EECE 5610, Fall 2025, Homework #5, due Nov. 14, turnitin

1. Problem 6.4.4 text
2. Problem 6.5.2 text
3. Problem 6.5.3 text
4. Problem 7.5.4 text
5. Problem 7.5.5 text

(a) Evaluate $C(z)$ if the input is to command a 20°C step in the output and $K = 10$. Note that the system input must be a step function with an amplitude 1.4 V. Why?
(b) Assuming the system to be stable, find the steady-state system output.
(c) Find the (approximate) time required for the system response to reach steady state.
(d) Simulate the system to verify the results in parts (b) and (c).

6.4-1. The block diagram of a control system of a joint in a robot arm is shown in Fig. P6.2-7. Let $T = 0.1$ s, $K = 10$, and $D(z) = 1$. The results of Problem 6.2-7 are useful in this problem if these results are available.
(a) Find the damping ratio ζ, the natural frequency ω_n, and the time constant τ of the open-loop system. If the system characteristic equation has two real zeros, find the two time constants. These values can be solved by inspection. Why?
(b) Repeat part (a) for the closed-loop system.
(c) Repeat parts (a) and (b) for the system with the sampler, digital controller, and data hold removed, that is, for the analog system.
(d) Use the results in parts (b) and (c) to find the percent overshoot in the step responses for the sampled-data closed-loop system and for the analog closed-loop system.

6.4-2. The block diagram of an attitude control system of a satellite is shown in Fig. P6.4-2. Let $T = 1$ s, $K = 100$, $J = 0.1$, $H_k = 0.02$, and $D(z) = 1$.
(a) Find the damping ratio ζ the natural frequency ω_n, and the time constant τ of the open-loop system. If the system characteristic equation has two real zeros, find the two time constants. These values can be solved by inspection. Why?
(b) Repeat part (a) for the closed-loop system.
(c) Repeat parts (a) and (b) for the system with the sampler, digital controller, and data hold removed, that is, for the analog system.
(d) The closed-loop sampled-data system is seen to be unstable and the closed-loop analog system is seen to be marginally stable. If the satellite is operated with each of these control systems, describe the resulting movement for both the sampled-data system and the analog system.

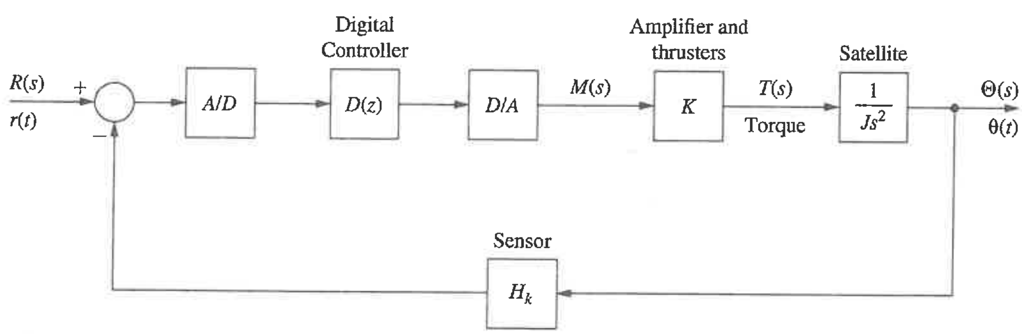

FIGURE P6.4-2 Block diagram for a satellite control system.

6.4-3. It is shown in Example 6.6 that the system of Fig. 6-2 has the parameters $\zeta = 0.250$, $\omega_n = 0.9191$, and $\tau = 4.36$ s.
(a) Find ζ, ω_n, and τ for the sample period $T = 0.5$.
(b) Repeat part (a) for $T = 0.1$.
(c) Repeat part (a) for the analog system, that is, the system with the sampler and data hold removed. Note that this case can be considered to be the limit as the sample period approaches zero.
(d) Give a table listing the three parameters as a function of sampling frequency $f_s = 1/T$. State the result of decreasing the sampling frequency on the parameters.

6.4-4. Consider the system of Fig. P6.4-4. This system is called a regulator control system, in which it is desired to maintain the output, $c(t)$, at a value of zero in the presence of a disturbance, $f(t)$. In this problem the disturbance is a unit step.

(a) With $D(z) = 1$ (i.e., no compensation) find the steady-state value of $c(t)$.
(b) For $f(t)$ to have no effect on the steady-state value of $c(kT)$, $D(z)$ should have a pole at $z = 1$. Let

$$D(z) = 1 + \frac{0.1z}{z - 1}$$

Determine the steady-state value of $c(kT)$.
(c) Repeat parts (a) and (b) with $T = 1$ s.
(d) Why does the value of sample period T have no effect on the steady-state response for a constant input?

