Advancing Spatial Reasoning in Large Language Models: An In-Depth Evaluation and Enhancement Using the StepGame Benchmark

Fangjun Li[1], David C. Hogg[1], Anthony G. Cohn[1,2]

[1]University of Leeds, UK [2]Alan Turing Institute, UK

Introduction

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10- hop story:
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3. Z presents right to X.
4. N is lower right of I.
5. S is positioned above J and to the right.
6. F is above T at 10 o'clock.
7. A is to the upper left of W.
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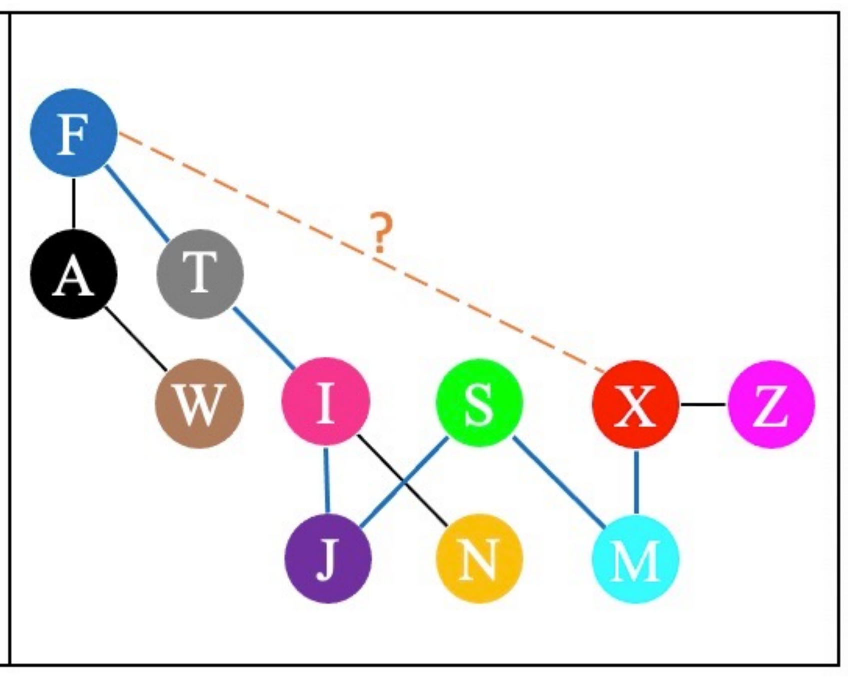

Solution to StepGame

Sentence-to-Relation Mapping + ASP Reasoner

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A is directly north east of B. →	A_upperright_B →	up_right("A", "B").
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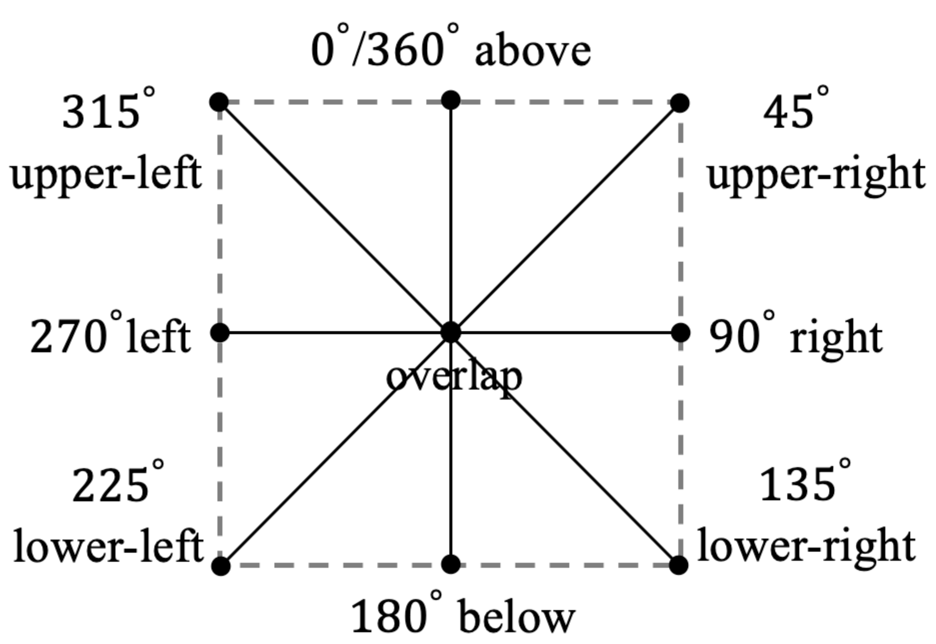

