FFR projection based on the factors of Taylor Rule and Treasury Yields with HMM

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Abstract

As we all know, in the last two years the Federal Reserve has raised interest rates multiple times (increasing the Federal Funds Rate), which has had a significant impact on global financial markets. The rate hikes have not only brought about the strength of the US dollar, which affects the Currency Markets, but also the pricing of global assets and the anchoring of sovereigns' credits, which makes the projection of the FFR very important. However, there are a number of obstacles to FFR forecasting: 1) lagging and insufficient publicly available data due to the low frequency of macro data releases; and 2) the susceptibility of market participants to historical data bias (e.g., being overly optimistic about FFR reductions). This study hopes to combine the Taylor Rule, commonly used by the FOMC, with the treasury yields to make a more forward-looking judgement on FFR.

Word Counts: 5912

Introduction

Recently, due to the FFR hikes, it has caused significant turbulence in the global financial markets. This brought about a short-term strength in the US dollar, which significantly influenced the FX market, and affected the pricing of the rest of the assets as well as the anchoring of sovereign credits. Many investors focused on the volatility of the FFR, the tone given by the Feds, and the official dot plots, in an attempt to make better predictions about the FFR to protect their portfolios or even make profits. Notably, the Taylor Rule is a popular approach for the Fed to build the relationship between the FFR and the macro conditions, which are explained mainly by Resource Gap measured by GDP and Inflation measured by PCE. Based on the Taylor Rule, the investors and institutions can make projections for the FFR with publicly available information.

According to the basic assumptions of time series modeling, forecasts at moment t cannot use future information, but only current moment as well as ever information [1]. However, in the actual publication of economics data, although different economic indicators reflect economic conditions over the same period, they cannot be regarded as variables with the same time stamp due to the difference in publication time. This is also the case in this study, although based on the Taylor Rule, the FFR can be calculated using the relevant macroeconomic indicators for this period, the corresponding macroeconomic indicators are published 1-2 days after the publication time of the FFR, and therefore it is impossible to use such future information for FFR determination in practice. As visualized in Fig.1, the FFR is released at the end of the quarter, but the corresponding macro data is released afterward. In reality, researchers generally use the previous period's macroeconomic data as the input factor for the Taylor Rule, however, this can lead to a lagged effect as the calculation of the FFR ignores the most recent period's economic situation. Many relevant financial institutions present alternative data for use, but this is not always publicly available and authoritative.

Figure 1. The release example of FFR and Macro Data

Moreover, especially the U.S. Gross Domestic Product (GDP) lags much more compared to the inflation data, as it has multiple revisions to provide more reliable predictions. The GDP revisions involve updating and refining previously released GDP data to provide a more accurate and comprehensive representation of economic activity. Revisions are a normal part of economic data reporting, as initial estimates are often based on incomplete information and preliminary data. The diagram below illustrates the release and adjustment of GDP throughout 2023. To explain, the initial GDP release on January 26th, 2023, is essentially a forecast, and it undergoes its first revision on February 23rd, 2023, which, nonetheless, remains a prediction. Only by March 30th, 2023, will the released GDP become more comparatively accurate. A comprehensive explanation of the modification process is put in Appendix 1.

Release Date	Time	Actual	Forecast	Previous
Nov 29, 2023 (Q3)	08:30			4.9%
Oct 26, 2023 (Q3)	07:30 P	4.9%	4.3%	2.1%
Sep 28, 2023 (Q2)	07:30	2.1%	2.1%	2.0%
Aug 30, 2023 (Q2)	07:30 P	2.1%	2.4%	2.0%
Jul 27, 2023 (Q2)	07:30 P	2.4%	1.8%	2.0%
Jun 29, 2023 (Q1)	07:30	2.0%	1.4%	2.6%
May 25, 2023 (Q1)	07:30 P	1.3%	1.1%	2.6%
Apr 27, 2023 (Q1)	07:30 P	1.1%	2.0%	2.6%
Mar 30, 2023 (Q4)	07:30	2.6%	2.7%	3.2%
Feb 23, 2023 (Q4)	08:30 P	2.7%	2.9%	3.2%
Jan 26, 2023 (Q4)	08:30 P	2.9%	2.6%	3.2%

Figure 2. Release and Modification of GDP in 2023

Although Taylor Rule has obstacles, it also proves the effectiveness of the Resource Gap and Inflation towards the FFR prediction, and the indicated linear relationship, which can be further studied by applying the rolling linear regression to make a more accurate estimation as the coefficients are ever-changing instead of setting preliminarily. In this study, we aim to embed the information from daily Treasury Yield data, achieved by applying the Hidden Markov Model with Gaussian Mixture Models (GMM-HMM), to make a timelier information projection. Combining this obtained information with the original Macro data, a rolling regression model is fitted with the historical data, creating a one-step-ahead prediction for the FFR rates. All the experiments are completed with Python, which is fully accessible on *GitHub*.

Literature Review

1. Federal Reserve Rate
   a. Monetary Policy

      The Federal Reserve, through its monetary policy adjustments, creates a favorable economic environment characterized by appropriate employment rates and stable prices [2]. When the aggregate demand lags the economy's production capacity, it increases unemployment and reduces inflation. To counter this, the Federal Open Market Committee (FOMC) intervenes by lowering interest rates and implementing an expansionary monetary policy to stimulate aggregate demand, helping stabilize the economy.

      Conversely, suppose demand for goods and services becomes excessively strong. In that case, it can lead to an unsustainable employment rate and increased inflation, leading the Federal Reserve to employ a contractionary monetary policy by elevating interest rates to guide economic activity back to normal levels. The procedure through which the FOMC enacts expansionary and contractionary monetary policies to achieve its goals can be summarized as shown in Fig.3 [3].

