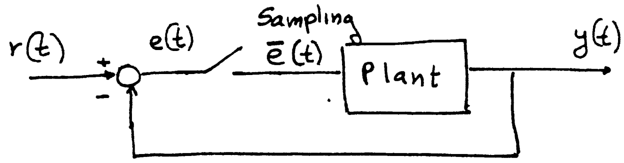
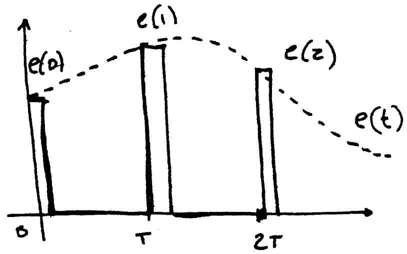
SAMPLING AND RECONSTRUCTION (Chapter 3)

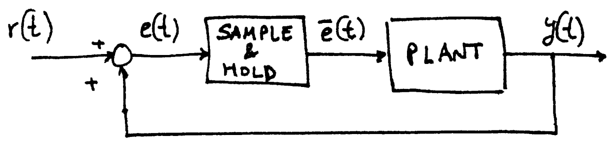
Suppose that we want to control a continuous time plant using a digital controller. We could have a loop of the form



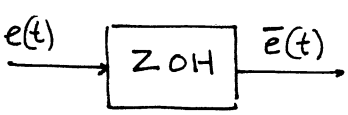
where $\bar{e}(t)$, the output of the sampler is of the form:

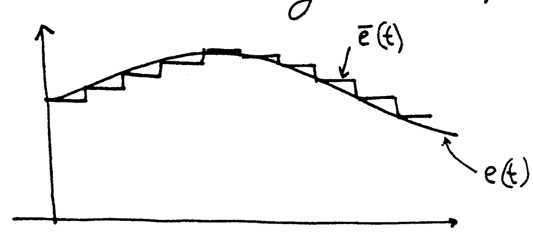


However: you don't want to apply a signal like this to the plant because of its high frequency components (which could excite high frequency resonant modes). Solution: use some sort of device to "reconstruct" the original signal:

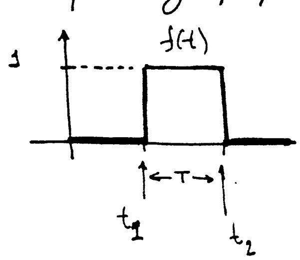


We could use as sample & hold device a zero-order hold that holds the output signal constant during the sampling period





Suppose that we want to write down $\bar{e}(t)$ in terms of $e(t)$. We will use the following property:



$$f(t) = u(t-t_1) - u(t-t_1-T)$$

$$F(s) = \frac{1}{s}\left[e^{-t_1 s} - e^{-(t_1+T)s}\right] = \frac{e^{-t_1 s}}{s}(1-e^{-Ts})$$

$$\bar{e}(t) = e(0)\left[u(t) - u(t-T)\right] + e(T)\left[u(t-T) - u(t-2T)\right] + \cdots$$

$\Downarrow$ (Laplace transform)

$$\bar{E}(s) = e(0)\left(\frac{1-e^{-Ts}}{s}\right) + e(T)\,e^{-Ts}\left(\frac{1-e^{-Ts}}{s}\right) + e(2T)\,e^{-2Ts}\left(\frac{1-e^{-2Ts}}{s}\right) + \cdots$$

$$\bar{E}(s) = \left(\frac{1-e^{-Ts}}{s}\right) * \left[e(0) + e(T)\,e^{-Ts} + e(2T)\,e^{-2Ts} + \cdots\right]$$

$$= \underbrace{\left(\frac{1-e^{-Ts}}{s}\right)} \cdot \underbrace{\sum_{0}^{\infty} e(kT)\,e^{-kTs}}$$

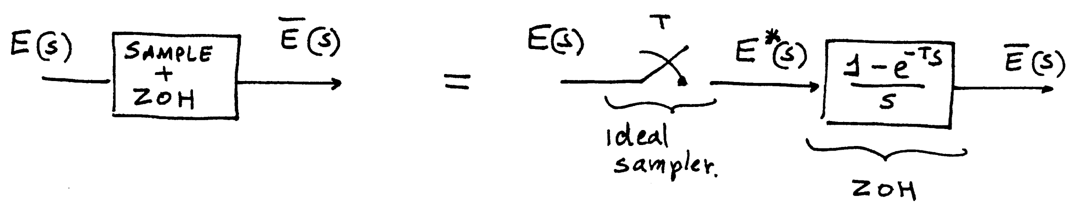
this is related only to the input $e(t)$ and T, but not the ZOH

This is independent of $e(t)$ an can be tought off as a transfer function associated with the Z.O.H

Thus it is convenient to represent the sample and hold operation as a combination of two operations

(a) an "ideal" sampler that generates $\sum_{0}^{\infty} e(kT)\,e^{-kTs}$ from $E(s)$

(b) a ZOH with transfer function $G_{ZOH}(s) = \dfrac{1-e^{-sT}}{s}$



We will call the intermediate variable $E^*(s)$ the "starred transform":

$$\boxed{\text{Definition:} \quad E^*(s) = \sum_{k=0}^{\infty} e(kT)\,e^{-kTs}}$$

Note: The ideal sampler $E(s) \xrightarrow{T} E^*(s)$ does not model a physical sampler and it does not have a Transfer function (because it is a many to one mapping: different $E(s)$ can yield the same $E^*(s)$)

The block $\boxed{\dfrac{1-e^{-Ts}}{s}}$ does not model a physical data hold.

However: The combination does model the operation of the combined sample & hold. and gives the correct mathematical description.

