Performance Modeling
of Computer Systems and Networks

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Memoryless property
and
probability distributions

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Memoryless property as lifetime

A random variable X is said to be memoryless if

$$Prob\{X > s+t \mid X > s\} = Prob\{X > t\} \quad \forall\, s,t > 0$$

<u>Example</u>
X is the lifetime of a lightbulb.
The property says that the probability that the lightbulb survives for at least another t seconds before burning out, given that the lightbulb has survived for s seconds already, is the same as the probability that the lightbulb survives at least t seconds independent of s.

Does this seem realistic for a lightbulb???

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Lifetime and failure rate

Distributions for which $Prob\{X > s + t | X > s\}$ goes down as s goes up are said to have *increasing failure rate*.
The device is more and more likely to fail as time goes on.

Example
A car's lifetime. The older a car is the less likely that it will survive another $t = 6$ years.

Distributions for which $Prob\{X > s + t | X > s\}$ goes up as s goes up are said to have *decreasing failure rate*.
The device is less likely to fail as time goes on.

Example
- UNIX job CPU lifetimes. The more CPU a job has used up so far, the more it is likely to use up.
- The same for computer chips. If they're going to fail, they'll do it early. That's why chip manufacturers test them for a long while.

Hazard rate function

Let X be a continuous random variable with probability density function $f(t)$ and cumulative distribution function $F(t) = Pr\{X < t\}$.
Then $r(t)$ is formally defined as:

$$r(t) \equiv \frac{f(t)}{\bar{F}(t)}$$

where $\bar{F}(t) = 1 - F(t) = Pr\{X > t\}$

Consider the probability that a t-year old item will fail during the next dt seconds:

$$Pr\{X \in (t, t+dt) | X > t\} = \frac{Pr\{X \in (t, t+dt)\}}{Pr\{X > t\}} \approx \frac{f(t)dt}{\bar{F}(t)} = r(t)dt$$

the istantaneous failure rate

Hazard rate function

If $r(t)$ is constant then $f(t)$ must be exponential
Indeed for the exponential

$$r(t)=\frac{f(t)}{\bar{F}(t)}=\frac{\lambda \cancel{e^{-\lambda t}}}{\cancel{e^{-\lambda t}}}=\lambda$$

We use the failure rate concept when we study scheduling

Why the remaining lifetime is so important?

CPU load balancing in a Network of Workstations

- It may help to migrate a job to a <u>less-loaded</u> workstation (one with fewer jobs) in order to improve mean response times
- migration can be expensive if the job has <u>a lot of "state"</u> that has to be migrated with the job (lots of memory).

two types of migration used in load balancing techniques:

- non-preemptive migration (NP) — only relocates "newborn" processes (initial placement, or remote execution)
- preemptive migration (P) — migrates processes that are already active (running) (active process migration)

1. Should we bother with P migration, or is NP enough?
2. If we are going to bother with P migration, which processes are worth migrating?
   That is, what is a good migration policy?

terminology:

a job's "size": its total CPU demand
a job's "age": its total CPU usage thus far
a job's "lifetime" refers to its total CPU requirement
a job's "remaining lifetime" refers to its remaining CPU requirement

Observe that commonly, at any point in time, you don't know the job's remaining lifetime, just it's current CPU age.

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Measurements of CPU requirements of Unix jobs

jobs lifetime for 3 months, only those jobs with CPU lifetimes greater than one second

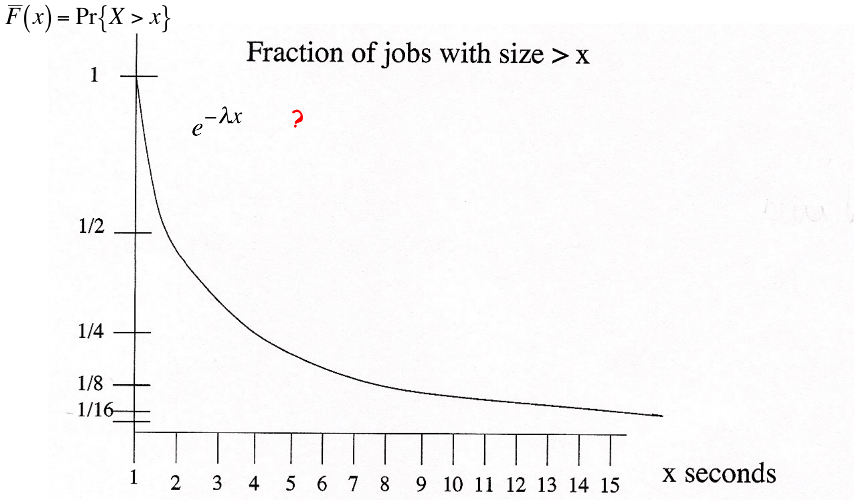

Figure 1: *Plot of measured distribution on non-log axes.*

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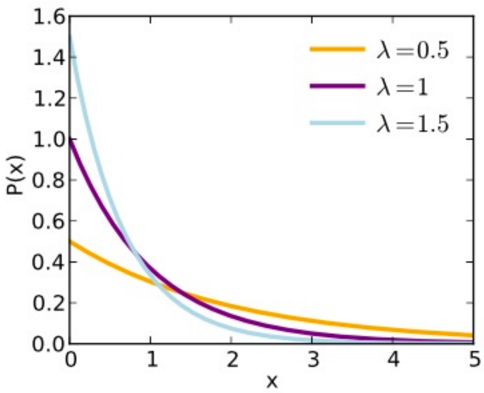
exponential distribution



$$f(x) = \lambda e^{-\lambda x}$$

1. For each unit step, the "quantity" is reduced of $e^{-\lambda}$

λ is the *decay constant*

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Analytical models

Measurements of CPU requirements of Unix jobs

jobs lifetime for 3 months, only those jobs with CPU lifetimes greater than one second

