

# Linear Algebra(MA2.101), Spring 2024, IIT Hyderabad

## Quiz 1

Total Marks: 15

Answer any three questions out of ~~five~~ <sup>four</sup>. Each question carries 5 marks.

✓ 1. Let

$$A = \begin{pmatrix} 3 & -6 & 2 & -1 \\ -2 & 4 & 1 & 3 \\ 0 & 0 & 1 & 1 \\ 1 & -2 & 1 & 0 \end{pmatrix}$$

For which values of  $Y = (y_1, y_2, y_3, y_4)^T$  do the system of equation  $AX = Y$  has a solution and under what conditions do the systems  $AX = Y$  don't have any solution? Use row-reduced echelon form of  $A$  to justify your answer. [5 marks]

✓ 2. Prove the following:

✓ (a) Suppose  $a \in \mathbb{F}$  and  $\vec{v} \in \mathbf{V}$ , where  $\mathbf{V}$  is a vector space defined over  $\mathbb{F}$ . If  $a\vec{v} = \vec{0}$ , then prove that either  $a = 0$  or  $\vec{v} = \vec{0}$ . [2 marks]

(b) For every  $\vec{v} \in \mathbf{V}$ ,  $-(-\vec{v}) = \vec{v}$ . [1 marks]

✓ (c) Every element in a vector space has a unique additive inverse. [1 marks]

✓ (d) A vector space has a unique additive identity. [1 marks]

3. If  $b \in \mathbb{F}$ , then the set  $\{(x_1, x_2, x_3, x_4) \in \mathbb{F}^4 : x_3 = 5x_4 + b\}$  is a subspace of  $\mathbb{F}^4$  if and only if  $b = 0$ . [5 marks]. *Prove*

✓ 4. Using elementary row operations, prove that  $A$ , where

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

is invertible if and only if  $ad - bc \neq 0$ . [5 marks]