EECE 5610, Fall 2025, Homework #4, Assigned 10/7/2025, Due 10/14/2025, Turnitin.

1. Problem 6.2.1 text
2. Problem 6.2.7 text
3. Problem 6.4.1 text
4. Problem 7.2.5 text
5. Problem 7.2.6 text

References and Further Readings

[1] C. L. Phillips and J. Parr, *Feedback Control Systems,* 5th ed. Upper Saddle River, NJ: Prentice-Hall, 2011.

[2] M. L. Dertouzos, M. Athans, R. N. Spann, and S. J. Mason, *Systems, Networks, and Computation: Basic Concepts.* Huntington, NY: R.E. Krieger Publishing Co., Inc., 1979.

[3] J. L. Melsa and S. K. Jones, *Computer Programs for Computational Assistance.* New York: McGraw-Hill Book Company, 1973.

[4] C. F. Gerald, *Applied Numerical Analysis,* 7th ed. Reading, MA: Addison-Wesley Publishing Company, Inc., 2007.

[5] J. A. Cadzow and H. R. Martens, *Discrete-Time and Computer Control Systems.* Upper Saddle River, NJ: Prentice-Hall, 1970.

[6] G. F. Franklin, J. D. Powell, and M. Workman, *Digital Control of Dynamic Systems,* 3d ed. Half Moon Bay, CA: Ellis-Kagle Press, 2006.

[7] E. I. Jury, *Theory and Application of the z-Transform Method.* Huntington, NY: R.E. Krieger Publishing Co., Inc., 1973.

[8] B. C. Kuo, *Digital Control Systems,* 2d ed. New York: Oxford University Press, 1995.

Problems

6.2-1. Consider the closed-loop system of Fig. P6.2-1.

(a) Calculate and plot the unit-step response at the sampling instants, for the case that $D(z) = 1$.

(b) Calculate the system unit-step response of the analog system, that is, with the sampler, digital controller, and data hold removed. Plot the response on the same graph with the results of part (a).

(c) For the system of Fig. P6.2-1, let $D(z) = 1$ and $T = 0.4$ s. Calculate the unit-step response and plot these results on the same graph used for parts (a) and (b).

(d) Use the system dc gains to calculate the steady-state responses for each of the systems of parts (a), (b), and (c). Why are these gains equal?

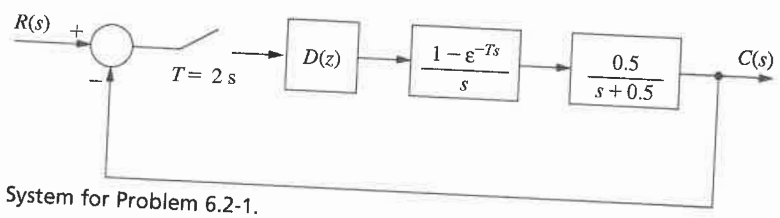

FIGURE P6.2-1 System for Problem 6.2-1.

6.2-2. Consider the system of Fig. P6.2-1, with $D(z) = 1$. Use the results of Problem 6.2-1 if available.

(a) Find the system time constant τ for $T = 2$ s.

(b) With the input a step function, find the time required for the system output $c(kT)$ to reach 98 percent of its final value, for $T = 2$ s. Recall that four time constants (4τ) are required.

(c) Repeat parts (a) and (b) for $T = 0.4$ s.

(d) Repeat parts (a) and (b) for the analog system, that is, for the system with the sampler-data-hold removed.

6.2-3. In Example 6.1 the response of a sampled-data system between sample instants was expressed as a sum of delayed step responses.

(a) Use this procedure to find the system output $y(t)$ of Fig. P6.2-1 at $t = 1$ s.

(b) Repeat part (a) at $t = 3$ s.

(c) The equation for $c(t)$ in part (a) and that for $c(t)$ in part (b) should give the same value at $t = 2$ s. Why? *Hint:* Consider the step response of the analog plant.