$\bar{F}(x) = \Pr\{X > x\}$

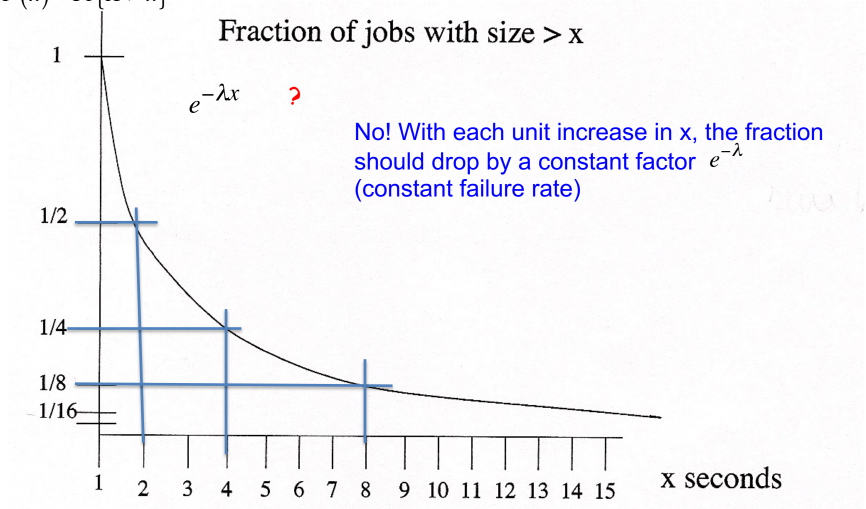

Figure 1: *Plot of measured distribution on non-log axes.*

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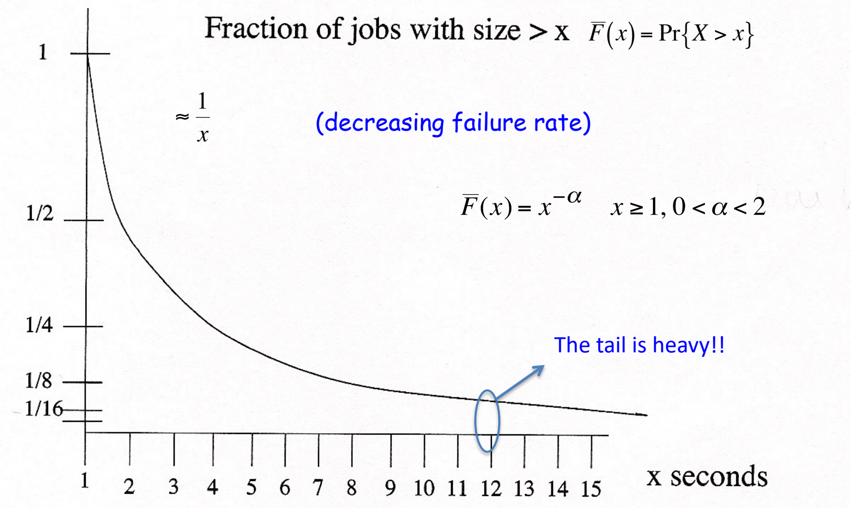

Figure 1: *Plot of measured distribution on non-log axes.*

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Pareto distributions

$$f(x) = \alpha k^{\alpha} x^{-\alpha-1} \quad k \leq x < \infty, 0 < \alpha < 2$$

α a measure of the distribution variability and of the "heavy-tailedness":

$\alpha \to 0$ +variability, +heavy

$\alpha \to 2$ -variability, -heavy

Problem: i-th moment is finite just for $\alpha > i$

$$E[X] = \frac{\alpha k}{\alpha - 1} \qquad \alpha > 1$$

$$var[X] = \frac{\alpha k^2}{(\alpha-1)^2(\alpha-2)} \qquad \alpha > 2$$

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Properties of Pareto distributions

Decreasing Failure Rate

The more cpu you have used so far, the more you will continue to use.

completely different from the exponential distribution, where your cpu usage after any point in time is completely independent of the amount of cpu used up to that point (memoryless property)

Infinite Variance

"Heavy-Tail Property"

A miniscule fraction of the very largest jobs comprise half of the load on the system.

For example, when $\alpha = 1.1$, the largest 1% of the jobs comprise 1/2 of the load.

Bounded Pareto distributions

the measured data has a *minimum* job lifetime and a *maximum* job lifetime.

Thus the measured data has all finite moments.
To model the measured data, we therefore want a Pareto distribution which has been truncated.

$$f(x) = \alpha x^{-\alpha-1} \frac{k^\alpha}{1-\left(\frac{k}{p}\right)^\alpha} \qquad k \le x \le p,\ 0 < \alpha < 2$$

all of the moments are finite

The actual measured squared coefficient of variation values were (obviously) finite and were between 25 and 49 !!!

$C^2 = \frac{var}{mean^2}$, expo=1

1. Should we bother with P migration, or is NP enough?
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   That is, what is a good migration policy?

the DFR property leads us to think that it may pay to migrate old jobs.
The reasoning is that although an old job may have high migration cost, because it has accumulated a lot of memory, if the job is really old then it has a high probability of using a lot more cpu in the future, which means that the cost of migration can be amortized over a very long lifetime.