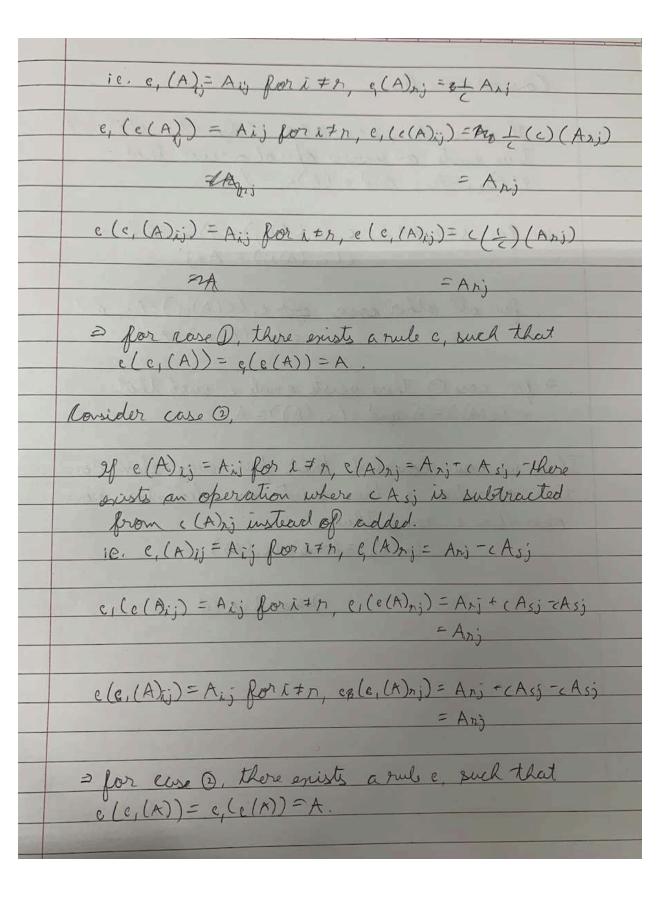
	ISHAAN ROMIL 2023/140/1	Page No. Date
(62)	To each elementary now operation c, there an elementary now operation c, such	corresponds
	$e_{i}(e(A)) = e(c_{i}(A)) = A$	
	for any matrixe A, Here e, is the same type of operation as	
A:	There exist three elementary row opera	Tions on an
	There exist three elementary row opera m×n matrix, A, over the field F: O multiplication of one row of A by a Scalar (
	ie. e(A) ij = Aij if if x + m, e(A) nj = cA	nj
(Times your rows, with a being any	or plus c
	I 7 & B	Scalar and
	ie. e(A) ij = Aij if i 7 r, e(A)nj = An	
<u></u>	3) interchange of two rows of A	
	ie. e(A)ij = Aij if i is different from s, e(A) nj = Abj, e(A) sj = Anj	both or and
	onsidering such operation a separate o	
Ro	onsider rase O,	
	If e (A) = Aij flor i + n and e(A) = c	Arj,
	an operation exists such that scalar, is multiply multiplied by a matri	14=1=0-1
	is a mati	y.



