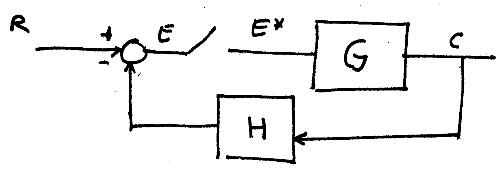
CLOSED LOOP SYSTEMS (Chapter 5)

So far (chapter 4) we have considered only open-loop systems. The next step is to look into what happens when you close the loop.

Example:



Want to find closed loop transfer function: $\Rightarrow$

$$E = R - HC = R - (HG)E^* \Rightarrow E^* = R^* - (HG)^* E^* \Rightarrow$$

$$E^* = \frac{R^*}{1+[HG]^*} \quad \text{and} \quad C = \frac{GR^*}{1+(HG)^*}$$

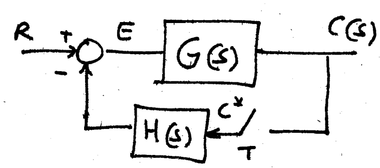
However, if we had selected C instead of E as variable we get

$$C = GE^*$$
$$E = R - HC \Rightarrow E^* = R^* - [HC]^* \Rightarrow C = GR^* - G[HC]^*$$
$$C^* = G^*R^* - G^*(HC)^*$$

Now we are stuck!! we can't proceed because we can't factor C^* out of $[HC]^*$.

Need a systematic way to avoid these problems. (even if a TF does not exist, we'd like to be able to compute the output)

Example 2:



$$E = R - HC^*$$
$$C = GR - GHC^*$$
$$C^* = (GR)^* - (GH)^* C^*$$

$$\Rightarrow C^* = \frac{(GR)^*}{1+(GH)^*} \quad \text{or} \quad C(z) = \frac{[GR](z)}{1+(GH)(z)}$$

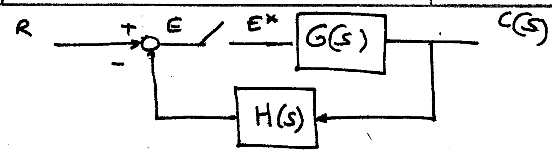
On the other hand, if we select E as a variable we have:

$$E = R - HC^*, \quad C = GE$$
$$C^* = (GE)^*$$

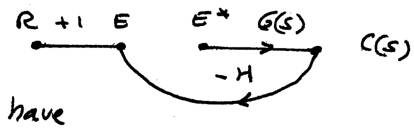
$$\Rightarrow E = R - H[GE]^* \quad \text{or} \quad E^* = R^* - H^*[GE]^* \quad \text{and}$$

stuck again!! (can't solve for E^*)

General Procedure



(1) Find the "original" flow graph:



Note that since the sampler does not have a T.F, it is not in the graph.

(2) Assign a variable to each <u>sampler output</u>

(3) Consider each sampler output as a <u>source</u>. Find the <u>sampler inputs</u> and the <u>systems outputs</u> in terms of each sampler output and the system input

(4) Take the starred transform of these equations and solve

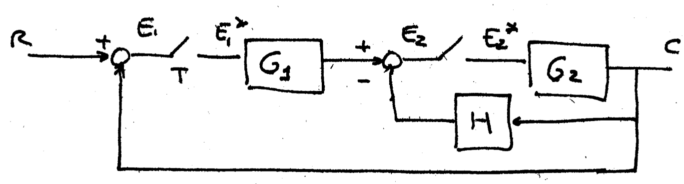
<u>Example</u>:

(2) sampler output: E^*
(3) sampler input: $E = R - (HG)\,E^*$
System output: $C = G(s)\,E^*$

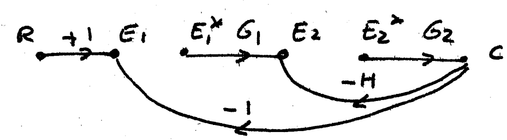
(4) $E^* = R^* - (HG)^* E^* \Rightarrow E^* = \dfrac{R^*}{1+(GH)^*}$,

$C^* = G^* E^* \Rightarrow C^* = \dfrac{G^*}{1+(GH)^*} R^*$

<u>Example 2</u>:



(1) "original" flow graph:



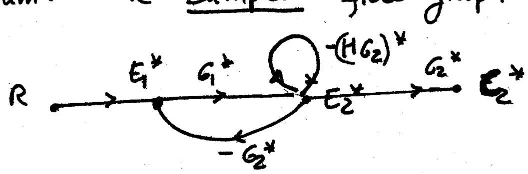
(2) variables: E_1^*, E_2^*

(3) equations:

$$\left.\begin{aligned} E_1 &= R - G_2 E_2^* \\ E_2 &= G_1 E_1^* - HG_2\, E_2^* \\ C &= G_2 E_2^* \end{aligned}\right. \Rightarrow \left.\begin{aligned} E_1^* &= R^* - G_2^* E_2^* \\ E_2^* &= G_1^* E_1^* - (HG_2)^* E_2^* \\ C^* &= G_2^* E_2^* \end{aligned}\right\} \text{need to solve these}$$

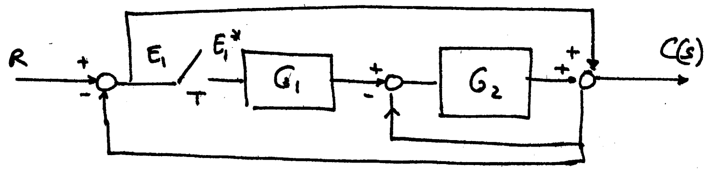
How do we solve these equations? (1) use Cramer's rule (tedious)
(2) Use Mason's formula

To use Mason's formula we need to find the corresponding flow graph diagram: the <u>sampled</u> flow graph

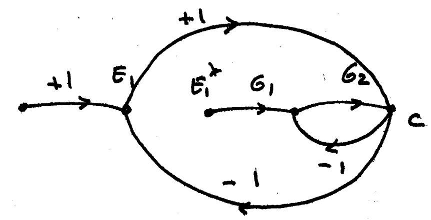


$$\Delta = 1 + G_1^* G_2^* + (H G_2)^* \Rightarrow \boxed{\frac{C^*}{R^*} = \frac{G_1^* G_2^*}{1 + G_1^* G_2^* + (G_2 H)^*}}$$

Example 3:



(1) original graph:



(2) variables: E_1^*

(3) need to find E_1, C as functions of R, E_1^* $\Rightarrow$ use Mason's formula on the "original" graph:

$$\Delta = 1 + 1 + G_2 = 2 + G_2$$

$$\frac{E_1}{R} = \frac{1 \cdot (1+G_2)}{2+G_2}; \quad \frac{E_1}{E_1^*} = \frac{-G_1 G_2}{2+G_2} \Rightarrow \boxed{E_1 = \frac{(1+G_2)R}{2+G_2} - \frac{G_1 G_2}{2+G_2} E_1^*} \quad (1)$$

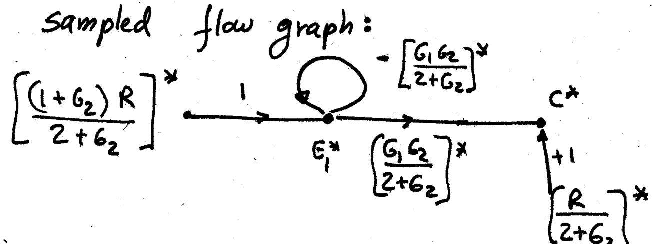
$$\frac{C}{R} = \frac{1}{2+G_2}; \quad \frac{C}{E_1^*} = \frac{G_1 G_2}{2+G_2} \Rightarrow \boxed{C = \frac{1}{(2+G_2)} R + \frac{G_1 G_2}{2+G_2} E_1^*} \quad (2)$$

(4) Take starred transforms of (1) and (2):

$$E_1^* = \left[\frac{(1+G_2)R}{2+G_2}\right]^* - \left[\frac{G_1 G_2}{2+G_2}\right]^* E_1^*$$

$$C^* = \left[\frac{R}{2+G_2}\right]^* + \left[\frac{G_1 G_2}{2+G_2}\right]^* E_1^*$$