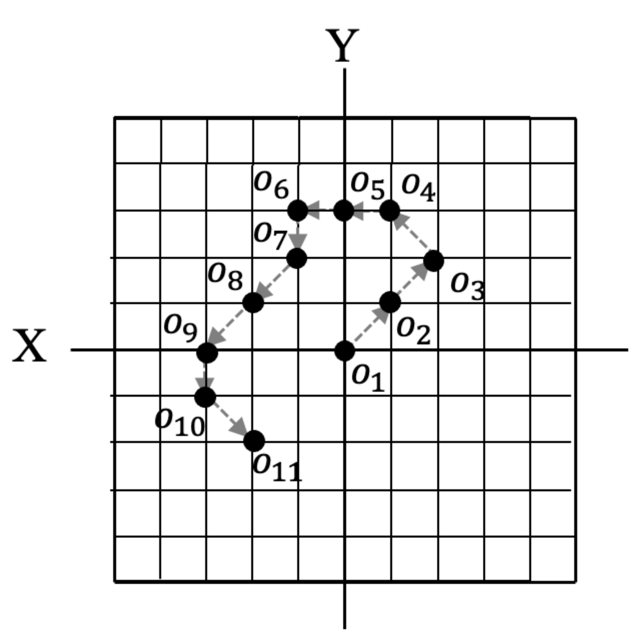

LLM + ASP

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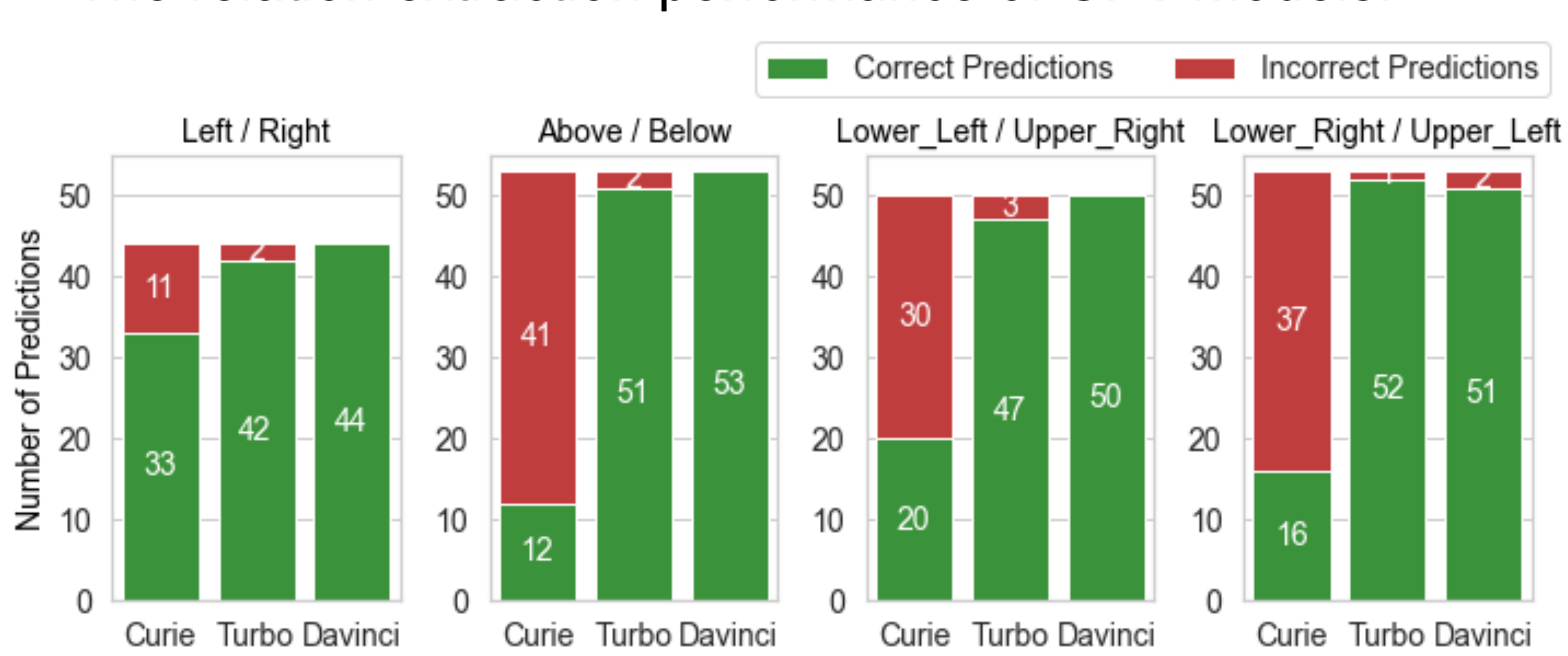

Results of LLMs for relation extraction + ASP Reasoner

	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9	k=10
Map+ASP	**100**	**100**	**100**	**100**	**100**	**100**	**100**	**100**	**100**	**100**
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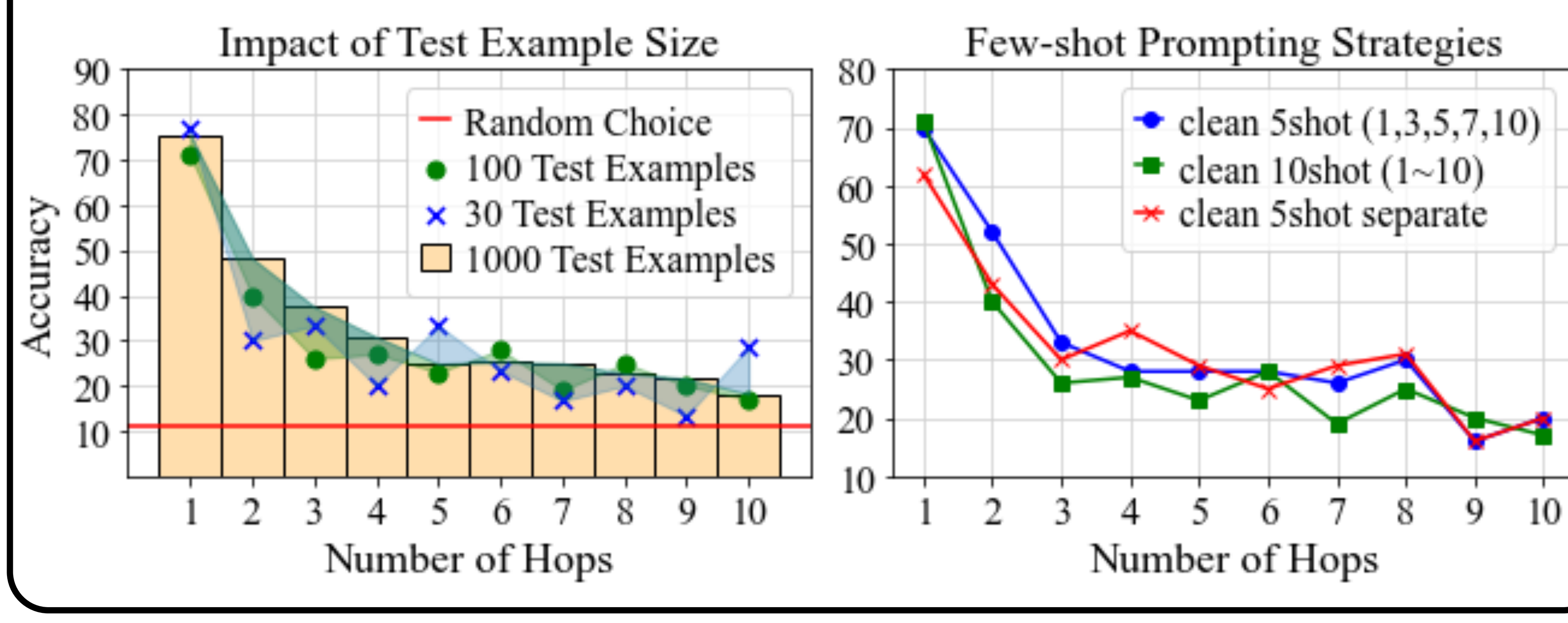
SOTA	92.6	89.9	89.1	93.8	92.9	91.6	91.2	90.4	89.0	88.3

Evaluation of GPT-3.5 Turbo on StepGame



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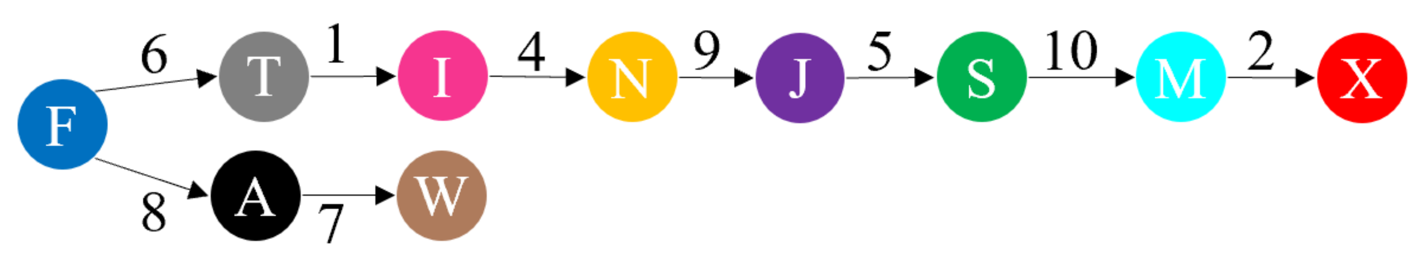

