Productivity and Efficiency Analysis

7) Productivity growth

b) Malmquist index

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Index theory

- Requires prices of all inputs and outputs
- Some problems with prices:
  - Opportunity cost of capital
  - Quality change (e.g., ICT capital, labor skills)
  - New goods and obsolete goods
  - Nonmarket goods (e.g., environmental bads, public goods)
  - Imperfect competition (e.g., natural monopolies)
  - Government interventions (e.g., taxes, subsidies, tariffs)

- When reliable price information is not available, we can use **shadow prices** (i.e., marginal rates of substitution; multiplier weights **β,γ**)

Shadow-price Fisher index

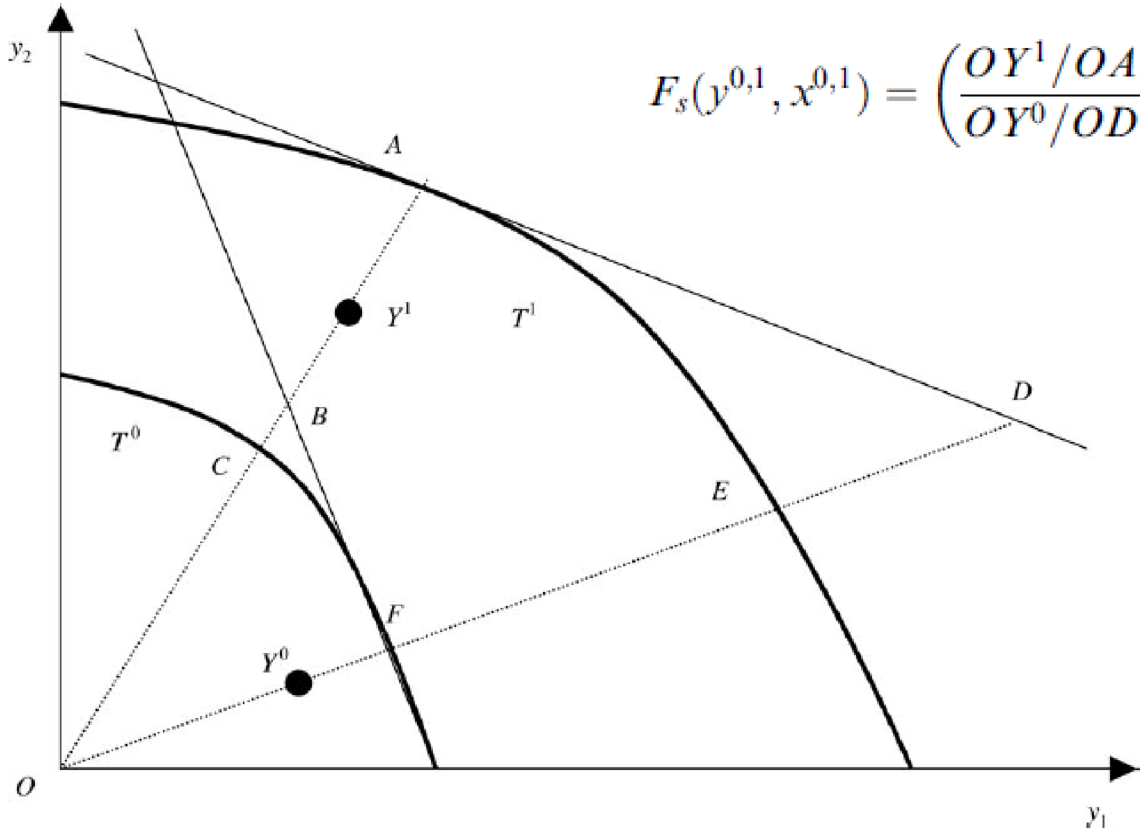
Recall that the Fisher TFP index can be stated as'

$$F_{\mathrm{TFP}}(p^{0,1},w^{0,1},y^{0,1},x^{0,1}) \equiv \frac{F_o(p^{0,1},y^{0,1})}{F_i(w^{0,1},x^{0,1})} = \prod_{j=0}^{1}\prod_{t=0}^{1}\left(\frac{p^j y^t}{w^j x^t}\right)^{t-1/2}.$$

