DIP: PDE _ Morphology

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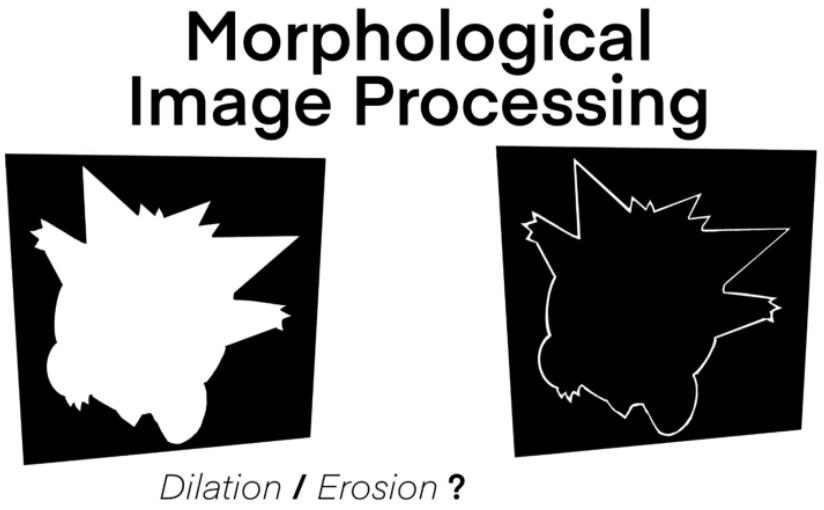
Morphological Image Processing

Morphology is a comprehensive set of image processing operations that process images based on shapes [1]. Morphological operations apply a structuring element to an input image, creating an output image of the same size.

In a morphological operation, the value of each pixel in the output image is based on a comparison of the corresponding pixel in the input image with its neighbors.



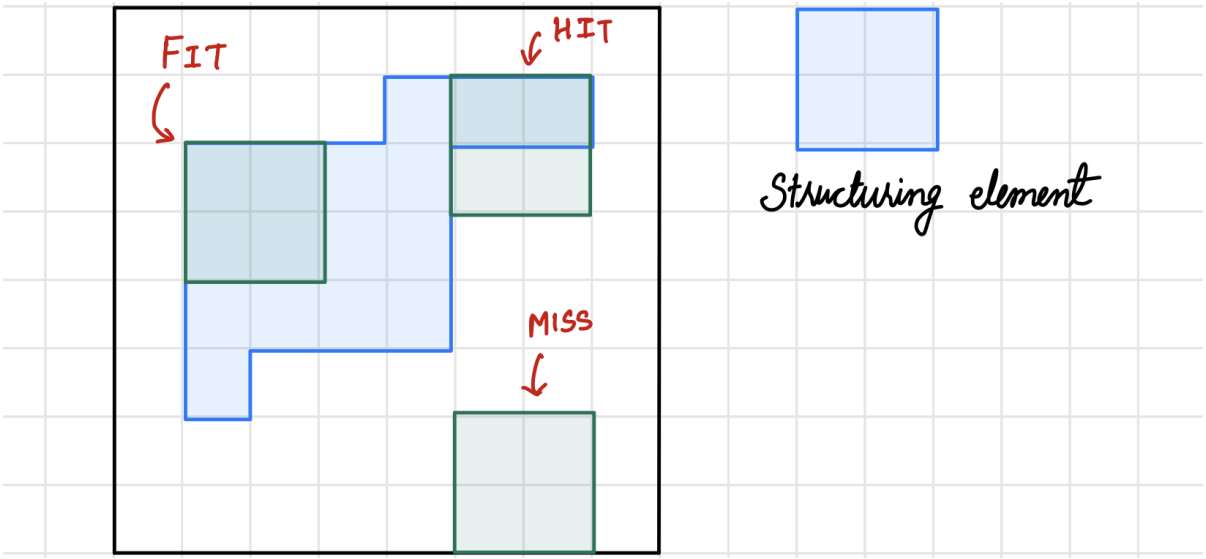
Terminologies

Structuring Element: It is a matrix or a small-sized template that is used to traverse an image. The structuring element is positioned at all possible locations in the image, and it is compared with the connected pixels. It can be of any shape.

Fit: When all the pixels in the structuring element cover the pixels of the object, we call it Fit.

Hit: When at least one of the pixels in the structuring element cover the pixels of the object, we call it Hit.

Miss: When no pixel in the structuring element cover the pixels of the object, we call it miss.



Dilation and Erosion

0	1	0
1	1	1
0	1	0

Two basic operations:

A is the image, B is the “structural element”, a mask akin to a kernel in convolution

Dilation : $A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \phi\}$

$$A \oplus B = \{z \mid [(\hat{B})_z \cap A] \subseteq A\}$$

(all shifts of B that have a non-empty overlap with A)

Erosion : $$A \Theta B = \{z \mid (B)_z \subseteq A\}$$

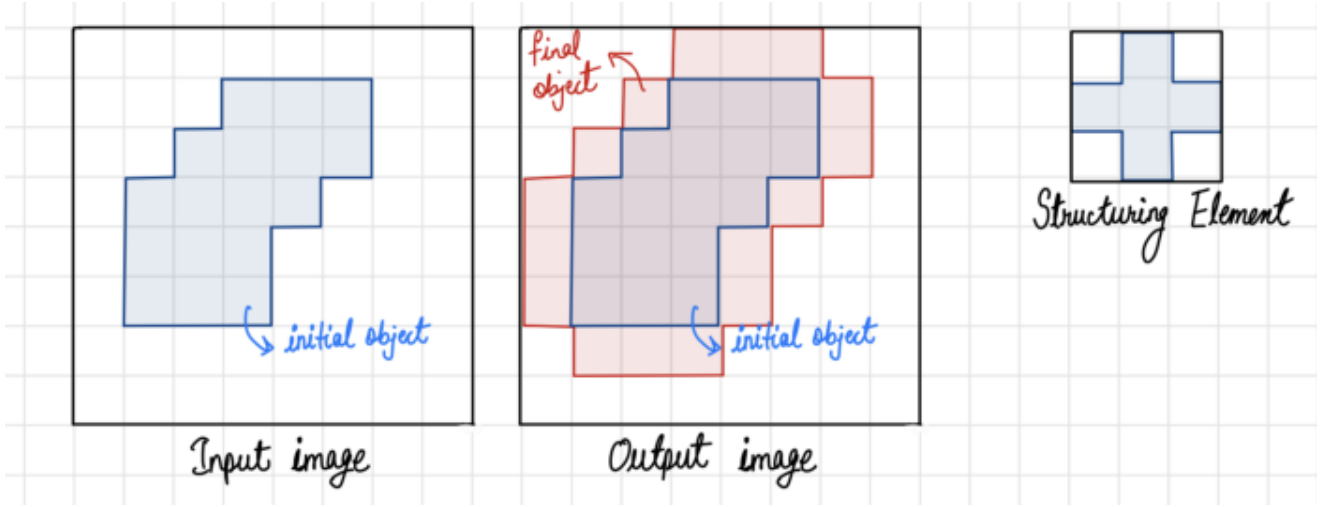
(all shifts of B that are fully contained within A)

1. Dilation

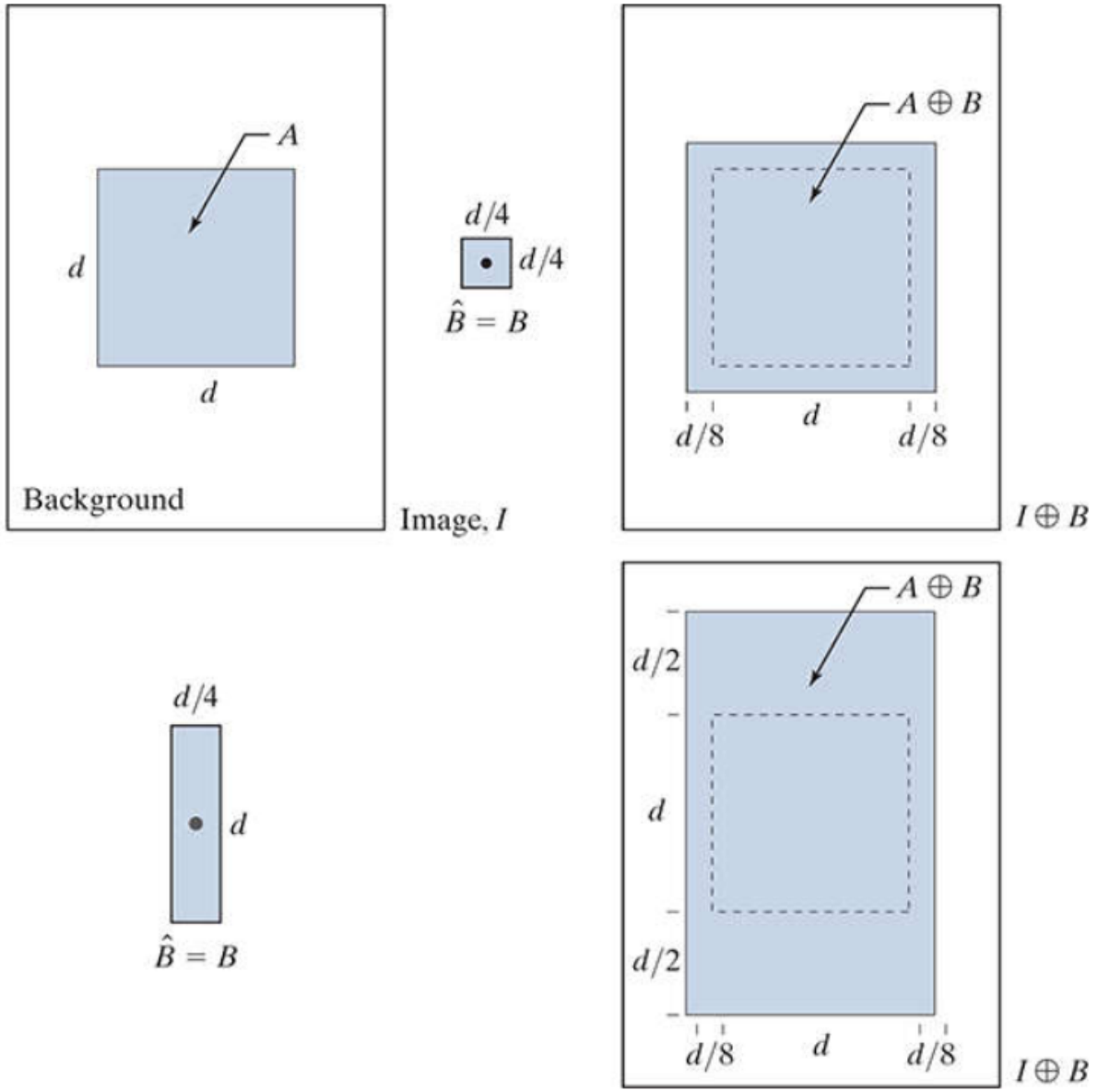
The output pixel values are calculated using the following equation.