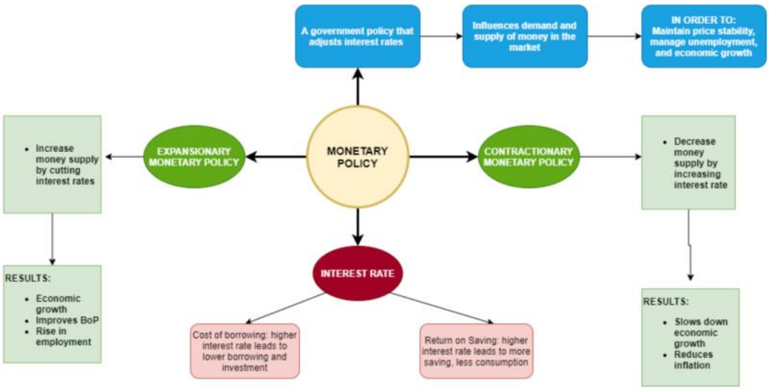

Figure 3. Federal Reserve Monetary Policy

b. FFR

The primary method to exert monetary policy is adjusting the federal funds rate (FFR) [4]. Banks maintain reserve balances at the Federal Reserve to fulfill unforeseen liquidity requirements, so they engage in borrowing and lending of reserves among one another based on their specific needs. The federal funds rate represents the interest rate at which banks engage in overnight borrowing, which plays a pivotal role in determining the expense of short-term credit.

To impact the federal funds rate, the FOMC can modify the interest rate applied to bank reserves. This adjustment leads to changes in the federal funds rate, aligning it with the FOMC's desired objectives and influencing the cost of short-term interbank credit. The empirical analysis for historical rate hikes is in Appendix 2, with the Effective FFR diagram shown in Fig. 4.

In recent contexts, the U.S. economy has been experiencing a robust recovery after economic disruption, possibly due to the COVID-19 pandemic. Annual inflation rates have risen above the Federal Reserve's target of 2%. As measured by the Consumer Price Index (CPI), inflation is 3.5%, and core inflation (excluding food and energy) is 2.8%. To address these economic conditions, the Federal Reserve announced an increase in the target FFR from 0.25% to 0.50% [5]. This is the first-rate hike in several years, signaling the central bank's confidence in the strength of the economic recovery.

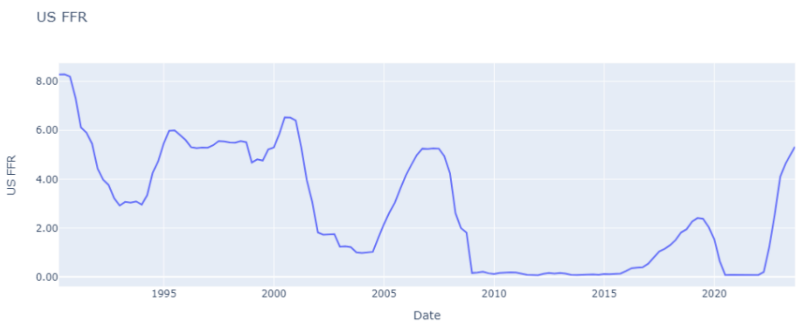

Figure 4. US Effective FFR Diagram

2. Taylor Rule

In a paper published in 1993, John Taylor showed how monetary policy in the United States from 1987 to 1992 was approximated by a formula that related the federal funds rate to three variables. The first variable is the inflation-adjusted long-run federal funds rate, the second is the deviation of current inflation from the 2% target set by the Federal Open Market Committee (FOMC), and the third is the percentage difference between actual GDP and its potential level. Taylor Rule takes the following general form, with the specific meanings of the indicators shown in the table below [6].

$$R_t^T = r_t^{LR} + \pi_t + 0.5(\pi_t - \pi^*) + 0.5(y_t - y^*)$$

Variable	Implication
FFR_t	Federal fund Rate
r_t^{LR}	Real Neutral Rate
π_t	Expected Inflation
π^*	Target Inflation
$y_t - y_t^\star$	Percent deviation between the current real GDP and the long-term linear trend in GDP

Table 1. Variables Explanation in Taylor Rule

The Taylor formula illustrates that when inflation surpasses the 2% target, the federal funds rate increases at a rate 1.5 times that of the inflation increase. Furthermore, if the GDP exceeds its potential level, the federal funds rate increases by 0.5 times the difference between it and its potential level.

The Taylor rule embodies the fundamental principles of monetary policy discussed earlier. Firstly, when the real long-term neutral federal funds rate, the actual and target inflation rates, and the real GDP level and its potential are all known, the adjustment based on the difference between the GDP and its potential level is zero, making FFR prediction feasible. Secondly, it advocates for higher FFR in response to rising inflation or increased resource utilization, and lower FFR when inflation subsides or resource utilization declines. This alignment corresponds with the Federal Reserve's dual mandate. Lastly, the equation dictates that the federal funds rate should be adjusted by more than a one-to-one ratio when inflation experiences upward or downward movements, a characteristic often referred to as the Taylor principle.

Federal Reserve officials and economists later introduced several alterations to the variables used in the Taylor Rule, aiming to provide a more accurate representation and interpretation of shifts in the real-world scenario and policy structure. As a result, numerous revised iterations emerged (Table. 2) [7].

Rules	Formula	Coefficient of Resource Gap	Coefficient of Inflation
Bernanke Rule	$FFR_t = r_t^{LR} + \pi_t + 0.5(\pi_t - \pi^\star) + (y_t - y^\star)$	1	0.5
Evans Rule	$FFR_t = r_t^{LR} + \pi_{t+1} + 0.5(\pi_t - \pi^\star) + 2(u_t - u^\star)$	2	0.5
Yellen Rule	$FFR_t = r_t^{LR} + \pi_{t+1} + 0.5(\pi_t - \pi^\star) + 2(u_t - u^\star)$	0.5	2
Bullard Rule	$FFR_t = \rho FFR_{t-1} + (1-\rho)[r_t^{LR} + \pi^\star + \beta_1(\pi_t - \pi^\star) + \beta_2(u_t - u^\star)]$	0.1	1.5

Table 2. Adjusted versions of the Taylor Rule

3. Treasury Yield

U.S. Treasury yield is the yield on U.S. government bonds, whose metric measures the return an investor can earn by purchasing U.S. government bonds. U.S. government bonds are bonds issued by the government to raise funds and are usually classified as having different maturities, including short-term, intermediate-term, and long-term bonds.

