2.10-2. Consider the system described by

$$\mathbf{x}(k+1) = \begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix}\mathbf{x}(k) + \begin{bmatrix} 1 \\ 1 \end{bmatrix}u(k)$$

$$y(k) = \begin{bmatrix} -2 & 1 \end{bmatrix}\mathbf{x}(k)$$

(a) Find the transfer function $Y(z)/U(z)$.

(b) Using any similarity transformation, find a different state model for this system.

(c) Find the transfer function of the system from the transformed state equations.

(d) Verify that $\mathbf{A}$ given and $\mathbf{A}_w$ derived in part (b) satisfy the first three properties of similarity transformations. The fourth property was verified in part (c).

Solution:

(a) $z\mathbf{I} - \mathbf{A} = \begin{bmatrix} z & -1 \\ 0 & z-3 \end{bmatrix}$; $\Delta = |z\mathbf{I} - \mathbf{A}| = z(z-3) = \Delta$

$$\frac{Y(z)}{U(z)} = \mathbf{C}[z\mathbf{I}-\mathbf{A}]^{-1}\mathbf{B} = \frac{1}{\Delta}[-2 \quad 1]\begin{bmatrix} z-3 & 1 \\ 0 & z \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{\Delta}[-2z+6 \quad z-2]\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{-z+4}{z(z-3)}$$

(b) $\mathbf{P} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$; $\mathbf{P}^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$

$$\mathbf{A}_w = \mathbf{P}^{-1}\mathbf{A}\mathbf{P} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$$

$$\mathbf{B}_w = \mathbf{P}^{-1}\mathbf{B} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\mathbf{C}_w = \mathbf{C}\mathbf{P} = [-2 \quad 1]\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = [-1 \quad 3]$$

$$\therefore \mathbf{w}(k+1) = \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}\mathbf{w}(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\mathbf{u}(k)$$

$$\mathbf{y}(k) = [-1 \quad 3]\mathbf{w}(k)$$

(c) $z\mathbf{I}-\mathbf{A}_w = \begin{bmatrix} z-2 & -2 \\ -1 & z-1 \end{bmatrix}$; $\Delta = |z\mathbf{I}-\mathbf{A}_w| = z^2-3z+2-2 = z(z-3)$

$$\frac{Y(z)}{U(z)} = \mathbf{C}_w[z\mathbf{I}-\mathbf{A}_w]^{-1}\mathbf{B}_w = \frac{1}{\Delta}[-1 \quad 3]\begin{bmatrix} z-1 & 2 \\ 1 & z-2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{\Delta}[-1 \quad 3]\begin{bmatrix} z-1 \\ 1 \end{bmatrix} = \frac{-z+4}{z(z-3)}$$

(d) $|z\mathbf{I}-\mathbf{A}| = \begin{vmatrix} z & 1 \\ 0 & z-3 \end{vmatrix} = z^2-3z$; $|z\mathbf{I}-\mathbf{A}_w| = \begin{vmatrix} z-2 & -2 \\ -1 & z-1 \end{vmatrix} = z(z-3)$

$$\therefore z_1 = 0,\ z_2 = 3$$

$$|\mathbf{A}| = \begin{vmatrix} 0 & 1 \\ 0 & 3 \end{vmatrix} = 0 = z_1 z_2;\ |\mathbf{A}_w| = \begin{vmatrix} 2 & 2 \\ 1 & 1 \end{vmatrix} = 0$$

$$\text{tr } \mathbf{A} = 3 = z_1 + z_2;\ \text{tr } \mathbf{A}_w = 3$$

2.10-3. Consider the system of Problem 2.10-2. A similarity transformation on these equations yields

$$\mathbf{w}(k+1) = \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix} \mathbf{w}(k) + \mathbf{B}_w u(k)$$

$$y(k) = \mathbf{C}_w \mathbf{x}(k)$$

(a) Find d_1 and d_2.

(b) Find a similarity transformation that results in the $\mathbf{A}_w$ matrix given. Note that this matrix is diagonal.

(c) Find $\mathbf{B}_w$ and $\mathbf{C}_w$.

