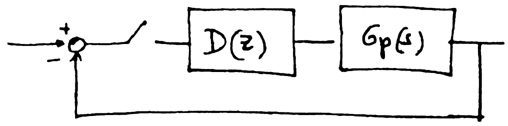
Compensation (Section 8.3)

In some cases using a P controller (ie $D(z) = k$) is not sufficient to achieve the performance specifications (sometimes even to stabilize the system)

In these cases we need to add one or more dynamic blocks to modify (hopefully improve) the behavior of the system. This process is called compensation



Assume for the time being that we use only first order compensators. Then

$$D(z) = k\left(\frac{z - z_0}{z - z_p}\right)$$

depending on the relative location of the pole and zero of the compensator we get different effects

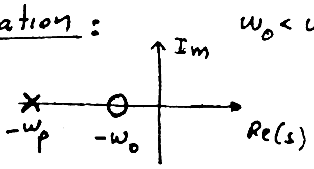
- Review of compensation for continuous time systems:

Recall from ECE 580 that for continuous time systems we have 2 basic types of compensation:

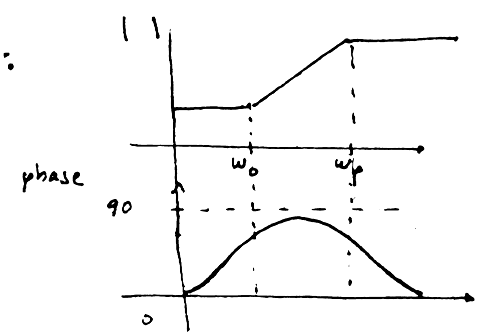
$$D(s) = A\frac{(1 + s/w_0)}{(1 + s/w_p)}$$ where A = DC gain; w_0, w_p location of zero & pole respectively

- Phase lead compensation: $w_0 < w_p$

Root Locus diagram



Bode plot:



← adds phase to the system, hence the name phase lead.

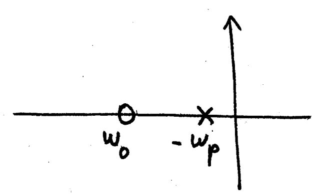
Effects of phase lead:

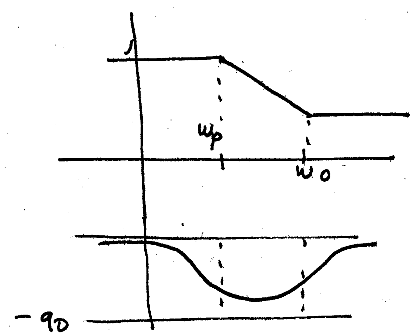
- Moves RL towards the left $\Rightarrow$ { stabilizing; yields faster systems; improves transient }

Potential drawback: increases high frequency gain

- **Phase lag compensation:**

$\omega_0 > \omega_p$





$\leftarrow$ adds $-$ phase to the system

Effects of phase lag:

Since it adds negative phase, it reduces the phase margin (equivalently, it moves the R.L. towards the right)

$\Rightarrow$ destabilizing effect

Phase lag: improves steady state but degrades stability

To minimize the destabilizing effect, usually ω_p and ω_0 are chosen very small, i.e the pole & zero are located both very close to the origin (but $\omega_0 \cong 10\,\omega_p$). Need to make sure that this does not introduce a very slow mode

- **Use of phase lead & lag compensators:**
  - Use phase lead when the RL needs to be moved towards the left to improve stability and/or transient
  - Use phase lag when it is necessary to improve steady state

Care must be exercised when using phase lag to keep the destabilizing effects to a minimum

- If you need to improve <u>both</u> transient & steady state you can combine a phase lead and a phase lag stage, leading to a <u>lead-lag</u> compensator.

- <u>Compensation of discrete time systems</u>:

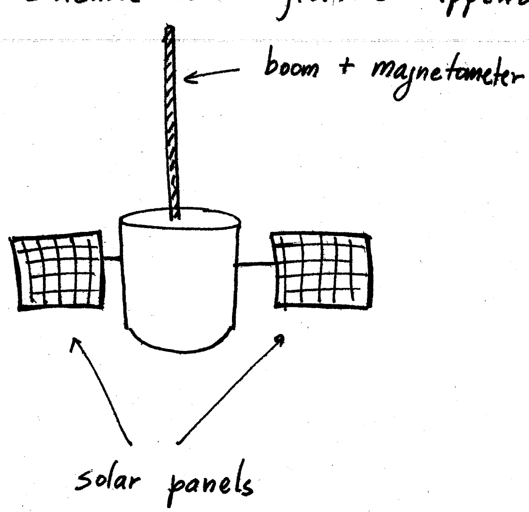
Q: How do we use these tools in discrete time systems?

A: (1) Use a bilinear transformation: $z = \dfrac{1 + \frac{wT}{2}}{1 - \frac{wT}{2}}$ to go to the "w-plane" and carry out the design there

(2) Alternative: derive the discrete time equivalent of phase lead and phase lag

- <u>Example of compensation in the w-plane</u>

Consider the problem of designing an attitude controller for a satellite with flexible appendages

