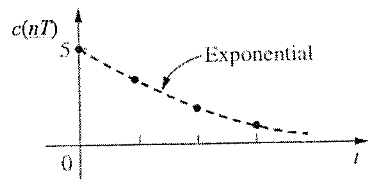
4.3-2

(a) $$G(z) = \frac{z-1}{z}\mathcal{Z}\left[\frac{5s}{s(s+0.1)}\right] = \frac{z-1}{z} \times \frac{5z}{z-\varepsilon^{-0.2}} = \frac{5(z-1)}{z-0.8187}$$

$$Y(z) = \left(\frac{z}{z-1}\right)\frac{5(z-1)}{z-0.8187} = \frac{5z}{z-0.8187}$$



$$\therefore c(nT) = 5(0.8187)^n$$

(b) $$M(s) = \frac{1}{s}$$

$$\therefore Y(s) = \frac{5s}{s(s+0.1)} = \frac{5}{s+0.1}, \quad \therefore y(t) = 5\varepsilon^{-0.1t}$$

$$\therefore y(nT) = y(2n) = 5\varepsilon^{-0.1(2n)} = 5(\varepsilon^{-0.2})^n = 5(0.8187)^n$$

(c) $$\text{dc gain} = G_p(s)\big|_{s=0} = \frac{5(0)}{0+0.1} = 0$$

$$\text{dc gain} = G(z)\big|_{z=1} = \frac{5(1-1)}{1-0.8187} = 0$$

(d) Yes - $C_{ss}(kT) = (1)(\text{dc gain}) = 0$ from (a), $\therefore$ dc gain $= 0$

$C_{ss}(t) = (1)(\text{dc gain}) = 0$ from (b), $\therefore$ dc gain $= 0$

4.3-7.

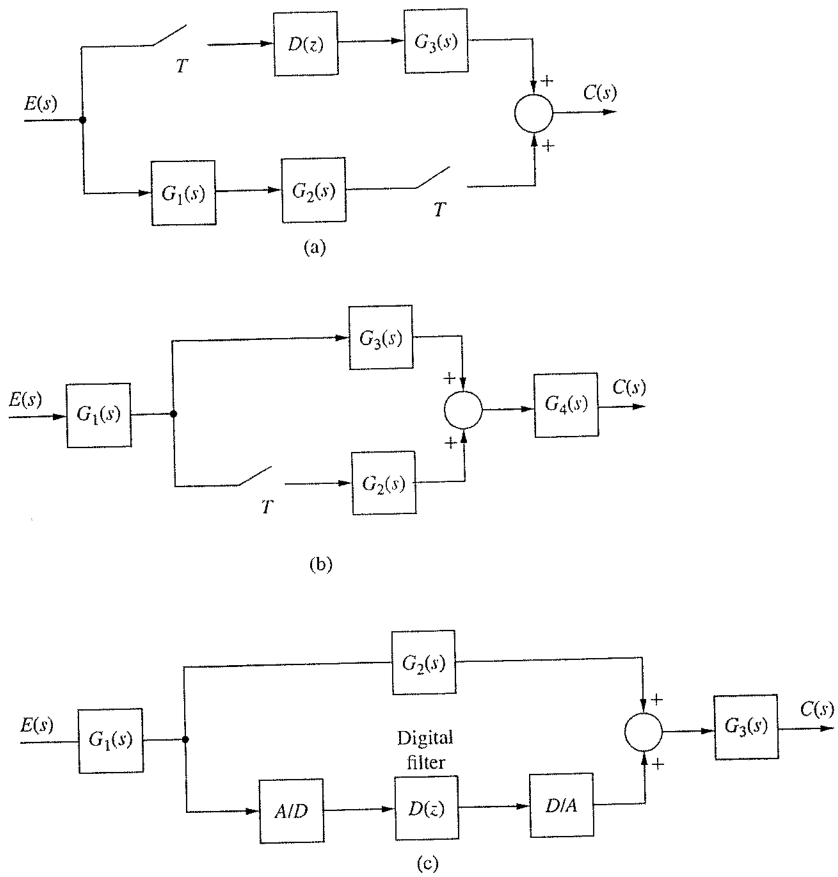


Fig. P4.3-7

(a) Express each $C(s)$ and $C(z)$ as functions of the input for the systems of Fig. P4.3-7.

