EECE 5610, Fall 2025 Homework #2, Assigned 09/19/2025, Due 09/26/2025, Turnitin.

1. Problem 2.2.2 text
2. Problem 2.2.3 text
3. Problem 2.6.3 text
4. Problem 2.7.1 text

- Write the difference equation relating $y(k+1)$, $y(k)$, and $x(k)$ for the left-side rule.
- Find the transfer function $Y(z)/X(z)$ for part (a).
- Write the difference equation relating $y(k+1)$, $y(k)$, and $x(k+1)$ for the right-side rule.
- Find the transfer function $Y(z)/X(z)$ for part (c).
- Express $y(k)$ as a summation on $x(k)$ for the left-side rule.
- Express $y(k)$ as a summation on $x(k)$ for the right-side rule.
- 2.2-2. The trapezoidal rule (modified Euler method) for numerical integration approximates the integral of a function $x(t)$ by summing trapezoid areas as shown in **Fig. P2.2-2**. Let $y(t)$ be the integral of $x(t)$.

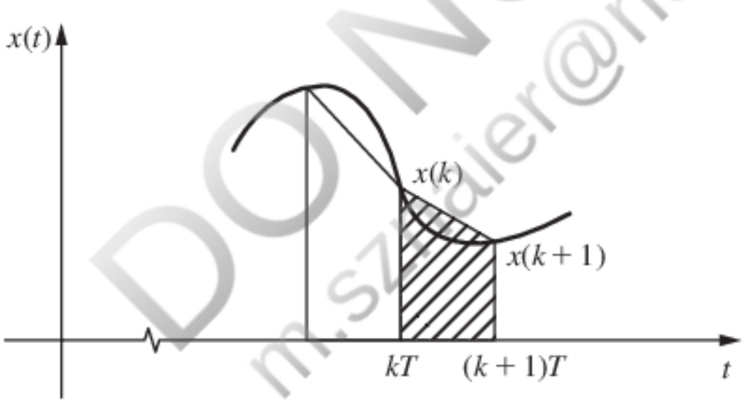

Figure P2.2-2 Trapezoidal rule for numerical integration.

Figure P2.2-2 Full Alternative Text

a. Write the difference equation relating $y[(k+1)T]$, $y(kT)$, $x[(k+1)T]$, and $x(kT)$ for this rule.

b. Show that the transfer function for this integrator is given by

$$\frac{(\ )}{(\ )} = \frac{(\ /2)(\ \ + 1)}{\ \ - 1}$$

- 2.2-3.

a. The transfer function for the right-side rectangular-rule integrator was found in **Problem 2.2-1** to be $Y(z)/X(z) = Tz/(z - 1)$. We would suspect that the reciprocal of this transfer function should yield an approximation to a differentiator. That is, if $w(kT)$ is a numerical derivative of $x(t)$ at $t = kT$,

$$\frac{W(z)}{X(z)} = \frac{z - 1}{Tz}$$

Write the difference equation describing this differentiator.

- Write the difference equation relating $y(k + 1)$, $y(k)$, and $x(k)$ for the left-side rule.
- Find the transfer function $Y(z)/X(z)$ for part (a).
- Write the difference equation relating $y(k + 1)$, $y(k)$, and $x(k + 1)$ for the right-side rule.
- Find the transfer function $Y(z)/X(z)$ for part (c).
- Express $y(k)$ as a summation on $x(k)$ for the left-side rule.
- Express $y(k)$ as a summation on $x(k)$ for the right-side rule.
- 2.2-2. The trapezoidal rule (modified Euler method) for numerical integration approximates the integral of a function $x(t)$ by summing trapezoid areas as shown in **Fig. P2.2-2**. Let $y(t)$ be the integral of $x(t)$.

