Performance Modeling of Computer Systems and Networks

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Batch Means

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- Two types of DES models: transient and steady-state
- For transient, construct interval estimates using *replication*
- For steady-state, obtain *point* estimate by simulating for a long time
- Can we obtain interval estimates for steady-state statistics?

→ *use method of batch means*

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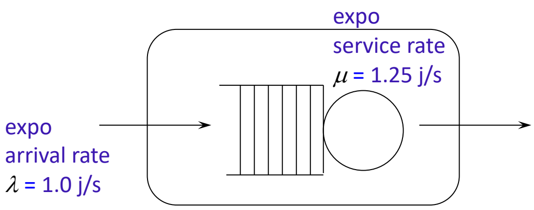
Example 8.4.1: Transient vs. Steady-State Estimates



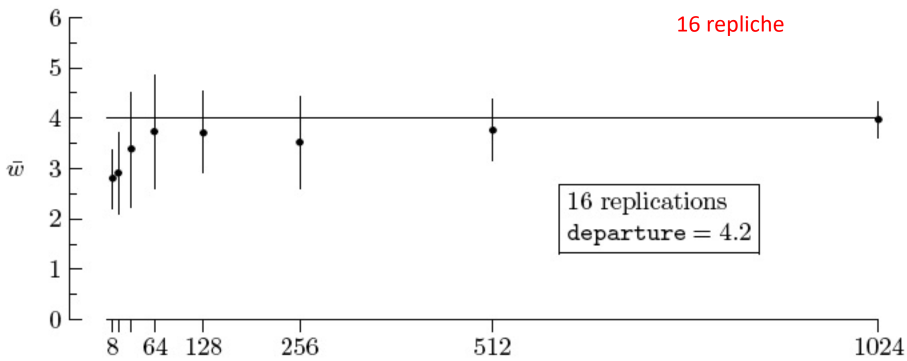
Analytically, utilization is 0.8 and expected steady-state wait is 4.0 s.

Can transient estimates be accurate steady-state estimates?

- Eliminate the initial state bias by setting departure to 4.2: the simulation begins in its *expected* steady-state condition
- Use 16 replications to construct transient interval estimates for 8, 16, 32, . . . , 1024 jobs

Example 8.4.1: Transient vs. Steady-State Estimates

16 repliche

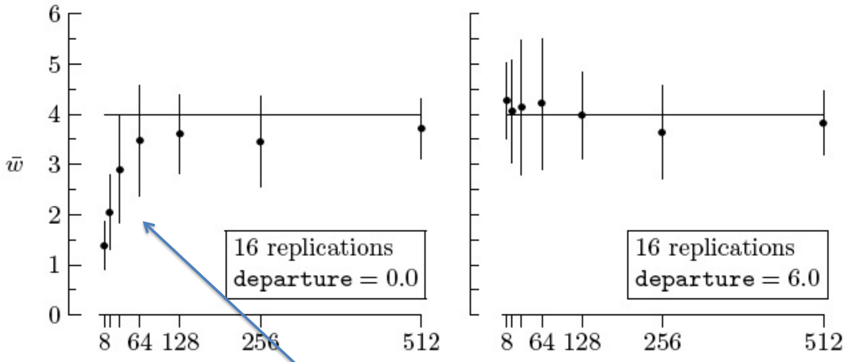


Finite-horizon interval estimates can be accurate steady-state estimates (provided the number of jobs is not small)

Example 8.4.2: Initial State Bias

Consider initial values of 0.0 and 6.0 for departure

16 repliche



initial state can have significant impact

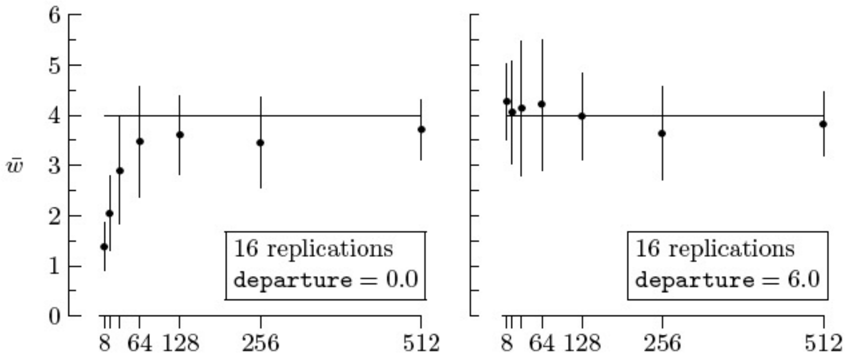
possible to choose an initial state that eliminates the initial state bias

Transient vs. Steady-State

Example 8.4.2: Initial State Bias

Consider initial values of 0.0 and 6.0 for departure

16 repliche



When the number of jobs is large, estimates are very good and essentially *independent* of the initial state

Interval Estimates for Steady-State

- Use replication-based transient interval estimates
- Each replication must correspond to a long simulated time period

Three issues:

- What is the initial state?
- What is the length of the simulated time?
- How many replications?