(d) Show that the statement in part (c) is true.

6.2-4. Shown in Fig. P6.2-4 is the block diagram of a temperature control system for a large test chamber. This system is described in Problem 1.6-1. Ignore the disturbance input for this problem.

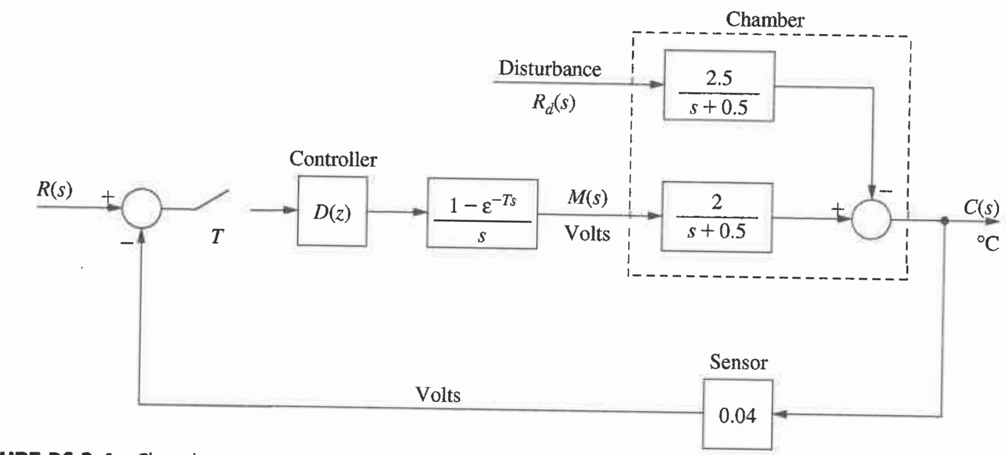

FIGURE P6.2-4 Chamber temperature control system.

(a) With $D(z) = 1$ and $T = 0.6$ s, evaluate and plot the system response if the input is to command a 10°C step in the output. Note that the system input must be a step function with an amplitude of 0.4 V. Why?

(b) Use the results of part (a) to plot the output of the zero-order hold.

(c) Solve for the steady-state output for part (a).

(d) Suppose that the gain of 2 in the plant is replaced with a variable gain K. What value does the output approach in the steady state as K becomes very large? Assume that the system remains stable as K is increased (an unrealistic assumption).

6.2-5. Consider the temperature control system of Problem 6.2-4 and Fig. P6.2-4.

(a) Let $T = 6$ s, and solve for the response to the input $R(s) = 0.4\ \mathrm{s}^{-1}$. Plot this response on the same graph with the response found in Problem 6.2-4. Note the effects of increasing T from 0.6 s to 6 s, when the plant has a time constant of 2 s.

(b) Find $c(t)$ for $0 \le t \le 6$ s. This response can be calculated without the use of the z-transform.

6.2-6. Consider the system of Fig. P6.2-4, with $D(z) = 1$. Use the results of Problems 6.2-4 and 6.2-5 if available.

(a) Find the system time constant τ for $T = 0.6$ s.

(b) With the input a step function, find the time required for the system output $c(kT)$ to reach 98 percent of its final value. Note that this time is approximately four time constants (4τ).

(c) Repeat parts (a) and (b) for $T = 6$ s.

(d) Repeat parts (a) and (b) for the analog system, that is, for the system with the sampler/data hold removed.

6.2-7. The block diagram of a control system of a joint in a robot arm is shown in Fig. P6.2-7. This system is discussed in Section 1.6. Let $T = 0.1$ s and $D(z) = 1$.