(b) List those transfer functions in Fig. P4.3-7 that contain the transfer function of a data hold.

Solution:

(a) (i) $C(z) = G_3(z)D(z)E(z) + \overline{G_1 G_2 E}(z)$

(ii) $C(z) = \overline{G_1 G_3 G_4 E}(z) + \overline{G_2 G_4}(z)\overline{G_1 E}(z)$

(iii) $C(z) = \overline{G_1 G_2 G_3 E}(z) + G_3^1(z) D(z) \overline{G_1 E}(z),$ $\quad G_3^1(z) = \left(\dfrac{1-\varepsilon^{-Ts}}{s}\right) G_3(s)$

(b) (i) $G_3(s)$

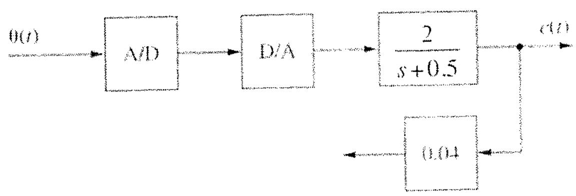
(ii) $G_2(s)$

(iii) $G_3^1(s)$

4.3-9

Solution:

(a)



(b) $G(z) = \dfrac{z-1}{z}\mathcal{Z}\left[\dfrac{2}{s(s+0.5)}\right] = \dfrac{z-1}{z}\dfrac{2}{0.5}\left[\dfrac{z(1-\varepsilon^{-0.5T})}{(z-1)(z-\varepsilon^{-0.5T})}\right]$

$$= \frac{4(1-\varepsilon^{-0.5T})}{z-\varepsilon^{-0.5T}}$$

(c) $G_p(0) = \text{dc gain} = \dfrac{2}{0.5} = 4, \therefore c_{ss}(kT) = 10(4) = 40°\text{C}$

(d) $G_d(0) = -\dfrac{2.5}{0.5} = -5, \;\therefore \Delta c_{ss}(kT) = (-5)(1) = -5°\text{C}$

(e) $C(s) = E^*(s)\left(\dfrac{1-\varepsilon^{-Ts}}{s}\right)\left(\dfrac{2}{s+0.5}\right) - \dfrac{2.5}{s+0.5}D(s)$

4.4-1. Example 4.3 calculates the step response of the system in Fig. 4-2. Example 4.4 calculates the step response of the same system preceded by a digital filter with the transfer function $D(z) = (2 - z^{-1})$. This system is shown in Fig. P4.4-1.

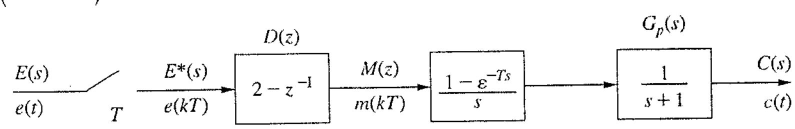


Fig. P4.4-1

(a) Solve for the output of the digital filter $m(kT)$.

(b) Let the response in Example 4.3 be denoted as $c_1(kT)$. Use the results in part (a) to express the output $c(kT)$ in Fig. P4.4-1 as a function of $c_1(kT)$.

(c) Use the response $c_1(kT)$ calculated in Example 4.3 and the results in part (b) to find the output $c(kT)$ in Fig. P4.4-1. This result should be the same as in Example 4.4.

(d) Use the response $C_1(z)$ calculated in Example 4.3 and the result in part (b) to find the output $C(z)$ in Fig. P4.4-1. This result should be the same as in Example 4.4.

Solution:

(a) $M(z) = (2 - z^{-1})E(z)$

$\therefore m(kT) = 2e(kT) - e[(k-1)T] = 2u(kT) - u[(k-1)T]$

$\therefore m(0) = 2,\ \ m(k) = 1,\ \ k \geq 1$

(b) $C(kT) = 2C_1(kT) - C_1[(k-1)T]$

(c) $C_1(kT) = (1 - \varepsilon^{-kT})u(kT)$

$C(kT) = 2(1 - \varepsilon^{-kT})u(kT) - u[(k-1)T] + \varepsilon^{-(k-1)T}u[(k-1)T]$

$$= \begin{cases} 0, & k = 0 \\ 1 - (2 - \varepsilon^{-T})\varepsilon^{-kT}, & k \geq 1 \end{cases}$$