(d) Find the transfer functions of both sets of state equations to verify the results of this problem.

Solution:

(a) Let z_1, z_2 be the characteristic value of $\mathbf{A}$. $d_1 = z_1, \quad d_2 = z_2$

$$z\mathbf{I} - \mathbf{A} = \begin{bmatrix} z & -1 \\ 0 & z-3 \end{bmatrix}, \quad \therefore |z\mathbf{I} - \mathbf{A}| = z(z-3); \ \therefore z_1 = 0, \ z_2 = 3$$

(b) $$(z_1\mathbf{I} - \mathbf{A})m_1 = \begin{bmatrix} 0 & -1 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} m_{11} \\ m_{21} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} -m_{21} = 0 \\ -3m_{21} = 0 \end{matrix}$$

$$\therefore m_{21} = 0, \text{ let } m_{11} = 1, \ \therefore m_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$(z_2\mathbf{I} - \mathbf{A})m_2 = \begin{bmatrix} 3 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} m_{12} \\ m_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow 3m_{12} - m_{22} = 0$$

$$\therefore \text{let } m_{12} = 1, \ m_{22} = 3, \ \therefore m_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$\therefore \mathbf{M} = \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}, \ |\mathbf{M}| = 3, \ \mathbf{M}^{-1} = \begin{bmatrix} 1 & -1/3 \\ 0 & 1/3 \end{bmatrix}$$

$$\mathbf{M}^{-1}\mathbf{A}\mathbf{M} = \begin{bmatrix} 1 & -1/3 \\ 0 & 1/3 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1/3 \\ 0 & 1/3 \end{bmatrix} \begin{bmatrix} 0 & 3 \\ 0 & 9 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix}$$

(c) $\mathbf{B}_w = \mathbf{M}^{-1}\mathbf{B} = \begin{bmatrix} 1 & -1/3 \\ 0 & 1/3 \end{bmatrix} = \begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix}$

$$\mathbf{C}_w = \mathbf{CM} = \begin{bmatrix} -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 \end{bmatrix}$$

$$\therefore \mathbf{w}(k+1) = \begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix} \mathbf{w}(k) + \begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix} \mathbf{u}(k)$$

$$\mathbf{y}(k) = \begin{bmatrix} -2 & 1 \end{bmatrix} \mathbf{w}(k)$$

(d) See Problem 2.10-2(a) for the first transfer function.

$$z\mathbf{I} - \mathbf{A}_w = \begin{bmatrix} z & 0 \\ 0 & z-3 \end{bmatrix}; \; |z\mathbf{I} - \mathbf{A}_w| = z(z-3) = \Delta$$

$$\frac{Y(z)}{U(z)} = \mathbf{C}_w[z\mathbf{I} - \mathbf{A}_w]^{-1}\mathbf{B}_w = \frac{1}{\Delta}\begin{bmatrix} -2 & 1 \end{bmatrix} \begin{bmatrix} z-3 & 0 \\ 0 & z \end{bmatrix} \begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix}$$

$$= \frac{1}{\Delta}\begin{bmatrix} -2z+6 & z \end{bmatrix} \begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix} = \frac{-\frac{4}{3}z + 4 + \frac{1}{3}z}{\Delta} = \frac{-z+4}{z(z-3)}$$

2.11-1. Consider a system with the transfer function

$$G(z) = \frac{Y(z)}{U(z)} = \frac{2}{z(z-1)}$$

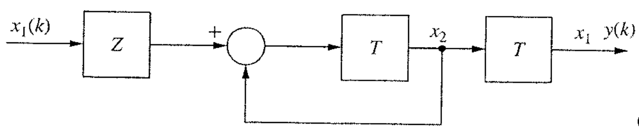
(a) Find three different state-variable models of this system.

(b) Verify the transfer function of each state model in part (a), using (2-84).