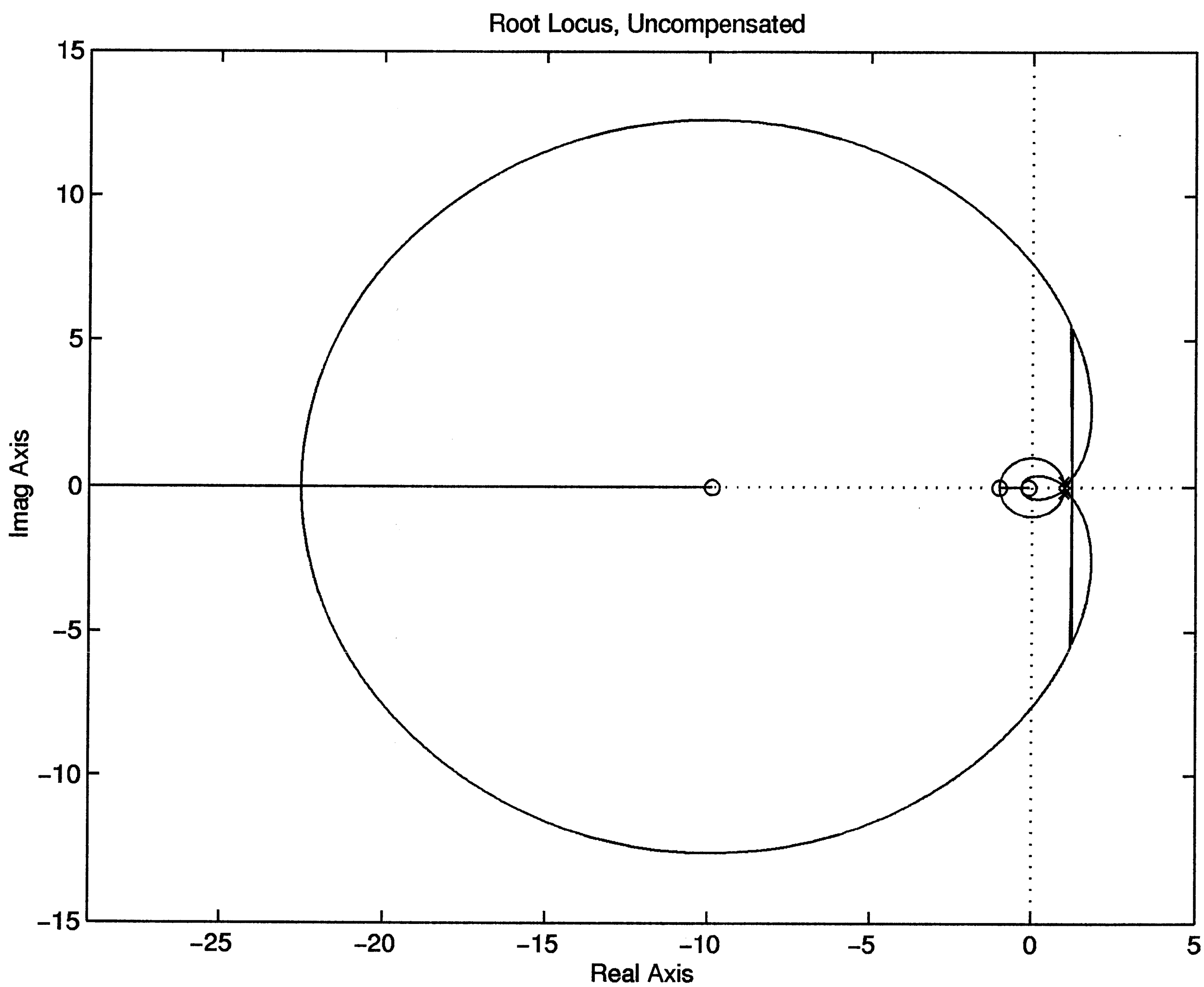


Due to the presence of the solar panels and the magnetometers the satellite cannot be modeled as a rigid body: we need to include at least the first flexible mode:

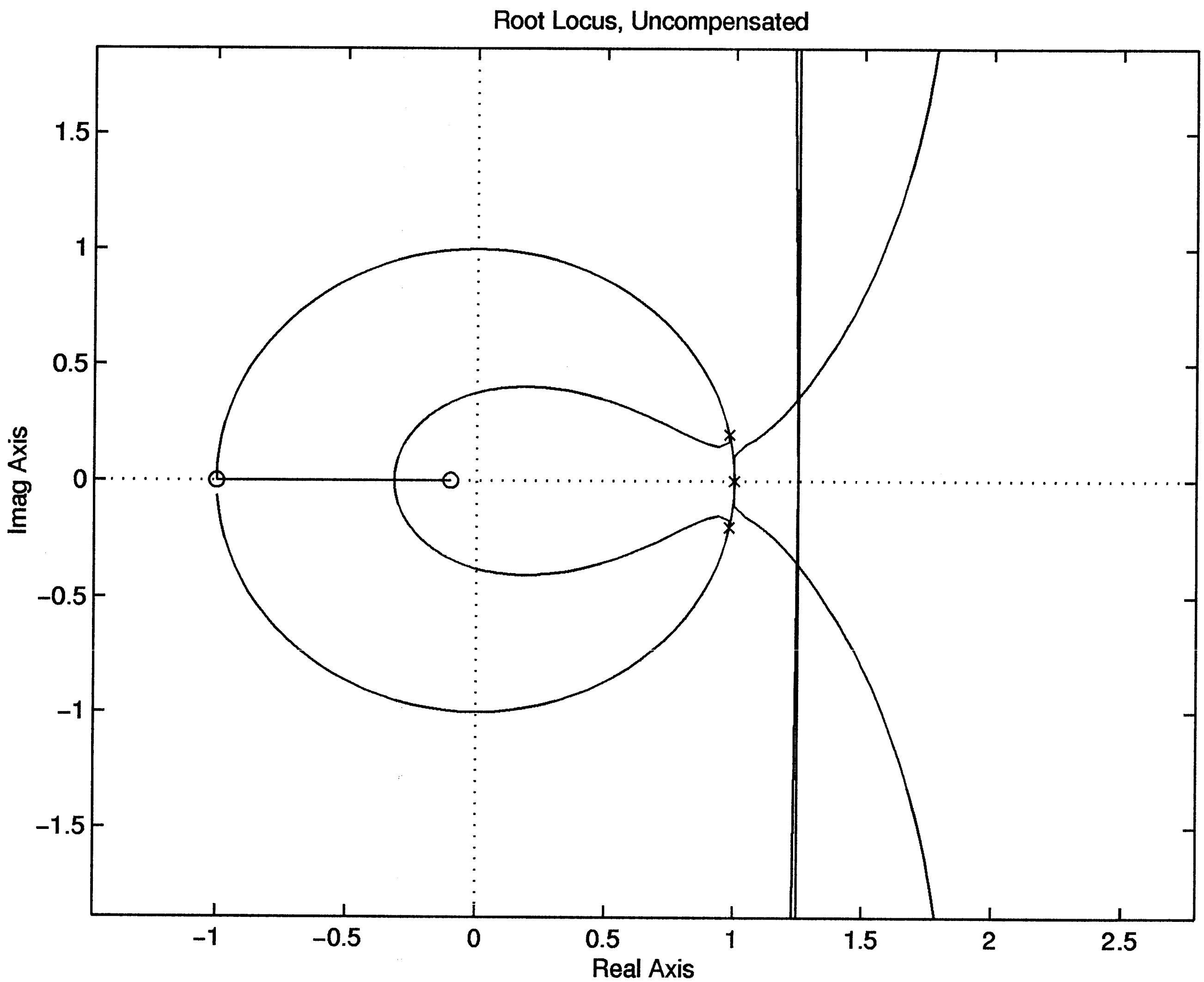
$$\Rightarrow G_p(s) = \frac{1}{s^2(s^2+1)}$$

s^2: rigid body mode; (s^2+1): first mode (freq ~ 1 Hz)