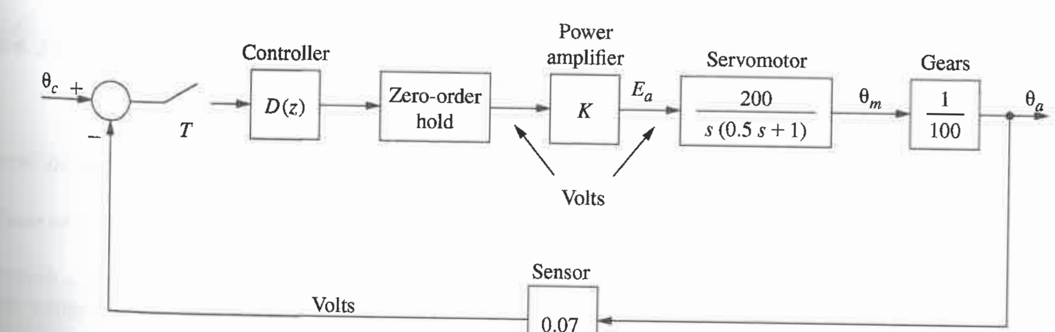

FIGURE P6.2-7 Robot arm joint control system.

(a) Evaluate $C(z)$ if the input is to command a 20°C step in the output and $K = 10$. Note that the system input must be a step function with an amplitude 1.4 V. Why?
(b) Assuming the system to be stable, find the steady-state system output.
(c) Find the (approximate) time required for the system response to reach steady state.
(d) Simulate the system to verify the results in parts (b) and (c).

6.4-1. The block diagram of a control system of a joint in a robot arm is shown in Fig. P6.2-7. Let $T = 0.1$ s, $K = 10$, and $D(z) = 1$. The results of Problem 6.2-7 are useful in this problem if these results are available.
(a) Find the damping ratio ζ, the natural frequency ω_n, and the time constant τ of the open-loop system. If the system characteristic equation has two real zeros, find the two time constants. These values can be solved by inspection. Why?
(b) Repeat part (a) for the closed-loop system.
(c) Repeat parts (a) and (b) for the system with the sampler, digital controller, and data hold removed, that is, for the analog system.
(d) Use the results in parts (b) and (c) to find the percent overshoot in the step responses for the sampled-data closed-loop system and for the analog closed-loop system.

6.4-2. The block diagram of an attitude control system of a satellite is shown in Fig. P6.4-2. Let $T = 1$ s, $K = 100$, $J = 0.1$, $H_k = 0.02$, and $D(z) = 1$.
(a) Find the damping ratio ζ the natural frequency ω_n, and the time constant τ of the open-loop system. If the system characteristic equation has two real zeros, find the two time constants. These values can be solved by inspection. Why?
(b) Repeat part (a) for the closed-loop system.
(c) Repeat parts (a) and (b) for the system with the sampler, digital controller, and data hold removed, that is, for the analog system.
(d) The closed-loop sampled-data system is seen to be unstable and the closed-loop analog system is seen to be marginally stable. If the satellite is operated with each of these control systems, describe the resulting movement for both the sampled-data system and the analog system.

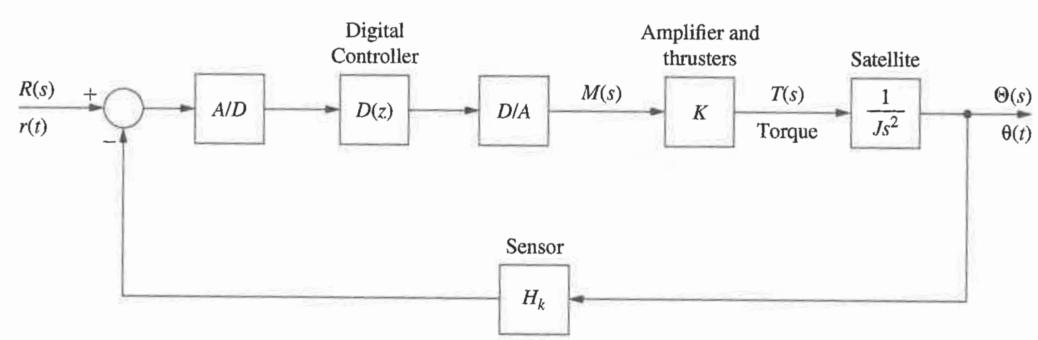

FIGURE P6.4-2 Block diagram for a satellite control system.