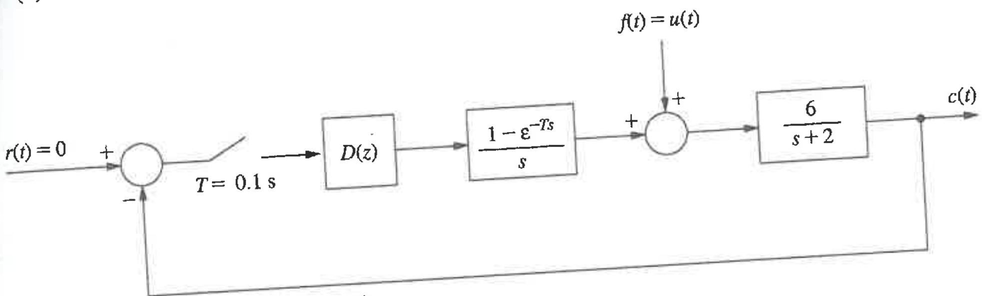

FIGURE P6.4-4 System for Problem 6.4-4.

6.4-5. Consider the sampled-data systems with the following characteristic equations:

(i) $z - 0.999 = 0$
(ii) $z - 0.99 = 0$
(iii) $z - 0.9 = 0$
(iv) $z + 0.9 = 0$
(v) $z^2 - 1.85z + 0.854 = 0$
(vi) $z^2 - 1 = 0$
(vii) $z^2 - 2z + 0.99 = 0$
(viii) $z^2 - 1.2z + 0.7 = 0$

(a) What can you determine about the system natural-response characteristics for each system, for $T = 0.1$ s?
(b) Repeat part (a) for $T = 1$ s.
(c) For a given characteristic equation as a function of z, which parameters of the transient response vary with the sample period T, and which are independent of T?

6.4-6. Consider the system of Fig. P6.2-1. Suppose that an ideal time delay of 0.2 s is added to the plant, such that the plant transfer function is now given by

$$G_p(s) = \frac{0.5\varepsilon^{-0.2T}}{s + 0.5}$$

(a) Find the time constant τ of the system if the time delay is omitted.
(b) Find the time constant τ of the system if the time delay is included.
(c) Repeat part (b) for a time delay of 1 s.
(d) What is the effect on the speed of response of the closed-loop system of adding time delay to the plant?

6.5-1. (a) Give the system type for the following systems, with $D(z) = 1$. It is not necessary to find the pulse transfer functions to find the system type. Why?

(i) Fig. P6.2-1.
(ii) Fig. P6.2-4.
(iii) Fig. P6.2-7.
(iv) Fig. P6.4-2.
(v) Fig. P6.4-4.

(b) It is desired that the systems of part (a) have zero steady-state error for a constant input. Give the required characteristics for each digital controller.
(c) It is desired that the systems of part (a) have zero steady-state error for a ramp input. Give the required characteristics for each digital controller.

21 **6.5-2.** Consider the system of Fig. P6.5-2. The digital filter is described by

$$m(kT) = e(kT) - 0.9e[(k-1)T] + m[(k-1)T]$$

(a) Find the system type.
(b) Find the steady-state response for a unit-step input, without finding $C(z)$.
(c) Find the approximate time for the system to reach steady state.
6 (d) Find the unit-step response for the system and verify parts (b) and (c).

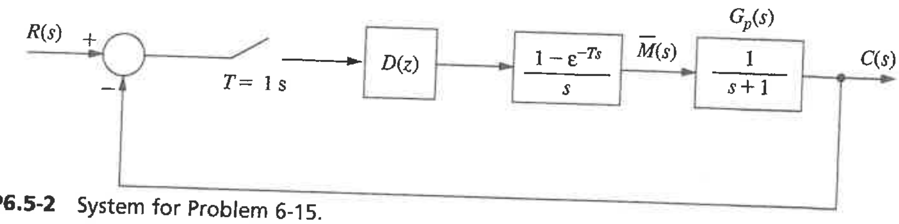

FIGURE P6.5-2 System for Problem 6-15.

6.5-3. Consider the system of Fig. P6.5-2, with the digital filter as described in Problem 6.5-2.
(a) Find the system type.
(b) Find the steady-state response for a unit ramp input, without finding $C(z)$.
(c) Find the approximate time for the system to reach steady state.
(d) Calculate the steady-state unit-ramp response for the system to verify parts (b) and (c).

6.5-4. Consider the system of Problem 6.5-2. Let the plant transfer function be given by

$$G_p(s) = \frac{s}{s+1}$$

Hence the dc gain of the plant is zero. The digital filter is as given in Problem 6.5-2.
(a) Find the system type.
(b) Find the steady-state response for a unit-step input, without finding $C(z)$.
(c) Find the approximate time for the system to reach steady state.
(d) Find $c(kT)$ and verify the results of parts (b) and (c).