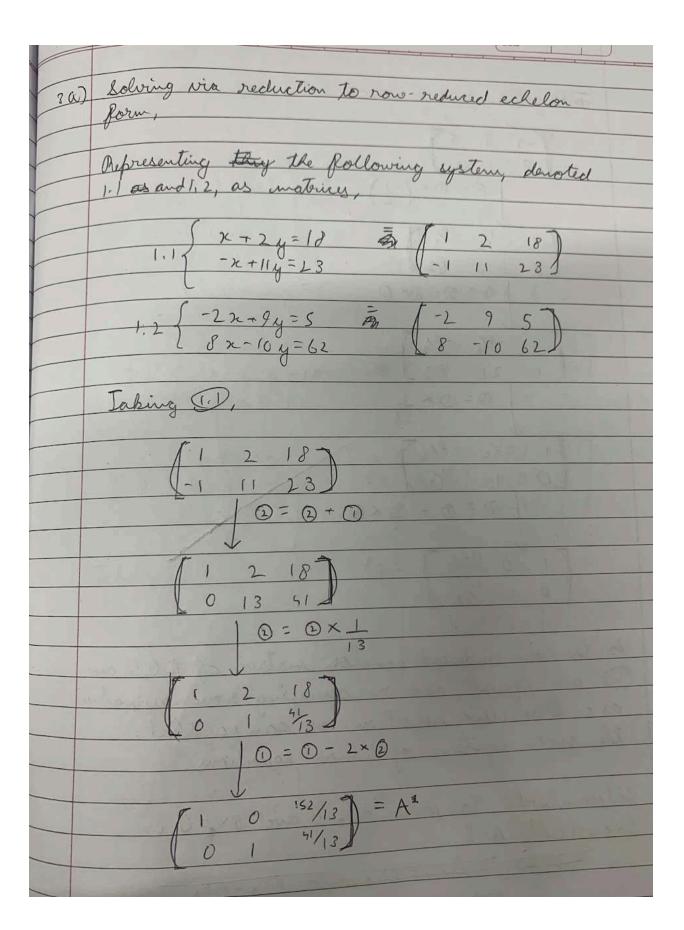
	Consider case 3,
Ē.v	If $e(A)_{ij} = A_{ij}$, $i \neq n$, $i \neq g$, $e(A)_{aj} = A_{aj}$, $e(A)_{sj} = A_{rj}$ those orinte an inverse operation such that $e(A)_{sj} = A_{rj}$ and $e(A)_{aj} = A_{sj}$
	there orinte an inverse operation such that
	e,(A) sj=Anj and c, (A) nj=Asj
-	ie. for $\lambda \neq \lambda$, $\lambda \neq \delta$, e , $(e(A)_{\delta j}) = A_{\delta j}$ and $e(e, (A)_{\delta j}) = A_{\delta j}$
	$e(c, (A)x_j) = Ax_j$
	0 00 +1.
	for all other cases, (e(A)ij)=Aij and
	e(c,(A)i;)=Aij
	2 Por car a the acist asul a real that
	⇒ for cose ③, There exists a rule e, such that e, (e(A)) = A and c(c, (A)) = A.
1	e, (E(R) - F) pro ((C, (R)) - 1.
	The contract Rose outre classication
1	Thus, we can conclude that for every elementary now operation e, there exists an elementary now
1	gour operation e, sold bross and surrendary from
+	operation c, such that
1	e, (e(A))=A=ac(e,(A))
1	
-	Hence proved
	Little and the state of the sta

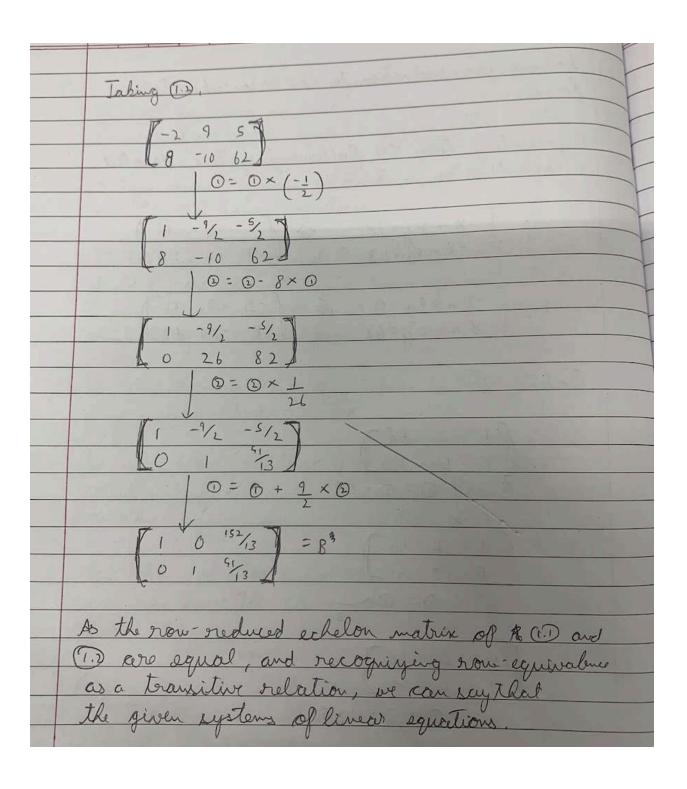
Q3. Find whether the given system of linear equations a) x + 2y = 18, -x + ||y| = 23 -2x + 9y = 5, 8x - 10y = 62b) x + y + y = 6, y + 2y = 5, x + 3y = 6 3x + 2y - y = 12, 3x + y - y = 10, y + y = 3 x + y - y = 6, 2x + y = 8, x - y = 2A: Two mxn matrices A and B are said to be now equirealent if they can be obtained from the other by a finite sequence of elementary row operations. Specifically, matrix A is said to be now-equivalent to matrix B if B can be obtained from A by a Rivite sequence of elementary now operations We know that now-equivalence is a equivalence relation. A equivalence relation is a binary relation which is reflexive, transitive, and symmetric. > now-equivalence is transitive. Thus, if matrix A is now-equivalent to a nowreduced echelon matrix, and B is also now-equivalent to the same now - greduced echelon matrix, we can Say that A is now equirement to B. to We can verify this as we know that for every elementary now operation e, there corresponds an elementary now operation es such that:

