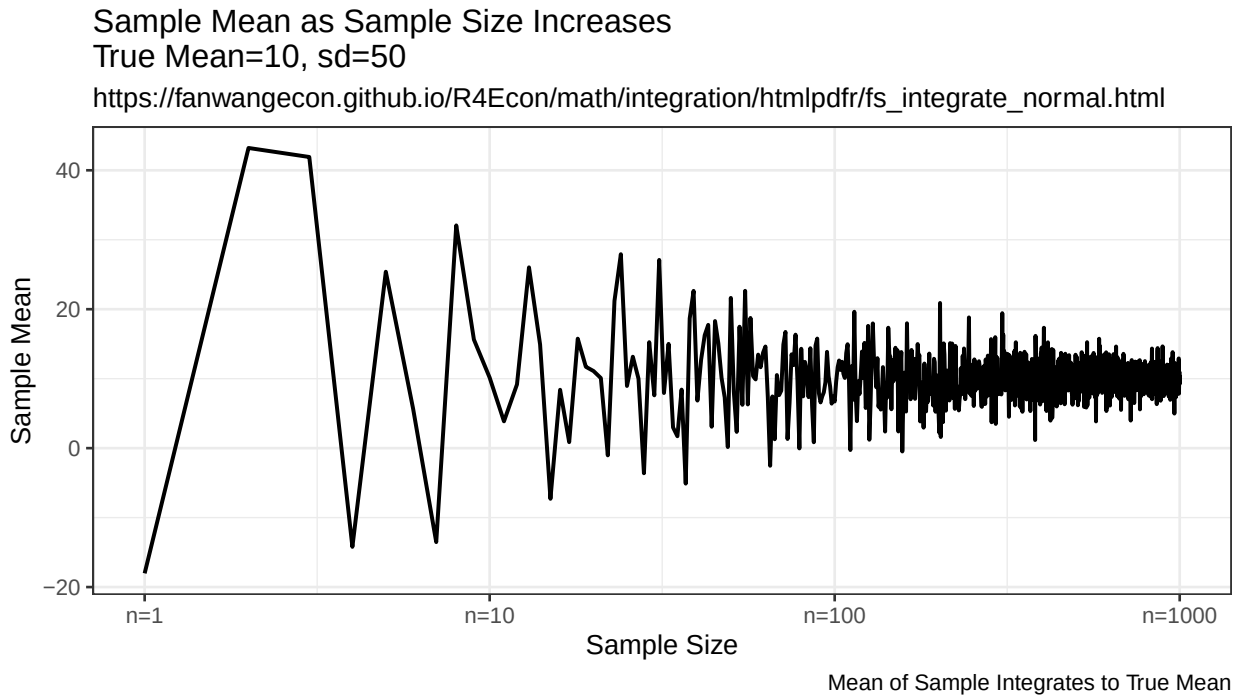
Some Common parameters

```
fl_eps_mean = 10
fl_eps_sd = 50
fl_cdf_min = 0.000001
fl_cdf_max = 0.999999
ar_it_draws <- seq(1, 1000)
```

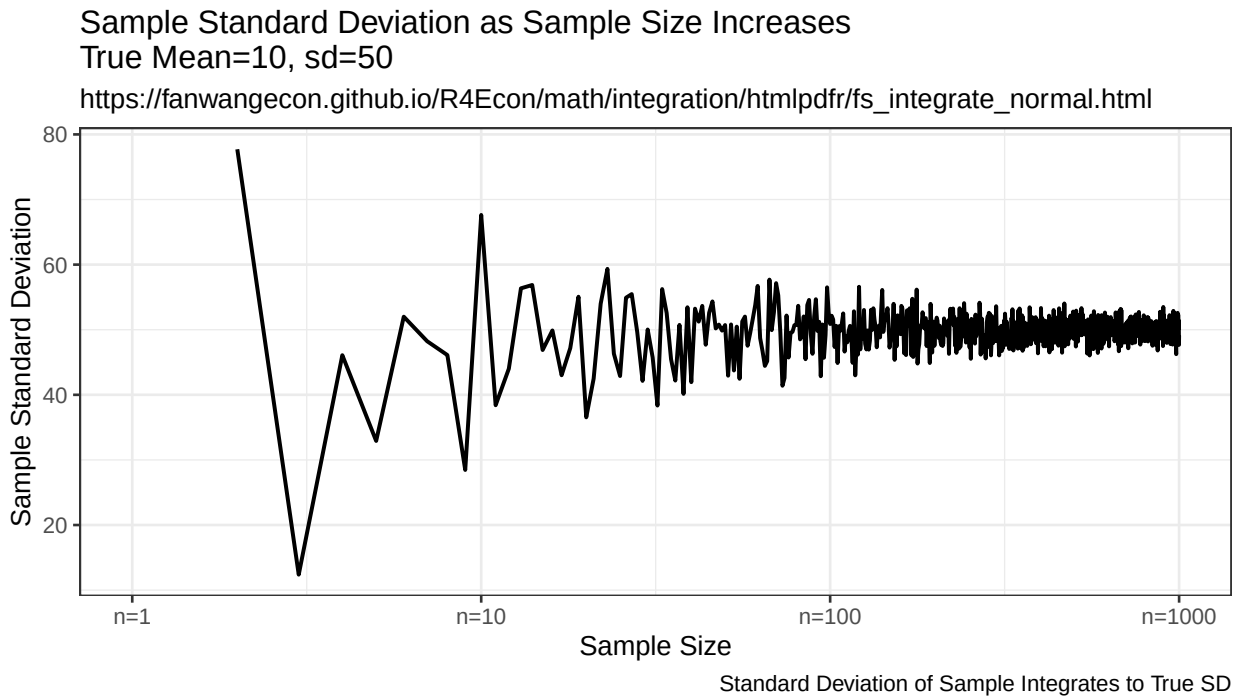
Randomly Sample and Integrate (Monte Carlo Integration) Compare randomly drawn normal shock mean and known mean. How does simulated mean change with draws. Actual integral equals to 10, as sample size increases, the sample mean approaches the integration results, but this is expensive, even with ten thousand draws, not very exact.

```
# Simulate Draws
set.seed(123)
ar_fl_means <-
  sapply(ar_it_draws, function(x)
    return(mean(rnorm(x[1], mean=fl_eps_mean, sd=fl_eps_sd))))
ar_fl_sd <-
  sapply(ar_it_draws, function(x)
    return(sd(rnorm(x[1], mean=fl_eps_mean, sd=fl_eps_sd))))

mt_sample_means <- cbind(ar_it_draws, ar_fl_means, ar_fl_sd)
colnames(mt_sample_means) <- c('draw_count', 'mean', 'sd')
tb_sample_means <- as_tibble(mt_sample_means)

