Marty Martin
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Homework 12

Problem 4.3.10

Question

The basketball team at State Tech, named the Fighting Logarithms, has a 70% foul-shooting percentage.

- (a) Write a formula for the exact probability that out of their next one hundred free throws, they will make between seventy-five and eighty, inclusive.
- (b) Approximate the probability asked for in part (a).

Solution

Part (a): Exact Probability

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$P(75 \leq X \leq 80) = \sum_{k=75}^{80} \binom{100}{k} (0.7)^k (0.3)^{100-k}$$

Part (b): Approximation using the Normal Distribution and Z-table

$$Z_{lower} = \frac{75 - 70}{4.5826} \approx 1.091$$

$$Z_{upper} = \frac{80 - 70}{4.5826} \approx 2.182$$

For Z = 1.09, the table gives a probability of 0.8621

For Z = 2.18, the table gives a probability of 0.9854

$$P(75 \leq X \leq 80) \approx 0.9854 - 0.8621 = 0.1233$$

Problem 4.3.18

Question

Suppose X_1, X_2, X_3, and X_4 are independent Poisson random variables, each with parameter $\lambda = 3$. Define $S = X_1 + X_2 + X_3 + X_4$.

- (a) Use the properties of Poisson processes to find the distribution of S.
- (b) Calculate the probability $P(13 \leq S \leq 14)$ exactly and approximate it using the Central Limit Theorem.

Solution

Part (a): Finding the Distribution of S

$$\lambda_S = 3 + 3 + 3 + 3 = 12$$

Part (b): Approximation using the Central Limit Theorem

$$Z_{13} = \frac{13 - 12}{\sqrt{12}} \approx 0.2887$$

$$Z_{14} = \frac{14 - 12}{\sqrt{12}} \approx 0.5774$$

$$P(Z \leq 0.29) \approx 0.6141 \quad \text{(for } Z_{13}\text{)}$$

$$P(Z \leq 0.58) \approx 0.7186 \quad \text{(for } Z_{14}\text{)}$$

$$P(13 \leq S \leq 14) \approx P(Z \leq 0.5774) - P(Z \leq 0.2887) = 0.7186 - 0.6141 = 0.1045$$

Problem 4.3.26

Question

The cross-sectional area of plastic tubing for use in pulmonary resuscitators is normally distributed with mean $\mu = 12.5$ mm^2 and standard deviation $\sigma = 0.2$ mm^2. When the area is less than 12.0 mm^2 or greater than 13.0 mm^2, the tube does not fit properly. If the tubes are shipped in boxes of one thousand, how many wrong-sized tubes per box can doctors expect to find?

Solution

$$Z_{low} = \frac{12.0 - 12.5}{0.2} = -2.5, \quad Z_{high} = \frac{13.0 - 12.5}{0.2} = 2.5$$

$$P(X < 12.0) = \Phi(-2.5), \quad \text{For } Z = -2.5, \text{ the table gives a probability of } 0.0062$$

$$P(X > 13.0) = 1 - \Phi(2.5), \quad \text{For } Z = 2.5, \text{ the table gives a probability of } 0.9938$$

$$P(\text{wrong size}) = P(X < 12.0) + P(X > 13.0) = 0.0062 + 0.0062 = 0.0124$$

$$E(\text{wrong-sized tubes}) = 1000 \times 0.0124 = 12.4$$

Problem 4.3.34

Question

Let $Y_1, Y_2, \ldots, Y_n$ be a random sample from a normal distribution where the mean is 2 and the variance is 4. How large must n be in order that

$$P(1.9 \leq \overline{Y} \leq 2.1) \geq 0.99$$

Solution

$$\text{Let } \overline{Y} \sim N\left(2, \frac{4}{n}\right)$$

$$Z = \frac{\overline{Y} - 2}{\sqrt{\frac{4}{n}}} = \frac{\overline{Y} - 2}{\frac{2}{\sqrt{n}}}$$

$$P\left(1.9 \leq \overline{Y} \leq 2.1\right) = P\left(\frac{1.9 - 2}{\frac{2}{\sqrt{n}}} \leq Z \leq \frac{2.1 - 2}{\frac{2}{\sqrt{n}}}\right)$$

$$= P\left(-\frac{0.1\sqrt{n}}{2} \leq Z \leq \frac{0.1\sqrt{n}}{2}\right)$$

$$P(-a \leq Z \leq a) = 0.99 \implies a \approx 2.576$$

$$\frac{0.1\sqrt{n}}{2} = 2.576 \implies \sqrt{n} = 51.52 \implies n = 2656.67$$

Problem 4.3.36

Question

The cylinders and pistons for a certain internal combustion engine are manufactured by a process that gives a normal distribution of cylinder diameters with a mean of 41.5 cm and a standard deviation of 0.4 cm. Similarly, the distribution of piston diameters is normal with a mean of 40.5 cm and a standard deviation of 0.3 cm. If the piston diameter is greater than the cylinder diameter, the former can be reworked until the two "fit." What proportion of cylinder-piston pairs will need to be reworked?

Solution

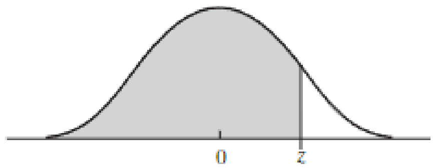
$$Z = Y - X \quad \text{where} \quad Y \sim N(40.5, 0.09) \quad \text{and} \quad X \sim N(41.5, 0.16)$$

$$\mu_Z = 40.5 - 41.5 = -1.0, \quad \sigma_Z^2 = 0.09 + 0.16 = 0.25, \quad \sigma_Z = 0.5$$

$$P(Z > 0) = 1 - P(Z \leq 0) = 1 - \Phi\left(\frac{0 - (-1.0)}{0.5}\right) = 1 - \Phi(2.0)$$

$$P(Z > 0) = 1 - 0.9772 = 0.0228$$

Table A.1 Cumulative Areas under the Standard Normal Distribution



z	0	1	2	3	4	5	6	7	8	9
−3.	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0126	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0238	0.0233
−1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0300	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0570	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0722	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2297	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Figure 1: Cumulative Areas under the Standard Normal Distribution (Part 1)

Table A.1 Cumulative Areas under the Standard Normal Distribution (*cont.*)

z	0	1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9278	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9430	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9648	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9700	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9762	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9874	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Figure 2: Cumulative Areas under the Standard Normal Distribution (Part 2)