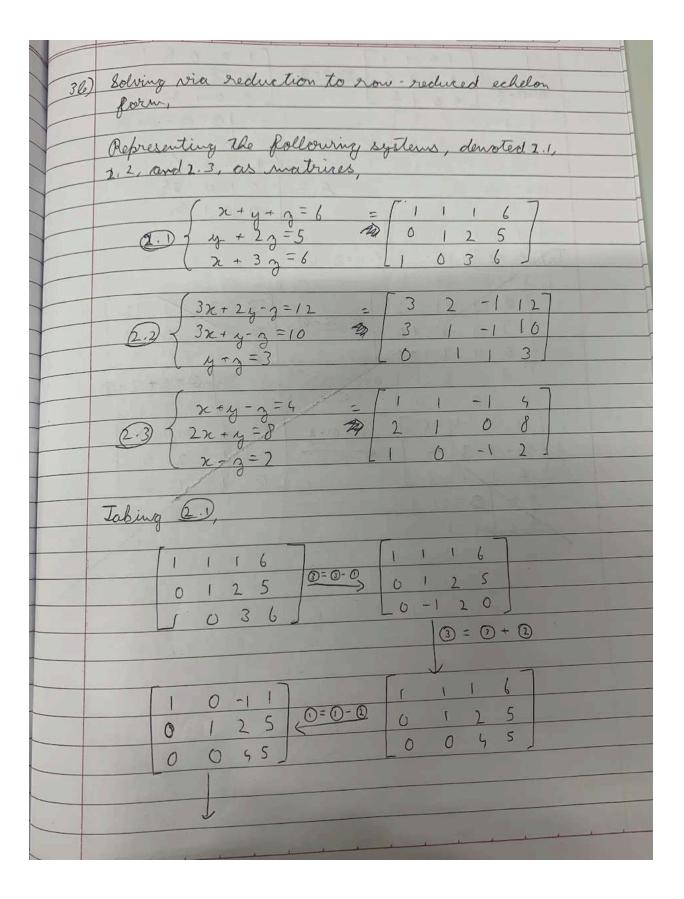
e(e,3(A))= e_2(e(A))=A.

Thus if the same as the now-reduced echelon matrix of B, B can be obtained from A via performing the inverse of the operations required to transform B to the now-reduced echelon matrix.

