Performance Modeling of Computer Systems and Networks

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Priority scheduling

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Service classes

- (Multimedia traffic)
- Quality of Service (QoS)
- Penalties

The proper scheduling policy can improve performance of a server tremendously.
It costs nothing to alter your scheduling policy (no money, no new hardware), so the performance gain comes for free.

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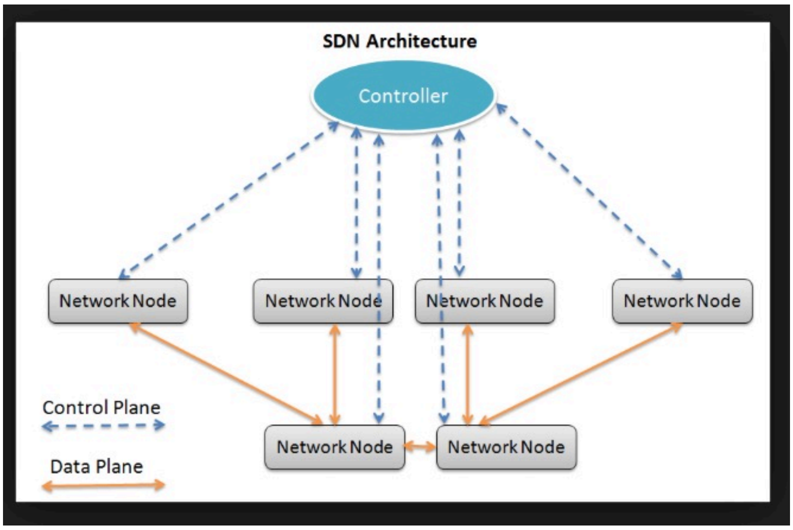
SDN

architettura per la realizzazione di reti di telecomunicazioni
il piano di controllo della rete e quello di trasporto dei dati sono separati logicamente



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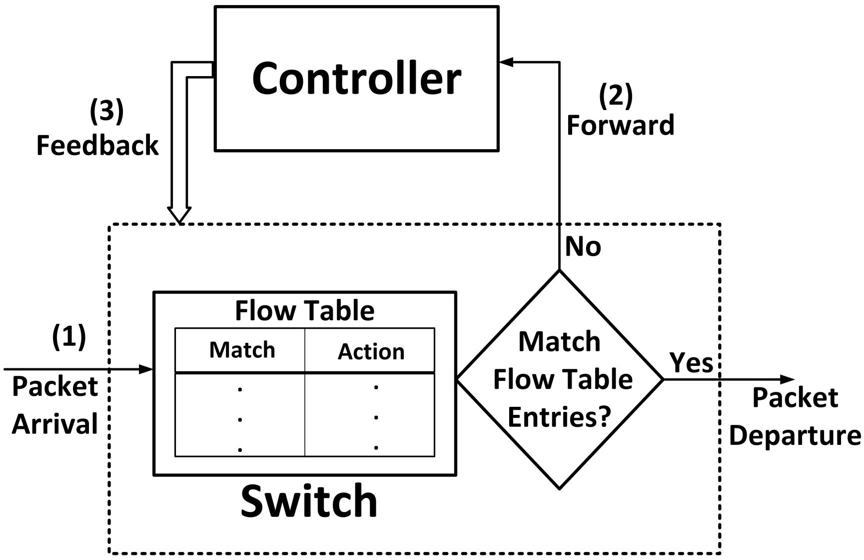

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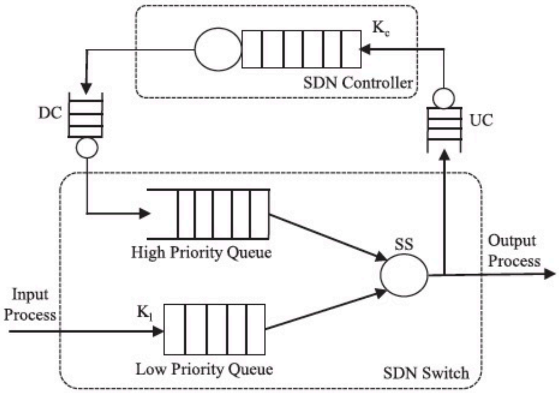

Fig. 1. The PQ-based SDN system architecture.