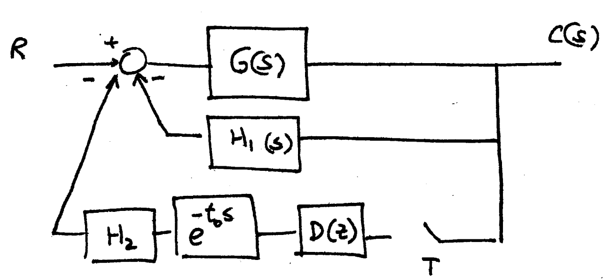
(5) sampled flow graph:



$$\Rightarrow \boxed{C^* = \left[\frac{R}{2+G_2}\right]^* + \frac{\left[\frac{G_1 G_2}{2+G_2}\right]^*}{1 + \left[\frac{G_1 G_2}{2+G_2}\right]^*} \left[\frac{(1+G_2)R}{2+G_2}\right]^*}$$

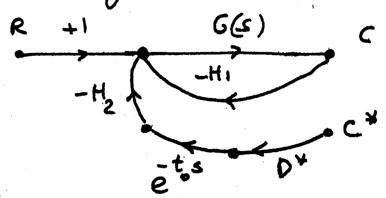
Note: No transfer function exists from $R(z)$ to $C(z)$ since you can't factor R^* out. This was expected since the continuous time input $R(s)$ goes into the block $G_2(s)$ without going through a sampler

Example 3:



A system containing a digital controller and non-negligible computation time:

(1) (Assume $0 \le t_0 < T$).



(2) variable: C^*

(3) $$C = \left(\frac{G}{1+GH_1}\right) R - \frac{G H_2 e^{-t_0 s} D^*}{1+GH_1} C^*$$

(4) $$C^* = \left[\frac{GR}{1+GH_1}\right]^* - \left(\frac{GH_2 e^{-t_0 s}}{1+GH_1}\right)^* D^* C^*$$

modified z transform

$$C(z) = \left[\frac{GR}{1+GH_1}\right](z) - \left[\frac{GH_2}{1+GH_1}\right](z,m) \cdot D(z)\, C(z)$$

with $t_0 = \Delta T$
$m = 1 - \Delta$
$mT = T - t_0$

Solving this equation for $C(z)$ yields:

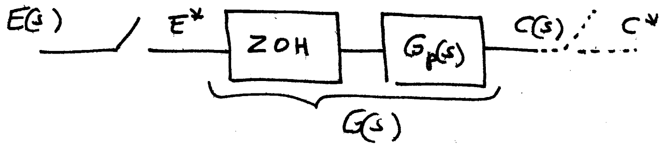
$$C(z) = \frac{\left[\frac{GR}{1+GH_1}\right](z)}{1 + \left[\frac{GH_2}{1+GH_1}\right](z,m)\, D(z)}$$

Idea of the system: wrap a stabilizing (continuous time) controller around the plant to guarantee stability.

Use the digital filter to recover performance

Summary

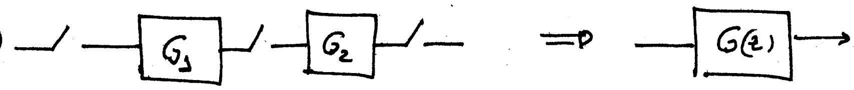
Pulse Transfer function for sample data systems:



$$C^* = G^*(s)\,E^*(s)$$

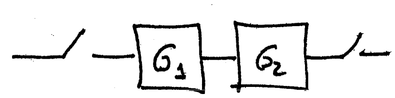
$$G(z) = \mathcal{Z}\,[G(s)]$$

If we have: (a)



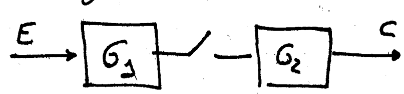
with $G(z) = G_1(z)\cdot G_2(z)$

(b)



$$\Rightarrow G(z) = \mathcal{Z}\,[G_1\cdot G_2] \neq G_1\cdot G_2$$

- Not all systems have a pulse transfer function:



This system <u>does not</u> have a pulse transfer function

- To get pulse T.F. you need to select as variables the inputs to the ideal samplers.