Solution:

(a) $G(z) = G_1(z)G_2(z) = \dfrac{2}{z^2 - z} = \dfrac{2z^{-2}}{1 - z^{-1}}$

(1)

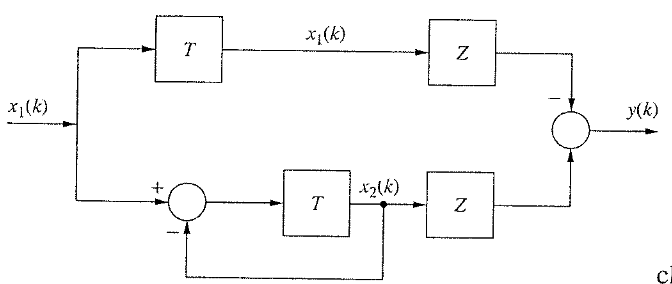


change Z to 2

$$\mathbf{x}(k+1) = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}\mathbf{x}(k) + \begin{bmatrix} 0 \\ 2 \end{bmatrix}u(k)$$

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix}\mathbf{x}(k)$$

(2) $G(z) = \dfrac{2}{z(z-1)} = \dfrac{-2}{z} + \dfrac{2}{z-1} = G_1(z) + G_2(z)$

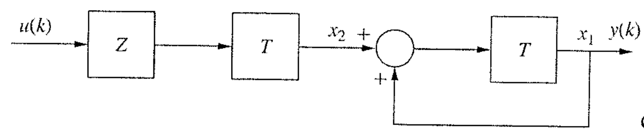


change Z to 2

$$\mathbf{x}(k+1) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\mathbf{x}(k) + \begin{bmatrix} 1 \\ 1 \end{bmatrix}u(k)$$

$$y(k) = \begin{bmatrix} -2 & 2 \end{bmatrix}\mathbf{x}(k)$$

(3)



change Z to 2

$$\mathbf{x}(k+1) = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}(k)$$

(b) (1) $z\mathbf{I} - \mathbf{A} = \begin{bmatrix} z & -1 \\ 0 & z-1 \end{bmatrix}$; $|z\mathbf{I} - \mathbf{A}| = z^2 - z = \Delta$

$$G(z) = \mathbf{C}[z\mathbf{I} - \mathbf{A}]^{-1}\mathbf{B} = \frac{1}{\Delta}\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} z-1 & 1 \\ 0 & z \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} z-1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \frac{2}{z(z-1)}$$

(2) $z\mathbf{I} - \mathbf{A} = \begin{bmatrix} z & 0 \\ 0 & z-1 \end{bmatrix}$; $|z\mathbf{I} - \mathbf{A}| = \Delta = z^2 - z$

$$G(z) = \mathbf{C}[z\mathbf{I} - \mathbf{A}]^{-1}\mathbf{B} = \frac{1}{\Delta}\begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} z-1 & 0 \\ 0 & z \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} 2z-2 \\ 2z \end{bmatrix} = \frac{2}{z(z-1)}$$

(3) $z\mathbf{I} - \mathbf{A} = \begin{bmatrix} z-1 & -1 \\ 0 & z \end{bmatrix}$; $|z\mathbf{I} - \mathbf{A}| = z^2 - z = \Delta$

$$G(z) = \mathbf{C}[z\mathbf{I} - \mathbf{A}]^{-1}\mathbf{B} = \frac{1}{\Delta}\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} z & 1 \\ 0 & z-1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} z & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \frac{2}{z(z-1)}$$

2.11-2. Consider a system described by the coupled difference equation

$$y(k+2) - v(k) = 0$$

$$v(k+1) + y(k+1) = u(k)$$

where $u(k)$ is the system input.

(a) Find a state-variable formulation for this system. Consider the outputs to be $y(k+1)$ and $v(k)$.
Hint: Draw a simulation diagram first.

(b) Repeat part (a) with $y(k)$ and $v(k)$ as the outputs.

(c) Repeat part (a) with the single output $v(k)$.

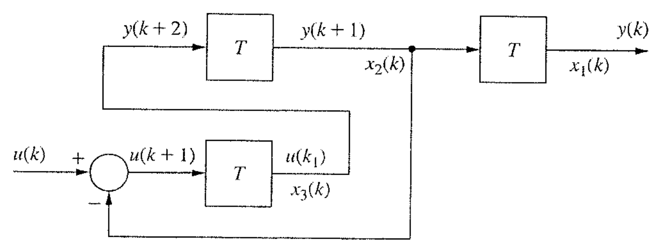
(d) Use (2-84) to calculate the system transfer function with $v(k)$ as the system output, as in part (c); that is, find $V(z)/U(z)$.