Miao W., Min G., Wu Y., Wang H. and Hu J., 2016. Performance Modelling and Analysis of Software-Defined Networking under Bursty Multimedia Traffic. *ACM Trans. Multimedia Comput. Commun. Appl.*, Vol. 12, No. 5s, Article 77.

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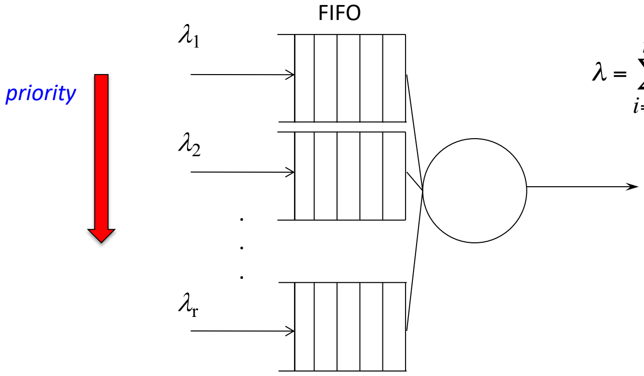
Priority classes



$$\lambda = \sum_{i=1}^{r} \lambda_i$$

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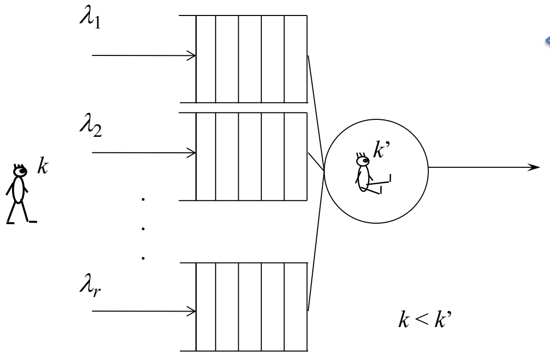
Priority classes



$k < k'$

abstract

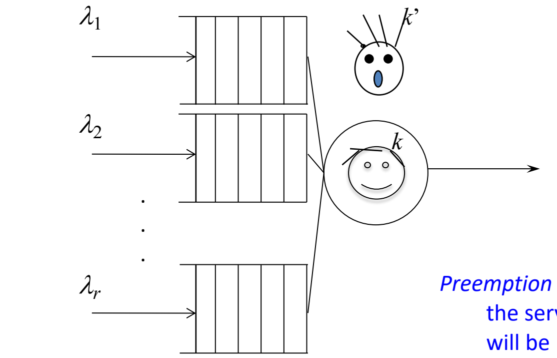
size based

preemptive

not-preemptive

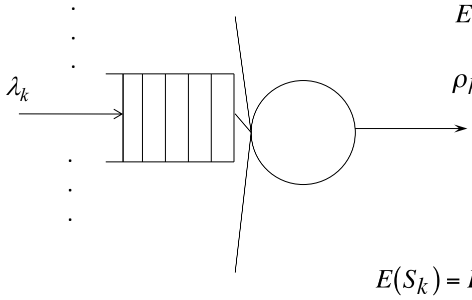
Priority classes with preemption

$k < k'$



Preemption without loss:
the service of the interrupted job will be resumed from the interruption point

