Productivity and Efficiency Analysis

6) Multiple outputs and bad outputs

6d) Parametric distance functions

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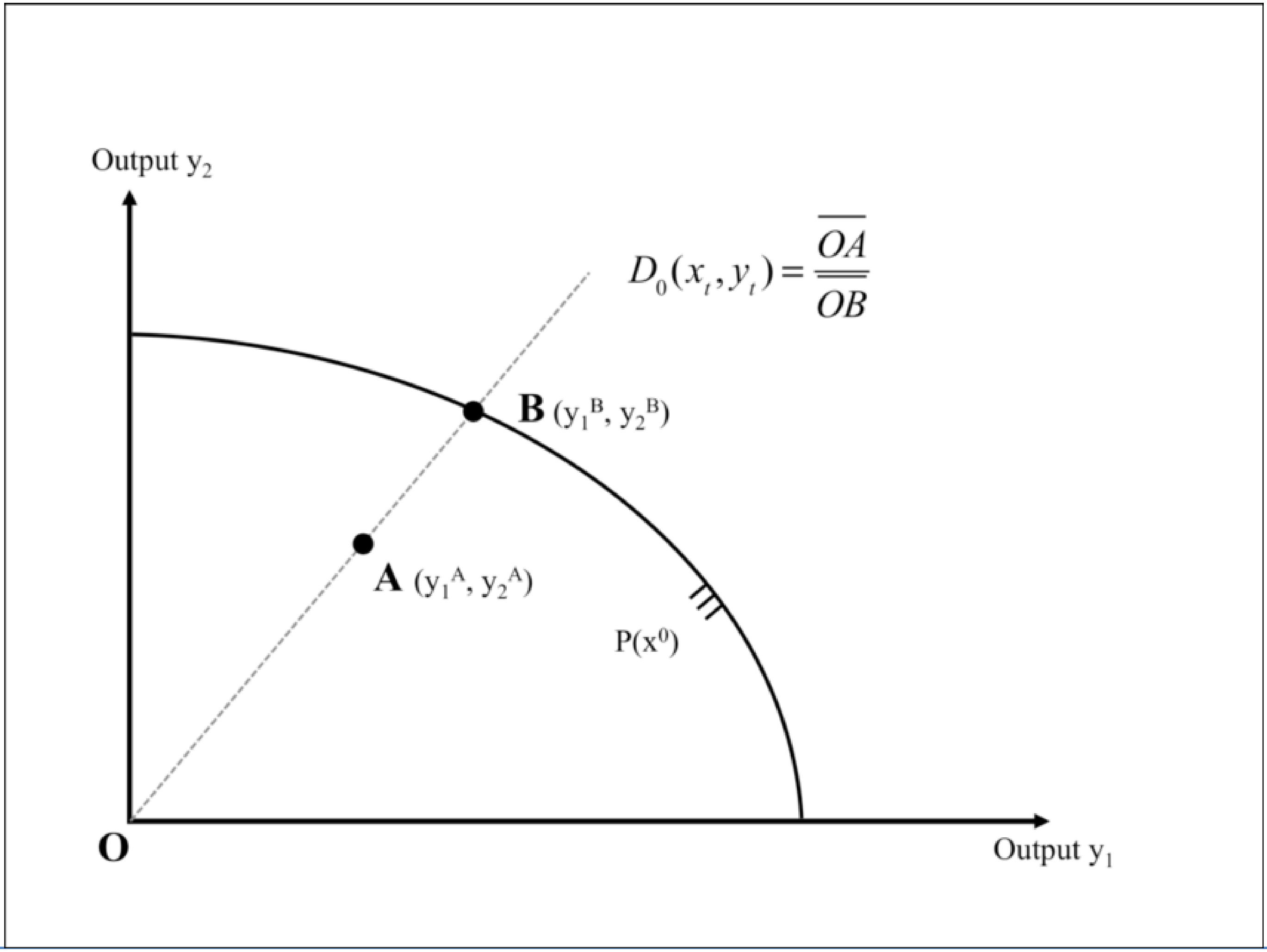
Output distance function $D_O(\mathbf{x,y})$



Source: Färe and Primont (1995)

Parametric distance functions

- It is common to parametrize the distance functions applying the usual functional forms such as Cobb-Douglas and translog
- The following discussion is based on the output distance function, but the same comments apply equally well to other distance functions as well as to the cost function

