Performance Modeling of Computer Systems and Networks

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Discrete-Event Simulation

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Multi-Stream Lehmer RNGs

- Typical DES models have many stochastic components
- Want a unique source of randomness for each component
- One (poor) option: multiple RNGs
- Better option: one RNG with multiple "streams" of random numbers
  one stream per stochastic component

We will partition output from our Lehmer RNG into multiple streams

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Case study ssq

Arrival and service processes

- two stochastic components: arrival and service
- allocate a different state variable to each

```
double GetService(void)
{
        return Uniform(1.0, 2.0);
}
```

```
double GetService(void)
{
        double s;
        static long x = 12345;
        PutSeed(x);
        s = Uniform(1.0, 2.0);
        GetSeed(&x);
        return (s);
}
```

- x represents the current state of the service process

Case study ssq

Arrival and service processes

- Arrival should have its own static variable, initialized differently

```
double GetArrival(void)
{
        static double arrival = START;
        arrival += Exponential(2.0);
        return (arrival);
}
```

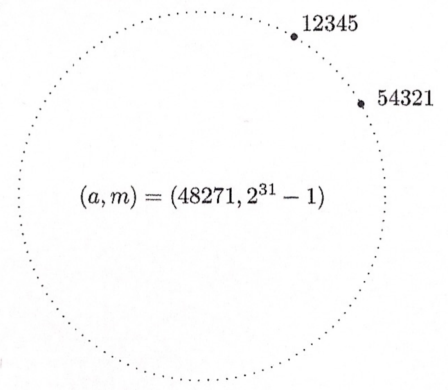
```
double GetArrival(void)
{
        static double arrival = START;
        static long x = 54321;
        PutSeed(x);
        arrival += Exponential(2.0);
        GetSeed(&x);
        return (arrival);
}
```

- x represents the current state of arrival process

The Modified Arrival and Service Processes

- As modified, arrival and service times are drawn from different streams of random numbers
- Provided the streams don't overlap → the processes are *uncoupled*
- Execution time cost is negligible

- Potential problem: assignment of initial seeds to produce *disjoint* streams
- If states are picked at whim, no guarantee of disjoint streams
- Some initial states may only be a few calls to Random apart!



Jump Multipliers

- We will develop a multi-stream version of rng

Theorem

Given $g(x) = ax \bmod m$ and integer j (1<j<m-1)

jump function: $g^j(x) = \left(a^j \; mod \; m\right) x \; mod \; m$

jump multiplier: $a^j \; mod \; m$

If *g(·)* generates $x_0, x_1, x_2, \ldots$ then $g^j(\cdot)$ generates $x_0, x_j, x_{2j}, \ldots$

- This theorem is the key to creating streams

Example 1

- If *m* = 31, *a* = 3 and *j* = 6, the jump multiplier is

$$a^j \; mod \; m = 3^6 \; mod \; 31 = 16$$

- If x_0 = 1, then $g(x) = 3x \; mod \; 31$ generates:

1, 3, 9, 27, 19, 26, 16, 17, 20, 29, 25, 13, 8, 24, 10, 30, 28, 22, 4, 12, 5, 15, 14, 11, 2, 6, 18, 23, 7, 21, 1, . . .

- The jump function $g^6(x) = 16x \; mod \; 31$ generates:

1, 16, 8, 4, 2, 1, . . .

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22, 4, 12, 5, 15, 14, 11, 2, 6, 18, 23, 7, 21, 1, . . .

- The jump function $g^6(x) = 16x \; mod \; 31$ generates:

1, 16, 8, 4, 2, 1, . . .

