

Analyzing Atkinson Family Utility (CES)

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Contents

Atkinson Family Utility	1
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Atkinson Family Utility

Go to the [RMD](#), [R](#), [PDF](#), or [HTML](#) version of this file. Go back to [fan's REconTools](#) Package, [R4Econ](#) Repository ([bookdown site](#)), or [Intro Stats with R](#) Repository.

How does the Atkinson Family utility function work? The Atkinson Family Utility has the following functional form.

$$V^{\text{social}} = (\alpha \cdot A^\lambda + \beta \cdot B^\lambda)^{\frac{1}{\lambda}}$$

Several key issues here:

1. V^{social} is the utility of some social planner
2. A and B are allocations for Alex and Ben.
3. α and β are biases that a social planner has for Alex and Ben: $\alpha + \beta = 1$, $\alpha > 0$, and $\beta > 0$
4. $-\infty < \lambda \leq 1$ is a measure of inequality aversion
 - $\lambda = 1$ is when the planner cares about weighted total allocations (efficient, Utilitarian)
 - $\lambda = -\infty$ is when the planner cares about only the minimum between A and B allocations (equality, Rawlsian)

What if only care about Alex? Clearly, if the planner only cares about Ben, $\beta = 1$, then:

$$V^{\text{social}} = (B^\lambda)^{\frac{1}{\lambda}} = B$$

Clearly, regardless of the value of λ , as B increases V increases.

What Happens to V when A or B increases? What is the derivative of V with respect to A or B ?

$$\frac{\partial V}{\partial A} = \frac{1}{\lambda} (\alpha A^\lambda + \beta B^\lambda)^{\frac{1}{\lambda}-1} \cdot \lambda \alpha A^{\lambda-1}$$

With just a little bit of simplification, we have:

$$\frac{\partial V}{\partial A} = (\alpha A^\lambda + \beta B^\lambda)^{\frac{1-\lambda}{\lambda}} \cdot \alpha A^{\lambda-1} > 0$$

It is important to note that $\frac{\partial V}{\partial A} > 0$. When $\lambda < 0$, $Z^\lambda > 0$. For example $10^{-2} = \frac{1}{100}$. And For example $0.1^{-\frac{3}{2}} = \frac{1}{0.1^{1.5}}$. Still Positive.

This might be surprising, because when $\lambda < 0$:

$$\text{if } \lambda < 0 \text{ then } \frac{d(\alpha A^\lambda + \beta B^\lambda)}{dA} = \alpha \lambda A^{\lambda-1} < 0$$

so if we did not have the outer $\frac{1}{\lambda}$ power, negative λ would lead to decreasing weighted sum. But:

$$\text{if } \lambda < 0 \text{ then } \frac{dG^{\frac{1}{\lambda}}}{dG} = \frac{1}{\lambda} \cdot G^{\frac{1-\lambda}{\lambda}} < 0$$

so when G is increasing and $\lambda < 0$, V would decrease. But when $G(A, B)$ is decreasing, due to increasing A when $\lambda < 0$, V will actually increase.