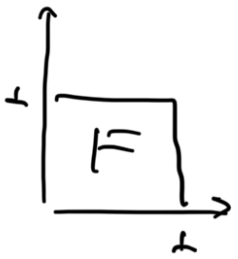
1. Geometric Transformations

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = A \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

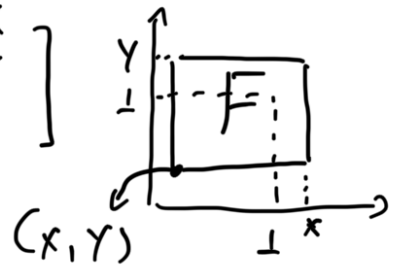
No change

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Translate (Tịnh tiến)

$$\begin{bmatrix} 1 & 0 & X \\ 0 & 1 & Y \\ 0 & 0 & 1 \end{bmatrix}$$

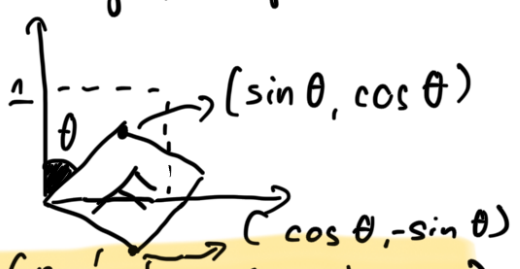


Scale about origin (Tỉ lệ quanh gốc toạ độ)

$$\begin{bmatrix} W & 0 & 0 \\ 0 & H & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

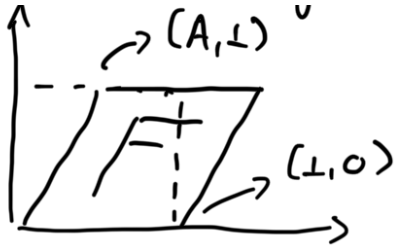
Rotate about origin (Quay quanh gốc toạ độ)

$$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



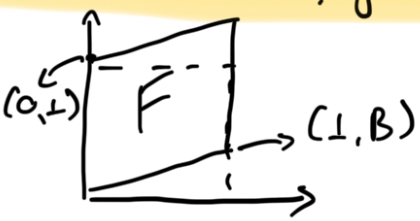
Shear in x direction (Biến dạng theo trục x)

$$\begin{bmatrix} 1 & A & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



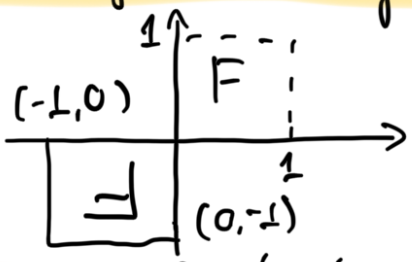
Shear in y direction (Biến dạng theo trục y)

$$\begin{bmatrix} 1 & 0 & 0 \\ B & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



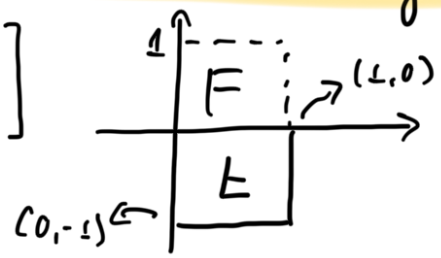
Reflect about origin (Đối xứng qua gốc tọa độ)

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



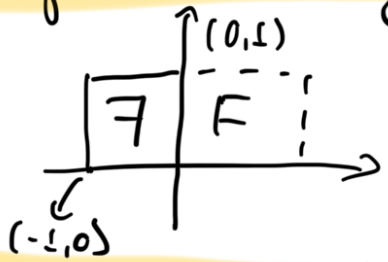
Reflect about x-axis (Đối xứng qua trục Ox)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Reflect about y-axis (Đối xứng qua trục Oy)

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Reflect about space (x=y) (Đối xứng qua đường y=x)

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2. Sharpening Spatial filters

Laplacian Filter

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$g(x,y) = f(x,y) + c[\nabla^2 f(x,y)]$$

kernel	Công thức $\nabla^2 f$
$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$	$\nabla^2 f(x,y) = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$
$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$	$\nabla^2 f(x,y) = \sum_{i=-1}^{1}\sum_{y=-1}^{1} f(x+i,y+i) - 9f(x,y) + f(x,y)$ $= f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)$ $+ f(x+1,y+1) + f(x+1,y-1) + f(x-1,y+1)$ $+ f(x-1,y-1) - 8f(x,y)$
$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$	$\nabla^2 f(x,y) = -[f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)] + 4f(x,y)$
$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$	$\nabla^2 f(x,y) = \sum_{\substack{(i,j)\in\{-1,0,1\}^2 \\ (i,j)\neq(0,0)}} -f(x+i,y+i) + 8f(x,y)$ $= -[f(x+1,y) + f(x-1,y) + f(x,y+1) +$ $f(x,y-1) + f(x+1,y+1) + f(x+1,y-1)$ $+ f(x-1,y+1) + f(x-1,y-1)] + 8f(x,y)$

Unsharp Masking & High boost Filtering

Blur the original image ⟶ Subtract the blurred image from the original (the result called mask) ⟶ Add the original with the mask

$$g_{mask}(x,y) = f(x,y) - \bar{f}(x,y)$$ → blurred image

$$g(x,y) = f(x,y) + k\, g_{mask}(x,y)$$

$\begin{cases} k = 1 : \text{unsharp masking} \\ k > 1 : \text{highboost Filtering} \end{cases}$

Gradient

$$\vec{\nabla} f(x,y) = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

$$|\vec{\nabla} f(x,y)| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

$G_x = f * K_x$, $G_y = f * K_y$

$G = \sqrt{G_x^2 + G_y^2}$

hoặc $G = |G_x| + |G_y|$

Tên toán tử	Kernel theo x	Kernel theo y
Sobel	$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$
Prewitt	$\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

Roberts	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

3. PDE _ Morphology

Dilation : $A \oplus B = \{ z \mid (\hat{B})_z \cap A \neq \emptyset \}$

$A \oplus B = \{ z \mid [\hat{B})_z \cap A] \subseteq A \}$

$P(\text{output}) = \begin{cases} 1 & \text{if hit} \\ 0 & \text{otherwise} \end{cases}$

Erosion : $A \ominus B = \{ z \mid (B)_z \subseteq A \}$

$p(\text{output}) = \begin{cases} 1 & \text{if fit} \\ 0 & \text{otherwise} \end{cases}$

Opening: $A \circ B = (A \ominus B) \oplus B$

Closing : $A \bullet B = (A \oplus B) \ominus B$

Hit or Miss Transformation

$A \otimes (B_1, B_2) = (A \ominus B_1) \cap (A^c \ominus B_2)$

Boundary Extraction

$B(A) = A - (A \ominus B)$

Hole Filling

$X_k = (X_{k-1} \oplus B) \cap A^c \quad k = 1, 2, 3, \ldots$

Until $X_k = X_{k-1}$

Filled = $A \cup X_k$