# Graph
# x-labels
x.labels <- c('n=1', 'n=10', 'n=100', 'n=1000')
x.breaks <- c(1, 10, 100, 1000)

# Shared Subtitle
st_subtitle <- paste0('https://fanwangecon.github.io/',
                      'R4Econ/math/integration/htmlpdfr/fs_integrate_normal.html')
```

```
# Shared Labels
slb_title_shr = paste0('as Sample Size Increases\n',
                       'True Mean=', fl_eps_mean,', sd=',fl_eps_sd)
slb_xtitle = paste0('Sample Size')

# Graph Results--Draw
plt_mean <- tb_sample_means %>%
  ggplot(aes(x=draw_count, y=mean)) +
  geom_line(size=0.75) +
  labs(title = paste0('Sample Mean ', slb_title_shr),
       subtitle = st_subtitle,
       x = slb_xtitle,
       y = 'Sample Mean',
       caption = 'Mean of Sample Integrates to True Mean') +
  scale_x_continuous(trans='log10', labels = x.labels, breaks = x.breaks) +
  theme_bw()
print(plt_mean)
```




```
plt_sd <- tb_sample_means %>%
  ggplot(aes(x=draw_count, y=sd)) +
  geom_line(size=0.75) +
  labs(title = paste0('Sample Standard Deviation ', slb_title_shr),
       subtitle = st_subtitle,
       x = slb_xtitle,
       y = 'Sample Standard Deviation',
       caption = 'Standard Deviation of Sample Integrates to True SD') +
  scale_x_continuous(trans='log10', labels = x.labels, breaks = x.breaks) +
  theme_bw()
print(plt_sd)
```

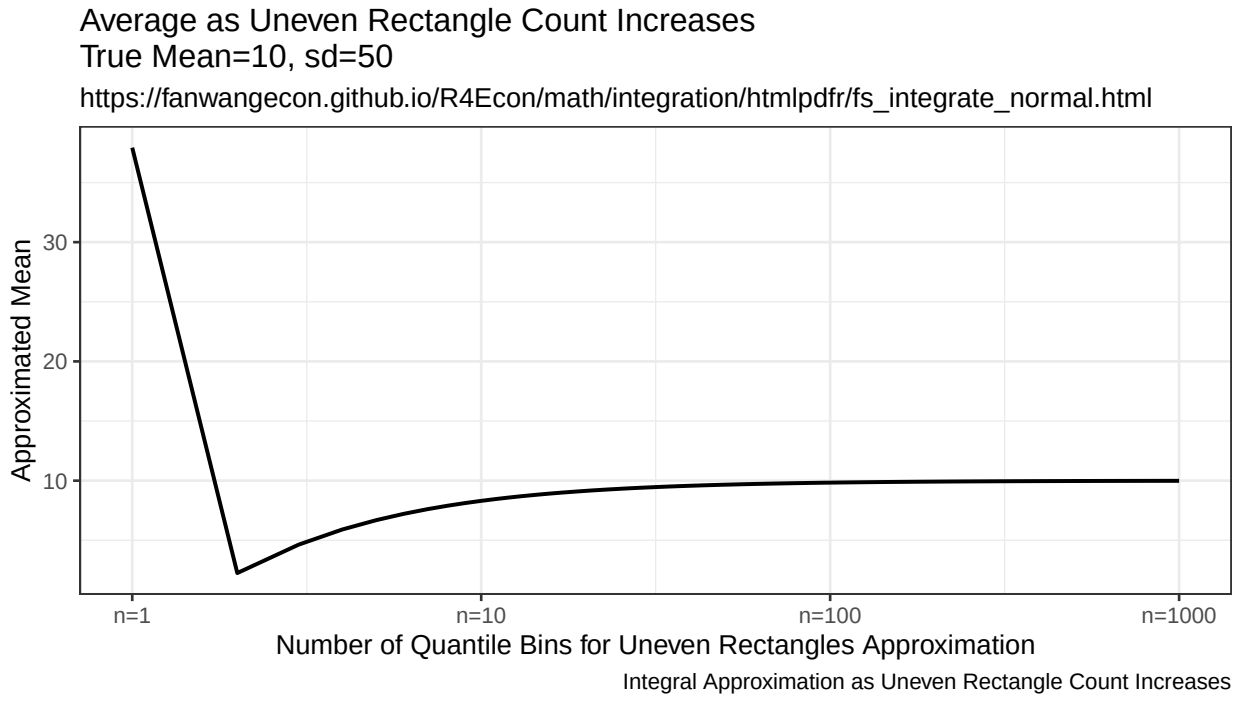


Integration By Symmetric Uneven Rectangle Draw on even grid from close to 0 to close to 1. Get the corresponding x points to these quantile levels. Distance between x points are not equi-distance but increasing and symmetric away from the mean. Under this approach, each rectangle aims to approximate the same area.

Resulting integration is rectangle based, but rectangle width differ. The rectangles have wider width as they move away from the mean, and thinner width close to the mean. This is much more stable than the random draw method, but note that it converges somewhat slowly to true values as well.