(e) Verify the transfer function $V(z)/U(z)$ in part (d) by taking the z-transform of the given system difference equations and eliminating $Y(z)$.

(f) Verify the transfer function $V(z)/U(z)$ in part (d) by using Mason's gain formula on the simulation diagram of part (a).

Solution:

(a)



$$\mathbf{x}(k+1)=\begin{bmatrix}0&1&0\\0&0&1\\0&-1&0\end{bmatrix}\mathbf{x}(k)+\begin{bmatrix}0\\0\\1\end{bmatrix}u(k)$$

$$y_0(k)=\begin{bmatrix}x_2(k)\\x_3(k)\end{bmatrix}=\begin{bmatrix}0&1&0\\0&0&1\end{bmatrix}\mathbf{x}(k);\ y_0(k)=\text{output}$$

(b) $\mathbf{x}(k+1)=$ same as (a)

$$y_0(k)=\begin{bmatrix}x_1(k)\\x_3(k)\end{bmatrix}=\begin{bmatrix}1&0&0\\0&0&1\end{bmatrix}\mathbf{x}(k)$$

(c) $\mathbf{x}(k+1)=$ same as (a)

$$y_0(k)=x_3(k)=\begin{bmatrix}0&0&1\end{bmatrix}\mathbf{x}(k)$$

(d) $$z\mathbf{I}-\mathbf{A}=\begin{bmatrix}z&-1&0\\0&z&-1\\0&1&z\end{bmatrix};\ |z\mathbf{I}-\mathbf{A}|=z^3-(-z)=z^3+z=\Delta$$

$$\mathrm{Cof}[z\mathbf{I}-\mathbf{A}] = \begin{bmatrix} z^2+1 & z^2 & 0 \\ z & z^2 & z \\ 1 & z & z^2 \end{bmatrix}; \ [z\mathbf{I}-\mathbf{A}]^{-1} = \frac{1}{\Delta}\begin{bmatrix} z^2+1 & z & 1 \\ z^2 & z^2 & z \\ 0 & z & z^2 \end{bmatrix}$$

$$\therefore \frac{Y_0(z)}{U(z)} = \mathbf{C}[z\mathbf{I}-\mathbf{A}]^{-1}\mathbf{B} = \frac{1}{\Delta}[0 \quad 0 \quad 1]\begin{bmatrix} z^2+1 & z & 1 \\ z^2 & z^2 & z \\ 0 & z & z^2 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

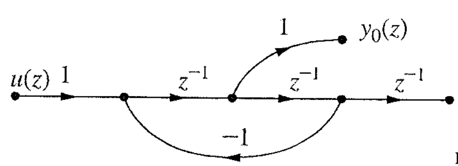
$$= \frac{1}{\Delta}\begin{bmatrix} 0 & z & z^2 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \frac{z^2}{z^3-z} = \frac{z}{z^2+1}$$

(e) $z^2Y(z) - V(z) = 0 \Rightarrow Y(z) = \dfrac{1}{z^2}V(z)$

$$zV(z) + zY(z) = zV(z) + \frac{1}{z}V(z) = U(z)$$

$$\therefore \frac{V(z)}{U(z)} = \frac{Y_0(z)}{U(z)} = \frac{1}{z+\frac{1}{z}} = \frac{z}{z^2+1}$$

(f) From (a):



make u and y capital letters

$$\therefore \frac{Y_0(z)}{U(z)} = \frac{z^{-1}}{1+z^{-2}} = \frac{z}{z^2+1}$$

3.4-2. Find $E^*(s)$ for each of the following functions. Express $E^*(s)$ in closed form.