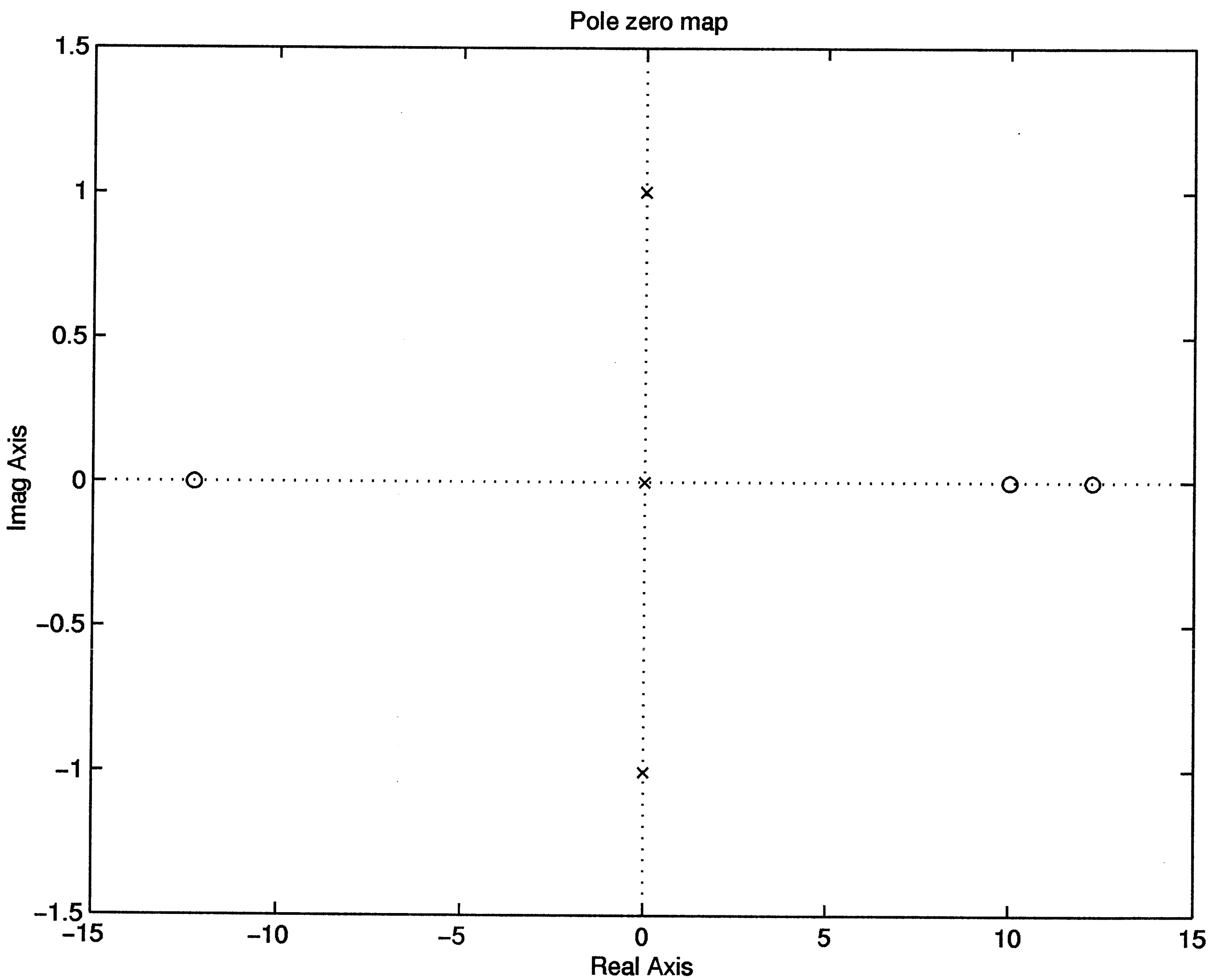
Goal: design a position controller to keep the satellite pointed at a desired location (with zero steady state error)



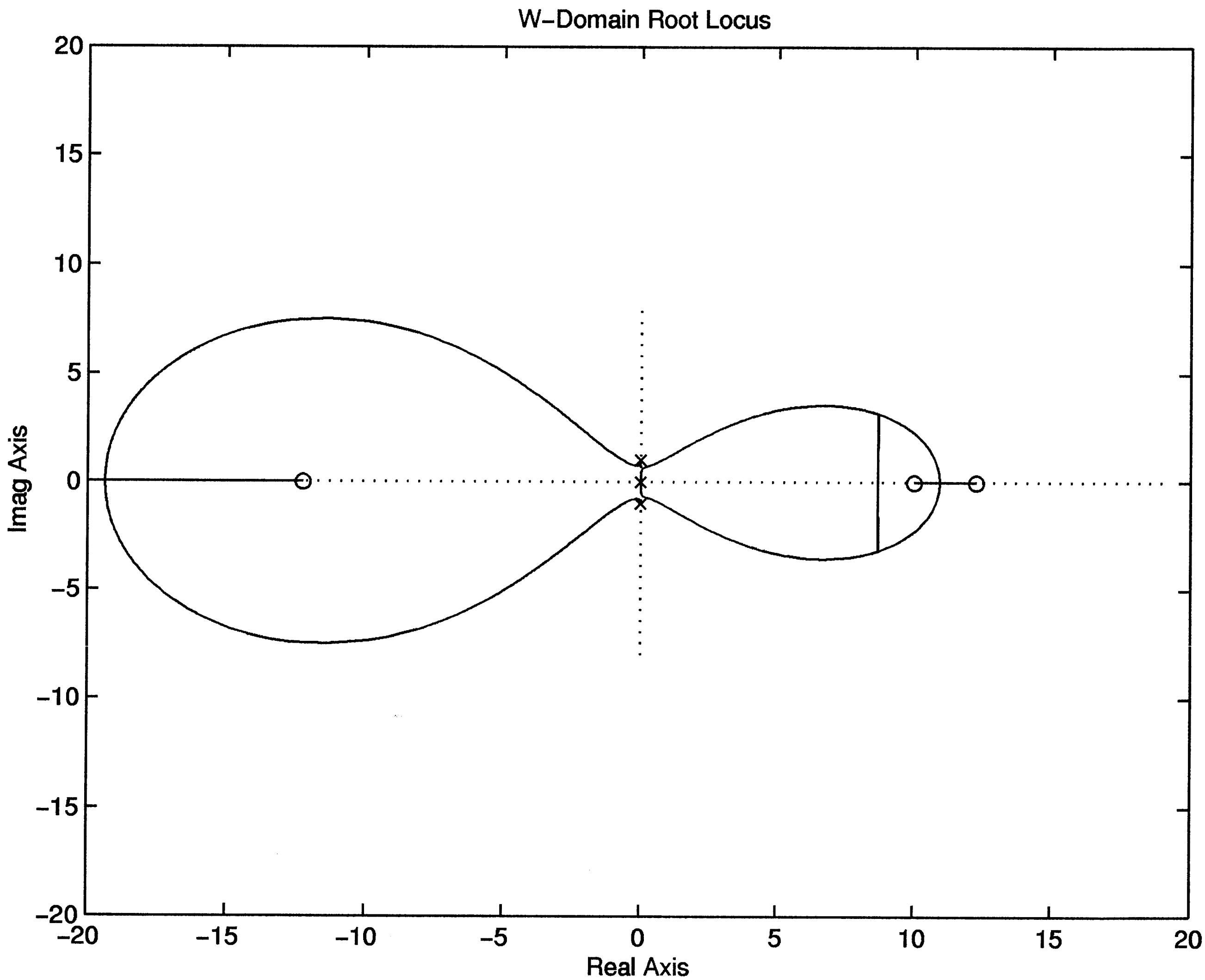
Root Locus, Uncompensated
15
10
5
0
-5
-10
-15
Imag Axis
-25
-20
-15
-10
-5
0
5
Real Axis