Kuosmanen, Post and Sipiläinen (2003) propose the shadow-price Fisher index that does not require price data:

$$F_s(y^{0,1},x^{0,1}) \equiv \prod_{j=0}^{1}\prod_{t=0}^{1}\left(\tilde{D}^j(y^t,x^t)\right)^{t-1/2},$$

Shadow-price Fisher index: illustration with 2 outputs



$$F_s(y^{0,1}, x^{0,1}) = \left(\frac{OY^1/OA}{OY^0/OD} \frac{OY^1/OB}{OY^0/OF} \right)^{1/2}.$$

Figure 1. Illustration of the shadow-price Fisher index.

Source: Kuosmanen, Post, Sipiläinen (2003) *JPA*

Shadow-price Fisher index

Kuosmanen, Post and Sipiläinen (2003)

THEOREM 2 ("The 2nd equivalence theorem"): *The following conditions are equivalent if the shadow price vectors determine unique relative prices:*

1. *Production vector* $(y^t, x^t), t \in \{0, 1\}$ *is allocatively efficient with respect to prices* (p^t, w^t) *and technology* T^t.

2. *The shadow-price Fisher index* $F_s(y^{0,1}, x^{0,1})$ *and the Fisher ideal index* $F_{\mathrm{TFP}}(p^{0,1}, w^{0,1}, y^{0,1}, x^{0,1})$ *are identical.*

If the two indices differ, which prices are more reliable: market prices or shadow prices?

Malmquist index

- Introduced by Caves, Christensen & Diewert (1982), *Econometrica*
- Can be stated in terms of Shephard's output distance functions (relative to CRS benchmark) as

$$M_y\left(\mathbf{x}^{0,1},\mathbf{y}^{0,1}\right) \equiv \left(\frac{D_y^{0,CRS}\left(\mathbf{x}^1,\mathbf{y}^1\right)}{D_y^{0,CRS}\left(\mathbf{x}^0,\mathbf{y}^0\right)}\cdot\frac{D_y^{1,CRS}\left(\mathbf{x}^1,\mathbf{y}^1\right)}{D_y^{1,CRS}\left(\mathbf{x}^0,\mathbf{y}^0\right)}\right)^{1/2},$$

- Remark: why this is not "Shephard index"?

Malmquist vs Shadow-price Fisher indices: illustration

$$F_s(y^{0,1}, x^{0,1}) = \left(\frac{OY^1/OA}{OY^0/OD}\,\frac{OY^1/OB}{OY^0/OF}\right)^{1/2}.$$

$$M(y^{0,1}, x^{0,1}) = \left(\frac{OY^1/OA}{OY^0/OE}\,\frac{OY^1/OC}{OY^0/OF}\right)^{1/2}.$$

