Productivity and Efficiency Analysis

7) Productivity growth

c) Application of Malmquist index

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Measurement of productivity growth

- Growth accounting
- Index numbers (Fisher, Törnqvist)

- Malmquist index and its many variants:
  - Shadow-price Fisher (Kuosmanen, Post, Sipiläinen, 2003)
  - Global Malmquist (Pastor and Lovell, 2005)
  - Malmquist-Luenberger using the DDF (Chung, Färe, Grosskopf, 1997)

Decomposing the Malmquist index

$$M_y\left(\mathbf{x}^{0,1},\mathbf{y}^{0,1}\right) = TECHCH \cdot PEFFCH \cdot SCH,$$

where

$$TECHCH \equiv \left(\frac{D_y^{0,CRS}(\mathbf{x}^0,\mathbf{y}^0)}{D_y^{1,CRS}(\mathbf{x}^0,\mathbf{y}^0)} \cdot \frac{D_y^{0,CRS}(\mathbf{x}^1,\mathbf{y}^1)}{D_y^{1,CRS}(\mathbf{x}^1,\mathbf{y}^1)}\right)^{1/2},$$

$$PEFFCH \equiv D_y^1\left(\mathbf{x}^1,\mathbf{y}^1\right)/D_y^0\left(\mathbf{x}^0,\mathbf{y}^0\right),$$

and

$$SCH \equiv \left(\frac{D_y^{1,CRS}(\mathbf{x}^1,\mathbf{y}^1)}{D_y^1(\mathbf{x}^1,\mathbf{y}^1)}\right) \bigg/ \left(\frac{D_y^{0,CRS}(\mathbf{x}^0,\mathbf{y}^0)}{D_y^0(\mathbf{x}^0,\mathbf{y}^0)}\right).$$

Typical DEA approach

- Yearly cross-sections $t = 1,\ldots,T$ modeled separately: sample of observations in period t represents the technology in period t
- Values of the *inter-period* distance function $D^t(\mathbf{x}^t, \mathbf{y}^t)$ are straightforward to calculate as DEA efficiency using data of period t
- This is enough for measuring efficiency change as

  $$\text{EFFCH} = D^{t+1}(\mathbf{x}^{t+1}, \mathbf{y}^{t+1}) \,/\, D^t(\mathbf{x}^t, \mathbf{y}^t)$$

- Scale efficiency change involves comparison of the CRS, VRS efficiency scores

Typical DEA approach

- Yearly cross-sections $t = 1,\ldots,T$

- To measure technical progress, we also need *intra-period* $D^{t-1}(\mathbf{x}^t, \mathbf{y}^t)$ and $D^{t+1}(\mathbf{x}^t, \mathbf{y}^t)$
- Example of DEA formulation (CRS, output orientation)

$$
\begin{aligned}
&\max \theta \\
&s.t. \\
&\mathbf{x}_i^t \geq \mathbf{X}^{t+1}\boldsymbol{\lambda} \\
&\theta \mathbf{y}_i^t \leq \mathbf{Y}^{t+1}\boldsymbol{\lambda} \\
&\boldsymbol{\lambda} \geq \mathbf{0}
\end{aligned}
$$

Typical DEA approach

- Yearly cross-sections $t = 1,\ldots,T$
- In some applications technical change is negative, or fluctuates heavily over time.
  - This is because yearly cross-sections are modeled independently, and DEA is sensitive to noise and outliers,
- Is it reasonable to assume technologies are “forgotten”? And reinvented again? In a period of one year?

Extended DEA approaches

- *Global Malmquist index*: pool all *T* cross-sections together, and measure distance relative to the "global" frontier
- Example

$$\max \theta$$

$$s.t.$$

$$\mathbf{x}_i^t \geq \mathbf{X}^{Pooled} \boldsymbol{\lambda}$$

$$\theta \mathbf{y}_i^t \leq \mathbf{Y}^{Pooled} \boldsymbol{\lambda}$$

$$\boldsymbol{\lambda} \geq \mathbf{0}$$

Parametric approach

- Similar to Solow (1957), consider the production model with Hicks neutral technical change

$$\ln y = \ln A(t) + \ln f(K,L) + \varepsilon$$

- Technical change can be estimated by introducing a linear (or polynomial) time trend. In the case of Cobb-Douglas production function

$$\ln y = \beta_0 + \beta_K \ln K + \beta_L \ln L + \alpha \cdot t + \varepsilon$$

Parametric Malmquist decomposition

$$\ln y - \ln f(K,L) = \ln A(t) + \varepsilon$$

$$y \,/\, f(K,L) = A(t)\cdot\exp(\varepsilon)$$

- Technical change

$$\ln A(t+1) - \ln A(t+1) \;=\; \alpha \quad \approx [A(t+1) - A(t)] / A(t)$$

$$A(t+1) \,/\, A(t) \;=\; \exp(\alpha)$$

- Efficiency change based on residuals e

$$e^{t+1} \,/\, e^{t}$$

- Productivity change = $\exp(\alpha) \cdot e^{t+1} / e^{t}$

Productivity estimation in StoNED

Two alternative approaches

- Estimate yearly cross-sections independently as in DEA (Cheng, Bjorndal, Bjorndal, 2015)
- Estimate a pooled model with technical change modeled using a parametric time trend (Kuosmanen, 2013)

Productivity estimation in StoNED

Two approaches

- Estimate yearly cross-sections independently as in DEA (Cheng, Bjorndal, Bjorndal, 2015)

- Cheng et al. show that distributional assumptions about the inefficiency term do not affect the estimated productivity change (Malmquist index), but do influence the decomposition to the subcomponents of technical change (frontier shift) and efficiency change (catching up)

Next lesson

7d) Empirical case: Green productivity growth