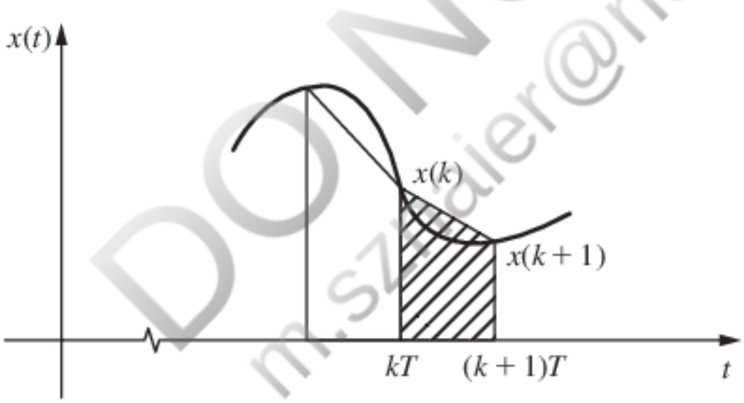

Figure P2.2-2 Trapezoidal rule for numerical integration.

Figure P2.2-2 Full Alternative Text

a. Write the difference equation relating $y[(k + 1)T]$, $y(kT)$, $x[(k + 1)T]$, and $x(kT)$ for this rule.

b. Show that the transfer function for this integrator is given by

$$\frac{(\ )}{(\ )} = \frac{(\ /2)(\ + 1)}{\ - 1}$$

- 2.2-3.

a. The transfer function for the right-side rectangular-rule integrator was found in **Problem 2.2-1** to be $Y(z)/X(z) = Tz/(z - 1)$. We would suspect that the reciprocal of this transfer function should yield an approximation to a differentiator. That is, if $w(kT)$ is a numerical derivative of $x(t)$ at $t = kT$,

$$\frac{W(z)}{X(z)} = \frac{z - 1}{Tz}$$

Write the difference equation describing this differentiator.

- The transfer function for the right-side rectangular-rule integrator was found in **Problem 2.2-1** to be $Y(z)/X(z) = Tz/(z - 1)$. We would suspect that the reciprocal of this transfer function should yield an approximation to a differentiator. That is, if $w(kT)$ is a numerical derivative of $x(t)$ at $t = kT$,

$$\frac{W(z)}{X(z)} = \frac{z - 1}{Tz}$$

  Write the difference equation describing this differentiator.

- Draw a figure similar to those in **Fig. P2.2-1** illustrating the approximate differentiation.

- Repeat part (a) for the left-side rule, where $W(z)/X(z) = T/(z - 1)$.

- Repeat part (b) for the differentiator of part (c).

- 2.3-1. Find the z-transform of the number sequence generated by sampling the time function $e(t) = t$ every T seconds, beginning at $t = 0$. Can you express this transform in closed form?

- 2.3-2.

  a. Write, as a series, the z-transform of the number sequence generated by sampling the time function $e(t) = t$ every T seconds, beginning at $t = 0$. Can you express this transform in closed form?
  b. Evaluate the coefficients in the series of part (a) for the case that $T = 0.05$ s.
  c. The exponential $e(t) = \varepsilon^{-bt}$ is sampled every $T = 0.2$ s, yielding the z-transform

$$E(z) = 1 + \left(\frac{1}{2}\right)z^{-1} + \left(\frac{1}{2}\right)z^{-2} + \left(\frac{1}{2}\right)z^{-3} + \cdots$$

Evaluate *b*.

-

2.3-3. Find the *z*-transforms of the number sequences generated by sampling the following time functions every *T* seconds, beginning at $t = 0$. Express these transforms in closed form.

a. $e(t) = \varepsilon^{-at}$
b. $e(t) = \varepsilon^{-(t-T)}u(t - T)$
c. $e(t) = \varepsilon^{-(t-5T)}u(t - 5T)$

-

2.4-1. A function $e(t)$ is sampled, and the resultant sequence has the *z*-transform

$$E(z) = \frac{z^3 - 2z}{z^4 - 0.9z^2 + 0.8}$$

Solve this problem using $E(z)$ and the properties of the *z*-transform.

a. Find the *z*-transform of $e(t - 2T)u(t - 2T)$.
b. Find the *z*-transform of $e(t + 2)u(t)$.
c. Find the *z*-transform of $e(t - T)\, u(t - 2T)$.

