Performance Modeling
of Computer Systems and Networks

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Performance Sensitivity to the Service time distribution

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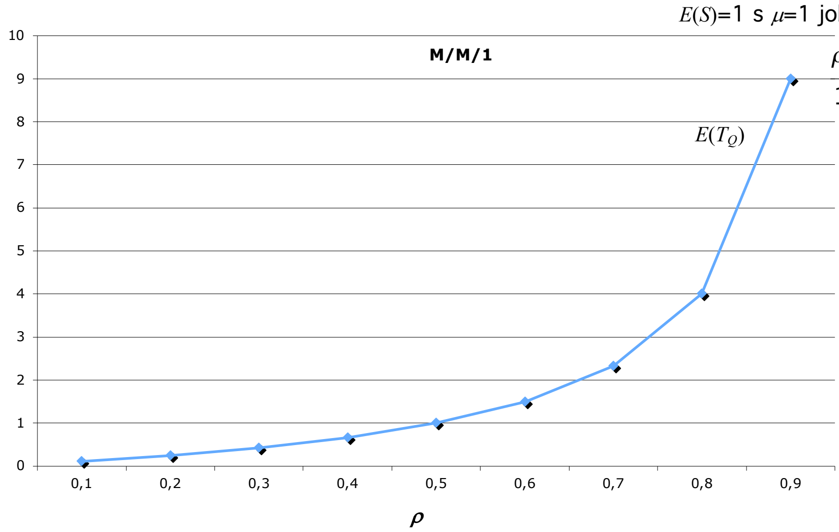
Analytical models
M/G/1

Expected waiting time as load grows

$E(S)$=1 s μ=1 job/s

$$\frac{\rho E(S)}{1-\rho}$$



What's happen if the service time distribution changes?

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The Khinchin Pollaczek equation (KP)

$$E(N_Q) = \frac{\rho^2}{2(1-\rho)}\left[1 + C^2\right], \qquad E(T_Q) = \frac{\rho}{1-\rho}\frac{C^2+1}{2}E(S)$$

$$C^2(S) = \frac{\sigma^2(S)}{E^2(S)}$$

Expected waiting time in an M/G/1 queue can be huge, even under very low utilization ρ, if C^2 is huge.

D $\longrightarrow$ $C^2=0$

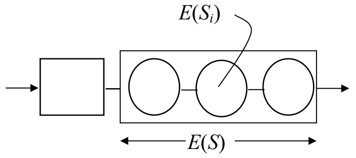
M $\longrightarrow$ $C^2=1$

$E_k \longrightarrow C^2 = \frac{1}{k}$

$H_2 \longrightarrow C^2 = g(p) = \frac{1}{2p(1-p)} - 1$

Expected waiting time as load grows: Erlang case

$E(S)$=0.5 s μ=2 job/s



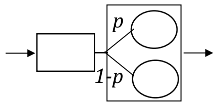
$$E(S_i) = \frac{0.5}{3} = 0.166666666 \quad \text{s}$$