Investors and economic observers often use Treasury yield as an indicator of risk and market expectations. Based on the risk-neutral interpretation, treasury yields equal the average value of expected future short rates [8]. A low Treasury yield may indicate market concerns about future economic uncertainty, while a high Treasury yield may reflect investor optimism about economic growth and inflation. In addition, Treasury yield is used to determine the pricing of other financial instruments, such as mortgage rates and corporate bonds. Treasury yields can reflect economic conditions, monetary and fiscal policies, and expectations about future economic activity, real interest rates, and inflation [9]. What can be agreed upon is that whenever macroeconomic data is released differently than the consensus, treasury yields always have a noticeable jump, indicating the influence of macroeconomic situations on treasury yields. In this research, we consider 6 U.S. treasury yields from Bloomberg according to the dataset coverage [10], and the remaining NAN values are forward-filled based on the previous dates' yield data. A visualization of the Treasury Yields is shown in Fig. 5:

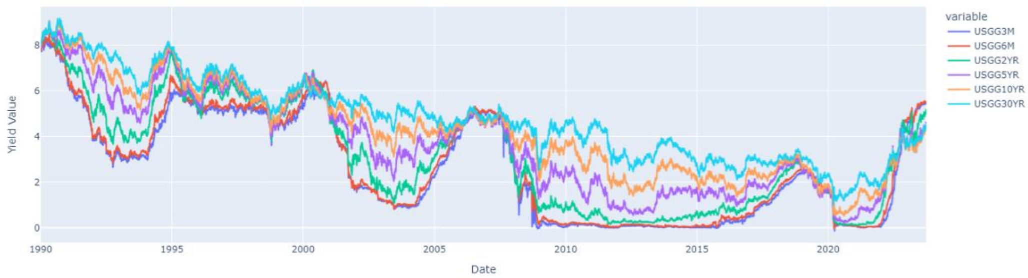

Figure 5. US Treasury Yields Diagram (Maturities including 3, 6 Months, 2, 5, 10, and 30 Year)

Additionally, the term structure of the treasury yields is also an essential dimension to understanding the economic situation. First, regarding the shape of the yield curve at a given point in time, broadly speaking, the curve should show an upward-sloping trend, but the rate of increase in interest rates decays as maturity increases. This shape is supported by the principles of Expectation, Bond Risk Premiums, and Convexity Bias, as stated by Ilmanen. [11] In history, researchers were interested in proposing models to fit the treasury yield curve better or analyzing the components from the yield curve, and there are indeed the components named level, slope, and curvature that are economically explainable to the structure of the yield curve [12]. By reviewing the yield curve in Fig. 3, we can visualize that the up and downshifts in the interest rate curve are essentially joint (as measured by level), and that, in most cases, the curve with high maturity lies on top (can be illustrated by slope). Various experimented models can prove the existence and significance of these three components, such as the Nelson-Siegel Model [13, 14]. However, this model required some essential tricks to determine the parameters in the model to provide a better simulation result. Another more straightforward method, which is named the Principal Component Analysis, is frequently used in Statistics and Data Science to reduce the dimensionally of the features and get the important ones, that is proved to be helpful in modeling these three factors [15]. From this method, the first three critical factors represent the level, slope, and curvature components in the yield curve, while the remaining are assumed to be noise and filtered out.

Indeed, treasury yields strongly correlate with FFR, especially in the short-term tenor parts, because the short-term rate is somewhat linked to the FFR. However, the rise of FFR doesn't necessarily provide evidence for the change in long-term yield. It can be concluded that this kind of situation could flatten the yield curve as the short side increases more than the long side. We perform a simple correlation measurement to the FFR and the monthly treasury yield rate in Fig. 6, and it is found that the correlation is very close to 1. Whereas, if we lag the treasury yield rate 1 year before and apply the correlation test, the relationship is still relatively stable. It seems that the treasury yield cannot be directly used as an instrument to project the FFR in the future.

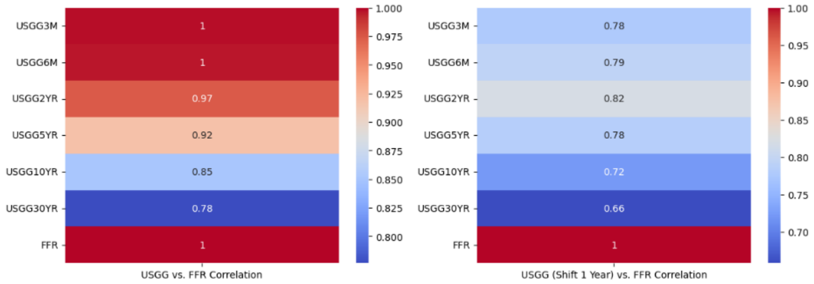

Figure 6. Correlation between the FFR and monthly Treasury yield (Left) v.s. Correlation between the FFR and monthly Treasury yield on the year before (Right)

Research Objective

1. Transform the treasury yield data into explanatory features, then feed them to the GMM-HMM to obtain the leading macroeconomic factors.
2. Construct the model based on the traditional Taylor Rule and generate new macro factors, that is a linear model:

$$FFR_t = \beta_{1,t} Resource\ Gap_{t-1} + \beta_{2,t} PCE_{t-1} + \boldsymbol{\beta_{k,t} Variable_k} + \epsilon_t$$

3. Adjust the incorporated information window to test the projection performance of the model.

Dataset Description

The Treasury yield data and FFR are downloaded from HKU Bloomberg Laboratory [10]. Note that the FFR is called effective FFR, which is further explained in Appendix 3. The Macro data of Real GDP, Potential GDP, and PCE are downloaded from the FRED Economic Data website [18]. The Resource GAP is measured by the logarithmic transformation of Real GDP minus the logarithmic transformation of Potential GDP. All the data are set to start from 1990-01-01.