Abstract priority without preemption



S_k

$E(S_k) = \dfrac{1}{\mu_k}$ $\quad \sigma^2(S_k)$

$\rho_k = \lambda_k E(S_k) \quad \rho = \sum_{i=1}^{r} \rho_i$

abstract

$$E(S_k) = E(S) = \frac{1}{\mu}, \quad \sigma^2(S_k) = \sigma^2(S), \forall k$$

$$\rho_k = \lambda_k E(S)$$

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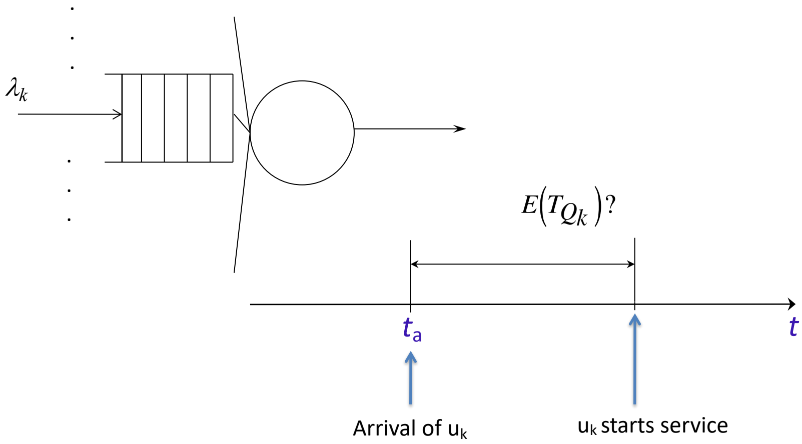
Abstract priority without preemption

local performance measures



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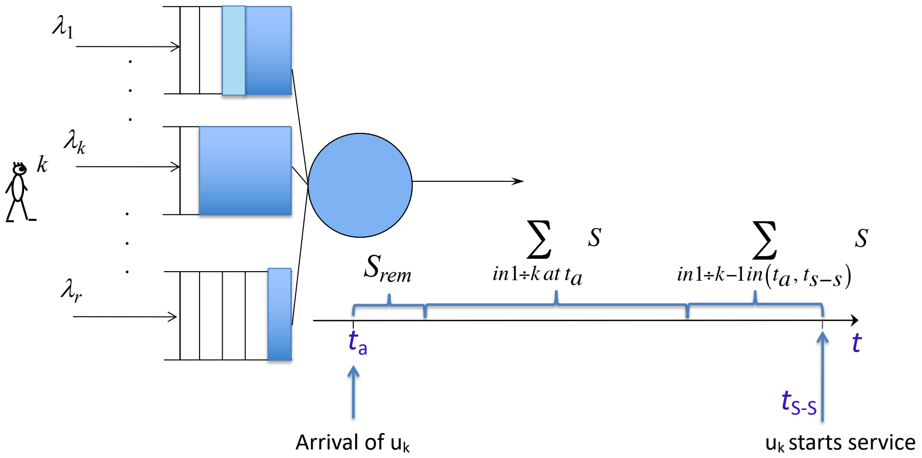
Abstract priority without preemption

local performance measures



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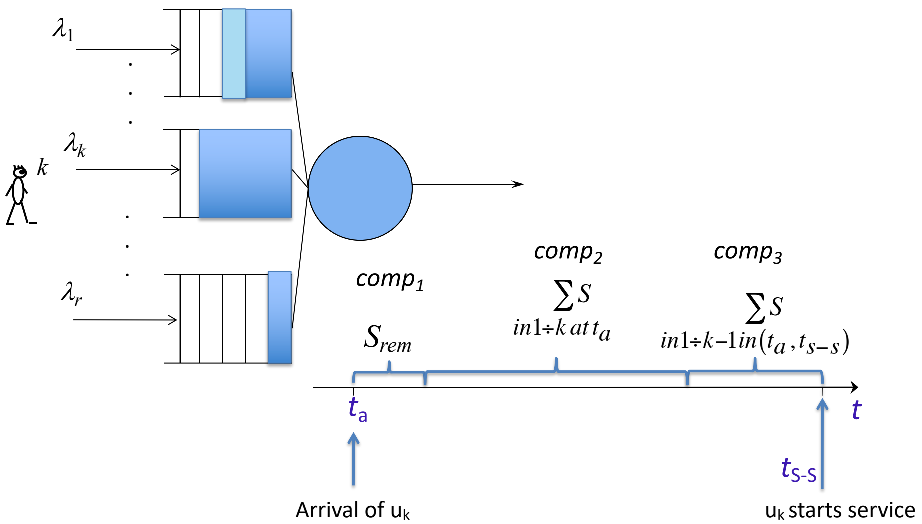
Analytical models
priority scheduling

