Homework 8

Problem 3.4.6

Question

Let n be a positive integer. Show that

$$f(y) = (n+2)(n+1)y^n(1-y), \quad 0 \leq y \leq 1,$$

is pdf.

Solution

Non-negativity

$$f(y) = (n+2)(n+1)y^n(1-y) \geq 0 \text{ for } 0 \leq y \leq 1$$

Total Integral Equals One

$$\int_0^1 y^n(1-y)\,dy = \frac{\Gamma(n+1)\Gamma(2)}{\Gamma(n+3)} = \frac{n! \cdot 1}{(n+2)!} = \frac{1}{(n+2)(n+1)}$$

$$\int_0^1 f(y)\,dy = (n+2)(n+1)\int_0^1 y^n(1-y)\,dy = (n+2)(n+1) \cdot \frac{1}{(n+2)(n+1)} = 1$$

Problem 3.4.9

Question

If the PDF for Y is given by

$$f_Y(y) = \begin{cases} 1 - |y| & \text{if } |y| \leq 1 \\ 0 & \text{if } |y| > 1 \end{cases}$$

find and graph $F_Y(y)$.

Solution

For $y < -1$

$$F_Y(y) = 0$$

For $-1 \leq y \leq 1$

$$F_Y(y) = \int_{-1}^{y} (1 - |t|) \, dt$$

$$F_Y(y) = \begin{cases} \int_{-1}^{y} (1 + t) \, dt & \text{if } -1 \leq y < 0 \\ \int_{-1}^{0} (1 + t) \, dt + \int_{0}^{y} (1 - t) \, dt & \text{if } 0 \leq y \leq 1 \end{cases}$$

For $y > 1$

$$F_Y(y) = 1$$

Problem 3.4.14

Question

In a certain country, the distribution of a family's disposable income, y, is described by the pdf $f_Y(y) = ye^{-y}$ for $y \geq 0$. Find $F_Y(y)$.

Solution

Verification of PDF

$$\text{Non-negativity: } f_Y(y) = ye^{-y} \geq 0 \text{ for } y \geq 0$$
$$\text{Integral equals 1: } \int_0^\infty ye^{-y}\,dy = 1$$

Calculation of CDF

$$F_Y(y) = \int_0^y te^{-t}\,dt$$
$$F_Y(y) = [-(t+1)e^{-t}]_0^y = -(y+1)e^{-y} + 1$$
$$F_Y(y) = 1 - (y+1)e^{-y}, \quad y \geq 0$$