

1. Find the QR factorization of the matrix

$$\begin{pmatrix} 1 & 1 & -4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & 0 \end{pmatrix}$$

[CO-2][5]

2. Obtain an orthogonal basis for the subspace R^3 spanned by the vectors

$$x = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, y = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix}, \text{ by Gram-Schmidt orthogonalisation process. Here the inner product is the}$$

standard inner product defined for R^n

[CO-2][5]

3. Find the eigen value and eigen vector of the matrix

$$\begin{pmatrix} 1 & 0 & 2 \\ 3 & -1 & 3 \\ 2 & 0 & 1 \end{pmatrix}$$

. Also find the characteristic polynomial, geometric multiplicity and algebraic multiplicity.

[CO-2][5+1+1+1]

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4. Show that the functions $f_1(x) = 1$, $f_2(x) = \sin x$, and $f_3(x) = \cos x$ are orthogonal in $C^0[-\pi, \pi]$, and then construct an orthonormal set of functions in $C^0[-\pi, \pi]$ with the help of these functions. (C^0 means the function is continuous)

[CO-2][5]

5. (a) Determine whether the given matrix is orthogonal. If it is, then find its inverse

$$\begin{pmatrix} 1/3 & 1/2 & 1/5 \\ 1/3 & -1/2 & 1/5 \\ -1/3 & 0 & 2/5 \end{pmatrix}$$

(b) If Q be a 2×2 orthogonal matrix, then Q must have the form $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ or $\begin{pmatrix} a & b \\ b & -a \end{pmatrix}$

[CO-2][4+3]

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