Productivity and Efficiency Analysis

3) Stochastic frontier analysis (SFA)

b) Basics of SFA

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Taxonomy of methods

based on Kuosmanen & Johnson (2010), Operations Research

		Parametric	Nonparametric	
			Local averaging	**Axiomatic**
Average curve		*OLS* Gauss (1795), Legendre (1805)	*Kernel regression* Nadaraya (1964), Watson (1964)	*Convex regression* Hildreth (1954), Hanson and Pledger (1976)
Frontier	**Deterministic (Sign constr.)**	*Parametric programming* Aigner and Chu (1968)	*Nonparametric programming* Post et al. (2002)	***DEA*** **Farrell (1957),** **Charnes et al. (1978)**
	Deterministic (2-stage)	*Corrected OLS* Winsten (1957) Greene (1980)	*Corrected kernel* Kneip and Simar (1996)	*Corrected CNLS* Kuosmanen and Johnson (2010)
	Stochastic	***SFA*** **Aigner et al. (1977)** **Meeusen and van den Broeck (1977)**	*Semi-nonparametric SFA* Fan, Li and Weersink (1996)	*StoNED* Kuosmanen and Kortelainen (2012)

Stochastic frontier model

$$\ln y_i = \alpha + \sum_{s=1}^{S} \beta_s \ln x_{si} - u_i + v_i, \quad i = 1,\ldots,n$$

where

y_i is output of firm i

β_s is **output elasticity** of input s

x_{si} is input s of firm i

u_i is inefficiency term of firm i

v_i is random noise term of firm i

Aigner, Lovell & Schmidt (1977): Formulation and estimation of stochastic frontier production function models, *Journal of Econometrics*

Meeusen and van den Broeck (1977) Efficiency estimation from Cobb-Douglas production functions with composed error, *International Economic Review*

Battese and Corra (1977) Estimation of a production frontier model: with application to the pastoral zone of Eastern Australia, *Australian Journal of Agricultural Economics*

Stochastic frontier model: common distributional assumptions

Random noise term v_i follows the normal distribution N with zero mean and a constant variance.

Inefficiency term u_i follows

- Half-normal distribution
- Exponential distribution
- Truncated normal distribution
- Gamma distribution

