

ECHOES AS ANCHORS: PROBABILISTIC COSTS AND ATTENTION REFOCUSING IN LLM REASONING

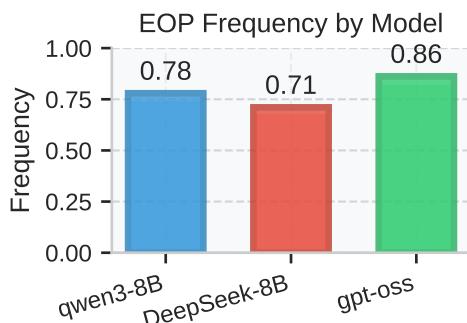
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ABSTRACT

011 Test-time compute allocation in large reasoning models (LRMs) is widely used
012 and has applications in mathematical problem solving, code synthesis, and plan-
013 ning. Recent work has addressed this problem by scaling self-consistency and par-
014 allel thinking, adding generic “thinking tokens” and prompting models to re-read
015 the question before answering. Unfortunately, these approaches either inject task-
016 agnostic tokens or mandate heuristics that do not explain—and often ignore—
017 the *spontaneous* repetition that many LRMs exhibit at the head of their internal
018 chains. In contrast, we analyze and harness the model’s tendency to restate the
019 question, which we term the *Echo of Prompt (EOP)*, as a front-loaded, compute-
020 shaping mechanism. We formalize its probabilistic cost by casting echo removal
021 as rejection-based conditioning and defining the *Echo Likelihood Gap* $\Delta\mathcal{L}$ as a
022 computable proxy. This provides the missing theoretical link that links early repe-
023 tition to likelihood gains and downstream accuracy. However, it does not by itself
024 specify how to exploit EOP. Consequently, we develop *Echo-Distilled SFT (ED-
025 SFT)* to instill an “echo-then-reason” pattern through supervised finetuning, and
026 *Echoic Prompting (EP)* to re-ground the model mid-trace without training. While
027 promising, quantifying benefits beyond verbosity is non-trivial. Therefore, we
028 conduct length and suffix-controlled likelihood analyses together with layer-wise
029 attention studies, showing that EOP increases answer to answer-prefix attention
030 in middle layers, consistent with an *attention refocusing* mechanism. We eval-
031 uate under identical decoding settings and compute budgets on GSM8K, MathQA,
032 Hendrycks-MATH, AIME24, and MATH-500 under identical decoding settings
033 and budgets, and find consistent gains over baselines.

1 INTRODUCTION

034
035
036
037
038 **User Query:** Determine the radius of a
039 cylindrical can given the area of a label...
040 and the height...
041 **Model’s Echo:** <think>Okay, let me see.
042 The problem is asking for the radius of a
043 cylindrical can. They give the area of the
044 label as a quadratic expression..., and the
045 height is also given...



046
047
048 Figure 1: An illustration of the Echo of Prompt (EOP). Left: An example of a model’s thinking
049 process starting with an echo of the user’s query. Right: The frequency of EOP across several
050 open-source models on the GSM8K dataset, as measured by our trained MLP probe (see §A.4).
051

052 Recent advancements in Large Language Models (LLMs) have demonstrated remarkable capabili-
053 ties in complex reasoning tasks, often mediated by a process known as Chain-of-Thought (CoT)
prompting (Wei et al., 2022; Kojima et al., 2022; Wang et al., 2023; Yao et al., 2023).

Inspired by the CoT paradigm, modern large *reasoning* models (LRMs) achieve strong performance on complex tasks by allocating significant test-time compute to think before answering (Wei et al., 2022; OpenAI, 2024; DeepSeek-AI et al., 2025; Qwen Team, 2025). A common yet underexplored phenomenon in their reasoning traces is the tendency to begin by repeating the user’s prompt (see Figure 1 and §A.3), a behavior we term the *Echo of Prompt* (EOP).

While uncontrolled repetition is a known failure mode (the “repeat curse,” (Yao et al., 2025)), explicit instructions to re-read or “look-twice” are known to improve performance (Xu et al., 2024; Zou et al., 2024). The spontaneous EOP that initiates a complex reasoning trace, however, has remained largely unanalyzed. This initial echo raises a critical question:

Is it a superfluous artifact of the training process, or does it serve a functional role in reasoning?

This paper confronts this question directly, presenting the first systematic study to isolate, analyze, and harness this emergent behavior as a powerful cognitive aid.

We hypothesize that the EOP serves as an intrinsic *attention-refocusing mechanism*, a learned strategy to ground subsequent reasoning steps in the salient details of the original query. To validate this, we provide a dual theoretical and empirical analysis:

1. **A Probabilistic Framework (§3).** We introduce a rejection sampling framework to formalize the EOP, defining the *Echo Likelihood Gap* to quantify its probabilistic cost.
2. **An Attention-Based Mechanistic Explanation (§3.3).** We uncover the underlying mechanism through attention analysis, showing that EOP serves to refocus the model’s attention, an act that correlates with correctness.
3. **Practical Methods and Empirical Validation (§4).** We translate this insight into two effective methods. **Echo-Distilled SFT (ED-SFT)** instills this behavior via fine-tuning, yielding significant performance gains that generalize across data distributions. Concurrently, **Echoic Prompting (EP)** provides a training-free inference strategy that re-grounds the model on the prompt, outperforming strong baselines.

Taken together, our findings reframe the EOP from a superficial flaw into a functional strategy for cognitive self-alignment. This work not only solves the puzzle of the initial repetition but also offers new insights into how models learn to structure their own thought processes for complex reasoning.

This paper makes three main contributions:

- We propose a novel probabilistic framework based on rejection sampling to quantify the cost of an echo, introducing the **Echo Likelihood Gap** ($\Delta\mathcal{L}$) as a core metric to measure the alignment between a model’s natural tendency and echo-free reasoning.
- We present two practical methods to leverage this phenomenon: **Echo-Distilled SFT (ED-SFT)**, a fine-tuning approach to instill echo behavior, and **Echoic Prompting (EP)**, a training-free inference technique that achieves similar gains by re-introducing the prompt.
- We provide a mechanistic explanation for the effectiveness of EOP. Through attention analysis, we demonstrate that echoing acts as an intrinsic refocusing mechanism, guiding the model to concentrate on critical problem details that are often overlooked.

2 RELATED WORK

Computation In Reasoning: Efficiency And Effectiveness. The scaling of test-time computation has demonstrably improved reasoning in LRMs, but often at the cost of significant overhead from long and sometimes redundant chains of thought—the “overthinking phenomenon” (Sui et al., 2025). One line of research tackles this by improving computational *efficiency*, developing methods like early exiting or step compression to reduce wasteful generation. A complementary approach, more aligned with our work, focuses on computational *effectiveness*. For instance, several studies have found that compelling a model to explicitly restate or re-read the input question can enhance reasoning (Xu et al., 2024; Mekala et al., 2024). These methods treat repetition as an instructed heuristic to re-align the model. Our work bridges these views by analyzing the *spontaneous* emergence of echoes, not as a heuristic to be added, but as an intrinsic, learned strategy that trades a small initial computational cost for more effective and focused downstream reasoning.

108 **Attention-Refocusing Mechanisms.** The challenge of maintaining focus over long contexts is a
 109 known issue in language models, often manifesting as a positional bias where information in the
 110 middle of a long input is under-utilized (Liu et al., 2024). This issue is analogous to *attention drift*
 111 in computer vision, where attention can shift away from salient regions during sequence generation
 112 (Cheng et al., 2017). To counteract these effects, various explicit mechanisms have been proposed.
 113 These range from model-level architectural changes and calibration methods that correct positional
 114 biases (Cheng et al., 2017; Hsieh et al., 2024), to inference-time interventions that re-weight at-
 115 tention or re-inject evidence to steer the model back to relevant information (Gu et al., 2024; Zou
 116 et al., 2024). Our work identifies a different, more intrinsic phenomenon: we show that the initial
 117 Echo of Prompt itself can serve a similar refocusing role, where the model *spontaneously* restates
 118 salient parts of the prompt to condition its subsequent generation, without any external guidance or
 119 modification.

3 THE PRICE OF AN ECHO: A PROBABILISTIC COST FRAMEWORK

123 This section formalizes the impact of prompt echoes using a probabilistic framework that goes be-
 124 yond simple text deletion. The core idea is to treat the presence or absence of an echo as a probabilis-
 125 tic event, which enables a formal definition of a hypothetical *echo-free* model and a measurement
 126 of the echo’s likelihood cost. The analysis proceeds in three parts: §3.1 introduces the rejection
 127 sampling framework, §3.2 measures the resulting likelihood cost, and §3.3 investigates the echo’s
 128 function as an attention-refocusing mechanism.

129 To ground this framework, the probabilistic and attention analyses in this section are performed on
 130 the GSM8K benchmark using DeepSeek-R1-Distill-Llama-8B. We use exact match accuracy as the
 131 primary task metric and log full model outputs for likelihood and attention analysis.

3.1 PROBLEM FORMULATION

135 This framework allows us to precisely quantify the effect of echo removal as a probabilistic condi-
 136 tioning event, laying the groundwork for the metric we introduce next.

137 Let $\mathbf{x} \in \mathcal{X}$ be an input prompt and $\mathbf{y} \in \mathcal{Y}$ be a generated output sequence (i.e., a reasoning trace).
 138 We consider a base large reasoning model parameterized by θ , which defines a conditional proba-
 139 bility distribution $\pi_\theta(\mathbf{y}|\mathbf{x})$ over possible output sequences.

