3.3-1. A signal $e(t)$ is sampled by the ideal sampler as specified by (3-3).

(a) List the conditions under which $e(t)$ can be completely recovered from $e^*(t)$, that is, the conditions under which *no* loss of information by the sampling process occurs.

(b) State which of the conditions listed in part (a) can occur in a physical system. Recall that the ideal sampler itself is not physically realizable.

(c) Considering the answers in part (b), state why we can successfully employ systems that use sampling.

Solution:

(a) 1. No frequencies in $e(t)$ greater than $w_s/2$.

2. An ideal low-pass filter follows the sampler.

(b) None

(c) The signal can be recovered to a sufficient degree of accuracy.

3.4-2. Find $E^*(s)$ for each of the following functions. Express $E^*(s)$ in closed form.

(a) $e(t) = \varepsilon^{at}$ (b) $E(s) = \dfrac{\varepsilon^{-2Ts}}{s-a}$

(c) $e(t) = \varepsilon^{a(t-2T)}u(t-2T)$ (d) $e(t) = \varepsilon^{a(t-T/2)}u(t-T/2)$

Solution:

(a) $E^*(s) = 1 + \varepsilon^{aT}\varepsilon^{-Ts} + \varepsilon^{2aT}\varepsilon^{-2Ts} + \mathrm{L} = 1 + \varepsilon^{(a-s)T} + [\varepsilon^{(a-s)T}]^2 + \mathrm{L}$

$$= \frac{1}{1-\varepsilon^{(a-s)T}}$$

(b) $e(t) = \varepsilon^{a(t-2T)}u(t-2T)$

$$E^*(s) = \varepsilon^{-2Ts} + \varepsilon^{aT}\varepsilon^{-3Ts} + \varepsilon^{2aT}\varepsilon^{-4Ts} + \cdots$$

$$= \varepsilon^{-2Ts}(1 + \varepsilon^{aT}\varepsilon^{-Ts} + \varepsilon^{2aT}\varepsilon^{-2Ts} + \cdots) = \frac{\varepsilon^{-2Ts}}{1-\varepsilon^{(a-s)T}}$$

(c) From (b), $E^*(s) = \dfrac{\varepsilon^{-2Ts}}{1-\varepsilon^{(a-s)T}}$

(d) $E^*(s) = \varepsilon^{aT/2}\varepsilon^{-Ts} + \varepsilon^{3aT/2}\varepsilon^{-2Ts} + \varepsilon^{5aT/2}\varepsilon^{-3Ts} + \cdots$

$$E^*(s) = \varepsilon^{aT/2}\varepsilon^{-Ts}(1 + \varepsilon^{aT}\varepsilon^{-Ts} + \varepsilon^{2aT}\varepsilon^{-2Ts} + \cdots)$$

$$= \frac{\varepsilon^{aT/2}\varepsilon^{-Ts}}{1-\varepsilon^{(a-s)T}}$$

3.4-4. Express the starred transform of $e(t-kT)\,u(t-kT)$, k an integer, in terms of $E^*(s)$, the starred transform of $e(t)$. Base your derivation on (3-3).

Solution:

$$E^*(s) = e(0) + e(T)\varepsilon^{-Ts} + e(2T)\varepsilon^{-2Ts} + \cdots = [e(t)]^*$$

Let: $e_1(t) = e(t-kT)u(t-kT)$

$$\therefore E_1^*(s) = e(0)\varepsilon^{-kTs} + e(T)\varepsilon^{-(k+1)Ts} + \cdots$$

$$= \varepsilon^{-kTs}[e(0) + e(T)\varepsilon^{-Ts} + \cdots] = \varepsilon^{-kTs}E^*(s)$$

$\therefore$ from (3-14),

$$E_1^*(s) = \varepsilon^{-kTs}\left[\sum_{\substack{\text{poles}\\ \text{of } E(\lambda)}} \text{residues of } \frac{E(\lambda)}{1-\varepsilon^{-T(s-\lambda)}}\right]$$

3.6-1. (a) Find $E^*(s)$, for $T = 0.1$ s, for the two functions below. Explain why the two transforms are equal, first from a time-function approach, and then from a pole–zero approach.

