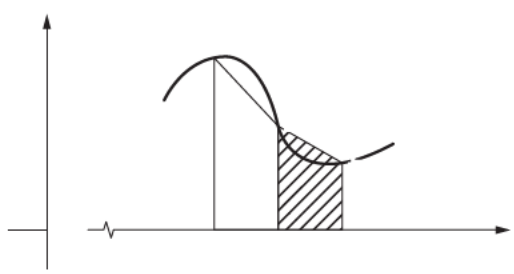
2.2-2. The trapezoidal rule (modified Euler method) for numerical



$$\frac{Y(z)}{X(z)} = \frac{(T/2)(z+1)}{z-1}$$

2.2-3. (a) The transfer function for the right-side rectangular-rule integrator was found in Problem 2.2-1 to be $Y(z)/X(z) = Tz/(z-1)$. We would suspect that the reciprocal of this transfer function should yield an approximation to a differentiator. That is, if $w(kT)$ is a numerical derivative of $x(t)$ at $t = kT$,

$$\frac{W(z)}{X(z)} = \frac{z-1}{Tz}$$

Write the difference equation describing this differentiator.

(b) Draw a figure similar to those in Fig. P2.2-1 illustrating the approximate differentiation.

(c) Repeat part (a) for the left-side rule, where $W(z)/X(z) = T/(z-1)$.

(d) Repeat part (b) for the differentiator of part (c).

2.6-3. Given the difference equation

$$x(k) - x(k-1) + x(k-2) = e(k)$$

where $e(k) = 1$ for $k \geq 0$.

(a) Solve for $x(k)$ as a function of k, using the z-transform. Give the values of $x(0), x(1)$, and $x(2)$.

(b) Verify the values $x(0)$, $x(1)$, and $x(2)$, using the power-series method.

(c) Verify the values $x(0)$, $x(1)$, and $x(2)$ by solving the difference equation directly.

(d) Will the final-value property give the correct value for $x(\infty)$?

2.7-1. (a) Find $e(0)$, $e(1)$, and $e(10)$ for

$$E(z) = \frac{0.1}{z(z-0.9)}$$

using the inversion formula.

(b) Check the value of $e(0)$ using the initial-value property.

(c) Check the values calculated in part (a) using partial fractions.

(d) Find $e(k)$ for $k = 0,\ 1,\ 2,\ 3,$ and 4 if $\mathfrak{z}[e(k)]$ is given by

$$E(z) = \frac{1.98z}{\left(z^2 - 0.9z + 0.9\right)\left(z - 0.8\right)\left(z^2 - 1.2z + 0.27\right)}$$

(e) Find a function $e(t)$ which, when sampled at a rate of 10 Hz $(T = 0.1s)$, results in the transform $E(z) = 2z/(z - 0.8)$.

(f) Repeat part (e) for $E(z) = 2z/(z + 0.8)$.

(g) From parts (e) and (f), what is the effect on the inverse z-transform of changing the sign on a real pole?