140 Our first step is to formally identify which sequences contain a Echo of Prompt. We define a pred-
 141 icate, implemented by a separately trained MLP (see §A.4), that partitions the output space \mathcal{Y} into
 142 two disjoint sets, $\mathcal{Y} = \mathcal{Y}_{\text{trim}} \cup \mathcal{Y}_{\text{echo}}$. Here, $\mathcal{Y}_{\text{trim}} \subset \mathcal{Y}$ is the set of all *trimmed* sequences that are
 143 deemed echo-free, and $\mathcal{Y}_{\text{echo}}$ is its complement. We can then define an indicator function:

$$\mathbf{1}_{\mathbf{y} \in \mathcal{Y}_{\text{trim}}} = \begin{cases} 1 & \text{if } \mathbf{y} \text{ is echo-free,} \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

147 Assuming the model has a non-zero probability of producing at least one echo-free trace ($Z_x > 0$),
 148 we define our target, the *trimmed distribution* $\tau_\theta(\mathbf{y}|\mathbf{x})$, as the base distribution π_θ conditioned on
 149 the event that the output is echo-free:

$$\tau_\theta(\mathbf{y}|\mathbf{x}) = \frac{\pi_\theta(\mathbf{y}|\mathbf{x}) \mathbf{1}_{\mathbf{y} \in \mathcal{Y}_{\text{trim}}}}{\sum_{\mathbf{y} \in \mathcal{Y}_{\text{trim}}} \pi_\theta(\mathbf{y}|\mathbf{x})}. \quad (2)$$

153 Here, the denominator Z_x is the partition function, which normalizes the distribution by summing
 154 the probabilities of all echo-free sequences under the base model π_θ :

$$Z_x = \sum_{\mathbf{y} \in \mathcal{Y}_{\text{trim}}} \pi_\theta(\mathbf{y}|\mathbf{x}) = \mathbb{E}_{\mathbf{y} \sim \pi_\theta(\cdot|\mathbf{x})} [\mathbf{1}_{\mathbf{y} \in \mathcal{Y}_{\text{trim}}}] . \quad (3)$$

159 This distribution represents our targeted behavior—a hypothetical model constrained to produce
 160 only echo-free outputs. However, directly computing τ_θ is intractable because the partition function
 161 Z_x requires summing over all possible echo-free sequences. This intractability motivates the use of
 rejection sampling to reason about and sample from τ_θ without needing to calculate Z_x explicitly.

162
 163 Table 1: Echo metrics on GSM8K. Averages over samples in each group, where N denotes the num-
 164 ber of samples. The Suffix-only Likelihood Gap measures the per-token log-likelihood difference
 165 on the reasoning suffix when conditioned with versus without the echo prefix.

Group	Echo Likelihood Gap (per-token)				Extended Echo Metrics			
	N	Mean $\overline{\Delta\mathcal{L}}$	Std $\sigma(\Delta\mathcal{L})$	Neg. ratio (%)	$\overline{\Delta\mathcal{L}}$ (per token)	$\Delta/\#\text{removed}$	Suffix-only gap	Avg. removed tokens
Correct	819	2.5231	0.7786	0.12	2.4614	0.01103	1.1449	219.7
Wrong	500	2.4421	0.7657	0.00	2.3666	0.01085	1.2938	218.7

171
 172 The rejection sampling view provides a principled way to think about echo suppression, but to
 173 measure its effect, we need a concrete metric. Our goal is to quantify how much *preference* the
 174 model shows for a raw, echo-containing trace versus its trimmed, echo-free counterpart.
 175

176 Given a raw trace y_{raw} and its echo-trimmed counterpart y_{trim} , we define the (length-normalized)
 177 average log-likelihood

$$\mathcal{L}(y) = \frac{1}{|y|} \sum_{t=1}^{|y|} \log \pi_\theta(y_t | x, y_{<t}) \quad (4)$$

180 (nats/token). The *Echo Likelihood Gap* is $\Delta\mathcal{L} = \mathcal{L}(y_{\text{raw}}) - \mathcal{L}(y_{\text{trim}})$. A positive $\Delta\mathcal{L}$ means the
 181 model *prefers* the echo-containing trace. Unless otherwise noted, we report nats/token.
 182

183 While our framework defines the echo-free distribution τ_θ , the partition function Z_x required to
 184 compute it is intractable. This motivates a practical, sample-based alternative: the Echo Likelihood
 185 Gap ($\Delta\mathcal{L}$). This metric serves as a direct proxy for the probabilistic cost of an echo by comparing
 186 the average log-likelihood of a generated trace (y_{raw}) against its trimmed counterpart (y_{trim}). A
 187 positive $\Delta\mathcal{L}$ indicates that the model assigns a higher likelihood to the sequence containing the
 188 echo, quantifying the *price* of this behavior on a per-sample basis.
 189

This leads to our central question: is there a positive relationship between this probabilistic cost and
 190 the model’s final reasoning accuracy? In other words, does *spending* probability on an echo lead to
 191 better performance? The following sections are dedicated to empirically validating this trade-off.
 192

193 3.2 THE ECHO LIKELIHOOD GAP IN PRACTICE

194 Table 1 and Figure 2 highlight our central empirical finding: a larger probabilistic investment in an
 195 Echo of Prompt (EOP) strongly correlates with correct final answers. To formalize this, we introduce
 196 two metrics: the overall Echo Likelihood Gap ($\Delta\mathcal{L}$), which measures the total probabilistic cost,
 197 and the Suffix-only Likelihood Gap ($\Delta\mathcal{L}_{\text{suffix}}$), which isolates the echo’s influence on subsequent
 198 reasoning.
 199

200 **Defining the Likelihood Gaps.** Our primary metric, the **Echo Likelihood Gap** ($\Delta\mathcal{L}$), is defined
 201 in §3 as the difference in average per-token log-likelihood between a raw trace y_{raw} and its trimmed
 202 counterpart y_{trim} . To isolate the echo’s influence on the subsequent reasoning steps, we introduce
 203 a more granular metric. For a raw trace y_{raw} composed of an echo prefix e and a reasoning suffix
 204 s (i.e., $y_{\text{raw}} = [e, s]$), we compare the model’s likelihood of generating s with and without the
 205 conditioning prefix e . The **Suffix-only Likelihood Gap** is defined as:

$$\Delta\mathcal{L}_{\text{suffix}} = \mathcal{L}(s | x, e) - \mathcal{L}(s | x), \quad (5)$$

207 where $\mathcal{L}(s | \cdot)$ is the average per-token log-likelihood of the suffix s under the given context.
 208 A positive value indicates that the echo prefix makes the subsequent reasoning trace appear more
 209 probable to the model.
 210

211 **Analysis of Results.** The overall Echo Likelihood Gap reveals a clear correlation with correctness.
 212 As shown in Table 1, the Correct group ($N = 819$) has a larger average gap than the Wrong group
 213 ($N = 500$): $\overline{\Delta\mathcal{L}} = 2.5231$ vs. 2.4421 . This positive difference (+0.0811 nats/token) indicates that
 214 a larger total probabilistic investment in echoing co-occurs with correct final answers. We further
 215 validate this relationship with logistic regression in the Appendix, confirming $\Delta\mathcal{L}$ as a significant
 positive predictor of correctness, even after controlling for trace length.

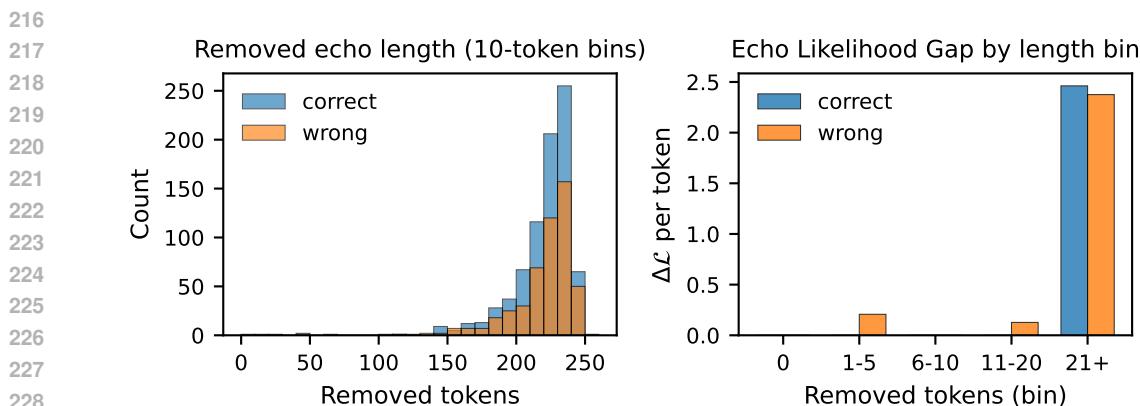


Figure 2: Echo metrics on GSM8K. **Left:** High-resolution histogram of removed echo-prefix lengths (10-token bins) for correct and wrong traces; most mass lies between roughly 200 and 240 tokens. **Right:** Echo Likelihood Gap $\Delta\mathcal{L}$ (per-token) stratified by removed-prefix length bin; the gap remains positive across all bins.

Interestingly, the *Suffix-only Likelihood Gap* is slightly larger for the Wrong group (1.2938 vs. 1.1449). While this seems counter-intuitive, it does not contradict our main finding. It suggests that while echoes make subsequent reasoning seem more plausible in general (as both values are positive), they may also act as a form of "confirmation bias," slightly strengthening the model's confidence in locally coherent but ultimately flawed reasoning paths. The determinative factor for correctness remains the overall likelihood trade-off captured by $\Delta\mathcal{L}$, as explained by the likelihood decomposition in §A.7.