- <u>Ideal sampler</u>:

Consider $E^*(s) = \sum_0^\infty e(kT)\, e^{-kTs}$

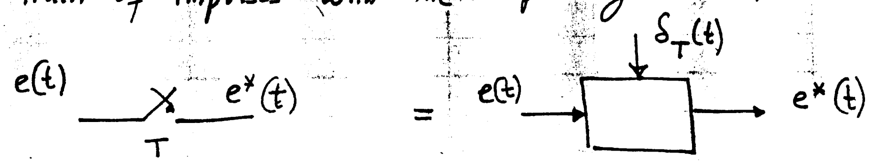
$$\Downarrow \mathcal{L}^{-1}$$

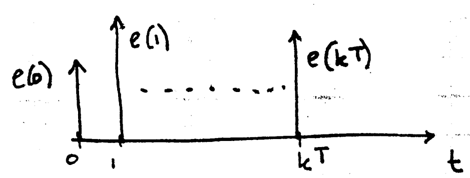
$$e^*(t) = \sum_0^\infty e(kT)\,\mathcal{L}^{-1}\left[e^{-kTs}\right] = \sum_0^\infty e(kT)\,\delta(t-kT) = \sum_0^\infty e(t)\,\delta(t-kT)$$

$$= e(t)\sum_0^\infty \delta(t-kT)$$

Let: $\delta_T = \sum_0^\infty \delta(t-kT)$ (a train of impulses)

$\Rightarrow$ The action of the ideal sampler can be tought off as modulating a train of impulses with the input signal $e(t)$





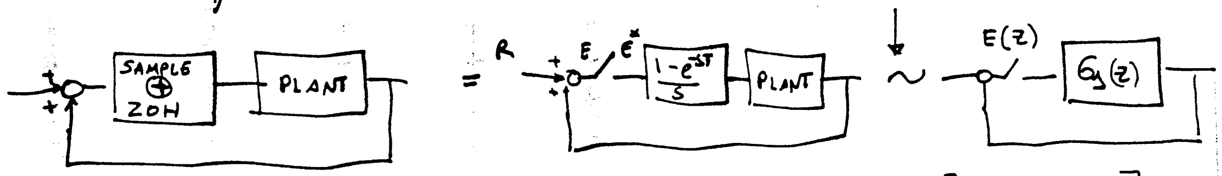
$\Rightarrow$ In the future we will think of $E^*(s)$ as the output of an ideal sampler.

- As mentioned before neither the ideal sampler nor the $\left[\frac{1-e^{-Ts}}{s}\right]$ block by themselves model the operation of a physical device. However, their <u>combination</u> does give the correct mathematical description of the sample & hold operation.

- Q: Why do we go through all this trouble?

A: We want to get a T.F model of the hold operation and an input-output model of the sample and hold suitable for using in combination with linear systems analysis tools, such as the z-transform

we'll see later



where $G_d(z) = Z\left[\left(\frac{1-e^{-sT}}{s}\right)\cdot \text{Plant}\right]$

Example: $e(t) = u(t)$

e^* ?

$$e(nT) = u(nT) = 1 \Rightarrow e^*(t) = \sum_{n=0}^{\infty} \overset{1}{\overset{\shortparallel}{e(nT)}}\,\delta(t-nT) = \sum_{n=0}^{\infty} \delta(t-nT)$$

$$\Updownarrow LT$$

$$E^*(s) = \sum_{0}^{\infty} e^{-nTs} = \frac{1}{1-e^{Ts}} \quad (\text{ROC: } |e^{-Ts}|<1)$$

(Note that the expression for $E^*(s)$ resembles that of $E(z)$, in fact, they are identical if we define $z = e^{sT}$. This is no accident, more on this later)

- How do we compute $E^*(s)$? (We'd like to have a closed form like in the example, rather than the infinite series)
- <u>Facts</u>:

Useful to find $E^*(s)$ → (a) $E^*(s) = \sum \left[\text{residues of } \left\{ E(\lambda) \frac{1}{1-e^{-T(s-\lambda)}} \right\} \text{ at poles of } E(\lambda) \right]$

Useful to determine properties of $E^*(s)$ → (b) $E^*(s) = \frac{1}{T}\left[\sum_{-\infty}^{+\infty} E(s+jn\omega_s) + \frac{e(0)^+}{2} \right]$ where $\omega_s = 2\frac{\pi}{T}$ = sampling frequency

The proof uses again Cauchy's theorem. Sketch of the proof:

$$e^*(t) = e(t)\,\delta_T(t) \qquad \text{where} \quad \delta_T(t) = \sum_{n=0}^{\infty} \delta(t-nT)$$

$$E^*(s) = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} E(\lambda)\,\Delta_T(s-\lambda)\,d\lambda \qquad \Delta_T(s) = \mathcal{L}\left[\delta_T(t)\right]$$

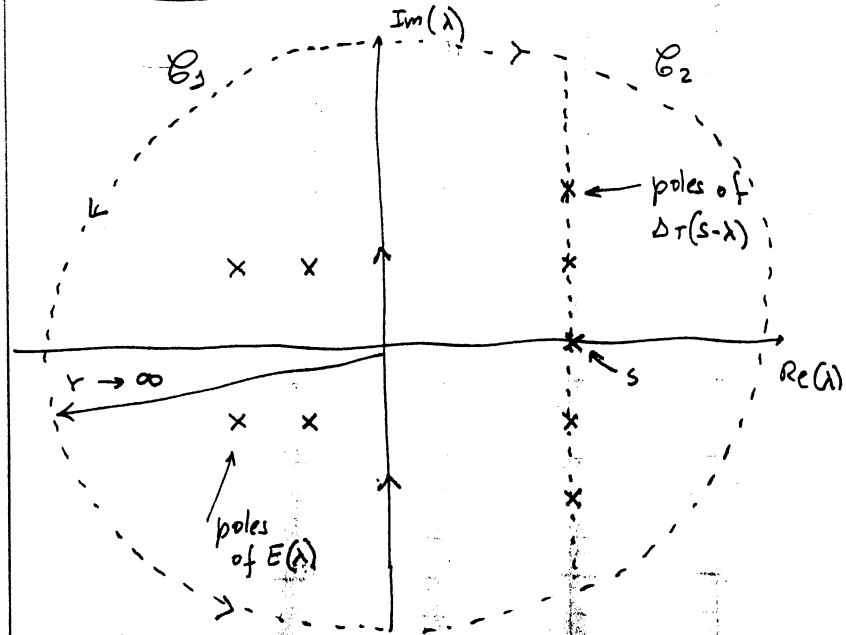
(Here we used the fact that product in time domain $\overset{\mathcal{L}}{\Longleftrightarrow}$ convolucion in s ("frequency") domain)

where c must be chosen such that all poles of $E(\lambda)$ are to <u>the left</u> and all poles of Δ_T are to <u>the right</u>

