Performance Modeling of Computer Systems and Networks

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The model for a service center: analytical results

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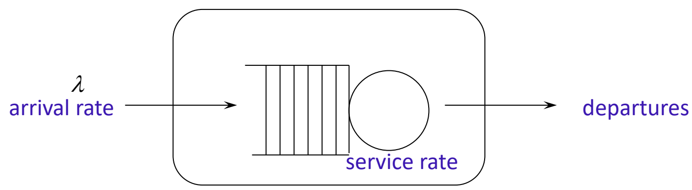
Server center



Little's law

$$E(T_S) = E(T_Q) + E(S) \qquad E(N_S) = \lambda E(T_S)$$

$$E(N_S) = E(N_Q) + \rho \qquad E(N_Q) = \lambda E(T_Q)$$

$$E(T_S) = \frac{E(N_S)}{\lambda}$$

$$E(T_Q) = \frac{E(N_Q)}{\lambda}$$

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The Kendall notation

A/S/m/B/N/D

Interarrival distribution

Service distribution

n° of servers

Queue capacity

Population size

Queue discipline

Distributions: D, M, E_k, H_2, G

Non-preemptive abstract scheduling

FIFO, LIFO-non-preemp, Random

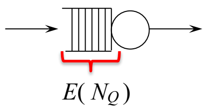
It seems like
- FIFO should have the best mean response time because jobs are serviced most closely to the time they arrive
- LIFO may make a job wait a very long time

all the above policies have exactly the *same* mean response time.

1930: The Khinchin Pollaczek equation (KP)

M/G/1 abstract scheduling

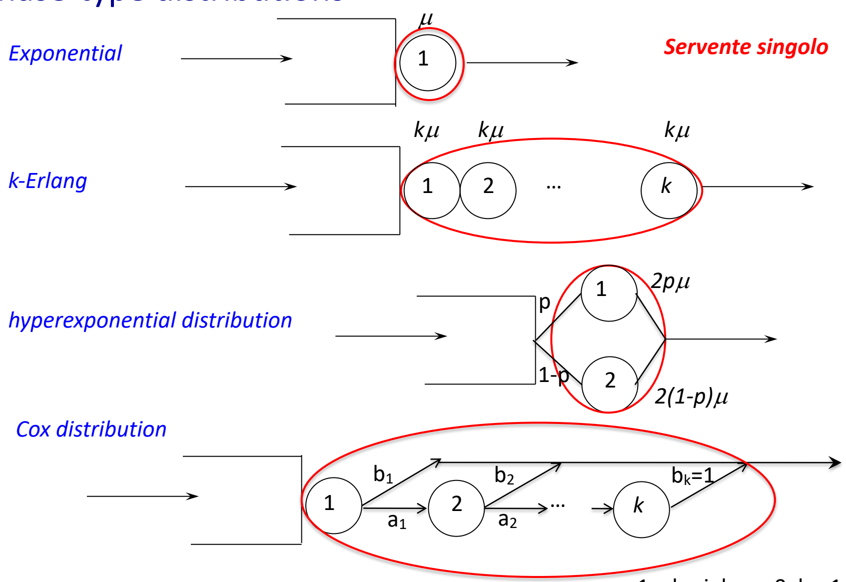
$$E(N_Q) = \frac{\rho^2}{2(1-\rho)}\left[1+\frac{\sigma^2(S)}{E(S)^2}\right]$$



$= C^2$
Squared coefficient of variation
Service time dispersion

1. Any service time distribution
2. Poisson arrivals
3. Abstract discipline (FIFO, LIFO, RAND…)

Phase-type distributions



$a_i = 1 - b_i$, $i<k$, $a_k=0$, $b_k=1$

The Khinchin Pollaczek equation (KP)

$$E(N_Q) = \frac{\rho^2}{2(1-\rho)}\left[1 + C^2\right]$$

The mean queue population grows as C^2

D $\longrightarrow$ $C^2=0$

E_k $\longrightarrow$ $C^2 = \frac{1}{k},\ k \geq 1$

M $\longrightarrow$ $C^2=1$

H_2 $\longrightarrow$ $C^2 = g(p) = \frac{1}{2p(1-p)} - 1$

$p = 0.6 \quad C^2 = 1.08\overline{3}$

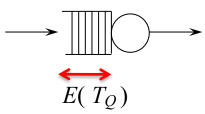
$p = 0.7 \quad C^2 = 1.38095$

$p = 0.8 \quad C^2 = 2.125$

$p = 0.9 \quad C^2 = 4.\overline{5}$

The Khinchin Pollaczek equation (KP)