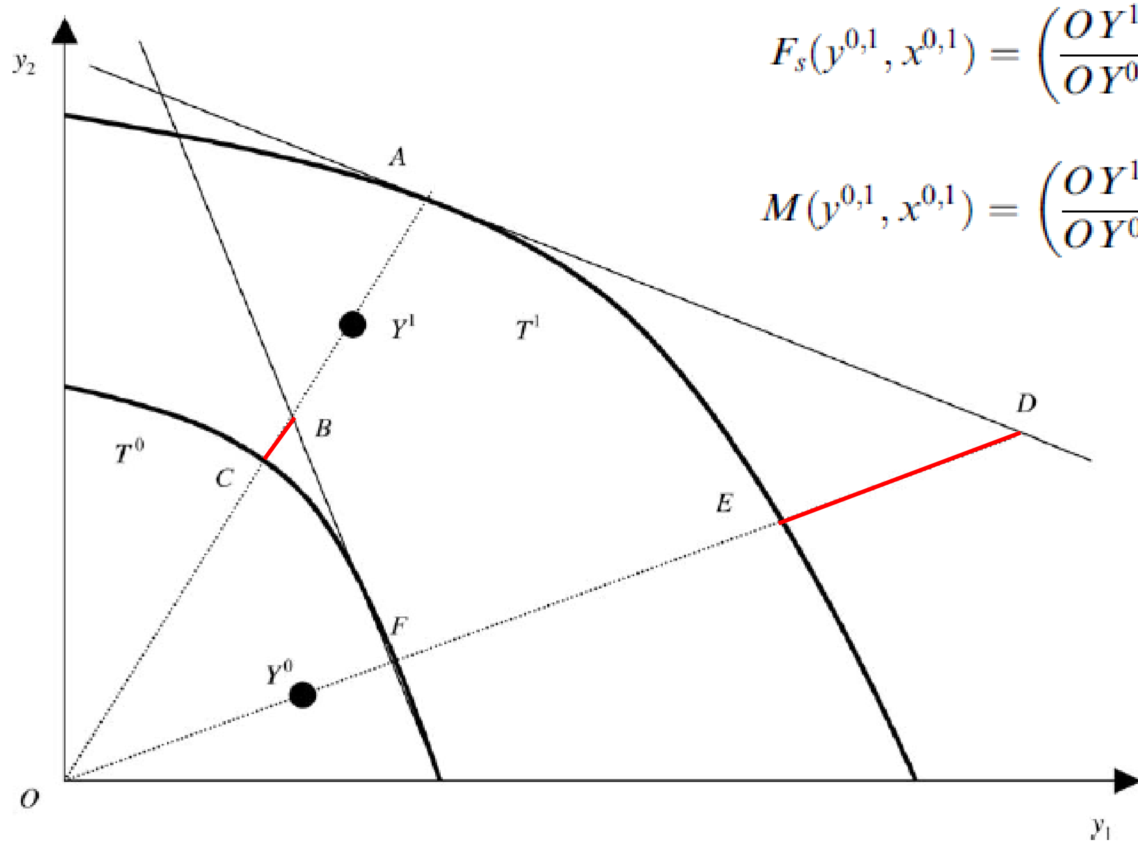


Figure 1. Illustration of the shadow-price Fisher index.

Source: Kuosmanen, Post, Sipiläinen (2003) *JPA*

Decomposing the Malmquist index

- Nishimizu and Page (1982), *Econ. J.*
- **Färe, Grosskopf, Norris, Zhang (1994) *AER***

$$M_y(\mathbf{x}^{0,1}, \mathbf{y}^{0,1}) = TECHCH \cdot PEFFCH \cdot SCH,$$

where

$$TECHCH \equiv \left(\frac{D_y^{0,CRS}(\mathbf{x}^0, \mathbf{y}^0)}{D_y^{1,CRS}(\mathbf{x}^0, \mathbf{y}^0)} \cdot \frac{D_y^{0,CRS}(\mathbf{x}^1, \mathbf{y}^1)}{D_y^{1,CRS}(\mathbf{x}^1, \mathbf{y}^1)} \right)^{1/2},$$

$$PEFFCH \equiv D_y^1(\mathbf{x}^1, \mathbf{y}^1) / D_y^0(\mathbf{x}^0, \mathbf{y}^0),$$

and

$$SCH \equiv \left(\frac{D_y^{1,CRS}(\mathbf{x}^1, \mathbf{y}^1)}{D_y^1(\mathbf{x}^1, \mathbf{y}^1)} \right) \Big/ \left(\frac{D_y^{0,CRS}(\mathbf{x}^0, \mathbf{y}^0)}{D_y^0(\mathbf{x}^0, \mathbf{y}^0)} \right).$$

Efficiency change:

(OA/OY1)/(OF/OY0)

