Homework 2

September 7, 2017

Due: September 14, 2017, 11:59 PM

Instructions

Your homework submission must cite any references used (including articles, books, code, websites, and personal communications). All solutions must be written in your own words, and you must program the algorithms yourself. If you do work with others, you must list the people you worked with. Submit your solutions as a PDF to the E-Learning at UF (http://elearning.ufl.edu/).

Your programs must be written in either MATLAB or Python. The relevant code to the problem should be in the PDF you turn in. If a problem involves programming, then the code should be shown as part of the solution to that problem. If you solve any problems by hand just digitize that page and submit it (make sure the problem is labeled).

If you have any questions address them to:

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- Sheng Zou (TA) – shengzou@ufl.edu

Question 1 – 15 points

Suppose $\mathbf{x}$ is a random vector with covariance matrix $\Sigma = \mathbf{x}^T\mathbf{x} = \frac{1}{4}\begin{bmatrix} 5 & \sqrt{3} \\ \sqrt{3} & 7 \end{bmatrix}$. Take the *eigendecomposition* of the matrix Σ in order to:

1.1 Find the eigenvalues, λ_1 and λ_2, of Σ.

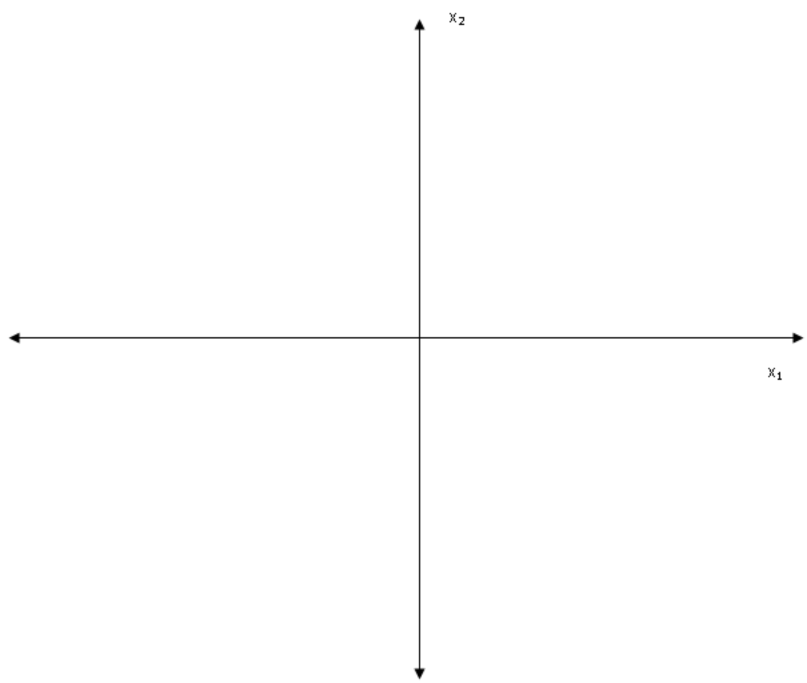
1.2 Verify that $\mathbf{v}_1 = \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} \sqrt{3} \\ -1 \end{bmatrix}$ are eigenvectors of Σ.

1.3 Find an *orthogonal* matrix $\mathbf{U}$ such that $\Sigma = \mathbf{U}\mathbf{D}\mathbf{U}^T$. *(Hint: use the spectral theorem).*

1.4 Suppose $\mathbf{y}$ is a random vector obtained by the principal component transform of $\mathbf{x}$. Write the formula for computing $\mathbf{y}$ from $\mathbf{x}$.

1.5 What is the covariance matrix of $\mathbf{y}$?

1.6 Plot the new coordinate system on the following graph and plot the curve describing the error of all points with a Mahalanobis distance of 1 from the origin.



