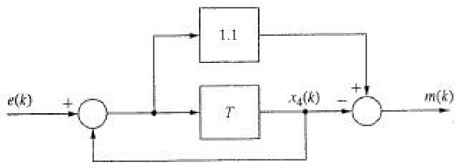
4.8-1

Solution:

$$D(z) = 1 + \frac{1.1z}{z-1} = \frac{1.1z-1}{z-1}$$



$$m(k) = -x_4(k) + 1.1[e(k) + x_4(k)] = 0.1x_4(k) + 1.1e(k)$$

$$\therefore \mathbf{x}(k+1) = A\mathbf{x}(k) + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} [0.1\, x_4(k) + 1.1e(k)]$$

and $x_4(k+1) = x_4(k) + e(k)$

$$\therefore \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ x_4(k+1) \end{bmatrix} = \begin{bmatrix} 0.9 & 1 & 2 & 0 \\ 0 & 0.9 & 0 & 0.1 \\ -1 & 0 & 0.5 & 0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \overbrace{\begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix}}^{x_e(k)} + \begin{bmatrix} 0 \\ 1.1 \\ 1.1 \\ 1 \end{bmatrix} e(k)$$

$$y(k) = [1 \quad 1.5 \quad 2.3 \quad 0]\, \mathbf{x}_e(k)$$

4.10-1

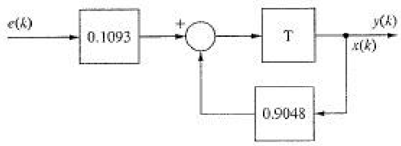
Solution:

(a) $(s+0.05)Y(s) = 0.1M(s), \quad \therefore G_p(s) = \dfrac{0.1}{s+0.05}$

$$G(z) = \frac{Y(z)}{E(z)} = \frac{z-1}{z}\mathfrak{z}\left[\frac{0.1}{s(s+0.05)}\right]$$

$$= \frac{z-1}{z}\left(\frac{0.1}{0.05}\right)\frac{z(1-\varepsilon^{-(0.05)(2)})}{(z-1)(z-0.9048)} = \frac{0.1903}{z-0.9048}$$

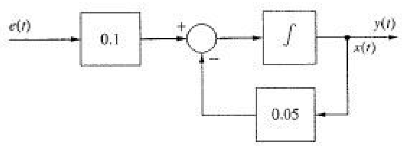
(b)



$$x(k+1) = 0.9048x(k) + 0.1093e(k)$$

$$y(k) = x(k)$$

(c)



$$\dot{x}(t) = -0.05x(t) + 0.1e(t)$$
$$y(t) = x(t)$$

(d) $(s\mathbf{I}-\mathbf{A})^{-1} = \dfrac{1}{s+0.05}, \therefore \mathbf{\Phi}(t) = \varepsilon^{-0.05t}$

$$\therefore \mathbf{A} = \mathbf{\Phi}(T) = 0.9048$$

$$\mathbf{B} = \mathbf{B}_c\int_0^2 \mathbf{\Phi}(\tau)d\tau = \mathbf{B}_c\int_0^2 \varepsilon^{-0.05\tau}d\tau$$

$$= \frac{0.1}{-0.05}\varepsilon^{-0.05\tau}\Big|_0^2 = 2[1-0.9048] = 0.1903$$

$$\therefore\ x(k+1) = 0.9048x(k) + 0.1093e(k)$$
$$y(k) = x(k)$$

(e) Same as (b).