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Results - Accuracy

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GPT-4	base	**100**	70	55	45	40	25	40	35	35	25
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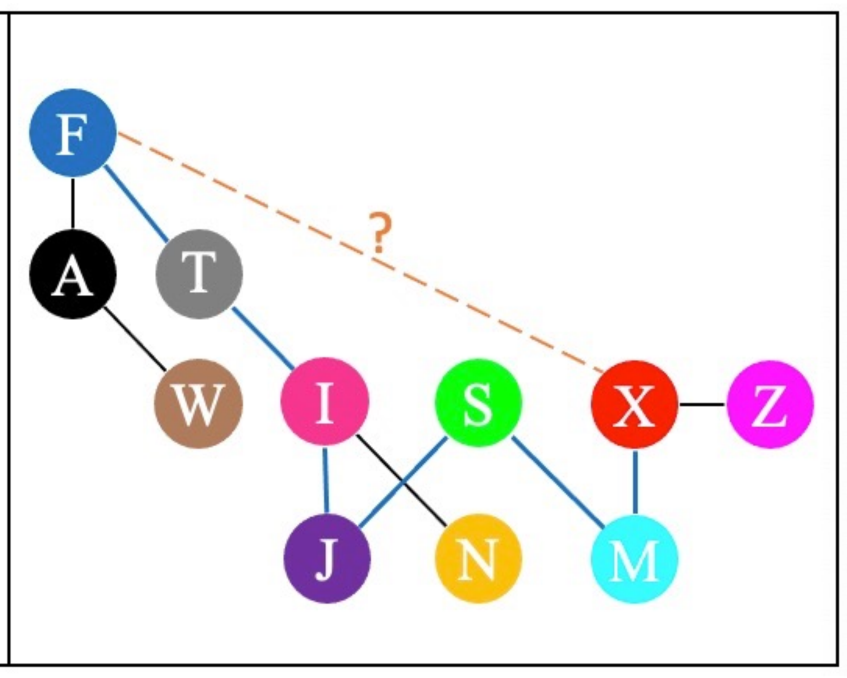

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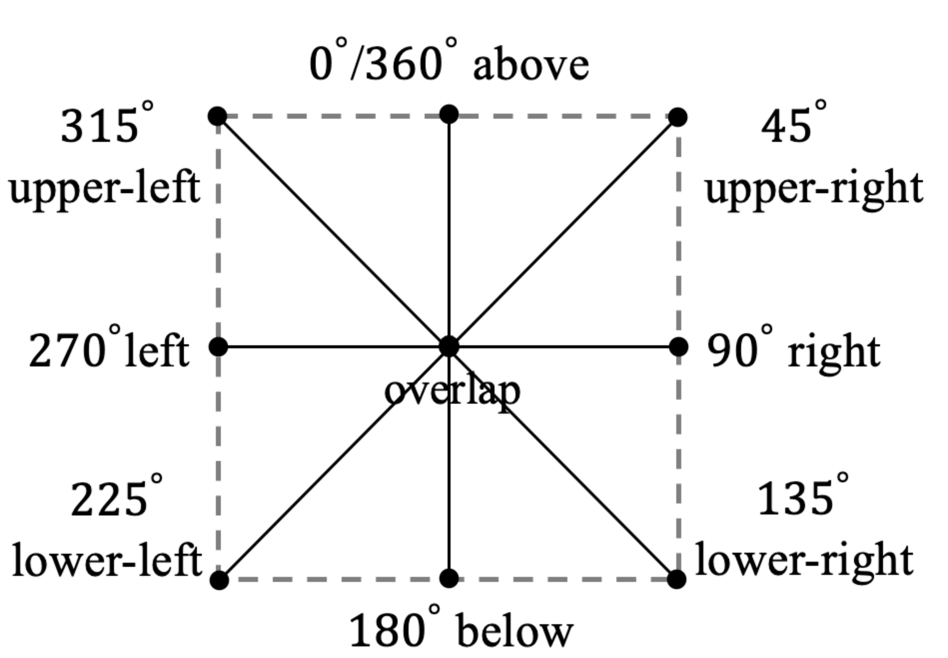