(i) $e_1(t) = \cos(4\pi t)$

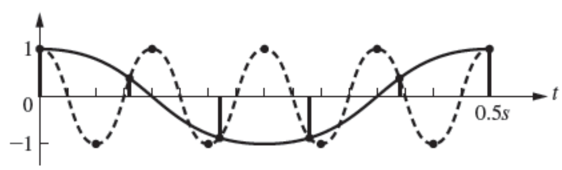
(ii) $e_2(t) = \cos(16\pi t)$

(b) Give a third time function that has the same $E^*(s)$.

Solution:

(a) (i) $\omega_1 = 4\pi,\ \omega_s = \dfrac{2\pi}{T} = 20\pi$ (ii) $\omega_2 = 16\pi = \dfrac{4\omega_s}{5} = 4\omega_1$

$\omega_1 = \dfrac{\omega_s}{5},\ T_1 = 0.5$

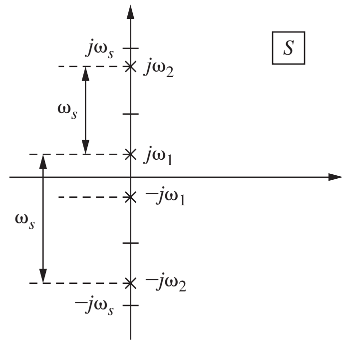


$\cos 72^\circ = 0.309 \quad \cos(4\times 72^\circ) = 0.309$

$\cos 144^\circ = -0.809 \quad \cos(4\times 144^\circ) = -0.809$

$\cos 216^\circ = -0.809 \quad \cos(4\times 216^\circ) = -0.809$

$\cos 280^\circ = 0.309 \quad \cos(4\times 280^\circ) = 0.309$



$$\omega_1 = \omega_s - \omega_2$$

$$-\omega_1 = -\omega_s + \omega_2$$

(b) $\omega_3 = \omega_s + \omega_1 = 20\pi + 4\pi = 24\pi \Rightarrow e_3(t) = \cos\,(24\pi t)$

3.7-8. A polygonal data hold is a device that reconstructs the sampled signal by the straight-line approximation shown in Fig. P3.7-8. Show that the transfer function of this data hold is

$$G(s) = \frac{\varepsilon^{Ts}\left(1 - \varepsilon^{-Ts}\right)^2}{Ts^2}$$

Is this data hold physically realizable?

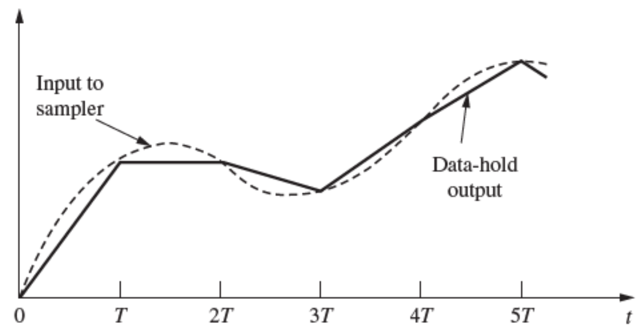
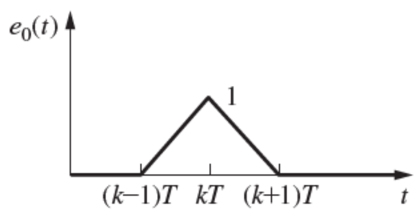


Fig. P3.7-8

Solution:

assume: input $= e_i(t) = \delta(t-kT), \ \therefore E_i(s) = \varepsilon^{-kTs}$

Then



$$\therefore e_0(t) = \frac{1}{T}\{t-(k-1)T\}u[t-(k-1)T] - \frac{2}{T}\{t-kT\}u[t-kT] + \frac{1}{T}\{t-(k+1)T\}u[t-(k+1)T]$$

$$\therefore E_0(s) = \frac{1}{Ts^2}\varepsilon^{-(k-1)Ts} - \frac{2}{Ts^2}\varepsilon^{-kTs} + \frac{1}{Ts^2}\varepsilon^{-(k+1)Ts}$$

$$\therefore G_{ha}(s) = \frac{E_0(s)}{E_i(s)} = \frac{\varepsilon^{Ts}}{Ts^2}(1-\varepsilon^{-Ts})^2$$

Not physically realizable, since an advance in time is required.

