Click-Through Rate prediction: TOP-5 solution for the Avazu contest

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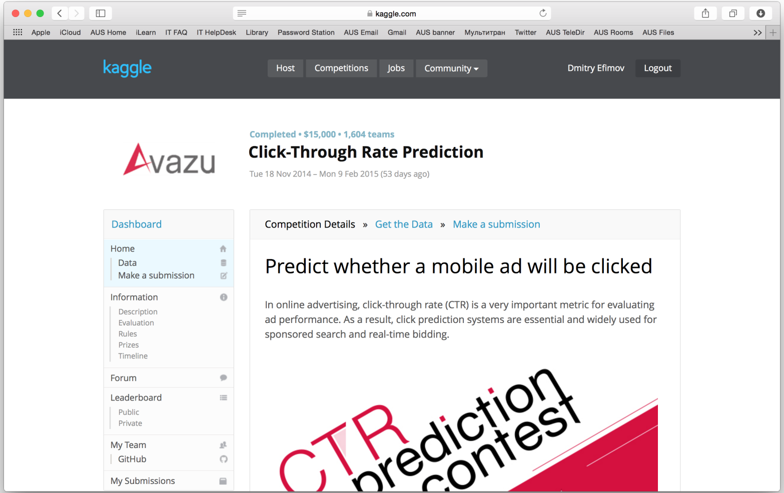
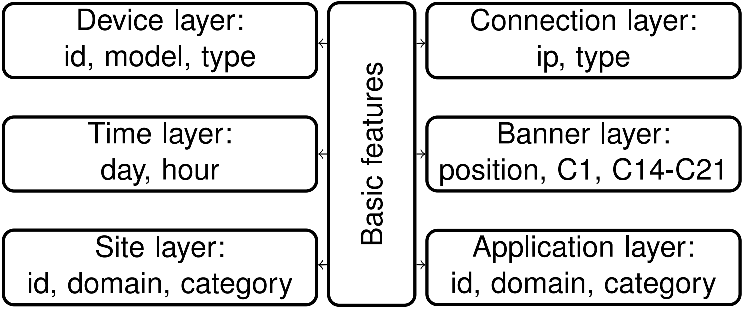
Outline

Competition



Provided data



Notations

$\mathbf{X}$: $m \times n$ design matrix

$$m_{train} = 40\,428\,967$$
$$m_{test} = 4\,577\,464$$
$$n = 23$$

$\mathbf{y}$: binary target vector of size m
$\mathbf{x^j}$: column j of matrix X
$\mathbf{x_i}$: row i of matrix X
$\sigma(\mathbf{z}) = \dfrac{\mathbf{1}}{\mathbf{1} + \mathbf{e}^{-\mathbf{z}}}$: sigmoid function

Evaluation

Logarithmic loss for $y_i \in \{0, 1\}$:

$$L = -\frac{1}{m}\sum_{i=1}^{m}\left(y_i \log(\hat{y}_i) + (1 - y_i)\log(1 - \hat{y}_i)\right)$$

$\hat{y}_i$ is a prediction for the sample i

Logarithmic loss for $y_i \in \{-1, 1\}$:

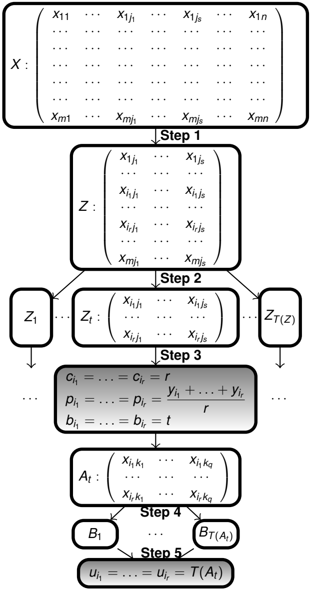
$$L = \frac{1}{m}\sum_{i=1}^{m}\log(1 + e^{-y_i p_i})$$

p_i is a raw score from the model
$\hat{y}_i = \sigma(p_i), \forall i \in \{1, \dots, m\}$

Feature engineering

- **Blocks:** combination of two or more features
- **Counts:** number of samples for different feature values
- **Counts unique:** number of different values of one feature for fixed value of another feature
- **Likelihoods:** $\min_{\theta_t} L$, where $\theta_t = P(y_i \mid x_{ij} = t)$
- **Others**