Abstract priority without preemption

local performance measures



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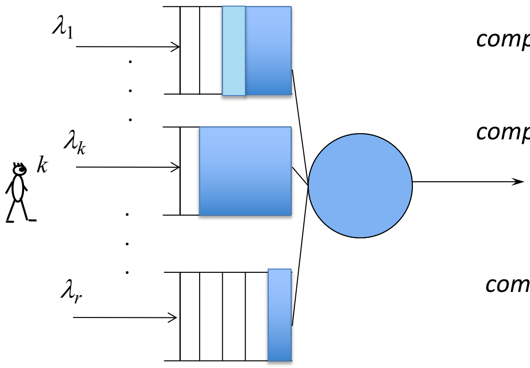
Abstract priority without preemption

local performance measures



comp₁: $E(S_{rem}) = \frac{\lambda}{2} E(S^2)$

comp₂: proportional to the load of queues 1:k $= \frac{1}{1 - \sum_{i=1}^{k} \rho_i}$

comp₃: proportional to the load of queues 1:k-1 $= \frac{1}{1 - \sum_{i=1}^{k-1} \rho_i}$

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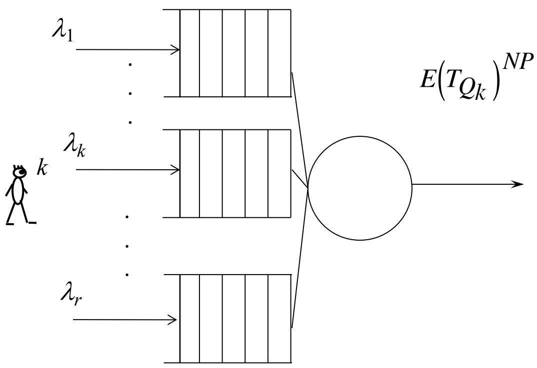
Analytical models
priority scheduling

Abstract priority without preemption

local performance measures



$$E(T_{Qk})^{NP\_priority} = \frac{\frac{\lambda}{2} E(S^2)}{\left(1 - \sum_{i=1}^{k} \rho_i\right)\left(1 - \sum_{i=1}^{k-1} \rho_i\right)}$$

$$E(T_{Qk}) \leq E(T_{Qk+1})$$

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Abstract priority without preemption

local performance measures

$$\boxed{E\left(T_{Q_k}\right) \le E\left(T_{Q_{k+1}}\right)}$$

$$\frac{\frac{\lambda}{2}E\left(S^2\right)}{\left(1-\sum_{i=1}^{k}\rho_i\right)\left(1-\sum_{i=1}^{k-1}\rho_i\right)} \le \frac{\frac{\lambda}{2}E\left(S^2\right)}{\left(1-\sum_{i=1}^{k}\rho_i\right)\left(1-\sum_{i=1}^{k+1}\rho_i\right)}$$

$$\frac{1}{\left(1-\sum_{i=1}^{k}\rho_i\right)\left(1-\sum_{i=1}^{k-1}\rho_i\right)} \le \frac{1}{\left(1-\sum_{i=1}^{k}\rho_i\right)\left(1-\sum_{i=1}^{k+1}\rho_i\right)}$$

$$\left(1-\sum_{i=1}^{k}\rho_i\right)\left(1-\sum_{i=1}^{k-1}\rho_i\right) \ge \left(1-\sum_{i=1}^{k}\rho_i\right)\left(1-\sum_{i=1}^{k+1}\rho_i\right)$$