$$\sigma^2(S) = \frac{1}{k}\left(\frac{1}{\mu}\right)^2 = 0.0833333$$

Expected waiting time as load grows: Hyperexponential case

$E(S)$=0.5 s μ=2 job/s

$2p\mu$=0.8 job/s



p=0.2

$2(1-p)\mu$=3.2 job/s

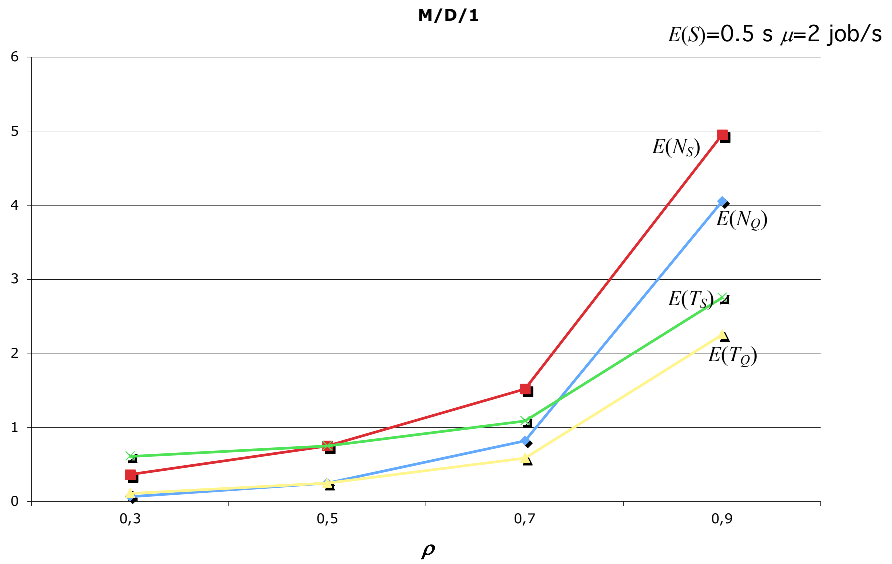
$$\sigma^2(S) = g(p)\left(\frac{1}{\mu}\right)^2 = 0.53125 \qquad g(p) = \frac{1}{2p(1-p)} - 1 = 2.125$$

The Khinchin Pollaczek equation (KP)

$$g(p) = \frac{1}{2p(1-p)} - 1 \qquad E(N_Q) = \frac{\rho^2}{2(1-\rho)}\left[1 + C^2\right], \quad E(T_Q) = \frac{\rho}{1-\rho}\,\frac{C^2+1}{2}\,E(S)$$

Service time	$E(N_Q)$	$E(T_Q)$
Determinisctic, M/D/1	$\frac{\rho^2}{2(1-\rho)}$	$\frac{\rho E(S)}{2(1-\rho)}$
Markovian, M/M/1	$\frac{\rho^2}{1-\rho}$	$\frac{\rho E(S)}{1-\rho}$
K-Erlang, M/E$_k$/1 $\sigma^2(S) = \frac{E(S)^2}{k}$	$\frac{\rho^2}{2(1-\rho)}\left(1+\frac{1}{k}\right)$	$\frac{\rho E(S)}{2(1-\rho)}\left(1+\frac{1}{k}\right)$
Hyperexpo, M/H$_2$/1 $\sigma^2(S) = E(S)^2 g(p)$	$\frac{\rho^2}{2(1-\rho)}\left(1+g(p)\right)$	$\frac{\rho E(S)}{2(1-\rho)}\left(1+g(p)\right)$

Expected waiting time as load grows: Deterministic case



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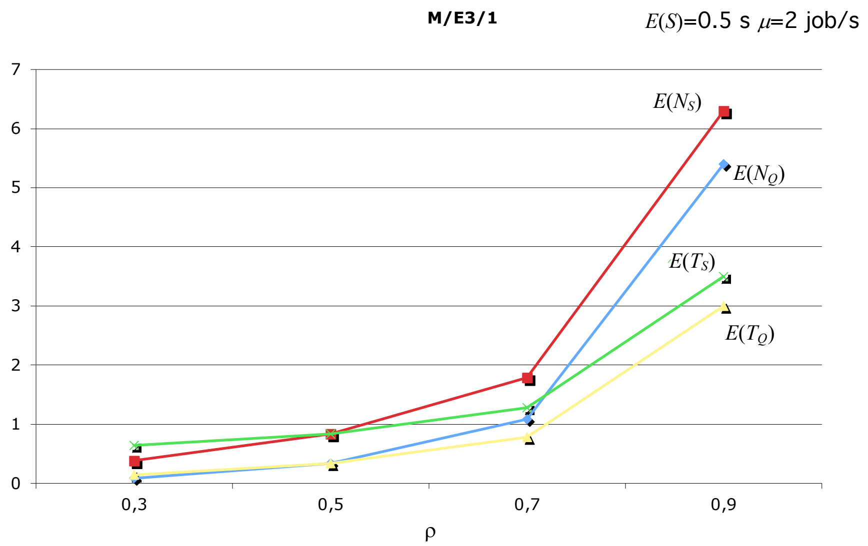
Expected waiting time as load grows: Erlang case



