FINM 36700 **Mark Hendricks**
Autumn 2024

Homework #6

Case: AQR's Momentum Funds (A) [9-211-025].

1 The Momentum Product

This section is not graded, and you do not need to submit your answers. But you are expected to consider these issues and be ready to discuss them.

1. What is novel about the AQR Momentum product under construction compared to the various momentum investment products already offered?

2. Name three reasons the momentum investment product will not exactly track the momentum index, (ie. why the strategy will have tracking error.)

3. When constructing the momentum portfolio, AQR ranks stocks on their returns from month $t-12$ through $t-2$. Why don't they include the $t-1$ return in this ranking?

2 Investigating Momentum

In this section, we empirically investigate some concerns regarding AQR's new momentum product.

On Canvas, find the data file, **"momentum_data.xlsx"**.

- The first tab contains the momentum factor as an excess return: $\tilde{r}^{\text{mom}}$.

- The second tab contains returns on portfolios corresponding to scored momentum deciles.
  - $r^{\text{mom}(1)}$ denotes the portfolio of stocks in the lowest momentum decile, the "losers" with the lowest past returns.
  - $r^{\text{mom}(10)}$ denotes the portfolio of stocks in the highest momentum decile.

- The third tab gives portfolios sorted by momentum and size.
  - r^{momSU} denotes the portfolio of small stocks in the top 3 deciles of momentum scores.
  - r^{momBD} denotes the portfolio of big-stocks in the bottom 3 deciles of momentum scores.

Note that the Fama-French momentum return, $\tilde{r}^{\text{mom:FF}}$, given in the first tab, is constructed by FF as,

$$\tilde{r}^{\text{mom:FF}} = \frac{1}{2}\left(r^{\text{momBU}} + r^{\text{momSU}}\right) - \frac{1}{2}\left(r^{\text{momBD}} + r^{\text{momSD}}\right) \tag{1}$$

1. **Is momentum still profitable?**

   The excess returns of lucrative trading strategies often disappear once the strategy is well-known. The first widely-cited paper on momentum was published in 1993. Have momentum returns or risk changed since then? The AQR case takes place at the end of 2008. Have momentum returns changed in 2009-2024?

   Investigate by filling out the summary statistics below for the full-sample and three sub-samples.

   (a) Using the data provided, fill in Table 1 with the appropriate stats for $\tilde{r}^{\text{mom:FF}}$.

   Table 1: Momentum performance over time.

Subsample	mean	vol	Sharpe	skewness	corr. to $\tilde{r}^m$	corr. to $\tilde{r}^v$
1927-2024						
1927-1993						
1994-2008						
2009-2024						

   (b) Has momentum changed much over time, as seen through these subsample statistics?

   (c) Does this data support AQR's argument that momentum is an important piece of the ideal portfolio? What if mean returns to momentum are in actuality near zero due to transaction costs—would there still be evidence here that momentum is valuable?

2. The case is about whether a long-only implementation of momentum is valuable.

   Construct your own long-only implementation:[1],

$$\tilde{r}^{\text{momU:FF}} = \frac{1}{2}\left(r^{\text{momBU}} + r^{\text{momSU}}\right) - r^f$$

   Note that this is following the FF approach of treating big and small stocks separately. This would be very similar to a scaled version of,

$$\tilde{r}^{\text{momU}} = \frac{1}{3}\left(r^{\text{mom(8)}} + r^{\text{mom(9)}} + r^{\text{mom(10)}}\right) - r^f$$

   For the question below, use the FF-style, $\tilde{r}_t^{\text{momU:FF}}$.

   (a) Fill out Table 2 for the data **in the period 1994-2024.**

   (b) Is long-only momentum as attractive as long-short momentum with respect to mean, volatility, and Sharpe Ratio?

[1] Note that we must subtract the risk-free rate to get the excess return of this portfolio, $\tilde{r}^{\text{momU}}$.

Table 2: Long-only momentum performance.

