# Homework 4

October 10, 2017

## Due: October 17, 2017, 11:59 PM EST

## Instructions

Your homework submission must cite any references used (including articles, books, code, websites, and personal communications). All solutions must be written in your own words, and you must program the algorithms yourself. If you do work with others, you must list the people you worked with. Submit your solutions as a PDF to the E-Learning at UF (http://elearning.ufl.edu/).

Your programs must be written in either MATLAB or Python. The relevant code to the problem should be in the PDF you turn in. If a problem involves programming, then the code should be shown as part of the solution to that problem. If you solve any problems by hand just digitize that page and submit it (make sure the problem is labeled).

If you have any questions address them to:


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- Sheng Zou (TA) – shengzou@ufl.edu

## Question 1 - 10 points

Assuming a univariate Gaussian data likelihood given $N$ i.i.d. data points:

$$p(\mathbf{X}|\mu) = \prod_{n=1}^{N} \mathscr{N}(x_n|\mu, \sigma^2) \tag{1}$$

and a Gaussian prior distribution on the mean:

$$p(\mu|\mu_0) = \mathscr{N}(\mu|\mu_0, \sigma_0^2) \tag{2}$$

with fixed variances ($\sigma^2$, $\sigma_0^2$, and $\sigma^2 \neq \sigma_0^2$), using the method of completing the square (in the exponent) show that the posterior distribution is given by:

$$p(\mu|X) = \mathscr{N}(\mu|\mu_N, \sigma_N^2) \tag{3}$$

where

$$\mu_n = \frac{\sigma^2}{N\sigma_0^2 + \sigma^2}\mu_0 + \frac{N\sigma_0^2}{N\sigma_0^2 + \sigma^2}\mu_{ML} \tag{4}$$

$$\frac{1}{\sigma_N^2} = \frac{1}{\sigma^2} + \frac{N}{\sigma^2} \tag{5}$$

and $\mu_{ML}$ is the maximum likelihood solution for $\mu$ given the $N$ data points.

Show all your work in the derivation.

## Question 2 - 10 points

Extend your solution to Question 1 to a multivariant Gaussian likelihood and Gaussian prior. Assuming a fixed covariance on the prior and the likelihood, derive the MAP solution for the mean vector given $N$ i.i.d. data points and a multivariate Gaussian prior on the mean.

## Question 3 - 10 points

- In our Binomial/Beta example in class, we computed the ML and MAP solutions for the $\mu$ parameter of the Binomial distribution iteratively with an increasing number of trials/random draws. Recall, the parameter $\mu$ represented the probability of heads.
- In this homework question, you will do the same sort of experiment for a random draws from a Gaussian distribution (i.e., a Gaussian data likelihood) with a Gaussian prior distribution on the mean parameters (assume a fixed known variance for the Gaussian likelihood and Gaussian prior).

- Using your solution to Question 1, write a script that iteratively draws one data point from the true Gaussian distribution (with known mean). Each iteration compute and ML solution and the MAP solution for the Gaussian mean. After each draw, update the prior distribution to be replaced with the posterior distribution from the previous draw (just like the Binomal/Beta example in class).

- In your solution, provide:

  - Display multiple sample runs of your code and include a description of what the code shows you about ML vs MAP solutions. Your discussion should illustrate that you understand ML and MAP concepts and their differences. Your discussion should answer, at a minimum, the following questions:

    * What happens when the prior mean is initialized to the wrong value? to the correct value?
    * What happens as you vary the prior variance from small to large?
    * What happens when the likelihood variance is varied from small to large?
    * How do the initial values of the prior mean, prior variance, and likelihood variance interact to effect the final estimate of the mean?