Methodologies

1. Principal Component Analysis (PCA) [19]

The goal of the PCA method is to reduce the dimensionality of the data without losing much meaningful information, which can even filter out the noise. In this study, the PCA is utilized to reduce the components of the yield curve from 6 original curves to 3 meaningful components that can be classified as Level, Slope, and Curvature. The detailed steps are shown below:

a) Get the covariance matrix of all yields data C.

b) Calculate the eigenvalues and eigenvectors of C.

c) Select some best eigenvalues, and project the original data to the corresponding eigenvectors. The eigenvectors represent the new dimensions to which the original features are projected, called the principal components. And the eigenvalues are the mode of the unknown projected vectors on the principal components.

From these three steps, we can obtain the three components on a daily basis, and they can also be combined to rebuild the yield curve. An example of the PCA method utilized for 1991 Q1 is shown in Fig. 7. The left diagram shows the cases of Level, Slope, and Curvature, which can be easily understood. The Level values for yield with different maturities are all greater than 0, meaning that the curve can move upwards or downwards simultaneously according to the Level change.

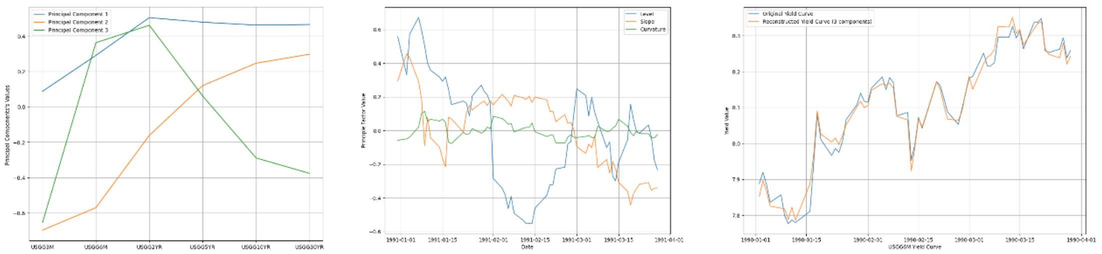

Figure 7. The values that the original 6 yield curves projected on the principal components (Left); The daily Level, Slope, and Curvature values provided by PCA during 1991 Q1 (Middle); Rebuilt Yield Values compared to Original Yield Values of USGG6M (Right)

2. Hidden Markov Model with Gaussian Mixture Models (GMM-HMM)

Under the assumption, macroeconomic conditions are affected by the newly released FFR, after which it continues to operate independently. The yield curve is, to some extent a reaction to the macro data, and when there are optimistic expectations about the economic trend, the yield curve will go down. The macroeconomic situation is positional during this period, but we can observe changes in the Treasury. The difference in Treasury yield is not only affected by GDP and Inflation, but it may also be affected by other macro data releases (e.g., non-farm payroll), Fed speeches, and changes in the political situation, and all of these are likewise outward forms of the macroeconomic situation. Therefore, this study aims to use the GMM-HMM to create new variables from the Treasury yield data that can describe the macroeconomic situation from another aspect.

For the HMM [20], we had the notations:

Set of hidden states: $H = \{h_1, h_2, \ldots, h_N\}$

Set of observable states: $O = \{o_1, o_2, \ldots, o_M\}$

The series of hidden states: $I = \{i_1, i_2, \ldots, i_T\},\ i_t \in H$

The series of observable states: $J = \{j_1, j_2, \ldots, j_T\},\ j_t \in O$

Parameters: $\lambda = (A, B, \Pi)$

Hidden States Transition Probability (how the hidden states from $t-1$ change to t):

$$A = (a_{ij})_{N \times N}$$

How the hidden states can come up with the observable states:

$$B = \left[b_j(k)\right]_{N\times M}$$

The initial states at $t = 0$, which is the probability q_i

$$II = [\pi(i)]_N,\ \pi(i) = p(i_1) = q_i$$

It is assumed that the series of hidden states follows Markov Assumption, that the current state at t is only dependent on the state at $t - 1$ and the constant hidden states transition probability matrix A, but not any of the prior state data. And the observable states at t is only determined by the hidden states at time t and the matrix B.

The above statements are for the general discrete HMM model, but in this study, the states are not discrete, but instead continuous. Hence, the discrete probability measurement cannot be directly used in this situation, and Gaussian Mixture Model can be applied instead. Gaussian Mixture Model [21] is a probabilistic model used for representing a mixture of multiple Gaussian distributions, which is an extension of the traditional Gaussian Model. It is a generative model that assumes all the data points follow a mixture of several Gaussian distributions with unknown parameters. It can be formulated step-by-step, and the final Gaussian Mixture Model is applied to substitute the probabilities in the discrete HMM model:

P.D.F. of Univariate Gaussian Model:

$$f(x|\mu,\ \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

P.D.F. of Multivariate Gaussian Distribution:

$$f(\boldsymbol{X}|\boldsymbol{\mu},\ \boldsymbol{\Sigma}) = \frac{1}{(2\pi)^{\frac{D}{2}}|\boldsymbol{\Sigma}|^{\frac{1}{2}}}\exp\left(-\frac{1}{2}(\boldsymbol{X}-\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{X}-\boldsymbol{\mu})\right)$$

P.D.F. of Gaussian Mixture Model:

$$f(\boldsymbol{X}) = \sum_{k=1}^{K}\pi_k \cdot f_k(\boldsymbol{X}|\boldsymbol{\mu_k},\ \boldsymbol{\Sigma_k})$$

where:

π_k is the weight of the k-th component, with $\sum_{k=1}^{K}\pi_k = 1$

$f_k(\boldsymbol{X}|\boldsymbol{\mu_k},\ \boldsymbol{\Sigma_k})$ is the P.D.F. of the k-th Multivariate Gaussian Distribution

The target of this study is to obtain the last projection of the hidden states, which is h_t, and then utilized as the variable for rolling regression fitting. To obtain the estimation of this value, the Viterbi Algorithm [22] is utilized and utilized in Python.