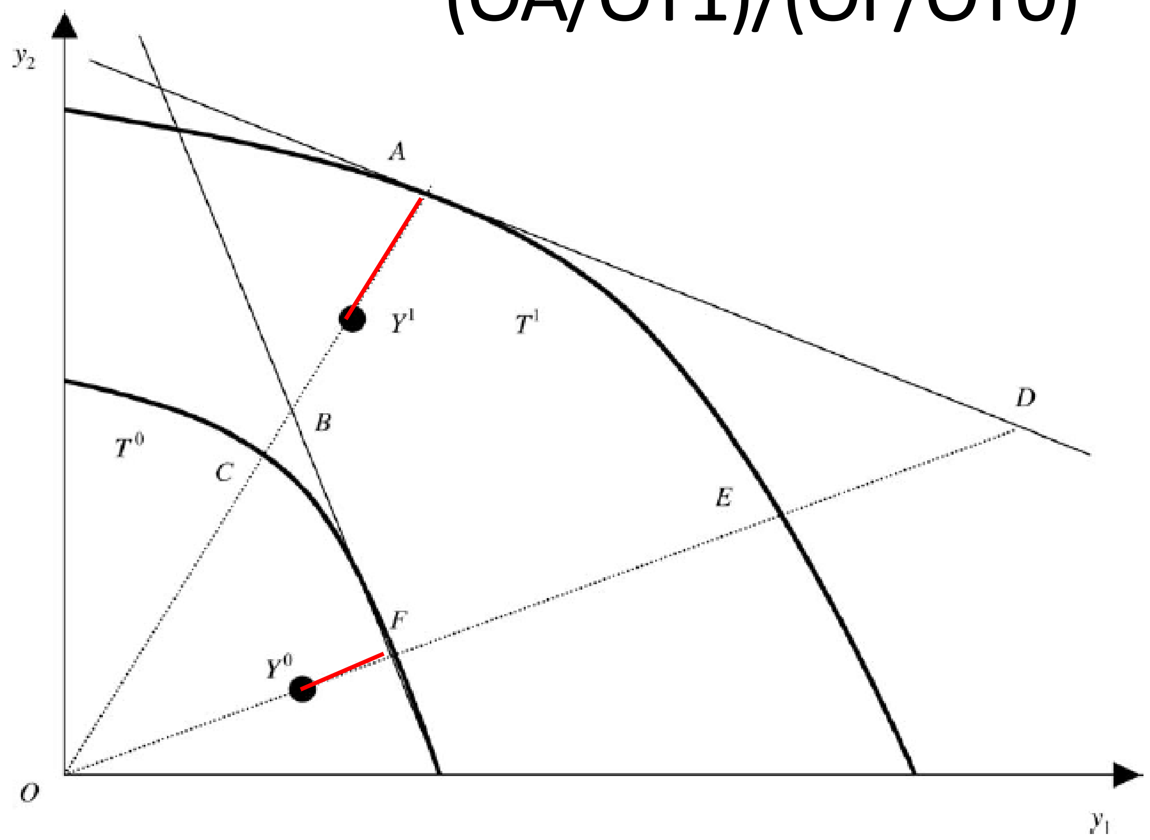


Figure 1. Illustration of the shadow-price Fisher index.

Technical change:

$$[(OA/OC)\cdot(OE/OF)]^{1/2}$$

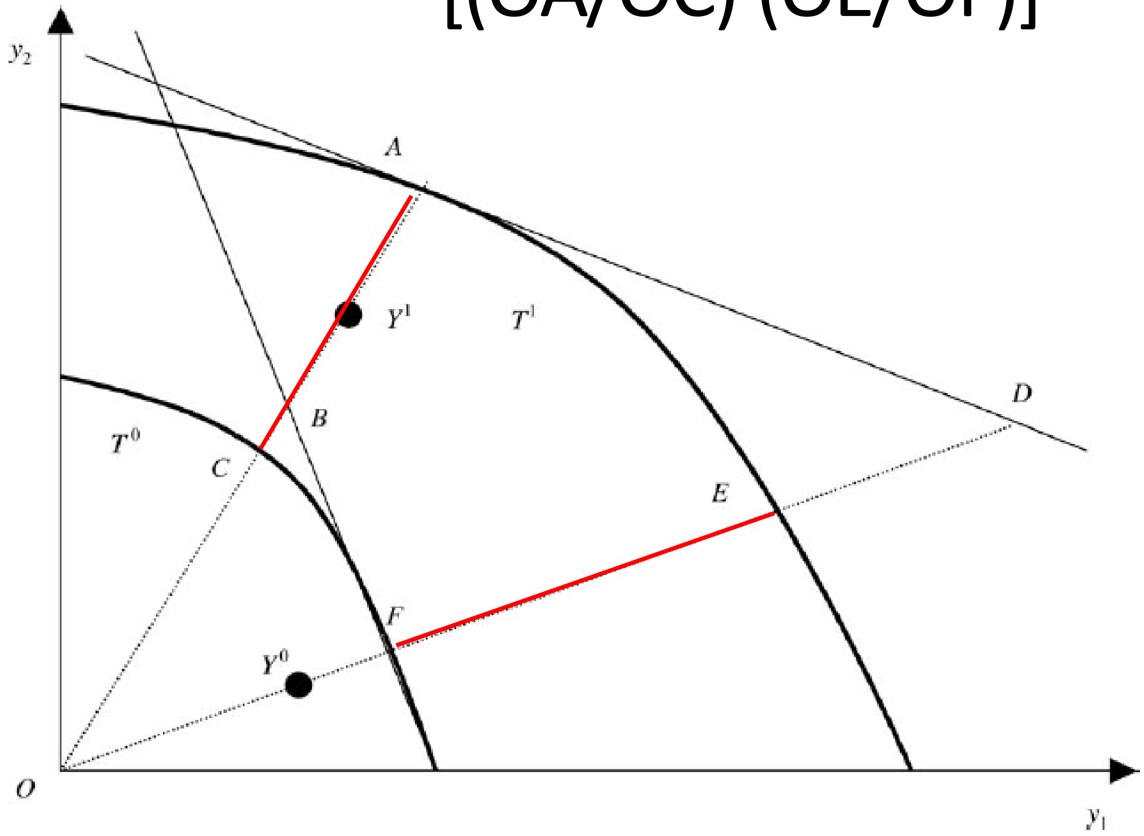


Figure 1. Illustration of the shadow-price Fisher index.

Scale efficiency change

- First introduced by Färe et al. (1994) *AER*
- Ray and Desli (1997) critique:
  - Why technical change is measured using CRS technology?
  - Propose alternative decomposition based on VRS technology, but then other components become less intuitive
- Several authors have further proposed their own solutions to this decomposition dilemma

Decomposition of the Fisher index

Proposition 2 *The Fisher ideal TFP index is the product of the technical efficiency ($\Delta TEff$), technical change ($\Delta Tech$), scale efficiency ($\Delta SEff$), allocative efficiency ($\Delta AEff$), and the price effect (ΔPE) components:*

$$F_{TFP} = \Delta TEff \cdot \Delta Tech \cdot \Delta SEff \cdot \Delta AEff \cdot \Delta PE, \tag{21}$$

where

$$\Delta TEff \equiv (\Delta ITEff \cdot \Delta OTEff)^{1/2} \tag{22}$$

$$\Delta ITEff \equiv D_x^1(\mathbf{x}^1, \mathbf{y}^1)/D_x^0(\mathbf{x}^0, \mathbf{y}^0) \tag{22a}$$

$$\Delta OTEff \equiv D_y^1(\mathbf{x}^1, \mathbf{y}^1)\Big/D_y^0(\mathbf{x}^0, \mathbf{y}^0) \tag{22b}$$

Source: Kuosmanen & Sipiläinen (2009) *JPA*

Next lesson

7c) Application of Malmquist index