Advanced Digital Control Project

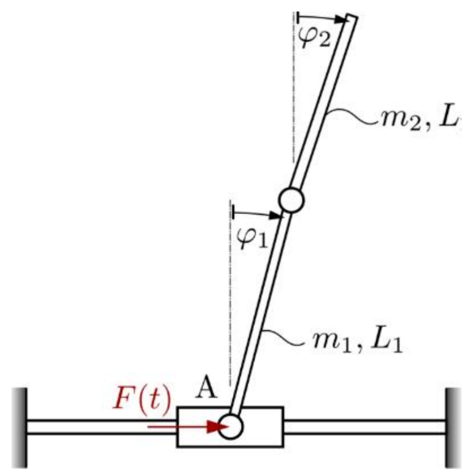
ECE 9705/9057 - Winter 2025

Western University

Electrical and Computer Engineering

Instructor: Moteaal Asadi Shirzi

Project Title:
Digital Control of a Double Inverted Pendulum System via State-Space Methods



Objective:
To design, analyze, and implement digital state-space controllers for the stabilization and trajectory tracking of a 2-DOF inverted pendulum. The project emphasizes state-space methods, including system modeling, controller design, observer design, and digital implementation.

Expected Learning Outcomes:

- Develop a deep understanding of state-space methods in digital control systems.
- Gain practical experience in modeling, analysis, and controller design for multivariable systems.
- Learn how to discretize continuous-time models and implement digital controllers.
- Enhance skills in simulation, analysis, and documentation.

This project emphasizes the application of state-space theory in a real-world, dynamic system and provides an excellent platform for mastering advanced digital control concepts.

Evaluation Criteria:

- Completeness and correctness of the state-space model and analysis.
- Quality of digital controller and observer designs.
- Performance of the controllers in simulation (stability, response characteristics, robustness).
- Clarity and depth of the report and presentation.

Deliverables:

1. **State-Space Model:**
    - Continuous-time and discrete-time models.
    - Eigenvalue analysis and controllability/observability checks.
2. **Controller and Observer Designs:**
    - Pole-placement, LQR, and observer results.
3. **Simulations:**
    - MATLAB/Simulink files with detailed annotations.
    - Plots showing system response for various scenarios.
4. **Report and Presentation:**
    - Report (PDF): Prepare a technical report (~10-20 pages) with clear explanation of the design process, results, and conclusions that includes:
        a. Detailed derivation of the state-space model.
        b. Step-by-step design of controllers and observers.
        c. Simulation results with analysis and comparison.
        d. Include code snippets and block diagrams where applicable.
        e. Highlight practical challenges and possible improvements.
    - MATLAB®/Simulink® files
    - A PowerPoint file containing a 5-minute video presentation. Students should be prepared to present their project and answer questions either in person, in class, or via email if requested.

Suggested Project Components:

1. System Modeling and State-Space Representation:

- **Model Derivation:**
    - Derive the nonlinear equations of motion for the 2-DOF inverted pendulum using the **Euler-Lagrange** or **Newton-Euler** approach.
    - Define the state variables, such as cart position, pendulum angles, and their velocities.
- **Linearization:**
    - Linearize the nonlinear equations around the unstable equilibrium (upright position) to obtain a linear time-invariant (LTI) model.
- **State-Space Formulation:**
    - Represent the system in state-space form
- Discretize the continuous-time state-space model using methods like **Zero-Order Hold (ZOH)**

2. Stabilizing State-Feedback Controller Design:

- **Pole Placement:**
    - Design a digital **state-feedback controller** by placing the poles of the closed-loop system at desired locations in the z-plane.

- **Optimal Control:**
    - Formulate and solve the **discrete-time Linear Quadratic Regulator (LQR)** problem to minimize a cost function.
- **Integral Action:**
    - Augment the state-space model with an integrator state to eliminate steady-state errors in tracking.

3. Observer Design:

- **Full-State Observer:**
    - Design a digital **Observer** for estimating all states when only partial states are measured.
- **Kalman Filter (Optional):**
    - Implement a **Kalman Filter** to handle noisy measurements and estimate states optimally.
- Integrate the observer with the state-feedback controller to form an **output-feedback controller**.

4. Simulation in MATLAB/Simulink:

- Implement the derived discrete-time state-space models, controllers, and observers in MATLAB/Simulink.
- Test the system with various initial conditions, disturbances, and reference trajectories.
- Evaluate:
    - Stability
    - Transient response (rise time, settling time, overshoot).
    - Robustness to parameter variations and external disturbances.