M/G/1 abstract scheduling



$$E(T_Q) = \frac{E(N_Q)}{\lambda} = \frac{\rho^2}{\lambda 2(1-\rho)}\left[1 + C^2\right] = \frac{\rho}{1-\rho}\,\frac{C^2+1}{2}\,E(S)$$

The Khinchin Pollaczek equation (KP)

$$g(p) = \frac{1}{2p(1-p)} - 1 \qquad E(N_Q) = \frac{\rho^2}{2(1-\rho)}\left[1+C^2\right], \quad E(T_Q) = \frac{\rho}{1-\rho}\frac{C^2+1}{2}E(S)$$

Service time	$E(N_Q)$	$E(T_Q)$
Deterministic, M/D/1	$\frac{\rho^2}{2(1-\rho)}$	$\frac{\rho E(S)}{2(1-\rho)}$
Markovian, M/M/1	$\frac{\rho^2}{1-\rho}$	$\frac{\rho E(S)}{1-\rho}$
K-Erlang, M/E_k/1 $\sigma^2(S) = \frac{E(S)^2}{k}$	$\frac{\rho^2}{2(1-\rho)}\left(1+\frac{1}{k}\right)$	$\frac{\rho E(S)}{2(1-\rho)}\left(1+\frac{1}{k}\right)$
Hyperexpo, M/H_2/1 $\sigma^2(S) = E(S)^2 g(p)$	$\frac{\rho^2}{2(1-\rho)}(1+g(p))$	$\frac{\rho E(S)}{2(1-\rho)}(1+g(p))$

Service time Sensitivity

$$E(N_Q)_D \le E(N_Q)_{E_k} \le E(N_Q)_M \le E(N_Q)_{H_2}$$

$$\sigma^2(N_Q)_D \le \sigma^2(N_Q)_{E_k} \le \sigma^2(N_Q)_M \le \sigma^2(N_Q)_{H_2}$$

By considering $E(N_S)= E(N_Q)+\rho$, the same order holds for the variable N_S

By considering the Little's equation, the same order can be derived for the mean times $E(T_S)$ and $E(T_Q)$, but just for the 1° order moment, not for the variance

Discipline Sensitivity

By definition, KP holds for any abstract service discipline, so

$$E(N_Q)_{\text{FIFO}} = E(N_Q)_{\text{LIFO}} = E(N_Q)_{\text{RAND}} = E(N_Q)_{\text{abstract}}$$
$$\sigma^2(N_Q)_{\text{FIFO}} = \sigma^2(N_Q)_{\text{LIFO}} = \sigma^2(N_Q)_{\text{RAND}} = \sigma^2(N_Q)_{\text{abstract}}$$

By considering $E(N_S)= E(N_Q)+\rho$, the same equalities hold for the variable N_S

By considering the Little's equation, the same holds for $E(T_S)$ and $E(T_Q)$,

$$E(T_Q)_{\text{FIFO}} = E(T_Q)_{\text{LIFO}} = E(T_Q)_{\text{RAND}} = E(T_Q)_{\text{abstract}}$$

Is $\sigma^2(T_Q)$ the same for all these policies?

Discipline Sensitivity

No!
LIFO can generate some extremely high response times because we have to wait for system to become empty to take care of that first arrival

$$\sigma^2(T_Q)_{\text{FIFO}} \le \sigma^2(T_Q)_{\text{RAND}} \le \sigma^2(T_Q)_{\text{LIFO}}$$