Pixel (output) = 1 {if HIT}

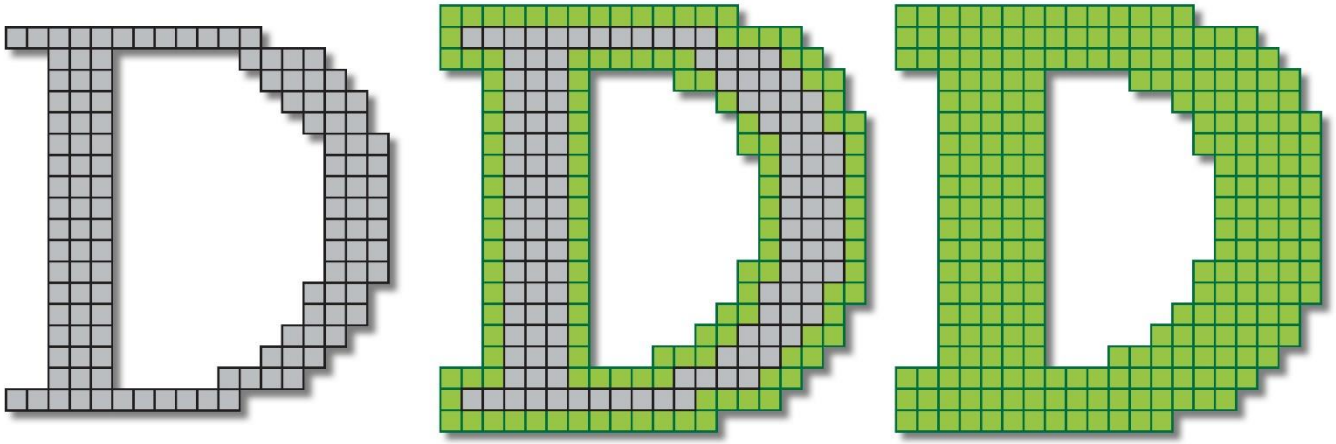
Pixel (output) = 0 {otherwise}



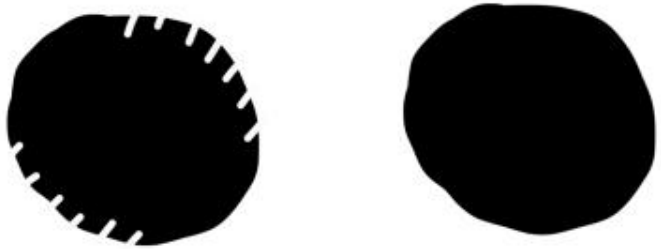
Dilation


A
d
d
Background
Image, I
d/4
d/4
$\hat{B} = B$
$A \oplus B$
d/8
d
d/8
$I \oplus B$
d/4
d
$\hat{B} = B$
$A \oplus B$
d/2
d
d/2
d/8
d
d/8
$I \oplus B$