(f) $G(z) = \dfrac{0.1093z^{-1}}{1-0.9048z^{-1}} = \dfrac{0.1093}{z-0.9048}$

MATLAB:

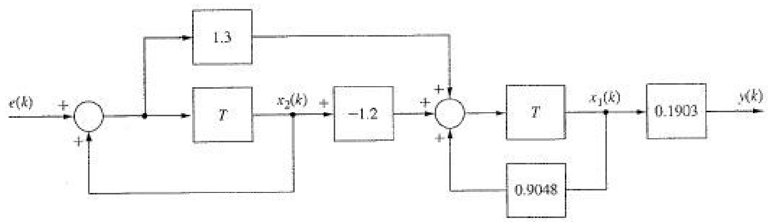
```
num=[0 0.1];

den=[1 0 .05];

[Ac,Bc,C,D]=tf2ss(num,den)

[A,B]=c2d(Ac,Bc,2)

[n,d]=ss2tf(A,B,C,D)

pause

Ac = -0.5; BC = 0.1;

[A,B]=c2d(Ac,Bc,2)
```

4.10-7

Solution:

$$D(z) = 1.2 + \frac{0.1z}{z-1} = \frac{1.3z-1.2}{z-1}$$

(a) $G(z) = \dfrac{0.1903}{z-0.9048}$



$$x_1(k+1) = 0.9408x_1(k) - 1.2x_2(k) + 1.3[e(k) + x_2(k)]$$

$$= 0.9408x_1(k) + 0.1x_2(k) + 1.3e(k)$$

$$x_2(k+1) = x_2(k) + e(k)$$

$$\therefore \mathbf{x}(k+1) = \begin{bmatrix} 0.9408 & 0.1 \\ 0 & 1 \end{bmatrix}\mathbf{x}(k) + \begin{bmatrix} 1.3 \\ 1 \end{bmatrix}e(k)$$

$$y(k) = \begin{bmatrix} 0.1903 & 0 \end{bmatrix}\mathbf{x}(k)$$

(b) Same filter simulation diagram as in (a).

From Problem 4.10-5(d):

$$\mathbf{x}(k+1) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} ½ \\ 1 \end{bmatrix} m(k)$$

$$m(k) = [-1.2 + 1.3] x_3(k) + 1.3e(k) = 0.1x_3(k) + 1.3e(k)$$

$$\therefore \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0.05 \\ 0 & 1 & 0.1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 0.65 \\ 1.3 \\ 1 \end{bmatrix} e(k)$$

$$y[k] = [K/J \quad 0 \quad 0] \mathbf{x}(k)$$

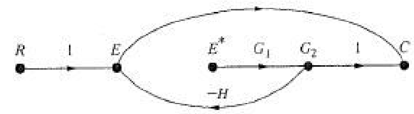
(c)

```
nd=[1.3 -1.2]; dd = [1 -1];

ng=[0 0.1903]; dg = [1 -0.9048];

n=conv(nd,ng)

d=conv(dd,dg)

pause

A = [.9408 .1;0 1]; B = [1.3; 1];

C = [0.1903 0]; D = 0;

[num,dem] = ss2tf(A,B,C,D)

nd=[1.3 -1.2]; dd = [1 -1];

ng=[0 0.5 0.5]; dg = [1 -2 1];

n=conv(nd,ng)

d=conv(dd,dg)

pause

A = [1 1 .05;0 1 .1;0 0 1]; B = [.65;1.3;1];

C = [1 0 0]; D = 0;

[num,dem] = ss2tf(A,B,C,D)
```

5.3-4

Solution:

(a)



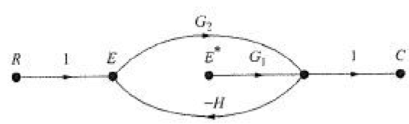
$$E = R - G_1 H E^*$$

$$\therefore E^* = \frac{R^*}{1+\overline{GH}^*}$$

$$C = G_2 R - G_1 G_2 H E^* + G_1 E^*$$

$$\therefore C(z) = \overline{G_2 R}(z) + \frac{G_1(z) - \overline{G_1 G_2 H}(z)}{1+\overline{G_1 H}(z)} R(z)$$

(b)