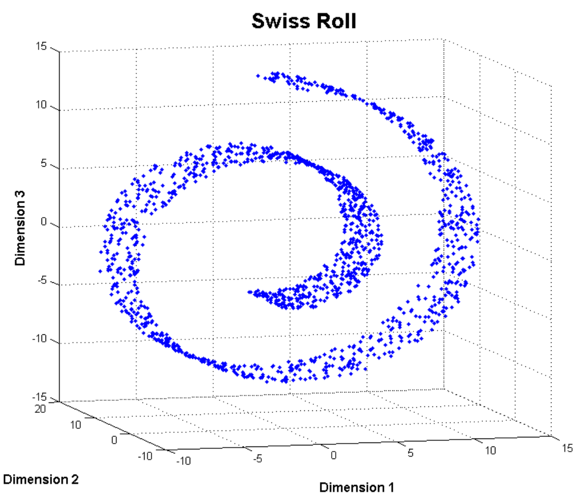
Question 2 – 10 points

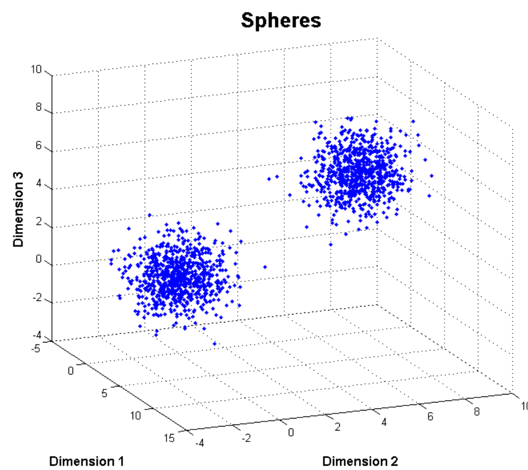
Compute a formula for $\mathbf{A}^k$, i.e., the matrix power operation defined as the matrix product of k copies of A, using *eigendecomposition*. Show the necessary steps of your derivation.

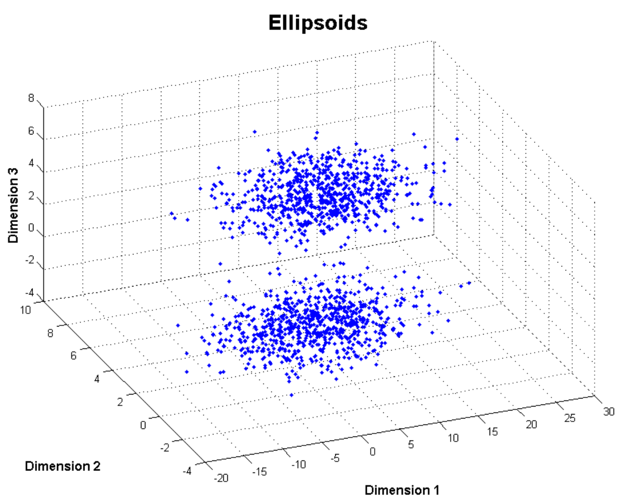
Question 3 – 15 points

For this question, please download all 3 datasets provided along with this assignment: *swissroll.txt*, *spheres.txt*, and *ellipsoid.txt*. Import these files into your programming software.
All datasets have 1500 samples and 3 features/dimensions. Now, consider **X** to be a data set (so **X** is of size 1500×3). For example, in MATLAB, you can use the following script to generate a plot of the dataset **X**:

```
figure,plot3(X(:,1),X(:,2),X(:,3),'.');grid on;
xlabel('Dimension 1','FontSize',12,'FontWeight','bold');
ylabel('Dimension 2','FontSize',12,'FontWeight','bold');
zlabel('Dimension 3','FontSize',12,'FontWeight','bold');
title('Swiss Roll','FontSize',20,'FontWeight','bold');
```







For each dataset:

4.1 Find the covariance matrix.

4.2 Find the eigenvectors and eigenvalues of the covariance matrix.

4.3 Find (and plot) the projection of the data points into the 2-D and 1-D principal components. After projecting the data into 2-D and 1-D, provide a short discussion (2-3 sentences) of the results for each data set that answers the following question: Does the projection preserve the "important" or "most informative" structure of the original data? Why or why not?