- I.e., the first sequence is $x_0, x_1, x_2, \ldots$; the second is $x_0, x_6, x_{12}, \ldots$

Example 1

m = 31, a = 3, x_0 = 1

1, 3, 9, 27, 19, 26, 16, 17, 20, 29, 25, 13, 8, 24, 10, 30, 28,
22, 4, 12, 5, 15, 14, 11, 2, 6, 18, 23, 7, 21, 1, . . .

x_0 = 1, 3, 9, 27, 19, 26,
x_6 = 16, 17, 20, 29, 25, 13,
x_{12} =8, 24, 10, 30, 28, 22,
x_{18} =4, 12, 5, 15, 14, 11,
x_{24} =2, 6, 18, 23, 7, 21,

- The jump function $g^6(x) = 16x \; mod \; 31$ generates:

1, 16, 8, 4, 2, 1, . . .

Using the jump function

- First, compute the jump multiplier $a^j \ mod\ m$ (one time cost)
- Then, $g^j(\cdot)$ allows jumping from x_0 to x_j to x_{2j} to …
- The user supplies ONE initial seed
- If *j* is chosen well, $g^j(\cdot)$ can "plant" additional initial seeds
- Each planted seed corresponds to a different stream
- Each planted seed is *separated* by j calls to `Random`

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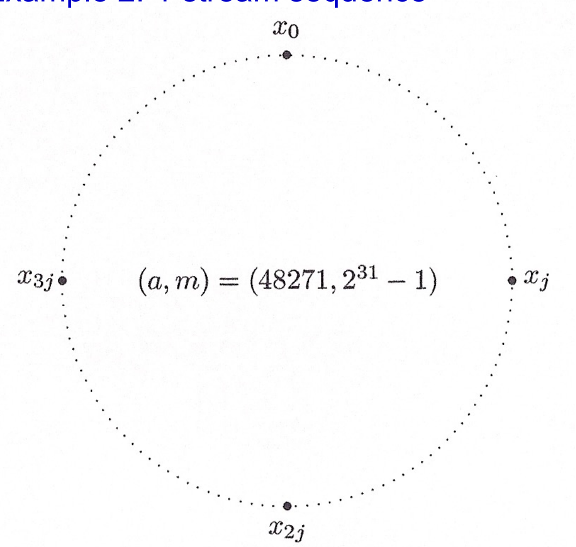
Example 2: 4-stream sequence



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An appropriate jump multiplier

- Consider 256 = 2^8 different streams of random numbers
- Partition the RNG output sequence into 256 disjoint subsequences of equal length
- Find the largest $j < 2^{31}/2^8 = 2^{23}$ such that the jump multiplier is modulus-compatible
- $g^j(x) = \left(48271^j \ mod\ m\right) x \ mod\ m$ can be implemented via algorithm 1 (2.2.1 in the book)
- Then $g^j(x)$ can be used to plant the other 255 initial seeds
- Possibility of stream overlap is minimized (though not eliminated!)

Algorithm 1

```
t = a * (x % q) - r * (x / q);          /* t = γ(x) */
if (t > 0)
        return (t);                     /* δ(x) = 0 */
else
        return (t + m);                 /*  δ(x) = 1 */
```


Maximal Modulus-Compatible Jump Multipliers

- *Maximal jump multiplier*: maximize the distance between streams, a^j mod m where j is the largest integer less than $\lfloor m/s \rfloor$, s number of streams, such that a^j mod m is modulus compatible

Example 2 (cont.)

# of streams s	$\lfloor m/s \rfloor$	jump size j	jump multiplier a^j mod m
1024	2097151	2082675	97070
512	4194303	4170283	44857
256	8388607	8367782	22925
128	16777215	16775552	40509

Library rngs

- Upward-compatible multi-stream replacement for rng
- Provides 256 streams, indexed 0 to 255 (0 is the default)
- Only one stream is active at any time
- 6 available functions:
  - `Random(void)`: to use the standard Lehmer generator
  - `PutSeed(long x)`: to set the state of the active stream
  - `GetSeed(long *x)`: to obtain the state of the active stream
  - `TestRandom(void)`: to test the implementation correctness
  - `SelectStream(int s)`: to define the active stream
  - `PlantSeeds(long x)`: "plants" one seed per stream