Sanity Checks. Before further analysis, we perform several checks to validate $\Delta\mathcal{L}$ as a metric. First, for traces without a detected echo, $\Delta\mathcal{L}$ is definitionally zero, as the raw and trimmed sequences are identical. Second, we confirm that $\Delta\mathcal{L}$ correlates positively with the number of removed tokens in the echo prefix, indicating that longer echoes correspond to a larger likelihood gap. Finally, the data in Table 1 confirms that the suffix-only likelihood gap on the shared reasoning trace remains positive, confirming the echo's influence extends beyond the prefix itself. These checks establish $\Delta\mathcal{L}$ as a robust measure of the echo's probabilistic impact.

Length- And Suffix-Controlled Analysis. To ensure this gap is not merely a length artifact, we conduct a length-stratified analysis. As shown in Figure 2, the $\Delta\mathcal{L}$ remains consistently positive across different trace lengths. This indicates that the Echo of Prompt (EOP)'s contribution is robust and also improves the model's scoring on the subsequent, shared reasoning steps.

Distribution Of Removed Echo Lengths. We further examine the length distribution of the removed echo prefixes (Figure 2, left). The distribution is heavy-tailed, with most prefixes falling between roughly 200 and 240 tokens (mean 219, median 226), confirming that the Echo of Prompt (EOP) constitutes a non-trivial segment of the generation that acts as a probabilistic sink. This distribution reveals that echo prefixes consistently consume substantial portions of the model's output budget, with most instances removing more than 200 tokens of echoed content.

3.3 UNVEILING THE MECHANISM: ECHOES AS ATTENTION REFOCUSING

To understand why prompt echoing is effective, we analyze the model's attention patterns during generation. We hypothesize that re-introducing the original prompt effectively refocuses the model's attention on the core problem statement, preventing drift during extended reasoning chains.

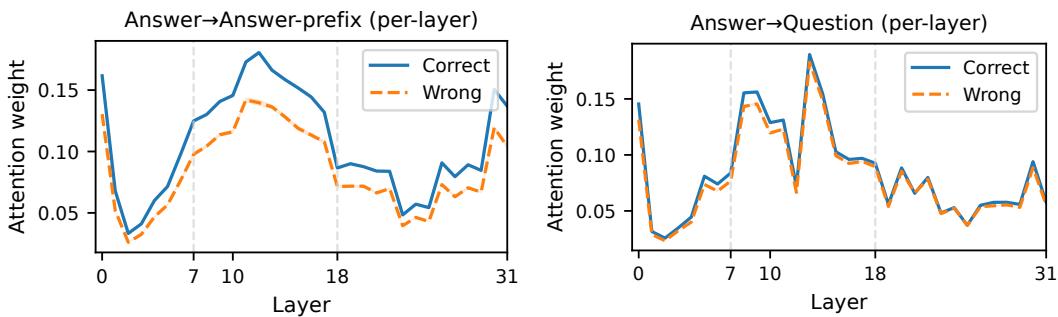
Attention Redistribution After Echo Removal. We investigate the mechanism underlying the Echo of Prompt's effectiveness, hypothesizing that it serves to refocus the model's attention. To test this, we compute two attention metrics on the original outputs: (i) attention from answer tokens

270 to the question tokens, and (ii) attention from answer tokens to the answer prefix. Formally, let
 271 $A^{(l)} \in \mathbb{R}^{T \times T}$ be the head-averaged attention matrix at layer l for a full sequence of T tokens. We
 272 define the average attention weight from a set of query tokens with indices \mathcal{S}_Q to a set of key tokens
 273 with indices \mathcal{S}_K as:

$$\text{Attn}^{(l)}(\mathcal{S}_Q \rightarrow \mathcal{S}_K) = \frac{1}{|\mathcal{S}_Q|} \sum_{i \in \mathcal{S}_Q} \sum_{j \in \mathcal{S}_K} A_{ij}^{(l)}. \quad (6)$$

277 For our *answer*→*answer-prefix* metric, \mathcal{S}_Q comprises the indices of all tokens in the generated
 278 reasoning trace, while \mathcal{S}_K contains the indices of the initial K tokens of that same trace. This metric
 279 quantifies the degree to which subsequent reasoning steps are grounded in the model’s own initial
 280 problem interpretation. For all results reported in this section, including the aggregate statistics
 281 in Table 2 and the layer-wise analysis in Figure 3, the prefix length is dynamically set to the per-
 282 sample echo length estimated by our MLP probe. This allows for a precise analysis of the *actual*
 283 echo’s effect. As the results show, correctly solved problems consistently exhibit stronger attention
 284 to the answer prefix than incorrect ones, supporting our *attention refocusing* hypothesis.

285 **Layer-Wise Attention Dynamics.** To further understand where the attention refocusing occurs
 286 within the model’s architecture, we conducted a fine-grained layer-wise analysis across all 32 layers.
 287 Figure 3 visualizes the attention weight distribution, revealing distinct patterns between correct and
 288 incorrect reasoning traces.



301 Figure 3: Layer-wise attention weight distribution on GSM8K (DeepSeek-R1-Distill-Llama-8B)
 302 for Left: *answer*→*answer-prefix* and Right: *answer*→*question*. The blue lines represent correct
 303 reasoning traces while orange lines represent incorrect ones. The attention refocusing effect is most
 304 pronounced in layers 7–18 for *answer*→*answer-prefix*, with correct traces maintaining consistently
 305 higher attention weights.

306 The layer-wise analysis localizes the EOP effect to the model’s reasoning bottleneck. We observe
 307 a **Middle-Layer Dominance** where the attention gap peaks in layers 7–18, consistent with findings
 308 that intermediate layers govern reasoning aggregation. Furthermore, the **Differential Impact**—
 309 where correct traces attend significantly more to the answer-prefix ($\Delta \approx 3\%$) than to the
 310 original question ($\Delta < 1\%$)—confirms that the echo acts as a distinct working memory anchor,
 311 actively refocusing the model on the problem statement during critical computation steps.

312 Interestingly, the early layers (1–6) show minimal differences between correct and incorrect groups,
 313 with both trajectories nearly overlapping. This suggests that low-level token processing remains
 314 largely unaffected. Crucially, as shown in Figure 3 Right, the *answer*→*question* attention remains
 315 closely matched across all layers, serving as a valuable negative control.

317 This confirms that the performance gain is not attributable to a simple, uniform increase in attention
 318 to the original question. Instead, the discriminative signal emerges in Figure 3 Left, where the diver-
 319 gence in *answer*→*answer-prefix* attention begins at layer 7. We report mean ± s.e.m. across sam-
 320 ples; focusing on layers 7–18 reveals a group difference of $\Delta(C - W) \approx 0.66\%$ for *answer*→*question*
 321 and a more substantial $\approx 2.87\%$ for *answer*→*answer-prefix*.

322 This supports our hypothesis that the Echo of Prompt acts as a cognitive scaffold for higher-level rea-
 323 soning; it is not merely about re-reading the question, but about anchoring the subsequent reasoning
 324 process to a stable internal representation, a mechanism that strongly correlates with correctness.

324
 325 Table 2: Average attention weights (%) on GSM8K (DeepSeek-R1-Distill-Llama-8B) with probe-
 326 estimated prefix length. Global statistics and layer-specific analysis showing attention from answer
 327 tokens to question and answer prefix.

Metric	Correct	Wrong	Diff (C-W)
<i>Global Statistics</i>			
Last-layer: answer → question	5.77%	5.54%	+0.23%
Last-layer: answer → answer-prefix	13.69%	10.41%	+3.28%
All-layers mean: answer → question	8.45%	8.00%	+0.45%
All-layers mean: answer → answer-prefix	10.64%	8.49%	+2.15%
<i>Peak Effect Layers (7-18)</i>			
Layers 7-18: answer → question	12.17%	11.51%	+0.66%
Layers 7-18: answer → answer-prefix	14.45%	11.58%	+2.87%

334
 335 Table 3: Layer-wise discriminability (Correct vs. Wrong) aggregated into **layer groups**. Metrics are
 336 computed on answer→answer-prefix and answer→question attention. The mid-layer group shows
 337 the strongest effect size (d) for answer→answer-prefix.

Layer Group (layers)	AUC↑ (Ans→Pref)	$d\uparrow$ (Ans→Pref)	AUC (Ans→Q)	d (Ans→Q)
Early (0–6)	0.716	0.820	0.628	0.482
Mid (7–18)	0.719	0.832	0.585	0.303
Late (19–31)	0.723	0.828	0.563	0.184

347 To ensure our findings are robust and not merely an artifact of the dynamically-set prefix length, we
 348 conducted an ablation study using fixed prefix lengths. The results, detailed in §A.5, confirm that
 349 the attention gap persists across several fixed prefix lengths, supporting our conclusion that EOP’s
 350 function is genuine attention refocusing.

