Productivity and Efficiency Analysis

7) Productivity growth

a) Growth accounting and index numbers

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Productivity and efficiency: basic concepts

Productivity *growth* depends on

- **Technical progress (Theme 7)**

- **Efficiency improvement**
  - Technical efficiency
  - Scale efficiency
  - Allocative efficiency

- **Structural change (Theme 8)**
  - Entry and exit of units
  - Reallocation of resources between units

Productivity and economic growth

Solow (1957) model:

- Two inputs, capital *K*, labor *L*, output quantity *Q*
- Hicks neutral technical change A(t)

  $Q = A(t) \cdot f(K,L)$
- Differentiating with respect to time *t*, we have growth decomposition

$$\frac{\dot{Q}}{Q} = \frac{\dot{A}}{A} + w_K \frac{\dot{K}}{K} + w_L \frac{\dot{L}}{L}.$$

$$w_k = \frac{\partial Q}{\partial K} \frac{K}{Q} \text{ and } w_L = \frac{\partial Q}{\partial L} \frac{L}{Q}$$

Growth accounting

Solow (1957) model:

- If K and L are paid their marginal products (competitive markets), then w_k and w_L are equal to the relative cost shares of *K* and *L.*
- Thus

$$\frac{\dot{Q}}{Q} = \frac{\dot{A}}{A} + w_K \frac{\dot{K}}{K} + w_L \frac{\dot{L}}{L}.$$

Real GDP growth
= Productivity growth + K growth + L growth

$\Rightarrow$ Productivity growth
= Real GDP growth – K growth – L growth

Index theory

Consider the growth of *M* outputs *y* from the base period by *s* to target period by *t*. Prices *p*

The Paasche (1874), Laspeyres (1871), and Fisher (1922, p. 234) output quantity indexes can be defined as follows using the quantity aggregates given in (10)–(13):

$$Q_P \equiv \sum_{i=1}^{M} p_i^t y_i^t \Big/ \sum_{j=1}^{M} p_j^t y_j^s, \tag{14}$$

$$Q_L \equiv \sum_{i=1}^{M} p_i^s y_i^t \Big/ \sum_{j=1}^{M} p_j^s y_j^s, \tag{15}$$

and

$$Q_F \equiv (Q_P Q_L)^{(1/2)}. \tag{16}$$

$$Q_P \times P_L = Q_L \times P_P = Q_F \times P_F = R^t / R^s.$$

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Source: Diewert and Nakamura (2003) *JPA*

Index theory

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$$Q_P \times P_L = Q_L \times P_P = Q_F \times P_F = R^t / R^s.$$

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Index theory

TFP index is the ratio of **output quantity index** and **input quantity index**. Denoting the base period by 0 and target period by 1, the **Fisher TFP index** can be stated as

$$F_{TFP}(\mathbf{p}^{0,1}, \mathbf{w}^{0,1}, \mathbf{y}^{0,1}, \mathbf{x}^{0,1}) \equiv \frac{F_y(\mathbf{p}^{0,1}, \mathbf{y}^{0,1})}{F_x(\mathbf{w}^{0,1}, \mathbf{x}^{0,1})},$$

where

$$F_y(\mathbf{p}^{0,1}, \mathbf{y}^{0,1}) \equiv \left(\frac{\mathbf{p}^0 \cdot \mathbf{y}^1}{\mathbf{p}^0 \cdot \mathbf{y}^0} \cdot \frac{\mathbf{p}^1 \cdot \mathbf{y}^1}{\mathbf{p}^1 \cdot \mathbf{y}^0} \right)^{1/2}$$

and

$$F_x(\mathbf{w}^{0,1}, \mathbf{x}^{0,1}) \equiv \left(\frac{\mathbf{w}^0 \cdot \mathbf{x}^1}{\mathbf{w}^0 \cdot \mathbf{x}^0} \cdot \frac{\mathbf{w}^1 \cdot \mathbf{x}^1}{\mathbf{w}^1 \cdot \mathbf{x}^0} \right)^{1/2}$$

Source: Kuosmanen and Sipiläinen (2009) *JPA*

Index theory

Törnqvist (1936) indexes are weighted geometric averages of growth rates for the microeconomic data (the quantity or price relatives). These indexes appear more complicated than the others defined so far, but are also widely used. It is the formula for the natural logarithm of a Törnqvist index that is usually shown. For the output quantity index, this is

$$\ell n\, Q_T = (1/2)\sum_{m=1}^{M}\left[\left(p_m^s y_m^s \Big/ \sum_{i=1}^{M} p_i^s y_i^s\right) + \left(p_m^t y_m^t \Big/ \sum_{j=1}^{M} p_j^t y_j^t\right)\right] \boxed{\ell n\left(y_m^t / y_m^s\right).} \quad (39)$$

Törnqvist, L. (1936). "The Bank of Finland's Consumption Price Index." *Bank of Finland Monthly Bulletin* 10, 1–8.

Source: Diewert and Nakamura (2003) *JPA*

Index theory

- Growth accounting and TFP indices remain widely used, particularly at macro level (countries).
- *No estimation needed!*

- Requires prices of all inputs and outputs
- Some known difficulties with prices:
  - Opportunity cost of capital is notoriously hard to measure
  - Quality change (e.g., ICT capital, labor skills)
  - New goods and obsolete goods
  - Nonmarket goods (e.g., environmental bads, public goods)
  - Imperfect competition (e.g., natural monopolies)
  - Government interventions (e.g., taxes, subsidies, tariffs)

Next lesson

7b) Malmquist index