(a) $e(t) = \varepsilon^{at}$ (b) $E(s) = \dfrac{\varepsilon^{-2Ts}}{s - a}$

(c) $e(t) = \varepsilon^{a(t-2T)}u(t - 2T)$ (d) $e(t) = \varepsilon^{a(t-T/2)}u(t - T / 2)$

Solution:

(a) $E^*(s) = 1 + \varepsilon^{aT}\varepsilon^{-Ts} + \varepsilon^{2aT}\varepsilon^{-2Ts} + L = 1 + \varepsilon^{(a-s)T} + [\varepsilon^{(a-s)T}]^2 + L$

$$= \frac{1}{1 - \varepsilon^{(a-s)T}}$$

(b) $e(t) = \varepsilon^{a(t-2T)}u(t - 2T)$

$$E^*(s) = \varepsilon^{-2Ts} + \varepsilon^{aT}\varepsilon^{-3Ts} + \varepsilon^{2aT}\varepsilon^{-4Ts} + \cdots$$

$$= \varepsilon^{-2Ts}(1 + \varepsilon^{aT}\varepsilon^{-Ts} + \varepsilon^{2aT}\varepsilon^{-2Ts} + \cdots) = \frac{\varepsilon^{-2Ts}}{1 - \varepsilon^{(a-s)T}}$$

(c) From (b), $E^*(s) = \dfrac{\varepsilon^{-2Ts}}{1 - \varepsilon^{(a-s)T}}$

(d) $E^*(s) = \varepsilon^{aT/2}\varepsilon^{-Ts} + \varepsilon^{3aT/2}\varepsilon^{-2Ts} + \varepsilon^{5aT/2}\varepsilon^{-3Ts} + \cdots$

$$E^*(s) = \varepsilon^{aT/2}\varepsilon^{-Ts}(1 + \varepsilon^{aT}\varepsilon^{-Ts} + \varepsilon^{2aT}\varepsilon^{-2Ts} + \cdots)$$

$$= \frac{\varepsilon^{aT/2}\varepsilon^{-Ts}}{1 - \varepsilon^{(a-s)T}}$$

3.4-6. Find $E^*(s)$, with $T = 0.5$ s, for

$$E(s) = \frac{\left(1 - \varepsilon^{-0.5s}\right)^2}{0.5s^2(s+1)}$$

Solution:

Consider $E_1(s) = \dfrac{1}{s^2(s+1)}$; Then $E^*(s) = E_1^*(s)[2(1-\varepsilon^{-Ts})^2]$

$$(\text{residue})\Big|_{\lambda=0} = \frac{d}{d\lambda}\left[\frac{1}{(\lambda+1)(1-\varepsilon^{-T(s-\lambda)})}\right]_{\lambda=0}$$

$$= \frac{-(1-\varepsilon^{-T(s-\lambda)}) - (-T\varepsilon^{-T(s-\lambda)})(\lambda+1)}{(\lambda+1)^2(1-\varepsilon^{-T(s-\lambda)})^2}\Bigg|_{\lambda=0} = \frac{-1+\varepsilon^{-Ts}+T\varepsilon^{-Ts}}{(1-\varepsilon^{-Ts})^2}$$

$$(\text{residue})_{\lambda=-1} = \left[\frac{1}{\lambda^2(1-\varepsilon^{-T(s-\lambda)})}\right]_{\lambda=-1} = \frac{1}{1-\varepsilon^{-T(s+1)}}$$

$$\therefore E_1^*(s) = \frac{1}{1-\varepsilon^{-T(s+1)}} + \frac{-1+\varepsilon^{-Ts}+T\varepsilon^{-Ts}}{(1-\varepsilon^{-Ts})^2}$$

$$\therefore E^*(s) = \frac{1-\varepsilon^{-Ts}-1+\varepsilon^{-Ts}+T\varepsilon^{-Ts}}{(1-\varepsilon^{-T(s+1)})(1-\varepsilon^{-Ts})^2}[2(1-\varepsilon^{-Ts})^2] = \frac{2T\varepsilon^{-Ts}}{1-\varepsilon^{-T(s+1)}}$$

3.6-1. (a) Find $E^*(s)$, for $T = 0.1$ s, for the two functions below. Explain why the two transforms are equal, first from a time-function approach, and then from a pole–zero approach.