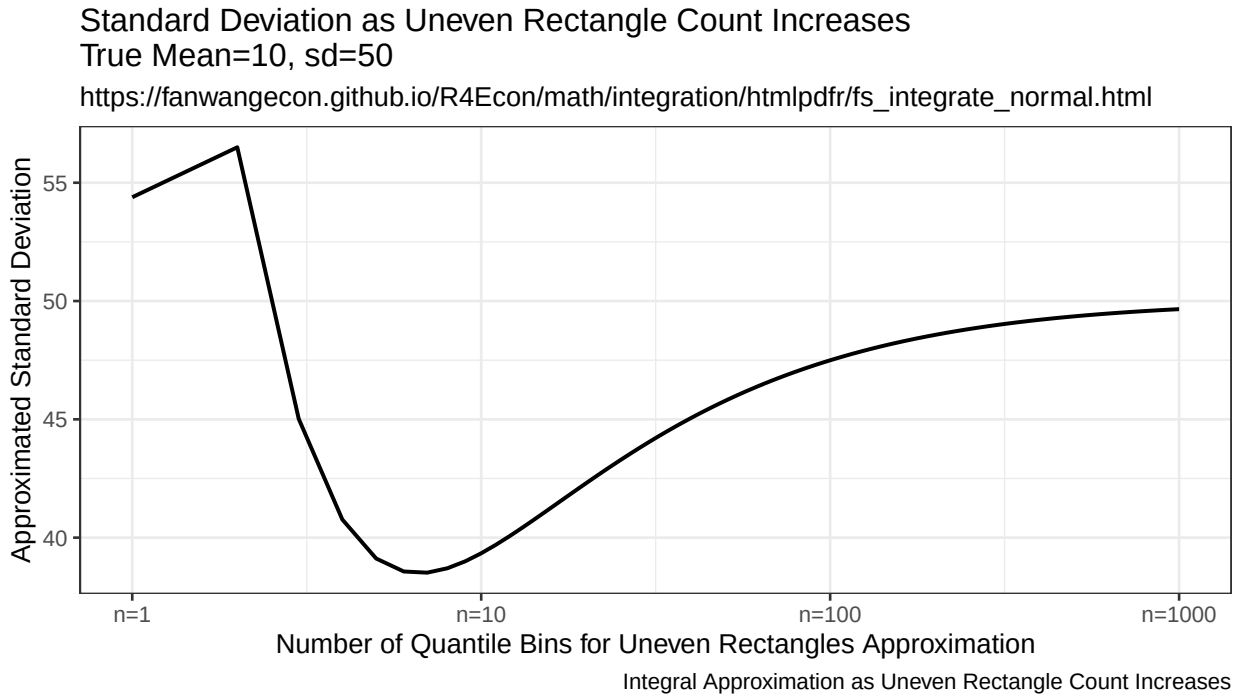
```
mt_fl_means <-
  sapply(ar_it_draws, function(x) {

    fl_prob_break = (fl_cdf_max - fl_cdf_min)/(x[1])
    ar_eps_bounds <- qnorm(seq(fl_cdf_min, fl_cdf_max,
                               by=(fl_cdf_max - fl_cdf_min)/(x[1])),
                           mean = fl_eps_mean, sd = fl_eps_sd)
    ar_eps_val <- (tail(ar_eps_bounds, -1) + head(ar_eps_bounds, -1))/2
    ar_eps_prb <- rep(fl_prob_break/(fl_cdf_max - fl_cdf_min), x[1])
    ar_eps_fev <- dnorm(ar_eps_val,
                        mean = fl_eps_mean, sd = fl_eps_sd)

    fl_cdf_total_approx <- sum(ar_eps_fev*diff(ar_eps_bounds))
    fl_mean_approx <- sum(ar_eps_val*(ar_eps_fev*diff(ar_eps_bounds)))
    fl_sd_approx <- sqrt(sum((ar_eps_val-fl_mean_approx)^2*(ar_eps_fev*diff(ar_eps_bounds))))

    return(list(cdf=fl_cdf_total_approx, mean=fl_mean_approx, sd=fl_sd_approx))
  })

mt_sample_means <- cbind(ar_it_draws, as_tibble(t(mt_fl_means)) %>% unnest())
colnames(mt_sample_means) <- c('draw_count', 'cdf', 'mean', 'sd')
```

```
tb_sample_means <- as_tibble(mt_sample_means)

# Graph
# x-labels
x.labels <- c('n=1', 'n=10', 'n=100', 'n=1000')
x.breaks <- c(1, 10, 100, 1000)