-

2.4-2. A function $e(t)$ is sampled, and the resultant sequence has the *z*-transform

$$E(z) = \frac{z - b}{z^2 - cz^2 + d}$$

Find the *z*-transform of $\varepsilon^{akT}e(kT)$. Solve this problem using $E(z)$ and the properties of the *z*-transform.

-

2.5-1. From **Table 2-3**,

$$\mathfrak{z}[\cos akT] = \frac{z(z - \cos aT)}{z^2 - 2z \cos aT + 1}$$

a. Find the conditions on the parameter a such that $\mathfrak{z}$ [cos akT] is first order (pole-zero cancellation occurs).
b. Give the first-order transfer function in part (a).
c. Find a such that $\mathfrak{z}$[cos akT] = $\mathfrak{z}$[$u(kT)$], where $u(kT)$ is the unit step function.

•

2.5-2. Find the z-transform, in closed form, of the number sequence generated by sampling the time function $e(t)$ every T seconds beginning at t = 0. The function $e(t)$ is

specified by its Laplace transform,

$$E(s) = \frac{2(1 - \varepsilon^{-5s})}{s(s + 2)}, \quad T = 1\,\text{s}$$

•

2.5-3. Use the residue method to find the z-transform of the following functions. Check your results by computer.

a. $E(s) = \frac{20}{(s+2)(s+5)}$
b. $E(s) = \frac{5}{s(s+1)}$
c. $E(s) = \frac{s+2}{s(s+1)}$
d. $E(s) = \frac{s+2}{s^2(s+1)}$
e. $E(s) = \frac{s^2+5s+6}{s(s+4)(s+5)}$
f. $E(s) = \frac{2}{s^2+2s+5}$

•

2.6-1. Solve the given difference equation for $x(k)$ using:

a. The sequential technique.
b. The z-transform.
c. Will the final-value theorem give the correct value of $x(k)$ as $k \to \infty$?

$$x(k) - 3x(k-1) + 2x(k-2) = e(k)$$

where

$$\begin{aligned} e(k) &= \begin{cases} 1, & k = 0, 1 \\ 0, & k \geq 2 \end{cases} \\ x(-2) &= x(-1) = 0 \end{aligned}$$

-

2.6-2. Given the difference equation

$$y(k+2) - \frac{3}{4}y(k+1) + \frac{1}{8}y(k) = e(k)$$

where $y(0) = y(1) = 0$, $e(0) = 0$, and $e(k) = 1$, $k = 1,2,\ldots$.

a. Solve for $y(k)$ as a function of k, and give the numerical values of $y(k)$, $0 \leq k \leq 4$.
b. Solve the difference equation directly for $y(k)$, $0 \leq k \leq 4$. to verify the results of part (a).
c. Repeat parts (a) and (b) for $e(k) = 0$ for all k, and $y(0) = l$, $y(l) = -2$.

-

2.6-3. Given the difference equation

$$x(k) - x(k-1) + x(k-2) = e(k)$$

where $e(k) = 1$ for $k \geq 0$.

a. Solve for $x(k)$ as a function of k, using the z-transform. Give the values of $x(0)$, $x(1)$, and $x(2)$.

b. Verify the values $x(0)$, $x(1)$, and $x(2)$, using the power-series method.
c. Verify the values $x(0)$, $x(1)$, and $x(2)$ by solving the difference equation directly.
d. Will the final-value property give the correct value for $x(\infty)$?