1994-2024	mean	vol	Sharpe	skew	corr. to $\tilde{r}^m$	corr. to $\tilde{r}^v$
Long-and-short, ($\tilde{r}^{\text{mom:FF}}$)						
Long-only ($\tilde{r}^{\text{momU:FF}}$)						

(c) Is long-only momentum as diversifying as long-short momentum with respect to market and value premia?

(d) Show a plot of the cumulative product of $1 + \tilde{r}^{\text{mom:FF}}$ and $1 + \tilde{r}^{\text{momU:FF}}$ over the 1994-2024 subsample.[2]

3. **Is momentum just data mining, or is it a robust strategy?**

Assess how sensitive the threshold for the "winners" and "losers" is in the results. Specifically, we compare three constructions:

- long the top 1 decile and short the bottom 1 deciles

$$\tilde{r}^{\text{momD1}} = r^{\text{mom}(10)} - r^{\text{mom}(1)}$$

- long the top 3 deciles and short the bottom 3 deciles

$$\begin{aligned}\tilde{r}^{\text{momD3}} =& \frac{1}{3}\left(r^{\text{mom}(8)} + r^{\text{mom}(9)} + r^{\text{mom}(10)}\right) - \frac{1}{3}\left(r^{\text{mom}(1)} + r^{\text{mom}(2)} + r^{\text{mom}(3)}\right) \\ =& \frac{1}{3}\sum_{k=8}^{10} r^{\text{mom}(k)} - \frac{1}{3}\sum_{k=1}^{3} r^{\text{mom}(k)}\end{aligned}$$

- long the top 5 deciles and short the bottom 5 deciles

$$\tilde{r}^{\text{momD5}} = \frac{1}{5}\sum_{k=6}^{10} r^{\text{mom}(k)} - \frac{1}{5}\sum_{k=1}^{5} r^{\text{mom}(k)}$$

(a) Compare all three constructions, (in the full-sample period,) by filling out the stats in the table below **for the period 1994-2024**.

(b) Do the tradeoffs between the 1-decile, 3-decile, and 5-decile constructions line up with the theoretical tradeoffs we discussed in the lecture?

(c) Should AQR's retail product consider using a 1-decile or 5-decile construction?

(d) Does $\tilde{r}^{\text{momD3}}$ have similar stats to the Fama-French construction in (1). Recall that construction is also a 3-decile, long-short construction, but it is segmented for small and large stocks. Compare the middle row of Table 3 with the top row of Table 2.

[2] This is not quite a cumulative return given that these are **excess** returns, which is why we must add 1 before taking the cumulative product.

Table 3: Robustness of momentum construction.

1994-2024	mean	vol	Sharpe	skewness	corr. to $\tilde{r}^m$	corr. to $\tilde{r}^v$
$\tilde{r}^{\text{momD1}}$						
$\tilde{r}^{\text{momD3}}$						
$\tilde{r}^{\text{momD5}}$						

4. **Does implementing momentum require trading lots of small stocks—thus causing even larger trading costs?**

   For regulatory and liquidity reasons, AQR is particularly interested in using larger stocks for their momentum baskets. (Though they will launch one product that focuses on medium-sized stocks.)

   Use the data provided on both small-stock "winners", r^{momSU}, and small-stock "losers", r^{momSD}, to construct a small-stock momentum portfolio,

   $$r_t^{\text{mom}S} = r_t^{\text{momSU}} - r_t^{\text{momSD}}$$

   Similarly, use the data provided to construct a big-stock momentum portfolio,

   $$r_t^{\text{mom}B} = r_t^{\text{momBU}} - r_t^{\text{momBD}}$$

   (a) Fill out Table 4 over the sample 1994-2024.

Table 4: Momentum of small and big stocks.

1994-2024	mean	vol	Sharpe	skewness	corr. to $\tilde{r}[m]$
All stocks, $\tilde{r}^{\text{mom:FF}}$					
Small stocks $r_t^{\text{mom}S}$					
Large stocks $r_t^{\text{mom}B}$					

   (b) Is the attractiveness of the momentum strategy mostly driven by the small stocks? That is, does a momentum strategy in large stocks still deliver excess returns at comparable risk?

5. **In conclusion,** what is your assessment of the AQR retail product? Is it capturing the important features of the Fama-French construction of momentum? Would you suggest any modifications?

