Definitions and General Principles

Inverse of a Matrix

- $(A^{-1})^{-1} = A$
- $(AB)^{-1} = B^{-1}A^{-1}$
- $AA^{-1} = A^{-1}A = I$

LDU Decomposition

- For a symmetric matrix A: $A = LDL^T$
- L: lower triangular with unit diagonal
- D: diagonal matrix

Vector Space Axioms

- Addition: commutativity, associativity, identity, inverses
- Scalar Multiplication: distributivity, compatibility, identity

Subspaces

- Closed under addition and scalar multiplication

Linear Dependence and Independence

- Dependent: $\exists$ scalars, not all zero, s.t. $a_1v_1 + \ldots + a_nv_n = 0$
- Independent: only solution is $a_1 = \ldots = a_n = 0$

Basis and Dimension

- Basis: linearly independent spanning set
- Dimension: number of vectors in a basis

General Principles for Subspaces

- Closed under vector addition
- Closed under scalar multiplication

Linear Transformation

- Preserves vector addition and scalar multiplication

Image and Kernel

- $\text{im}(A)$: span of column vectors of A
- $\ker(A)$: $\{x \in \mathbb{R}^n : Ax = 0\}$

Basis Transformation

- Unique representation of a vector in terms of basis vectors

Key Theorems and Results

Problem-Solving Strategies