Note: Quantum Physics in One Dimension

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1 Basic Concepts fot interacting quantum systems

1.1 Weak Interacting particles

1.1.1 Fermi liquid and free fermions

Anti-symmetric wave function: $c_k^\dagger(c_k)$ the (de)creation operator be anti commute with each other. Hamiltonian

$$H = \sum_k \epsilon_k c_k^\dagger c_k$$

and the Fermi energy in condensed matter system is about $\epsilon_F \sim 1\text{eV} \sim 12000\text{K}$. For cold atom systems, $\epsilon_F \sim 100\text{nK}$. Density of states:

$$D(E) = \sum_k \delta(E - \epsilon_k)$$

Average:

$$\langle \cdots \rangle = Z^{-1} Tr\left[e^{-\beta H} \cdots\right]$$

Spechual Function. The correlation function:

$$\langle GS|e^{iHt} c_{x,t} e^{-iHt} c_{0,0}^\dagger |GS\rangle$$

Green Function:

$$G(x,t) = -i\theta(t)\langle [c_{x,t}, c_{0,0}^\dagger]\rangle$$

Probability to find an excitation:

$$A(k,\omega) = -\frac{1}{\pi} ImG(k,\omega)$$

(Kallen-Lehmmann) and the Green Function is

$$G(k,\omega) = \frac{1}{\omega - \xi_k + i\delta} \quad \delta = 0^+$$

$\uparrow$ is the retarded Green Function.
Effect of interaction in metals $U \sim E_F$ so the fermi surface no longer exist. Landau Quasi particles–dressed electrons, redefine parameters, effective mass, free quasi particles. $n(\epsilon)$ of quasi particles has a jump at E_F.

In 2d and 3d, FLT can get rid of interaction.

1.1.2 Interacting Boson system

BEC:$\langle b_k^\dagger b_k\rangle = N\delta_{k,0}$ and $n_k = N_0\delta_{k,0} + n^{\text{regular}}(k)$.
Bogoliubov approximation:

$$H = \sum_k \xi_k b_k^\dagger b_k + U\sum_{kk'q} b_{k+q}^\dagger b_{k'-q}^\dagger b_{k'} b_k$$

assume that there is macroscopic occupation in $k = 0$ and the Hamiltonian will be quadratic.

1.2 Strong Correlation System

A theory on lattice, tight binding model.

$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_j$$

transform into momentum space and the result will be

$$H = \sum_k (-2t \cos k) c_k^\dagger c_k$$

in which the lattice constant $a = 1$.

1.3 One Dimensional System

2 One dimensional systems; Tomonaga-Luttinger liquid

3 Experimental realizations

4 Disorder and other perturbation

5 Beyond Tomonaga-Luttinger liquid