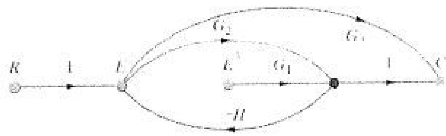


$$E = \frac{R}{1+G_2 H} - \frac{G_1 H}{1+G_2 H} E^* \Rightarrow E^* = \frac{\left[\dfrac{R}{1+G_2 H}\right]^*}{1+\left[\dfrac{G_1 H}{1+G_2 H}\right]^*}$$

$$C = \frac{G_2 R}{1+G_2 H} + \frac{G_1}{1+G_2 H} E^*$$

$$\therefore C(z) = \left[\frac{G_2R}{1+G_2H}\right](z) + \frac{\left[\frac{G_1}{1+G_2H}\right](z)}{1+\left[\frac{G_1H}{1+G_2H}\right](z)}\left[\frac{R}{1+G_2H}\right](z)$$

(c)



$$E = R - H\left[G_1E^* + G_2E\right] \Rightarrow E = \frac{R}{1+G_2H} - \frac{G_1H}{1+G_2H}E^*$$

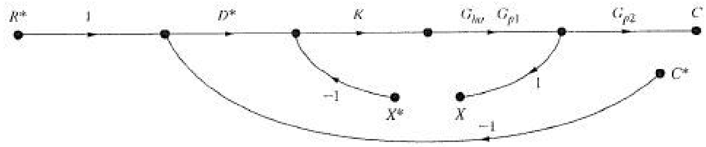
$$E^* = \frac{\left[\frac{R}{1+G_2H}\right]^*}{1+\left[\frac{G_1H}{1+G_2H}\right]^*}$$

$$C = G_2E + G_1E^* + G_2E = 2G_2E + G_1E^*$$

$$= \frac{2G_2R}{1+G_2H} + \frac{G_1\left[\frac{R}{1+G_2H}\right]^*}{1+\left[\frac{G_1H}{1+G_2H}\right]^*}$$

$$\therefore C(z) = \left[\frac{2G_2R}{1+G_2H}\right](z) + \frac{G_1(z)\left[\frac{R}{1+G_2H}\right](z)}{1+\left[\frac{G_1H}{1+G_2H}\right](z)}$$

5.3-7



Let: $G_{h0}\,G_{p1} = G_1$

$G_{h0}G_{p1}G_{p2} = G_2$

$X = -KG_1X^* + D^*\,\mathrm{KG}_1(R^* - C^*)$

$X(z) = \dfrac{D(z)KG_1(z)}{1+KG_1(z)}\left[R(z) - C(z)\right]$

$C = -KG_2X^* + KD^*G_2(R^* - C^*)$

$C(z) = -KG_2(z)\left[\dfrac{KD(z)G_1(z)}{1+KG_1(z)}\right]\left[R(z) - C(z)\right] + KD(z)G_2(z)\left[R(z) - C(z)\right]$

$\therefore C(z)\left[1+KG_1(z)\right] = -KG_2(z)D(z)KG_1(z)\left[R(z) - C(z)\right]$

$\qquad\qquad + D(z)KG_2(z)\left[1+KG_1(z)\right]\left[R(z) - C(z)\right]$

$\therefore C(z) = \dfrac{KD(z)G_2(z)R(z)}{1+KG_1(z)+KD(z)G_2(z)}$

5.3-11

Solution:

(a) $C(s) = G(s)R(s) - G(s)H(s)\varepsilon^{-0.1Ts}D^*(s)C^*(s)$

$\therefore C(z) = \overline{GR}(z) - \overline{GH}(z,m)\big|_{m=0.9} D(z)C(z)$

$\therefore C(z) = \dfrac{GR(z)}{1 + D(z)\overline{GH}(z, 0.9)}$

(b) $C(z)$ same as in (a).

5.3-14

Solution:

(a) $\theta_{max}(t) = 360°$, maximum sensor output $= (0.02)(360°) = 7.2\text{ V}$

$\therefore$ input for $360° = 7.2\text{ V}$

input for $0° = 0\text{ V}$

$\therefore$ need A/D range $0 - 7.2\text{ V}$, $\therefore$ choose $0 - 10\text{ V}$ range.