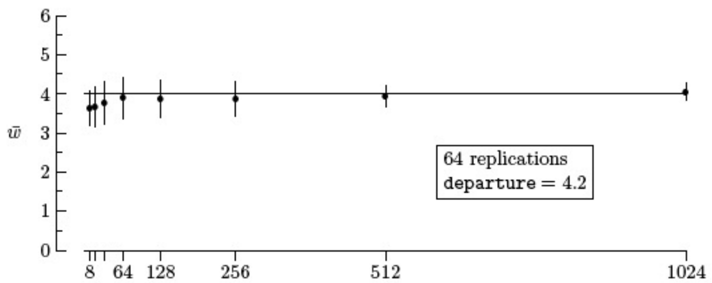
Previous example provides insight into first two issues

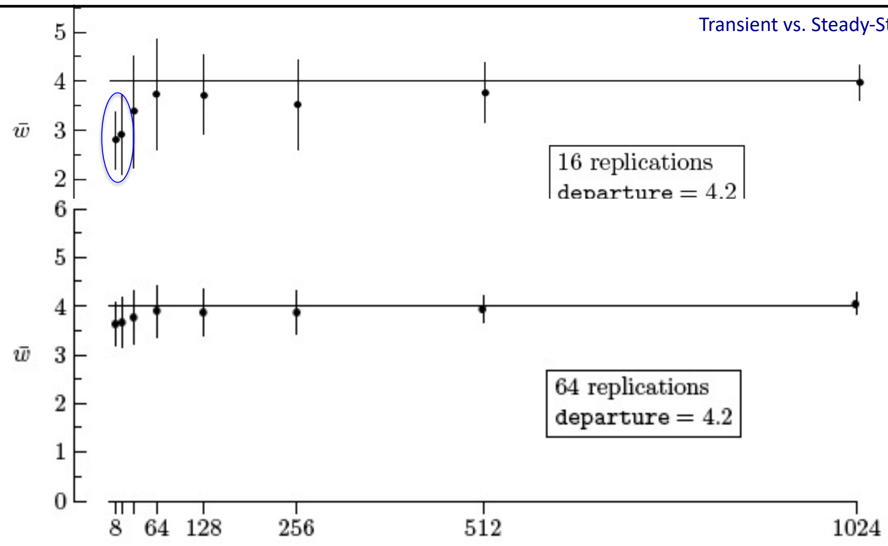
Example 8.4.3: Increase the Number of Replications

Repeat the previous experiments using 64 replications

All other parameters remain the same

64 repliche





- The interval estimates have been cut approximately in half
- Initial state bias eliminated by using departure = 4.2

Even for a small number of jobs!

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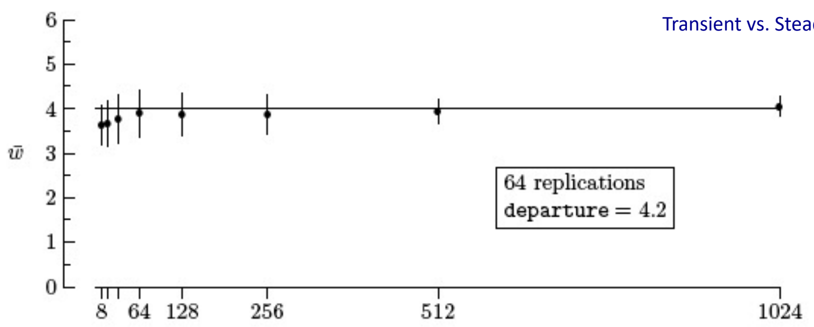

- The interval estimates have been cut approximately in half
- Initial state bias eliminated by using departure = 4.2

Even for a small number of jobs!

