

Electronic Structure from the top down

Predicting the behavior of materials starting from high temperature

Using Dynamical Mean Field theory

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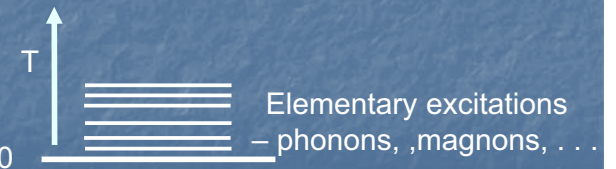
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Electronic Structure from the bottom up (starting at low T)

Key words:
Order
Energy

Ground State – $T=0$



In principle -- well-understood

Advantage – most straightforward to treat ordered systems

Goal for this talk

In much of the field of electronic structure, all the theory and methods for calculation are formulated at zero temperature

This is sufficient for much of the work. I am NOT trying to criticize present calculations!!

But consider the goal that electronic structure calculations can predict the transition temperature of ferromagnetic Fe --- and many other cases where temperature is crucial

Electronic Structure from the top down (starting at high T)

In principle -- well-understood

Advantage – Higher T – less correlation, shorter range

High temperature
($T \gg \text{energy scale } /k_B$)

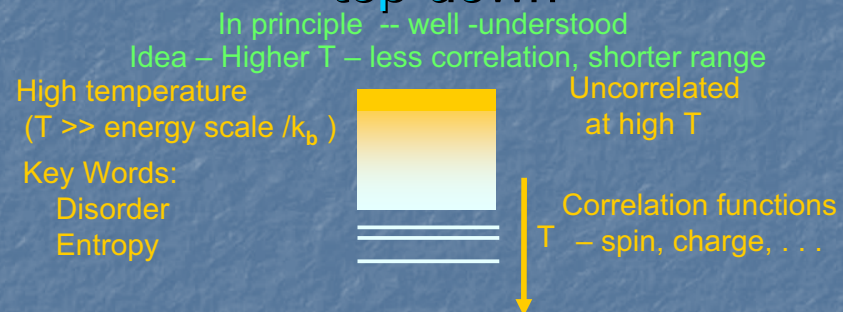
Key Words:
Disorder
Entropy



Uncorrelated
at high T

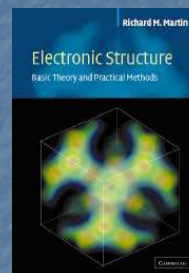
Correlation functions
 T – spin, charge, ...

Electronic Structure from the top down



Nice Description by Walter Kohn
 "Nearsighted"

The big picture in perspective "Nearsightedness"



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Locality and linear scaling $O(N)$ methods

Nearsightedness

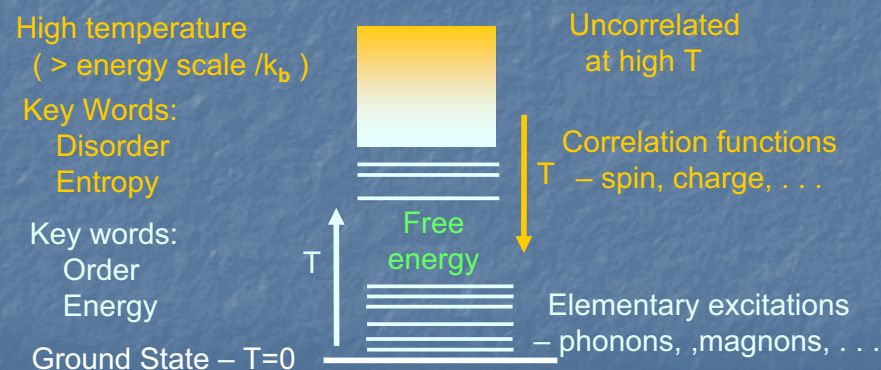
W. Kohn

Throwing out k-space

V. Heine

- Nearsightedness is a property of a many-body system . . .
- In insulators and metals at non-zero temperature, . . .