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Abstract priority without preemption

local performance measures

$$\cancel{\left(1-\sum_{i=1}^{k}\rho_i\right)}\left(1-\sum_{i=1}^{k-1}\rho_i\right) \ge \cancel{\left(1-\sum_{i=1}^{k}\rho_i\right)}\left(1-\sum_{i=1}^{k+1}\rho_i\right)$$

$$\cancel{1}-\sum_{i=1}^{k-1}\rho_i \ge \cancel{1}-\sum_{i=1}^{k+1}\rho_i$$

$$\sum_{i=1}^{k+1}\rho_i \ge \sum_{i=1}^{k-1}\rho_i \qquad \rho_i \ge 0, \forall i$$

$$\boxed{E\left(T_{Q_k}\right) \le E\left(T_{Q_{k+1}}\right)}$$

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Abstract priority without preemption

local performance measures

$$E(T_{S_k}) = E(T_{Q_k}) + E(S) \qquad E(T_{S_k}) \leq E(T_{S_{k+1}})$$

$$E(N_{Q_k}) = \lambda_k E(T_{Q_k})$$

$$E(N_{S_k}) = \lambda_k E(T_{S_k}) \qquad E(N_{S_k}) = E(N_{Q_k}) + \rho_k$$

Abstract priority without preemption

global performance measures

And the “global” performance?

$$E(T_Q)^{NP\_priority} = E(E(T_{Q_k})) = \sum_{k=1}^{r} p_k E(T_{Q_k})$$

$$p_k = \frac{\lambda_k}{\lambda}$$

and similarly for $E(T_S)^{NP\_priority}$

$$E(T_S)^{NP\_priority} = E(T_Q)^{NP\_priority} + E(S)$$

Abstract priority without preemption

$$\lambda_k = p_k \lambda$$

$$\rho_k = \lambda_k E(S) = p_k \lambda E(S) = p_k \rho$$

priority vs no-priority

How are the performance improved in respect of a simple abstract scheduling not-considering the priority classes?

$$E(T_{Q_k})^{NP\_priority} = \frac{\frac{\lambda}{2}E(S^2)}{\left(1-\sum_{i=1}^{k}\rho_i\right)\left(1-\sum_{i=1}^{k-1}\rho_i\right)} \quad ? \quad E(T_Q)^{KP} = \frac{\frac{\lambda}{2}E(S^2)}{1-\rho}$$

The highest priority class:

$$E(T_{Q_1})^{NP\_priority} = \frac{\frac{\lambda}{2}E(S^2)}{(1-\rho_1)} \leq E(T_Q)^{KP}$$

priority vs no-priority

How are the performance improved in respect of a simple abstract scheduling not-considering the priority classes?

The lowest priority class:

$$E\left(T_{Q_r}\right)^{NP\_priority} = \frac{\frac{\lambda}{2}E\left(S^2\right)}{(1-\rho)\left(1-\sum_{i=1}^{r-1}\rho_i\right)} \geq E\left(T_Q\right)^{KP}$$

And what about the “global” performance?

$$E\left(T_Q\right)^{NP\_priority} = E\left(T_Q\right)^{KP}$$

$$\Downarrow$$

$$E\left(T_S\right)^{NP\_priority} = E\left(T_S\right)^{KP}$$

Analytical models
priority scheduling

priority vs no-priority

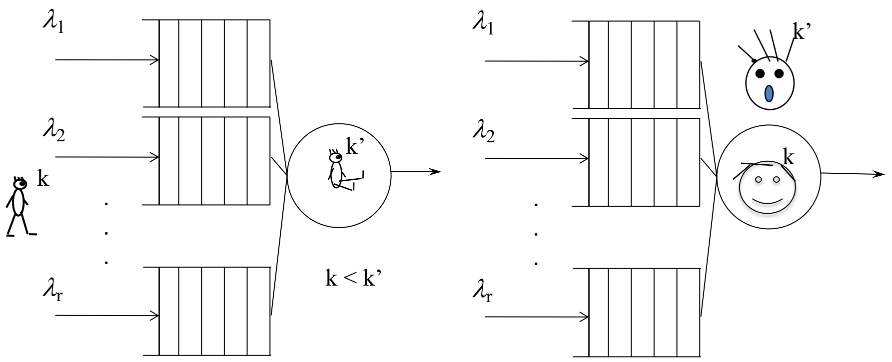
$$E\left(T_Q\right)^{NP\_priority} = E\left(E\left(T_{Q_k}\right)\right) = \sum_{k=1}^{r} p_k E\left(T_{Q_k}\right) = E\left(T_Q\right)^{KP}$$