- The initial state bias that seemed to be evident before (16 replications) was natural small-sample variation
- Note: we were able to avoid initial state bias only because we already knew E[w]

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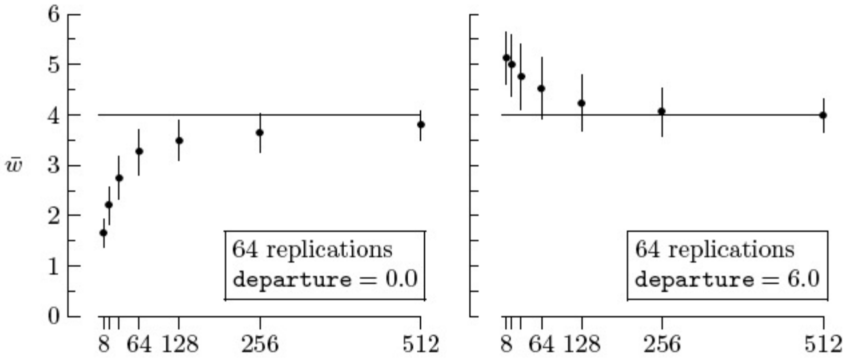

That departure = 6.0 seemed right was a false impression based on misleading small-sample statistics

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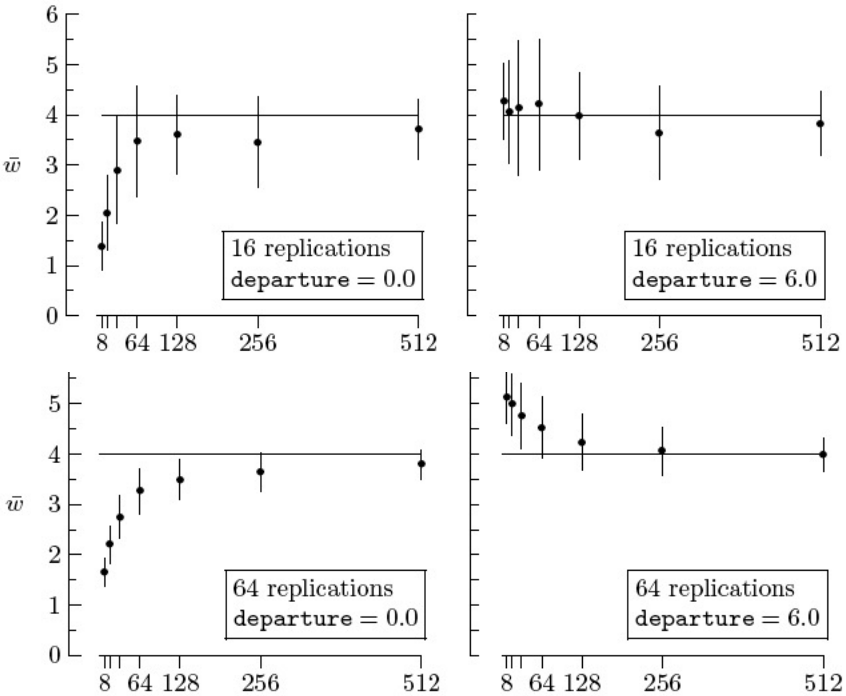

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Summary

- Want interval estimates for steady-state
- Replicated transient statistics can be used
- However, initial bias problem
- Need technique that avoids the initial bias problem

Method of Batch Means

- Previously, each replication was initialized with same state
- Gives initial bias problem

Batch means:

- Make one long run and partition into batches
- Compute an average statistic for each batch
- Construct an interval estimate using the batch means

- Initial state bias is eliminated
- State at the beginning of each batch is the state at the end of previous batch

Algorithm 8.4.1: Method of Batch Means

Consider a sequence of samples $x_1, x_2, \ldots, x_n$

1. Select a batch size $b > 1$
2. Group the sequence into k batches

$$\underbrace{x_1, x_2, \cdots, x_b}_{\text{batch } 1}, \underbrace{x_{b+1}, x_{b+2}, \ldots, x_{2b}}_{\text{batch } 2}, \underbrace{x_{2b+1}, x_{2b+2}, \ldots, x_{3b}}_{\text{batch } 3}, \ldots$$

and for each calculate the batch mean

$$\bar{x}_j = \frac{1}{b} \sum_{i=1}^{b} x_{(j-1)b+i} \qquad j = 1, 2, \ldots, k$$