-

2.6-4. Given the difference equation

2.6-4. Given the difference equation

$$x(k+2) + 3x(k+1) + 2x(k) = e(k)$$

where

$$e(k) = \begin{cases} 1, & k = 0 \\ 0, & \text{otherwise} \end{cases}$$
$$x(0) = 1$$
$$x(1) = -1$$

a. Solve for $x(k)$ as a function of k.
b. Evaluate $x(0)$, $x(1)$, $x(2)$, and $x(3)$ in part (a).
c. Verify the results in part (b) using the power-series method.
d. Verify the results in part (b) by solving the difference equation directly.

-

2.6-5. Given the difference equation

$$x(k+3) - 2.2x(k+2) + 1.57x(k+1) - 0.36x(k) = e(k)$$

where $e(k) = 1$ for all $k \geq 0$ and $x(0) = x(1) = x(2) = 0$.

a. Write a digital computer program that will calculate $x(k)$. Run this program solving for $x(3), x(4), \ldots, x(25)$.
b. Using the sequential technique, check the values of $x(k)$, $0 \leq k \leq 5$.
c. Use the z-transform and the power-series method to verify the values $x(k)$, $0 \leq k \leq 5$.

-

2.6-6. Given the MATLAB program

```
>>s1 = 0; e = 0; for k = 0:5 s2 = e - s1; m = 0.5*s2 - s1; s1 =
s2; [k,m] e = e + 1; end
```

that solves the difference equation of a digital controller.

a. Find the transfer function of the controller.
b. Find the *z*-transform of the controller input.
c. Use the results of parts (a) and (b) to find the inverse *z*-transform of the controller output.
d. Run the program to check the results of part (c).

- 2.7-1.

a. Find $e(0)$, $e(1)$, and $e(10)$ for

using the inversion formula.
b. Check the value of $e(0)$ using the initial-value property.
c. Check the values calculated in part (a) using partial fractions.
d. Find $e(k)$ for $k = 0, 1, 2, 3$, and 4 if $\mathcal{Z}[e(k)]$ is given by

e. Find a function $e(t)$ which, when sampled at a rate of 10 Hz ($T = 0.1$ *s*), results in the transform $E(z) = 2z/(z - 0.8)$.
f. Repeat part (e) for $E(z) = 2z/(z + 0.8)$.
g. From parts (e) and (f), what is the effect on the inverse *z*-transform of changing the sign on a real pole?

- 2.7-2. For the number sequence $\{e(k)\}$,

a. Apply the final-value theorem to $E(z)$.

b. Check your result in part (a) by finding the inverse z-transform of $E(z)$.

c. Repeat parts (a) and (b) with $E(z) = z/(z-1)^2$.

c. Repeat parts (a) and (b) with $E(z) = z/(z-1)^2$.

d. Repeat parts (a) and (b) with $E(z) = z/(z-0.9)^2$.

e. Repeat parts (a) and (b) with $E(z) = z/(z-1.1)^2$.

•

2.7-3. Find the inverse z-transform of each $E(z)$ below by the four methods given in the text. Compare the values of $e(k)$, for $k = 0, 1, 2,$ and 3, obtained by the four methods.

a. $E(z) = \frac{0.5z}{(z-1)(z-0.6)}$

b. $E(z) = \frac{0.5}{(z-1)(z-0.6)}$

c. $E(z) = \frac{0.5(z+1)}{(z-1)(z-0.6)}$

d. $E(z) = \frac{z(z-0.7)}{(z-1)(z-0.6)}$

e. Use MATLAB to verify the partial-fraction expansions.

•

2.7-4. Find the inverse z-transform of each $E(z)$ below by any method.

a. $E(z) = \frac{0.5z^2}{(z-1)^2(z-0.6)}$

b. $E(z) = \frac{0.5}{(z-1)(z-0.6)^2}$

c. $E(z) = \frac{0.5z(z-0.7)}{(z-1)(z-0.6)^2}$

d. $E(z) = \frac{0.5z(z-0.7)}{(z-1)(z-0.59)(z-0.61)}$

•