351
 352 **Layer-Wise Discriminability.** To quantify where attention refocusing emerges, we compute
 353 layer-wise discriminability between Correct and Wrong groups using AUC and Cohen’s d . Specifi-
 354 cally, for each layer l , we treat the attention scores from the Correct traces ($\mathcal{A}_C^{(l)}$) and Wrong traces
 355 ($\mathcal{A}_W^{(l)}$) as two distributions. The Area Under the ROC Curve (AUC) measures how well attention at
 356 a given layer classifies a trace as correct. We also compute Cohen’s d to quantify the effect size:

$$d^{(l)} = \frac{\mu(\mathcal{A}_C^{(l)}) - \mu(\mathcal{A}_W^{(l)})}{s_p^{(l)}}, \quad (7)$$

360 where μ denotes the mean and $s_p^{(l)}$ is the pooled standard deviation for layer l . We also aggregate
 361 layers into three **groups** to analyze broader trends. Table 3 shows that the **mid-layer group (7-18)**
 362 **exhibits the strongest effect size on answer→answer-prefix** (Cohen’s $d=0.832$), with its AUC
 363 being comparable to the late-layer group. In contrast, the *answer→question* discriminability remains
 364 significantly lower across all groups, serving as a negative control.

366 These statistics, combined with the attention trajectories (Figure 3), strongly indicate that EOP’s pri-
 367 mary mechanism is to refocus representations within the mid layers (layers 7 through 18), anchoring
 368 subsequent reasoning to the answer prefix.

370 4 EMPIRICAL VALIDATION

372 4.1 ECHO REINSERTION AS A CAUSAL INTERVENTION

374 We construct an interventional experiment that starts from *failed* GSM8K completions produced by
 375 several models (**DeepSeek-R1-Distill-Llama-8B**, **Qwen3-8B**, and the non-reasoning **Qwen3-8B-Base**). For each wrong example we (i) truncate an echo-free trace to 50% of its tokens to obtain
 376 a shared prefix, (ii) resume generation either directly (**echo-free**) or after inserting the template
 377 phrase “now I need to look back at the question again:” (**echo reinsertion**), and (iii) score the new

378
 379 Table 4: Echo reinsertion ablation on the wrong-subset GSM8K traces for several models. We
 380 report exact-match accuracy (%) when continuing from the same echo-free prefix with and without
 381 a forced echo phrase.

Model	Echo-free EM (%)	Echo reinsertion EM (%)
DeepSeek-R1-Distill-Llama-8B	15.85	26.22
Qwen3-8B	21.34	29.27
Qwen3-8B-Base (no CoT)	10.56	10.56

386
 387 Table 5: Supervised fine-tuning with distilled CoT data improves mathematical reasoning.
 388 –ED-SFT denotes models fine-tuned on our EOP distilled dataset. –normal-SFT refers to the
 389 normal CoT distilled dataset. Base is the pretrained model; the unmarked variant is instruction-tuned.
 390 Best scores in each column are in **bold**.

Model	GSM-8K		MathQA	Hendrycks-MATH
	Strict EM	Flex EM		
<i>Qwen3-8B-Base</i>				
Base	79.4	80.5	31.0	0.76
+ ED-SFT	94.2 (+3.4)	94.2 (+3.4)	58.8 (+11.8)	10.0 (+8.2)
+ normal-SFT	90.8	90.8	47.0	1.8
<i>Qwen3-8B (Instruct Version)</i>				
Base	87.49	88.1	49.2	0.8
+ ED-SFT	93.1 (+2.8)	93.4 (+3.3)	53.7 (+1.9)	6.1 (+1.1)
+ normal-SFT	90.3	90.1	51.8	5.0
<i>DeepSeek-Distill-Llama-8B</i>				
Base	67.6	66.1	31.6	0.38
+ ED-SFT	78.2	79.7 (+0.2)	34.8 (+3.4)	3.0 (+2.24)
+ normal-SFT	80.5	79.5	31.4	0.76

405
 406 completions with the standard GSM8K exact-match script. Both branches see identical questions,
 407 prefixes, decoding parameters, and random seeds, isolating the causal impact of the injected echo.
 408

409 The intervention confirms that echoes are *causally helpful* for reasoning-capable models: forcing a
 410 short echo before resuming the chain yields sizable absolute EM gains (+10.4 and +7.9 points for
 411 DeepSeek-R1-Distill-Llama-8B and Qwen3-8B, respectively), while the non-reasoning base model
 412 shows no improvement. This null result for the base model is expected, as it lacks the instruction-
 413 following and reasoning priors typically acquired via RLHF to utilize the re-injected context in a
 414 zero-shot manner. This interpretation is reinforced by our ED-SFT results (Table 5), where the
 415 base model exhibits the largest relative improvement (+3.4 points) when the reasoning capability
 416 and echo strategy are instilled simultaneously. Qualitatively, we observe the reinsertion branch
 417 revisiting the original quantities and constraints (Figure 9), whereas the echo-free branch continues
 418 the drifting reasoning that led to the initial error. This interventional evidence complements our
 419 correlational analyses (§3) and grounds the attention refocusing hypothesis in an explicit cause-and-
 420 effect experiment.

421 4.2 PERFORMANCE GAINS FROM ECHO-DISTILLED SFT (ED-SFT)

422 Having established that Echo of Prompt correlates with improved reasoning performance, we investi-
 423 gate whether this behavior can be systematically instilled through targeted training. Our Supervised
 424 Fine-Tuning (SFT) methodology is inspired by recent work (Team, 2025).

425 The core hypothesis is that **explicitly training models on echo-prefixed traces enhances their**
 426 **problem-solving approach**: the echo phase forces deeper engagement with problem constraints
 427 and establishes a stronger foundation for subsequent reasoning steps.

428
 429 **Methodology.** We develop **Echo-Distilled SFT (ED-SFT)**, a supervised fine-tuning method that
 430 embeds this echo-then-reason pattern as a learned behavior. We first construct a shared pool of
 431 high-quality teacher traces on GSM8K by querying a capable teacher model, gpt-oss-120B,

432 with a standard CoT prompt that wraps the reasoning in a single `<think>` block and requires the
 433 final answer to be a plain value. We automatically verify that the final answer exactly matches the
 434 ground-truth solution and discard any trace that fails this check. This pool of verified (*question*,
 435 *CoT*, *answer*) triples is the common source for both ED-SFT and the **normal-SFT** baseline.

436 To obtain **ED-SFT** data, we encourage an explicit echo-then-reason pattern at the head of the trace.
 437 We train a small MLP probe to detect whether an early Echo of Prompt segment is present. For
 438 traces flagged as missing EOP, we call `gpt-oss-120B` once more with an edit instruction that
 439 *minimally* inserts a short echo-style opening that restates the question (e.g., “Okay, let me see. The
 440 problem is asking: …”) while preserving the subsequent reasoning and final answer. Traces that
 441 already contain an echo are kept unchanged. After editing, we re-run the automatic checker and drop
 442 any example whose final answer no longer matches the gold label. The resulting ED-SFT dataset
 443 therefore differs from the standard CoT pool only by the presence of an initial echo segment.

444 For the **normal-SFT** baseline we again start from the same verified teacher traces but *remove* the
 445 echo segment while keeping the remainder of the reasoning untouched. Because the MLP probe is
 446 a binary EOP detector rather than a span localizer, we delegate span selection to the teacher: when
 447 the probe predicts EOP presence, we prompt `gpt-oss-120B` to delete the echo prefix under a
 448 “do not change the reasoning or final answer” instruction, and we discard any sample whose answer
 449 changes. This yields paired ED-SFT and normal-SFT corpora that are nearly identical token-wise
 450 and differ primarily in the presence or absence of the initial echo. On GSM8K, the inclusion of the
 451 echo prefix results in a longer average sequence length for ED-SFT compared to normal-SFT (175
 452 vs. 136 tokens).

453 **Experimental Setup.** To test the echo strategy’s effectiveness at different stages of model training,
 454 we fine-tuned models from two families: Qwen3 (8B) and Deepseek-distill-Llama-8B. For
 455 the Qwen3 family, we experimented on two distinct versions: the pre-trained base model (**Qwen3-8B-Base**) and the final, fully instruction-tuned model (**Qwen3-8B**). For each model, we applied
 456 our SFT procedure to produce both -ED-SFT and -normal-SFT variants. All fine-tuning runs
 457 use the same optimizer (AdamW), learning-rate schedule, batch size, maximum sequence length,
 458 and number of training steps; the only difference is whether the training traces come from the
 459 echo-augmented (ED-SFT) or echo-trimmed (normal-SFT) versions of the same teacher CoTs. We
 460 evaluated all models on a suite of mathematical reasoning benchmarks (GSM8K, MathQA, and
 461 Hendrycks-MATH) to assess generalization under distribution shift, as fine-tuning was performed
 462 *only* on the GSM8K training set with 7k samples.

463 **Results.** As shown in Table 5, SFT with our distilled data yields substantial and consistent
 464 performance improvements. Crucially, these gains appear on both the pre-trained base and the
 465 fully-aligned instruction-tuned models. Fine-tuning the base model (**Qwen3-8B-Base-echo-SFT**)
 466 achieves a remarkable gain of +3.4 points on GSM-8K, while fine-tuning the already capable
 467 instruction-tuned model (**Qwen3-8B-echo-SFT**) still provides a solid boost of +2.8 points.

468 **Cross-Model Generalization.** The effectiveness of Echo-Distilled SFT extends across different
 469 model architectures. For **DeepSeek-distill-lama-8B**, we observe consistent improvements, with
 470 particularly strong gains on benchmarks that differ distributionally from GSM8K, such as MathQA
 471 (+3.4 points) and Hendrycks-MATH (+2.24 points). The consistent success on both base and instruct
 472 models strongly suggests that the Echo of Prompt (EOP) is a fundamental and transferable cognitive
 473 alignment strategy, not merely an artifact of existing instruction tuning.