3. Compute $\bar{x}$ and s of batch means $\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_k$

Algorithm 8.4.1: Method of Batch Means

4. Pick a *level of confidence* $1 - \alpha$ (typically $\alpha = 0.05$)
5. Calculate the critical value $t^* = \text{idfStudent}(k - 1, 1 - \alpha/2)$
6. Calculate the interval endpoints $\bar{x} \pm t^* s/\sqrt{k-1}$

- $(1 - \alpha) \times 100\%$ confident that the true *unknown* steady-state mean lies in the interval
- Provided b is large, true *even if the sample is autocorrelated*

Effect of Batch Parameters

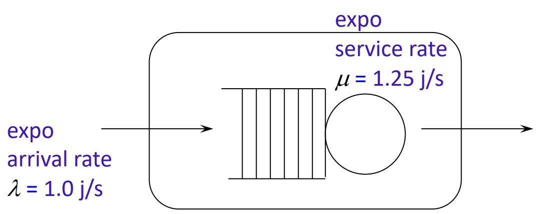
Provided no points are discarded:

$$\bar{x} = \frac{1}{k}\sum_{j=1}^{k}\bar{x}_j = \frac{1}{n}\sum_{i=1}^{n}x_i$$

$$\underbrace{x_1, x_2, \ldots, x_b}_{\bar{x}_1}, \underbrace{x_{b+1}, x_{b+2}, \ldots, x_{2b}}_{\bar{x}_2}, \underbrace{x_{2b+1}, x_2, \ldots, x_{3b}}_{\bar{x}_3}, \ldots\ldots\ldots\ldots, \underbrace{\ldots, x_n}_{\bar{x}_k}$$

- Choice of (b, k) has *no* impact on the *point* estimate
- Only the *width* of the *interval* estimate is affected

Example 8.4.5: Effect of (b, k)



Consider the queue is initially idle, use ssq2 to generate n = 32768 consecutive waits

Using batch means with different (b, k):

(b, k)	$(8, 4096)$	$(64, 512)$	$(512, 64)$	$(4096, 8)$
$\bar{w}$	3.94 ± 0.11	3.94 ± 0.25	3.94 ± 0.29	3.94 ± 0.48

- Note that 3.94 is independent of (b, k)
- Width of the interval estimate is not

Is the Method of Batch Means Valid?

For interval estimation, the batch means must be *iid Normal*

1. Are the batch means *Normal*?

   As b increases, mean of b RVs tends to Normal

2. Is the data actually independent?

   Autocorrelation (Section 4.4) becomes zero if b is large

Therefore, as *b* increases, method of batch means becomes increasingly more valid

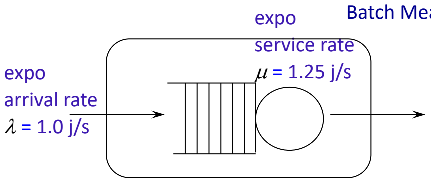
Guidelines for Choosing (*b*, *k*)

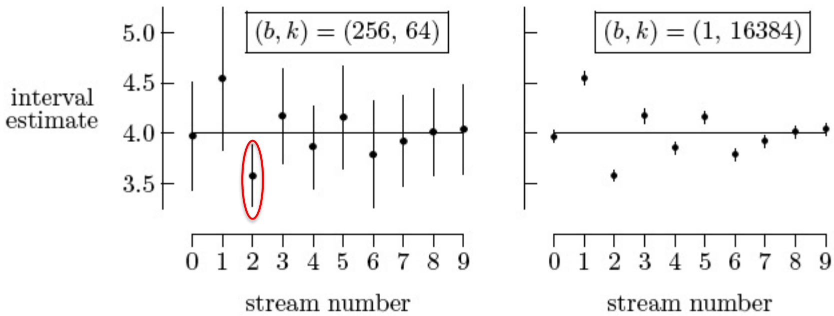
- Note: If *b* is too large, *k* will be small giving wide interval estimates
- Number of batches *k*:
  - Avoid small-sample variation
  - $k \geq 32$; k = 64 is recommended
- Batch size *b*:
  - Want to ensure (approximate) independence
  - *b* should be at least twice the autocorrelation "cut-off" lag (Section 4.4)

(See example 8.4.6)

Example 8.4.5: Effect of (b, k)

Produce 10 interval estimates using batch means





An actual coverage of 90%

Not batching:
Intervals are too small
An actual coverage of 30%