Productivity and Efficiency Analysis

6) Multiple outputs and bad outputs

6a) Synergies of joint production

Timo Kuosmanen

Aalto University School of Business

https://people.aalto.fi/timo.kuosmanen

Importance of multi-product firms

- Bernard, Redding & Schott (2010) *AER*:

  Data of U.S. Manufacturing Censuses 1972-1997

Prevalence of Firms Producing Multiple Products, Industries and Sectors

Type of Firm	Percent of Firms	Percent of Output	Mean Products, Industries or Sectors per Firm
Single-Product	59	9	1
Multiple-Product	41	91	4.0
Multiple-Industry	29	87	3.1
Multiple-Sector	13	76	2.5

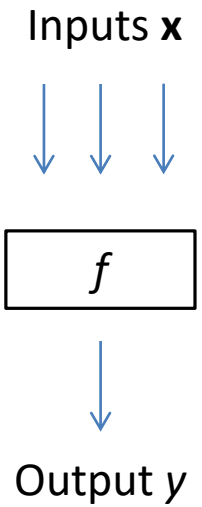
Notes: Table displays a breakdown of firms according to whether they produce multiple products (five-digit SIC categories), multiple industries (four-digit SIC categories) and multiple sectors (two-digit SIC categories). Columns two and three summarize the distribution of firms and output, respectively. Column four reports the mean number of products for multiple-product firms, the mean number of industries for multiple-industry firms and the mean number sectors for multiple-sector firms. Results are based on the pooled 1972 to 1997 sample.

Production function

Classic econometric model of production:

$\ln y = \ln f(\mathbf{x}) + \varepsilon$

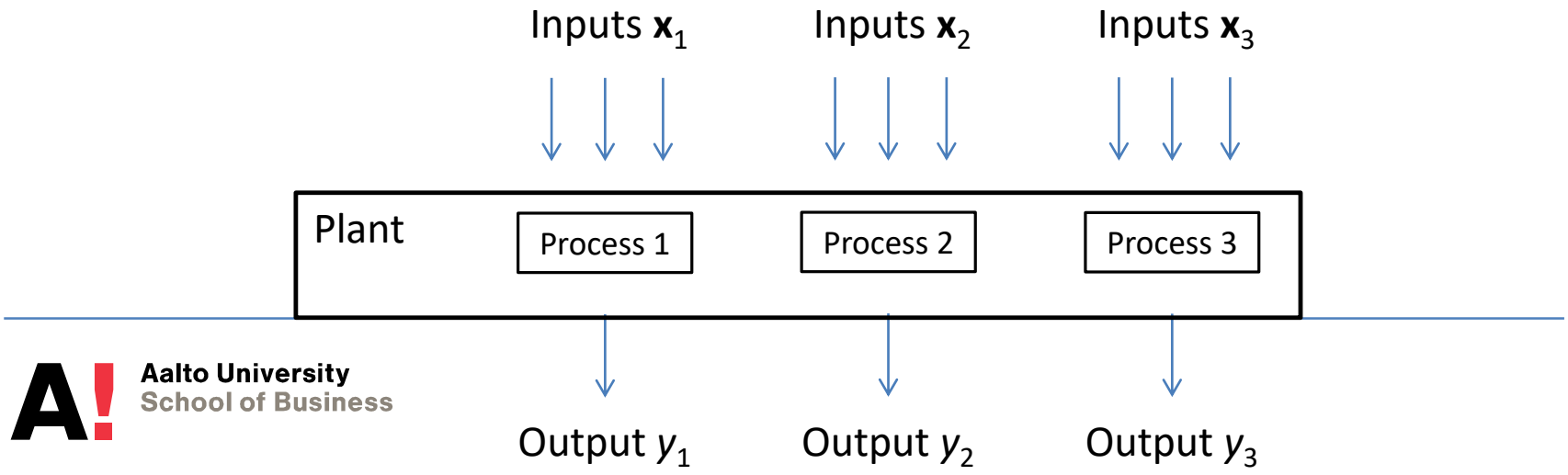
where
y is univariate output
f is production function
$\mathbf{x}$ is vector of inputs
ε is error term (noise – inefficiency)



Modeling multi-product firms as parallel processes

- Foster, Haltiwanger & Syverson (2008) *AER*:

"Because plants' factor inputs are not reported separately by product but rather at the plant level, we must for multi-product plants apportion the share of inputs used to make our product of interest. Operationally, we make this adjustment by dividing the plant's reported output of the product of interest by that product's share of plant sales."