3. Rolling Regression

Rolling Regression approach is a statistical technique used in time-series analysis, which involves estimating regression relationships over sequential subsamples, where the subsamples move forward through time [23]. This allows for the examination of how the regression coefficients change over time as new observations are included, and it will make use of all the available information but not the future information, which aligns with the research requirements.

In the study, after the new macro variables generated from GMM-HMM, they are incorporated into the rolling regression model with the original Macro data together, and fit the multivariate linear regression model each quarter. The projection of the FFR is made one quarter ahead of the fitted model, and all the projections are joint to form the predicted FFR series. The formulas are:

$$Y_t = \beta_t X_t + \epsilon_t$$

$$X_t = X_{t-1}\ append\ x_t$$

The min_data_points parameter in this study is set to be 12, indicating the data points number to fit the regression model is at least 12.

Results Analysis

It is essential to confirm all the data matching in this study, as the aim is to provide a practical forecasting model that can be directly used in real situations. After obtaining the dataset, this study rearranged the time index of FFR to be the start of next month, which represented the FFR of the end of last quarter, as required by Python that the index of explanatory variables and response variables were required to be matched. For instance:

- Macro_df data (Macro information dataset contains both the original macro variables and the generated macro variables): 2023-04-01 represents the Macro data for Quarter 2.
- FFR_df data (FFR Dataset): 2023-10-01 represents the FFR for Quarter 3.
- Actual FFR release: Q2 on 2023-06-30, Q3 on 2023-09-30.

Hence the model only included Macro_df data of 2023-04-01 and the information of treasury yields from 2023-06-30 to 2023-09-29 to do the modeling.

1. Traditional Taylor Rule

   By applying the Taylor Rule formula to the obtained dataset as described before, FFR could be predicted sequentially and hence compared to the real FFR. In order to evaluate the prediction performance, the Mean Square Error (MSE) metric was used, with the formula $MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \widehat{y_i})^2$. From Fig. 8, it could be clearly observed that the projection had captured the trend of the real FFR, but the value was not accurate, and even always overestimated the real FFR. By the MSE measurement, the calculated MSE of the Traditional Taylor Rule was 10.13, which was the benchmark of this research to break.

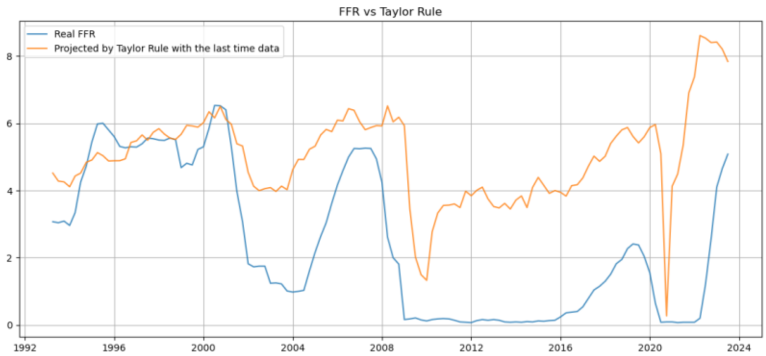

Figure 8. Plot of Real FFR and Projected FFR based on Taylor Rule

2. Rolling Regression Taylor Rule

   Here the rolling regression model with the two variables, Resource Gap and PCE, was fitted to predict the Federal Funds rate sequentially, which was shown in Fig. 9. It could be seen that the prediction was much closer to the real FFR than Traditional Taylor Rule, as the model was fitted in a rolling basis that considers all the information prior to the prediction. The MSE was 1.0258, which was much lower than the Traditional Taylor Rule prediction.

$$FFR_t = \beta_{1,t} Resource\ Gap_{t-1} + \beta_{2,t} PCE_{t-1} + \epsilon_t$$

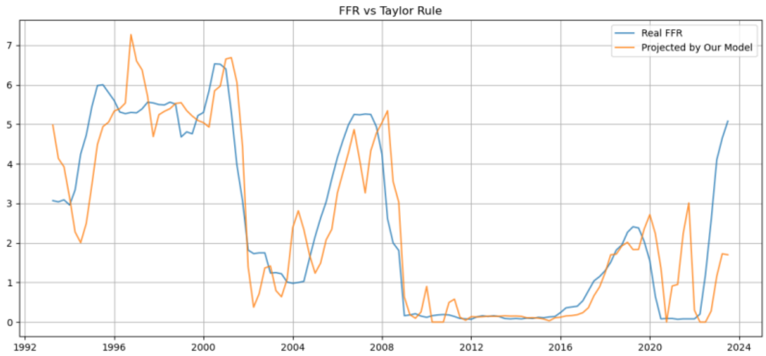

Figure 9. Plot of Real FFR and Projected FFR based on Rolling Regression Model

3. Rolling Regression with variables from GMM-HMM

After obtaining the daily Level, Slope, and Curvature Values from PCA method, they were used to construct the hidden states of the Macroeconomy. For GMM-HMM, it was required to assume how many possible hidden states are there, hence the algorithm would determine which one is the most likely. In this study, it was assumed that the hidden states were “Better”, “Flat”, and “Worse”, meaning that there were totally 3 types of hidden states. And the Level, Slope, and Curvature values were input separately to the GMM-HMM, where they represented the observable states. For instance, there were Levels from 1990-01-02 to 1990-03-29, and the FFR was released on 1990-03-30, which meant that the hidden states were calculated for the prediction of this FFR. And from the model, we had the predicted states to be [2 2 2 2 2 2 2 2 2 2 2 2 2 2 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1] for this quarter, with 2 represents “Better”, 1 represents “Flat”, and 0 represents “Worse”, and in this case the last “Flat” value was only taken to represent the macro condition.

The regression model was fitted in forward selection manner, by comparing the reduction of MSE after adding Level, Slope, or Curvature:

$$FFR_t = \beta_{1,t} Resource\ Gap_{t-1} + \beta_{2,t} PCE_{t-1} + \boldsymbol{\beta_{k,t} Variable_k} + \epsilon_t$$

With several testing, it was found that only the Level variable contributes to the MSE reduction, as shown in Fig. 10. The MSE was 1.0177, which was slightly lower than the previous rolling regression model performance.