Translog cost function

the translog multiple-output cost function for K inputs and L outputs,

$$\ln C = \alpha + \sum_{k=1}^{K} \beta_k \ln w_k + \frac{1}{2} \sum_{k=1}^{K} \sum_{m=1}^{K} \gamma_{km} \ln w_k \ln w_m$$

$$+ \sum_{s=1}^{L} \delta_s \ln y_s + \frac{1}{2} \sum_{s=1}^{L} \sum_{t=1}^{L} \phi_{st} \ln y_s \ln y_t$$

$$+ \sum_{k=1}^{K} \sum_{s=1}^{L} \theta_{ks} \ln w_k \ln y_s,$$

- Source: Greene (2008)

Translog distance function

- Perelman and Santini (2009) *EJOR*

Translog output distance function

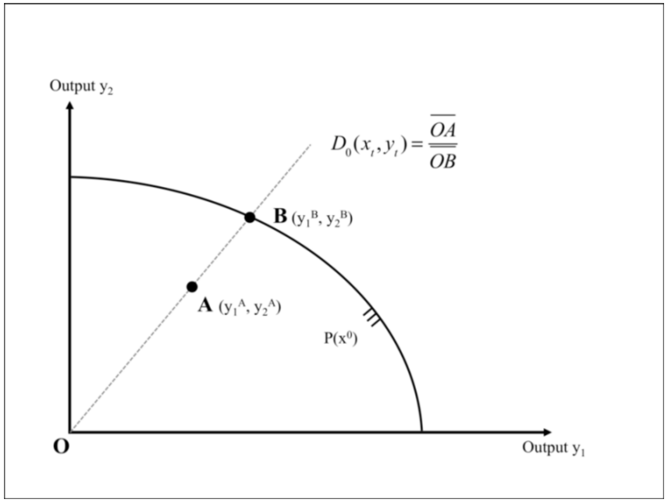
$$\ln D_{Oi}(x,y) = \alpha_0 + \sum_{m=1}^{M} \alpha_m \ln y_{mi} + \frac{1}{2}\sum_{m=1}^{M}\sum_{n=1}^{M} \alpha_{mn} \ln y_{mi} \ln y_{ni}$$
$$+ \sum_{k=1}^{K} \beta_k \ln x_{ki} + \frac{1}{2}\sum_{k=1}^{K}\sum_{l=1}^{K} \beta_{kl} \ln x_{ki} \ln x_{li}$$
$$+ \sum_{k=1}^{K}\sum_{m=1}^{M} \delta_{km} \ln x_{ki} \ln y_{mi}, \quad i = 1, 2, \ldots, N,$$

Translog distance function

- Perelman and Santini (2009) *EJOR*

Translog output distance function

$$\ln D_{Oi}(x,y) = \alpha_0 + \sum_{m=1}^{M} \alpha_m \ln y_{mi} + \frac{1}{2}\sum_{m=1}^{M}\sum_{n=1}^{M} \alpha_{mn} \ln y_{mi} \ln y_{ni} + \sum_{k=1}^{K} \beta_k \ln x_{ki} + \frac{1}{2}\sum_{k=1}^{K}\sum_{l=1}^{K} \beta_{kl} \ln x_{ki} \ln x_{li} + \sum_{k=1}^{K}\sum_{m=1}^{M} \delta_{km} \ln x_{ki} \ln y_{mi}, \quad i = 1,2,\ldots,N,$$



Symmetry:

$$\alpha_{mn} = \alpha_{nm}, \quad m,n = 1,2,\ldots,M, \text{ and}$$
$$\beta_{kl} = \beta_{lk}, \quad k,l = 1,2,\ldots,K,$$

Linear homogeneity:

$$\sum_{m=1}^{M} \alpha_m = 1,$$
$$\sum_{n=1}^{M} \alpha_{mn} = 0, \quad m = 1,2,\ldots,M, \text{ and}$$
$$\sum_{m=1}^{M} \delta_{km} = 0, \quad k = 1,2,\ldots,K.$$

How output isoquants of translog technology actually look like?

