1	ASSIGNMENT 6 ISHAAN ROMIL Page No. Date
61)	Let m and n be positive integers and F be a field. Let m and n be positive integers and F be a field. Let m and n be positive integers and F be a field. Let m and n be positive integers and F be a field.
A:	suppose the suppose of the suppose o
	As dimension of W, denoted dim W, is less than or equal to m: dim W & m, There are at least m- vectors &, &, &, which can be relected in wester that the vectors span W.
	Let A be mxn matrix with now vectors &, &, on in W which is equivalent to a now-reduced eghelon matrix which is now-equivalent to A,
	denoted as R. Then, the now space of Rig W. Assume R be any now-reduced echelon materia which has W as its now space.
S C	Let \$1, \$2, \$1,, \$1 be non-zero victors of R and uppose the leading non-zero entry of \$i occurs in olumn \$i, i=1,, \$1. The vectors \$1,, \$1 form basis for W.
99	t has been observed that if $\beta = (b_1, \dots, b_2)$ is
	B=c,p,+,+(np)

and This, the unique expression for B as a linear combination B= & Brifo. [: B= & copi] Ner are given Rij = 0 if i > 5, j = 1. This B=(0,..., bbg,...ba) Egura les + 0] This also demonstrates that the leading non-yero element of Boccurs in column hs. Note also that for each hs, &=1,..., n, there exists a vector in wwhich has a non-gero he coordinate, namely ps. From these assertions, it can be derived that R is uniquely determined by w. To describe R in terms of w, Consider Consider rell vertors $\beta = (b_1, b_2, \dots, b_d)$ in W. If B & O, the first non-zero occurence of B must ie. B=(0,0,0,...,0,bx,...,br),bx ±0. come in redumn A. 3.

Let h, by ..., by he positive integers to such
that there exists some \$70 in with first mon-yero
that there exists some \$70 in with Efirst mon-yero

decurence of which occurs in column to.

Arrange the aforementioned by, 1 ± x = n in an

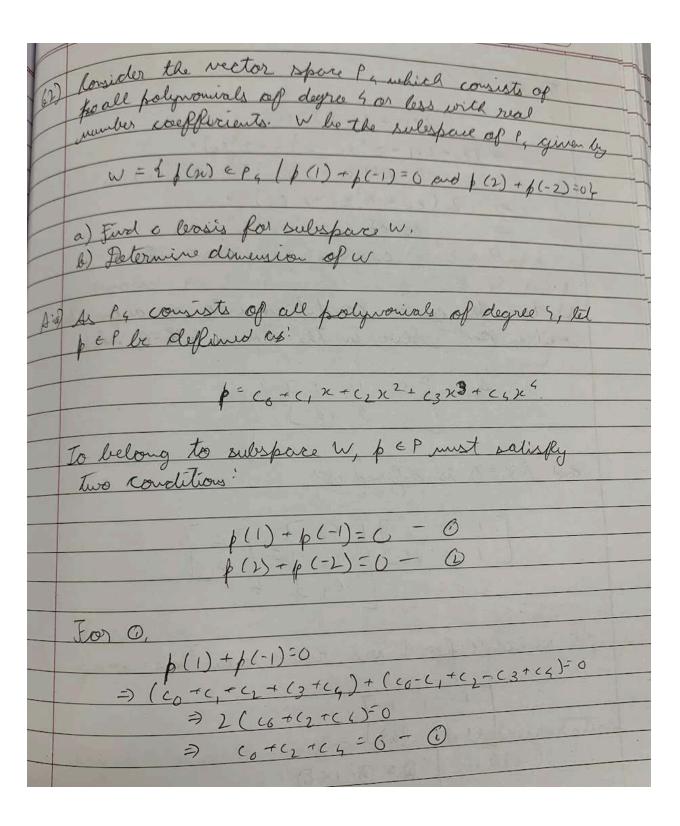
ascending order, B, 'b24n'... cbn.