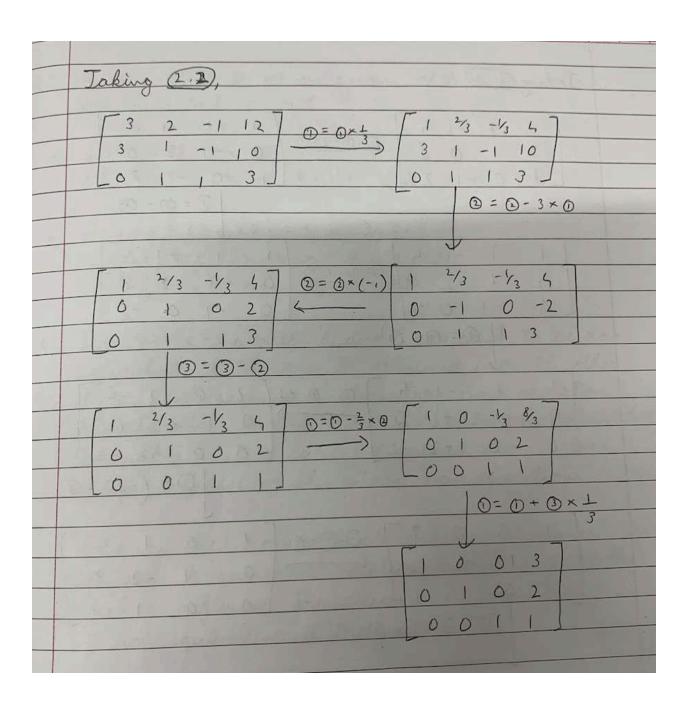
A c c c row-reduced c c c c c c B achelon matrix

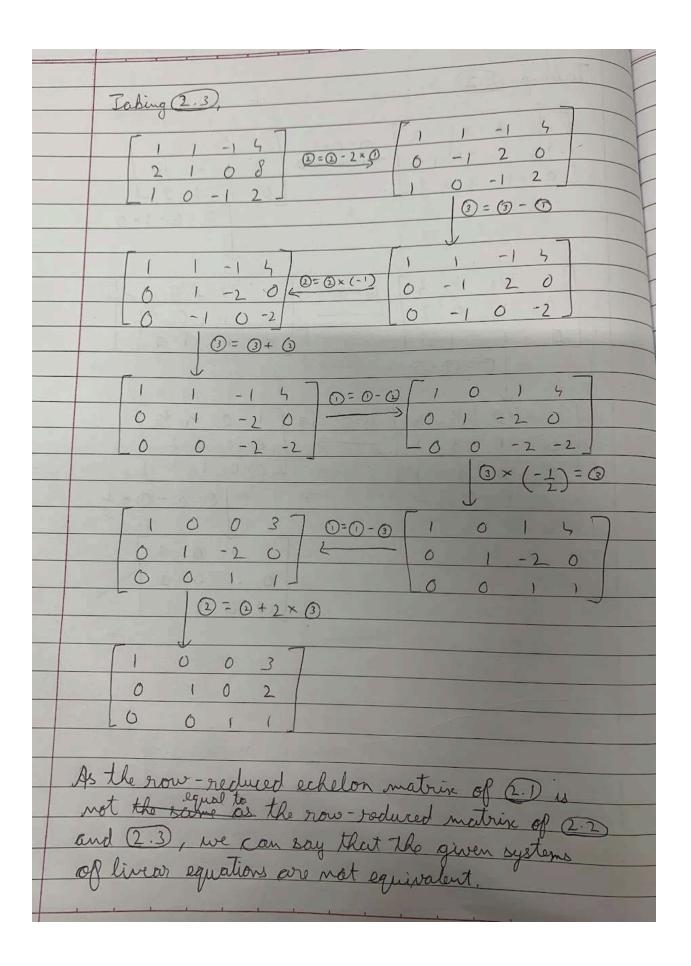






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Orave that every matrix has a now-reduced A mxn matrix is called now-reduced if it satisfies the Rollowing two conditions: @ The 1st non- gero entry in each non- yero now of R is equal to 1 @ each column of R which contains the leading mongero entry of some now has all its other tements entries equal to O (yero). Let there exist an mxn matrix R. Consider the Riest row of R. There are two cases for The entries present in R: Case 1: Every entry of the Pirst now of R is O Rose 2: There exists a non-yero entry in the first now of R. Consider Case 1: If every entry of the first now of R is yero, then condition @ is met for the first now of R and we can consider now 2. Consider Lase 2: If now I contains a non-yero entry, consider & b, where & is the smallest positive integer j

