An Introduction to Second Quantization and Path Integral in Statistical Mechanics

Fang Xie, *Department of Physics, Tsinghua University*

April 24, 2016

1 Introduction

Second Quantization is the best way to describe the many-body quantum systems. We can use the creation-annihilation operators to get a lot of interesting results by this method. In this article I will follow the formalism of Pathria's book and show how to use second quantization to get the statistical properties of boson liquid and fermi liquid.

2 Second quantization of Bosons and Fermions

Actually second quantization is the quantization of fields. So we need to define the creation-annihilation operators of the boson or fermion field:

$$[a_i, a_j^\dagger] = \delta_{ij} \tag{1}$$

$$[a_i, a_j] = 0 \tag{2}$$

$$\left[a_i^\dagger, a_j^\dagger\right] = 0 \tag{3}$$

in which i, j means some eigenvalue of some operators. For Fermions the commutation relation is altered by anti-commutation relation. Since the commutation relation is just like the upper and lower operators of Harmonic oscillator, we can find that

$$N = \sum_i a_i^\dagger a_i$$

is the operator of particle number. Now introduce the **one-body operator**: for any one-body operator $\mathcal{O}_1$, its second-quantized form under its eigenbasis is:

$$\hat{O} = \sum_i o_i a_i^\dagger a_i \tag{4}$$

and o_i is the eigenvalue of first-quantized operator O_1. Now if we do an representation transformation to basis that are not the eigenvectors of O_1, then the second quantized operator will be:

$$\hat{O} = \sum_{ij} \langle i|O_1|j\rangle\, a_i^\dagger a_j$$

For example, the second quantized Hamiltonian is

$$H = \sum_{mtk\sigma} a_{\mathbf{k}\sigma}^\dagger a_{\mathbf{k}\sigma} \quad \text{Free particle system}$$

or

$$H = -t \sum_{\langle ij\rangle} a_i^\dagger a_j \quad \text{(Tight-binding approximation)}$$

3