Productivity and Efficiency Analysis

5) Contextual variables

a) What are z-variables and where to put them?

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What are contextual variables z?

- Explanatory variables that influence firm performance, but are not inputs or outputs as such.
- Often contextual variables are binary dummy variables {0,1} or percentages (%).

What are contextual variables z?

- Contextual variables are often (but not always) **external factors** that are beyond the control of firms
  - Examples: competition, regulation, weather, location
  - Need to adjust efficiency estimates for operating environment
  - Policy makers may influence the operating environment

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- Contextual variables are often (but not always) **external factors** that are beyond the control of firms
  - Examples: competition, regulation, weather, location
  - Need to adjust efficiency estimates for operating environment
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- Contextual variables can also be **internal factors**
  - Examples: management practices, ownership
  - Better understanding of the impacts of internal factors can help the firm to improve performance

Contextual variables and incentives

- In the application to electricity distribution firms, internal factors such as outsourcing or ownership may influence efficiency, but such factors are irrelevant for incentive regulation.
- Since the firms cannot relocate, there is a need to adjust for the external operating environment:
  - In the 3rd regulation period, the z-variable was the % of underground cabled middle-voltage network.
  - In the 4th and 5th regulation periods, the z-variable is the % of connection points per user.
- Firms can influence underground cabling, but they cannot affect the connections per user.

Where to put the z's?

Four basic approaches:

- In the production function f (technology)
- In the inefficiency term u
- In the noise term (heteroscedasticity)
- Model **z**'s separately

Contextual variables as inputs

$$y_i = f(\mathbf{x}_i, \mathbf{z}_i) - u_i + v_i$$

This is the classic approach in DEA (Banker and Morey, 1986).

But are contextual variables **z** substitutable with inputs **x**?

Do contextual variables satisfy the maintained axioms such as free disposability and convexity?

Contextual variables in the inefficiency term

$$y_i = f(\mathbf{x}_i) - u_i(\mathbf{z}_i) + v_i$$

This is the most standard approach in the parametric SFA literature.

Some parametrizations used are difficult to interpret.

Doubly-heteroscedastic model (Hadri, 1999)

$$y_i = f(\mathbf{x}_i) - u_i(\mathbf{z}_i) + v_i(\mathbf{z}_i)$$

It is nearly impossible to tell if heteroscedasticity of the composite error term is specifically due to the inefficiency term u or the noise term.

Identification relies on the asymmetric distribution of u (parametric distributional assumptions)

Note: if heteroscedasticity is in noise v, but we ignore it, then inefficiency term u captures it.

Semi-nonparametric approach

$$y_i = f(\mathbf{x}_i) + \boldsymbol{\delta}'\mathbf{z}_i - u_i + v_i$$

Contextual variables influence the output, but we do not specify if they are part of the technology or inefficiency term. Both interpretations are equally valid:

$$y_i = [f(\mathbf{x}_i) + \boldsymbol{\delta}'\mathbf{z}_i] - u_i + v_i$$
$$= f(\mathbf{x}_i) + [\boldsymbol{\delta}'\mathbf{z}_i - u_i] + v_i$$

Next lesson

5b) SFA modeling of z-variables