Note that $\Delta_T(s) = \mathcal{L}\left[\delta_T(t)\right] = \sum_{0}^{\infty} e^{-nTs} = \frac{1}{1-e^{sT}} \Rightarrow$ poles at $e^{-Ts} = 1$

$$\Rightarrow s = \pm j\frac{2\pi n}{sT} = \pm j\omega_s n$$

so that $\Delta_T(s)$ has an <u>infinite</u> number of poles, all on the $j\omega$ axis, spaced ω_s



$\Delta_T(s)$ poles at $s = \pm jn\omega_s$

$\Delta_T(s-\lambda)$ poles at $\lambda = s \pm jn\omega_s$

Closing the contour with $\mathcal{C}_1$ (a semicircle in the LHP with radius $r \to \infty$) yields the first equality. If, on the other hand we close the contour with $\mathcal{C}_2$ (semicircle in RHP, $r \to \infty$) we obtain the second formula

Example 1: $\quad e(t) = u(t) \Rightarrow E(\lambda) = \frac{1}{\lambda}; \quad E(\lambda)\frac{1}{1-e^{-T(s-\lambda)}} = \frac{1}{\lambda(1-e^{-T(s-\lambda)})}$

$$\sum_{\text{poles of } E(\lambda)} \text{Res} = \sum_{\lambda=0} \text{Res} = \frac{1}{1-e^{-Ts}} \quad \#$$

Example 2: $\quad$ Suppose that $f(t) = 1 - e^{-t} \Rightarrow F^*(s) = \sum_{k=0}^{\infty} (1-e^{-kT}) e^{-kTs}$

$$= \sum_0^\infty e^{-kTs} - \sum_0^\infty e^{-k(s+1)T} = \boxed{\frac{1}{1-e^{-Ts}} - \frac{1}{1-e^{-(s+1)T}}}$$

Alternatively: $\quad F(s) = \frac{1}{s} - \frac{1}{(s+1)} = \frac{1}{s(s+1)}$, poles at $s=0$, $s=-1$

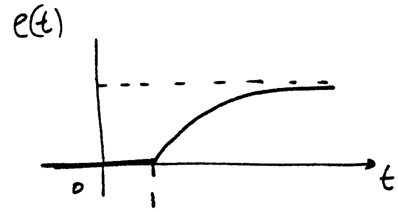
$$\sum_{\substack{\lambda=0\\ \lambda=-1}} \text{Res}\left\{F(\lambda)\frac{1}{1-e^{-T(s-\lambda)}}\right\} = \sum_{\substack{\lambda=0\\ \lambda=-1}} \text{Res} \frac{1}{\lambda(\lambda+1)} \cdot \frac{1}{(1-e^{-T(s-\lambda)})} = \frac{1}{1-e^{-Ts}} - \frac{1}{1-e^{-T(s+1)}} \quad \#$$

(same as before)

Example 3:

A function with a time-delay. This example will become relevant later on when we will look into the effects of sampling:

Let $e(t) = \left[1 - e^{-(t-1)}\right] u(t-1)$ (i.e: $e(t) = (1 - e^{-t})$ delayed by 1 second)



$e(k) = \left[1 - e^{-(0.5k-1)}\right] \quad k \geq 2; \qquad e(k) = 0 \quad k = 0, 1$

From the definition we have:

$$E^*(s) = \Sigma\, e(k)\, e^{-kTs} = \sum_{k=2}^{\infty} \left(1 - e^{-(0.5k-1)}\right) e^{-kTs} = \frac{e^{-2Ts}}{1 - e^{-Ts}} - \frac{e^{1} \cdot e^{-2(0.5+Ts)}}{1 - e^{-(0.5+Ts)}}$$

$$= \frac{e^{-s}}{1 - e^{-0.5s}} - \frac{\cancel{e^{1}} \cancel{e^{-1}} \cdot e^{-s}}{1 - e^{-0.5(s+1)}} = \boxed{\frac{(1 - e^{-0.5})\, e^{-1.5s}}{\left(1 - e^{-0.5s}\right)\left(1 - e^{-0.5(s+1)}\right)}} \quad \#$$

Now let's try our residues formula:

$$E(s) = \frac{e^{-s}}{s(s+1)} \Rightarrow E^*(s) = \sum_{\substack{\lambda=0 \\ \lambda=-1}} Res \left\{ \frac{e^{-\lambda}}{\lambda(\lambda+1)} \frac{1}{1 - e^{-T(s-\lambda)}} \right\} =$$

$$= \frac{1}{1 - e^{-Ts}} + \frac{e^{1}}{(-1)} \cdot \frac{1}{1 - e^{-T(s+1)}} = \boxed{\frac{1}{1 - e^{-0.5s}} - \frac{e^{1}}{1 - e^{-0.5(s+1)}}}$$

Surprise! we got <u>different</u> answers

Q: What went wrong here?

A: <u>A techical point</u>: the "proof" of the residues formula is <u>not</u> valid for systems having time delays

The reason is that $e^{-sT} \not\to 0$ on the infinite portion of the contour B_1 and thus we <u>can't</u> close the contour and compute the $\int$ using residues

<u>Solution</u> (a) don't use the residues formula for systems with delays Not too convenient. It defeats the whole purpose of introducing the * transform!

