Productivity and Efficiency Analysis

3) Stochastic frontier analysis (SFA)

a) Parametric approach

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Taxonomy of methods

based on Kuosmanen & Johnson (2010), Operations Research

		Parametric	Nonparametric: Local averaging	Nonparametric: Axiomatic
Average curve		*OLS* Gauss (1795), Legendre (1805)	*Kernel regression* Nadaraya (1964), Watson (1964)	*Convex regression* Hildreth (1954), Hanson and Pledger (1976)
Frontier	**Deterministic (Sign constr.)**	*Parametric programming* Aigner and Chu (1968)	*Nonparametric programming* Post et al. (2002)	***DEA*** **Farrell (1957),** **Charnes et al. (1978)**
	Deterministic (2-stage)	*Corrected OLS* Winsten (1957) Greene (1980)	*Corrected kernel* Kneip and Simar (1996)	*Corrected CNLS* Kuosmanen and Johnson (2010)
	Stochastic	*SFA* Aigner et al. (1977) Meeusen and van den Broeck (1977)	*Semi-nonparametric SFA* Fan, Li and Weersink (1996)	*StoNED* Kuosmanen and Kortelainen (2012)

Cobb-Douglas model

$$y_i = A \prod_{s=1}^{S} x_{si}^{\beta_s} \cdot \exp(v_i)$$

$$\ln y_i = \ln A + \sum_{s=1}^{S} \beta_s \ln x_{si} + v_i, \quad i = 1, \ldots, n$$

where

y_i is output of firm i

β_s is **output elasticity** of input s

x_{si} is input s of firm i

v_i is random noise term of firm i

Cobb and Douglas (1928) A Theory of Production. *American Economic Review* 18, 139-165.

Cobb-Douglas production function: properties

- Coefficients β_s are output elasticities.
- Scale elasticity is equal to $\Sigma\beta_s$
- Under CRS, $\Sigma\beta_s = 1$

- Elasticity of substitution between any two inputs is equal to 1 by construction.

Translog model

$$\ln y_i = \alpha + \sum_{s=1}^{S} \beta_s \ln x_{si} + 0.5 \sum_{r=s}^{S} \sum_{s=1}^{S} \gamma_{rs} \ln x_{ri} \ln x_{si} + v_i$$

where

y_i is output of firm i

β_s are the first-order paramerers

γ_{rs} are the second-order paramerers

x_{si} is input s of firm i

v_i is random noise term of firm i

Christensen, Jorgenson and Lau (1973) Transcendental logarithmic production frontiers, *Review of Economics and Statistics*

Deterministic parametric frontier model

$$\ln y_i = \alpha + \sum_{s=1}^{S} \beta_s \ln x_{si} - u_i, \quad i = 1,\ldots,n$$

where

y_i is output of firm i

β_s is **output elasticity** of input s

x_{si} is input s of firm i

u_i is inefficiency term of firm i

Aigner and Chu (1968) On estimating the industry production function, *American Economic Review.*

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Parametric programming

$$\min \sum_{i=1}^{n} \left(\ln y_i - \alpha - \sum_{s=1}^{S} \beta_s \ln x_{si} \right)^2$$

subject to

$$\ln y_i - \alpha - \sum_{s=1}^{S} \beta_s \ln x_{si} \leq 0$$

Quadratic programming problem with linear inequality consraints

Aigner and Chu (1968) On estimating the industry production function, *American Economic Review*.