Dilation

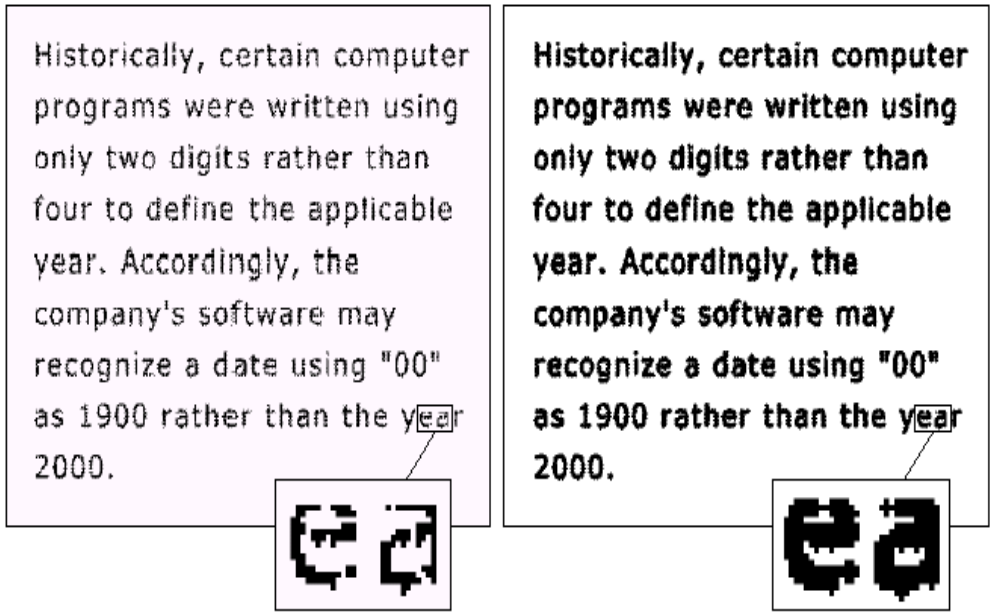


can repair breaks



can repair intrusions

Dilation

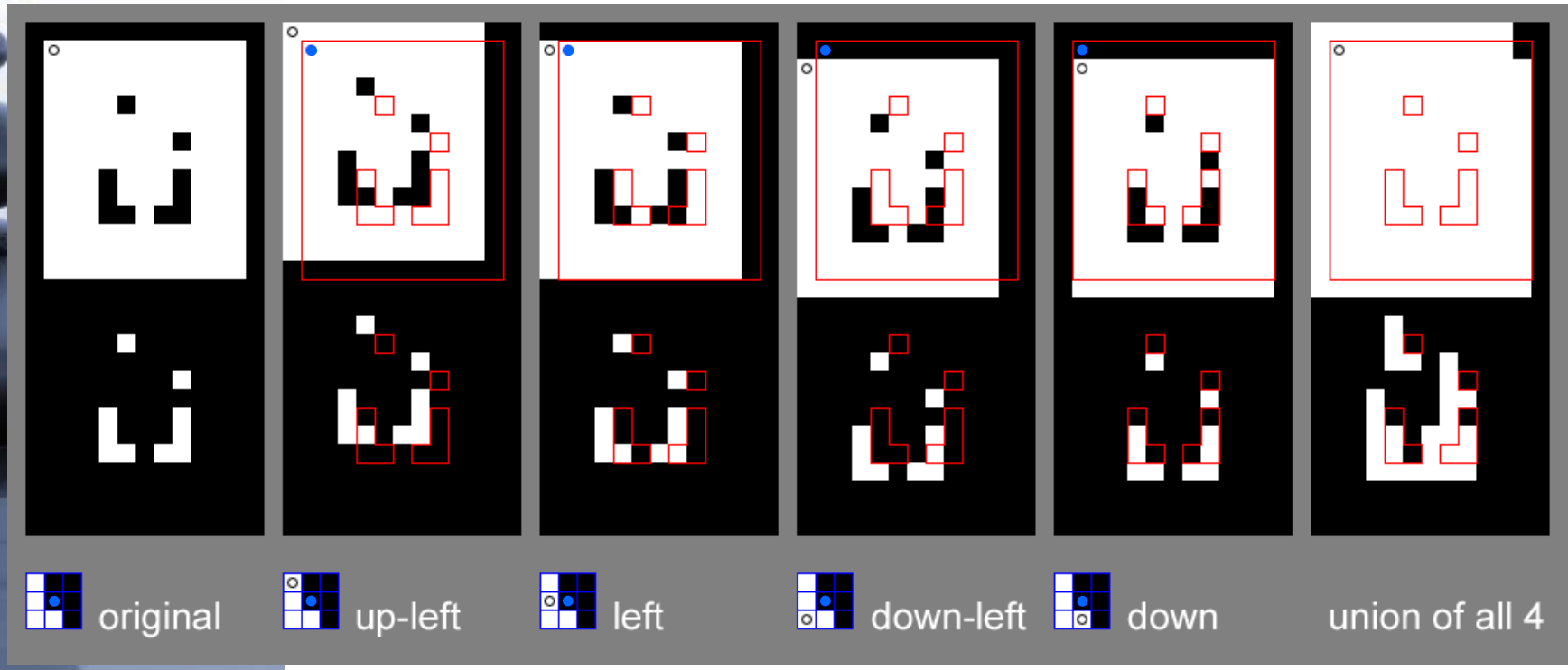


a c
b

FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

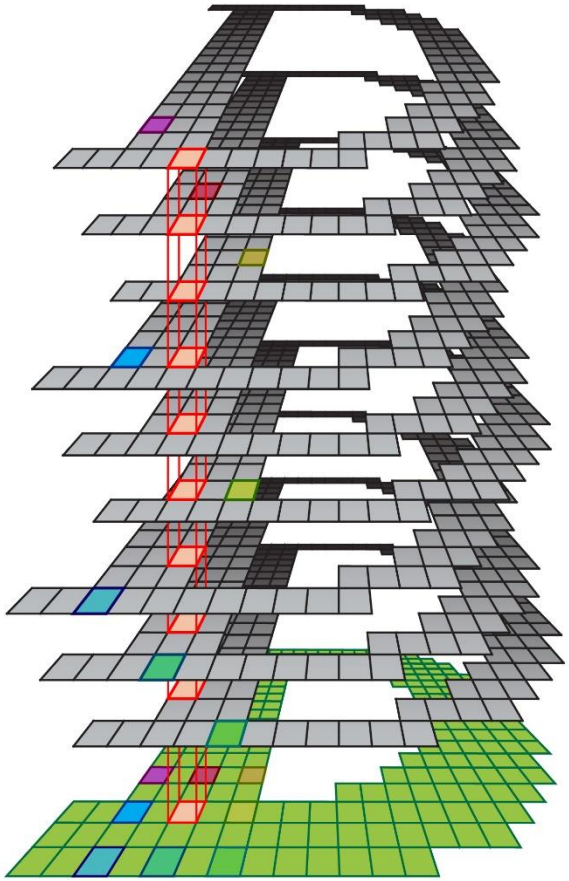
0	1	0
1	1	1
0	1	0

Dilation through Image Shifting



The red outlines indicate the positions of the features in the original images.

Dilation through Image Shifting

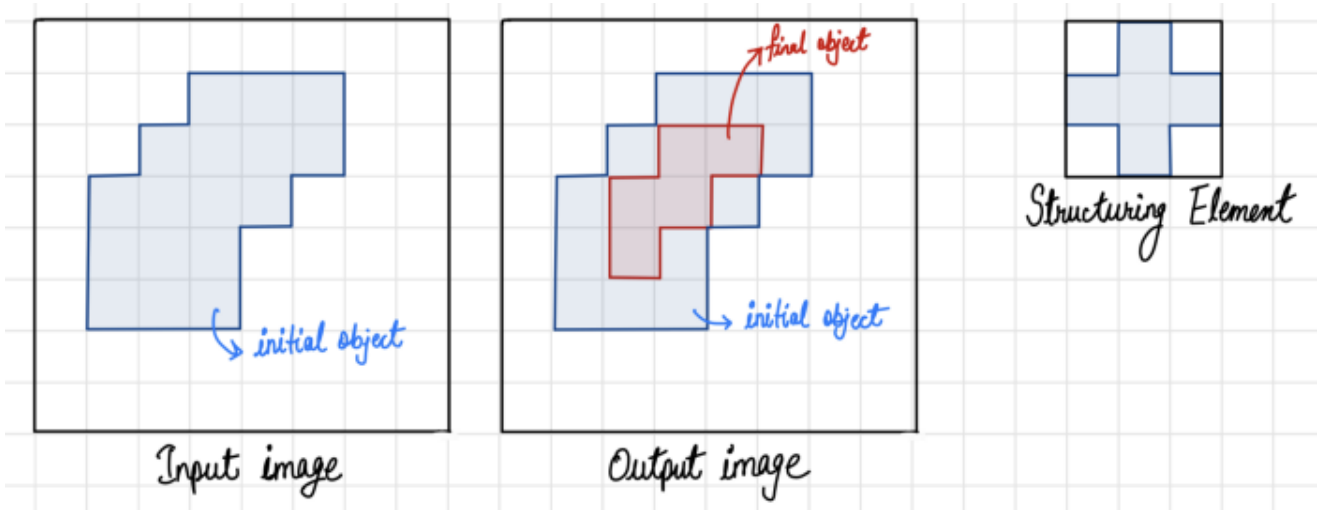


2. Erosion

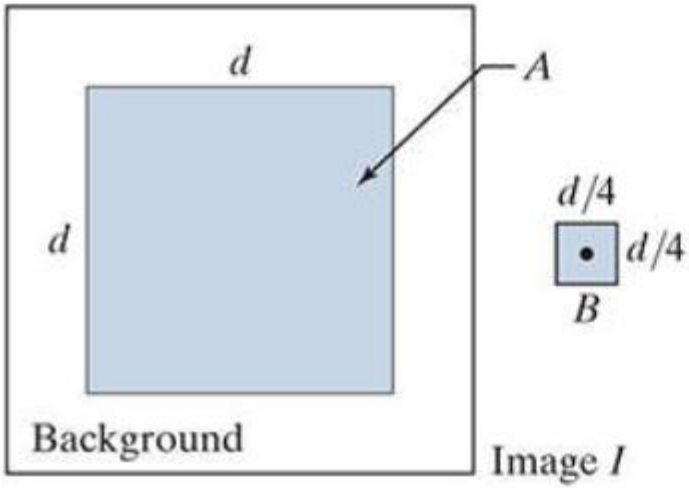
The output pixel values are calculated using the following equation.