474 **Mechanistic Alignment With Attention Analysis.** The success of ED-SFT can be understood
 475 through the lens of our attention analysis (§3.3). The layer-wise attention patterns reveal that models
 476 trained with echo-prefixed traces naturally develop stronger attention connections in middle layers
 477 (7-18), where we observed the most significant differences between correct and incorrect reasoning
 478 (1.73% increase in answer→answer-prefix attention). This suggests that ED-SFT effectively instills
 479 the attention refocusing mechanism we identified in our analysis, teaching models to leverage these
 480 critical layers for maintaining problem-relevant attention throughout the reasoning process.

481 To further substantiate this, we analyzed the *answer*→*answer-prefix* attention gap (Correct–Wrong)
 482 specifically within the critical mid-layer block (layers 7–18) across our model variants. This targeted
 483 metric confirms that **ED-SFT** most effectively strengthens this mechanism, exhibiting the largest

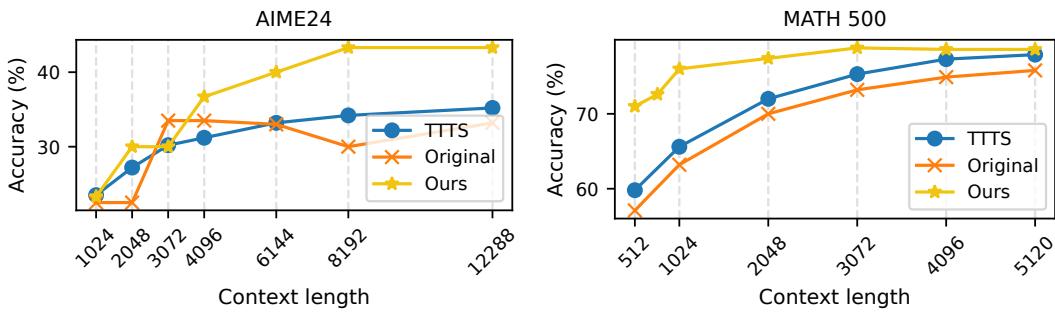


Figure 4: Echoic Prompting (EP) vs. TTTS on AIME24 (left) and MATH-500 (right).

attention gap (+3.20 pp) compared to the base model (+1.90 pp) and the normal SFT variant (+2.40 pp). This finding provides direct evidence that ED-SFT successfully instills the desired refocusing behavior where it is most impactful. Full statistics are provided in §A.2.

4.3 ECHOIC PROMPTING (EP): A TRAINING-FREE ENHANCEMENT

The Echoic Prompting (EP) Method. Our proposed **Echoic Prompting (EP)** strategy is a training-free method designed to enhance reasoning capabilities at inference time. The core idea is to re-engage the model by echoing the original prompt. Specifically, after the model produces an initial reasoning chain, we append a reminder phrase such as *look back at the question again* followed by the original question itself. This intervention encourages the model to revisit the problem’s context and continue generating a more grounded response. Unlike methods that inject generic, task-agnostic stimuli, EP re-grounds the model with task-specific context from the original query and shows consistent gains over 2 math reasoning datasets following TTTS’s settings.

Experimental Setup. To evaluate the effectiveness of EP, we compare it against a strong baseline, Thinking Token based Test-time Scaling (TTTS) (Qian et al., 2025), which artificially inserts generic *thinking tokens* (e.g., *So*, *Hmm*) to spur reasoning. For a fair comparison, we reproduce TTTS following its official implementation. Both methods are evaluated on the **DeepSeek-R1-Distill-Llama-8B** model, using the vLLM backend with deterministic decoding (temperature=0.0).

Results. As shown in Figure 4, our EP approach consistently and substantially outperforms TTTS across both AIME24 and MATH-500. The performance gains are robust under identical decoding and budget settings. This indicates that re-grounding the model on the input via a natural echo of the prompt is more effective than injecting generic, artificial thinking tokens.

5 CONCLUSION

In this work, we systematically investigated the Echo of Prompt (EOP), the spontaneous tendency of large reasoning models to repeat a user’s query before generating a solution. We introduced a probabilistic framework based on rejection sampling to define and measure the *Echo Likelihood Gap*. Through detailed attention analysis, we provided a mechanistic explanation, demonstrating that EOP serves to refocus attention on task-critical information, particularly in model’s middle layers.

To harness this phenomenon, we proposed two practical methods: *Echo-Distilled SFT (ED-SFT)*, which instills behavior through fine-tuning, and *Echoic Prompting (EP)*, a training-free inference technique. Across multiple mathematical reasoning benchmarks, both methods demonstrated consistent performance gains over strong baselines, validating EOP as a beneficial cognitive primitive.

Ultimately, our work advocates for a shift in alignment research from validating final outputs to cultivating beneficial thought processes. This provides a framework to understand emergent behaviors like EOP, paving the way for more interpretable and robust AI systems. Future work could engineer dynamic echo mechanisms across diverse tasks and model scales, bridging the gap between emergent phenomena and deliberate cognitive design.

540 **ETHICS STATEMENT**

541
 542 All authors adhere to the ICLR Code of Ethics. Our research focuses on understanding and im-
 543 proving the reasoning capabilities of large language models, a foundational scientific goal. The
 544 datasets used for fine-tuning and evaluation, such as GSM8K and MathQA, are standard public
 545 benchmarks in the field. The synthetic data used for SFT was generated by a large proprietary
 546 model GPT-OSS-120B, and the annotations for our MLP probe were assisted by GPT-4.1, as
 547 detailed in our LLM Usage Disclosure in §A.1. We acknowledge that the models used in this study
 548 may inherit biases from their original, opaque training data. Our work does not introduce new ap-
 549 plications with foreseeable negative societal impacts. We believe that a deeper understanding of
 550 emergent behaviors like the Echo of Prompt contributes to the development of more transparent and
 551 reliable AI systems.

552
 553 **REPRODUCIBILITY STATEMENT**

554
 555 To ensure the reproducibility of our findings, we commit to releasing our source code, including
 556 scripts for data processing, training, and evaluation, upon publication. Our work relies on pub-
 557 licly available models, including the Qwen3 and DeepSeek series, and standard benchmarks such as
 558 GSM8K, MathQA, and Hendrycks-MATH. The methodology for our Echo-Distilled SFT data gen-
 559 eration is detailed in §A.8. The design and training of the MLP probe used for echo detection are
 560 fully described in §A.4, and all experimental hyperparameters and evaluation settings are detailed
 561 in §4. The theoretical claims in §3 are self-contained within the paper.

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648 **A APPENDIX**
 649

650 **A.1 THE USE OF LARGE LANGUAGE MODELS**
 651

652 In accordance with ICLR 2026 policy, we disclose the usage of Large Language Models (LLMs) in
 653 this research. Our use of LLMs is primarily in three areas:

- 654
- 655 **Writing Assistance:** We utilized LLMs to improve the clarity, grammar, and overall readability
 of the manuscript. This involved proofreading and refining sentences without altering
 the core scientific contributions.
 - 658 **Data Annotation:** As detailed in §A.4, GPT-4.1 was employed to annotate our Chain-of-
 Thought (CoT) dataset. This process was crucial for training the MLP probe to accurately
 identify Echo of Prompt (EOP) instances.
 - 661 **Synthetic Data Generation:** The CoT dataset for Supervised Fine-Tuning (SFT), dis-
 cussed in §A.8, was generated using the gpt-oss-120B model. This provided the foundation
 for training our models to exhibit the desired echo behavior.
- 664

665 **A.2 ADDITIONAL CROSS-MODEL ATTENTION STATISTICS**
 666

667 To supplement the analysis in §4, Table 6 provides the detailed mid-layer (layers 7–18) attention
 668 gap statistics for the answer→answer-prefix metric across model variants. The gap is computed as
 669 the difference in average attention percentage points (pp) between correct and incorrect reasoning
 670 traces. A larger positive value indicates stronger within-model discriminability.

672 Table 6: Mid-layer (layers 7–18) Ans→Pref gap (pp, Correct–Wrong). Values are for within-model
 673 discriminability; do not use for cross-model ranking.

675 Model / Setting	676 Mid-layer gap (pp)
676 Qwen3-8B-Base	1.90
677 Echo-SFT	3.20
678 Normal-SFT	2.40

680 **A.3 ON THE ORIGINS OF THE ECHO OF PROMPT**

683 While the precise mechanisms underlying the formation of the Echo of Prompt (EOP) are not yet
 684 fully understood, we note its appearance in related emergent LLM-reasoning phenomena, such as
 685 the initial COT prompting (Wei et al., 2022) and the "aha moment" observed in DeepSeek-R1-zero
 686 (DeepSeek-AI et al., 2025). We hypothesize that EOP is an emergent behavior that arises from the
 687 model's implicit need to ground its reasoning process in the problem statement. By restating the
 688 prompt, the model may be reinforcing its internal representation of the task, thereby improving its
 689 focus on relevant information for subsequent reasoning steps.

690 **A.4 MLP PROBE FOR ECHO DETECTION**

692 To operationalize our probabilistic framework, we require a reliable method to detect echo prefixes.
 693 We train a lightweight two-layer MLP probe for this binary classification task.

695 **Data and Annotation.** Training data are sampled from the `am_0.9M_1k.jsonl` subset of
 696 AM-DeepSeek-R1-Distilled-1.4M (Zhao et al., 2025). Each example consists of a (question,
 697 think_content) pair. Labels are generated using a hybrid approach: we prompt GPT-4.1 (OpenAI,
 698 2024) with a deterministic rubric to identify semantic repetition and its approximate boundary. This
 699 boundary is then refined using sentence-level semantic similarity to correct for formatting artifacts.
 700 To validate annotation quality, a random subset of 200 annotations was manually reviewed, showing
 701 over 96% agreement with the final labels. We release the prompt template and parsing code in our
 repository.

