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Chapter 1

Axioms

1.0.1 Axiom 1.1

Closure under addition: For all $X, Y \in \mathbb{Z}$*, the sum* $X + Y$ *is also in* $\mathbb{Z}$*.*

1.0.2 Axiom 1.2

Existence of an additive identity: There exists an element $0 \in \mathbb{Z}$*, such that for every* $X \in \mathbb{Z}$*,* $X + 0 = X$*.*

1.0.3 Axiom 1.3

Existence of additive inverses: For every $X \in \mathbb{Z}$*, there exists an element* $-X \in \mathbb{Z}$ *such that* $X + (-X) = 0$*.*

1.0.4 Axiom 1.4

Commutativity of addition: For all $X, Y \in \mathbb{Z}$*,* $X + Y = Y + X$*.*

1.0.5 Axiom 1.5

Associativity of addition: For all $X, Y, Z \in \mathbb{Z}$*,* $(X + Y) + Z = X + (Y + Z)$*.*

Chapter 2

Definitions

2.0.1 Definition: Varmint 1

Definition for Varmint 1 goes here.

2.0.2 Definition: Varmint 2

Definition for Varmint 2 goes here.

Chapter 3

Propositions

3.0.1 Proposition 1.1

Let X, Y, and Z be elements of the set of integers $\mathbb{Z}$. This proposition demonstrates several foundational algebraic properties of integers:

3.0.1 part i Commutativity of addition: $X + Y = Y + X$.

3.0.1 part ii Associativity of addition: $(X + Y) + Z = X + (Y + Z)$.

3.0.1 part iii Existence of additive identity: $X + 0 = X$.

3.0.1 part iv Existence of additive inverse: For every X, there exists an integer $-X$ such that $X + (-X) = 0$.

3.0.2 Proof of Proposition 1.1

We now prove each part of Proposition 1.1:

Proof of part (3.0.1 part i). We prove the commutativity of addition. Let $X, Y \in \mathbb{Z}$. Consider:

(1) $X + Y = Y + X$ by the definition of commutative property in $\mathbb{Z}$

This concludes the proof of commutativity of addition. $\square$

Proof of part (3.0.1 part ii). We prove the associativity of addition. Let $X, Y, Z \in \mathbb{Z}$. Consider:

(1) $(X + Y) + Z = X + (Y + Z)$ by the definition of associative property in $\mathbb{Z}$

This concludes the proof of associativity of addition. $\square$