Stochastic frontier model: common distributional assumptions

- Illustration

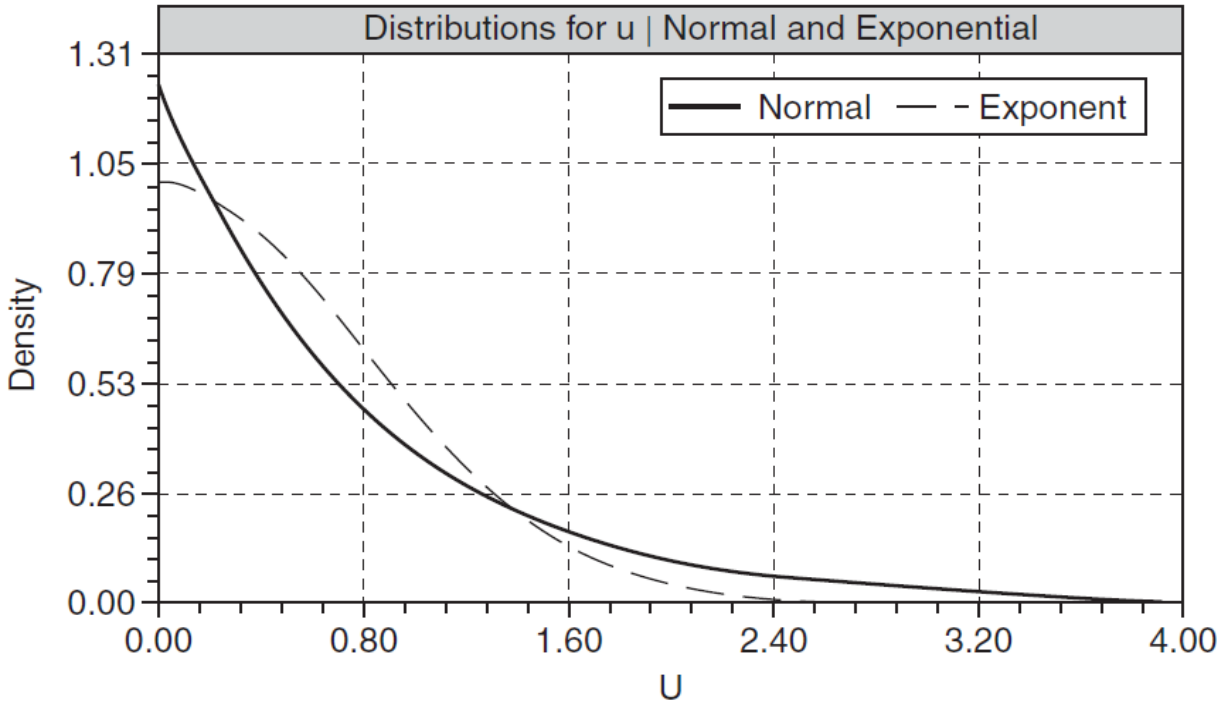


Figure 2.5. Half-Normal and Exponential Distributions

Source: Greene (2008) The Econometric Approach to Efficiency Analysis, in Fried et al. (Eds.) *The Measurement of Productive Efficiency and Productivity Growth*

