

# Performance Modeling of Computer Systems and Networks

*Prof. Vittoria de Nitto Personè*

## Multiserver and Priority scheduling

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Analytical models  
priority scheduling

### Assumptions:

- Arrival rate  $\lambda$  j/s random
- Average demand  $Z=4 \times 10^5$  oxat, expo, do not know size

### Possible configurations:

- 1 server of capacity  $C=10^6$  oxat/s
- Dual-core of  $C/2$  each one

### QoS requirements:

- Average waiting  $T_Q < 0.15$  s
- For at least 35% of arrivals average response time  $T_S < 0.5$  s

*Def.*

$$E(S) = Z/C = 0.4 \text{ s}$$

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## QoS requirements:

- Average waiting  $T_Q < 0.15$  s

Analytical models  
priority scheduling

$$\lambda = 1 \text{ j/s}, E(S) = 0.4 \text{ s} \quad \longrightarrow \quad \rho = 0.4$$

- 1 server of capacity  $C=10^6$  owerat/s

$$E(T_Q) = 0.26 \text{ s}$$

$$E(T_Q)^{\text{Abstract-P}} = 0.2243 \text{ s}$$

- Dual-core of  $C/2$  each one

$$E(S_i) = \frac{Z}{\frac{C}{2}} = 2 \frac{Z}{C} = 2E(S) = 0.8 \text{ s}$$

$$E(T_Q)_{\text{Erlang}} = \frac{P_Q E(S)}{1 - \rho} = 0.15238 \text{ s}$$

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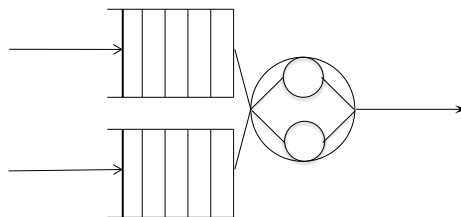
## QoS requirements:

- Average waiting  $T_Q < 0.15$  s

Analytical models  
priority scheduling

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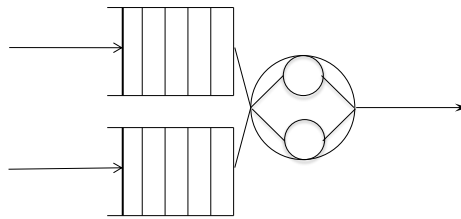


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## Multiserver with priority classes



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Analytical models  
the multiserver queue

## The Erlang formula

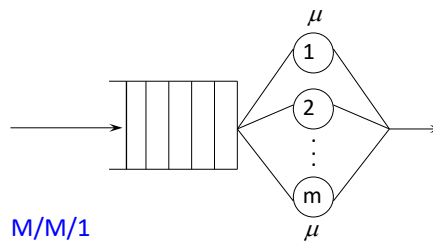
**M/M/m**

$$E(T_Q)_{Erlang} = \frac{P_Q E(S)}{1 - \rho}$$

**M/M/1**

$$E(T_Q)_{KP} = \frac{\rho E(S)}{1 - \rho} = \frac{E(S_{rem})}{1 - \rho}$$

$$E(S) = \frac{E(S_i)}{m}$$



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### Multiserver with priority classes

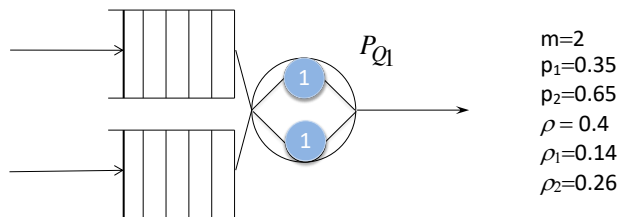
$$E(T_Q) = p_1 \frac{\rho_1 E(S)}{(1-\rho_1)} + p_2 \frac{\rho E(S)}{(1-\rho)(1-\rho_1)}$$



$$E(T_Q) = p_1 \frac{P_{Q1} E(S)}{(1-\rho_1)} + p_2 \frac{P_Q E(S)}{(1-\rho)(1-\rho_1)}$$

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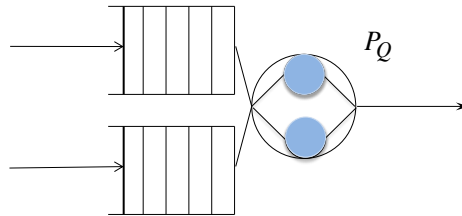
### Multiserver with priority classes



$$P_{Q1} = \text{Erlang}(\rho_1) = 0.03438$$

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## Multiserver with priority classes



$$P_{Q1} = \text{Erlang}(\rho_1) = 0.03438 \quad P_Q = 0.22857$$

$$E(T_Q) = p_1 \frac{P_{Q1} E(S)}{(1-\rho_1)} + p_2 \frac{P_Q E(S)}{(1-\rho)(1-\rho_1)} = 0.12077$$

QoS requirements:

- Average waiting  $T_Q < 0.15$  s !!

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QoS requirements:

- For at least 35% of arrivals average response time  $T_S < 0.5$  s

Analytical models  
priority scheduling

$$\lambda = 1 \text{ j/s}, E(S) = 0.4 \text{ s} \quad \longrightarrow \quad \rho = 0.4$$

- 1 server of capacity  $C=10^6$  operat/s

$$E(T_Q) = 0.26 \text{ s}$$

- Dual-core of  $C/2$  each one

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## QoS requirements:

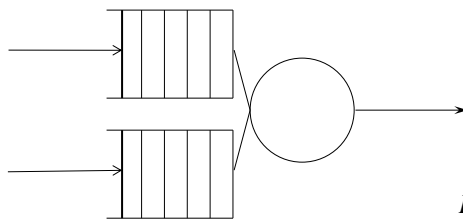
- For at least 35% of arrivals average response time  $T_S < 0.5$  s

Analytical models  
priority scheduling

$$\lambda = 1 \text{ j/s}, E(S) = 0.4 \text{ s} \quad \Rightarrow \quad \rho = 0.4$$

- 1 server of capacity  $C=10^6$  operat/s

## Abstract-P



$$\begin{aligned} p_1 &= 0.35 \\ p_2 &= 0.65 \\ \rho &= 0.4 \\ \rho_1 &= 0.14 \\ \rho_2 &= 0.26 \end{aligned}$$

$$E(T_{S1}) = 0.4651162$$

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