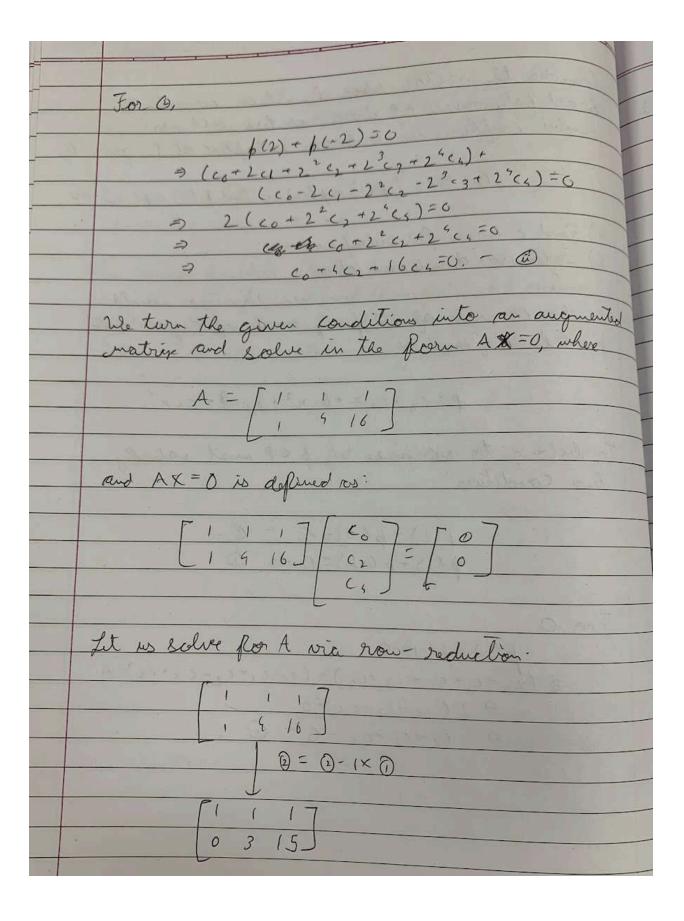
For each positive integer by there will be one and
only one vector ps in w such that the b & coordinate
of ps is I and the b; the coordinate of ps = 0 (i ± b).

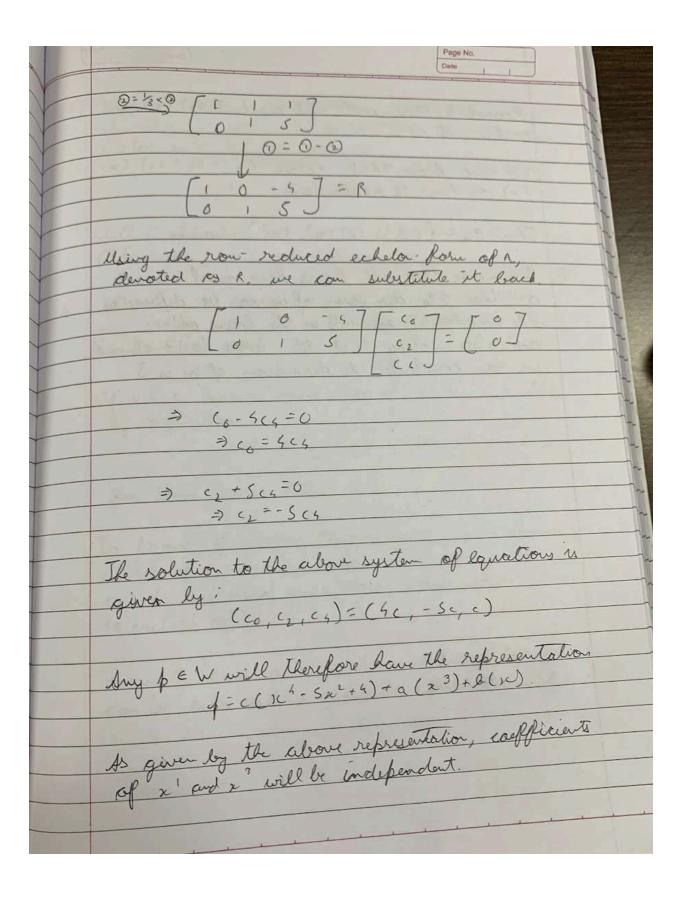
Then, R is n'n matrix which has rown vectors

b, p, p, ..., ph, O, O, ..., O.