Electronic Structure The big picture



Theoretical formulation

Partition Function --- $Z = \sum_i \exp(-\beta E_i), \quad \beta = 1/kT$
 $= \text{Tr} \exp(-\beta H)$

Free energy -- $F = -kT \ln Z$

Starting from low T --- more difficult as T increases
 → more eigenstates required

Starting from high T --- more difficult as T decreases
 → longer correlation lengths required

Electronic Temperature scales

Natural --- Hartree/ $k_B \sim 200,000$ K

Large effects of valence electronic excitations in typical materials

--- $eV/k_B \sim 8,000$ K

Good reasons to ignore effect of temperature for electronic excitations in many materials for ordinary T

Practical reasons to ignore effect of temperature for electronic excitations

Questions for audience

Electronic Properties where temperature matters

My list

Magnetism – spins are really electrons

$T_c \sim 1\text{K} - 1000\text{K}$ sets scale for magnetism

Other scales – band width \sim exchange \sim eV

Metal – insulator transitions

Transition metal oxides

Kondo Effect -- magnetic impurities in metals

T_K can be $\sim 10^{-3}$ K

Also can be $\sim 10^3$ K

-- Can cause change in volume by $\sim 15\%$ (Ce)

In every case there are also high energy scales – band widths – exchange energies – multiplet energies --

Electronic Temperature scales

Why is it difficult to treat electronic states in “real materials” at non-zero T ?

Difficult to carry out trace --- $Z = \text{Tr} \exp(-\beta H)$

Feasible for small number of states

Applies to a “real material” only if one can establish that a small number of states is relevant for the interesting properties

Questions for audience

Low energy excitations in magnetic system

Magnetic moments form on atomic sites – disordered at high temperature



Moments order at a critical temperature to form magnetic order

Ferromagnet



Antiferromagnet



Model: Heisenberg Spin Model – appropriate for Lanthanides – reasonable for 3d transition metals (Fe, Ni, ...)

Periodic Table

[illegible]

Transition & Rare Earth Elements

More Localized

Empty Shell Partially Filled Shell Filled Shell

4f 57 La Ce Pr Nd Pm Sm Eu Gd Tb Dy Ho Er Tm Yb 71 Lu

5f 89 Ac Th Pa U Np Pu Am Cm Bk Cf Es Fm Md No 103 Lw

3d 20 Ca 21 Sc 22 Ti 23 V 24 Cr 25 Mn 26 Fe 27 Co 28 Ni 29 Cu 30 Zn 31 Ga

4d 38 Sr 39 Y 40 Zr 41 Nb 42 Mo 43 Tc 44 Ru 45 Rh 46 Pd 47 Ag 48 Cd 49 In

5d 56 Ba 57 La 72 Hf 73 Ta 74 W 75 Re 76 Os 77 Ir 78 Pt 79 Au 80 Hg 81 Th

■ magnetic
 ■ enhanced
 ■ superconducting

From J. L. Smith

Periodic Table

Fractional filling → Strong correlation effects

1 H																	2 He						
3 Li	4 Be																	5 B	6 C	7 N	8 O	9 F	10 Ne
		Transition metals																					
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar						
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr						
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe						
55 Cs	56 Ba	57 La	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Th	82 Pb	83 Bi	84 Po	85 At	86 Rn						
87 Fr	88 Ra	89 Ac	Lanthanides - Actinides																				
		58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu								
		90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lw								

Need method to treat strong correlation on each atom
and
Correlation between sites that determines order, phase transitions, ...

What is “U” in “LDA + U”

How to treat the localized states?

Selection of a few orbitals

Partially filled --- “Open Shell”

Two energy scale -- strong interactions -- U

low energy excitations -- magnetic moments

U is a screened interaction – must be determined

No! – depends upon choice of the localized state

Reasonable choices give reasonable, useful results

This is the problem in making LDA+U quantitative

Important – the same problem limits other methods such as DMFT discussed later!

LDA+U

In LDA + U the system is treated as ordered
Assumes the system is ordered – e.g.,
antiferromagnetic order

This is correct for $T=0$
very valuable results – e.g., correcting the
usual density functionals (like LDA) which often
predict a metal when the correct state is an
ordered antiferromagnetic insulator.

What about disordered states?

Characteristic issues and applications

What about “U” in disordered` states at high temperature?

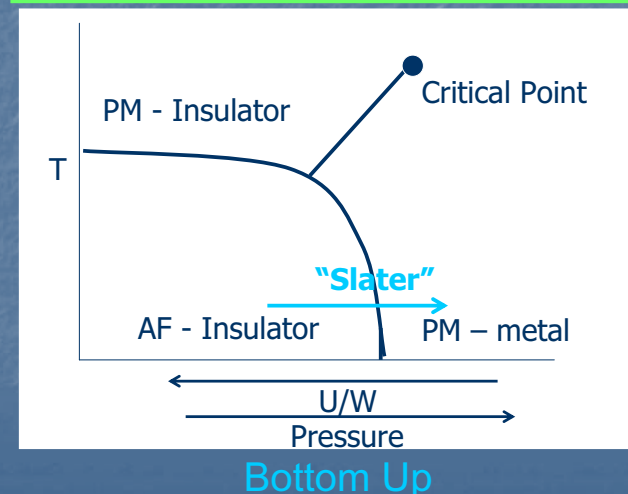
U does not go away with disorder
Large energy scale
Determined by atomic properties

This is where DMFT provides a
systematic approach

Starting from High T and working down!