Feature engineering (algorithm 1)



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function SPLITBYROWS(M)

get partition of the matrix M into $\{M_t\}$ by rows such that each M_t has identical rows

Require:

$J = \{j_1, \dots, j_s\} \subset \{1, \dots, n\}$

$K = \{k_1, \dots, k_q\} \subset \{1, \dots, n\} \backslash J$

$Z \leftarrow (x_{ij}) \subset X,\ i \in \{1, 2, \dots, m\},\ j \in J$

for $I = \{i_1, \dots, i_r\} \in$ SPLITBYROWS(Z)

$c_{i_1} = \dots = c_{i_r} = r$

$$p_{i_1} = \dots = p_{i_r} = \frac{y_{i_1} + \dots + y_{i_r}}{r}$$

$b_{i_1} = \dots = b_{i_r} = t$

$A_t = (x_{ik}) \subset X,\ i \in I,\ k \in K$

$T(A_t) = \text{size}(\text{SPLITBYROWS}(A_t))$

$u_{i_1} = \dots = u_{i_r} = T(A_t)$

Feature engineering (algorithm 2)

function $\textsc{splitByRows}(M)$

get partition of the matrix M into $\{M_t\}$ by rows such that each M_t has identical rows

Require:

parameter $\alpha > 0$

$J \leftarrow (j_1, \ldots, j_s) \subset \{1, \ldots, n\}$

increasing sequence $V \leftarrow (v_1, \ldots, v_l) \subset \{1, \ldots, s\}$, $v_1 < s$

$f_i \leftarrow \dfrac{y_1 + \ldots + y_m}{m}, \forall i \in \{1, \ldots, m\}$

for $v \in V$ **do**

$J_v = \{j_1, \ldots, j_v\}$, $Z = (x_{ij}) \subset X$, $i \in \{1, 2, \ldots, m\}$, $j \in J_v$

for $I = \{i_1, \ldots, i_r\} \in \textsc{splitByRows}(Z)$ **do**

$c_{i_1} = \ldots = c_{i_r} = r$

$p_{i_1} = \ldots = p_{i_r} = \dfrac{y_{i_1} + \ldots + y_{i_r}}{r}$

$w = \sigma(-c + \alpha)$ - weight vector

$f_i = (1 - w_i) \cdot f_i + w_i \cdot p_i, \forall i \in \{1, \ldots, m\}$

FTRL-Proximal model

Weight updates:

$$w_{i+1} = \arg\min_{w} \left(\sum_{r=1}^{i} g_r \cdot w + \frac{1}{2} \sum_{r=1}^{i} \tau_r ||w - w_r||_2^2 + \lambda_1 ||w||_1 \right) =$$

$$= \arg\min_{w} \left(w \cdot \sum_{r=1}^{i} (g_r - \tau_r w_r) + \frac{1}{2} ||w||_2^2 \sum_{r=1}^{i} \tau_r + \lambda_1 ||w||_1 + \text{const} \right),$$

$$\text{where } \sum_{r=1}^{i} \tau_{rj} = \frac{\beta + \sqrt{\sum_{r=1}^{i} \left(g_{rj}\right)^2}}{\alpha} + \lambda_2, \ j \in \{1, \ldots, N\},$$

$\lambda_1, \lambda_2, \alpha, \beta$ - parameters, $\tau_r = (\tau_{r1}, \ldots, \tau_{rN})$ - learning rates,

g_r - gradient vector for the step r

FTRL-Proximal Batch model

Require: parameters α, β, λ_1, λ_2

$z_j \leftarrow 0$ and $n_j \leftarrow 0$, $\forall j \in \{1, \dots, N\}$

for $i = 1$ to m **do**

receive sample vector x_i and let $J = \{j | x_{ij} \neq 0\}$

for $j \in J$ **do**

$$w_j = \begin{cases} 0 & \text{if } |z_j| \leqslant \lambda_1 \\ -\left(\frac{\beta + \sqrt{n_j}}{\alpha} + \lambda_2\right)^{-1} \left(z_j - \text{sign}(z_j)\lambda_1\right) & \text{otherwise} \end{cases}$$