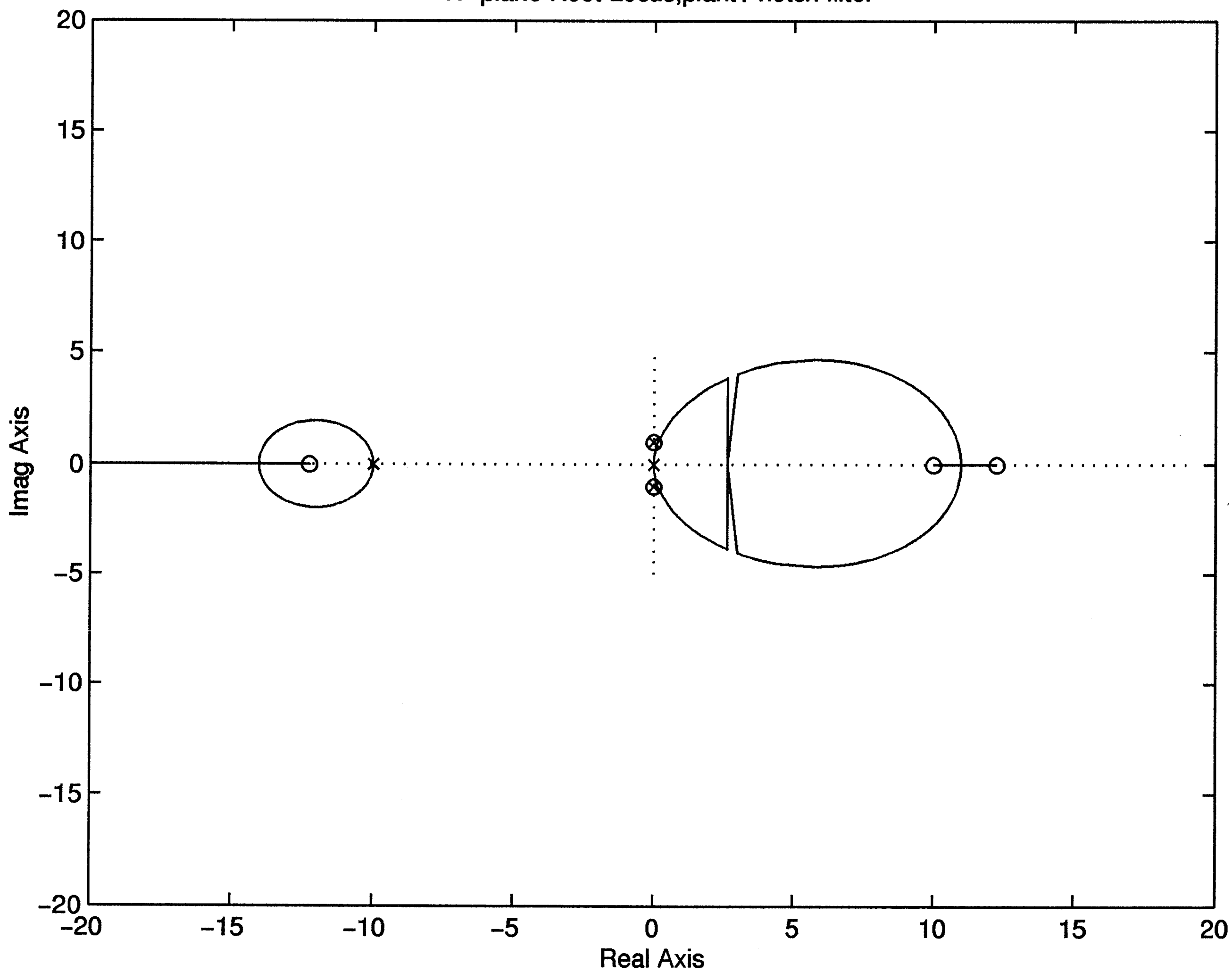
Root Locus, Uncompensated
1.5
1
0.5
0
−0.5
−1
−1.5
Imag Axis
−1
−0.5
0
0.5
1
1.5
2
2.5
Real Axis