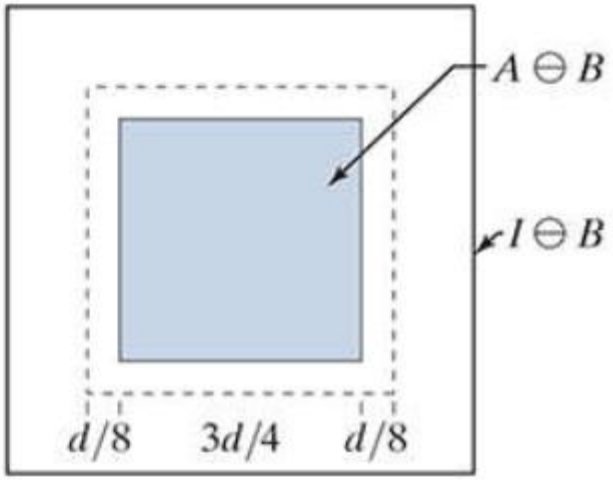
Pixel (output) = 1 {if FIT}

Pixel (output) = 0 {otherwise}

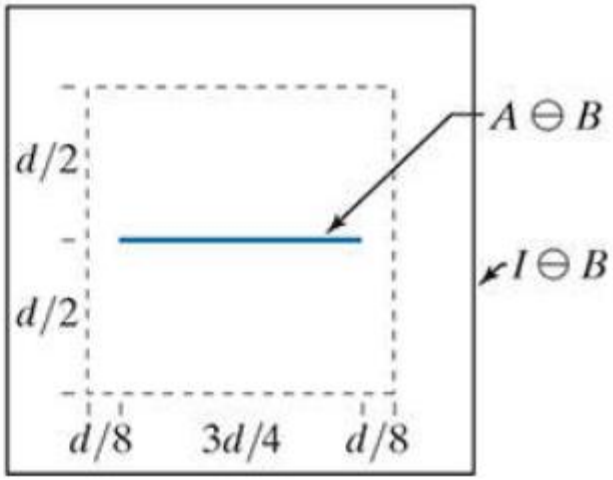


Erosion

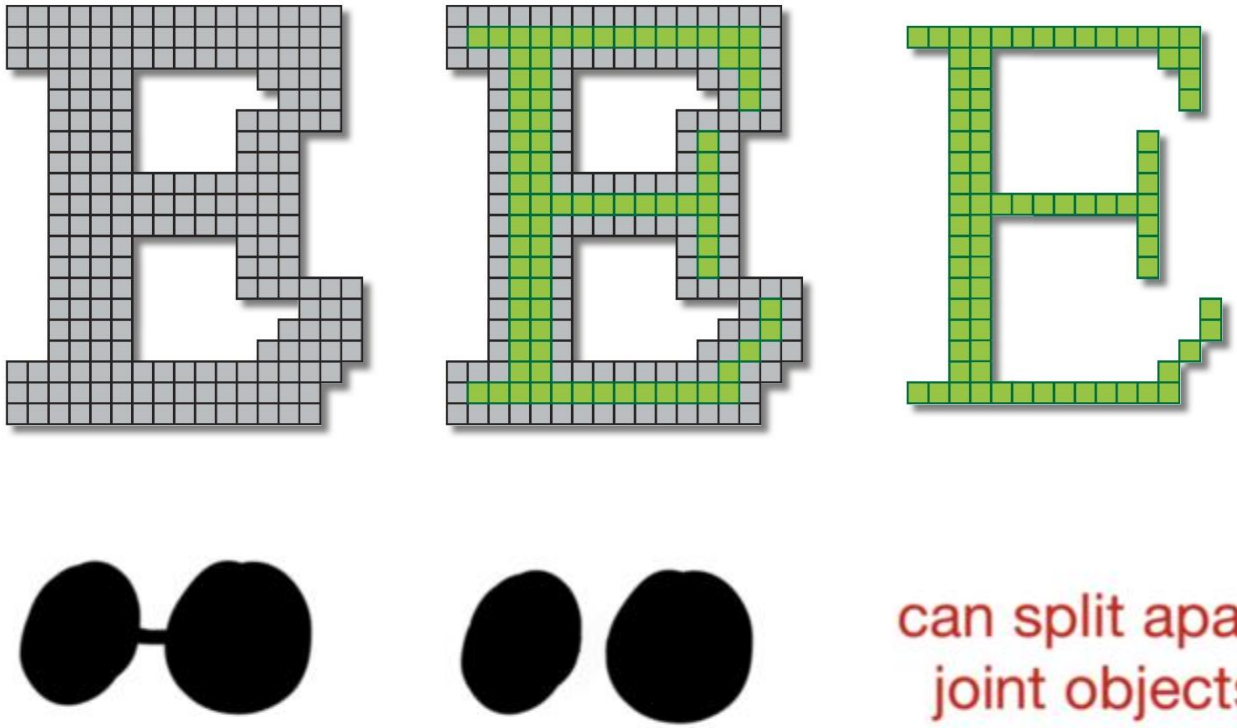

d
A
d
d/4
d/4
B
Background
Image I


A ⊖ B
I ⊖ B
d/8
3d/4
d/8


d/4
d
B

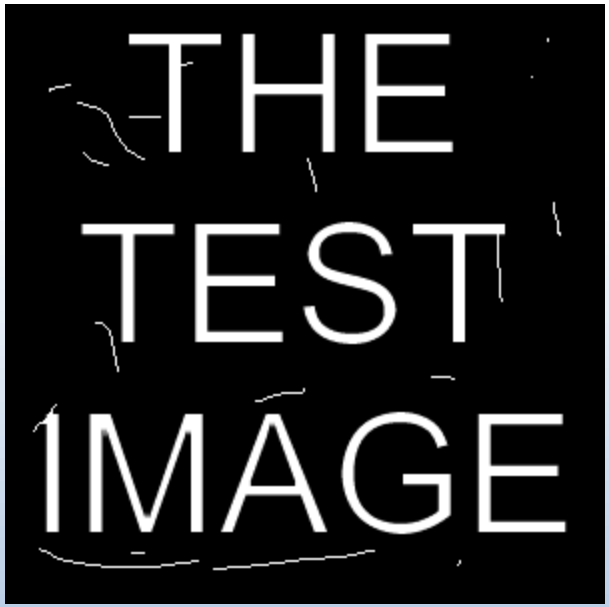

A ⊖ B
I ⊖ B
d/2
d/2
d/8
3d/4
d/8

Erosion



Erosion

Original image



Eroded image

Erosion

Eroded once

Eroded twice

Opening and Closing

Opening : smooth the contour of an object, breaks narrow isthmuses, and eliminates thin protrusions

$$A \circ B = (A \Theta B) \oplus B$$

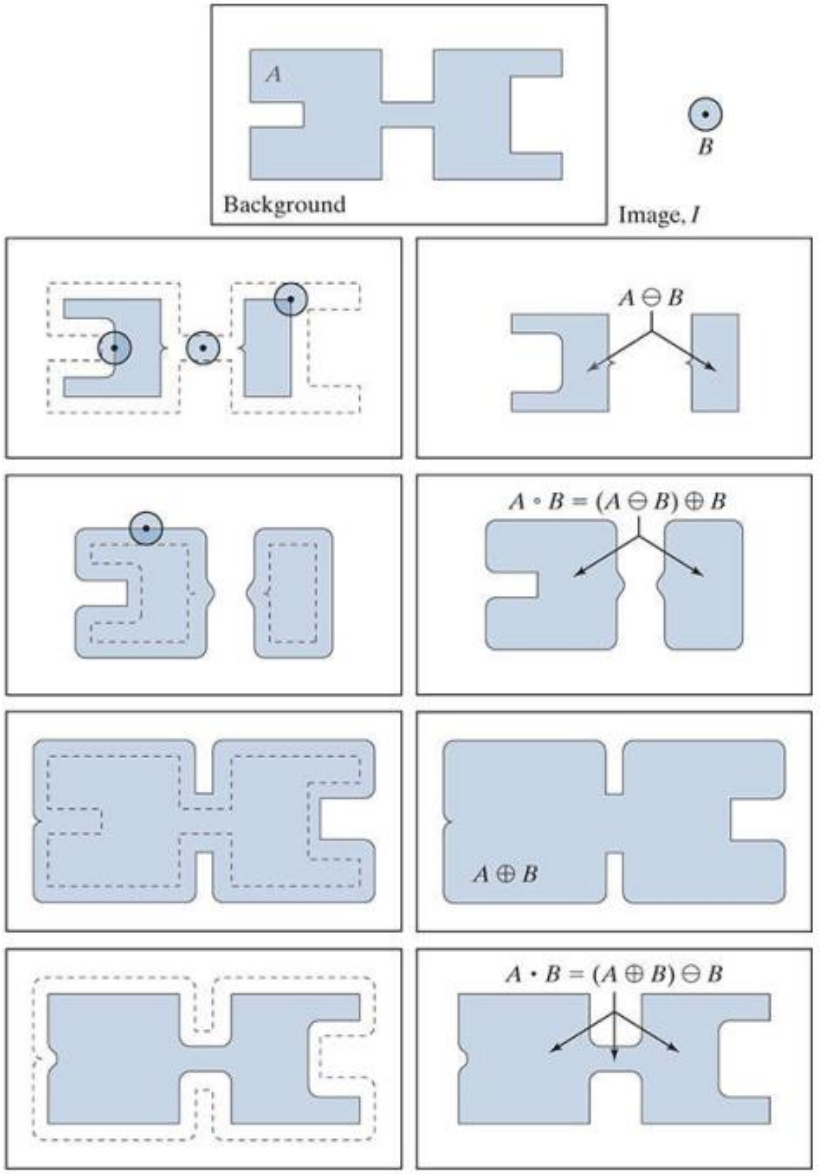
Closing : smooth sections of contours but, as opposed to opening, it generally fuses narrow breaks and long thin gulfs, eliminates small holes, and fills gaps in the contour

