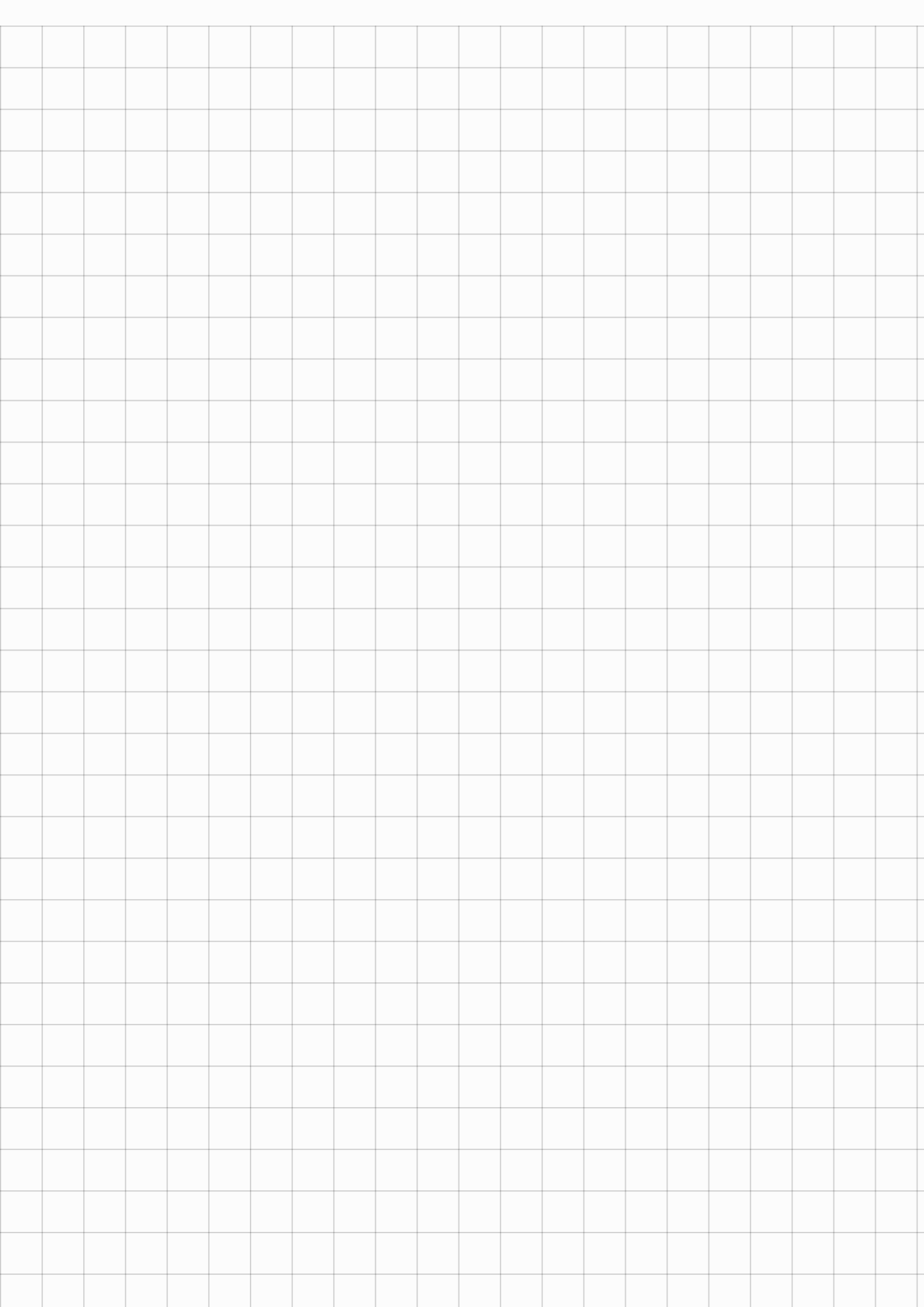


1. Problem 2.2.2 text

2. Problem 2.2.3 text

3. Problem 2.6.3 text

4. Problem 2.7.1 text



## Problem 2.2.2

2.2-2. The trapezoidal rule (modified Euler method) for numerical integration approximates the integral of a function  $x(t)$  by summing trapezoid areas as shown in Fig. P2.2-2. Let  $y(t)$  be the integral of  $x(t)$ .