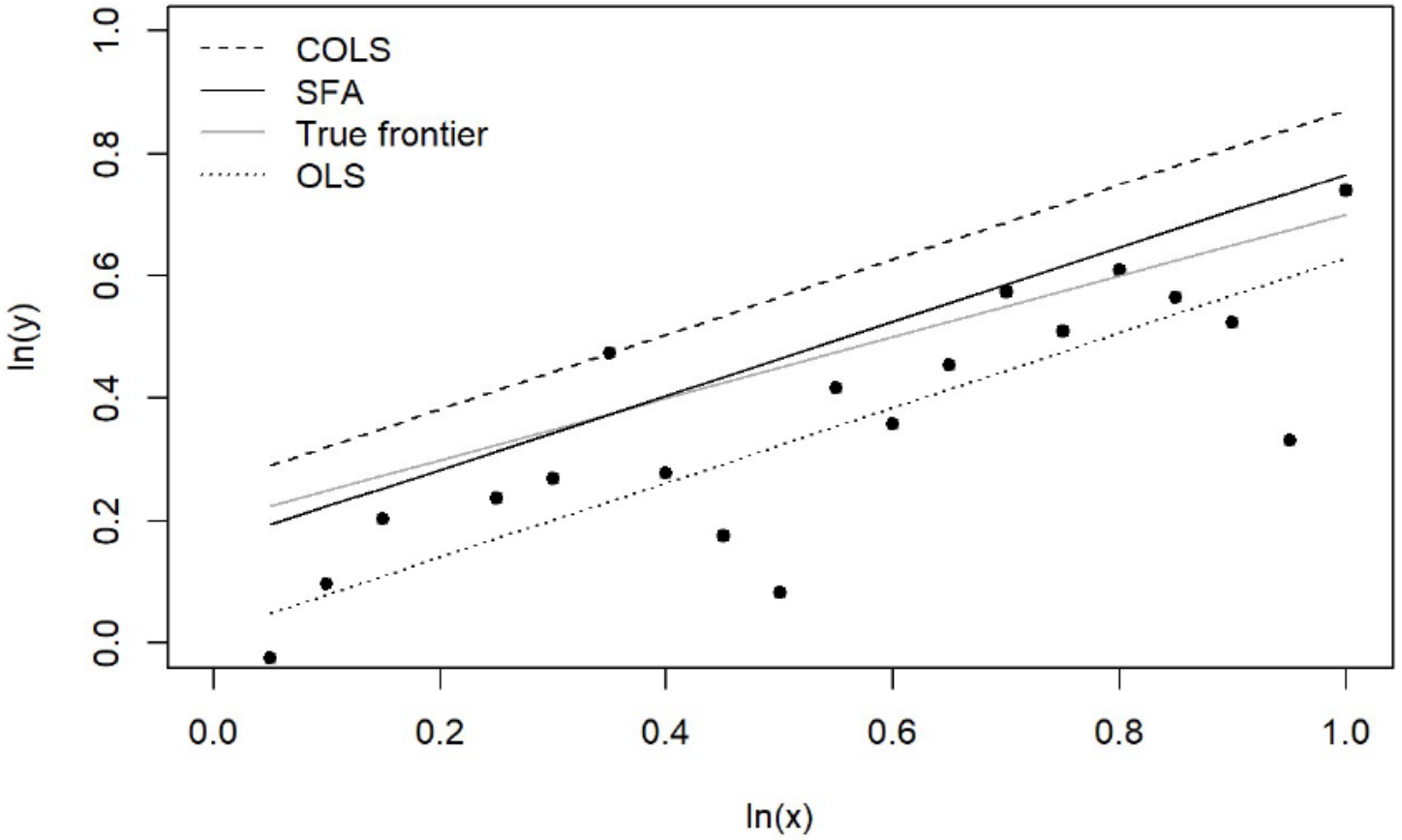
Two ways to estimate the SFA model

Modified OLS (MOLS):

1. Estimate coefficients β by OLS.
2. Estimate parameters of u,v using OLS residuals (method of moments or pseudolikelihood)
3. Adjust the intercept α (similar to COLS)

Maximum likelihood: Estimate all model parameters jointly in one step.

Illustration



Estimating inefficiency

- In the stochastic frontier model, distance to frontier depends on both inefficiency and noise.
- Inefficiency u_i is a random variable. It is impossible to distinguish realization u_i from noise v_i.

Estimating inefficiency

- Illustration

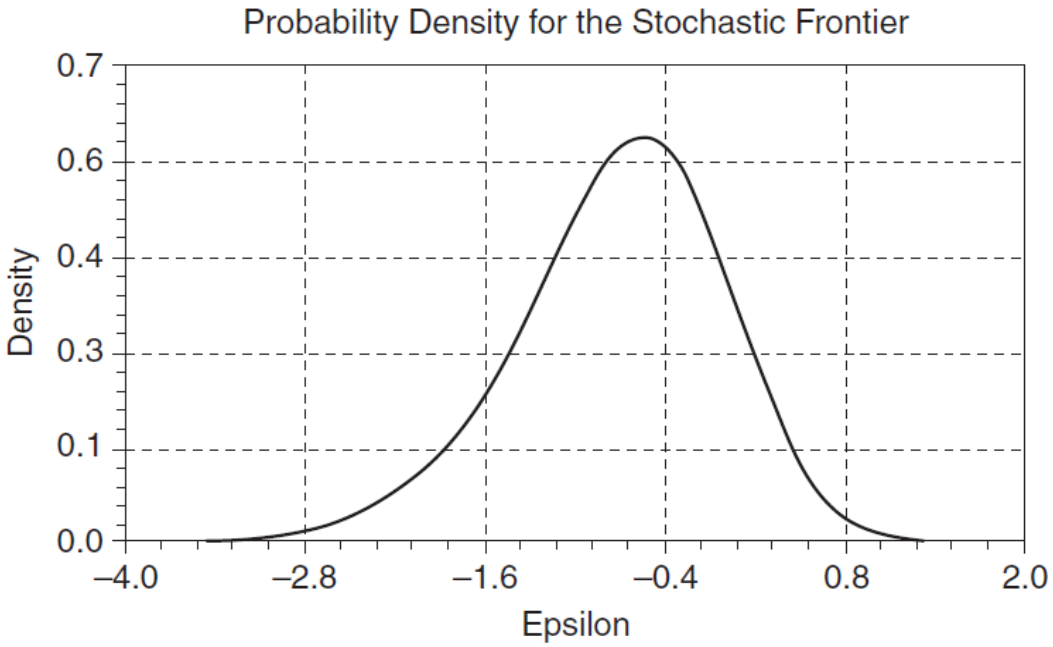


Figure 2.4. Density of a Normal Minus a Half-Normal

Source: Greene (2008) The Econometric Approach to Efficiency Analysis, in Fried et al. (Eds.) *The Measurement of Productive Efficiency and Productivity Growth*

