Productivity and Efficiency Analysis

4) Unified approach: StoNED

b) StoNED estimation

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Taxonomy of methods

based on Kuosmanen & Johnson (2010), Operations Research

		Parametric	Nonparametric: Local averaging	Nonparametric: Axiomatic
Average curve		*OLS* Gauss (1795), Legendre (1805)	*Kernel regression* Nadaraya (1964), Watson (1964)	*Convex regression* Hildreth (1954), Hanson and Pledger (1976)
Frontier	**Deterministic (Sign constr.)**	*Parametric programming* Aigner and Chu (1968)	*Nonparametric programming* Post et al. (2002)	***DEA*** **Farrell (1957),** **Charnes et al. (1978)**
	Deterministic (2-stage)	*Corrected OLS* Winsten (1957) Greene (1980)	*Corrected kernel* Kneip and Simar (1996)	*Corrected CNLS* Kuosmanen and Johnson (2010)
	Stochastic	***SFA*** **Aigner et al. (1977)** **Meeusen and van den Broeck (1977)**	*Semi-nonparametric SFA* Fan, Li and Weersink (1996)	***StoNED*** **Kuosmanen and Kortelainen (2012)**

Unified frontier model

$$y_i = f(\mathbf{x}_i) - u_i + v_i, \quad i = 1,\ldots,n$$

where

y_i is output of firm i

f is frontier production function

$\mathbf{x}_i$ is input vector of firm i

u_i is asymmetric inefficiency term of firm i

v_i is random noise term of firm i

Kuosmanen, Johnson & Saastamoinen (2014) Stochastic nonparametric approach to efficiency analysis: A Unified Framework, in J. Zhu (Ed) *Handbook on DEA Vol. 2*, Springer.

Conditional mean

$$y_i = f(\mathbf{x}_i) - u_i + v_i$$

$$E(y_i|\mathbf{x}_i) = f(\mathbf{x}_i) - E(u_i)$$

$$= f(\mathbf{x}_i) - \mu$$

$$= g(\mathbf{x}_i)$$

- If the frontier production function f is monotonic increasing and concave (+CRS), then so is the average-practice production function g.
- Function g can be consistently estimated by convex regression.

Stepwise StoNED estimation

1) Estimate the average-practice production function g by convex nonparametric least squares (CNLS).