(b) Modify the formula:

It can be shown that if the delay is an <u>integer</u> number of periods then:

$$E^*(s) = \left[e^{-kTs} \underset{\substack{\uparrow \\ \text{non delayed} \\ \text{signal}}}{E_1(s)}\right]^* = e^{-kTs} \sum_{\substack{\text{at poles} \\ \text{of } E_1}} \left\{ \text{Res } E_1(\lambda) \frac{1}{1 - e^{-T(s-\lambda)}} \right\}$$

Applying this modified formula to our earlier example we get

$$E_1(s) = \frac{1}{s(s+1)} \qquad \sum_{\substack{\lambda=0 \\ \lambda=-1}} \text{Res } \frac{1}{\lambda(\lambda+1)} \cdot \frac{1}{\left(1 - e^{-T(s-\lambda)}\right)}$$

$$= \frac{1}{\left(1 - e^{-Ts}\right)} - \frac{1}{\left(1 - e^{-T(s+1)}\right)}$$

$$\Rightarrow E^*(s) = e^{-2Ts}\left[\frac{1}{1 - e^{-0.5s}} - \frac{1}{1 - e^{-0.5(s+1)}}\right] = e^{-s}\left[\frac{1}{1 - e^{-0.5s}} - \frac{1}{1 - e^{-0.5(s+1)}}\right]$$

which coincides with our earlier result

Properties of $E^*(s)$

1) $E^*(s)$ is <u>periodic</u>, with period $T_s = jw_s = j\frac{2\pi}{T}$

proof:

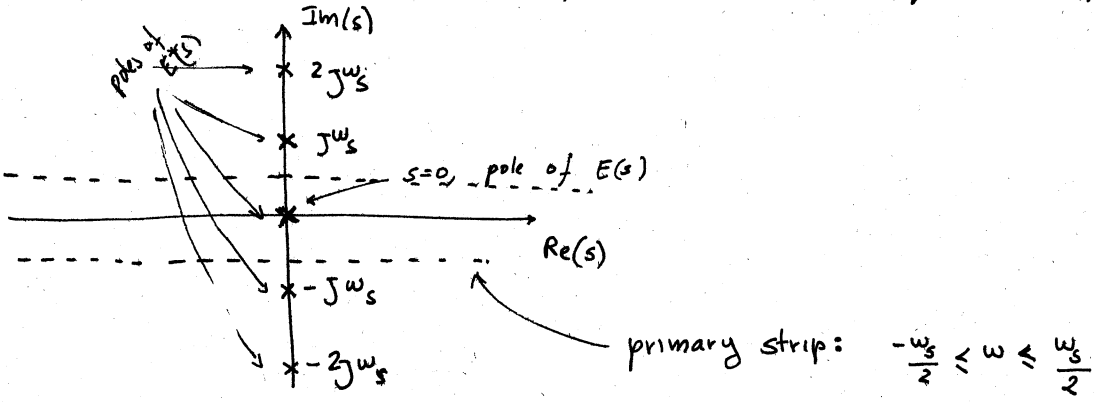
$$E^*(s) = \sum_0^\infty e(kT)\, e^{-skT}$$

$$E^*(s+jw_s) = \sum_0^\infty e(kT)\, e^{-k\left[s+j\frac{2\pi}{T}\right]T} = \sum_0^\infty e(kT)\, e^{-skT} \underbrace{e^{-jk\frac{2\pi}{T}\cdot T}}_{1} = E^*(s) \quad \#$$

2) If $E(s)$ has a pole at $s=s_1 \Rightarrow E^*(s)$ <u>has poles at $s = s_1 + jm\,w_s$</u> $m = 0, \pm 1 \ldots$

<u>Note</u>: some property <u>does not</u> apply to zeros of $E^*(s)$

Example: assume that $E(s)$ has a pole at $s=0 \Rightarrow E^*(s)$ poles at $s_m = \pm j n w_s$



proof: $E^*(s) = \frac{1}{T}\sum_{n=-\infty}^{n=+\infty} E(s+jnw_s) = \frac{1}{T}\left[E(s) + E(s+jw_s) + \cdots\right]$

if $E(s)$ has a pole at $s=s_1$, the first term contributes a pole at $s=s_1$

second $s = s_1 - jw_s$

n^{th} . . . $s = s_1 - jnw_s$

<u>Example</u> 2: Recall that we have shown that:

$$F(s) = \frac{1}{s}\cdot\frac{1}{(s+1)} \Leftrightarrow F^*(s) = \frac{1}{1-e^{-sT}} - \frac{1}{1-e^{-T(s+1)}}$$

$\Downarrow$ poles at $s=0$, $s=-1$

$\Downarrow$ poles at $s = \pm jn\frac{2\pi}{T}$ $(e^{-jTs}=1)$, $s = -1 \pm jn\frac{2\pi}{T}$