6.4-3. It is shown in Example 6.6 that the system of Fig. 6-2 has the parameters $\zeta = 0.250$, $\omega_n = 0.9191$, and $\tau = 4.36$ s.
(a) Find ζ, ω_n, and τ for the sample period $T = 0.5$.
(b) Repeat part (a) for $T = 0.1$.
(c) Repeat part (a) for the analog system, that is, the system with the sampler and data hold removed. Note that this case can be considered to be the limit as the sample period approaches zero.
(d) Give a table listing the three parameters as a function of sampling frequency $f_s = 1/T$. State the result of decreasing the sampling frequency on the parameters.

6.4-4. Consider the system of Fig. P6.4-4. This system is called a regulator control system, in which it is desired to maintain the output, $c(t)$, at a value of zero in the presence of a disturbance, $f(t)$. In this problem the disturbance is a unit step.

(a) With $D(z) = 1$ (i.e., no compensation) find the steady-state value of $c(t)$.
(b) For $f(t)$ to have no effect on the steady-state value of $c(kT)$, $D(z)$ should have a pole at $z = 1$. Let

$$D(z) = 1 + \frac{0.1z}{z - 1}$$

Determine the steady-state value of $c(kT)$.
(c) Repeat parts (a) and (b) with $T = 1$ s.
(d) Why does the value of sample period T have no effect on the steady-state response for a constant input?

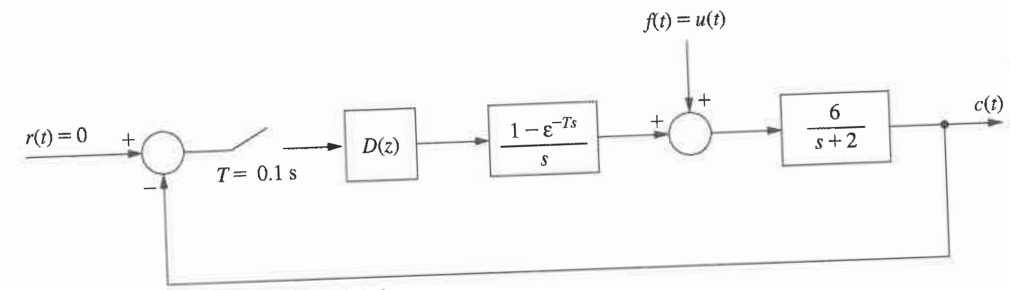

FIGURE P6.4-4 System for Problem 6.4-4.

6.4-5. Consider the sampled-data systems with the following characteristic equations:

(i) $z - 0.999 = 0$	**(v)** $z^2 - 1.85z + 0.854 = 0$
(ii) $z - 0.99 = 0$	**(vi)** $z^2 - 1 = 0$
(iii) $z - 0.9 = 0$	**(vii)** $z^2 - 2z + 0.99 = 0$
(iv) $z + 0.9 = 0$	**(viii)** $z^2 - 1.2z + 0.7 = 0$

(a) What can you determine about the system natural-response characteristics for each system, for $T = 0.1$ s?
(b) Repeat part (a) for $T = 1$ s.
(c) For a given characteristic equation as a function of z, which parameters of the transient response vary with the sample period T, and which are independent of T?

6.4-6. Consider the system of Fig. P6.2-1. Suppose that an ideal time delay of 0.2 s is added to the plant, such that the plant transfer function is now given by

$$G_p(s) = \frac{0.5\varepsilon^{-0.2T}}{s + 0.5}$$

(a) Find the time constant τ of the system if the time delay is omitted.
(b) Find the time constant τ of the system if the time delay is included.
(c) Repeat part (b) for a time delay of 1 s.
(d) What is the effect on the speed of response of the closed-loop system of adding time delay to the plant?