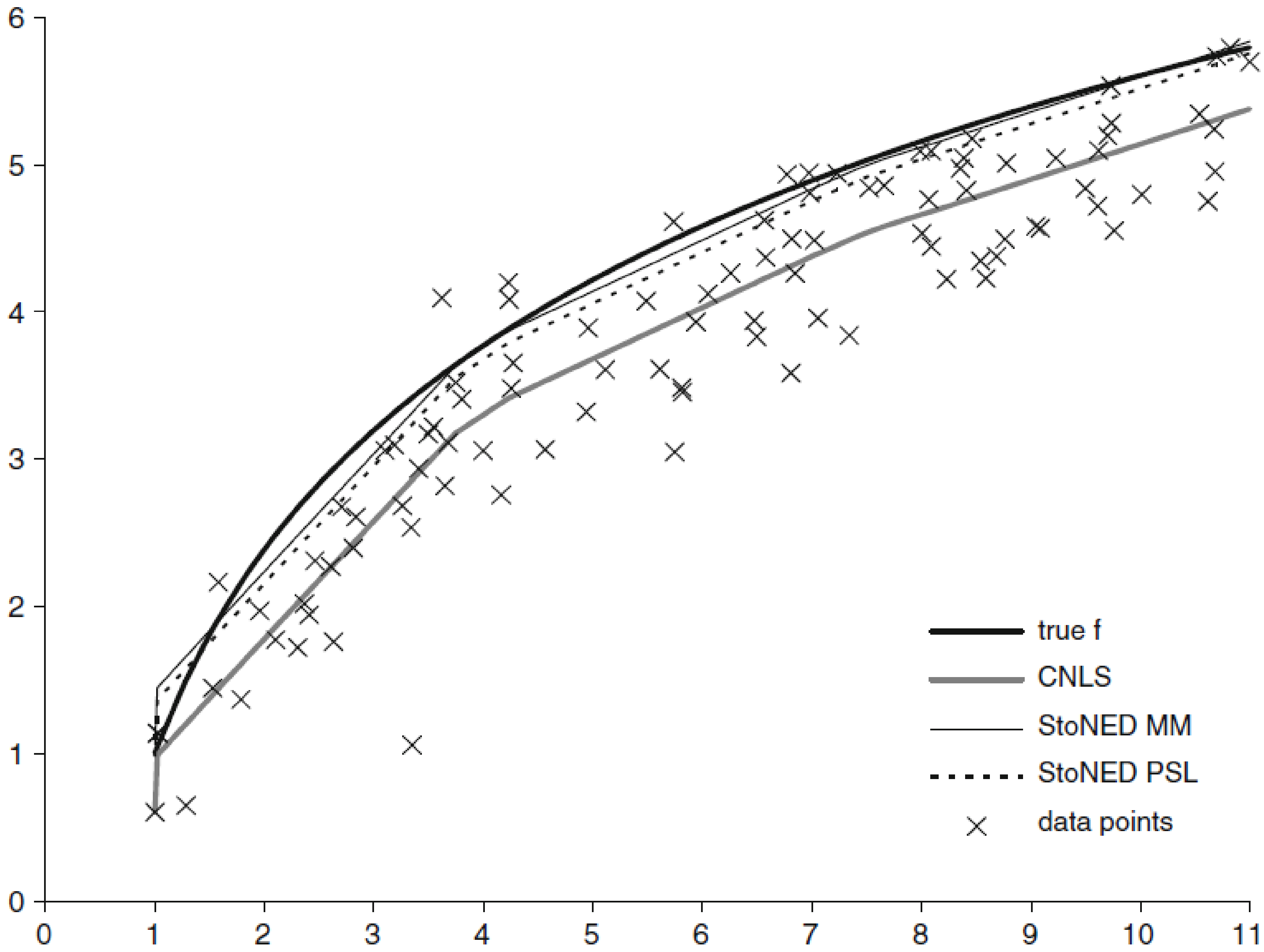
2) Estimate the expected inefficiency E(u) based on the CNLS residuals, assuming u is half-normally or exponentially distributed.

3) Estimate the frontier production function as

$$\hat{f}^{StoNED}(\mathbf{x}) = \hat{g}_{\min}^{CNLS}(\mathbf{x}) + \hat{\mu}$$

4) Apply JLMS formula to estimate firm-specific efficiency.

StoNED estimation: Illustration



What if there is heteroscedasticity?

In step 1, we estimate the conditional mean

$$g(\mathbf{x}_i) = E(y_i \,|\mathbf{x}_i) = f(\mathbf{x}_i) - E(u_i \,|\mathbf{x}_i).$$

Note: severe heteroscedasticity can violate concavity.

In step 2, we apply some SFA parametrization that allows for heteroscedasticity of u with respect to $\mathbf{x}$.

In step 3, we estimate the frontier as

$$\hat{f}^{StoNED}(\mathbf{x}_i) = \hat{g}^{CNLS}(\mathbf{x}_i) + E(u_i \,\big|\mathbf{x}_i, \hat{\mu}_i\ , \hat{\sigma}_{u,i}).$$

Sequential estimation approach

CNLS

↓

SFA

↓

DEA

↓

Frontier, efficiency scores, targets, benchmarking, RTS, etc.