6.5-5. For the system of Fig. 6-13, $G_p(s) = 20(s+5)/(s(s+4)(s+6))$, with $T = 0.05$ and $D(z)$ implementing the following difference equation

$$m(k) = 10e(k) - 16e(k-1) - 6.3e(k-2) + m(k-1)$$

(a) Find the system type.
(b) Plot the unit-step response.
(c) Find the rise time, overshoot, and settling time.

6.5-6. Repeat Problem 6.5-5 with $T = 0.02$.
(a) Find the system type.
(b) Plot the unit-step response.
(c) Find the rise time, overshoot, and settling time.
(d) Compare your results with those of Problem 6.5-5 and explain any differences.

6.6-1. Consider the system of Fig. P6.5-2, with $D(z) = 1$.
(a) With the sampler and zero-order hold removed, write the system differential equation.
(b) Using the rectangular rule for numerical integration with the numerical integration increment equal to 0.25 s, evaluate the unit-step response for $0 \le t \le 1.5$ s.
(c) Repeat part (b) with the sampler and zero-order hold in the system, and with a sample period $T = 0.5$ s.
(d) Solve for the exact unit-step responses for parts (b) and (c), and compare results.

6.6-2. Consider the numerical integration of the differential equation (6-24) in Section 6.6. Applying the rectangular rule results in the difference equation (6-25), which can be expressed as

$$x[(k+1)H] = x(kH) + H[-x(kH)]$$

(e) Comparing the ranges of K from parts (b) and (d), give the effects on stability of adding sampling to the analog system.

.5-4. Consider the robot arm joint control system of Fig. P7.5-4. This system is described in Problem 1.5-4. For this problem, $T = 0.1$ s and $D(z) = 1$. It was shown in Problem 6-7 that

$$\mathfrak{z}\left[\frac{1-\varepsilon^{-Ts}}{s}\frac{4}{s(s+2)}\right] = \frac{0.01873z + 0.01752}{(z-1)(z-0.8187)}$$

(a) Write the closed-loop system characteristic equation.
(b) Use the Routh–Hurwitz criterion to determine the range of K for stability.
(c) Check the results of part (b) using the Jury test.
(d) Determine the location of all roots of the characteristic equation in both the w-plane and the z-plane for the value of $K > 0$ for which the system is marginally stable.
(e) Determine both the s-plane frequency and the w-plane frequency at which the system will oscillate when marginally stable, using the results of part (d).
(f) Show that the frequencies in part (e) satisfy (7-10).

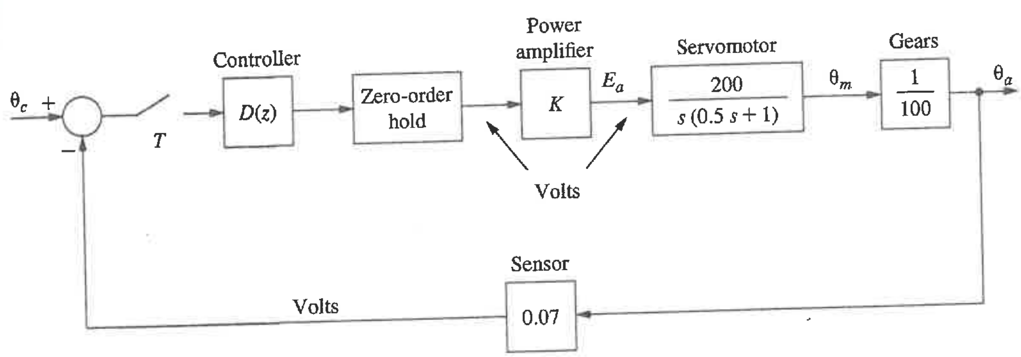

FIGURE P7.5-4 Robot arm joint control system.

7.5-5. Consider the antenna control system of Fig. P7.5-5. This system is described in Problem 1.5-1. For this problem, $T = 0.05$ s and $D(z) = 1$. It was shown in Problem 5.3-15 that

$$\mathfrak{z}\left[\frac{1-\varepsilon^{-Ts}}{s}\frac{20}{s(s+6)}\right] = \frac{0.02268z + 0.02052}{(z-1)(z-0.7408)}$$

(a) Write the closed-loop system characteristic equation.
(b) Use the Routh–Hurwitz criterion to determine the range of K for stability.
(c) Check the results of part (b) using the Jury test.
(d) Determine the location of all roots of the characteristic equation in both the w-plane and the z-plane for the value of $K > 0$ for which the system is marginally stable.
(e) Determine both the s-plane frequency and the w-plane frequency at which the system will oscillate when marginally stable, using the results of part (d).
(f) Show that the frequencies in part (e) satisfy (7-10).