Production functions with “no synergies”

- De Loecker, Goldberg, Khandelwal & Pavcnik (2016) *Econometrica*

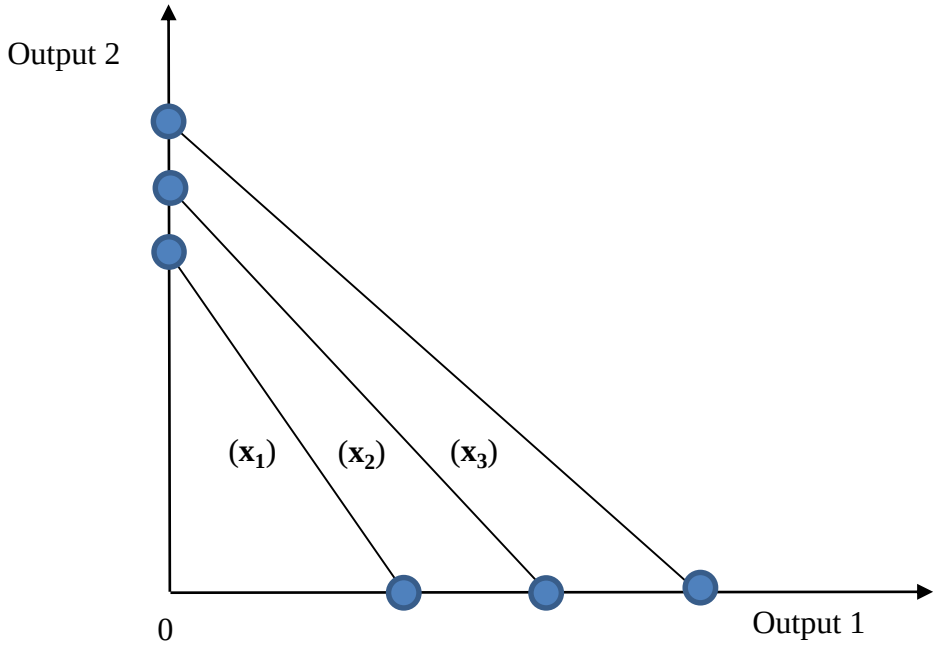
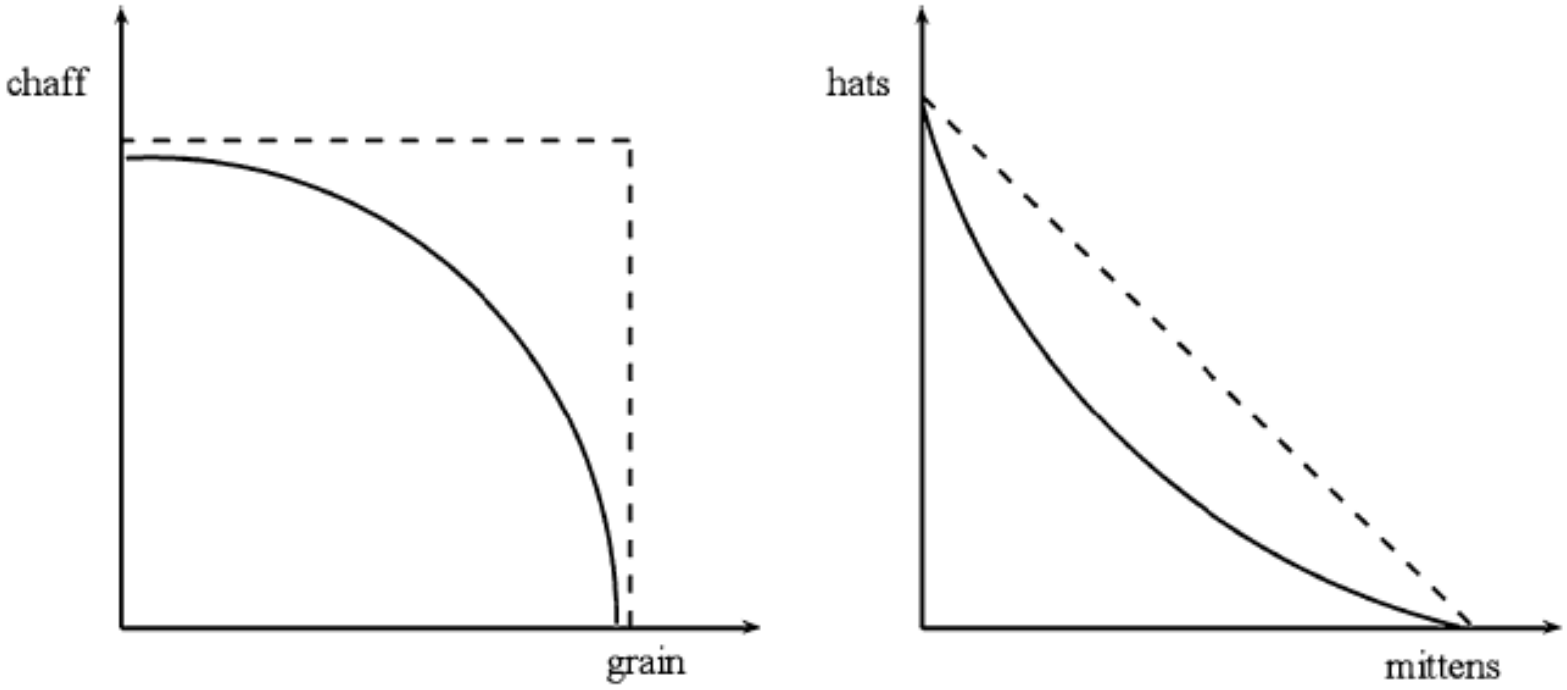