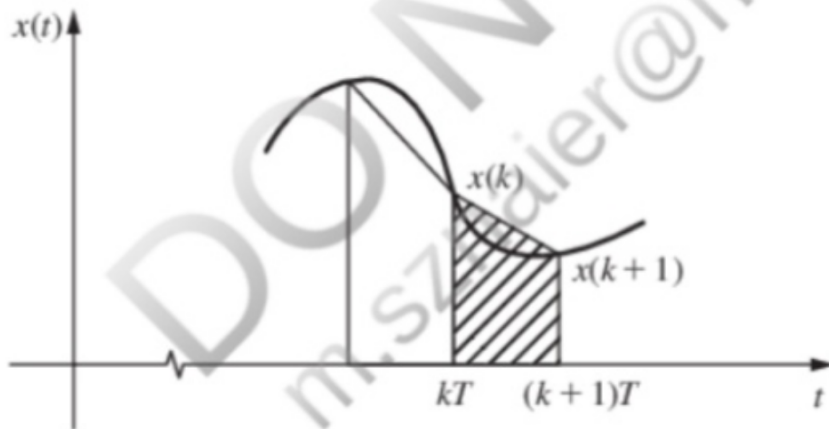


Figure P2.2-2 Trapezoidal rule for numerical integration.

### Figure P2.2-2 Full Alternative Text

- Write the difference equation relating  $y[(k+1)T]$ ,  $y(kT)$ ,  $x[(k+1)T]$ , and  $x(kT)$  for this rule.
- Show that the transfer function for this integrator is given by

$$\frac{Y(s)}{X(s)} = \frac{(s/2)(s+1)}{s^2 - 1}$$



## Problem 2.2.3

2.2-3.

- a. The transfer function for the right-side rectangular-rule integrator was found in **Problem 2.2-1** to be  $Y(z)/X(z) = Tz/(z - 1)$ . We would suspect that the reciprocal of this transfer function should yield an approximation to a differentiator. That is, if  $w(kT)$  is a numerical derivative of  $x(t)$  at  $t = kT$ ,

$$\frac{W(z)}{X(z)} = \frac{z - 1}{Tz}$$

Write the difference equation describing this differentiator.

- b. Draw a figure similar to those in **Fig. P2.2-1** illustrating the approximate differentiation.

- c. Repeat part (a) for the left-side rule, where  $W(z)/X(z) = T/(z - 1)$ .

- d. Repeat part (b) for the differentiator of part (c).



## Problem 2.6.3

2.6-3. Given the difference equation

$$x(k) - x(k-1) + x(k-2) = e(k)$$

where  $e(k) = 1$  for  $k \geq 0$ .

- Solve for  $x(k)$  as a function of  $k$ , using the z-transform. Give the values of  $x(0)$ ,  $x(1)$ , and  $x(2)$ .
- Verify the values  $x(0)$ ,  $x(1)$ , and  $x(2)$ , using the power-series method.
- Verify the values  $x(0)$ ,  $x(1)$ , and  $x(2)$  by solving the difference equation directly.
- Will the final-value property give the correct value for  $x(\infty)$ ?





## Problem 2.7.1

•

**2.7-1.** (a) Find  $e(0)$ ,  $e(1)$ , and  $e(10)$  for

$$E(z) = \frac{0.1}{z(z - 0.9)}$$

using the inversion formula.

(b) Check the value of  $e(0)$  using the initial-value property.

(c) Check the values calculated in part (a) using partial fractions.

(d) Find  $e(k)$  for  $k = 0, 1, 2, 3$ , and 4 if  $\mathcal{Z}[e(k)]$  is given by

$$E(z) = \frac{1.98z}{(z^2 - 0.9z + 0.9)(z - 0.8)(z^2 - 1.2z + 0.27)}$$

(e) Find a function  $e(t)$  which, when sampled at a rate of 10 Hz ( $T = 0.1s$ ), results in the transform  $E(z) = 2z/(z - 0.8)$ .

(f) Repeat part (e) for  $E(z) = 2z/(z + 0.8)$ .

(g) From parts (e) and (f), what is the effect on the inverse  $z$ -transform of changing the sign on a real pole?