Input Features. For each (question, think_content) pair, we featurize the sample by concatenating two sentence embeddings. The first embedding represents the full question. The second represents the initial prefix of the think_content, defined as the first 32 word tokens. Both are encoded using a SentenceTransformer model (Qwen3-Embedding-0.6B). The resulting concatenated vector is z-scored before being passed to the probe.

Architecture and Training. The probe is a two-layer MLP with a 32-dimensional hidden layer and ReLU activation, mapping the concatenated embedding to a single logit. We train the model using weighted binary cross-entropy on logits (sigmoid + BCE computed in a numerically stable form; implemented via PyTorch’s `BCEWithLogitsLoss`), where the positive class weight is set to the ratio of negative to positive samples in the training set to handle class imbalance. Optimization is performed with AdamW (learning rate 10^{-4} , weight decay 0.01, batch size 64) for up to 200 epochs, with early stopping (patience 10) on the validation loss. The dataset is split into training (70%), validation (15%), and testing (15%) sets.

Evaluation and Usage. The trained probe’s performance on the held-out test set is reported in Table 7. The high AUROC and F1-Score confirm its reliability for identifying echoes. During inference for our main experiments (e.g., attention analysis), this probe is used as a predicate to identify and measure echo prefixes. It is not used to score task correctness. For truncation, we use a calibrated threshold on the sigmoid output with a hysteresis scheme (initial threshold 0.6, drop threshold 0.15) to ensure stable prefix detection.

Table 7: MLP probe performance on the held-out test set. The probe reliably identifies echo prefixes, justifying its use in our framework.

Metric	Accuracy	Precision	Recall	F1-Score	AUROC
Value	0.912	0.921	0.908	0.914	0.963

Reproducibility and licensing. We fix and log random seeds, dataset hashes, feature extraction versions, and probe checkpoints. Data originate from AM-DeepSeek-R1-Distilled-1.4M (Zhao et al., 2025) (subset am_0.9M_1k.jsonl). The GPT-4.1 annotator is referenced in (OpenAI, 2024). Scripts to reproduce this pipeline are provided in our code release.

A.5 ABLATION STUDY ON FIXED PREFIX LENGTHS

To verify that the observed attention refocusing is not merely a byproduct of the echo’s length, we performed an ablation study. Instead of using the dynamically estimated echo length from our MLP probe, we re-computed the $\text{answer} \rightarrow \text{answer-prefix}$ attention metric using several fixed prefix lengths (K). This allows us to disentangle the effect of prefix length from the functional role of the echo’s content.

Methodology. We repeated the layer-wise attention analysis from §3.3 with fixed prefix lengths of $K \in \{32, 64, 128\}$ tokens for all samples. For each value of K , we calculated the average attention from all answer tokens to the first K answer tokens, separately for the Correct and Wrong groups.

Results. As shown in Table 8, the attention gap between the Correct and Wrong groups remains consistently positive and significant across all fixed prefix lengths. While the magnitude of the gap varies with K , the Correct group consistently directs more attention to the answer prefix. This demonstrates that the attention refocusing effect is a robust phenomenon and not just an artifact of longer echoes co-occurring with correct answers.

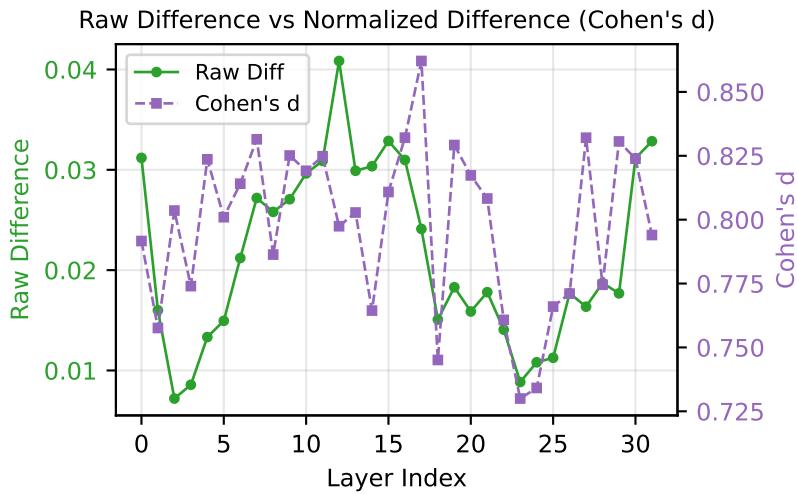
A.6 VERIFICATION OF ATTENTION NORMALIZATION

To ensure that the observed differences in attention weights between correct and incorrect traces are not artifacts of absolute weight fluctuations across layers, we performed a normalization analysis. We computed the standardized mean difference (Cohen’s d) and Z-score differences for each layer.

756
 757 Table 8: Ablation study on fixed prefix lengths for *answer*→*answer-prefix* attention. Attention
 758 weights are averaged across all layers. The positive difference (Correct – Wrong) persists for all
 759 values of K .

Prefix Length (K)	Correct (%)	Wrong (%)	Difference (%)
32 tokens	10.61	8.42	2.19
64 tokens	16.78	13.94	2.83
128 tokens	20.51	19.43	1.08

760
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 766 Figure 5 presents a dual-axis comparison of the raw attention difference (Correct – Wrong) and the
 767 normalized Cohen’s d effect size for the *answer*→*answer-prefix* metric. The two curves track each
 768 other closely, with the normalized effect size consistently exceeding 0.75 in the critical middle layers
 769 (7–18) and peaking at 0.86. This confirms that the attention refocusing signal is robust to layer-
 770 specific magnitude variations and represents a statistically significant difference in model behavior.



789
 790 Figure 5: Comparison of raw attention difference (Correct – Wrong) and normalized effect size
 791 (Cohen’s d) across layers for the *answer*→*answer-prefix* metric. The strong alignment between the
 792 raw and normalized metrics confirms that the mid-layer refocusing peak is a robust phenomenon.

A.7 LINKING $\Delta\mathcal{L}$ TO CORRECTNESS: A LIKELIHOOD DECOMPOSITION

793
 794
 795
 796 Let $\pi_\theta(y | x)$ be the base model and $\tau_\theta(y | x) = \pi_\theta(y | x) \mathbf{1}_{y \in \mathcal{Y}_{\text{trim}}} / Z_x$ the trimmed distribution
 797 with $Z_x > 0$. For a raw trace $y_{\text{raw}} = [e, s]$ and its trimmed counterpart $y_{\text{trim}} = s$, define the
 798 per-token log-likelihood $\mathcal{L}_\pi(y | x) = \frac{1}{|y|} \sum_t \log \pi_\theta(y_t | x, y_{<t})$, and the Echo Likelihood Gap
 799 $\Delta\mathcal{L} = \mathcal{L}_\pi(y_{\text{raw}} | x) - \mathcal{L}_\pi(y_{\text{trim}} | x)$.

800 Because $\log \tau_\theta(y_{\text{trim}} | x) = \log \pi_\theta(y_{\text{trim}} | x) - \log Z_x$, we have, for $n = |y_{\text{trim}}|$,

$$\mathcal{L}_\tau(y_{\text{trim}} | x) = \mathcal{L}_\pi(y_{\text{trim}} | x) - c(x, n) = \mathcal{L}_\pi(y_{\text{raw}} | x) - \Delta\mathcal{L} - c(x, n),$$

801
 802 where the “constant” shift is $c(x, n) = \frac{1}{n} \log Z_x$. Taking conditional expectations with respect to
 803 the correctness label $G \in \{\text{C}, \text{W}\}$ yields

$$\mathbb{E}[\mathcal{L}_\tau | G] = \mathbb{E}[\mathcal{L}_\pi(y_{\text{raw}} | x) | G] - \mathbb{E}[\Delta\mathcal{L} | G] - \mathbb{E}[c(x, n) | G].$$

810 Therefore,

$$\begin{aligned} & \mathbb{E}[\mathcal{L}_\tau | C] - \mathbb{E}[\mathcal{L}_\tau | W] \\ &= \underbrace{\left(\mathbb{E}[\mathcal{L}_\pi(y_{\text{raw}} | x) | C] - \mathbb{E}[\mathcal{L}_\pi(y_{\text{raw}} | x) | W] \right)}_{\text{controlled by length/suffix stratification}} \\ &\quad - \left(\mathbb{E}[\Delta \mathcal{L} | C] - \mathbb{E}[\Delta \mathcal{L} | W] \right) \\ &\quad - \underbrace{\left(\mathbb{E}[c(x, n) | C] - \mathbb{E}[c(x, n) | W] \right)}_{\approx 0 \text{ if } x \text{ and } n \text{ are matched}}. \end{aligned}$$

821 Under matched prompts x and matched (or stratified) lengths n , the first and last terms are negligible,
822 so the group difference in \mathcal{L}_τ is approximately the *negative* of the group difference in $\Delta \mathcal{L}$.

823 A.8 COT DISTILLATION PIPELINE

825 Our Chain-of-Thought (CoT) distillation pipeline, used to create the datasets for Echo-Distilled SFT
826 (ED-SFT), follows a teacher-student approach grounded in a *single* shared pool of teacher traces.
827 We first use a highly capable teacher model (`gpt-oss-120B`) to generate reasoning traces for the
828 training questions (e.g., from the GSM8K training set) with a standard CoT prompt that wraps the
829 reasoning in a `<think>` block and enforces an exact-match final answer. Any trace whose final
830 answer does not match the gold label is discarded, yielding a pool of verified (`question`, `CoT`,
831 `answer`) triples.