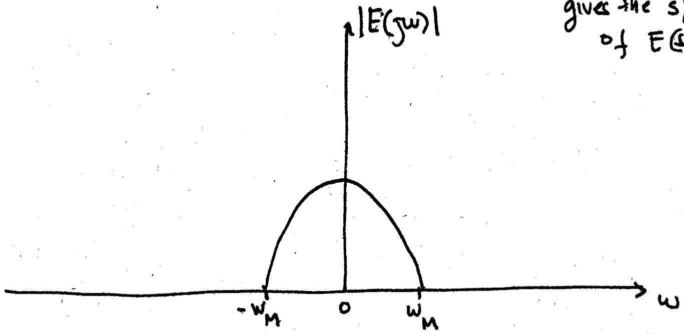
- <u>Spectrum of a Sampled Signal</u>

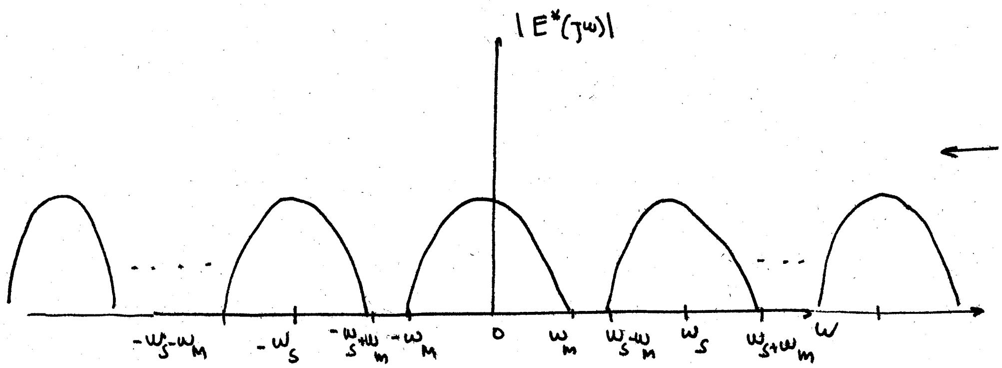
$e(t)$ ⟋ $e^*(t)$

We'd like to relate the spectrum (i.e. fourier transform) of $e(t)$ and $e^*(t)$. This will become relevant when we discuss how to <u>reconstruct</u> (if possible) $e(t)$ from $e^*(t)$.

Let $$e^*(t) = \sum_{k=-\infty}^{k=+\infty} e(t)\,\delta(t-kT)$$

$$E^*(s) = \frac{1}{T}\sum_{-\infty}^{+\infty} E(s+jn\omega_s) = \frac{1}{T}\Big[\underbrace{E(s)}_{\text{gives the spectrum of } E(s)} + E(s+j\omega_s) + \cdots \underbrace{E(s+jn\omega_s)}_{\text{gives the spectrum of } E\text{, shifted by } n\omega_s} + \cdots\Big]$$

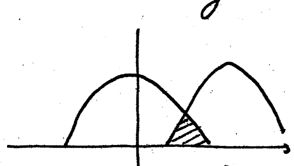




← original spectrum replicated at $\omega_s, 2\omega_s, \ldots$

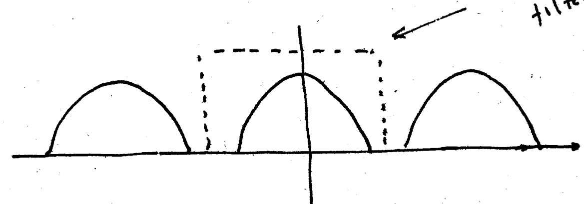
From these plots it follows that we can recover $E(s)$ from $E^*(s)$ <u>only</u> if the highest frequency present in $e(t)$ is smaller than $\frac{\omega_s}{2}$ (the "Nyquist frequency")

We can't have any overlapping:



can't recover

$\omega_M > \frac{\omega_s}{2}$



can put an ideal filter here

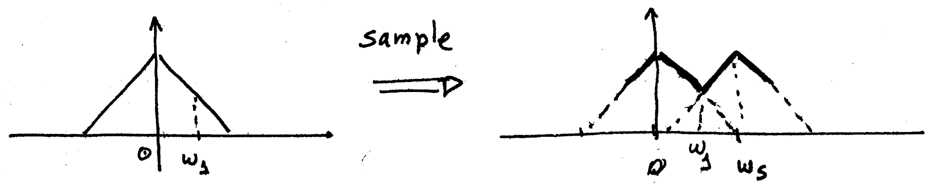
can recover

$\omega_m < \frac{\omega_s}{2}$

This is the celebrated <u>Shannon's Sampling Theorem</u>:

- A function $e(t)$ which contains no frequency component higher than f_0 is <u>uniquely</u> determined by the values of $e(t)$ at any set of sampling points spaced $T = \frac{1}{2f_0}$

If $e(t)$ has components above the Nyquist frequency we have the following situation:

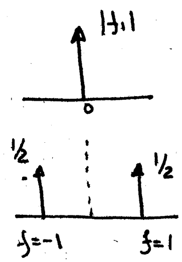


In the sampled signal the contributions from the frequencies w_1 and $w_2 = w_1 - w_s$ <u>both</u> show up at w_1. This phenomenon is called <u>aliasing</u>

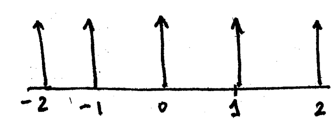
<u>Implications</u>: 2 sinusoids of different frequencies appear at the same place when sampled $\Rightarrow$ can't tell them appart

Example: $f_1(t) = 1$

$f_2(t) = \cos 2\pi t$

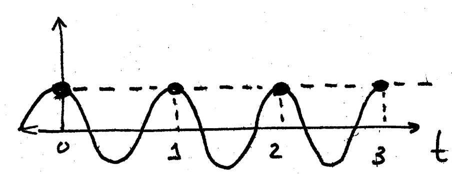


If sampled at $f_s = 1\ Hz$ both yield:

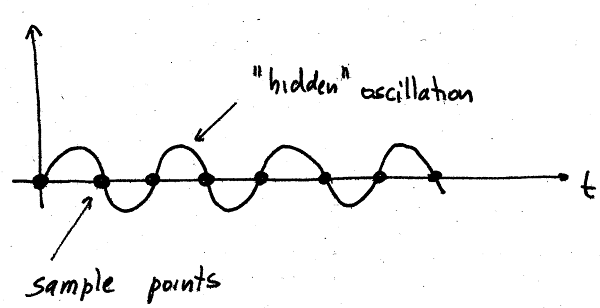


(the same spectrum!)