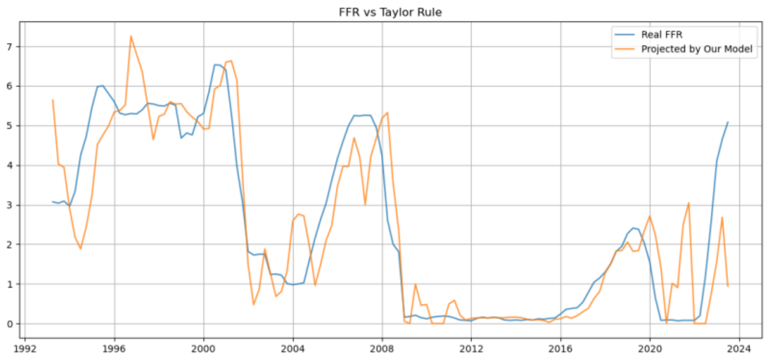

Figure 10. Plot of Real FFR and Projected FFR based on Level and Macro Data Rolling Regression Model

The Level variable accounted for the main variance from the PCA method, as it was the first component that had greatest eigenvalue, hence it was reasonable that only this variable had provided sufficient information of the macro condition. It could be interpreted as the average shift of the yield curve, meaning the market consensus corresponds to the macro situation.

4. Limitations

Although the rolling regression method with/without GMM-HMM could significantly reduce the MSE compared to the traditional Taylor Rule method, it had multiple limitations. Firstly, some of the variables were not statistically significant, which had p-value greater than 0.5, no matter considering the Level variable or not. The often happened for the Resource Gap and Level variable, but the PCE variable was generally significant, which could also explain the function of FFR to fight against inflation. Additionally, as shown in Fig. 9 and Fig. 10, the recent FFR projection was not aligned with the real FFR (significantly underestimated), meaning that the current yield curve was so abnormal for the model to make accurate projection. Moreover, the GMM-HMM model could not provide a stable states prediction, which could be illustrated as this example:

Predicted Hidden States from iteration 1: [2 2 2 2 2 2 2 2 2 2 2 2 2 2 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0]

Predicted Hidden States from iteration 2: [1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2]

Generally, the GMM-HMM which had strong assumptions to the distributions, required a large amount of data to provide a good distribution estimation, as it contained many density functions and probabilities inside. But in our case, there were only at most 63 data points to estimate such complicated distribution, which was hard to converge to a global minimum.

In the Extension part, it is aimed at using other methods to do the hidden states prediction, and also solve the Research Objective 3.

Extensions

1. Kalman Filter Method instead of GMM-HMM

The Kalman Filter is a recursive algorithm for estimating the state of a Linear Dynamic System (LDS) from a series of noisy measurements, which is built based on Linear Algebra and Hidden Markov Model [27, 28]. It assumes a system, which can be understood as the Hidden States of HMM, and it can be impacted by the input information and noise to predict the next new State. As here we don't go into the details of this algorithm, the Kalman Filter Method just has two main steps:

a) Prediction: Predict the current state of the system according to the previous state and the system dynamics.
b) Update: Obtain the current state prediction according to another measurement and combine with the prediction from the previous state to have a new estimate. The optimization process aims to minimize the uncertainty for the prediction.

This study also tries to use the Kalman Filter to Reformulate the Macro, as the system in this Kalman Filter is assumed to be the actual Macro situation. For example, if the correlation of the previous state and the current state is assumed to be 0.05 (which is a trial number), we have the following diagram generated from the Kalman Filter with the Level component:

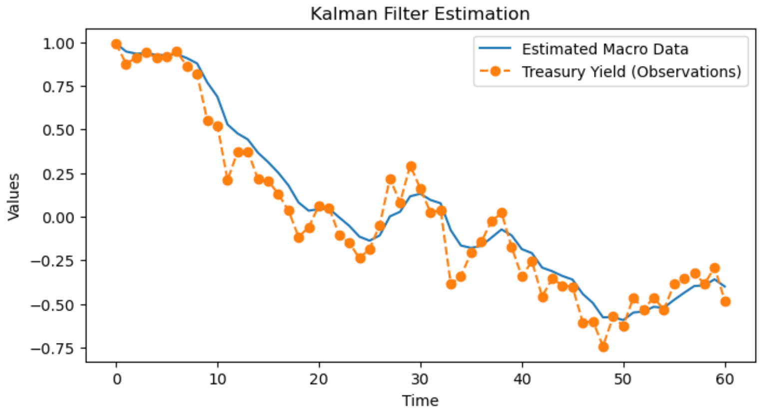

Figure 11. Level Component and its Kalman Filter Series with 0.05 Correlation Setting

From the diagram, it can be illustrated that the estimated macro condition is really close to the Level data, which seems to be the de-noise series of the Level. If it is indeed the Macro condition, then it should have great benefits to the rolling regression fitting. However, as it is so close to the Level components, it might also be affected by many other factors that represent the properties of Level but not the actual Macro, which is unfortunately unknown and hard to verify. If the correlation is further reduced to 0.0001, Fig. 12 can show another estimated macro condition that is more deviate from the Level series. However, it is still impossible to determine what is a good correlation assumption for this model.