$r=2$

$$E\left(T_Q\right) = p_1 E\left(T_{Q_1}\right) + p_2 E\left(T_{Q_2}\right) = p_1 \frac{\frac{\lambda}{2}E\left(S^2\right)}{(1-\rho_1)} + p_2 \frac{\frac{\lambda}{2}E\left(S^2\right)}{(1-\rho)(1-\rho_1)}$$

$$= \frac{\lambda}{2}E\left(S^2\right)\left[\frac{p_1}{(1-\rho_1)} + \frac{p_2}{(1-\rho)(1-\rho_1)}\right] = \frac{\lambda}{2}E\left(S^2\right)\frac{p_1(1-\rho)+p_2}{(1-\rho)(1-\rho_1)} = \frac{\frac{\lambda}{2}E\left(S^2\right)}{1-\rho}$$

Preemptive Priority

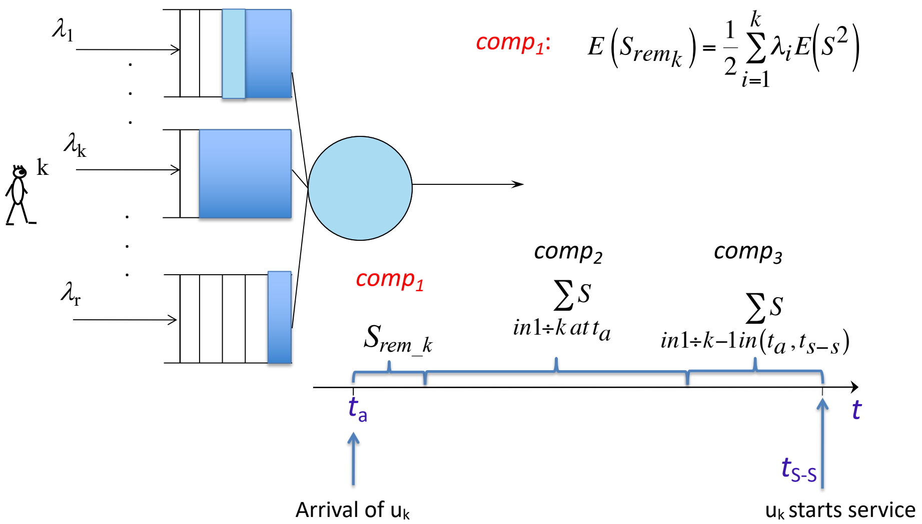


Preemption without loss:
the service of the interrupted job will be resumed from the interruption point

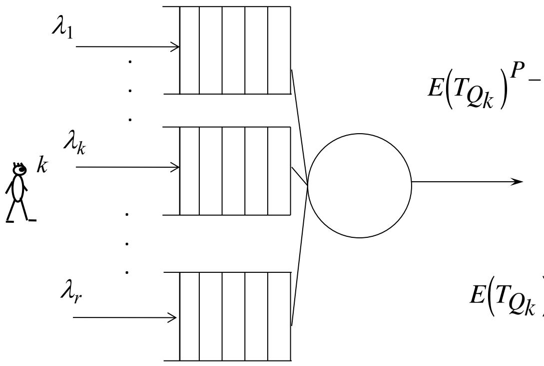
Preemptive Priority

local performance measures

comp₁: $E\left(S_{rem_k}\right) = \frac{1}{2}\sum_{i=1}^{k}\lambda_i E\left(S^2\right)$