With 20/20 hindsight this is not surprising:

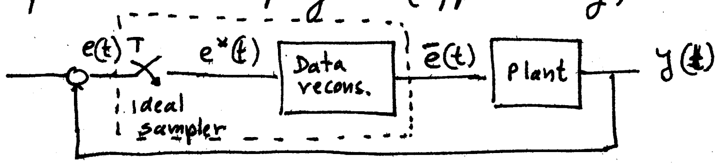


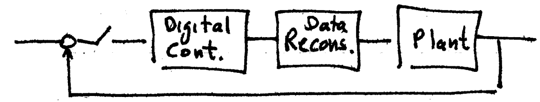
Related phenomenon: <u>Hidden oscillations</u>: We can have a signal that <u>does not</u> show up at all when sampled



• Data Reconstruction:

Dual operation to sampling: (approximately) reconstruct a signal from its samples



(We want to analyze this as a first step to: 

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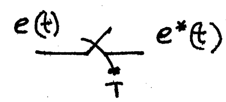
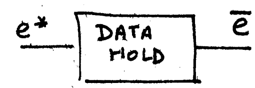
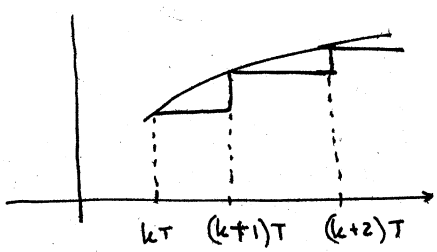
We already analyzed the first half: 

Now we will analyze the second half: 

Simplest approximation: zero order hold

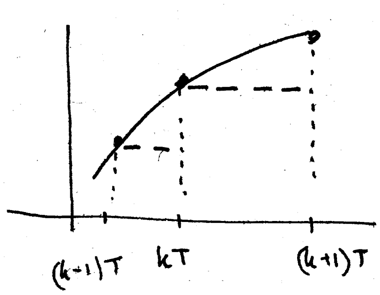


Better idea (perhaps): Try to get a tighter fit by using a polynomial approximation.

Recall from calculus that we can approximate any signal (with arbitrary accuracy) by considering enough terms of its Taylor series expansion:

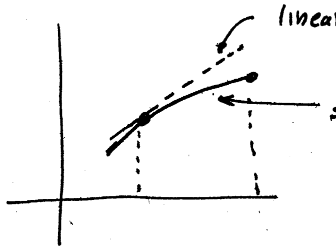
$$e(t) = e(kT) + e'(kT)(t-kT) + e''(kT)\frac{(t-kT)^2}{2} + \cdots$$

If we use only the first term we get back the zero order approximation $\left(e(t) = e(kT) \quad kT \le t < (k+1)T \right)$

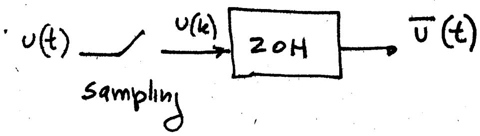


If we consider the first <u>two</u> terms we get a first order hold:

$$e(t) = e(kT) + e'(kT)(t-kT)$$

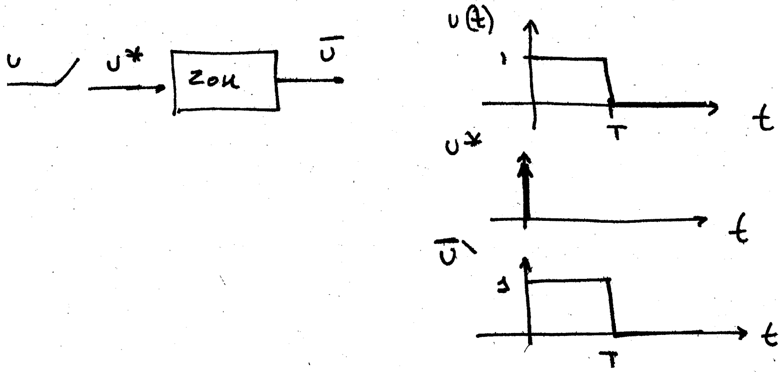


- <u>Transfer function for a zero order hold</u> (ZOH)



Recall (from EECE5580) that we can get the T.F by finding the <u>impulse</u> response and then taking its Laplace transform.

Since the ZOH keeps the output clamped at the input value for one period we have:



impulse response of ZOH

$$g(t) = u(t) - u(t-T)$$

$$\Downarrow \mathcal{L}$$

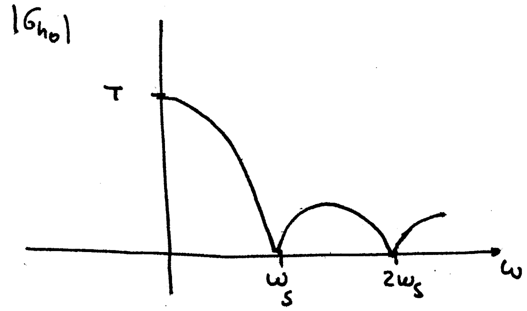
(same as before) $$\boxed{G_{h_0}(s) = \frac{1-e^{-sT}}{s}}$$

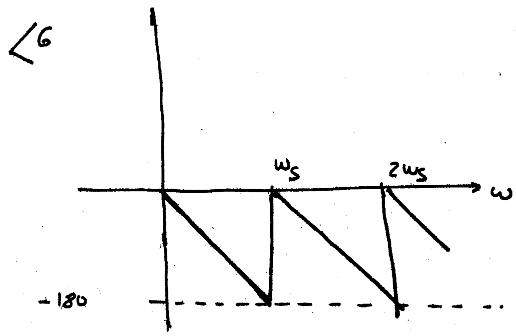
- <u>Frequency response of a ZOH</u>:

$$G_{h_0}(j\omega) = \frac{1-e^{-j\omega T}}{j\omega} = \frac{\left(e^{j\frac{\omega T}{2}} - e^{-j\omega T/2}\right)Te^{-j\frac{\omega T}{2}}}{2j\frac{\omega T}{2}} = Te^{-j\frac{\omega T}{2}}\left(\frac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}}\right)$$