# Shared Labels
slb_title_shr = paste0('as Uneven Rectangle Count Increases\n',
                       'True Mean=', fl_eps_mean,', sd=',fl_eps_sd)
slb_xtitle = paste0('Number of Quantile Bins for Uneven Rectangles Approximation')

# Graph Results--Draw
plt_mean <- tb_sample_means %>%
  ggplot(aes(x=draw_count, y=mean)) +
  geom_line(size=0.75) +
  labs(title = paste0('Average ', slb_title_shr),
       subtitle = st_subtitle,
       x = slb_xtitle,
       y = 'Approximated Mean',
       caption = 'Integral Approximation as Uneven Rectangle Count Increases') +
  scale_x_continuous(trans='log10', labels = x.labels, breaks = x.breaks) +
  theme_bw()
print(plt_mean)
```

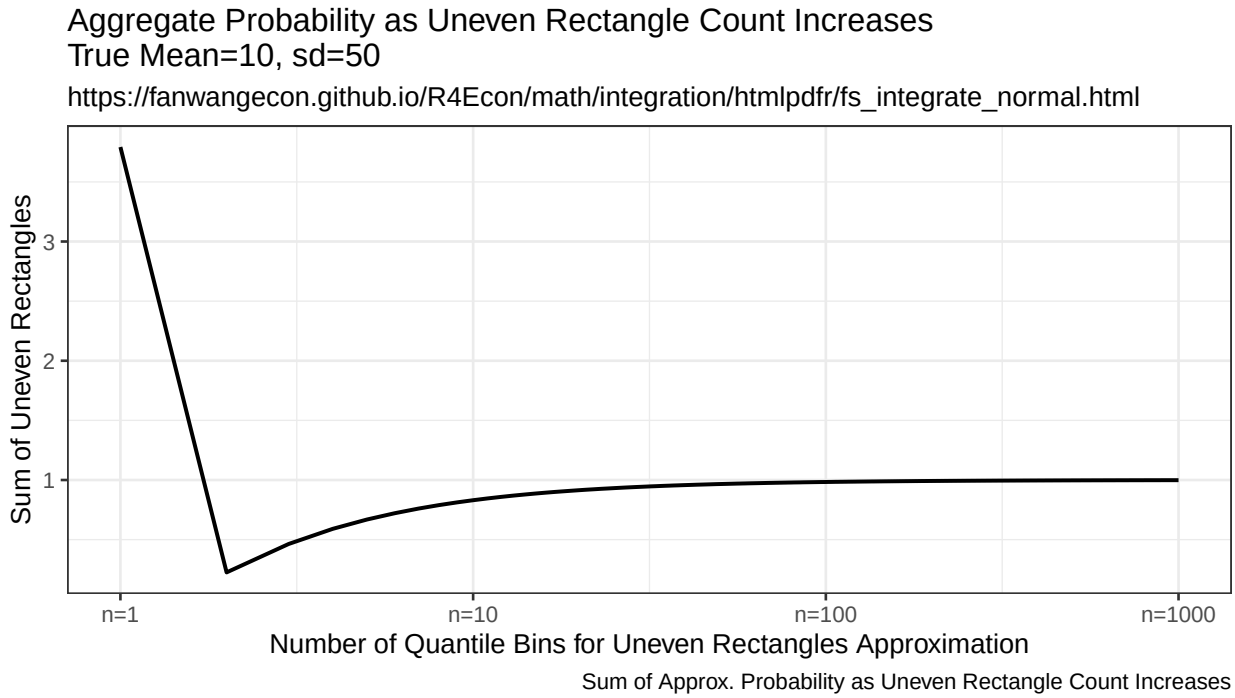



```
plt_sd <- tb_sample_means %>%
  ggplot(aes(x=draw_count, y=sd)) +
  geom_line(size=0.75) +
  labs(title = paste0('Standard Deviation ', slb_title_shr),
       subtitle = st_subtitle,
```

```
       x = slb_xtitle,
       y = 'Approximated Standard Deviation',
       caption = 'Integral Approximation as Uneven Rectangle Count Increases') +
  scale_x_continuous(trans='log10', labels = x.labels, breaks = x.breaks) +
  theme_bw()
print(plt_sd)
```



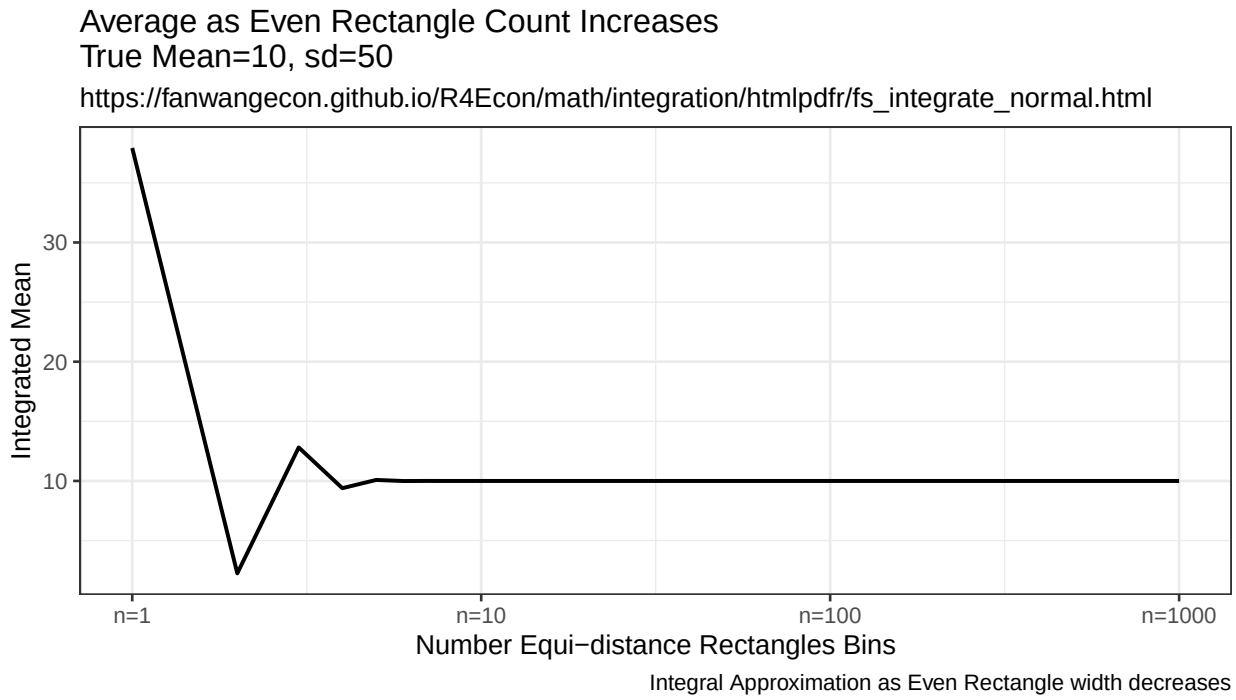

```
plt_cdf <- tb_sample_means %>%