832 From this pool we derive two closely matched SFT datasets. For the **ED-SFT** dataset, we encourage
833 an explicit echo-then-reason pattern: we train an MLP probe to detect whether an early Echo of
834 Prompt segment is present, and for traces predicted to be echo-free we ask `gpt-oss-120B` to
835 minimally insert a short echo-prefix that restates the question while preserving the existing reasoning
836 and answer. Traces that already contain an echo are kept as-is. For the **normal-SFT** baseline, we
837 again start from the same verified traces but, when the probe predicts EOP presence, we prompt the
838 teacher to delete the echo-prefix under a “do not change the reasoning or final answer” instruction.
839 In both directions we re-run answer checking and drop any edited example whose final answer
840 changes.

841 This procedure yields paired ED-SFT and normal-SFT corpora that are nearly identical token-wise
842 and differ primarily in the presence or absence of the initial echo. As reported in the main text, on
843 GSM8K the inclusion of the echo prefix results in longer average sequences for ED-SFT compared
844 to normal-SFT (175 vs. 136 tokens).

845 A.9 TOKEN-WISE ATTENTION SIGNIFICANCE

847 To verify that the attention refocusing effect is not driven by positional bias or a few outlier tokens,
848 we performed a token-wise analysis of the attention weights. Figure 6 shows the average attention
849 weights for the first 32 answer tokens towards the answer-prefix (left) and the question (right),
850 comparing Correct and Wrong groups.

852 We conducted a Welch’s t-test at each token position. For `answer → answer-prefix`, the Correct group
853 shows consistently higher attention, with significant differences ($p < 0.05$) at 10 out of 32 positions.
854 Conversely, for `answer → question`, the Wrong group attends significantly more to the question at 22
855 out of 32 positions. This confirms that successful reasoning involves a systematic shift of attention
856 from the original question to the model’s own echoed representation.

857 A.10 WORD-LEVEL ATTENTION CASE STUDY

859 To visualize which specific parts of the echo are attended to, we aggregated token-level attention into
860 word-level scores. Figure 7 shows a heatmap of attention from the reasoning trace to the echo prefix
861 for a representative correct solution to a GSM8K problem (“Janet’s ducks”). The model focuses
862 most intensely on the key numerical entities and constraints (e.g., “16”, “eggs”, “3”, “13”) within the
863 echo, rather than on function words. This supports the hypothesis that the echo serves as a semantic
864 anchor for critical problem details.

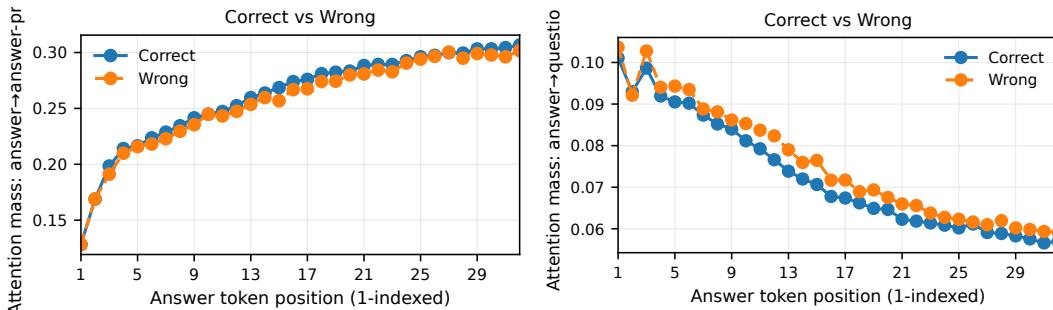


Figure 6: Token-wise average attention weights for the first 32 answer tokens. Left: Attention to Answer-Prefix. Right: Attention to Question. Shaded regions indicate standard error. The Correct group (blue) consistently attends more to the prefix, while the Wrong group (orange) attends more to the question.

Ducks	lay	16	eggs	per	day,	She	eats	3	eggs
for	breakfas so	she	has	16	-	3	=	<<16-3=13>>13	

Figure 7: Word-level attention heatmap from the reasoning trace to the echo prefix (mid-layers 7-18). Darker red indicates higher attention. The model selectively attends to key quantities (numbers of eggs, dollars) in the echoed prompt.

A.11 INFORMATION FLOW ANALYSIS

We further investigated how information propagates through the model layers using an information flow route analysis. Figure 8 visualizes the attention-based routing of information for a single answer token (orange triangle at the top right). In the visualization, blue nodes represent question tokens, and green nodes represent echo/prefix tokens. In Correct traces, backward attribution paths from answer tokens repeatedly route through the echo-prefix tokens before reaching the rest of the prompt, whereas in Wrong traces these paths more often terminate in the question region or fail to reach the key numerals (e.g., idx 1151/1001 vs. 594). This suggests that the echo acts as a recurrent internal hub that integrates and refines information before it is used in the final generation.

A.12 LOGISTIC REGRESSION ANALYSIS OF $\Delta\mathcal{L}$

To quantify the predictive power of the Echo Likelihood Gap ($\Delta\mathcal{L}$) on reasoning correctness, we fitted a logistic regression model on the 1,319 GSM8K samples used in our analysis. We predicted the binary correctness outcome $Y \in \{0, 1\}$ using $\Delta\mathcal{L}$ and the length of the trimmed echo (L_{echo}) as predictors:

$$\text{logit}(P(Y = 1)) = \beta_0 + \beta_1 \Delta\mathcal{L} + \beta_2 L_{\text{echo}} \quad (8)$$

The regression results (Table 9) indicate that $\Delta\mathcal{L}$ is a statistically significant positive predictor of correctness ($p \approx 0.022$), with a coefficient $\beta_1 \approx 0.24$. This implies that for every 1.0 nat/token increase in the likelihood gap, the odds of a correct answer increase by a factor of $\exp(0.24) \approx 1.27$, confirming that the model’s probabilistic preference for the echo is meaningfully associated with task success.

A.13 ANALYSIS OF EOP-PRESENT VS. EOP-ABSENT TRACES

To disentangle the effect of the Echo of Prompt (EOP) from general model capabilities, we compared traces where the model spontaneously produced an echo (EOP-present) versus those where it did not (EOP-absent). As shown in Table 10, the EOP-present group has a higher overall accuracy (63.8% vs 57.2%). Furthermore, even when controlling for the final outcome (Correct or Wrong), traces

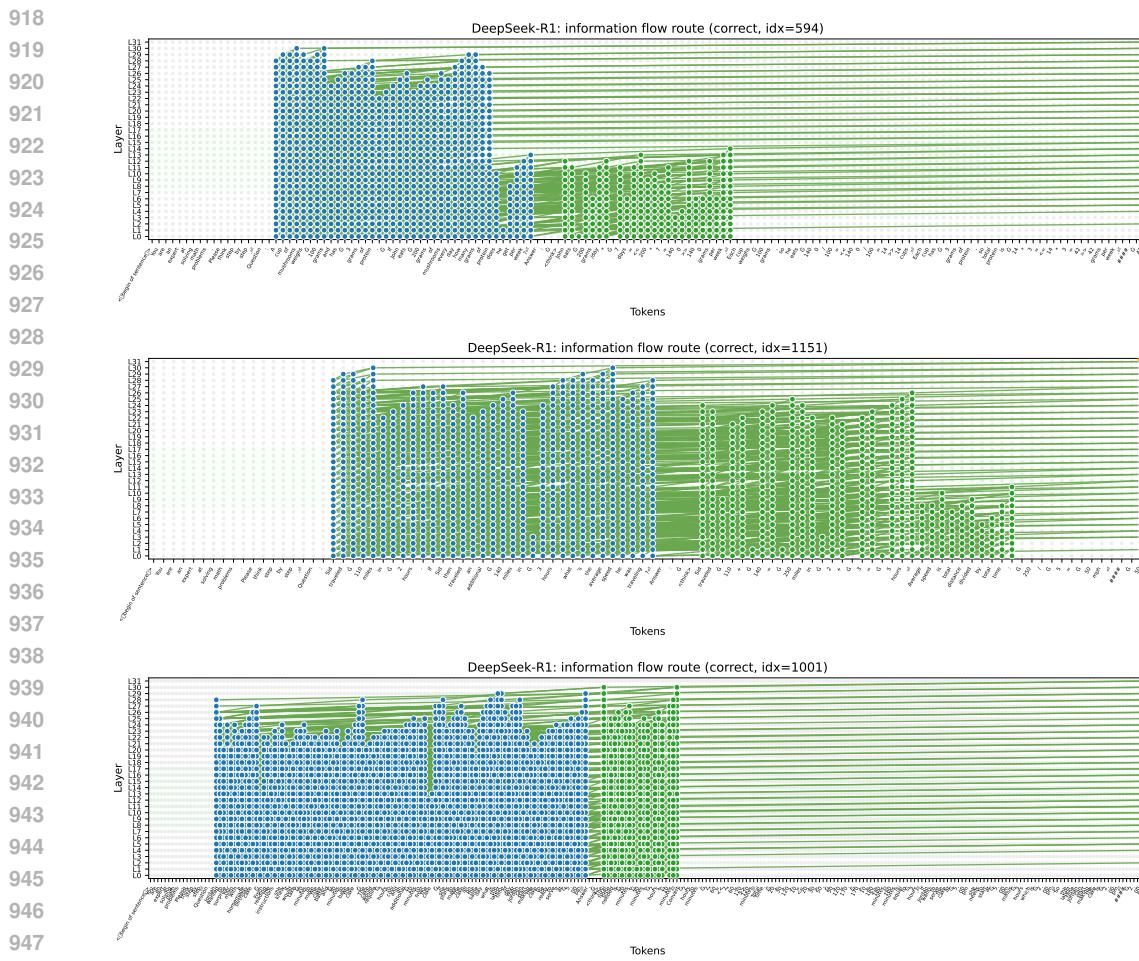


Figure 8: Information flow visualization for generated answer tokens across three different examples. The graphs show the primary attention routes through the model layers. In correct traces, information flows significantly through the Echo of Prompt (EOP) tokens in the middle layers, acting as a bridge between the question and the answer.