Preemptive Priority



$$E\left(T_{Q_k}\right)^{P\_priority} = \frac{\frac{1}{2}\sum_{i=1}^{k}\lambda_i E\left(S^2\right)}{\left(1-\sum_{i=1}^{k}\rho_i\right)\left(1-\sum_{i=1}^{k-1}\rho_i\right)}$$

$$E\left(T_{Q_k}\right)^{P\_priority} \le E\left(T_{Q_{k+1}}\right)^{P\_priority}$$

$$E\left(T_{Q_k}\right)^{P\_priority} \le E\left(T_{Q_k}\right)^{NP\_priority}$$

$$E\left(T_{Q}\right)^{P\_priority} \le E\left(T_{Q}\right)^{NP\_priority} = E\left(T_{Q}\right)^{KP}$$

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Preemptive Priority

$$\frac{\frac{1}{2}\sum_{i=1}^{k}\lambda_i E\left(S^2\right)}{\left(1-\sum_{i=1}^{k}\rho_i\right)\left(1-\sum_{i=1}^{k-1}\rho_i\right)} \le \frac{\frac{1}{2}\sum_{i=1}^{k+1}\lambda_i E\left(S^2\right)}{\left(1-\sum_{i=1}^{k}\rho_i\right)\left(1-\sum_{i=1}^{k+1}\rho_i\right)}$$

$$\frac{\sum_{i=1}^{k}\lambda_i}{\left(1-\sum_{i=1}^{k}\rho_i\right)\left(1-\sum_{i=1}^{k-1}\rho_i\right)} \le \frac{\sum_{i=1}^{k+1}\lambda_i}{\left(1-\sum_{i=1}^{k}\rho_i\right)\left(1-\sum_{i=1}^{k+1}\rho_i\right)}$$

$$\frac{\sum_{i=1}^{k}\lambda_i}{\left(1-\sum_{i=1}^{k}\rho_i\right)\left(1-\sum_{i=1}^{k-1}\rho_i\right)} \le \frac{\sum_{i=1}^{k+1}\lambda_i}{\left(1-\sum_{i=1}^{k}\rho_i\right)\left(1-\sum_{i=1}^{k-1}\rho_i\right)}$$

$$\left(1-\sum_{i=1}^{k}\rho_i\right)\left(1-\sum_{i=1}^{k-1}\rho_i\right) \ge \left(1-\sum_{i=1}^{k}\rho_i\right)\left(1-\sum_{i=1}^{k+1}\rho_i\right)$$

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Preemptive Priority

$$E(T_Q)^{X\_priority} = E\left(E(T_{Qk})\right) = \sum_{k=1}^{r} p_k E(T_{Qk})$$
$$= p_1 E(T_{Q1}) + p_2 E(T_{Q2}) + \ldots + p_r E(T_{Qr})$$

$$E(T_Q)^{NP\_priority} = p_1 E(T_{Q1}) + p_2 E(T_{Q2}) + \ldots + p_r E(T_{Qr})$$

$$E(T_Q)^{P\_priority} = p_1 E(T_{Q1}) + p_2 E(T_{Q2}) + \ldots + p_r E(T_{Qr})$$

$$E(T_Q)^{P\_priority} \le E(T_Q)^{NP\_priority} = E(T_Q)^{KP}$$

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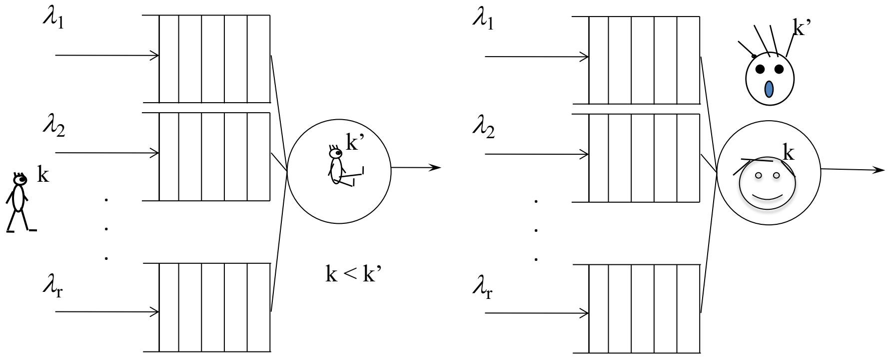
Analytical models
priority scheduling