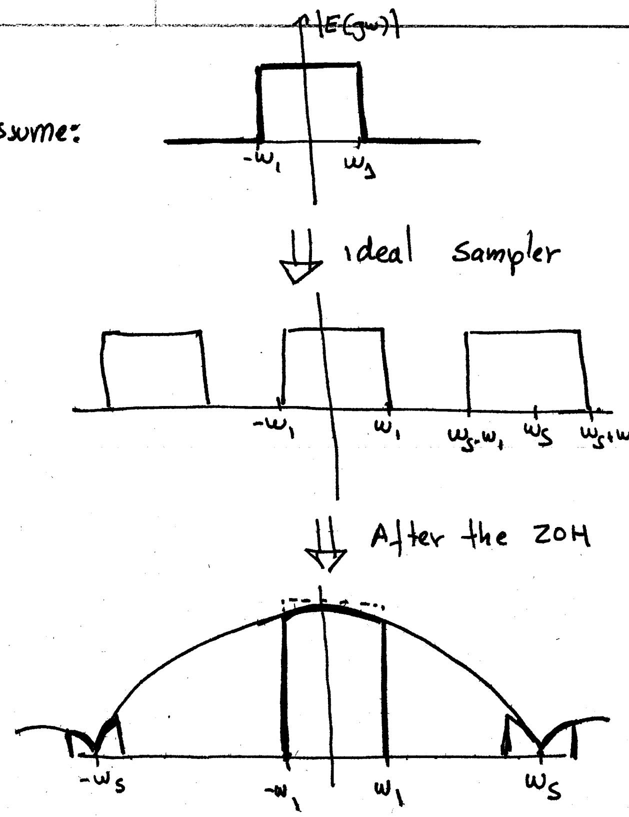
recall that $T = \frac{2\pi}{\omega_s} \Rightarrow$

$$G_{h_0}(j\omega) = T\left[\frac{\sin \pi\left(\frac{\omega}{\omega_s}\right)}{\pi\left(\frac{\omega}{\omega_s}\right)}\right]e^{-j\left(\pi\frac{\omega}{\omega_s}\right)} \Rightarrow \begin{cases} |G_{h_0}| = T\cdot\left|\operatorname{sinc} \pi\left(\frac{\omega}{\omega_s}\right)\right| \\ \angle G_{h_0} = -\pi\frac{\omega}{\omega_s} + \theta \quad \begin{cases} \theta = 0 & \text{if } \sin\pi\left(\frac{\omega}{\omega_s}\right) > 0 \\ \pi & \text{if } \sin\pi\left(\frac{\omega}{\omega_s}\right) < 0 \end{cases} \end{cases}$$





Assume:



If $\omega_1 \ll \omega_s$ the high frequency components of E^* fall close to zeros of $G_{h_0} \Rightarrow$ Zero order hold works like a low pass filter and we recover $E(s)$

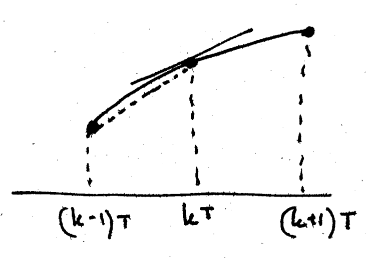
- First order hold: Use the first 2 terms of the polynomial expansion

$$\bar{e}(t) = e(kT) + e'(kT)(t - kT) \qquad kT \leq t < (k+1)T$$

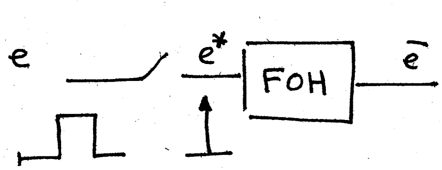
Problem: $e'(kT)$ is not readily available

Solution: approximate: $e'(kT) \cong \dfrac{e(kT) - e(k-1)T}{T}$

(as in the hw problem 2.17)



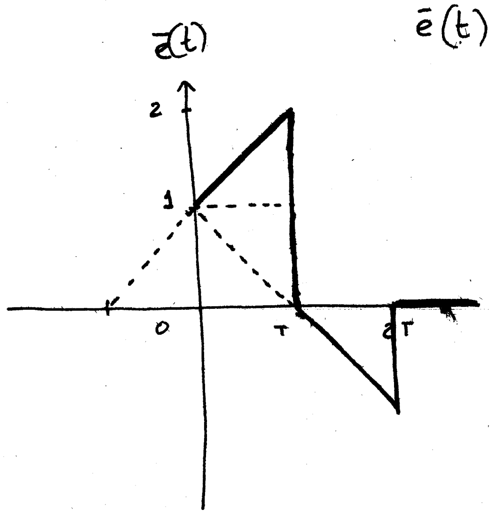
Note that you need memory to accomplish this.



$e(t) = 1$ at $t = 0$, 0 $t > 0$
$e^*(t) = \delta(t)$

Desired output: $\bar{e}(t) = 1 + e'(0)\,t = 1 + \frac{t}{T} \qquad 0 \le t < T$

$\bar{e}(t) = 0 + e'(T)(t-T) = 0 - \frac{(t-T)}{T} = 1 - \frac{t}{T} \qquad T \le t < 2T$



$$e(t) = \left(1+\frac{t}{T}\right)\left[u(t) - u(t-T)\right] + \left(1 - \frac{t}{T}\right)\left[u(t-T) - u(t-2T)\right]$$

$$= \left(1+\frac{t}{T}\right)u(t) \quad - 2\frac{t}{T}\,u(t-T) - \left(1 - \frac{t}{T}\right)u(t-2T)$$

$$= \left(1+\frac{t}{T}\right)u(t) - 2\frac{t}{T}\,u(t-T) + \left(\frac{t-2T}{T}\right)u(t-2T) + u(t-2T)$$

$$= \left(1+\frac{t}{T}\right)u(t) - 2\left(\frac{t-T}{T}\right)u(t-T) + \left(\frac{t-2T}{T}\right)u(t-2T) + u(t-2T) - 2u(t-T)$$

$$\Rightarrow \quad E(s) = \frac{1}{s}\left(1 - 2e^{-sT} + e^{-2sT}\right) + \frac{1}{Ts^2}\left(1 - 2e^{-Ts} + e^{-2sT}\right)$$

$$= \left(\frac{1+Ts}{Ts^2}\right)\left(1 - e^{-Ts}\right)^2 = \boxed{\left(\frac{1+sT}{T}\right)\left(\frac{1-e^{-sT}}{s}\right)^2}$$