$$A \bullet B = (A \oplus B) \Theta B$$

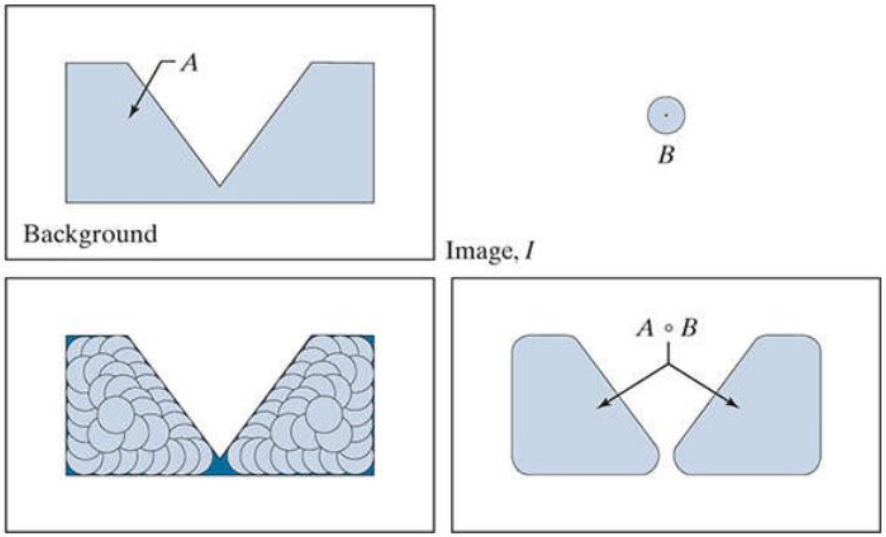
Opening and Closing

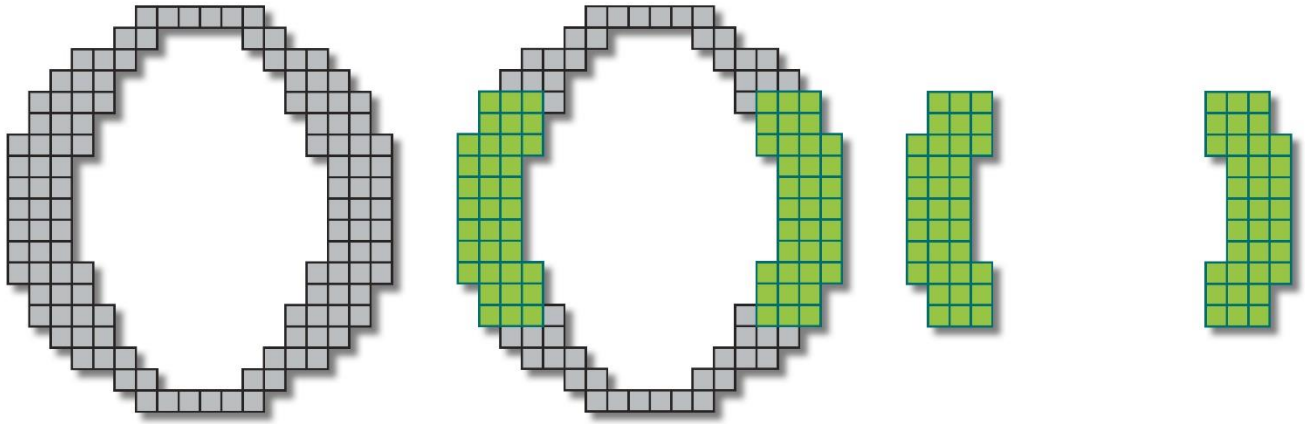
Opening

Closing

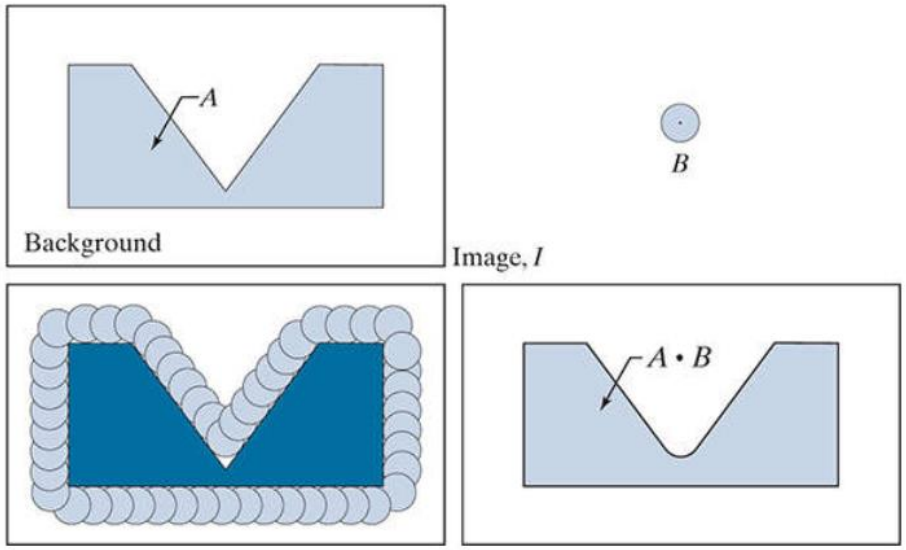


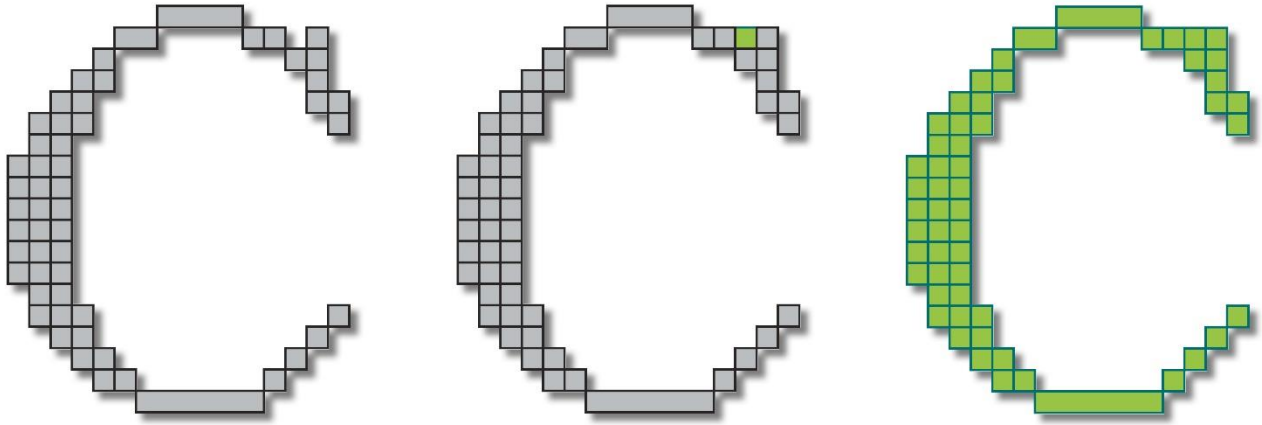
Opening





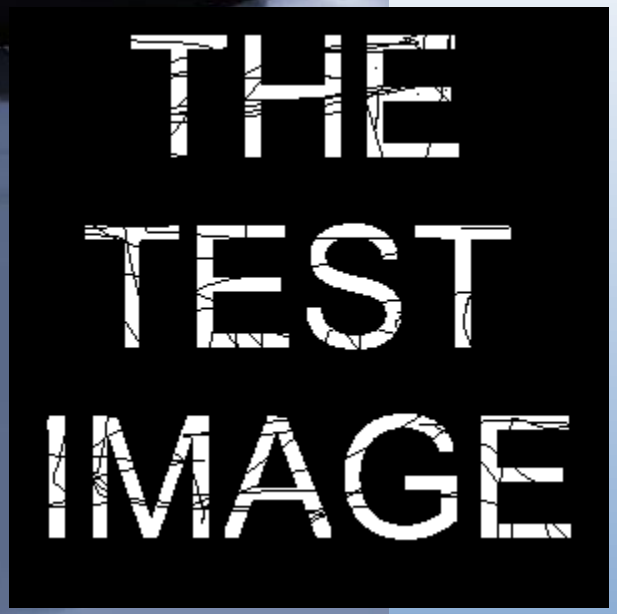
Closing





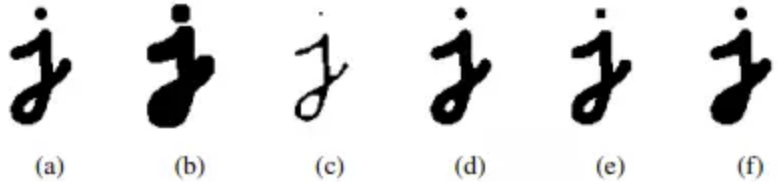
Opening and Closing

OPENING: The original image eroded twice and dilated twice (opened). Most noise is removed

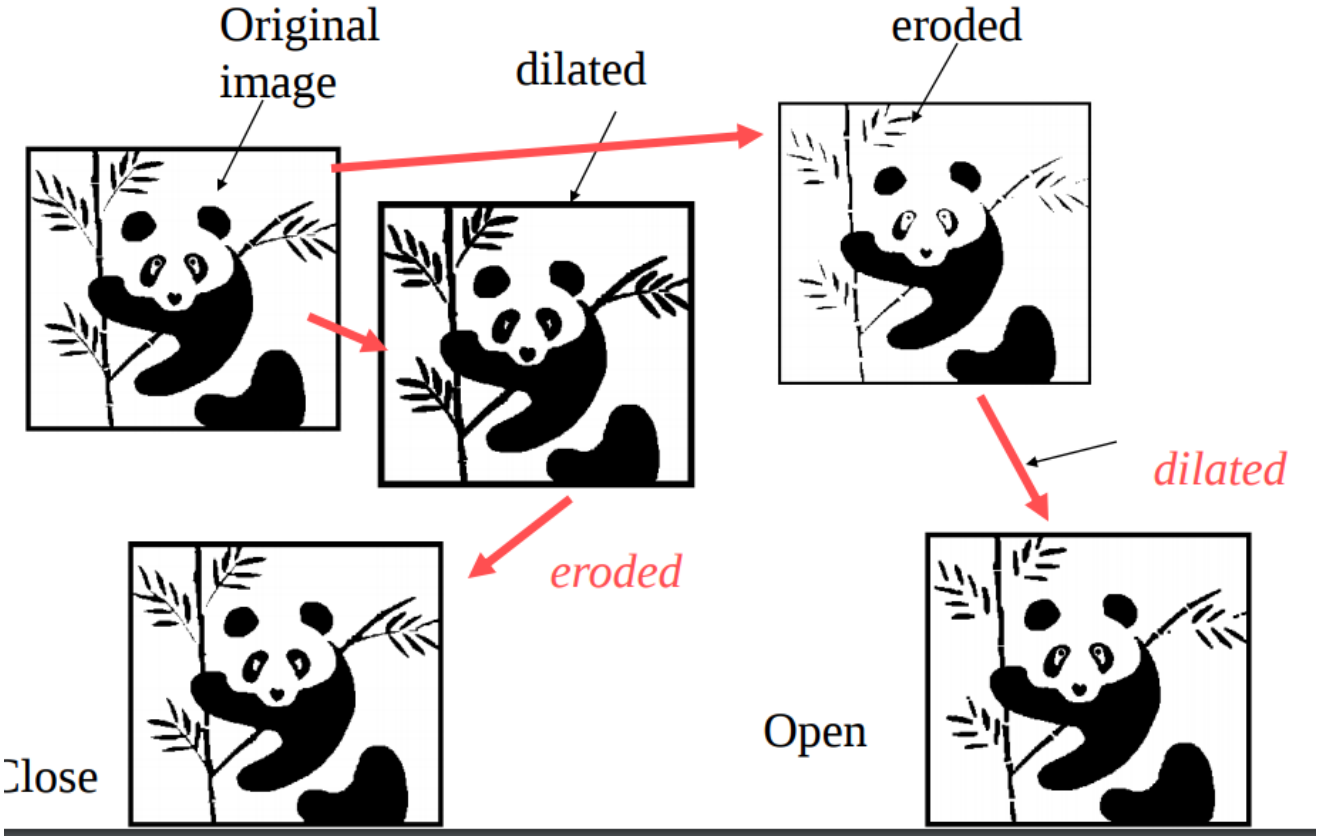


CLOSING: The original image dilated and then eroded. Most holes are filled.