6.5-1. (a) Give the system type for the following systems, with $D(z) = 1$. It is not necessary to find the pulse transfer functions to find the system type. Why?
(i) Fig. P6.2-1.
(ii) Fig. P6.2-4.
(iii) Fig. P6.2-7.
(iv) Fig. P6.4-2.
(v) Fig. P6.4-4.
(b) It is desired that the systems of part (a) have zero steady-state error for a constant input. Give the required characteristics for each digital controller.
(c) It is desired that the systems of part (a) have zero steady-state error for a ramp input. Give the required characteristics for each digital controller.

In summary, we see from both the constant M circles of Fig. 7-24 and the Nichols chart of Fig. 7-26 that a peaking in the closed-loop frequency response can occur only if the open-loop frequency response passes near the -1 point. Hence any control system that has a significant resonance also has small stability margins. Conversely, any system that has small stability margins will exhibit a significant resonance in its time response.

7.11 SUMMARY

In this chapter a number of techniques for analyzing the stability of discrete-time systems have been presented. It has been shown that many of the methods used in the analysis of continuous-time systems are applicable to sampled-data systems also. The chapter contains a number of examples, and the use of the same system in many of the examples throughout the chapter provides a common thread and basis for comparison among the various stability analysis techniques. Many of the analysis techniques presented in this chapter will be extended to design in Chapter 8.

References and Further Readings

[1] C. L. Phillips and J. Parr, *Feedback Control Systems,* 5th ed. Upper Saddle River, NJ: Prentice-Hall, 2011.

[2] E. I. Jury, *Theory and Application of the z-Transform Method.* Huntington, NY: R.E. Krieger Publishing Co., Inc., 1973.

[3] G. F. Franklin, J. D. Powell, and A. Emami-Naeini, *Digital Control of Dynamic Systems,* 6th ed. Upper Saddle River, NJ: Prentice-Hall, 2011.

[4] R. C. Dorf and R. H. Bishop, *Modern Control Systems,* 12th ed. Upper Saddle River, NJ: Prentice-Hall, 2010.

[5] C. R. Wylie and L.C. Barrett, *Advanced Engineering Mathematics,* 6th ed. New York: McGraw-Hill Book Company, 1995.

[6] H. M. James, N. B. Nichols, and R. S. Phillips, *Theory of Servomechanisms.* New York: McGraw-Hill Book Company, 1947.

[7] J. A. Cadzow and H. R. Martens, *Discrete-Time and Computer Control Systems.* Upper Saddle River, NJ: Prentice-Hall, 1970.

[8] W. R. Evans, *Control System Dynamics.* New York: McGraw-Hill Book Company, 1954.

[9] G. F. Franklin, J. D. Powell, and M. Workman, *Digital Control of Dynamic Systems,* 3d ed. Half Moon Bay, CA: Ellis-Kagle Press, 2006.

Problems

7.2-1. Assume that for the system of Fig. 7-1, the system closed-loop transfer-function pole p_1 is repeated such that the system characteristic equation is given by

$$(z - p_1)^r(z - p_{r+1})(z - p_{r+2})\cdots(z - p_n) = 0$$

where r is an integer. Show that the requirement for system stability is that the magnitudes of all poles of the closed-loop transfer function are less than unity, that is, $|p_i| < 1, i = 1, r + 1, r + 2, \ldots, n$.

7.2-2. The system of Example 7.1 and Fig. 7-3 has two samplers. The system characteristic equation is derived in Example 7.1 as

$$1 + G_1(z)G_2(z) + \overline{G_2H}(z) = 0$$

Show that the same characteristic equation is obtained by opening the system at the second sampler.