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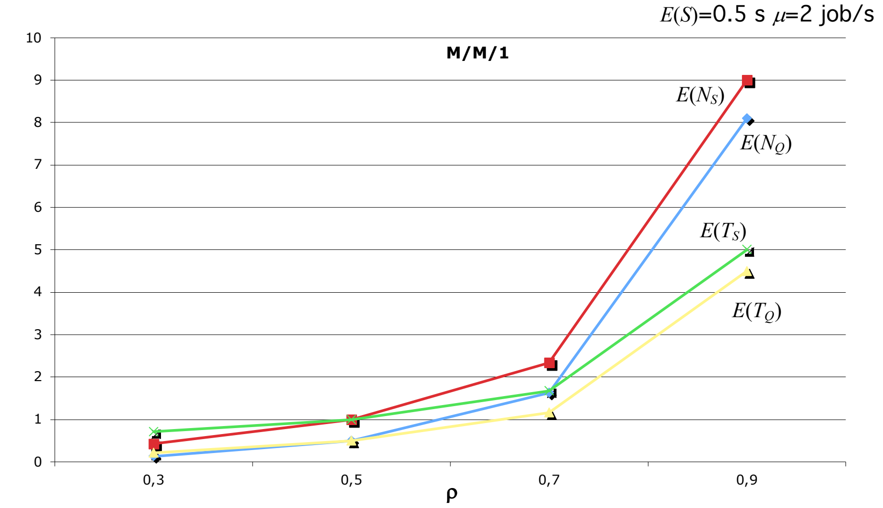
Expected waiting time as load grows: Exponential case

$E(S)$=0.5 s μ=2 job/s



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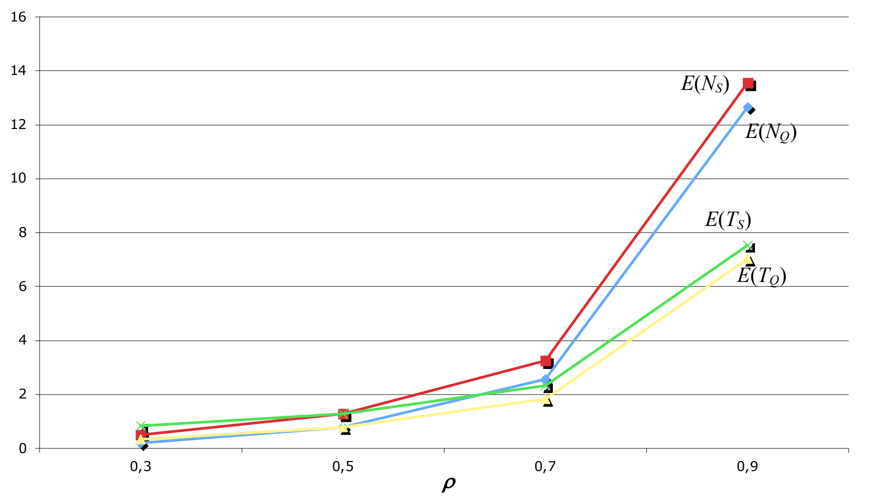
Expected waiting time as load grows: Hyperexponential case