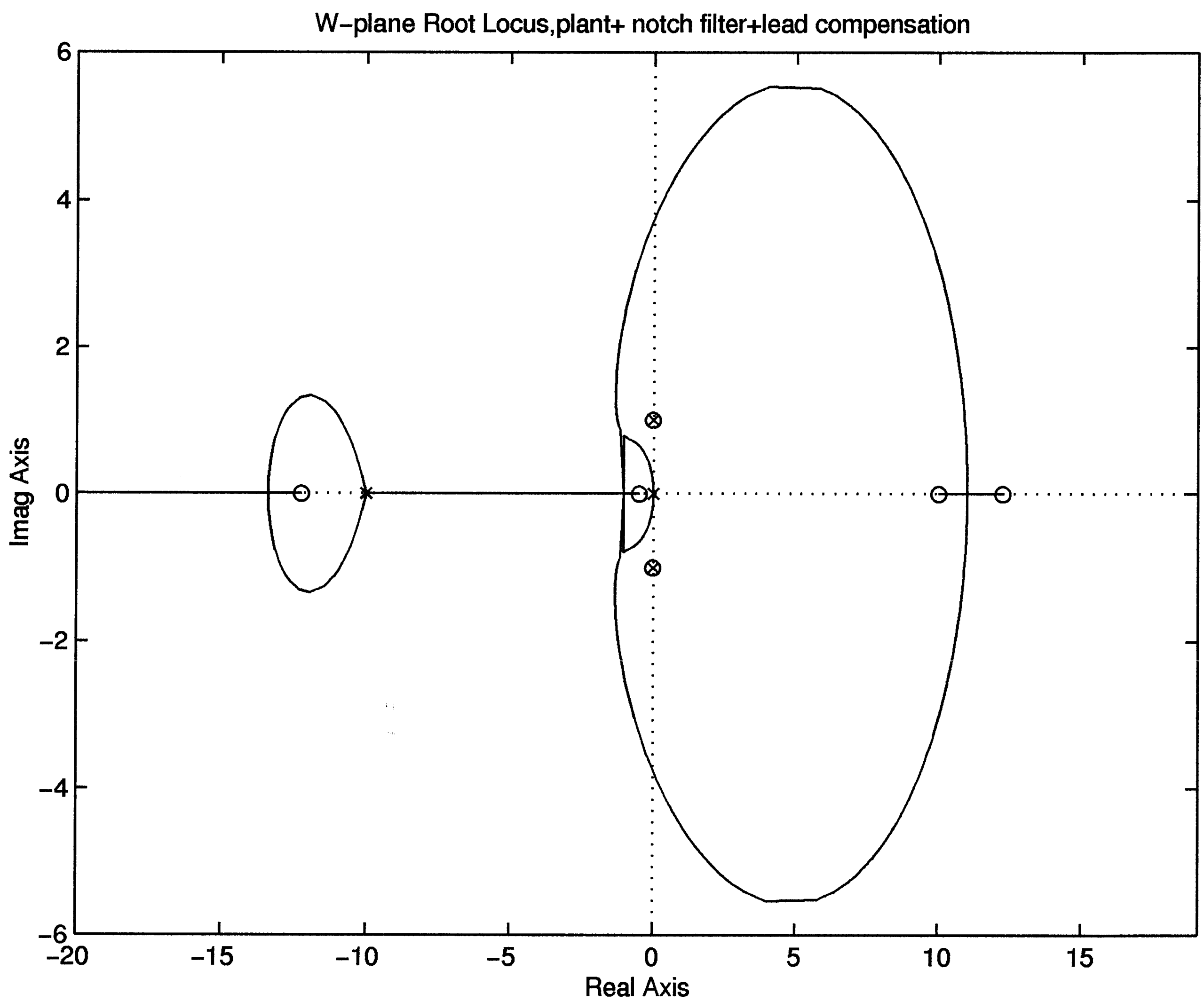


Pole zero map
1.5
1
0.5
0
-0.5
-1
-1.5
Imag Axis
-15
-10
-5
0
5
10
15
Real Axis

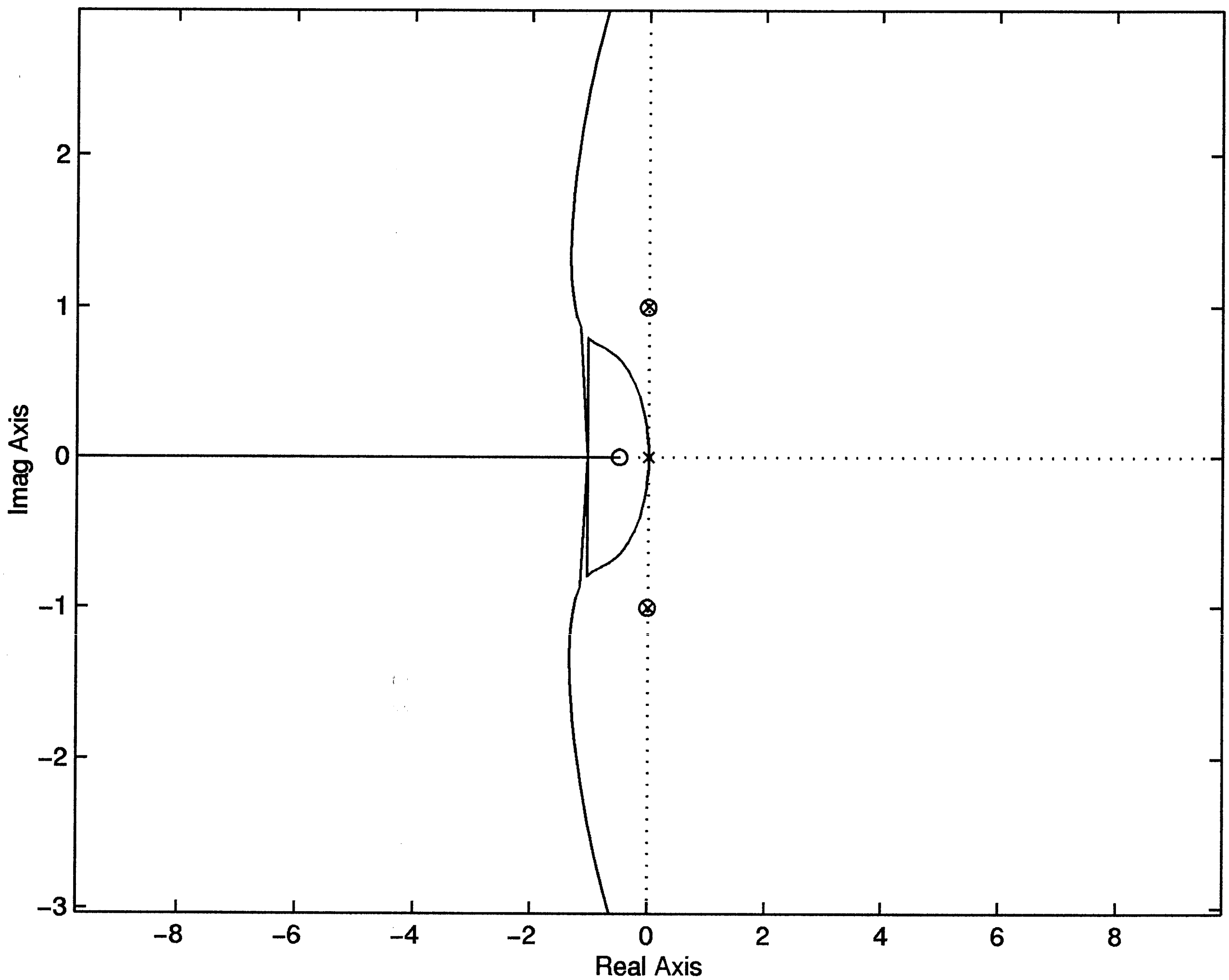

W-Domain Root Locus
20
15
10
5
0
-5
-10
-15
-20
Imag Axis
-20
-15
-10
-5
0
5
10
15
20
Real Axis