(b) From (a), sensor output $= (0.02)(70°) = 1.4\text{ V}$

$\therefore$ input $= 1.4\text{ V}$

(c) (a) maximum sensor output $= (0.02)(\pm 180°) = \pm 3.6\text{ V}$

$\therefore$ choose $\pm 5\text{ V}$ range

(b) From (b), 1.4V

(d) $G(z) = \mathcal{Z}\left[\dfrac{1-\varepsilon^{-Ts}}{s} G_p(s)\right]$

$$M^*(s) = D^*\left[R^* - KG^* H_k M^*\right] \Rightarrow M^* = \frac{D^* R^*}{1 + KD^* G^* H_k}$$

$$C = KG^* M^* \Rightarrow C(z) = \frac{KD(z)G(z)}{1 + KD(z)G(z)H_k} R(z)$$

(e) $G(z) = \dfrac{z-1}{z}\mathcal{Z}\left[\dfrac{1}{Js^3}\right] = \dfrac{z-1}{z}\dfrac{(1)^2 z(z+1)}{0.1(2)(z-1)^3} = \dfrac{5(z+1)}{(z-1)^2}$

$$\therefore \frac{KD(z)G(z)}{1 + KD(z)G(z)H_k} = \frac{2[5(z+1)]}{z^2 - 2z + 1 + 10(z+1)0.02}$$

$$\therefore \text{system transfer function} = \frac{10(z+1)}{z^2 - 1.8z + 1.2}$$

5.4-1

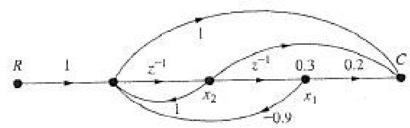
Solution:

(a)
$$\frac{C(z)}{R(z)} = \frac{1+0.3z^{-1}+0.2z^{-2}}{1-z^{-1}+0.9z^{-2}}$$

$$(1-z^{-1}+0.9z^{-2})C(z) = (1+0.3z^{-1}+0.2z^{-2})R(z)$$

$$\therefore c(k) = c(k-1) - 0.9c(k-2) + r(k) + 0.3r(k-1) + 0.2r(k-2)$$

(b)



Let: $y(k) = c(k)$
$u(k) = r(k)$

$$\mathbf{x}(k+1) = \begin{bmatrix} 0 & 1 \\ -0.9 & 1 \end{bmatrix}\mathbf{x}(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u(k)$$

$$y(k) = [0.2-0.9 \quad 0.3+1]\mathbf{x}(k) + u(k) = [-0.7 \quad 1.3]\mathbf{x}(k) + u(k)$$

(c)
$$z\mathbf{I}-\mathbf{A} = \begin{bmatrix} z & -1 \\ 0.9 & z-1 \end{bmatrix}, \quad |z\mathbf{I}-\mathbf{A}| = \Delta = z^2 - z + 0.9$$

$$\therefore [z\mathbf{I}-\mathbf{A}]^{-1} = \frac{1}{\Delta}\begin{bmatrix} z-1 & 1 \\ -0.9 & z \end{bmatrix}$$

$$\frac{Y(z)}{U(z)} = \mathbf{C}[z\mathbf{I}-\mathbf{A}]^{-1}\mathbf{B} + \mathbf{D} = \frac{1}{\Delta}[-0.7 \quad 1.3]\begin{bmatrix} z-1 & 1 \\ -0.9 & z \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} + 1$$

$$= \frac{1}{\Delta}[-0.7 \quad 1.3]\begin{bmatrix} 1 \\ z \end{bmatrix} + 1 = \frac{1.3z-0.7}{z^2-z+0.9} + 1 = \frac{z^2+0.3z+0.2}{z^2-z+0.9}$$

(d) A = [0 1;−.9 1]; B = [0;1]; C = [−0.7 1.3]; D = 1; [num, den] = ss2tf(A, B, C, D)