7.2-3. (a) The unit-step response of a discrete system is the system response $c(k)$ with the input $r(k) = 1$ for $k \geq 0$. Show that if the discrete system is stable, the unit-step response, $c(k)$, approaches a constant as

$k \rightarrow \infty$. [Let $T(z)$ be the closed-loop system transfer function. Assume that the poles of $T(z)$ are distinct (no repeated poles).]

(b) Find the conditions on the closed-loop system transfer function $T(z)$ such that the unit-step response approaches zero as $k \rightarrow \infty$.

(c) The discrete unit-impulse response of a discrete system is the system response $c(k)$ with the input $r(k) = 1$ for $k = 0$ and $r(k) = 0$ for $k \geq 1$. Show that if the discrete system is stable, the unit impulse response, $c(k)$, approaches zero. [Let $T(z)$ be the closed-loop system transfer function. Assume that the poles of $T(z)$ are distinct (no repeated poles).]

7.2-4. Consider a sampled-data system with $T = 0.5$ s and the characteristic equation given by

$$(z - 0.9)(z - 0.8)(z^2 - 1.9z + 1.0) = 0$$

(a) Find the terms in the system natural response.

(b) A discrete LTI system is stable, unstable, or marginally stable. Identify the type of stability for this system.

(c) The natural response of this system contains an undamped sinusoidal response term of the form $A\cos(\omega kT + \theta)$. Find the frequency ω of this term.

7.2-5. Consider the system of Fig. P7.2-5 with $T = 1$ s. Let the digital controller be a variable gain K such that $D(z) = K$. Hence $m(kT) = Ke(kT)$.

(a) Write the closed-loop system characteristic equation.

(b) Determine the range of K for which the system is stable.

(c) Suppose that K is set to the lower limit of the range in part (b) such that the system is marginally stable. Find the natural-response term that illustrates the marginal stability.

(d) Repeat part (c) for the upper limit of the range of K.

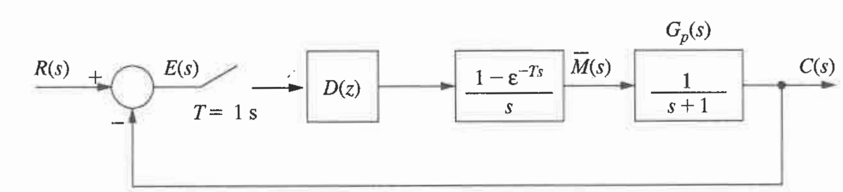

FIGURE P7.2-5 System for Problem 7.2-5.

7.2-6. Consider the system of Fig. P7.2-5, and let the digital controller be a variable gain K such that $D(z) = K$. Hence $m(kT) = Ke(kT)$.

(a) Write the closed-loop system characteristic equation as a function of the sample period T.

(b) Determine the ranges of $K > 0$ for stability for the sample periods $T = 1$ s, $T = 0.1$ s, and $T = 0.01$ s.

(c) Consider the system with all sampling removed and with $G_p(s) = K/(s + 1)$. Find the range of $K > 0$ for which the analog system is stable.

(d) Comparing the ranges of K from parts (b) and (c), give the effects on stability of reducing the sample period T.

7.3-1. Consider the general bilinear transformation

$$z = \frac{1 + aw}{1 - aw}$$

where a is real and nonzero.

(a) Show that this function transforms the stability boundary of the z-plane into the imaginary axis in the w-plane.

(b) Find the relationship of frequency in the s-plane to frequency in the w-plane.

(c) Find the stable region of the w-plane for $a < 0$ and for $a > 0$.

7.5-1. Given below are the characteristic equations of certain discrete systems.

(i) $z^2 - 1.1z + 0.3 = 0$ **(ii)** $z^2 - z + 0.25 = 0$

(iii) $z^2 - 0.1z - 0.3 = 0$ **(iv)** $z^2 - 0.25 = 0$