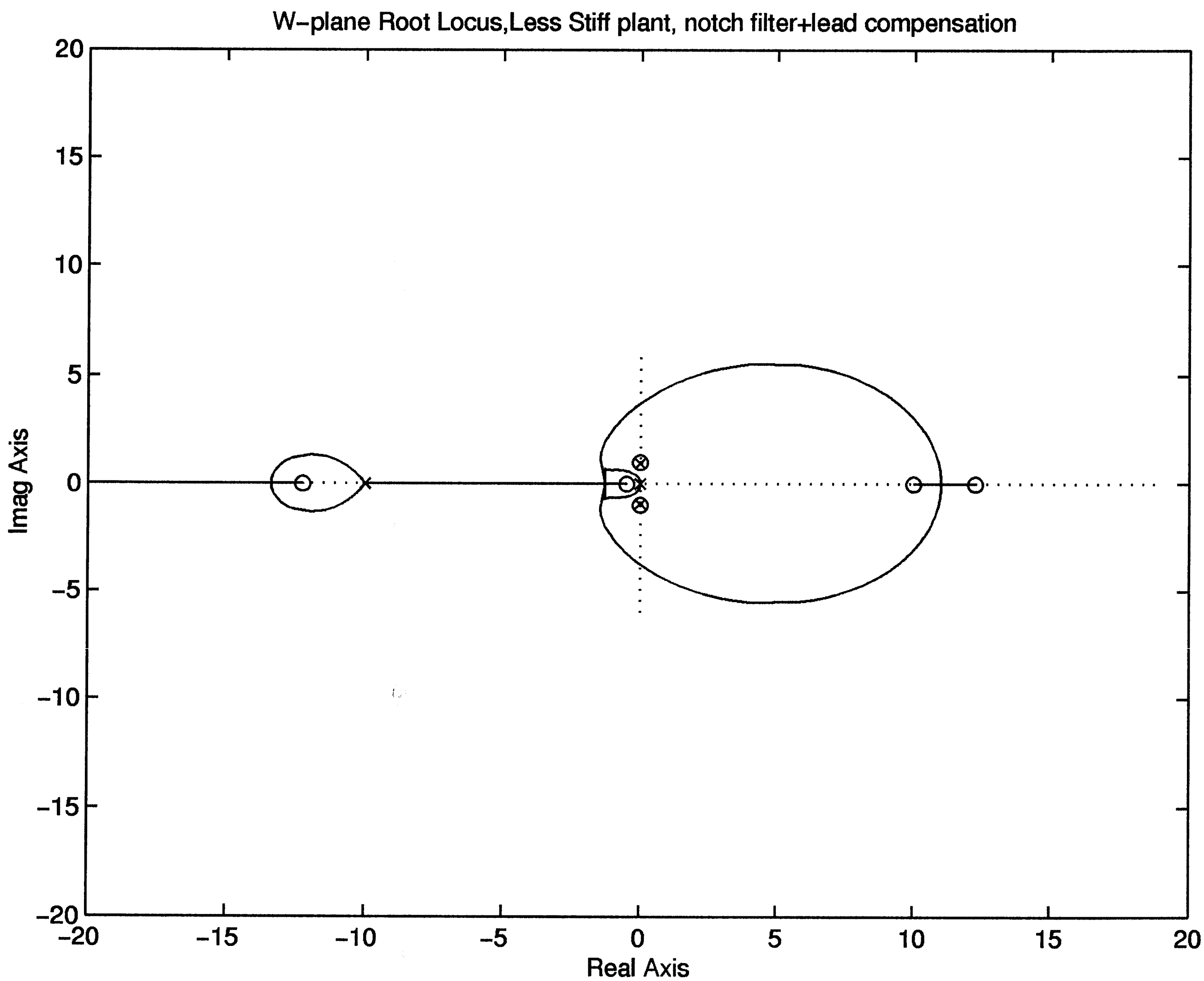
W-plane Root Locus,plant+ notch filter



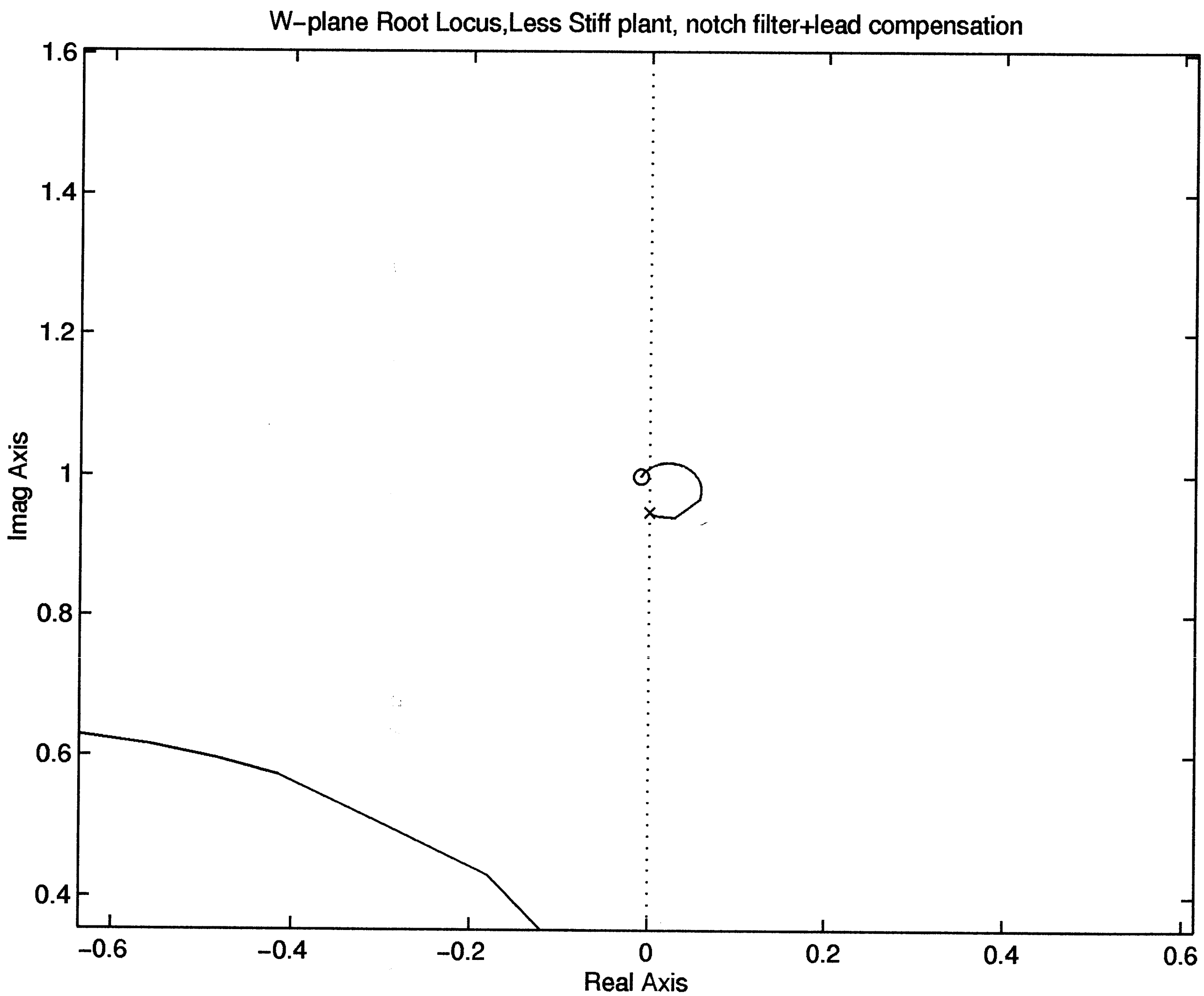

W-plane Root Locus,plant+ notch filter+lead compensation
6
4
2
0
-2
-4
-6
Imag Axis
-20
-15
-10
-5
0
5
10
15
Real Axis

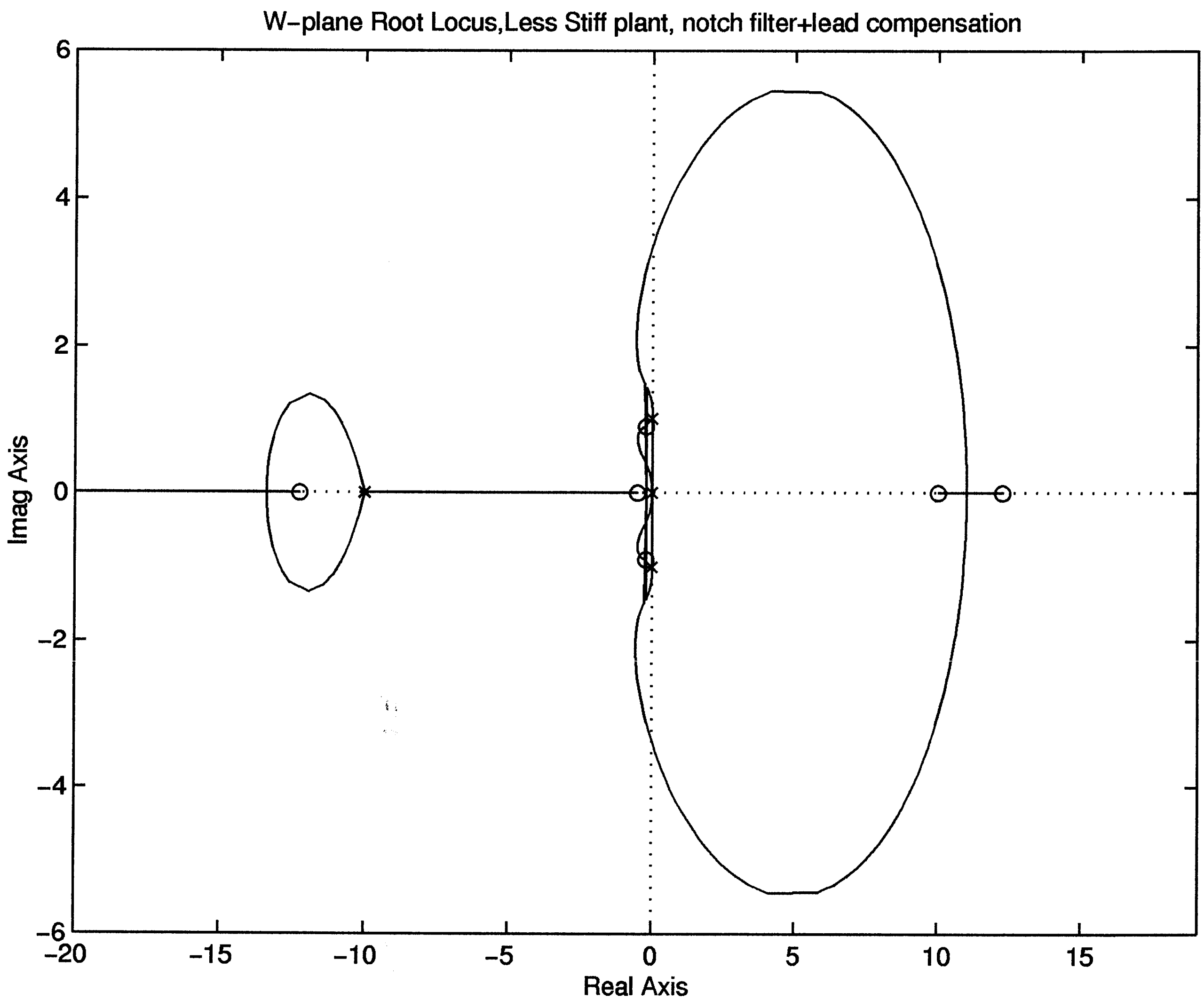
W–plane Root Locus,plant+ notch filter+lead compensation