Preemptive Priority

Due to preemption, the service time of a job of class k may increase for the arrivals of higher priority classes



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Preemptive Priority

Due to preemption, the service time of a job of class k may increase for the arrivals of higher priority classes

Virtual service time $E(S_{virt\_k})$

$$E(S_{virt\_k}) = \frac{E(S)}{1-\sum_{i=1}^{k-1}\rho_i}$$

$$E(T_{S_k})^{P\_priority} = E(T_{Q_k})^{P\_priority} + E(S_{virt\_k})$$

$$\wedge| \qquad\qquad |\vee \qquad ?$$

$$E(T_{S_k})^{NP\_priority} = E(T_{Q_k})^{NP\_priority} + E(S)$$

Preemptive Priority

global response time

$$\boxed{E(T_{Q_k})^{P\_priority} + E(S_{virt\_k})}$$

And what about the "global" performance?

$$E(T_S)^{P\_priority} = E(E(T_{S_k})) = \sum_{k=1}^{r} p_k E(T_{S_k})$$

$$E(T_S)^{P\_priority} = \sum_{k=1}^{r} p_k\left[E(T_{Q_k}) + E(S_{virt\_k})\right]$$

$$= \sum_{k=1}^{r} p_k E(T_{Q_k}) + \sum_{k=1}^{r} p_k E(S_{virt\_k})$$

$$= E(T_Q)^{P\_priority} + \sum_{k=1}^{r} p_k E(S_{virt\_k})$$

Analytical models
priority scheduling

preemption vs no-preemption

$$E(T_S)^{P\_priority} = E(T_Q)^{P\_priority} + \sum_{k=1}^{r} p_k E(S_{virt\_k})$$

$\wedge\!|$ $\qquad$ $|\!\vee$

$$E(T_S)^{NP\_priority} = E(T_Q)^{NP\_priority} + E(S) = E(T_S)^{KP}$$

?

In general

$$E(T_S)^{P\_priority} \quad ? \quad E(T_S)^{KP}$$

For exponential service time

$$E(T_S)^{P\_priority} = E(T_S)^{KP}$$

!!!

Analytical models
priority scheduling

preemption vs no-preemption

$r = 2$

$$E(T_S)^{P\_priority} = p_1 E\left(T_{S_1}\right) + p_2 E\left(T_{S_2}\right)$$

$$= p_1 \left[\frac{\frac{\lambda_1}{2} E\left(S^2\right)}{(1-\rho_1)} + E(S) \right] + p_2 \left[\frac{\frac{\lambda}{2} E\left(S^2\right)}{(1-\rho)(1-\rho_1)} + \frac{E(S)}{1-\rho_1} \right]$$

from the expo assumption

$$= p_1 \left[\frac{\rho_1 E(S)}{(1-\rho_1)} + E(S) \right] + p_2 \left[\frac{\rho E(S)}{(1-\rho)(1-\rho_1)} + \frac{E(S)}{1-\rho_1} \right]$$

$$= E(S) \left\{ p_1 \left[\frac{\rho_1 + 1 - \rho_1}{(1-\rho_1)} \right] + p_2 \left[\frac{\rho + (1-\rho)}{(1-\rho)(1-\rho_1)} \right] \right\}$$

preemption vs no-preemption

$r = 2$

$$E(T_S)^{P\_priority} = p_1 E\left(T_{S_1}\right) + p_2 E\left(T_{S_2}\right)$$

$$= E(S)\left[\frac{p_1}{(1-\rho_1)} + \frac{p_2}{(1-\rho)(1-\rho_1)}\right]$$

$$= E(S)\frac{p_1(1-\rho) + p_2}{(1-\rho)(1-\rho_1)} \quad = \frac{E(S)}{1-\rho} \quad = E(T_S)^{KP}$$