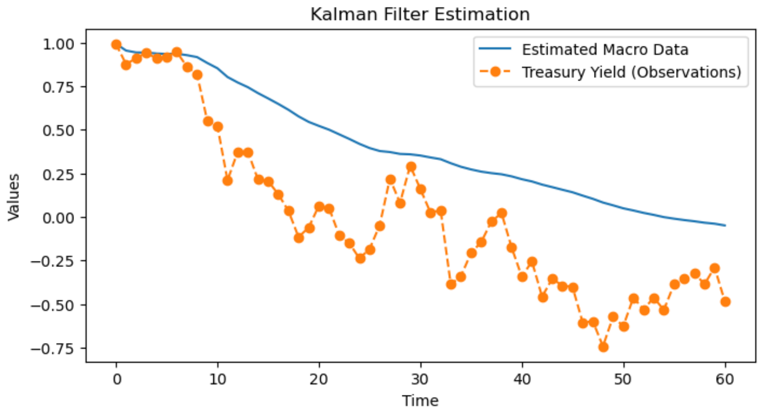

Figure 12. Level Component and its Kalman Filter Series with 0.0001 Correlation Setting

Then, models with different correlation assumptions are tried to generate the system states for constructing the rolling regression model. For example, the case with correlation 0.0001 is shown in Fig. 13, and with the MSE to be 1.06. This plot is provided with the Level variable generated from Kalman Filter, but not the Slope and Curvature, as these two components still perform weak after multiple experiments. What can be observed is that the curve doesn't get closer to the real FFR generally, and especially deviates more from the real in the earlier period, but the projection of FFR in the recent periods are more accurate. This can be explained by the dramatic fluctuations in treasury yield over the last two years and the market's sensitivity to it. In the past treasury yield has been relatively stable and therefore not necessarily affected by more macro factors, instead it is various microeconomic or geopolitical news that can have a sudden impact. However in the last two years due to the dramatic impact of the FFR's large interest rate hikes on the market, investors and politicians alike have been very concerned about the factors that could affect the next FFR forecast, and as a result, daily trading in treasury yields has been able to reflect to a great extent the state of the macro-economy as well as people's expectations of the macro-economic situation. In terms of why this rate hike may have had such an explanation, it may have stemmed from the fact that information is now more easily communicated, as well as more active trading activity reflected in greater trading volumes in the markets.

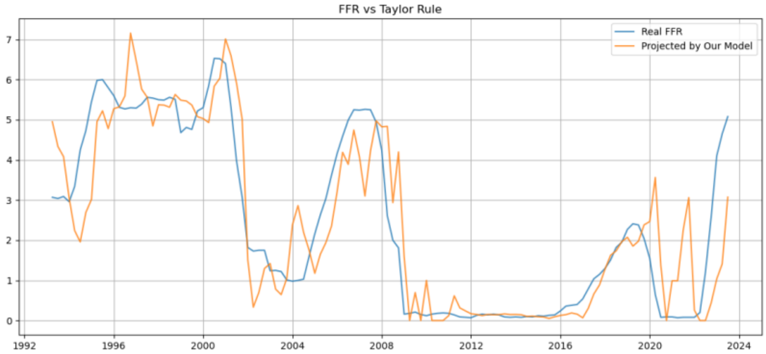

Figure 13. Plot of Real FFR and Projected FFR based on Level (generated from Kalman Filter Method with correlation to be 0.0001) and Macro Data Rolling Regression Model

However, in terms of the MSE value, which doesn't outperform the rolling regression model with GMM-HMM generated variable, the effectiveness of Kalman Filter remains doubted. Perhaps this method can give future researchers a direction to consider other observable series instead of treasury yield to make better prediction models.

2. Adjusting Information Window for longer FFR projection

Previously, the GMM-HMM model was used to generate variables that are only 1 day previous to the release of FFR, which is not very practical to build relevant trade strategies or risk management systems. Hence, this section tries to provide FFR projection based on the variables generated from variables that generated by data from further back, and also to see whether this projection can have performance similar to or even better than the previous GMM-HMM.

To begin with, the variables with 21 trading days ahead of FFR release instead of 1 day ahead of FFR release are tried, and the result is shown in Fig. 14, with the MSE of 0.9538 that is even less than the previous GMM-HMM rolling regression model. And we also have the plot of 42 trading days ahead of the FFR release, which is presented in Fig. 15, with the MSE to be 1.07511 that is greater than the previous one. With the case of 21 trading days ahead that provides MSE lower than the original one, it means that the markets can based on this model to make the FFR prediction 21 days ahead of the FFR release more accurate than only 1 day ahead, and hence provides enough time for the market to develop effective bet on the change on the FFR one month later. It is a surprising result as the Markov Assumption tells that the new states can reflect the full information of the previous states, but since the information now only provides one dimension, which is its magnitude, it is not determined that whether this should strictly follow the Markov Assumption or not. But this result should be validated further with more data and better parameter tuning for the GMM-HMM to make robust predictions.

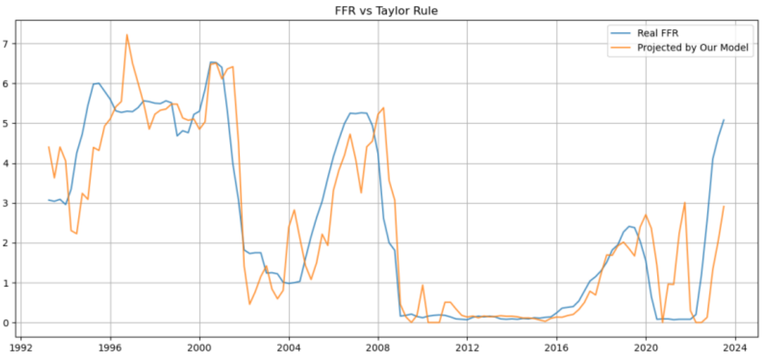

Figure 14. Plot of Real FFR and Projected FFR based on Level (generated from GMM-HMM with 21 days ahead of real FFR release) and Macro Data Rolling Regression Model

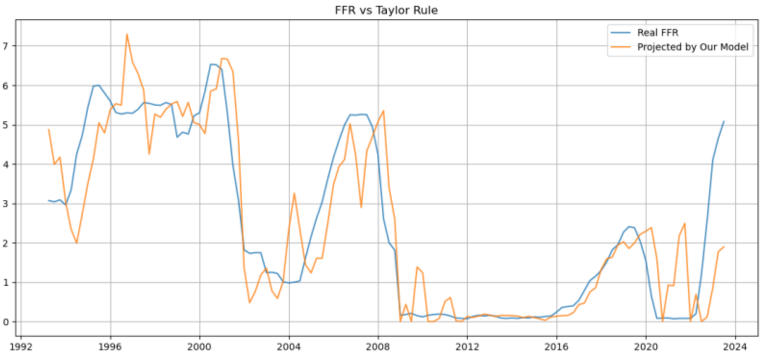

Figure 15. Plot of Real FFR and Projected FFR based on Level (generated from GMM-HMM with 42 days ahead of real FFR release) and Macro Data Rolling Regression Model

Recommendations

Since the release of macro data has the characteristic of lag, however, the integration of the Hidden Markov Model (GMM-HMM), can use daily released data, treasury yield, to project the Federal Fund Rate (FFR), offering a series of benefits for the general public, policymakers, statistics agencies, financial institutions, and individual investors.