  ggplot(aes(x=draw_count, y=cdf)) +
  geom_line(size=0.75) +
  labs(title = paste0('Aggregate Probability ', slb_title_shr),
       subtitle = st_subtitle,
       x = slb_xtitle,
       y = 'Sum of Uneven Rectangles',
       caption = 'Sum of Approx. Probability as Uneven Rectangle Count Increases') +
  scale_x_continuous(trans='log10', labels = x.labels, breaks = x.breaks) +
  theme_bw()
print(plt_cdf)
```



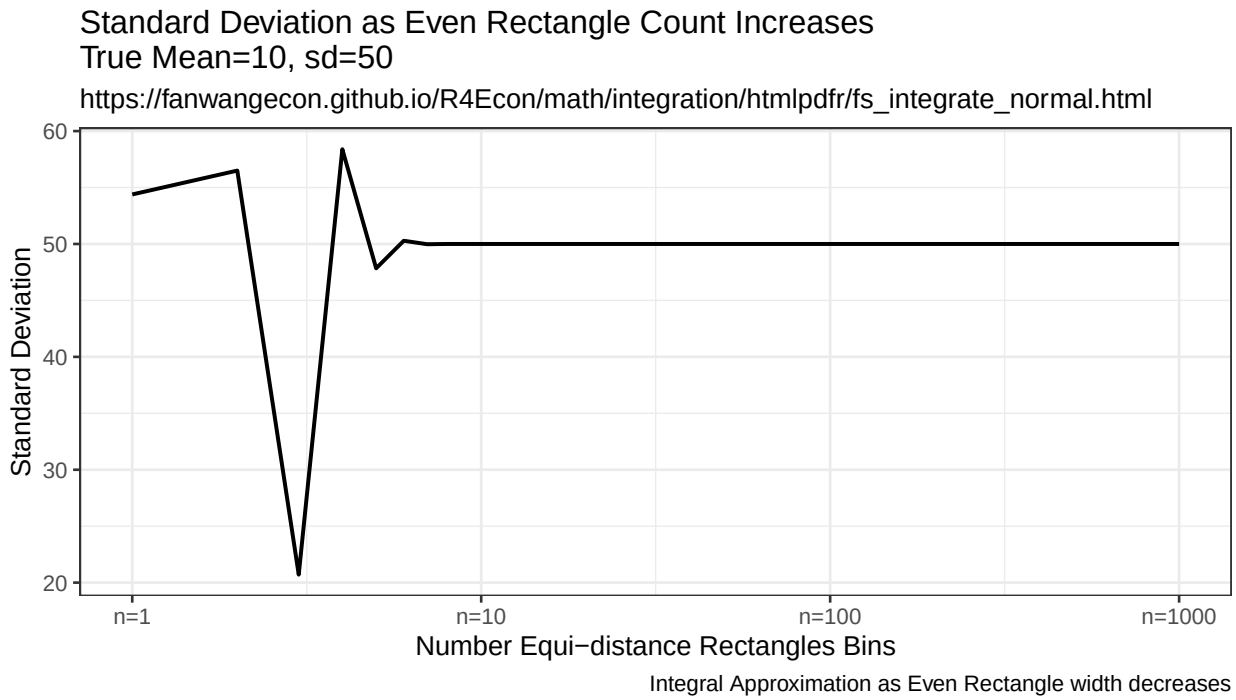
Integration By Constant Width Rectangle (Trapezoidal rule) This is implementing even width recentagle, even along x-axix. Rectangle width are the same, height is $f(x)$. This is even width, but uneven area. Note that this method approximates the true answer much better and more quickly than the prior methods.

```
mt_fl_means <-
  sapply(ar_it_draws, function(x) {

    fl_eps_min <- qnorm(fl_cdf_min, mean = fl_eps_mean, sd = fl_eps_sd)
    fl_eps_max <- qnorm(fl_cdf_max, mean = fl_eps_mean, sd = fl_eps_sd)
    fl_gap <- (fl_eps_max-fl_eps_min)/(x[1])
    ar_eps_bounds <- seq(fl_eps_min, fl_eps_max, by=fl_gap)
    ar_eps_val <- (tail(ar_eps_bounds, -1) + head(ar_eps_bounds, -1))/2
    ar_eps_prb <- dnorm(ar_eps_val, mean = fl_eps_mean, sd = fl_eps_sd)*fl_gap