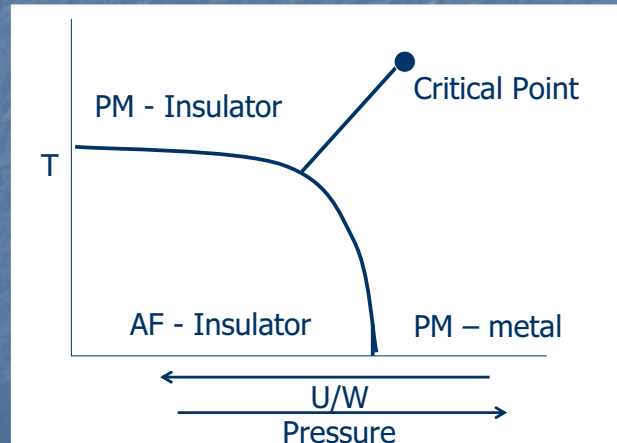
Metal-Insulator Transition

Classic Case --- V_2O_3



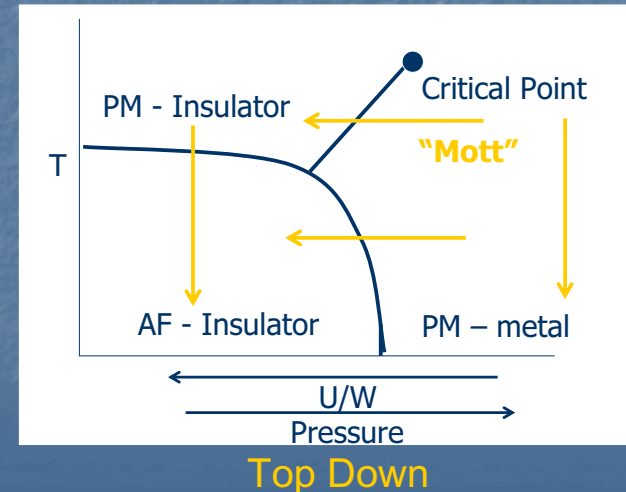
Metal-Insulator Transition

Classic Case --- V_2O_3



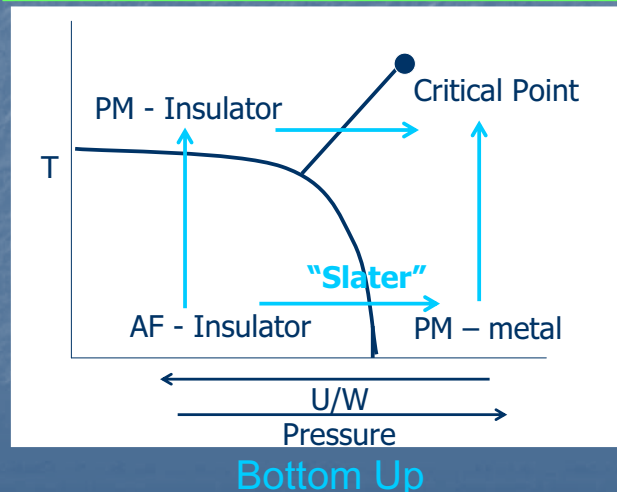
Metal-Insulator Transition

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Metal-Insulator Transition

Classic Case --- V_2O_3

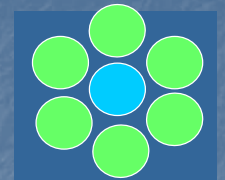


How to treat Correlation?

- Atom in a "bath" of surrounding atoms

Treat correlation on central atom

Treat bath as independent electrons



- Old idea – solve for one atom with some method of averaging over the environment

• Curie-Weiss Mean field for magnetism

- Applied idea for spins in a mean field that is the thermal average of neighboring spins -- each spin is in a "bath" that is an average effect on the spin
- Increased correlation as T is decreased – leads to a phase transition at some T (Must have ordered structure at T=0 to have zero entropy)

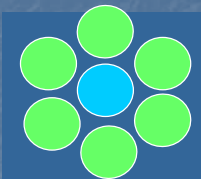
How to treat Correlation?