for which Rij 70. In other words, R. K is The first element in the first row which has a nongero value. Multiply now I by At such that his is now equal to 1, their once more satisfying condition 6. Further, to satisfy condition @, for each row i = 1, add (-Rip) times now 1 to now 1. This ensures that every other entry in column & aside from Rib 15 0, allowing us to consider cond the second row Considering now 2, we can effect it into the same cases as now 1: Loss 1' row 2 consists only of entries which are yero. Case 2: now 2 consists of at least one non yero Consider Cose 1: If every entry is zero, we leave now 2 unchanged as it satisfies condition a and thus, allows us to consider the next row. Rousider Rose 2! If the row contains a non-yero entry, we multiply from 2 by a scalar such that the leading man yero element becomes in a similar method to case 2 of the first now. However It is important to note

that if now! contained a leading non-gero entry in column b, count b, count b, count contain the leading nonzero entry of now 2 as all the entry of column b, except the non-gero entry in row! have been changed to 0 via elementary now transformations. Taking the column of the leading nonzero element of row 2, denoted b, we arrange all entries of column b, as 0 via the same technique applied in case 2 of the first now. We can now consider further nows.

For successive row, the same technique is followed as was followed for row 2. It is important to note that the entries of row i in column his - that is, the leading non-yero elements of row is will not change because of successive elementary row operations. We will also leave every the entry in columns to a purchanged, where to represent away prior row. The aforementioned columns unchanged so as to meet condition @ of the definition of a row-reduced matrix.

Following this method for all rows of matrix of R, we can construct a now reduced matrix of

Therefore, for any every matrix can be reduced to a now-reduced form and has a now-reduced form which can be neached by the above steps.

Here proved.

65) Every m & matrix QS) Prove that every MXN matrix A is now equipolent to a now-reduced echelon matrix Via 64, we know that A is now-equivalent to a now reduced matrix. Taking the now-reduced matrix form of A, we can perform a finite number of row interchanges to achieve a now reduced echelon matrix An mxn matrix R is said to be reduced if a now reduced echelon matrix if it satisfies the following conditions @ Ris now-reduced. () every now of R which has all its entries O roccurs before below every now which has a non-yero sutros entry. @ if rows I, ... , or are the non-zero rows of R and if the leading non- yoro entry of now i occurs in column bi, i=1, or, then ki < by < ... & Br. tirst, interchange the row of the row reduced matrix in such a way that they are located at the end. To do this assume there are a rows containing nongero element and I rows in consisting of only zeroes

This leads to three cases: 0 k=0 @ n=0 3 k70, 270 Rossider rase O. Arrange the rows in such a way that of the The rows with leading nongero elements Towards The left are placed towards the top of matrix A. Mathematically, Consider set R = { Aij } such that all elements of set & are leading nonzero elements of the matrix A Via the elementary row operation of row interchange, place the first row where it is the now containing Aij, where i represents The smallest possible value of j'in any element of set R. Next, consider the & A;";", where j" is the column number which is The second lovest in set R. Via now interchange, make i" The second now. Repeat this process for all or nows until a now reduced echelon waters is reached.

Consider case () If all elements of matrix A are yero, A is already in now-reduced echelon form. Consider case 3, thing the elementary row operation of now interchange interchange every row containing all zero elements such that they are located at the bottom of @.

Jo dothis, replace row b; with the lowest nonzero row, where i represents the topmost I ou containing all zero elements, Perform this interchange successively for every one of the & rows starting Next, perform the same operations as listed in case I for the remaining unsorted rows - ic. The now which contain mongero elements. Therefore, it is possible to reduce any arbitrary mxn matrix to a now-reduced echelon matrix, and Thus, any or every parletrary mxn matrix A is equivalent to a now reduced exhelor matrix. Henre proved