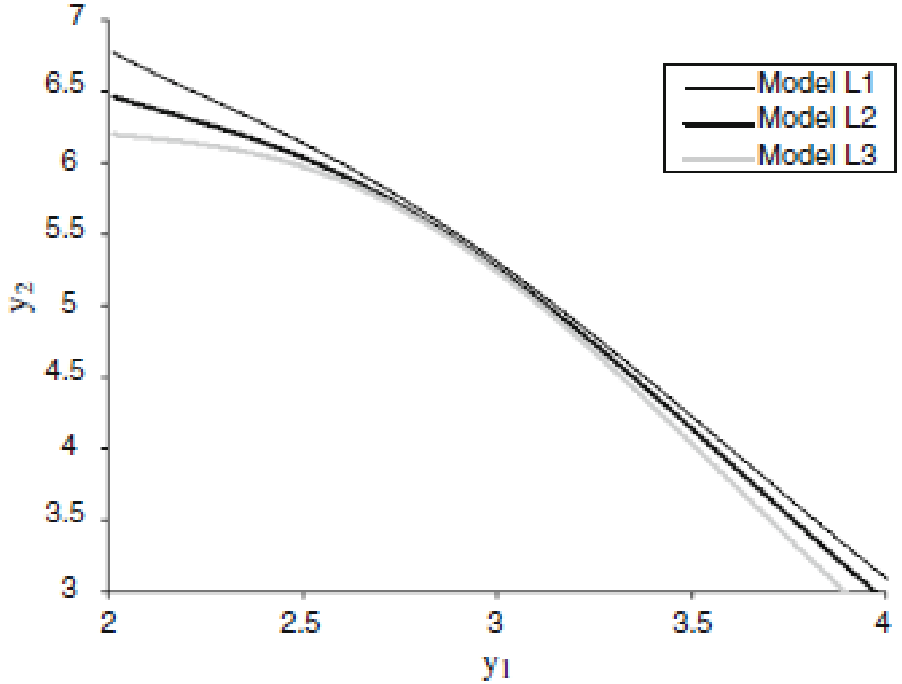
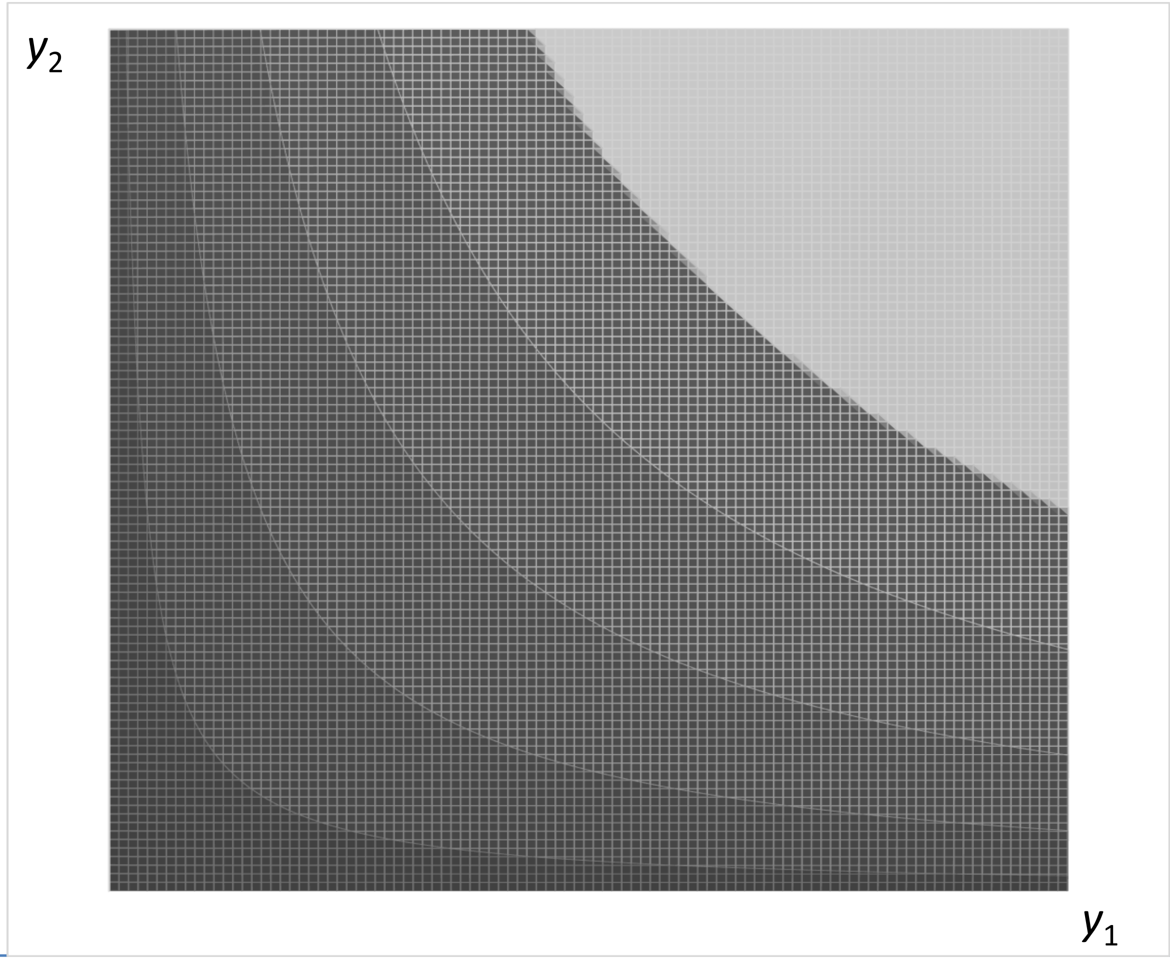