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- **Old idea** – solve for one atom with some method of averaging over the environment



• Gutzwiller – Hubbard – 1960's

- Applied idea at $T=0$ ---- open shell systems
- Correlation included in a clever way – Each atom treated as strongly correlated
Assume no correlation with surrounding atoms
- Kinetic energy - W – reduced due to restriction of hopping by the price of the repulsive interactions
- **Metal-insulator transition** (bootstrap effect)

Characteristic energy scales shown by spectra of electron excitations

Evolution with increasing interaction U :

• Top: Small U - Band-like spectrum

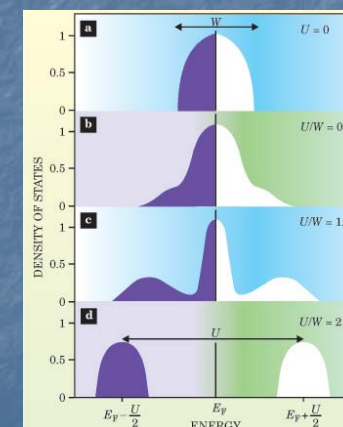
- $W \sim E_{\text{Fermi}} \sim 10 \text{ eV} \sim 10^5 \text{ K}$
- Increases with pressure

• Bottom: Large $U \gg W$

- - Localized, correlated atom-like
- Two types of excitations
- High energy $\sim U \sim 10 \text{ eV} \sim 10^5 \text{ K}$
- Increases for more localized states
- Low energy magnetic $\sim J \sim 10^2 \text{ K}$
- Decreases for more localized states

• Middle: Multiple energy scales

- High energy $\sim U \sim 10^5 \text{ K}$
- Increases for more localized states
- Intermediate $\sim 10^{-3} \text{ K} - 10^5 \text{ K}$
- Can vary over range – low to high



Kotliar and Vollardt,
Physics Today 2004

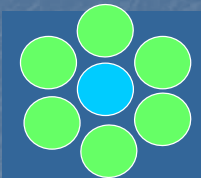
How to treat Correlation?

- **Atom** in a “**bath**” of surrounding atoms

Treat correlation on central atom

Treat bath as independent electrons

- **Old idea** – solve for one atom with some method of averaging over the environment



• Dynamical Mean Field Theory – 1980's - Present

- Applied idea at high T – decreasing to low T
- Each atom treated as strongly correlated
Neglect of correlation with surrounding atoms – **Justified at high T**
- Kinetic energy - W – reduced ... due to restriction of hopping –
---- **Treated by Self-Consistent Green's function**
- **Add correlation between sites as T decreases**
- **Phase Transitions -- Metal-insulator transition, magnetism,**

Dynamical Mean Field Theory

- Basic idea – Exact calculation for local system
In presence of “dynamical mean field”

- What is dynamical?

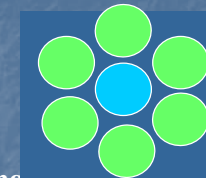
- Electrons can enter and leave site
- Dynamics comes from the dependence upon the time difference for entering, leaving

• Spectra of the site – which can be calculated exactly
(or to a good approximation) **Easier at high T !**

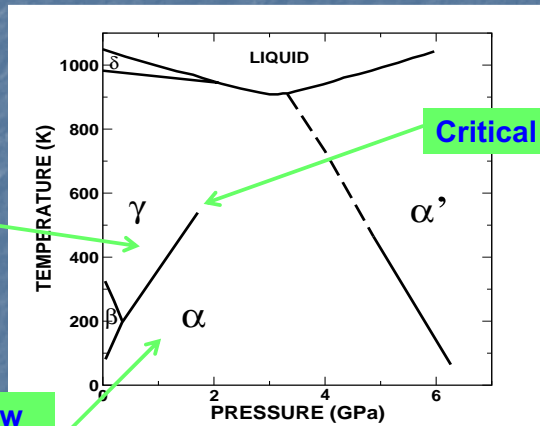
- But in a crystal the bath is due to other atoms that are the same as the central atom

• DMFT results from the requirement that the spectrum of the bath are consistent with spectrum of the site

- The limitation is that correlations between sites are ignored
- These can be included using clusters - expensive in computer time!