M/H2/1

$E(S)$=0.5 s μ=2 job/s



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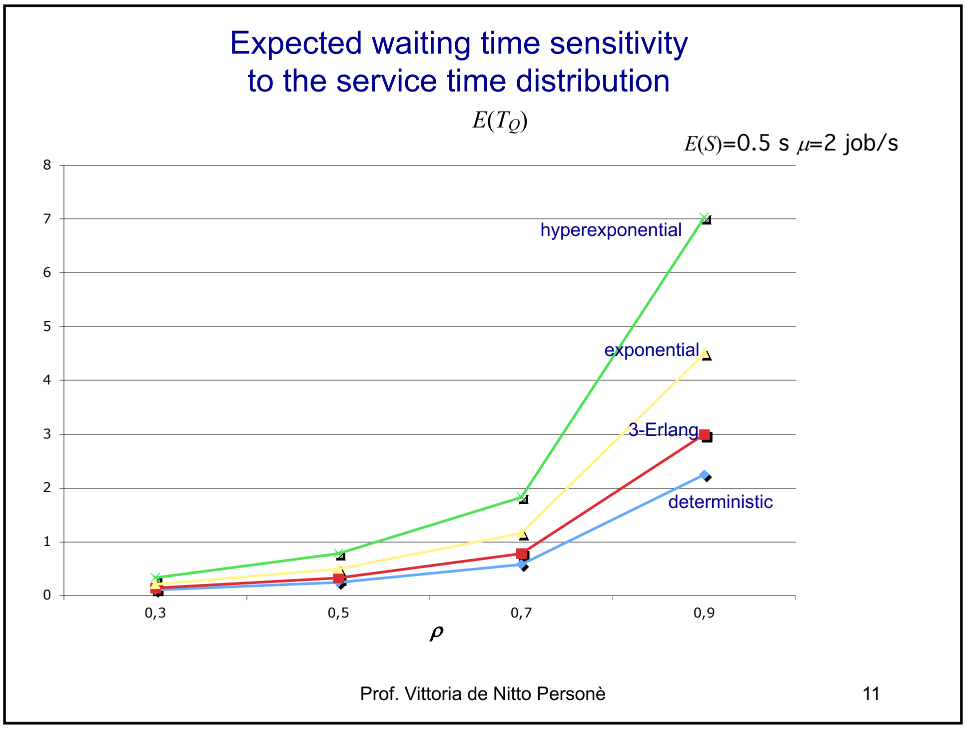

Expected waiting time sensitivity
to the service time distribution
$E(T_Q)$
$E(S)$=0.5 s μ=2 job/s
8
7
6
5
4
3
2
1
0
hyperexponential
exponential
3-Erlang
deterministic
0,3
0,5
0,7
0,9
ρ
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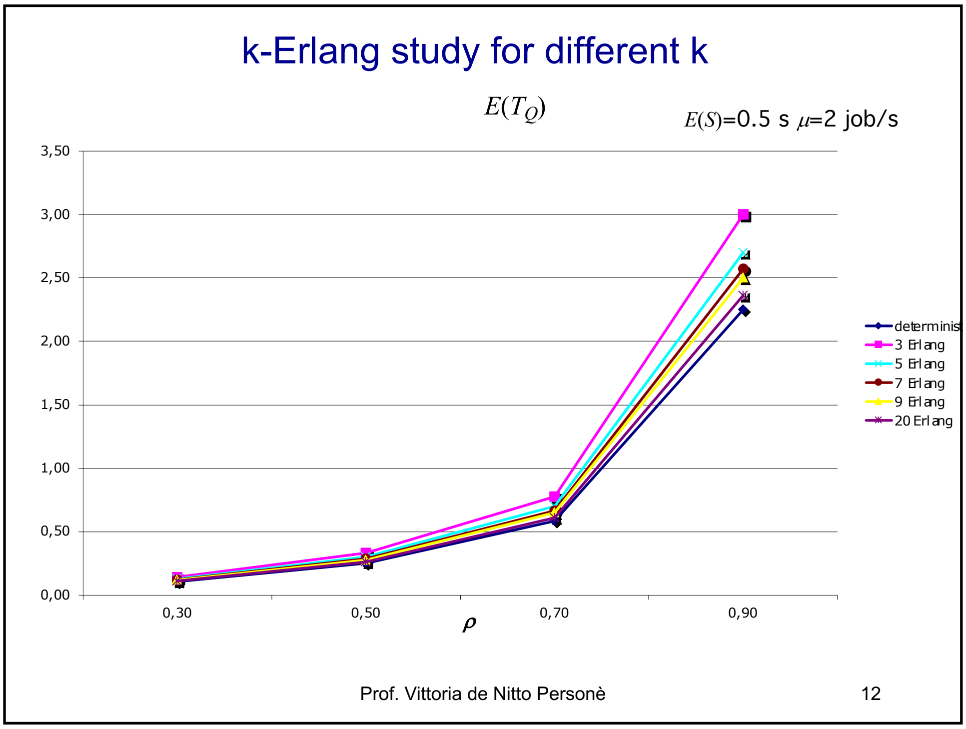