(i) $e_1(t) = \cos(4\pi t)$

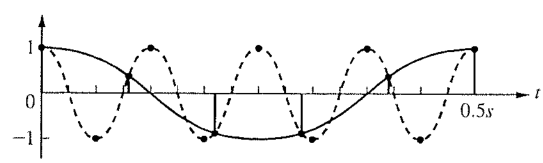
(ii) $e_2(t) = \cos(16\pi t)$

(b) Give a third time function that has the same $E^*(s)$.

Solution:

(a) (i) $\omega_1 = 4\pi,\ \omega_s = \dfrac{2\pi}{T} = 20\pi$ (ii) $\omega_2 = 16\pi = \dfrac{4\omega_s}{5} = 4\omega_1$

$\omega_1 = \dfrac{\omega_s}{5},\ T_1 = 0.5$

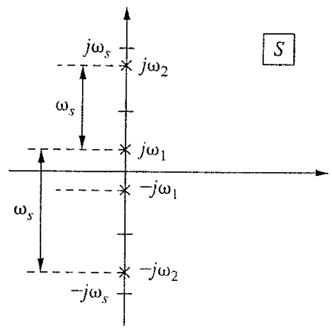


$\cos 72^\circ = 0.309 \quad \cos(4\times 72^\circ) = 0.309$

$\cos 144^\circ = -0.809 \quad \cos(4\times 144^\circ) = -0.809$

$\cos 216^\circ = -0.809 \quad \cos(4\times 216^\circ) = -0.809$

$\cos 280^\circ = 0.309 \quad \cos(4\times 280^\circ) = 0.309$



$$\omega_1 = \omega_s - \omega_2$$

$$-\omega_1 = -\omega_s + \omega_2$$

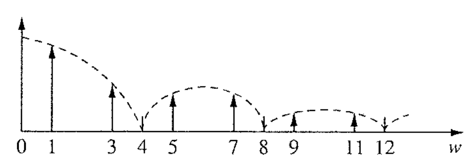
(b) $\omega_3 = \omega_s + \omega_1 = 20\pi + 4\pi = 24\pi \Rightarrow e_3(t) = \cos\,(24\pi t)$

3.7-5. A signal $e(t) = 4\sin 7t$ is applied to a sampler/zero-order-hold device, with $\omega = 4$ rad/s.

(a) What is the frequency component in the output that has the largest amplitude?
(b) Find the amplitude and phase of that component.
(c) Sketch the input signal and the component of part (b) versus time.
(d) Find the ratio of the amplitude in part (b) to that of the frequency component in the output at $\omega = 7$ rad/s (the input frequency).

Solution:

(a) Frequencies in $\bar{e}(t)$: 7; $4 \pm 7 = -3, 10$; $8 \pm 7 = 1, 15$; $12 \pm 7 = 5, 19$



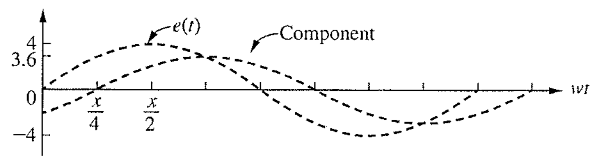
$\omega = 1$

(b)

$$\frac{1}{T} G_{h0}(j1) = \frac{\sin\left(\frac{\pi(1)}{4}\right)}{\pi(1)/4} e^{-j\pi/4} = 0.900 \angle -45^\circ$$

$\therefore$ component $= (4)(0.9)\sin(t - 45^\circ) = 3.6\sin(t - 45^\circ)$

(c)



(d) $\frac{1}{T}G_{h0}(j7) = \frac{\sin\left(\pi \frac{7}{4}\right)}{\pi \frac{7}{4}} e^{-j\pi \frac{7}{4}} = (-0.129)e^{-j315^\circ} = 0.129\angle -135^\circ$

$\therefore$ component $= 4(0.129)\sin(7t - 135^\circ) = 0.516\sin(7t - 135^\circ)$

$\text{ratio} = \frac{3.6}{0.516} = 6.977 \approx 7$