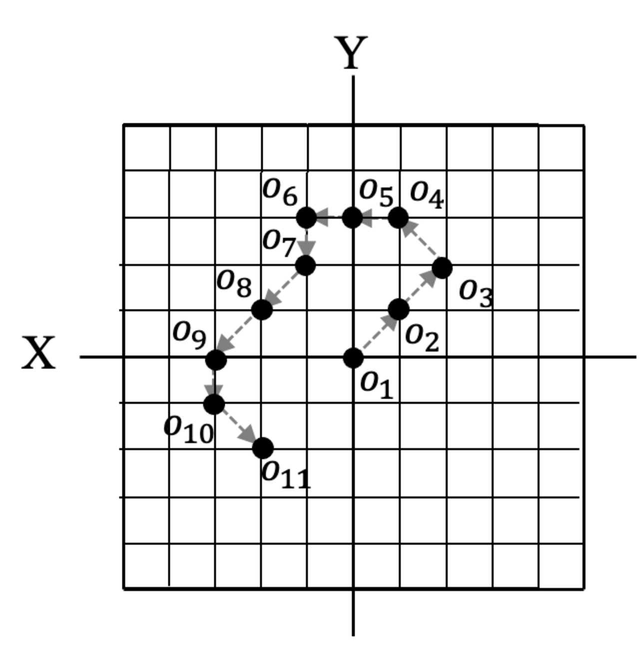

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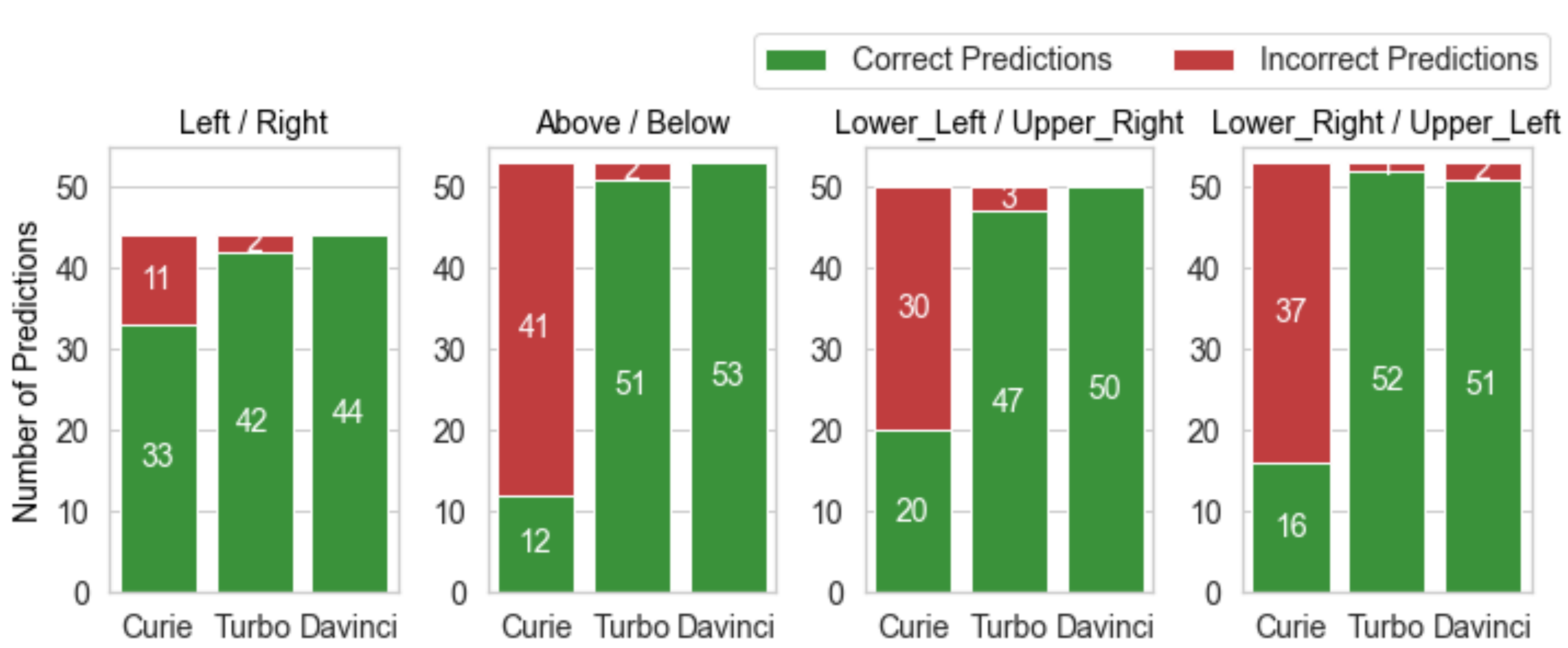

Accuracy results of LLMs for relation extraction + ASP Reasoner

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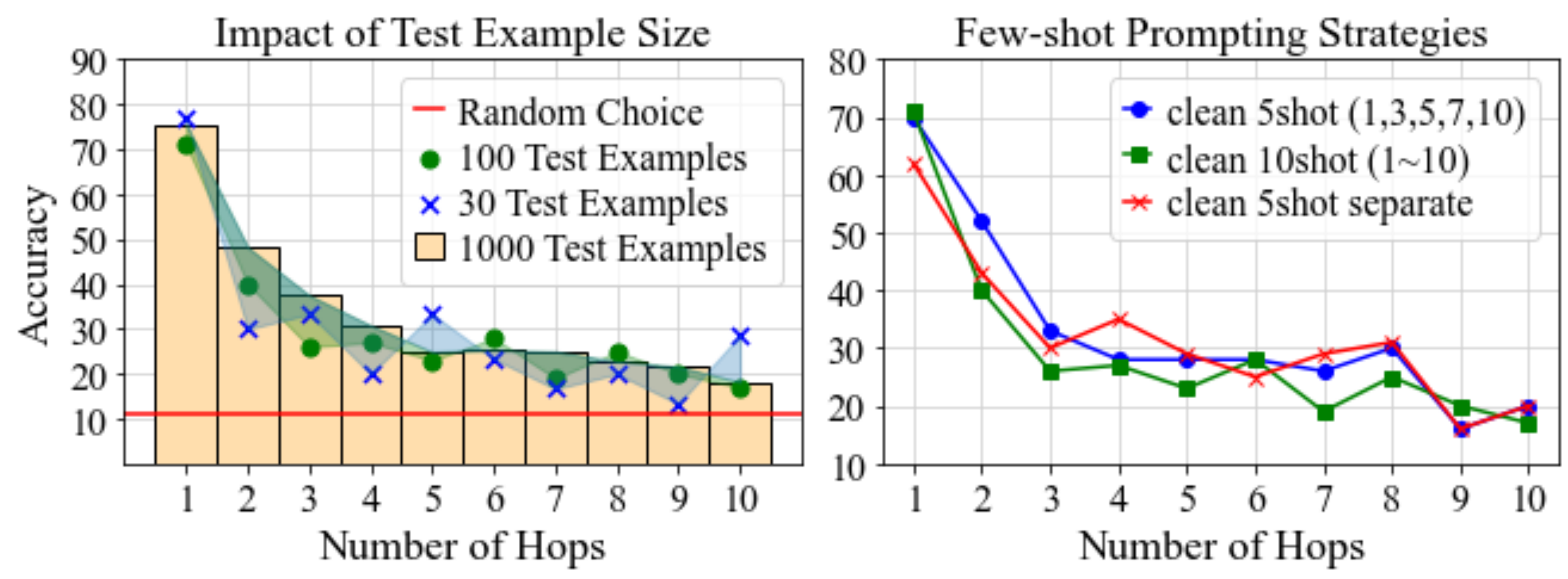
Evaluation of GPT Models on Rectified StepGame



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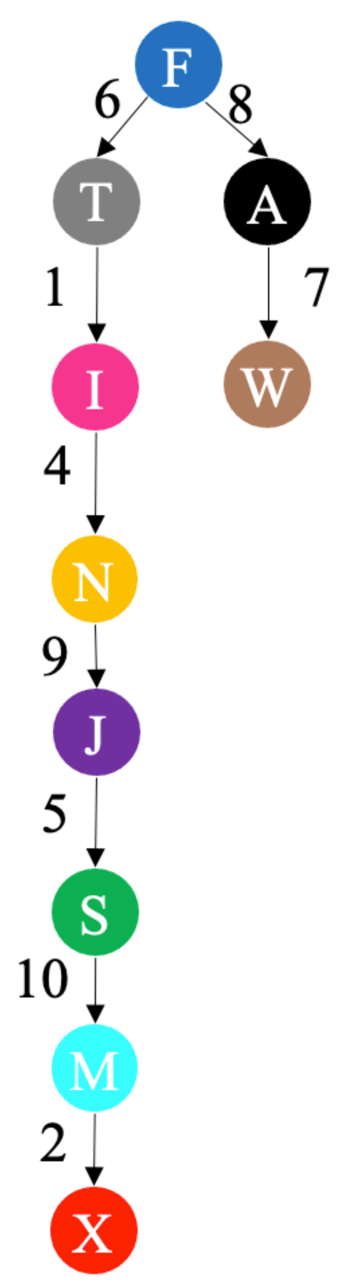
Link X and F