4.3-11. The antenna positioning system described in Section 1.5 and Problem 1.5-1 is depicted in Fig. P4.3-11. In this problem we consider the yaw angle control system, where $\theta(t)$ is the yaw angle. The angle sensor (a digital shaft encoder and the data hold) yields $v_o(kT) = \left[0.4\ \theta(kT)\right]$, where the units of $v_o(t)$ are volts and $\theta(t)$ are degrees. The sample period is $T = 0.05$ s.

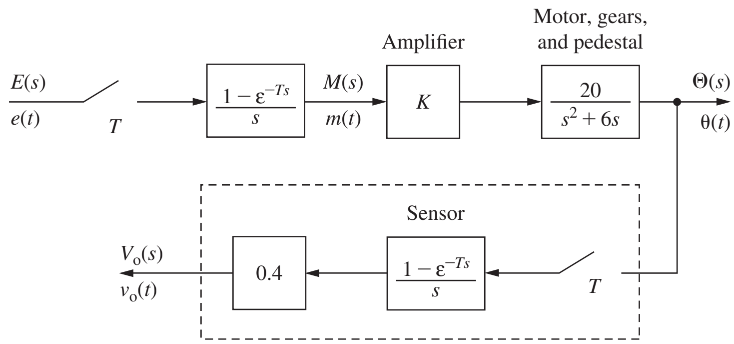


Fig. P4.3-11

(a) Find the transfer function $\Theta(z)/E(z)$.

(b) The yaw angle is initially zero. The input voltage $e(t)$ is set equal to 10 V at $t = 0$, and is zero at each sample period thereafter. Find the steady-state value of the yaw angle.

(c) Note that in part (b), the coefficients in the partial-fraction expansion add to zero. Why does this occur?

(d) The input voltage $e(t)$ is set to a constant value. Without solving mathematically, give a description of the system response.

(e) Suppose in part (d) that you are observing the antenna. Describe what you would see.

Solution:

(a) $$KG(z) = \frac{z-1}{z} K \mathfrak{z}\left[\frac{20}{s^2(s+6)}\right]$$

$$= \left(\frac{z-1}{z}\right)\frac{20K}{36}\left[\frac{z(0.3-1+0.7408)+(1-0.7408-0.2222)}{(z-1)^2(z-\varepsilon^{-6T})}\right]$$

$$= K\left[\frac{0.02268z+0.02052}{(z-1)(z-0.7408)}\right]$$

(b) $E(z) = 10$

$$\therefore \frac{C(z)}{z} = \frac{10K(0.02268z+0.02052)}{z(z-1)(z-0.7408)} = \frac{0.277K}{z} + \frac{1.67K}{z-1} + \frac{-1.94K}{z-0.7408}$$

$$c(kT) = 0.277K\delta(k) + 1.67K - 1.94K(0.7408)^k$$

$$\therefore c_{ss}(kT) = 1.67K$$

(c) In (b), $$C(z) = \frac{az+\dots}{z^2+\dots} = az^{-1}+\dots = c(0)+c(T)z^{-1}+\dots$$

$$\therefore c(0) = 0$$

In $c(kT)$ in (b), $c(0) = K[0.277 + 1.67 - 1.94] \approx 0$

(d) A constant voltage is applied to the motor. Thus the motor speed increases to a constant value, and the shaft angle $\theta(t)$ is a ramp voltage.

(e) From (d), the antenna rotates at a constant rpm.