Your Document Title

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March 23, 2024

Contents

Proof Methods

If then statements

Format:

Proof. **If A, then B:**

1. **Assume A.**
2. **Show that assuming A leads to B.**
3. **Therefore, B is concluded from A.**

□

Example:

Proof. **If $m = 1$, then $m + 0 = 1$.**

1. **Assume $m = 1$.**
2. **Considering $m = 1$, we have $1 + 0 = 1$.**
3. **This simplifies to $1 = 1$, which is true.**

□

If then types

Different types of implications and their meaning:

- $A \Rightarrow B$: "If it is Wednesday, Dr. Beck will get a cup of coffee from the student union."
- $B \Rightarrow A$ (Converse): "If Dr. Beck got a cup of coffee from the student union, then it is Wednesday."
- $A \Leftrightarrow B$ (Bi-conditional): "It is Wednesday if and only if Dr. Beck got a cup of coffee from the student union."
- $\neg B \Rightarrow \neg A$ (Contrapositive): "If Dr. Beck did not get a cup of coffee from the student union, then it is not Wednesday."

Induction Proof

Format:

Proof. **Prove that $F(x)$ is true for all $x \in A$:**

1. **Base case:** Show $F(a)$ is true, where a is the smallest element in set A.
2. **Induction step:** Assume $F(k)$ is true for an arbitrary $k \in A$. Show that $F(k) \Rightarrow F(k+1)$.
3. **Therefore, $F(x+1)$ is true for all $x \in A$.**

□

Example:

Proof. **For all $n \in \mathbb{N}$, $n = n$:**

1. **Base case ($n = 1$):** $1 = 1$ is true.
2. **Induction step:** Assume $n = n$ is true for an arbitrary natural number n. Show that this implies $n + 1 = n + 1$.
3. By the induction hypothesis, $n = n$. Adding 1 to both sides, $n + 1 = n + 1$, which holds true.

□

Proof by contradiction

Format:

Proof. **Prove that A is true by contradiction:**

1. Assume **not** A.
2. Show that this assumption leads to a contradiction (something that we know is false).
3. Therefore, A must be true.

$\square$

Different Negations

1. **AND $\Rightarrow$ OR:** If A and B, then **not** A or **not** B.

   Example: Dr. Beck is 5 ft tall and single $\Rightarrow$ Dr. Beck is **not** 5 ft tall or is **not** single.
2. **OR $\Rightarrow$ AND:** If A or B, then **not** A and **not** B.

   Example: Dr. Beck will drink a coffee or it is Wednesday $\Rightarrow$ Dr. Beck will **not** drink a coffee and it is **not** Wednesday.
3. **If, then $\Rightarrow$ AND:** If A, then B implies **not** A and **not** B.

   *If it is Monday, then Dr. Beck is on campus $\Rightarrow$ It is **not** Monday and Dr. Beck is **not** on campus.*
4. **For all $\Rightarrow$ There exists:** For all m, A is true implies there exists an m, A is **not** true.

   *For all $m \in \mathbb{Z}$, m is even $\Rightarrow$ There exists $m \in \mathbb{Z}$, m is **not** even.*
5. **There exists $\Rightarrow$ For all:** There exists an m, A is true implies for all m, A is **not** true.

   There exists an $m \in \mathbb{Z}$, $m + 1 = 0.5 \Rightarrow$ For all $m \in \mathbb{Z}$, $m + 1 \neq 0$.

Example:

Proof. **There is no $x \in \mathbb{N}$ that satisfies the equation $1 - x = 0 \cdot x$.**

1. Assume by way of contradiction that such an x exists in $\mathbb{N}$.
2. Since $x \neq 0$ for any $x \in \mathbb{N}$, cancelling x from both sides of the equation $1 - x = 0 \cdot x$ leads to $0 = 1$.
3. Since $0 \neq 1$ is a true mathematical contradiction, the initial statement is proven to be true by contradiction.

$\square$