Illustration of output isoquants under the assumption of “no synergies”

Positive vs. negative returns to scope



Source: Pope & Johnson (2013) *JPA*

Production theory: output sets *P*(x)

$$P(\mathbf{x}) = \{\mathbf{q}: \mathbf{x} \text{ can produce } \mathbf{q}\}$$

The properties of the output set can be summarised as follows. For each $\mathbf{x}$, the output set $P(\mathbf{x})$ is assumed to satisfy:

(i) $\mathbf{0} \in P(\mathbf{x})$: nothing can be produced from a given set of inputs (i.e., inaction is possible);

(ii) non-zero output levels cannot be produced from zero levels of inputs;

(iii) $P(\mathbf{x})$ satisfies strong disposability of outputs: if $\mathbf{q} \in P(\mathbf{x})$ and $\mathbf{q}^* \leq \mathbf{q}$ then $\mathbf{q}^* \in P(\mathbf{x})$;[2]

(iv) $P(\mathbf{x})$ satisfies strong disposability of inputs: if $\mathbf{q}$ can be produced from $\mathbf{x}$, then $\mathbf{q}$ can be produced from any $\mathbf{x}^* \geq \mathbf{x}$;[3]

(v) $P(\mathbf{x})$ is closed;

(vi) $P(\mathbf{x})$ is bounded; and

(vii) $P(\mathbf{x})$ is convex.

Source: Coelli, Prasada Rao, O'Donnell & Battese (2005)

Production theory: output sets *P*(x)

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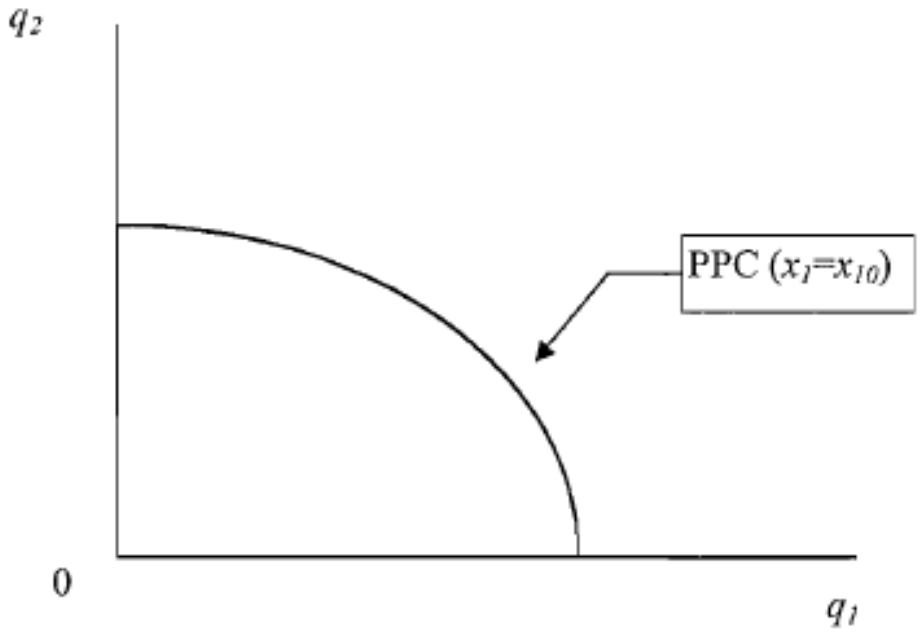

Figure 3.1 Production Possibility Curve

Source: Coelli, Prasada Rao, O'Donnell & Battese (2005)

Next lesson

6b) Distance functions and DEA