Frequency response:

$$G_{h_1}(j\omega) = \left(\frac{1+j\omega T}{T}\right)\left(\frac{1-e^{-j\omega T}}{j\omega}\right)^2$$

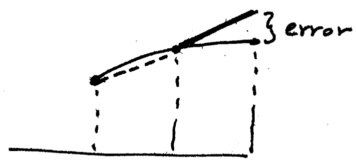
For low frequences ($\omega \ll \omega_s$) FOH has less phase lag (but more amplitude distortion)

For large ω, ZOH has less phase lag

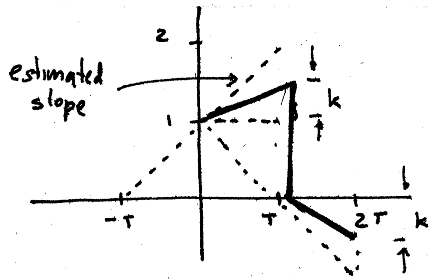
In practice we almost always use the ZOH due to the increased hardware complexity entailed by higher order holds. If the steps from the ZOH have detrimental effects on the plant the solution is to add a low-pass filter to the output of the ZOH (with time constant of the order of the sampling period)

• Fractional order holds:

A first order hold (FOH) performs a direct linear extrapolation from one sampling interval to the next.



⇒ We may attempt to reduce the error by using only a fraction of the slope

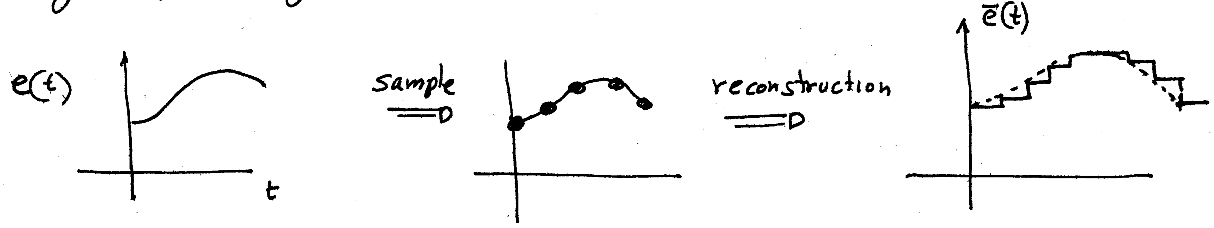


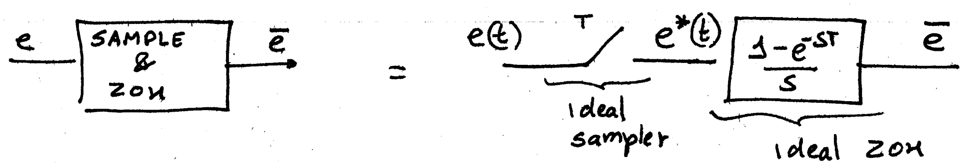
for $k=0$ you get ZOH
for $k=1$ you get FOH

$$G_{hk}(s) = \left(1 - k e^{-Ts}\right)\left(\frac{1 - e^{-Ts}}{s}\right) + \frac{k}{Ts^2}\left(1 - e^{-Ts}\right)^2$$

Summary:

Analysis of sampling and reconstruction:



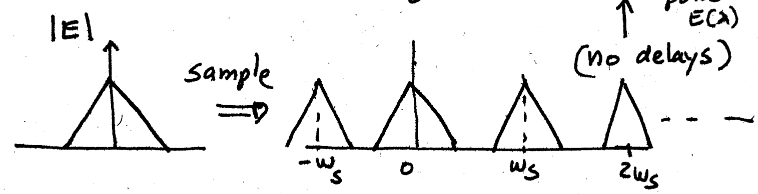


Ideal sampler $\underset{E}{e} \xrightarrow{T} \underset{E^*}{e^*}$

$$e^*(t) = \sum_0^\infty e(kT)\,\delta(t-kT) = e(t)\cdot\delta_T(t)$$

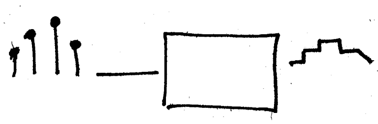
$$E^*(s) = \sum_0^\infty e(kT)\,e^{-kTs} \underset{\substack{\uparrow \\ \text{(no delays)}}}{=} \sum_{\substack{\text{poles}\\ E(\lambda)}} \text{Res}\left\{E(\lambda)\frac{1}{(1-e^{-T(s-\lambda)})}\right\}$$

Spectrum of $E^*(s)$:



Can reconstruct E from E^* <u>iff</u> $\omega_1 \le \frac{\omega_s}{2}$ (Shannon's Theorem)

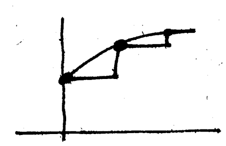
Reconstruction:



Polynomial interpolation: $e_n(t) = e(T) + e'(T)(t-T) + \cdots$

<u>ZOH:</u> $e_n(t) = e(kT) \qquad kT \le t < (k-1)T$

$$G_{ZOH} = \frac{1-e^{-sT}}{s}$$



F.OH: $e_n(t) = e(kT) + e'(kT)(t-kT)$

$$G_{FOH} = \left(\frac{1-e^{-sT}}{s}\right)^2\left(\frac{1+sT}{T}\right)$$