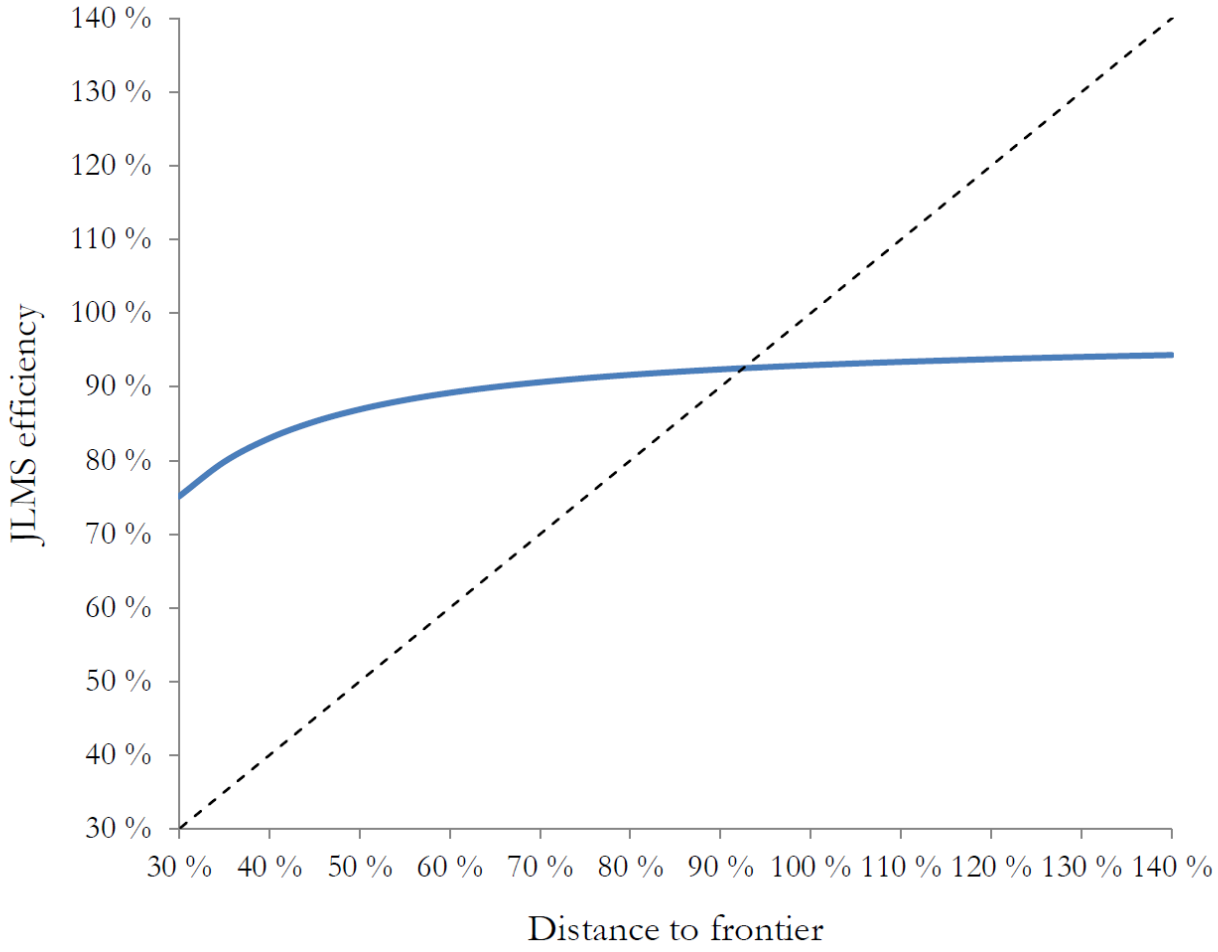
Estimating inefficiency

- The distributional assumptions allow us to estimate the expected value of inefficiency loss $E(u_i)$ based on the skewness of the residuals.
- Firm-specific SFA efficiency estimates are based on the JLMS conditional expectation $E(u_i | e_i)$.

Jondrow, Lovell, Materov and Schmidt (1982) On the estimation of technical inefficiency in the stochastic frontier production function model. *Journal of Econometrics*.

SFA efficiency

Illustration of the JLMS transformation of regression residual to efficiency:



Next lesson

3c) Application of SFA