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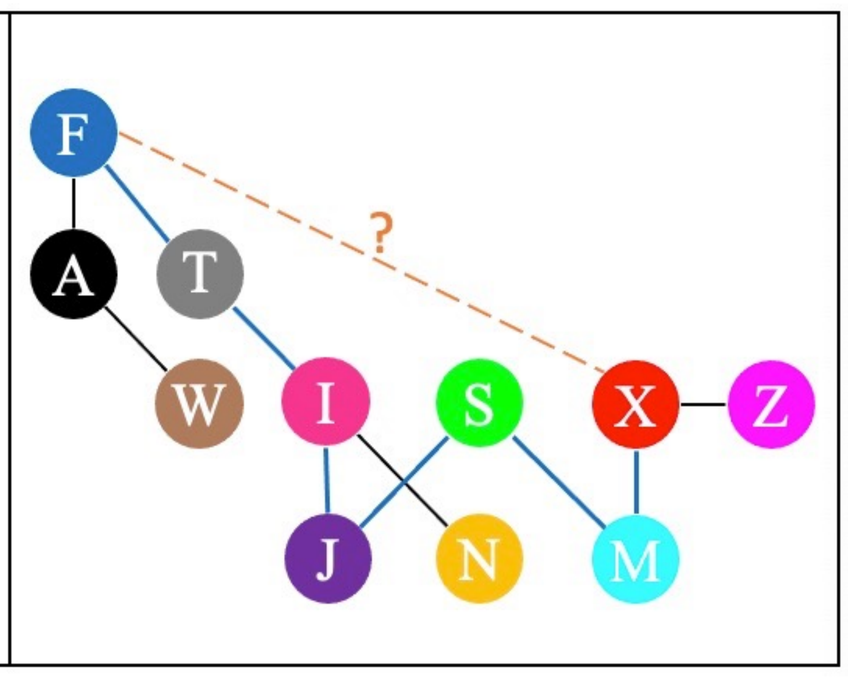

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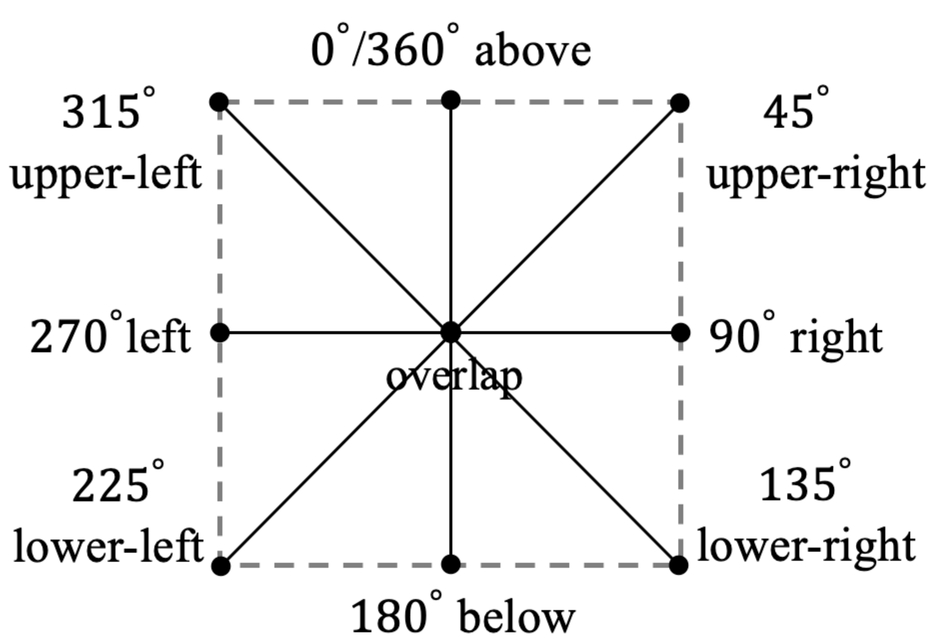