(d) Example 4.3, $C_1(kT) = \dfrac{z\,(1-\varepsilon^{-T})}{(z-1)(z-\varepsilon^{-T})}$

$$\therefore C(z) = 2C_1(z) - z^{-1}C_1(z) = \frac{2z\,(1-\varepsilon^{-T})}{(z-1)(z-\varepsilon^{-T})} - \frac{1-\varepsilon^{-T}}{(z-1)(z-\varepsilon^{-T})}$$

$$= \frac{(2z-1)(1-\varepsilon^{-T})}{(z-1)(z-\varepsilon^{-T})}$$

4.4-5. Consider the system of Fig. P4.4-5. The filter transfer function is $D(z)$.

Fig. P4.4-5

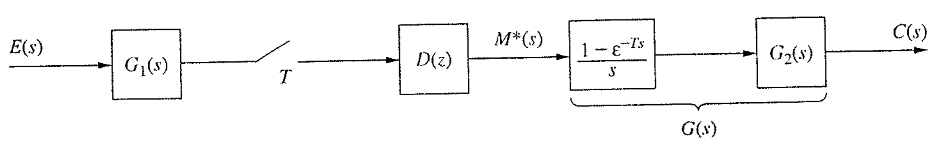
(a) Express $C(z)$ as a function of E.

(b) A discrete state model of this system does not exist. Why?

(c) What assumptions concerning $e(t)$ must be made in order to derive an approximate discrete state model?

Solution:

(a)



$$C(s) = G(s)M^*(s)$$

$$C(z) = G(z)M(z) = G(z)D(z)\ \overline{G_1E}(z)$$

(b) Cannot factor $E(z)$ from $\overline{G_1E}(z)$.

(c) Assume that $e(t)$ changes so slowly that the system can be accurately approximated by placing a sampler/data-hold in front of $G_1(s)$. Then

$$C(z) = G(z)\ D(z)\ \mathfrak{z}\left[\frac{1-\varepsilon^{-Ts}}{s}\ G_1(s)\right]E(z).$$

4.6-2. For the thermal test chamber of Problems 4.3-9 and 4.6-1, let $T = 0.6\text{ s}$. Suppose that a proportional-integral (PI) digital controller with the transfer function

$$D(z) = 1.2 + \frac{0.1z}{z-1}$$

is inserted between the sampler and the zero-order hold in Fig. P4.3–9.

(a) With $d(t) = 0$ and $e(t) = u(t)$, solve for $c(kT)$, with the time delay of 2 s omitted.

(b) Explain what happens to the temperature in the chamber in part (a). Is this result physically possible?

(c) In Fig. P4.3-9, calculate $m(kT)$, the signal that controls the valve in the steam line, for the inputs of part (a).

(d) Considering the physical characteristics of a valve, what happens to the temperature in the physical test chamber?

Solution:

(a) $D(z) = 1.2 + \dfrac{0.1z}{z-1} = \dfrac{1.3z-1}{z-1}$; From Prob, 4-21(a) : $G(z) = \dfrac{1.037}{z-0.7408}$

$$\therefore \frac{C(z)}{z} = \frac{1.037(1.3z-1)}{(z-1)^2(z-0.7408)} = \frac{1.20}{(z-1)^2} + \frac{0.5705}{z-1} + \frac{-0.5705}{z-0.7408}$$

$$\therefore C(z) = \frac{2(0.6z)}{(z-1)^2} + \frac{0.5705z}{z-1} + \frac{-0.5705z}{z-0.7408}$$

$$\therefore c(kT) = 2kT + 0.5705[1-0.7408^k]$$

(b) The temperature increases without limit-*Not* physically realizable.

(c) $$\frac{M(z)}{z} = \frac{1.3z-1}{(z-1)^2} = \frac{1.3(z-1)}{(z-1)^2} + \frac{0.3}{(z-1)^2} = \frac{0.3}{(z-1)^2} + \frac{1.3}{z-1}$$

$$\therefore m(kT) = kT + 1.3$$

(d) The value will reach full open. Then $m(kT)$ is affectively constant and the temperature will settle to a constant value.