k-Erlang study for different k
$E(T_Q)$
$E(S)$=0.5 s μ=2 job/s
3,50
3,00
2,50
2,00
1,50
1,00
0,50
0,00
0,30
0,50
0,70
0,90
ρ
determinis
3 Erlang
5 Erlang
7 Erlang
9 Erlang
20 Erlang
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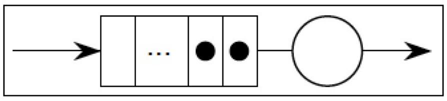
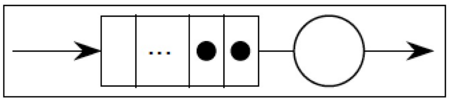
A TP system accepts and processes a stream of transactions, mediated through a (large) buffer:



- Transactions arrive "randomly" at some specified rate
- The TP server is capable of servicing transactions at a given service *rate*

Q: If both the arrival rate and service rate are doubled, what happens to the mea response time?



- The arrival rate is 15tps
- The mean service time per transaction is 58.37ms

Q: What happens to the mean response time if the arrival rate increases by 10%?

$$E(T_Q) = \frac{\rho}{1-\rho}\frac{C^2+1}{2}E(S)$$

$$E(T_{Q'}) = \frac{\rho'}{1-\rho'}\frac{C^2+1}{2}E(S)$$

$$\frac{E(T_Q)}{E(T_{Q'})} \cong 0,27 \cong \frac{1}{3,7}$$

Heavy tail distributions properties

esponenziale $\longrightarrow$ memoryless
failure rate costante

Heavy tail $\longrightarrow$ failure rate decrescente

(Pareto: $r(x) = \alpha / x$, $x > 1$)

Where they are

Jobs Unix

Sizes files websites $\alpha \approx 1.1$

Internet topology

Packet n° IP flows 1% → 50%

Natural phenomena

Pareto

$$f(x) = \alpha k^{\alpha} x^{-\alpha-1} \quad k \le x < \infty$$

α , parametro di forma

$$E[X] = \frac{\alpha k}{\alpha - 1} \quad \alpha > 1$$

$$\sigma^2[X] = \frac{\alpha k^2}{(\alpha-1)^2(\alpha-2)} \quad \alpha > 2$$

Bounded Pareto

$$f(x) = \alpha x^{-\alpha-1} \frac{k^{\alpha}}{1-\left(\frac{k}{p}\right)^{\alpha}} \quad k \le x \le p, 0 < \alpha < 2$$

(Vilfredo Pareto, 15 July 1848 – 19 August 1923, economista e sociologo)

Pareto

$$E(T_Q) = \frac{\rho}{1-\rho}\frac{C^2+1}{2}E(S)$$

$$C^2(S) = \frac{\sigma^2(S)}{E^2(S)}$$

$$E[T_Q] = \frac{\rho E[S]}{1-\rho}\frac{1+\alpha(\alpha-2)}{2\alpha(\alpha-2)}$$

