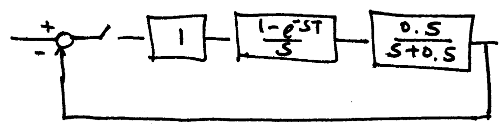
6.1)




(a) NO sample & hold:

$$G_{cl} = \frac{0.5}{s+1} \Rightarrow C(s) = \frac{0.5}{s(s+1)} = 0.5\left(\frac{1}{s} - \frac{1}{s+1}\right)$$

$$\Rightarrow \boxed{c(t) = 0.5\left(1 - e^{-t}\right)}$$

(b) with S & H, $T = 0.4s$

$$\mathcal{Z}\left[\frac{1-e^{-sT}}{s}\,\frac{0.5}{(s+0.5)}\right] = \left(\frac{z-1}{z}\right)\frac{z}{(z-1)}\,\frac{1-e^{-0.5\cdot 0.4}}{(z - e^{-0.2})} = \frac{0.1813}{z - 0.8187}$$

$$G_{cl}(z) = \frac{0.1813}{z - 0.6374} \Rightarrow C(z) = G_{cl}(z)\,R(z) = G_{cl}(z)\,\frac{z}{z-1}$$

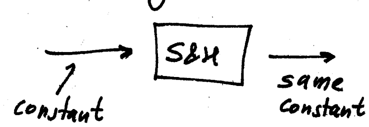
$$C(z) = \frac{0.1813\,z}{(z-1)(z-0.6374)} = \frac{z}{2}\left[\frac{1}{z-1} - \frac{1}{z-0.6374}\right]$$

$$c(k) = 0.5\cdot\left[1 - [0.6374]^k\right] = 0.5\left[1 - e^{-0.45k}\right]$$

$$\boxed{c(kT) = 0.5\left(1 - e^{-0.45k}\right)}$$

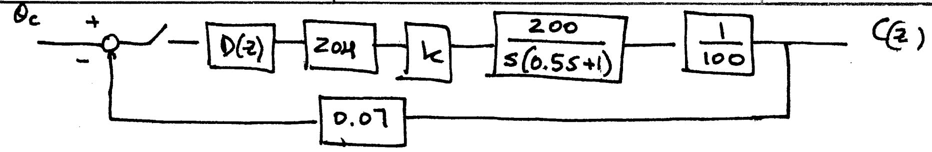
c) continuous time DC gain $= G_{ce}(0) = 0.5$ } same
discrete time DC gain $= G_{ce}(z)\big|_{z=1} = 0.5$

In steady state the sample & hold does <u>not have any effect</u>




$\Rightarrow$ <u>if</u> both systems <u>are stable</u> they ought to have the same DC gain

6.7)




$T = 0.1$
$D = 1$

Want $C_{ss}(z) = 20^\circ \Rightarrow \Theta_{ss} = 0.07 \cdot 20 = 1.4$ Volts

From problem 5.13 we have: $\frac{C(z)}{\Theta(z)} = \frac{\frac{z-1}{z} k D G}{1 + \frac{z-1}{z} (0.07) k D G}$ with $G(z) = \frac{z\,[0.0187z + 0.0175]}{(z-1)^2 (z - 0.8187)}$

$\Rightarrow$ if $k = 10 \Rightarrow \frac{C(z)}{\Theta(z)} = \frac{0.1873z + 0.1752}{z^2 - 0.8056z + 0.831} \Rightarrow$ poles at $\boxed{\begin{aligned} z_{1,2} &= 0.9030 \pm j\,0.1249 \\ &= 0.9116 \angle \pm 0.1374 \end{aligned}}$

(since $|z_{1,2}| < 1 \Rightarrow$ stable)

equivalent s plane poles $z = e^{sT} \Rightarrow s = \frac{1}{T} \log z \Rightarrow s = -0.9256 \pm j\,1.3741$

$\Rightarrow$ Time constant $\tau = \frac{1}{|\sigma|} = 1.08$ s $\Rightarrow \boxed{T_s \cong 4.3 \text{ sec}}$

$C(z) = G_{cl}(z)\,\Theta(z) = \left(\frac{0.1873z + 0.1752}{z^2 - 0.8056z + 0.831}\right) \cdot 1.4 \frac{z}{z-1} \Rightarrow C_{ss} = \lim_{z \to 1} (z-1)\,C(z) = \boxed{20}$ #

(Note: can get the same result using the fact that cont time DC gain = discrete time DC gain)

6.8) open loop: $G_{ol} = k \left[\frac{0.0187z + 0.0175}{(z-1)(z-0.8187)}\right] \Rightarrow$ poles at:

$z_1 = 1 \overset{z = e^{sT}}{\Longleftrightarrow} s = 0$

$z = 0.8187 \qquad s = -2$

$\Rightarrow$ (a) $\tau_1 = \frac{1}{0} = \infty$; $\tau_2 = \frac{1}{2} = 0.5$

(b) Closed loop: from 6.7 we have $z_{1,2} = 0.9116 \angle \pm 0.1374$

$\Rightarrow \tau_d = \frac{-T}{\log|z|} = 1.08$, $\omega_{n_d} = \frac{1}{T}\sqrt{\ln^2 r + \theta^2} = 1.657$, $\zeta_d = \frac{-\ln r}{\sqrt{\ln^2 r + \theta^2}} = 0.558$

(c) Analog system: $G_{cl} = \frac{40}{s^2 + 2s + 2.8}$

$\Rightarrow \omega_n = \sqrt{2.8} = 1.68$, $\omega_n \zeta = 1 \;//\; \zeta = \frac{1}{\omega_n} = 0.598$, $\tau = \frac{1}{\zeta \omega_n} = 1$

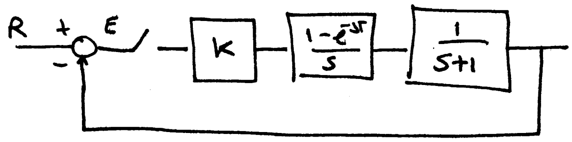
(d)

| | Sampled Data | Analog |
|---|---|---|
| $T_s$ | 4.3 sec | 4 sec |
| $M_p =$ | 12% | 9.6% |

$\left(M_p = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}\right)$

$\Rightarrow$ sampling <u>reduces</u> stability, increases $M_p$ & $T_s$

7.6)




a) $G_1(z) = \mathcal{Z}\left[\frac{1}{s(s+1)}\right] = \frac{z(1-e^{-T})}{(z-1)(z-e^{-T})} \Rightarrow G(z) = \left(\frac{z-1}{z}\right) G_1(z) = \frac{1-e^{-T}}{z-e^{-T}}$

$\Rightarrow$ Char. eq: $1 + \frac{k(1-e^{-T})}{z-e^{-T}} = 0 \Leftrightarrow \boxed{z - e^{-T} + k(1-e^{-T}) = 0}$

b) For stability we need poles in $|z|<1 \Rightarrow |e^{-T} - k(1-e^{-T})| < 1 \Leftrightarrow$

$-1 < e^{-T} - k(1-e^{-T}) < 1$

left hand side ineq. yields: $K < \frac{1+e^{-T}}{1-e^{-T}}$

RHS yields: $-k(1-e^{-T}) < 1-e^{-T} \Rightarrow -k < 1 \Rightarrow k > -1$

$\Rightarrow \boxed{-1 < K < \frac{1+e^{-T}}{1-e^{-T}}}$

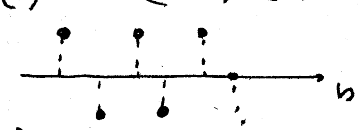
c) if $k=-1$ then we have a closed loop pole at $z=1$, i.e.

$G_{ce}(z) = -\frac{(1-e^{-T})}{z-1}$, $\Rightarrow$ impulse response: $c(0) = 0$, $c(n) = -(1-e^{-T})$, $n>0$

(note: since $c(n)$ constant $\Rightarrow \sum_0^\infty |c(n)| \to \infty$ <u>NOT</u> BIBO STABLE)

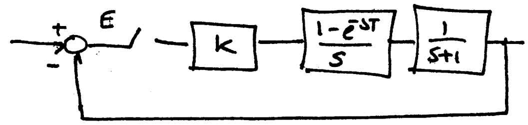
d) if $K = \frac{1+e^{-T}}{1-e^{-T}}$ we have a closed loop pole at $z=-1$, i.e.

$G_{ce} = \frac{1+e^{-T}}{z+1} \Rightarrow$ impulse response: $c(0) = 0$, $c(n) = (1+e^{-T})\cdot(-1)^{(n-1)}$ $n>0$




(again $|c(n)|$ constant $\Rightarrow \sum_0^\infty |c(n)| \to \infty$, NOT BIBO STABLE)

7.8)

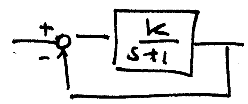



b) From problem 7.6 we have: $z - e^{-T} + k(1-e^{-T}) = 0$

stable $\Leftrightarrow$ $-1 < k < \frac{1+e^{-T}}{1-e^{-T}}$ $\Rightarrow$

| increasing T | | decreasing stability |
|---|---|---|
| $T = 0.01$ | | $-1 < k < 200$ |
| $T = 0.1$ | | $-1 < k < 20$ |
| $T = 1$ | | $-1 < k < 2.16$ |

c) with no sample & hold:




char eq: $s + 1 + k = 0$, pole at $s = -(1+k)$ $\Rightarrow$ stable for all $-1 < k < \infty$

d) As $T \uparrow$ the upper range of stability for $k \downarrow$.
(If $T \to 0$ $\Rightarrow$ upper range of stability for $k \to$ range of cont. time system)

<u>Sampling reduces stability</u>