Corrected OLS

Step 1: Solve the unconstrained least squared problem by OLS:

$$\min \sum_{i=1}^{n} \left(\ln y_i - \alpha - \sum_{s=1}^{S} \beta_s \ln x_{si} \right)^2$$

Step 2: Adjust the intercept to envelop all observations:

$$\hat{\alpha}^{COLS} = \max_i \left(\ln y_i - \sum_{s=1}^{S} \beta_s \ln x_{si} \right)$$

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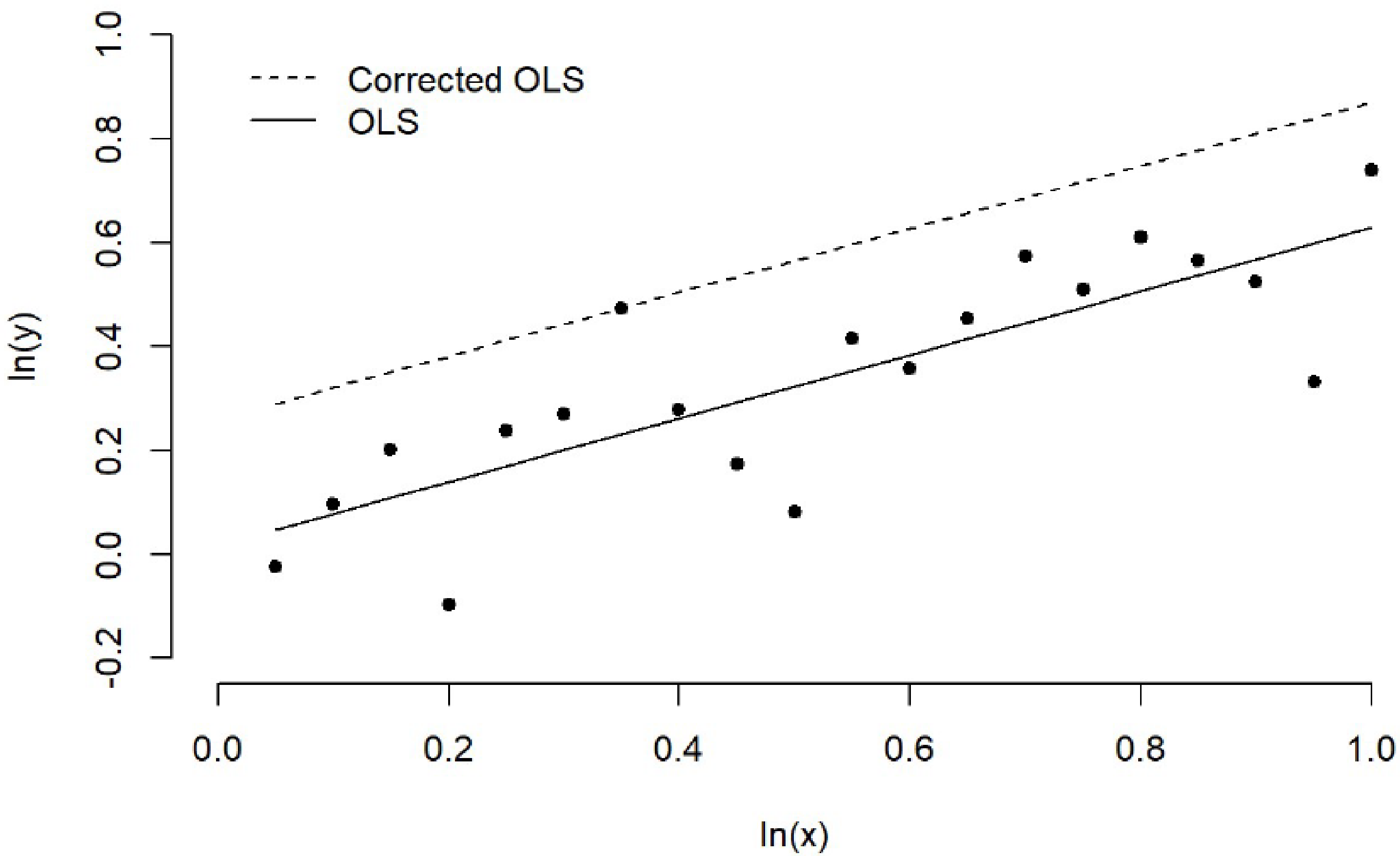
Illustration



Next lesson

3b) Basics of SFA