$\alpha > 2$

Pareto study as load grows

$E(S)$=0.5 s μ=2 job/s

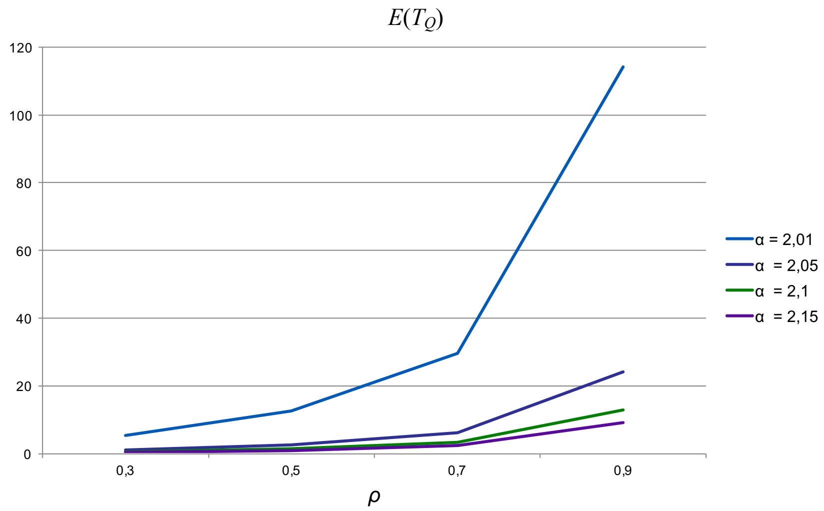
$E(T_Q)$

ro list	α = 2,01	α = 2,05	α = 2,1	α = 2,15	determ	3-Erlang	expo	hyper
0,3	5,437633262	1,152439024	0,617346939	0,439368771	0,107	0,142	0,213	0,333
0,5	12,68781095	2,68902439	1,44047619	1,025193798	0,25	0,333	0,5	0,781
0,7	30	6,274390244	3,361111111	2,392118863	0,583	0,778	1,167	1,823
0,9	114,1902985	24,20121951	12,96428571	9,226744186	2,25	3	4,5	7,031

k=0.2512 k=0.2619

$$E[S] = \frac{\alpha k}{\alpha-1} \Rightarrow k = \frac{\alpha-1}{\alpha}E[S]$$

Pareto study as load grows



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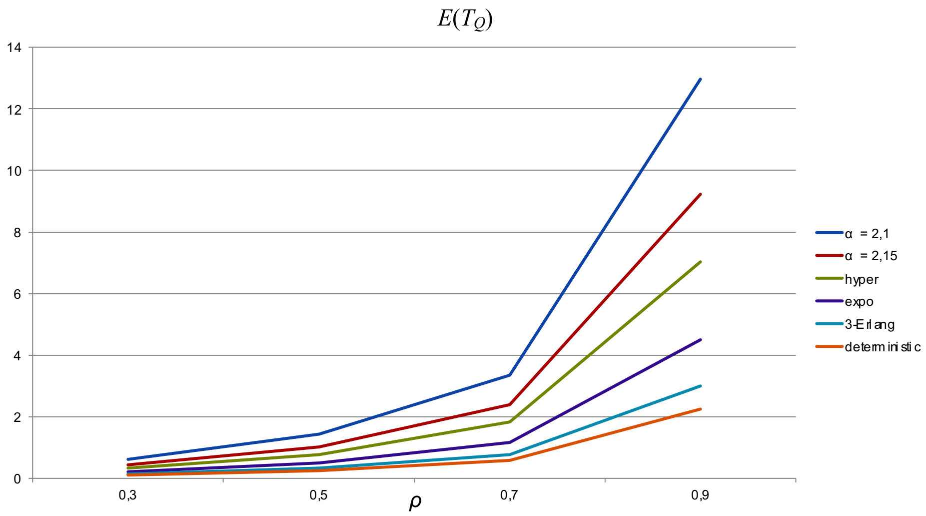
Pareto study as load grows



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