The Risk of Stocks in the Long-Run: The Barnstable College Endowment [HBS 9-296073].

3 Barnstable's Analysis

This section is not graded, and you do not need to submit your answers.
We may discuss these problems later, but they will not be covered in Midterm 2.

1. Barnstable's Philosophy
   (a) What has Barnstable's investment strategy been in the past?
   (b) Explain the logic behind their view that stocks are safer in the long run.
   (c) What assumptions underly Barnstable's belief in the long-run safety of stocks?

2. Two Proposals
   (a) Describe the two proposals Barnstable is considering for how to take advantage of their view regarding the long-run safety of stocks.
   (b) How is the trust different from simply shorting the risk-free rate to take a levered position in stocks?
   (c) Do these proposals take the same bet on long-run stock performance? In what outcomes will they have different returns?
   (d) Do the two proposals differ in their risk?

3. Do you recommend a direct investment in the S&P, the trust or the puts?

4 Estimating Underperformance

This section is required and is to be submitted..
This section is fair game for Midterm 2.

Data

- Use the returns on the S&P 500 (r^m) and 1-month T-bills, (r^f) provided in **"barnstable_analysis_data.xlsx"**.
- Barnstable's estimates of mean and volatility are based on the subsample of 1965 to 1999.
- We consider this subsample, as well as 2000-2024, as well as the full sample of 1926-2024.
- We only have data through part of 2024, but no adjustment is needed for the fact that you have only the partial year–just use what you have.

1. Summary Statistics

   (a) Report the following (annualized) statistics.

		1965-1999		2000-2024		1926-2024	
		mean	vol	mean	vol	mean	vol
levels	r^m						
	$\tilde{r}^m$						
	r^f						
logs	r^m						
	$\tilde{\mathrm{r}}^m$						
	r^f						

   (b) Comment on how the full-sample return stats compare to the sub-sample stats. Comment on how the level stats compare to the log stats.

2. Recall the following...

   - If $x \sim \mathcal{N}\left(\mu_x, \sigma_x^2\right)$, then

$$\Pr\left[x < \ell\right] = \Phi_{\mathcal{N}}\left(L\right)$$
$$L = \frac{c - \mu_x}{\sigma_x}$$

   where $\Phi_{\mathcal{N}}$ denotes the standard normal cdf.

- Remember that cumulative log returns are simply the sum of the single-period log returns,

$$\mathrm{r}^m_{t,t+h} \equiv \sum_{i=1}^{h} \mathrm{r}^m_{t+i}$$

- It will be convenient to use and denote sample averages. We use the following notation for an h-period average ending at time $t+h$,

$$\bar{\mathrm{r}}^m_{t,t+h} = \frac{1}{h} \sum_{i=1}^{h} \mathrm{r}^m_{t+i}$$

Calculate the probability that the cumulative market return will fall short of the cumulative risk-free return.[3]

$$\Pr\left[R^m_{t,t+h} < R^f_{t,t+h}\right] \tag{2}$$

To analyze this analytically, convert the probability statement above to a probability statement about mean log returns.

(a) Calculate (2) using the subsample 1965-1999.

(b) Report the precise probability for $h = 15$ and $h = 30$ years.

(c) Plot the probability as a function of the investment horizon, h, for $0 < h \leq 30$ years.

3. Use the sample 1965-2024 to reconsider the 30-year probability. As of the end of 2024, calculate the probability of the stock return underperforming the risk-free rate over the next 30 years. That is, $R^m_{t,t+h}$ underperforming $R^f_{t,t+h}$ for $0 < h \leq 30$.

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4. Let's consider how things turned out relative to Barnstable's 1999 expectations.

   What was the probability (based on the 1999 estimate of μ,) that the 25-year market return, $R^m_{t,t+25}$, would be smaller than that realized in 2000-2024? Note that we are asking about the market return, not the excess market return. Continue using the 1965-1999 sample standard deviation for σ.

[3] Note that this is essentially what Barnstable is estimating, but with the 1965-1999 sample estimate that the risk-free rate was 6% during that period.