



Random Number Generators

- ssq1 and sis1 require input data from an outside source
- The usefulness of these programs is limited by amount of available data:
 - What if more data needed?
 - What if the model changed?
 - What if the input data set is small or unavailable?

Random number generator

- It produces real values between 0.0 and 1.0
- The output can be converted to random variate via mathematical transformations

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Performance Modeling of Computer Systems and Networks

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Random Number Generators

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Random Number Generators

Historically there are three types of generators

- table look-up generators (1950)
- hardware generators
- algorithmic (software) generators

Algorithmic generators are widely accepted because they meet all of the following criteria:

- randomness output passes all reasonable statistical tests of randomness
- controllability able to reproduce output, if desired
- portability able to produce the same output on a wide variety of computer systems
- efficiency fast, minimal computer resource requirements
- documentation theoretically analyzed and extensively tested

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Random Number Generators

Algorithmic Generators

- An ideal random number generator produces output such that each value in the interval 0.0 < u < 1.0 is equally likely to occur
- A *good* random number generator produces output that is (almost) statistically indistinguishable from an ideal generator

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Random Number Generators

Conceptual Model

- Choose a *large* positive integer m>0. This defines the set $\chi_m = \{1,2,...m-1\}$
- Fill a (conceptual) urn with the elements of χ_m
- Each time a random number u is needed, draw an integer x "at random" from the urn and let u =x/m
- Each draw *simulates* a sample of an independent identically distributed sequence of *Uniform*(0, 1)
- The possible values are 1/m, 2/m, ... (m-1)/m
- It is important that *m* be large so that the possible values are densely distributed between 0.0 and 1.0

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Random Number Generators

Conceptual Model

- 0.0 and 1.0 are impossible

 This is important for some random variates
- for practical reasons, we will draw without replacement

 If *m* is large and the number of draws is small relative
 to *m*, then the distinction is largely irrelevant

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Random Number Generators Lehmer Generators

Lehmer Generator

- is defined in terms of two fixed parameters:
 - modulus m, a fixed large prime integer
 - multiplier a, a fixed integer in χ_m
- the possible values are 1/m, 2/m, ... (m-1)/m

The integer sequence $x_0,\,x_1,\,\dots$ is defined by the iterative equation

$$x_{i+1} = g(x_i)$$

with

 $g(x) = ax \mod m$

 $x_0 \in \chi_m$ is called *initial seed*

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Random Number Generators Lehmer Generators

- Because of the mod operator, $0 \le g(x) < m$
- 0 must not occur
 - since m is prime, $g(x) \neq 0$ if $x \in \chi_m$
 - if $x_0 \in \chi_m$, then $x_i \in \chi_m$ for all $i \ge 0$
- IF the multiplier and prime modulus are chosen properly, a Lehmer generator is statistically indistinguishable from drawing from χ_m with replacement
- NOTE, there is nothing random about a Lehmer generator

pseudo-random generator

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Random Number Generators *Lehmer Generators*

Parameter Considerations

- the choice of *m* is dictated, in part, by system considerations
 - on a system with 32-bit 2's complement integer arithmetic, 2³¹-1 is a natural choice (it is prime!)
 - with 16-bit or 64-bit integer representation, the choice is not obvious (the maxes are not prime)
 - in general, we want to choose m to be the largest representable prime integer
- Given m, the choice of a must be made with great care

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Random Number Generators Lehmer Generators

- For a chosen (a, m) pair, does the function g(·) generate a full-period sequence?
- If a full period sequence is generated, how random does the sequence appear to be?
- Can ax mod m be evaluated efficiently and correctly?
 - Integer overflow can occur when computing ax

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Random Number Generators Lehmer Generators

Full Period Multipliers

- If we pick any initial seed $x_0 \in \chi_m$ and generate the sequence x_0 , x_1, x_2, \ldots then x_0 will occur again
- Further x_0 will reappear at index p that is either m-1 or a divisor of m-1

We are interested in choosing full-period (FP) multipliers where p = m-1

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