Beare proved.







Meanwhile there will out a relation between coefficient of x', x', x'.

We also know that vectors (x'-5x'+4), (x')

(x) are linearly independent.

Thus, Bw = f(x'-5x'+4), (x2), (x2)}.

Boxnowing that all basis have equal elements and that the dimension of W can be defend as the number of elements in the basis of w, and also noting that the basis has 3 elements.

We have conclude the dimension of W is 3.

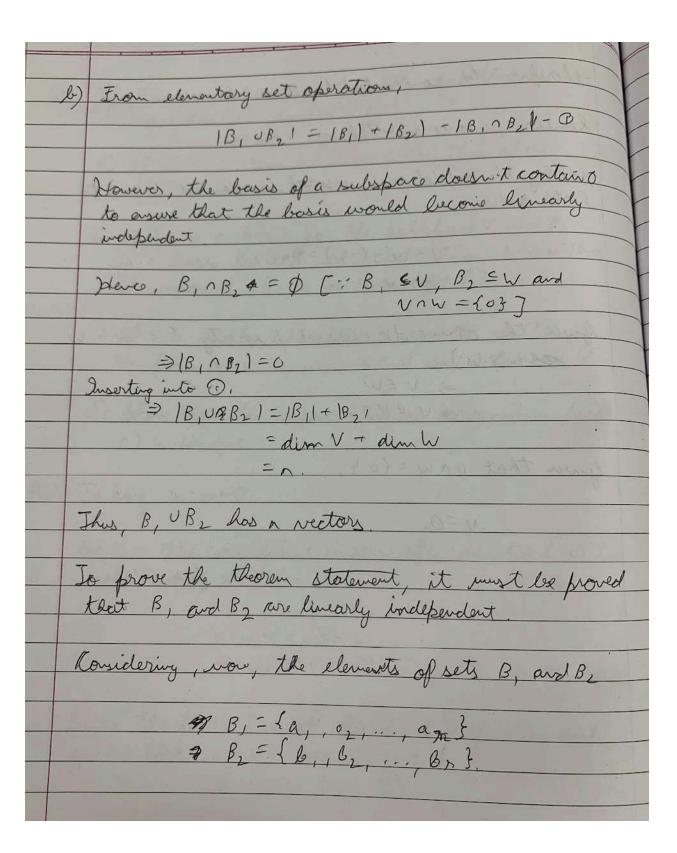
Consider a vector NCV, where V is a linear continution of vectors in Ser ie. V=d, v, + d, v2 +d3 v, +d4 v, Is coefficients of overtor in a linear combination car be zero. Rousidering the case d = 0 and thereby making a linear combination of v, v2, V3, we obtain the equation U=d, V, +d, V2 -d3 V3 + OV4 - 0 flor any arbitrary d, drid 3. (2) Roan be used to thorse claim that since coeff. of 14=6, the vector v is essentially a linear combination of {v, ,v2, v3 } Since Lv, v, v3 & has already been established Es & spanning set of V, any VEV can thereflore be expressed as v in 3. Thus, every NEV is a linear combination of vectors of Sz, and by extension, Sz is a spanning set of v. Therefore, adding another vector to spanning set Sz for some subspace V.

Say results in spanning set Sz for some subspace V.