Policymakers are recommended to accept the model for its capacity to process daily released treasury yield data, which provide a precise and real-time indicator of market flow. Unlike the usual reliance on quarterly macro statistics, this method allows policymakers to make timely and well-informed decisions. The additional information provided by the GMM-HMM model's capacity to detect latent states within the Treasury yield data enables policymakers to identify specific market situations that could impact the FFR. It is therefore advised that policymakers consider the incorporation of the HMM model, making use of its real-time capabilities and sophisticated comprehension of market dynamics to create monetary policies that are more flexible and successful.

Statistical agencies can use GMM-HMM model to forecast frameworks. The model's ability to handle high-frequency Treasury yield data mitigates the lag present in conventional macroeconomic indicators, which is the basis for this proposal. Statistics agencies can generate more accurate and valuable economic research, analysis, and forecasting datasets. The capacity of the GMM-HMM model to present a more complex picture of the economic environment allows statistics agencies to produce timelier insights and accurately represent the underlying market dynamics.

General investors are offered a significant alternative to understand the complexities of financial markets. As a result, investors have greater access to information sources and a deeper comprehension of upcoming interest rate adjustments and broader economic trends. The adaptable nature of the financial markets is caught by the dynamic and responsive aspects of the model, which enable investors to make well-informed decisions in response to shifting economic conditions.

To summarize, our research using the HMM model to project FFR presents a novel approach to policymakers, statistical agencies, and general investors. Policymakers are advised to use the model's real-time capabilities to make more intelligent and agile financial judgments. Statistical agencies are recommended to use this modeling method to generate faster and more accurate datasets, improving economic assessments. Finally, ordinary investors who accept the FFR estimates offered by the GMM-HMM model can utilize its dynamic and responsive character to negotiate the intricacies of financial markets more adeptly.

Conclusion

This study attempts to apply the GMM-HMM model to the Treasury yield data after PCA transformation in order to fit a macroeconomic condition measurement that is closer to the date of publication of the FFR. This study also hopes to improve the forecasting of the Taylor Rule with this data and therefore applies rolling regression to make forecasts of the FFR. and ensure that no possible future information is applied. After the experiments, it was found that the rolling regression model can provide exceptionally better performance than the Traditional Taylor Rule, with the MSE reduced from 10.13 to 1.0258. And the study observed that the only practical components provided by PCA method is the Level variable, which can help the rolling regression model reduce the MSE further to 1.0177. Moreover, the study applied Kalman Filter method and also tried to reduce the window size so as to make more forward projection in the extension part, and it was found that we could even have more accurate projection of FFR with the data one month ahead, with the MSE to be 0.9538. These extensions could potentially inspire future researchers to dive more into this area by applying the models more comprehensively, and applied other alternative but publicly available datasets. The study finally provided suggestions to the general public, policymakers, statistics agencies, financial institutions, and individual investors so that they could leverage on this proposed model to make better FFR projections.

Appendix

1. **The U.S. GDP Revision Details**

   GDP revision is to improve the accuracy and precision of GDP estimates. The process, which was started with the quarterly release of preliminary GDP figures, is based on a variety of data sources, such as economic indicators and surveys. But given the lack of some lagging indicators, these preliminary estimates might be inaccurate. Data collection continues after the first publication and includes revised business surveys, tax returns, and more economic data. The updated GDP estimates are then made available on a regular basis in subsequent phases, such as preliminary and final estimates for particular quarters or years. Furthermore, to reassess GDP data for a number of years, periodic historical revisions are carried out to consider modifications to methodologies or the addition of new data sources. Through a thorough and iterative process, GDP estimates are guaranteed to be in line with the most recent and accurate data available, giving the public, businesses, and policymakers a more sophisticated understanding of economic performance over time [24].

2. FFR hikes empirical analysis

In response to the 2008 economic crisis and subsequent economic recession, the Federal Open Market Committee lowered the target for the federal funds rate from 5.25% in mid-September 2007 to near zero by the end of December 2008 (See Fig. 4) [25]. This rate reduction was part of the Fed's strategy to stimulate economic activity and provide liquidity to the financial system. The goal was to make borrowing cheaper for banks, businesses, and consumers to encourage spending, investment, and lending. Together with various monetary policies such as buying back government securities, the market responded by purchasing large-scale assets, consequently fostering economic growth, job generation, and a gradual resurgence of inflation toward 2% [3]. Notably, in December 2015, the Federal Open Market Committee initiated the process of increasing the target for the federal funds rate, transitioning from its near-zero level to a more conventional rate. Subsequently, in October 2017, the Federal Open Market Committee embarked on the gradual reduction of its securities holdings, marking another substantial step toward the normalization of monetary policy [5]. As part of this shift, the Committee conveyed that future adjustments in the federal funds rate would serve as the primary mechanism for altering the overall stance of monetary policy.

3. Definition of Effective Federal Funds Rate

The federal funds market consists of domestic unsecured borrowings in U.S. dollars by depository institutions from other depository institutions and certain other entities, primarily government-sponsored enterprises. The effective federal funds rate (EFFR) is calculated as a volume-weighted median of overnight federal funds transactions reported in the FR 2420 Report of Selected Money Market Rates [26].

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