    fl_cdf_total_approx <- sum(ar_eps_prb)
    fl_mean_approx <- sum(ar_eps_val*ar_eps_prb)
    fl_sd_approx <- sqrt(sum((ar_eps_val-fl_mean_approx)^2*ar_eps_prb))

    return(list(cdf=fl_cdf_total_approx, mean=fl_mean_approx, sd=fl_sd_approx))
  })

mt_sample_means <- cbind(ar_it_draws, as_tibble(t(mt_fl_means)) %>% unnest())
colnames(mt_sample_means) <- c('draw_count', 'cdf', 'mean', 'sd')
tb_sample_means <- as_tibble(mt_sample_means)

# Graph
# x-labels
x.labels <- c('n=1', 'n=10', 'n=100', 'n=1000')
x.breaks <- c(1, 10, 100, 1000)
```

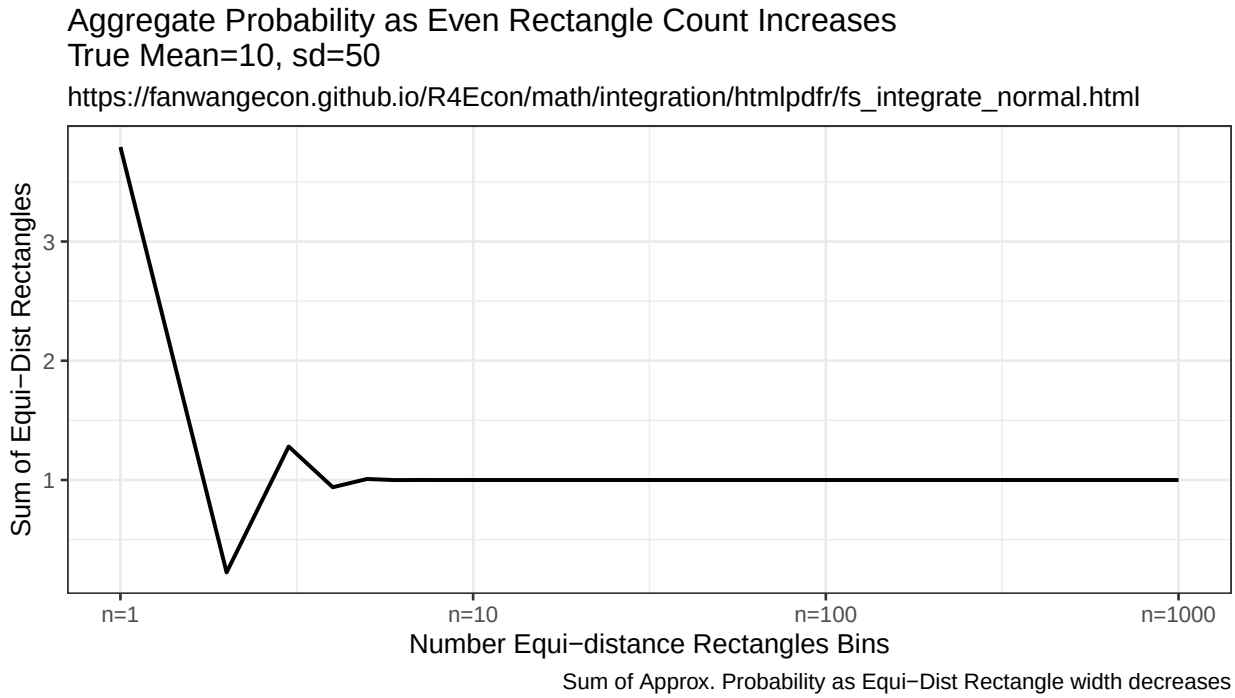
```
# Shared Labels
slb_title_shr = paste0('as Even Rectangle Count Increases\n',
                       'True Mean=', fl_eps_mean,', sd=',fl_eps_sd)
slb_xtitle = paste0('Number Equi-distance Rectangles Bins')

# Graph Results--Draw
plt_mean <- tb_sample_means %>%
  ggplot(aes(x=draw_count, y=mean)) +
  geom_line(size=0.75) +
  labs(title = paste0('Average ', slb_title_shr),
       subtitle = st_subtitle,
       x = slb_xtitle,
       y = 'Integrated Mean',
       caption = 'Integral Approximation as Even Rectangle width decreases') +
  scale_x_continuous(trans='log10', labels = x.labels, breaks = x.breaks) +
  theme_bw()
print(plt_mean)