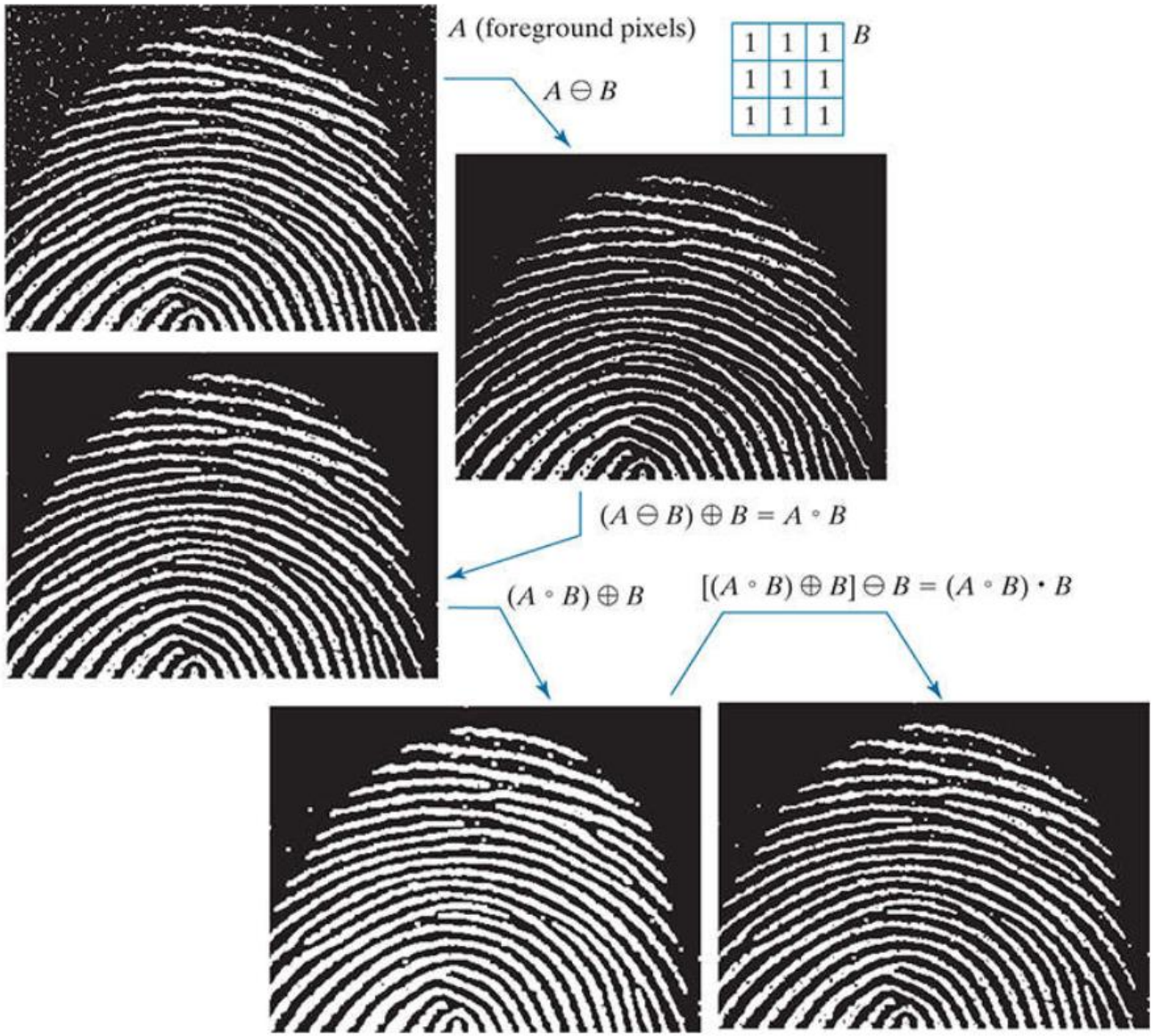
Summary



a-original image; b-dilation; c-erosion; e-opening; f-closing



Example


A (foreground pixels)
B
1 1 1
1 1 1
1 1 1
$A \ominus B$
$(A \ominus B) \oplus B = A \circ B$
$(A \circ B) \oplus B$
$[(A \circ B) \oplus B] \ominus B = (A \circ B) \bullet B$

Hit-or-Miss Transformation

The hit-or-miss transformation of an image A by B is denoted by $A \circledast B$.

B is a pair of structuring elements $B=(B_1,B_2)$ rather than a single element.

$B1$: set of elements of B associated with an object

$B2$: set of elements of B associated with the background

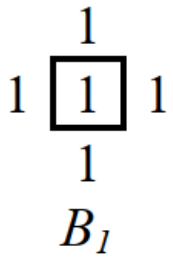
The hit-or-miss transform is defined as follows:

$$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$$

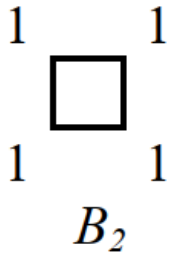
This transform is useful in locating all pixel configurations that match the B_1 structure (i.e a hit) but do not match that of B_2 (i.e. a miss). Thus, the hit-or-miss transform is used for shape detection.

Example

0 1 0 1 1 1 0 1 0 Shap	0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 1 0 0 1 1 1 1 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 1 0 0 0 0 0 0 1 0 0 1 1 1 0 0 0 0 1 1 1 0 0 1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 Image A



B_1



B_2

$A \ominus B_1 =$

$$\begin{matrix}
0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0
\end{matrix}$$

$A^c =$

$$\begin{matrix}
1&1&1&1&1&1&1&1&1&1&1\\
1&1&0&1&1&1&1&1&1&1&1\\
1&1&0&1&1&0&0&0&0&1&1\\
1&0&0&0&1&1&1&1&1&1&1\\
1&1&0&1&1&1&1&0&0&1&1\\
1&1&1&1&0&1&1&0&0&0&1\\
1&1&1&0&0&0&1&1&0&1&1\\
1&1&1&1&0&1&1&1&1&1&1\\
1&1&1&1&1&1&1&1&1&1&1
\end{matrix}$$

$A^c \ominus B_2 =$

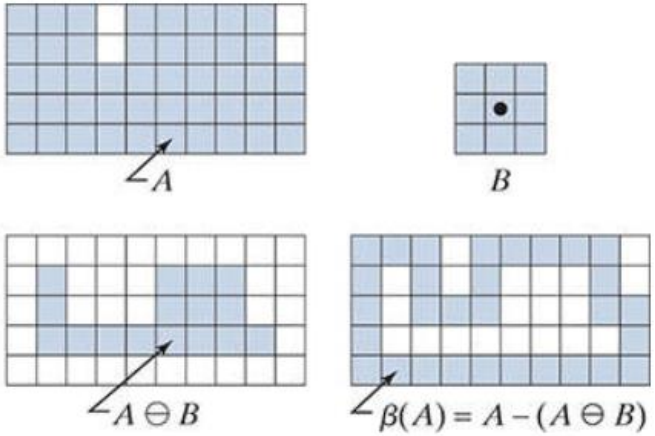
$$\begin{matrix}
1&0&1&0&1&1&1&1&1&1&1\\
1&0&1&0&0&0&0&0&0&0&1\\
0&0&0&0&0&1&1&1&1&1&1\\
1&0&1&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&1\\
1&1&1&0&1&0&0&0&0&0&0\\
1&1&0&0&0&0&0&0&1&0&1\\
1&1&1&0&1&0&1&1&1&1&1
\end{matrix}$$

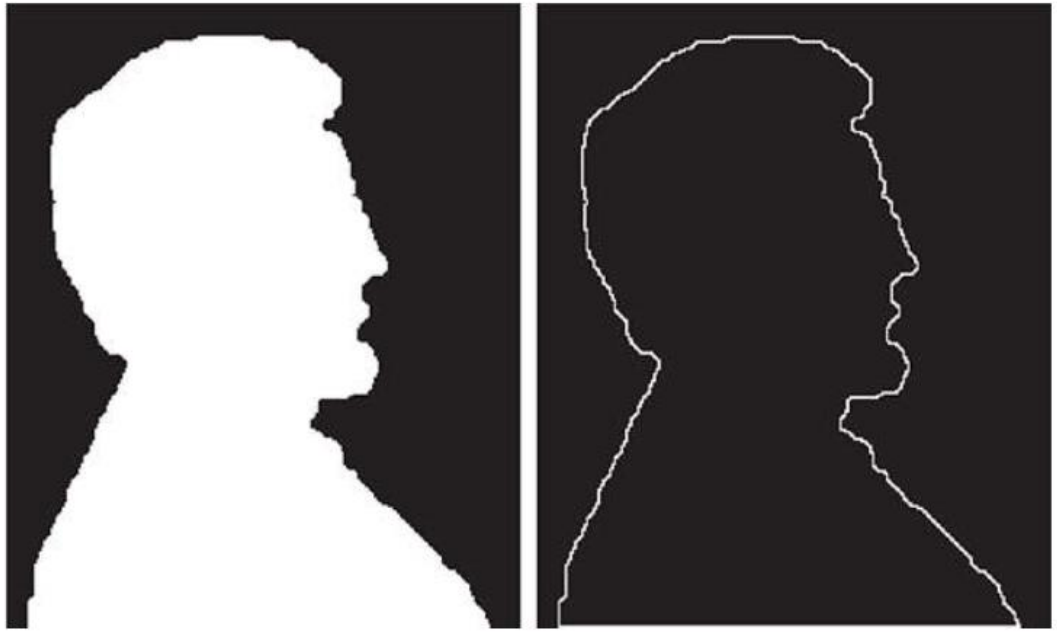
$A \circledast B =$

$$\begin{matrix}
0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0\\
0&0&\mathbf{1}&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&\mathbf{1}&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0
\end{matrix}$$