Here froved

Let Vand W be subspaced of R' such That Valu=
{0} and dem (V) + dim(W)=1. a) of v=w=0, then vevandweW, Than show that v=0 and n=0. b) If B, is a basis for the subspace V and B?
is a basis for the subspace W, Then show that the union B, UB, is a basis for R^ c) If x is in R?, show that x can be written in the form x-V+W, when V EV and w EW. d) Show that the representation obtained in pair c) is unique Ala) Jaking V+w=Q Adding the ordditine inverse of v, denoted (-v) To look sider V+w+(-V)=-V => W=-V,-0 given the properties of fields and speces, and given that $v \in V$, — $v \in V$ => WEV (by O) given that w = W and Vnw \$ {0}, it can

		Date	- 11
therefore !	be concluded that	t w = 0.	-
Tolk and the	AND DESCRIPTION	who were the state of the state	
	once more taking		
11: 11.	= d-liture =	of w, devoted (-u) to	
Adding the	addine musica	W, devoted (-w) to	
both sees			
The services	V+W+(-w)==	€-w	
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himer the	above discusses	& property Cicwe w	
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	⇒ V eV∩W	81 : 1 . 8 M E 81 . FR . E - 1	
	I will milk of Vin	N. K.	
liver that	t vnw={0},	2 8 14 M 2 1 4 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	
goo	F. St. The street	THE A RESERVE WAY AS	
	v=0.	MA IA BOR JAN WALL	4
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W. V. V.	An off harman	Here proved	
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Knowing that the subspace of a basis contains the set of all linear continuations of the nectors of the basis. Applying that in the content of v we know that I contains all linear combinations of B, vectors but you of the combination of B, vertors. They, the vectors of B, are not linear combinations of B. wectors. To prove lonear trotopart independent nature of B. and B2, offerd adjoin b, to B, B, Ufle, 3= {a, a2, ..., ax, b, 3 This represent a linearly independent set as B, is not a linear combination of any element of => B, U l b, 3 as & linearly independent set

Mert, adjoin by to B,
B, USb, 62 3= { 0,000,, ex, b, b, 5
Assume that the about I linear combination of B, U & b, b, 3 blongs to B.
B, U16, 16, 3 tolongs 12
Then,
$\begin{array}{c} (a_1 + c_2 a_2 + + c_n a_n + c_n c_1 b_n = b_2 - c_n c_1 b_n \\ \Rightarrow (a_1 + c_2 a_2 + + c_n a_n a_n = b_2 - c_n c_1 b_n \end{array}$
Setting the right hand side equal Tow (win the closure property of vector spaces),
(10,+c202++ CDOREW
However, by definition,
Market Strategic
(,010c202 t + cnon € V.
=> c, a, - c, o2 + (rar = Vn w = fo}
7 C101-C202 + + CNON =0
$= c_1 = c_2 = c_3 = \dots = c_n = 0.$
The fitte country of them made to:
The fitte equation @ then reduces to: $C_{n-1}B_1 = B_2$
which is obrejously false as B, and B, are linearly judipendent.
parper au

as any victor from R' can be expressed as

a linear combination of 8, UP;

a linear combination of 8, UP;

x = c, c, +c, a, + a, ... + c, a, + c, c, b, a, ... + c, b,

x = c, c, +c, a, + a, ... + c, a, b, a, ... + c, b,

which are disjoint sets and 8, UP; is the

union of obigont sets, examining and applitting it

c, 0, + c, c, + ... + c, a, ay = v & lin v - a

ant,

c, 0, + c, c, + ... + c, a, ay = v & lin v - a

lubstituting and and a m.

but v + w = 2, x & V, w & W.

Date This statement can be proved sice contradiction. ssume that the & representation is not unique. ie. n=v,+w,=v,+wz Add (- u, -w,) to both sidy, V, -V2 = W2-W, As v, -v2 & V and w, -v2 & W, the Two ran su sides can only be equal of U, - V2 and w, - w2 ∈ V1 w = 0 ie. V, -V2=0 and w, - w2=0 Jujevy and with Here, by contradiction, n=vz+wz is a unique representation, Here provede