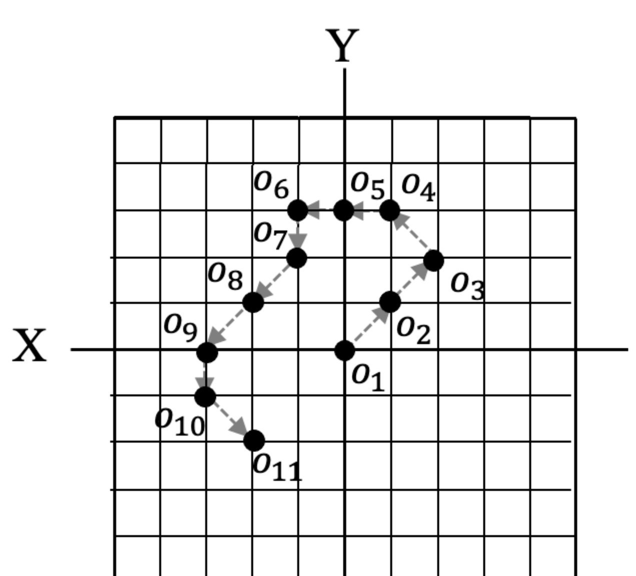

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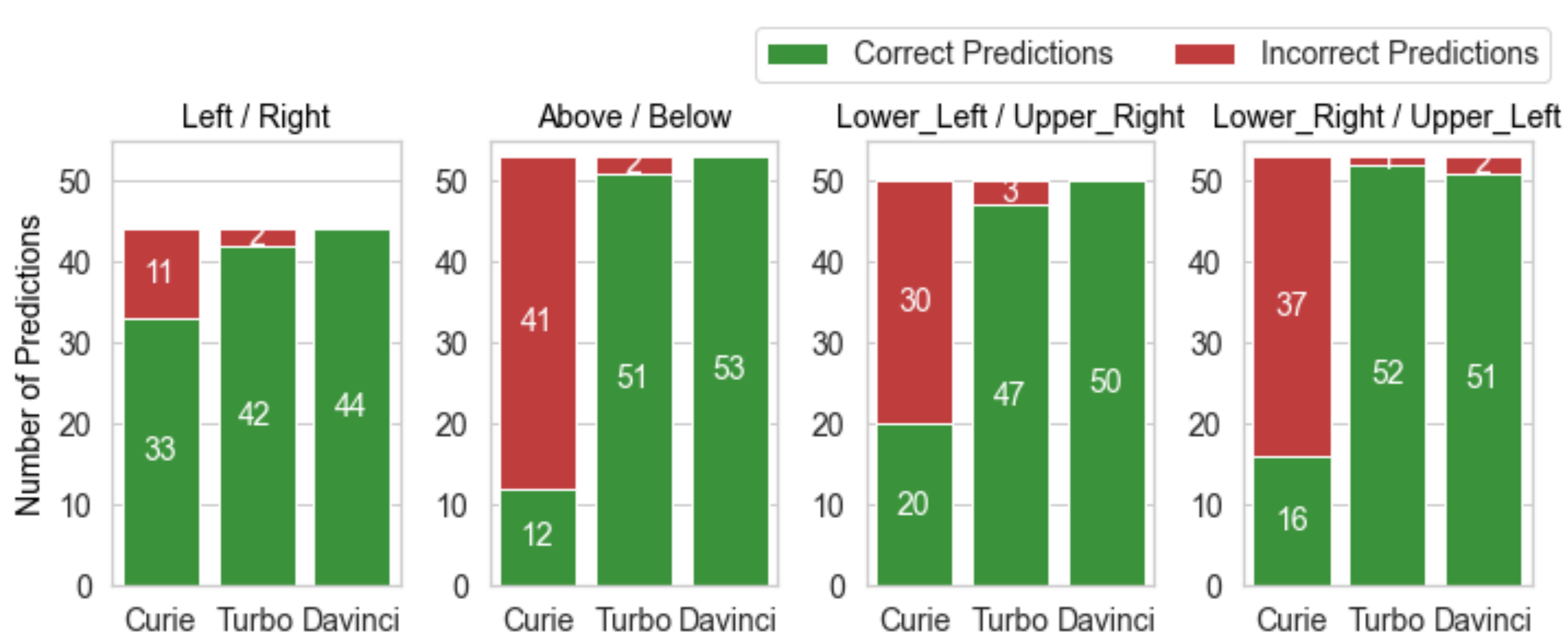

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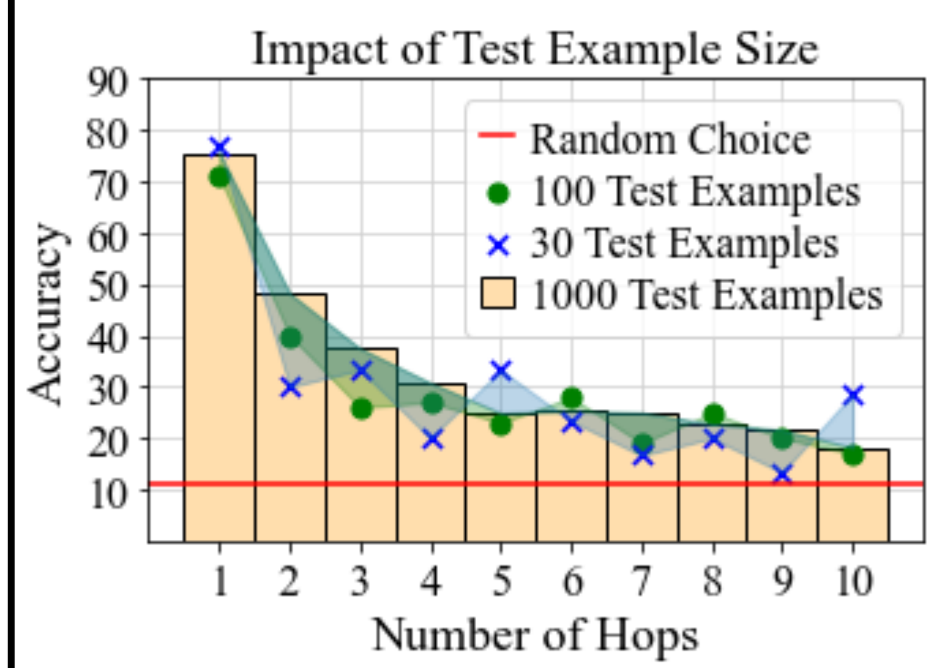