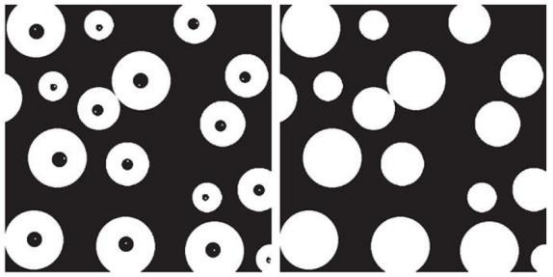
3. Boundary Extraction

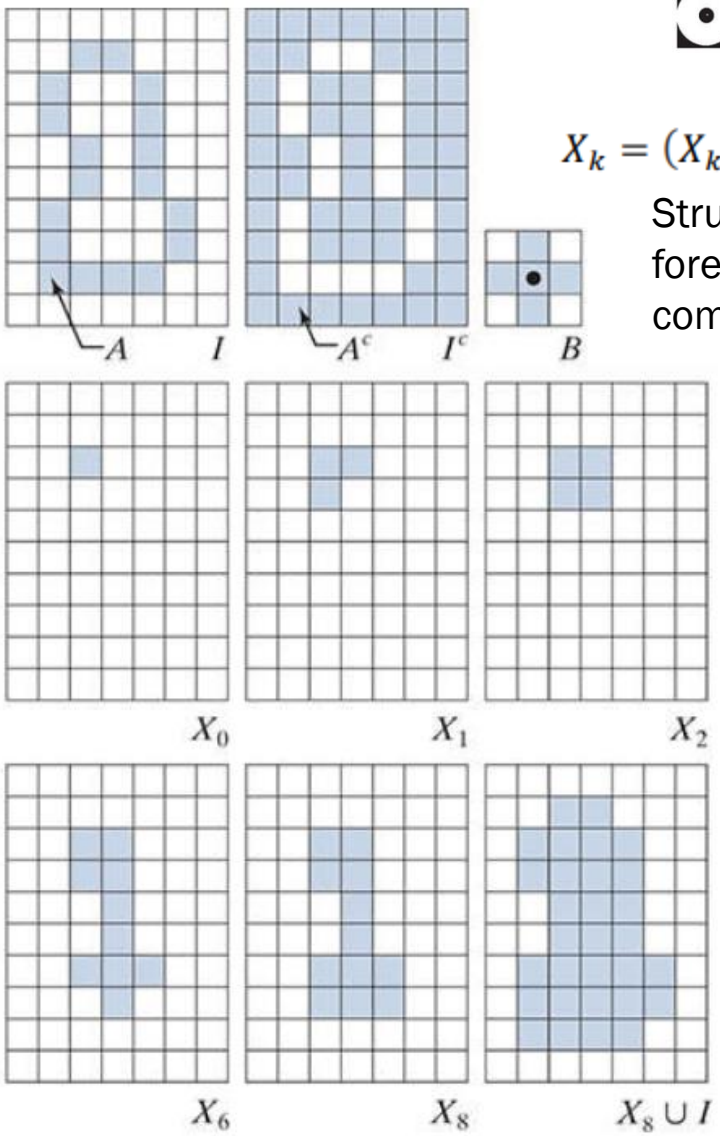
$$\beta(A) = A - (A \ominus B)$$





4. Hole Filling



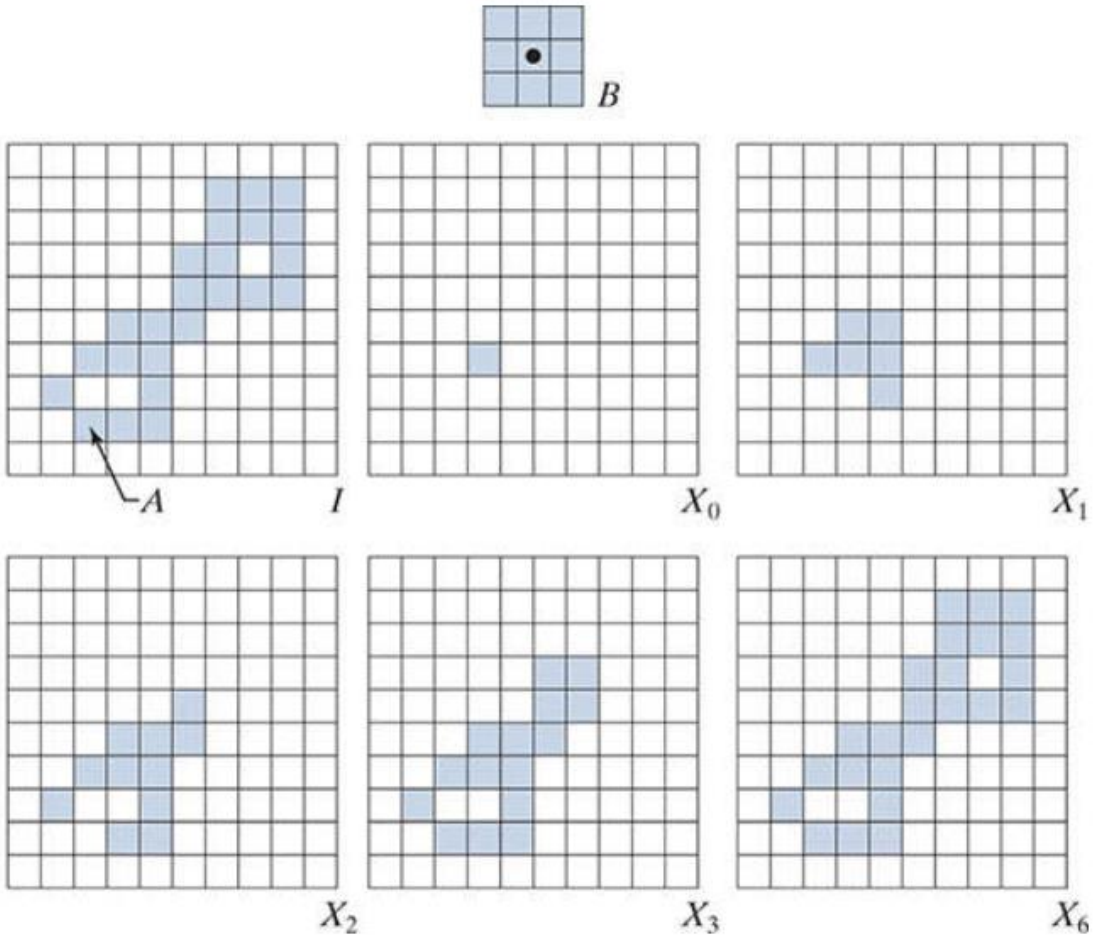


$$X_k = (X_{k-1} \oplus B) \cap I^c \quad k = 1, 2, 3, \ldots$$

Structuring element B. Only the foreground elements are used in computations

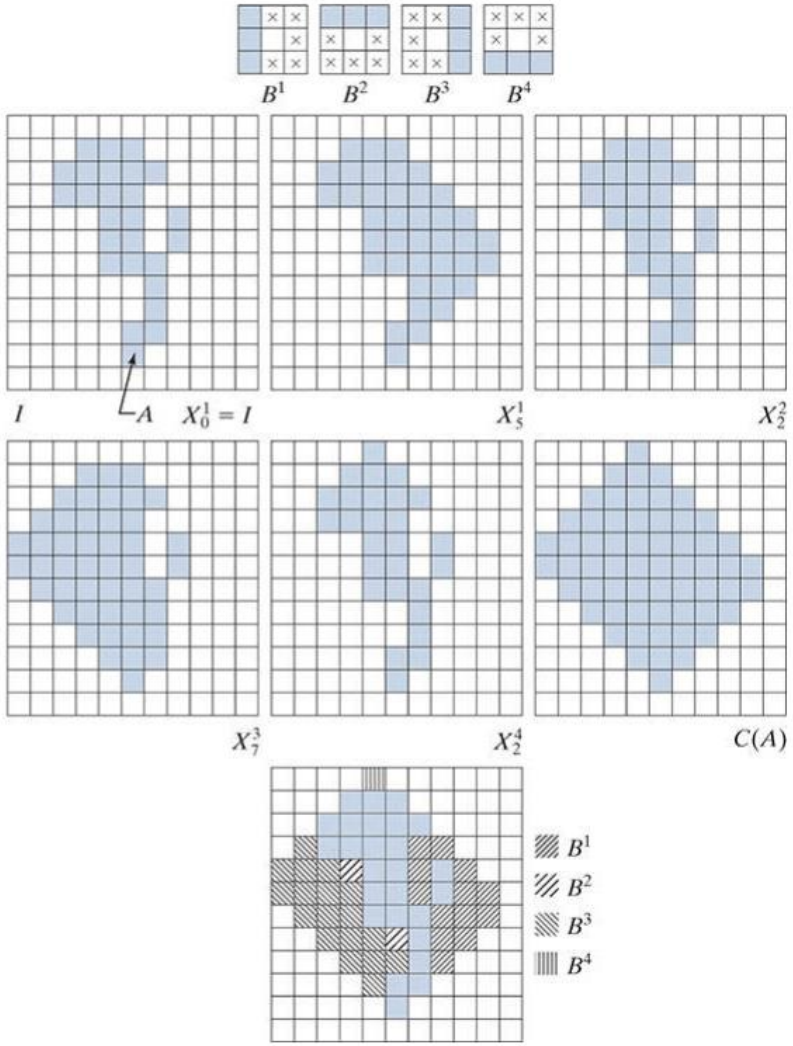
5. Extraction of Connected Components

$$X_k = (X_{k-1} \oplus B) \cap I \qquad k = 1, 2, 3, \ldots$$



6. Convex Hull

$$X_k^i = \left(X_{k-1}^i \circledast B^i\right) \cup X_{k-1}^i \quad i = 1,\ 2,\ 3,\ 4 \quad \text{and} \quad k = 1,\ 2,\ 3,\ \ldots$$