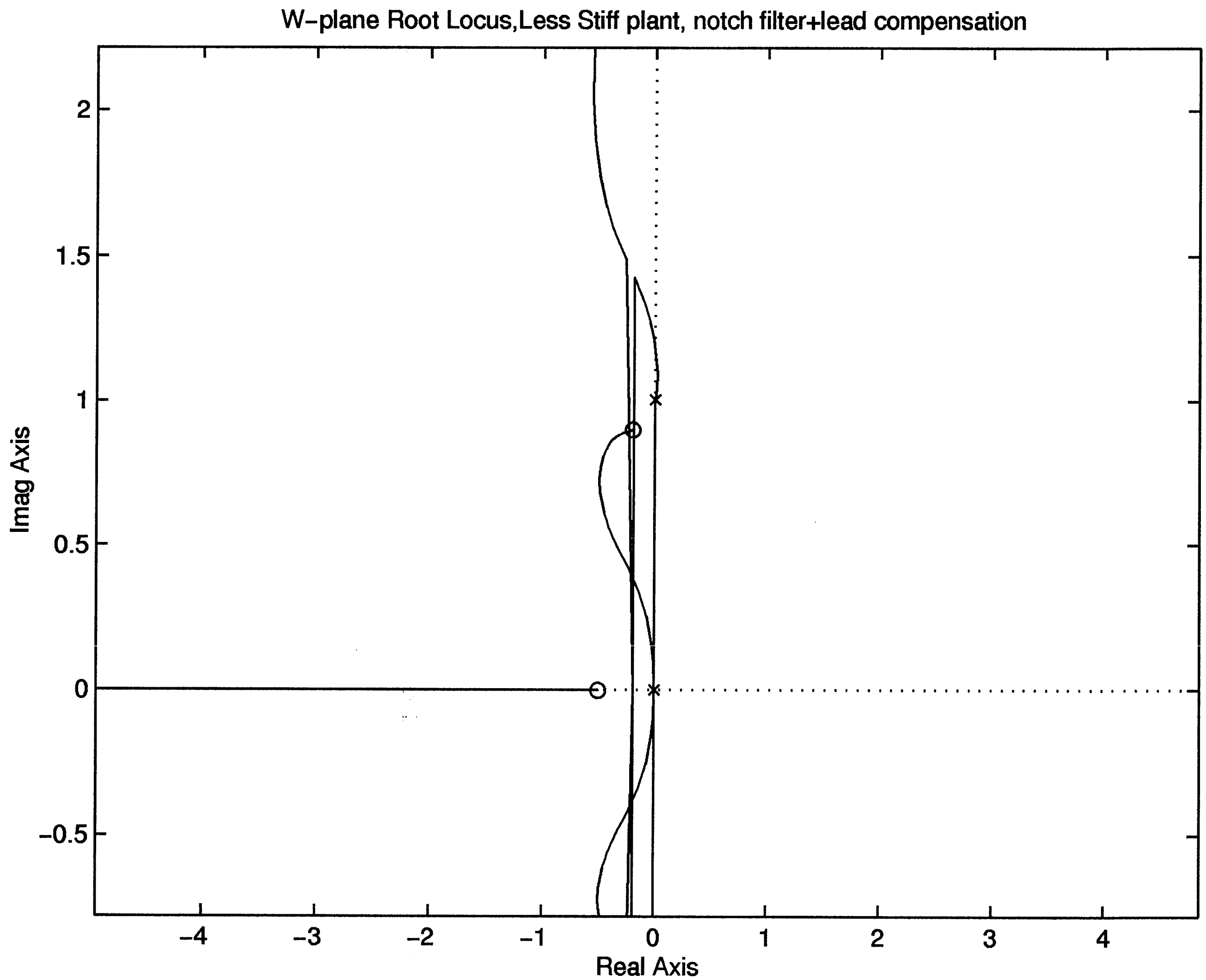


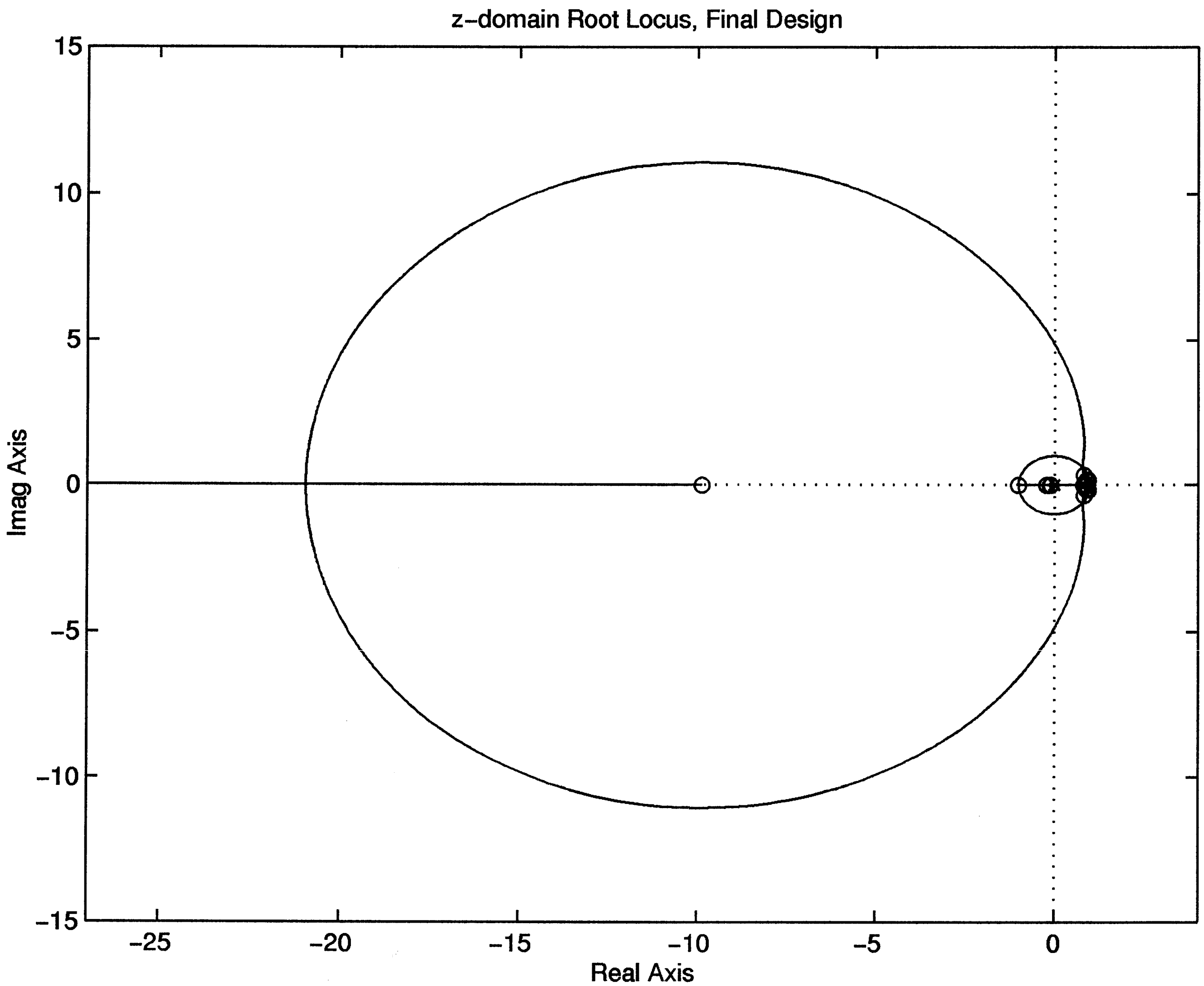


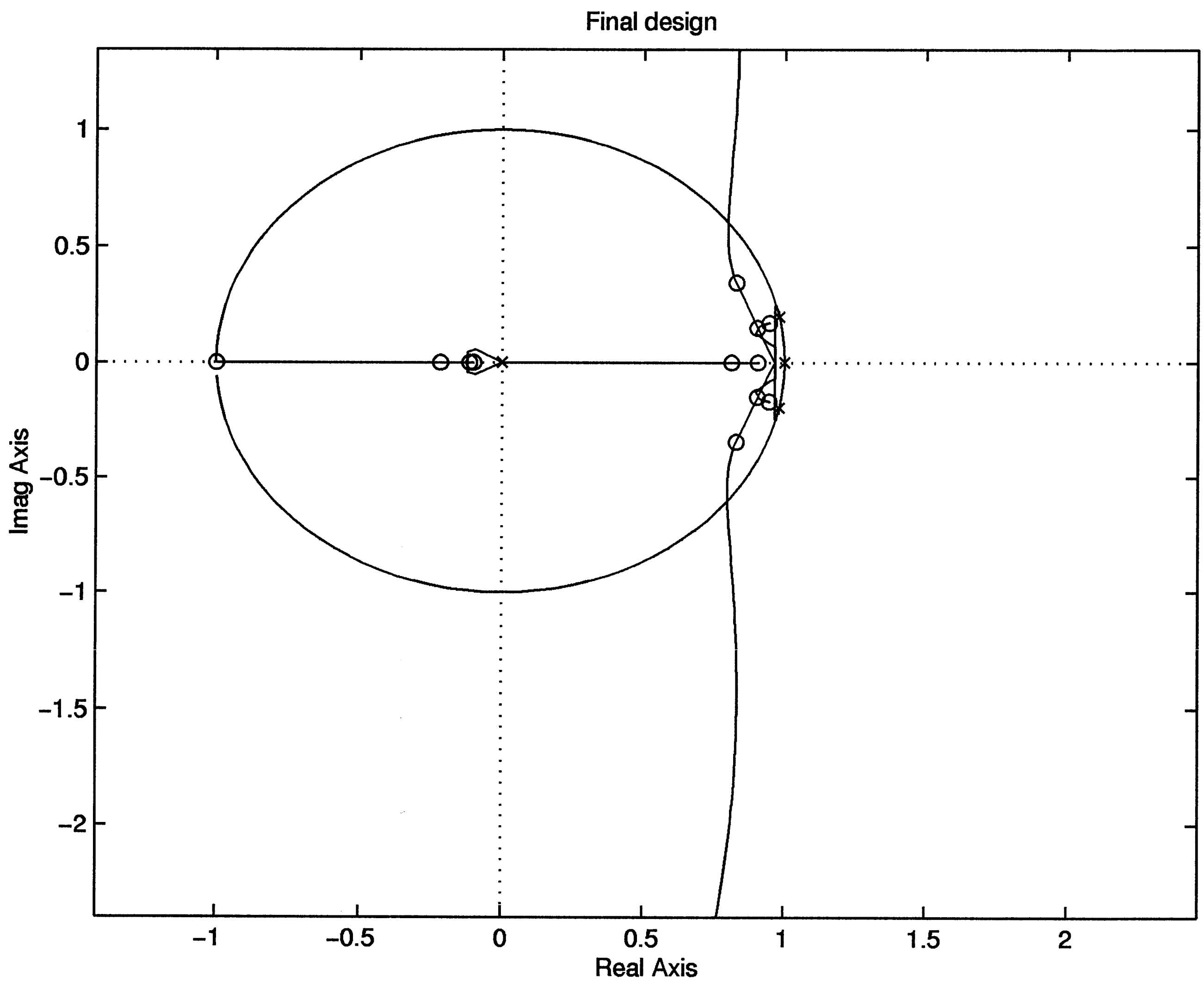
W–plane Root Locus,Less Stiff plant, notch filter+lead compensation
20
15
10
5
0
−5
−10
−15
−20
Imag Axis
−20
−15
−10
−5
0
5
10
15
20
Real Axis


W–plane Root Locus,Less Stiff plant, notch filter+lead compensation
1.6
1.4
1.2
1
0.8
0.6
0.4
Imag Axis
–0.6
–0.4
–0.2
0
0.2
0.4
0.6
Real Axis


W–plane Root Locus,Less Stiff plant, notch filter+lead compensation
6
4
2
0
–2
–4
–6
Imag Axis
–20
–15
–10
–5
0
5
10
15
Real Axis


W-plane Root Locus,Less Stiff plant, notch filter+lead compensation
Imag Axis
Real Axis
2
1.5
1
0.5
0
-0.5
-4
-3
-2
-1
0
1
2
3
4


z–domain Root Locus, Final Design
15
10
5
0
−5
−10
−15
Imag Axis
−25
−20
−15
−10
−5
0
Real Axis


Final design
1
0.5
0
−0.5
−1
−1.5
−2
Imag Axis
−1
−0.5
0
0.5
1
1.5
2
Real Axis