Unified frontier approach

- **Modelling phase**
  - Specify inputs x, outputs y, contextual variables z
  - Shape constraints: monotonicity, convexity, RTS
  - Parametric assumptions

- **Estimation phase**

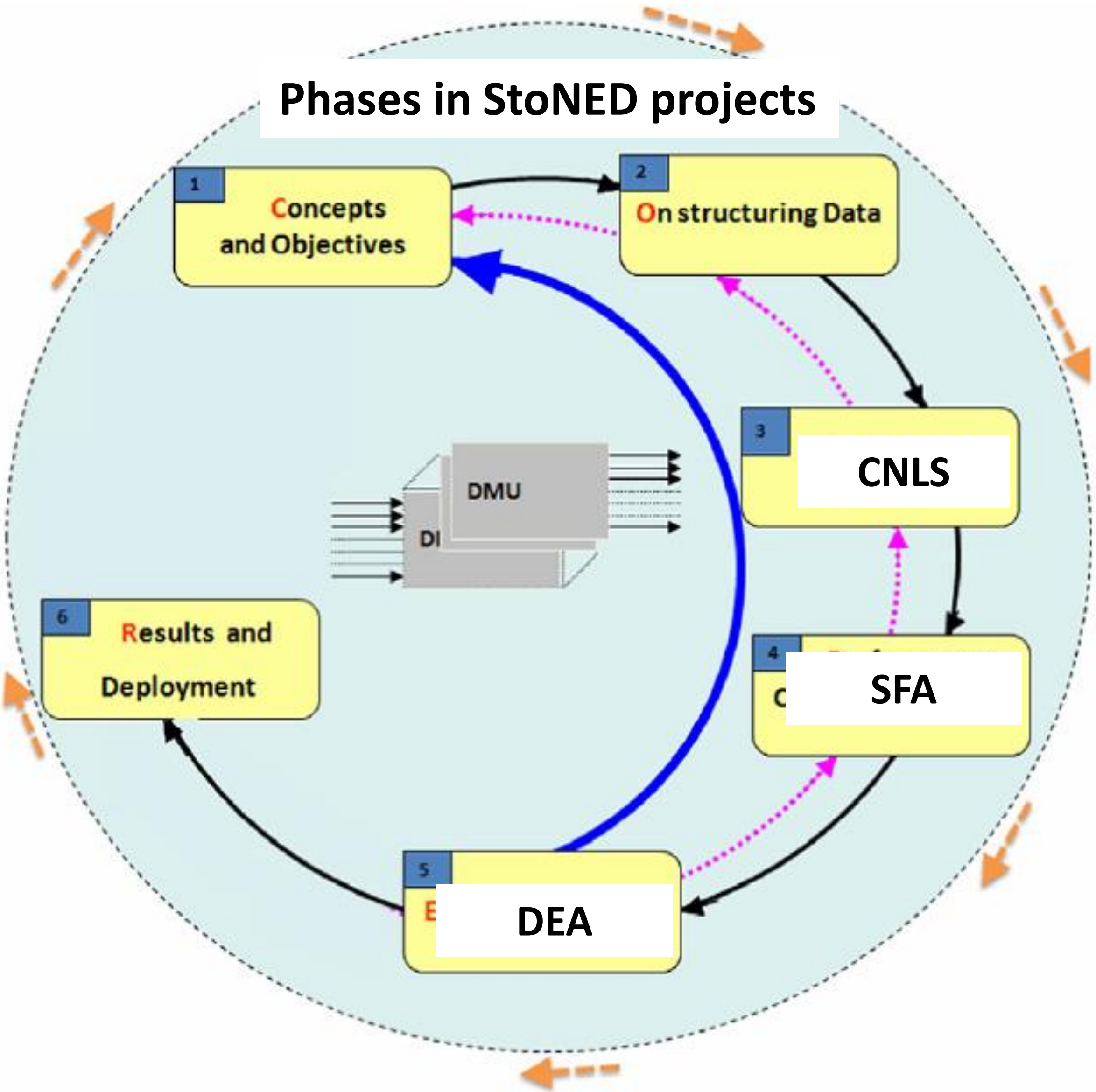
  1) Convex regression => conditional mean

  2) SFA analysis on residuals => expected inefficiency

- **Efficiency analysis phase**

  3) DEA analysis on fitted (x*, y*)
  - Alternative efficiency metrics
  - Benchmarking
  - Performance targets
  - RTS, scale elasticity

StoNED as a process innovation



Adapted from the COOPER framework by Emrouznejad & De Witte (2010) *EJOR*

Next lesson

4c) Convex regression