Phase Diagram Anomalous Lanthanide -- Ce



γ - High volume

α - Low volume

Critical Point

15% volume change at low T!
No change in symmetry – fcc metals

Electronic Spectrum Anomalous Lanthanide -- Ce

Photoemission

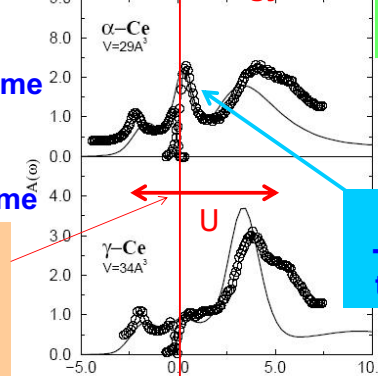
Low volume

High volume

Split bands due to interaction U
- Much wider than DFT bands

Fermi energy

Inverse Photoemission

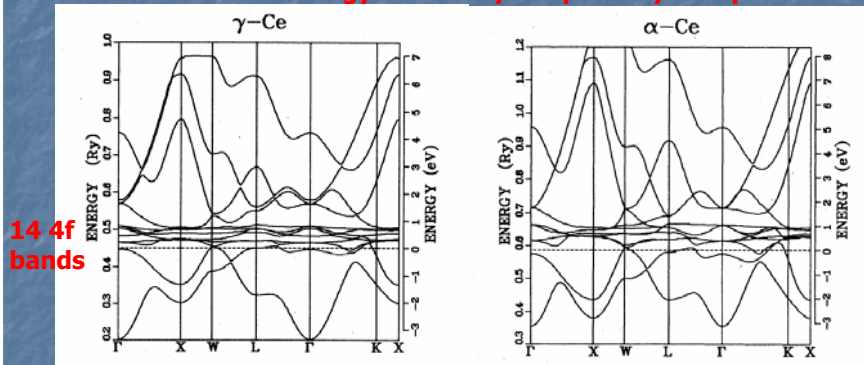


“Kondo” peak
- Much narrower than DFT bands

Cannot be explained by the usual KS theory
“LDA + DMFT” calculation explains:
Phase diagram, Driven by entropy of moments at high T,
Electronic spectrum
McMahan, et al., 2005

Calculated Bands of Ce Local Density Approximation

There are 7 4f bands (not counting spin). The 4f bands MUST be at the Fermi energy since they are partially occupied



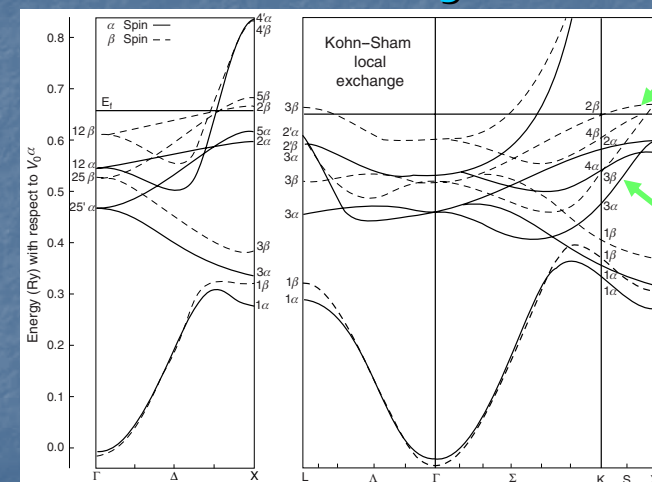
14 4f bands

Large volume Gamma phase -- Magnetic
(experimental fact)

Low volume Alpha phase
Non-magnetic
(experimental fact)

From: W. E. Pickett, et al., PRB 23, 1266 (1981).

T=0 Approach -- Spin polarized bands in ordered magnetic state - Ni



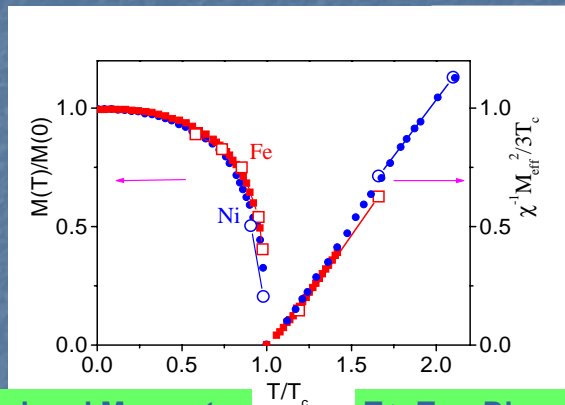
Spin down

~ 1.5 eV

Spin up

What happens for $T > T_c$ -- where spin polarization = 0 ?