Table 9: Logistic regression results predicting correctness on GSM8K.

Predictor	Coefficient (β)	Std. Error	P-value
Intercept (β_0)	0.12	0.15	0.423
Echo Likelihood Gap (β_1)	0.24	0.11	0.022
Echo Length (β_2)	0.001	0.0005	0.089

with an EOP exhibit stronger attention refocusing (Answer → Answer-Prefix attention) than those without. This suggests that the presence of an echo actively facilitates the attention mechanism that supports correct reasoning.

A.14 PROMPT TEMPLATES FOR TEACHER CoT AND EDITING

For reproducibility, we list the main prompt templates used to generate and edit teacher traces.

Standard CoT generation prompt. To obtain the initial pool of verified CoTs, we query `gpt-oss-120B` with a simple reasoning prompt that separates internal thinking from the final answer:

972
 973 Table 10: Comparison of EOP-present and EOP-absent traces on GSM8K (DeepSeek-R1-Distill-
 974 Llama-8B). EOP presence is associated with higher accuracy and stronger attention refocusing.

975 Group	976 N	977 Accuracy (%)	978 Last-layer Ans→Ans-prefix Attn
EOP-present	985	63.8	–
EOP-absent	334	57.2	–
<i>Conditioned on Outcome:</i>			
Correct, EOP-present	628	–	0.1374
Correct, EOP-absent	191	–	0.1328
Wrong, EOP-present	357	–	0.1046
Wrong, EOP-absent	143	–	0.0987

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 985 You are solving math problems. Structure your entire
 986 thought process within a single pair of <think> and </think>
 987 tags. After you've finished thinking, provide the final,
 988 concise answer on a new line. The final answer should be a
 989 plain value.

990
 991 **Echo-insertion prompt for ED-SFT.** When the MLP probe predicts that a trace lacks an early
 992 Echo of Prompt, we ask the teacher to minimally insert an echo-style opening that repeatedly brings
 993 the question back into focus. The high-level instruction is:

994 You are solving math problems. Think out loud naturally.
 995 To ensure you fully understand the problem, you must repeat
 996 the question or key parts of it multiple times throughout
 997 your reasoning process before you start solving. For
 998 instance, you might re-read it to confirm details or after a
 999 few steps of calculation to ensure you are on track. Start
 1000 by repeating the problem, then reason step by step. Wrap
 1001 the entire internal thinking process with a single pair of
 1002 <think> and </think> tags, and put the final answer after
 1003 the thinking. The final answer should be a concise plain
 1004 value (number if applicable). At the very beginning of
 1005 your <think>, start with the following opening line and then
 1006 continue the original reasoning. Do not change the final
 1007 answer.

1008 The opening line is randomly chosen from a small set of naturalistic variants, e.g.:

- 1009 • Okay, let me see. The problem is asking: [QUESTION]
- 1010 • Alright, so the question is: [QUESTION]
- 1011 • Let me understand this problem. We have: [QUESTION]
- 1012 • So the problem states: [QUESTION]

1013 where [QUESTION] is replaced with the original GSM8K question text.

1014
 1015 **Echo-removal prompt for normal-SFT.** For traces where the MLP probe predicts the presence
 1016 of an early EOP, we construct the normal-SFT counterpart by asking the teacher to remove the
 1017 echo-prefix while preserving all later reasoning steps and the final answer:

1018 You are given a math question and a chain-of-thought
 1019 solution that begins by repeating or paraphrasing the
 1020 question. Rewrite the reasoning so that it no longer
 1021 repeats the question at the beginning. Keep all subsequent
 1022 reasoning steps and the final answer exactly the same. Do
 1023 not change the logic or the final answer; only remove the
 1024 initial echo segment. Wrap the internal thinking in <think>
 1025 and </think> as before.

	Benchmark	Size / Format	Domain / Source	Description (from original papers)
1026	Hendrycks-MATH	12,500 problems; open-ended; step-by-step solutions	Competition mathematics (algebra, geometry, number theory, probability, etc.)	“A new dataset of 12,500 challenging competition mathematics problems. Each problem in MATH has a full step-by-step solution which can be used to teach models to generate answer derivations and explanations.” The evaluation follows the <code>hendrycks_math</code> group in the LM Evaluation Harness, which decomposes MATH into diverse subsets (<code>_algebra</code> , <code>_counting_and_prob</code> , <code>_geometry</code> , <code>_intermediate_algebra</code> , <code>_num_theory</code> , <code>_prealgebra</code> , <code>_precalc</code>), covering a wide range of mathematical skills beyond our training distribution.
1027	GSM8K	8.5K problems; free-form answers; word problems	Grade-school math word problems	“A dataset of 8.5K high quality linguistically diverse grade school math word problems. We find that even the largest transformer models fail to achieve high test performance, despite the conceptual simplicity of this problem distribution.”
1028	MathQA	37K problems; multiple choice	Multiple math domains; derived from AQuA operation programs	“A large-scale dataset of 37k English multiple-choice math word problems covering multiple math domain categories by modeling operation programs corresponding to word problems in the AQuA dataset.”

Table 11: Benchmarks used for our evaluation.

A.15 ILLUSTRATIVE EXAMPLES OF ECHO-THEN-REASON PATTERN

We provide two illustrative examples as case studies to demonstrate the Echo-then-Reason pattern, see Figure 9

In these examples, we compare outputs from models with and without the initial echo phase. The echo-prefixed response (green) correctly identifies that the yellow marbles are identical and thus treats pairs of yellow marbles as a single unique group, leading to the correct count of 7. In contrast, the direct response (red) incorrectly treats the yellow marbles as distinguishable, overcounting the combinations and arriving at 12.

A.16 SFT EVALUATION BENCHMARKS

In this section, we clarify our terminology regarding generalization. While we initially referred to performance on MathQA and Hendrycks-MATH as “out-of-domain generalization”, we acknowledge that all tested datasets fall within the broader domain of mathematics. However, they represent significant **distributional shifts** in terms of difficulty, topic coverage, and problem format compared to the GSM8K training set. Therefore, we adopt the terms **distributional generalization** or **robustness to distribution shift** to more precisely describe these experiments. Table 11 summarizes the characteristics of each benchmark, highlighting these differences.

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Question

Tom has a red marble, a green marble, a blue marble, and three identical yellow marbles. How many different groups of two marbles can Tom choose?

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Model output with Echo-then-Reason (Correct)

<think> The problem asks: "Tom has a red marble, a green marble, a blue marble, and three identical yellow marbles. How many different groups of two marbles can Tom choose?" First, repeat the key details: Tom has one red marble, one green marble, one blue marble, and three yellow marbles that are identical. He wants to form groups of two marbles. To avoid counting identical groups multiple times, consider the distinct colors: red, green, blue, and yellow. There are four distinct colors. Now, let's count the pairs systematically to avoid overcounting: - Red can be paired with green, blue, or yellow, giving 3 pairs. - Green can be paired with blue or yellow (red-green is already counted), giving 2 new pairs. - Blue can be paired with yellow (red-blue and green-blue are already counted), giving 1 new pair. - Two yellow marbles can be paired. Since they are identical, this is 1 more pair. Adding these up: $3 + 2 + 1 + 1 = 7$ pairs. Thus, the number of different groups of two marbles Tom can choose is 7. </think> The answer is 7.

Model output without Echo (Incorrect)

To determine the number of different groups of two marbles that Tom can choose, we need to consider all possible combinations of the marbles he has. Tom has a red marble (R), a green marble (G), a blue marble (B), and three identical yellow marbles (Y). The total number of marbles is 6, but since the yellow marbles are identical, we need to account for this in our combinations. We can use the combination formula $\binom{n}{k}$ to find the number of ways to choose 2 marbles out of 6, but we need to adjust for the identical yellow marbles. Let's break it down: 1. Choose 2 marbles from the 3 yellow marbles: This is not possible since we can't choose 2 identical marbles from a set of identical items. 2. Choose 1 yellow marble and 1 non-yellow marble: We can choose 1 yellow marble in $\binom{3}{1} = 3$ ways, and we can choose 1 non-yellow marble in $\binom{3}{1} = 3$ ways. So, the total number of ways to choose 1 yellow and 1 non-yellow marble is $3 \times 3 = 9$. 3. Choose 2 non-yellow marbles: We can choose 2 non-yellow marbles from the 3 non-yellow marbles (R, G, B) in $\binom{3}{2} = 3$ ways. Adding these up, the total number of different groups of two marbles that Tom can choose is $0 + 9 + 3 = 12$. The number of different groups of two marbles that Tom can choose is 12.

Figure 9: Qwen3-8B-Base model outputs illustrating the Echo-then-Reason pattern. Top: the math question from Hendrycks-MATH dataset. Middle (green): Qwen3-8B-ED-SFT's correct answer. Bottom (red): Qwen3-8B-Base that immediately jumps to calculation without echoing, resulting in the wrong answer.