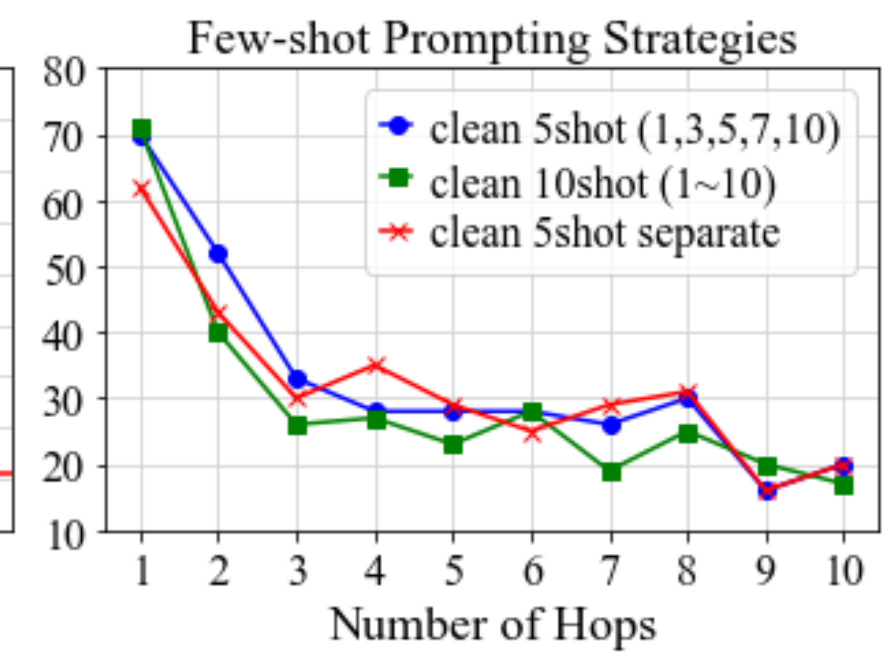

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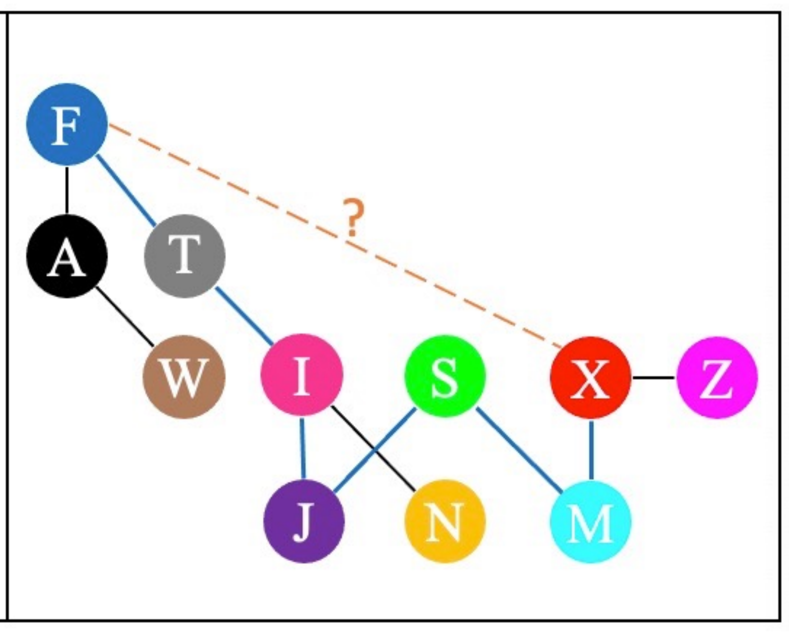

Template Errors in StepGame

Mapping	Original Incorrect Statement
BB_right_AA	AA and BB are parallel, and AA on the right of BB. AA and BB are parallel, and AA is to the right of BB. AA and BB are horizontal and AA is to the right of BB AA and BB are both there with the object AA is to the right of object BB.
BB_below_AA	AA is placed at the bottom of BB. AA is at the bottom of BB and is on the same vertical plane. AA presents below BB.
AA_lowerleft_BB	BB is there and AA is at the 10 position of a clock face. BB is positioned below AA and to the left. .
BB_upperright_AA	Object A is above object BB and to the right of it, too. AA is diagonally to the upper right of BB.
AA_lowerright_BB	AA is to the right and above BB at an angle of about 45 degrees.
BB_upperleft_AA	BB is to the right and above AA at an angle of about 45 degrees. BB is diagonally left and above BB.

	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9	k=10
Clean	7.64	15.03	20.87	26.39	32.54	37.66	41.71	47.20	51.50	**54.29**
Noise	20.43	30.19	34.59	48.18	57.13	61.14	63.60	69.45	72.84	**74.21**

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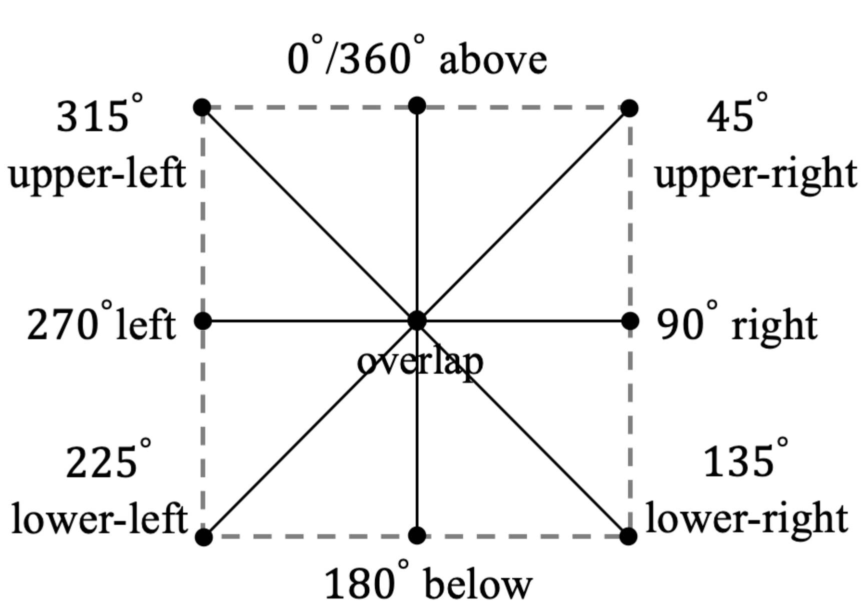
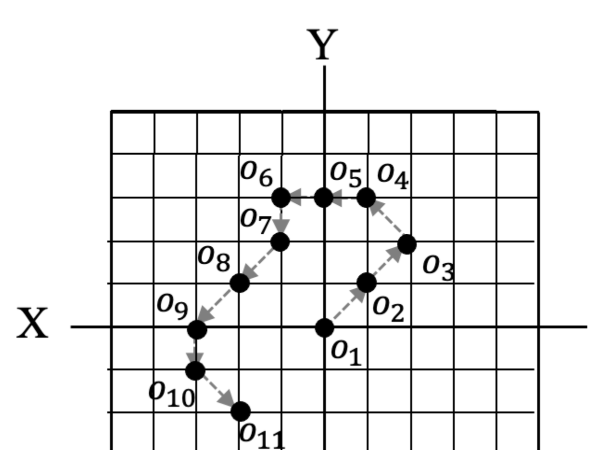