```




```
plt_sd <- tb_sample_means %>%
  ggplot(aes(x=draw_count, y=sd)) +
  geom_line(size=0.75) +
  labs(title = paste0('Standard Deviation ', slb_title_shr),
       subtitle = st_subtitle,
       x = slb_xtitle,
       y = 'Standard Deviation',
       caption = 'Integral Approximation as Even Rectangle width decreases') +
  scale_x_continuous(trans='log10', labels = x.labels, breaks = x.breaks) +
  theme_bw()
print(plt_sd)
```




```
plt_cdf <- tb_sample_means %>%
  ggplot(aes(x=draw_count, y=cdf)) +
  geom_line(size=0.75) +
  labs(title = paste0('Aggregate Probability ', slb_title_shr),
       subtitle = st_subtitle,
       x = slb_xtitle,
       y = 'Sum of Equi-Dist Rectangles',
       caption = 'Sum of Approx. Probability as Equi-Dist Rectangle width decreases') +
  scale_x_continuous(trans='log10', labels = x.labels, breaks = x.breaks) +
  theme_bw()
print(plt_cdf)
```


Aggregate Probability as Even Rectangle Count Increases
True Mean=10, sd=50
https://fanwangecon.github.io/R4Econ/math/integration/htmlpdfr/fs_integrate_normal.html
Sum of Equi−Dist Rectangles
3
2
1
n=1
n=10
n=100
n=1000
Number Equi−distance Rectangles Bins
Sum of Approx. Probability as Equi−Dist Rectangle width decreases