T ≠ 0 -- Magnetism of Fe and Ni calculated from electronic structure theory



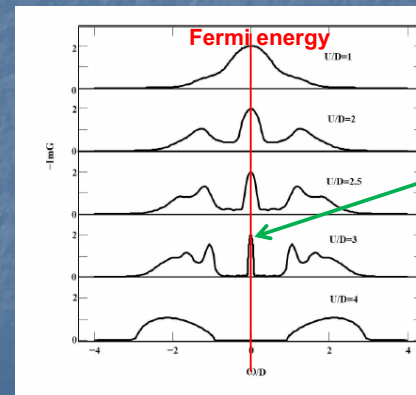
$T < T_c$ -- Ordered Moments
 $T=0$ Moment scaled to DFT
 Close to experiment

$T > T_c$ -- Disordered Local Moments
 Curie-Weiss $\chi^{-1} \sim (T - T_c)$

“LDA + DMFT” calculations Lichtenstein, et al., 2001

The problem in making DMFT quantitative U is not uniquely defined!

Simple model for bands - half-filled - D = band width
 for independent electrons – U = interaction



$U=0$ metal

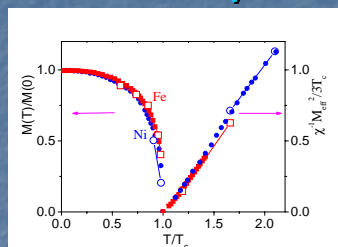
Low energy
 spin
 excitations

Large U
 insulator

Magnetism of Fe and Ni calculated from electronic structure theory

Quantitative successes:
 Ordered Moment
 Local Moments at high T

What about T_c ?



	Calculated	Experiment
Ni	700	631 K
Fe	1900	1031

Why is Fe off so much?

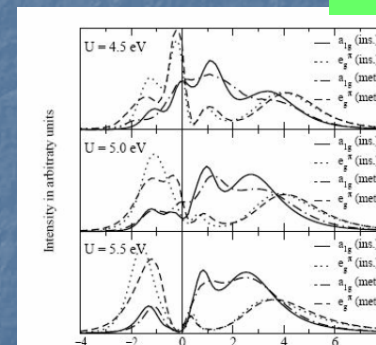
Spin wave fluctuations reduce T_c in Fe – not taken into account by the localized calculation

Lichtenstein, et al., 2001

Metal insulator transition Qualitative explanation from DMFT

SrVO₃
 Metal or insulator?

Photoemission clearly
 shows correlation effects
 – peak at Fermi energy



“LDA + DMFT” calculations --- the results are very sensitive to the value of U !

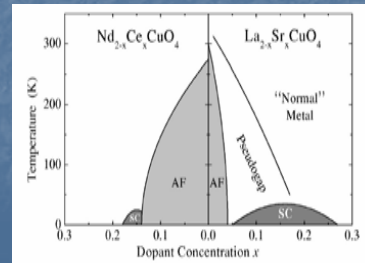
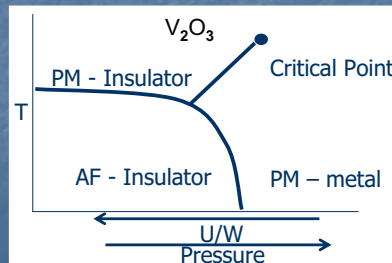
Lichtenstein, et al., 2001

Electronic Structure

The big picture

- Starting from Low T or high T:
- The limits – small U/W, large U/W - are relatively straightforward to understand and to treat
- The intermediate cases – “strongly correlated” – multiple energy scales -- are the great challenges

“Classic” examples



Conclusions

- Starting from Low T or high T
- Low T approach completely justified in many cases
 - Si – no electronic excitations below 1.1 eV $\sim 10,000K$
 - Phonons are the important excitations – use $T=0$ phonons to calculate specific heat – hard problem at melting requires high T approach for atomic motion
- High T approach completely justified in many cases
 - Magnetism at temperature \sim Curie temperature limits
 - “Kondo effect” – low energy crossover from zero entropy state at $T=0$ to thermally fluctuating spins at $T>T_K$ that is often very low $\sim 1-10 K$