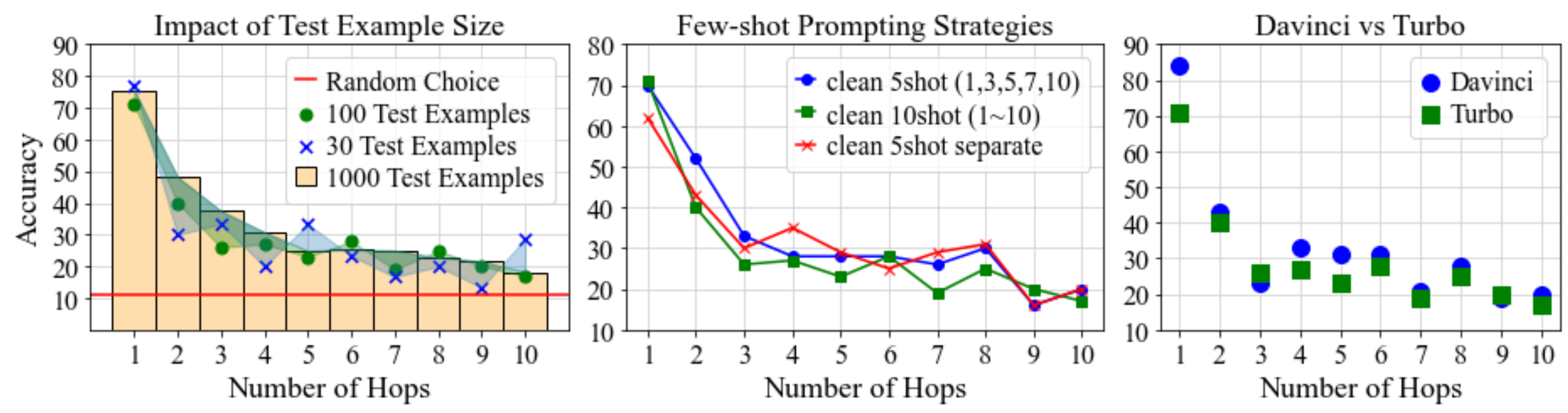


LLM + ASP

LLM for relation extraction + ASP Reasoner

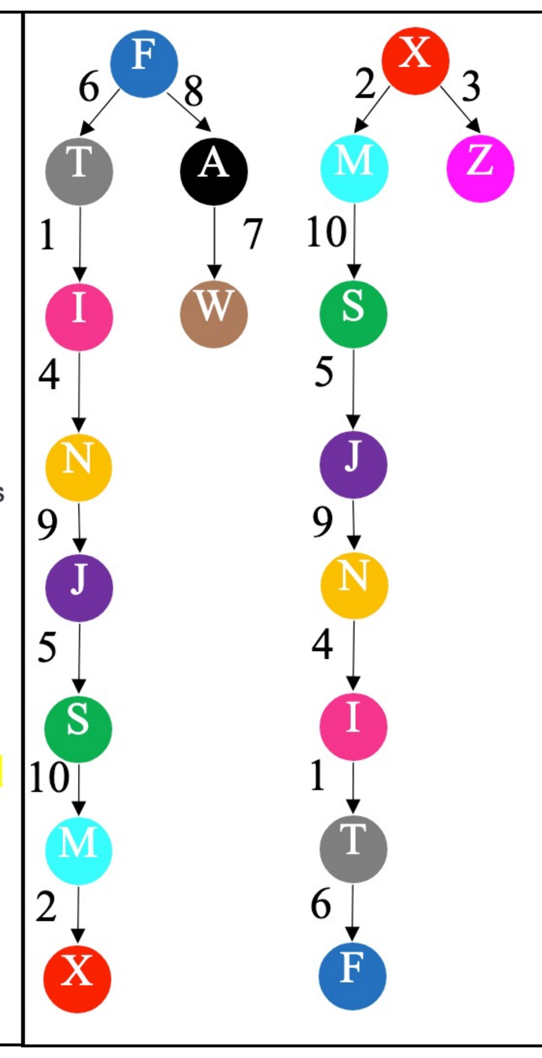
	left/ right	above /below	lower_left/ upper_right	lower_right/ upper_left
total	44	53	50	53
text-curie-001	11	41	30	37
text-davinci-003	**0**	**0**	**0**	2
gpt-3.5-turbo	2	2	3	**1**

Evaluation of GPT Models on Revised StepGame



Methods

Given a story about spatial relations among objects, answer the relation between two queried objects. **Possible relations are:** overlap, above, below, left, right, upper-left, upper-right, lower-left, and lower-right. If a sentence in the story is describing **clock-wise** information, then 12 denotes above, 1 and 2 denote upper-right, 3 denotes right, 4 and 5 denote lower-right, 6 denotes below, 7 and 8 denote lower-left, 9 denote left, 10 and 11 denote upper-left. If the sentence is describing **cardinal directions**, then north denotes above, east denotes right, south denotes below, and west denotes left. In all the spatial relations, assume that all agents occupy a position on a grid point of equally spaced points in the vertical and horizontal directions and that agents occupy the nearest grid point consistent with the spatial relation. The **offsets** of 9 spacial relations: offset(overlap) = (0,0); offset(top) = (0,1); offset(down) = (0,-1); offset(left) = (-1,0); offset(right) = (1,0); offset(top_left) = (-1,1); offset(top_right) = (1,1); offset(down_left) = (-1,1); offset(down_right) = (1,1).



Reasoning:
Suppose F is at (0,0). Link X and F using the relations in the story.
Start from F. According to 6. F is above T at 10 o'clock. So T is to the lower-right of F. T= F+ offset(lower-right) = (0,0)+(1,-1)=(1,-1).
Then search for T. According to 1. T is slightly off center to the top left and I is slightly off center to the bottom right. So I is to the lower-right of T. I= T+ offset(lower-right) = (1,-1)+(1,-1)=(2,-2).
Then search for I. According to 4. N is lower right of I. So N is to the lower-right of I. N= I+ offset(lower-right) = (2,-2)+(1,-1)=(3,-3).
Then search for N. According to 9. N is sitting in the right direction of J. So J is to the left of N. J= N+ offset(left) = (3,-3)+(-1,0)=(2,-3).
Then search for J. According to 5. S is positioned above J and to the right. So S is to the upper-right of J. S= J+ offset(upper-right) = (2,-3)+(1,1)= (3,-2).
Then search for S. According to 10. M is placed at the lower right of S. So M is to the lower-right of S. M= S+ offset(lower-right) = (3,-2)+(1,-1)=(4,-3).
Then search for M. According to 2. M is at the bottom of X. So X is above M. X= M+ offset(above) = (4,-3)+(0,1)=(4,-2). Come to X, ended. F(0,0), X(4,-2), so X is to the lower-right of F.
Answer: lower-right

Algorithm 1: Our ToT Approach

Require: LLM, input x

$S_0 \leftarrow Init(x)$
$i \leftarrow 1$
while no $s_f \in S_{i-1}$ has arrived at o_t **do**
  $S_i' \leftarrow \{s \cdot c \mid c \in G(s,j) \wedge ChainExtn(c) \wedge s \in S_{i-1}\}$
  if $S_i' = \emptyset$ **then return** failure
  $S_i \leftarrow select(b, \{\langle s, y\rangle \mid s \in S_i' \wedge y = \Sigma_1^n \sigma(V(s))\})$
  $i = i + 1$
end while
return $Link(s_f)$

Results

		k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9	k=10
Map+ASP		**100**	**100**	**100**	**100**	**100**	**100**	**100**	**100**	**100**	**100**
Curie+ASP		46	43	42	59	67	67	57	56	58	61
Davinci+ASP		**100**	**100**	99	**100**	**100**	99	**100**	**100**	**100**	**100**
SOTA		92.6	89.9	89.1	93.8	92.9	91.6	91.2	90.4	89.0	88.3
Turbo	base	62	43	30	35	29	25	29	31	16	20
	CoT	/	34	40	36	28	28	26	31	25	24
	ToT_CoT	/	/	35	35	25	45	15	40	40	35
Davinci	base	77	42	21	26	25	30	23	23	22	22
	CoT	/	48	53	46	46	48	40	45	41	32
	ToT_CoT	/	/	65	50	45	60	50	50	55	50
GPT-4	base	100	70	55	45	40	25	40	35	35	25
	CoT	/	**80**	75	**95**	85	85	**90**	80	60	65
	ToT_CoT	/	/	**85**	85	**90**	**90**	85	**90**	**100**	**95**