predict $\hat{y}_i = \sigma(x_i \cdot w)$ using the w_j and observe y_i

if $y_i \in \{0, 1\}$ **then**

for $j \in J$ **do**

$g_j = \hat{y}_i - y_i$ - gradient direction of loss w.r.t. w_j

$\tau_j = \frac{1}{\alpha}\left(\sqrt{n_j + {g_j}^2} - \sqrt{n_j}\right)$

$z_j = z_j + g_j - \tau_j w_j$

$n_j = n_j + {g_j}^2$

Performance

Description	Leaderboard score
dataset is sorted by *app id, site id, banner pos, count1, day, hour*	0.3844277
dataset is sorted by *app domain, site domain, count1, day, hour*	0.3835289
dataset is sorted by *person, day, hour*	0.3844345
dataset is sorted by *day, hour* with 1 iteration	0.3871982
dataset is sorted by *day, hour* with 2 iterations	0.3880423

Factorization Machine (FM)

Second-order polynomial regression:

$$\hat{y} = \sigma\left(\sum_{j=1}^{n-1}\sum_{k=j+1}^{n} w_{jk}x^j x^k\right)$$

Low rank approximation (FM):

$$\hat{y} = \sigma\left(\sum_{j=1}^{n-1}\sum_{k=j+1}^{n} (v_{j1},\ldots,v_{jH})\cdot(v_{k1},\ldots,v_{kH})x^j x^k\right)$$

H is a number of latent factors

Factorization Machine for categorical dataset

Assign set of latent factors for each pair level-feature:

$$\hat{y}_i = \sigma\left(\frac{2}{n}\sum_{j=1}^{n-1}\sum_{k=j+1}^{n}(w_{x_{ij}k1},\ldots,w_{x_{ij}kH})\cdot(w_{x_{ik}j1},\ldots,w_{x_{ik}jH})\right)$$

Add regularization term: $L_{reg} = L + \frac{1}{2}\lambda||w||^2$

The gradient direction:

$$g_{x_{ij}kh} = \frac{\partial L_{reg}}{\partial w_{x_{ij}kh}} = -\frac{2}{n}\cdot\frac{y_i e^{-y_i p_i}}{1+e^{-y_i p_i}}\cdot w_{x_{ik}jh} + \lambda w_{x_{ij}kh}$$

Learning rate schedule: $\tau_{x_{ij}kh} = \tau_{x_{ij}kh} + \left(g_{x_{ij}kh}\right)^2$

Weight update: $w_{x_{ij}kh} = w_{x_{ij}kh} - \alpha\cdot\sqrt{\tau_{x_{ij}kh}}\cdot g_{x_{ij}kh}$

Ensembling

Model	Description	Leaderboard score
ftrlb1	dataset is sorted by *app id, site id, banner pos, count1, day, hour*	0.3844277
ftrlb2	dataset is sorted by *app domain, site domain, count1, day, hour*	0.3835289
ftrlb3	dataset is sorted by *person, day, hour*	0.3844345
fm	factorization machine	0.3818004
ens	$\text{fm}^{0.6} \cdot \text{ftrlb1}^{0.1} \cdot \text{ftrlb2}^{0.2} \cdot \text{ftrlb3}^{0.1}$	0.3810447

Final results

Place	Team	Leaderboard score	Difference between the 1st place
1	4 Idiots	0.3791384	—
2	Owen	0.3803652	0.32%
3	Random Walker	0.3806351	0.40%
4	Julian de Wit	0.3810307	0.50%
5	**Dmitry Efimov**	**0.3810447**	**0.50%**
6	Marios and Abhishek	0.3828641	0.98%
7	Jose A. Guerrero	0.3829448	1.00%

Future work

- apply the batching idea to the Factorization Machine algorithm
- find a better sorting for the FTRL-Proximal Batch algorithm
- find an algorithm that can find better sorting without cross-validation procedure

References

- H.Brendan McMahan et al. ”Ad click prediction: a view from the trenches.” *In KDD*, Chicago, Illinois, USA, August 2013.
- Wei-Sheng Chin et al. ”A learning-rate schedule for stochastic gradient methods to matrix factorization.” *In PAKDD*, 2015.
- Michael Jahrer et al. ”Ensemble of collaborative filtering and feature engineered models for click through rate prediction.” *In KDD Cup*, 2012
- Steffen Rendle. ”Social network and click-through prediction with factorization machines.” *In KDD Cup*, 2012

Thank you! Questions???

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