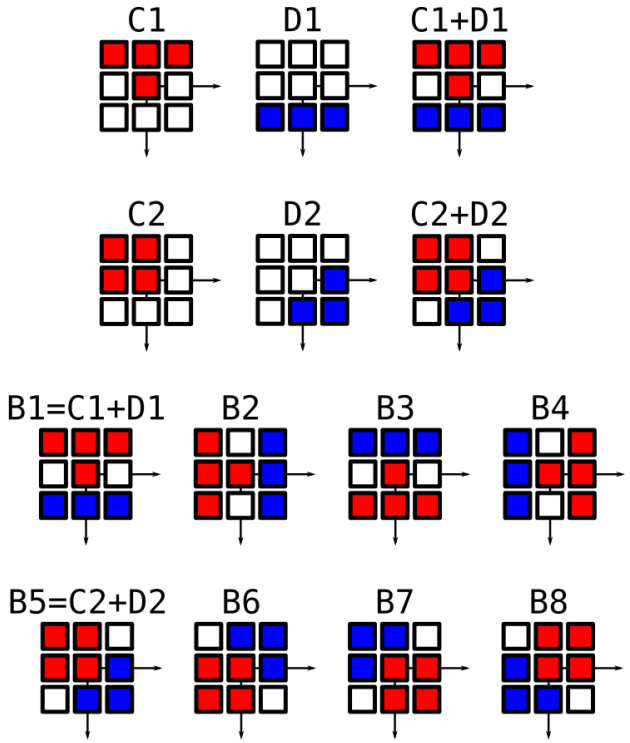
7. Thinning

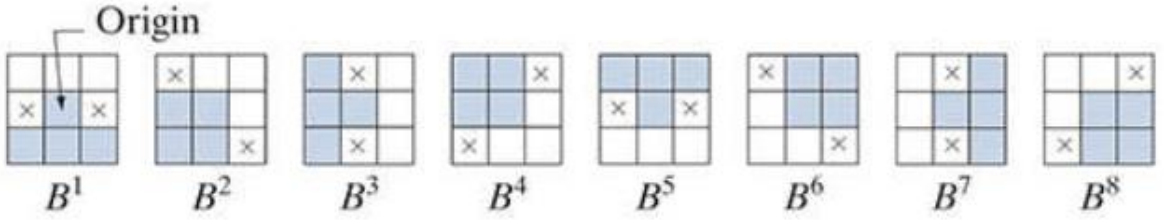
$$A \otimes B = A - (A \circledast B)$$

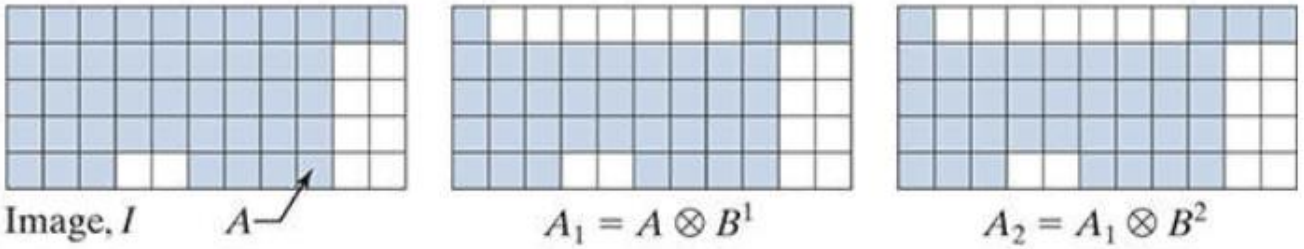
$$= A \cap (A \circledast B)^c$$

$$\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$$

$$A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$$

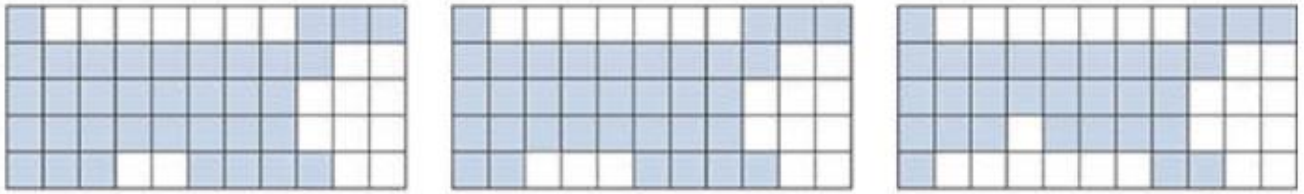






$A_1 = A \otimes B^1$

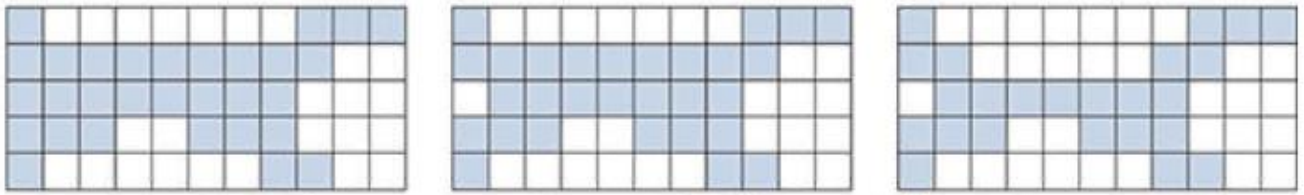
$A_2 = A_1 \otimes B^2$



$A_3 = A_2 \otimes B^3$

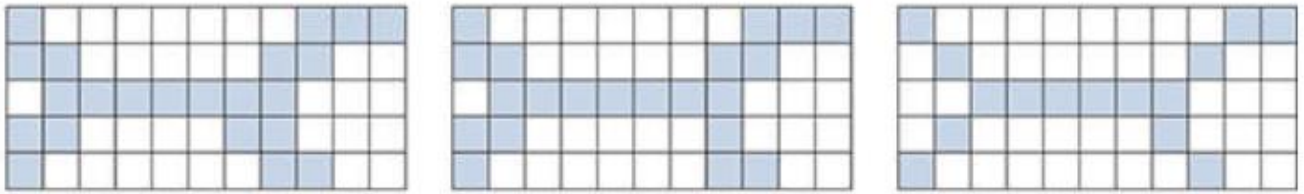
$A_4 = A_3 \otimes B^4$

$A_5 = A_4 \otimes B^5$



$A_6 = A_5 \otimes B^6$

$A_7 = A_6 \otimes B^7$ $(A_8 = A_7 \otimes B^8 = A_7)$ $A_9 = A_8 \otimes B^1$



$A_{12} = A_{11} \otimes B^4$
$(A_{11} = A_{10} = A_9)$

$A_{14} = A_{13} \otimes B^6$
No more changes after this.

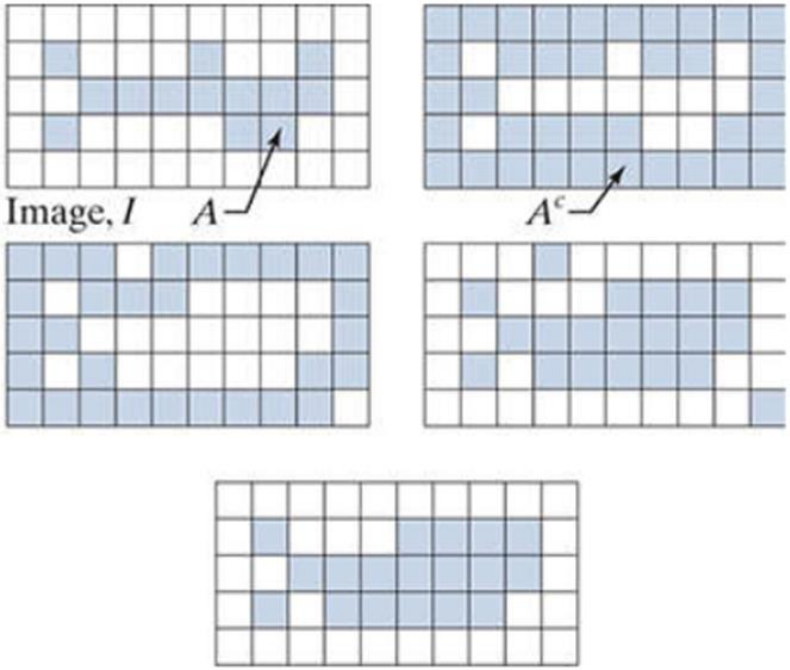
A_{14} converted to
m-connectivity.

8. THICKENING

$$A \odot B = A \cup (A \circledast B)$$

$$A \odot \{B\} = \left(\left(\ldots \left(\left(A \odot B^1 \right) \odot B^2 \right) \ldots \right) \odot B^n \right)$$

The structuring elements used for thickening have the same form as those shown in Thinning, but with all 1's and 0's interchanged.



Exercises

Gradient Morphology

$$\text{Gradient}(A) = (A \oplus B) - (A \ominus B)$$

Top-Hat Transform

$$\text{TopHat}(A) = A - (A \circ B)$$

Black-Hat Transform

$$\text{BlackHat}(A) = (A \bullet B) - A$$

Morphological Reconstruction

$$R(A, B) = \bigcup_{k=0}^{\infty} (A \cap B)$$

Exercises

H-Maxima and H-Minima

$$H\text{-Maxima}(A) = \{p \mid A(p) \geq H + \max(A)\}$$

$$H\text{-Minima}(A) = \{p \mid A(p) \leq \min(A) - H\}$$

Geodesic Dilation and Erosion

$$D_G^1(A, B) = (A \oplus B) \cap C$$

$$E_G^1(A, B) = (A \ominus B) \cup C$$

Distance Transform

Questions? More Information?

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