Fig. 1 I. True frontiers of the output set; polynomial technologies. II. True frontiers of the output set; translog technologies

Source: Färe, Martins-Filho & Vardanyan (2010) *JPA*

Output isoquants of translog technology

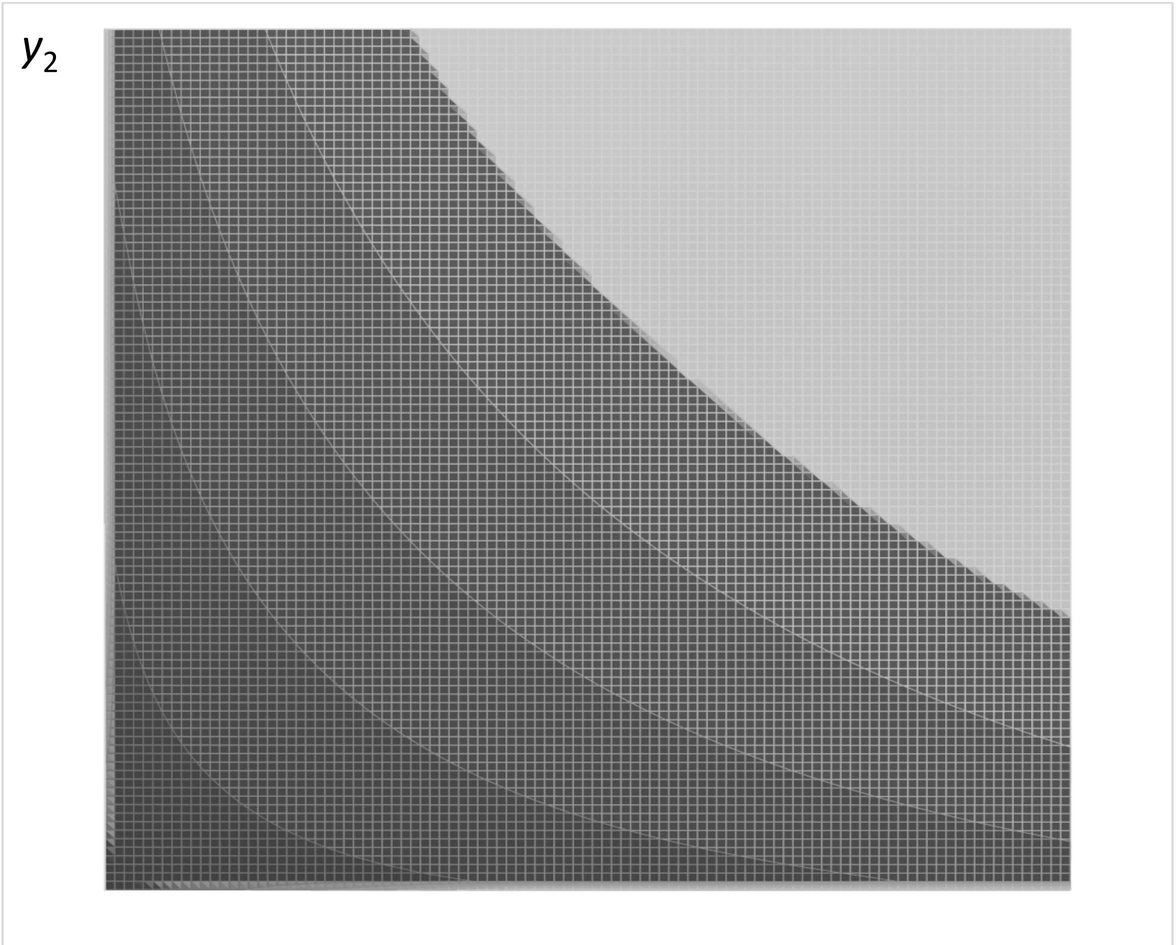


$\alpha_{11} = 0$

(Cobb-Douglas)

Note: homog. and symmetry imply $\alpha_{12} = 0 = \alpha_{21}$ and $\alpha_{22} = 0$.

Output isoquants of translog technology

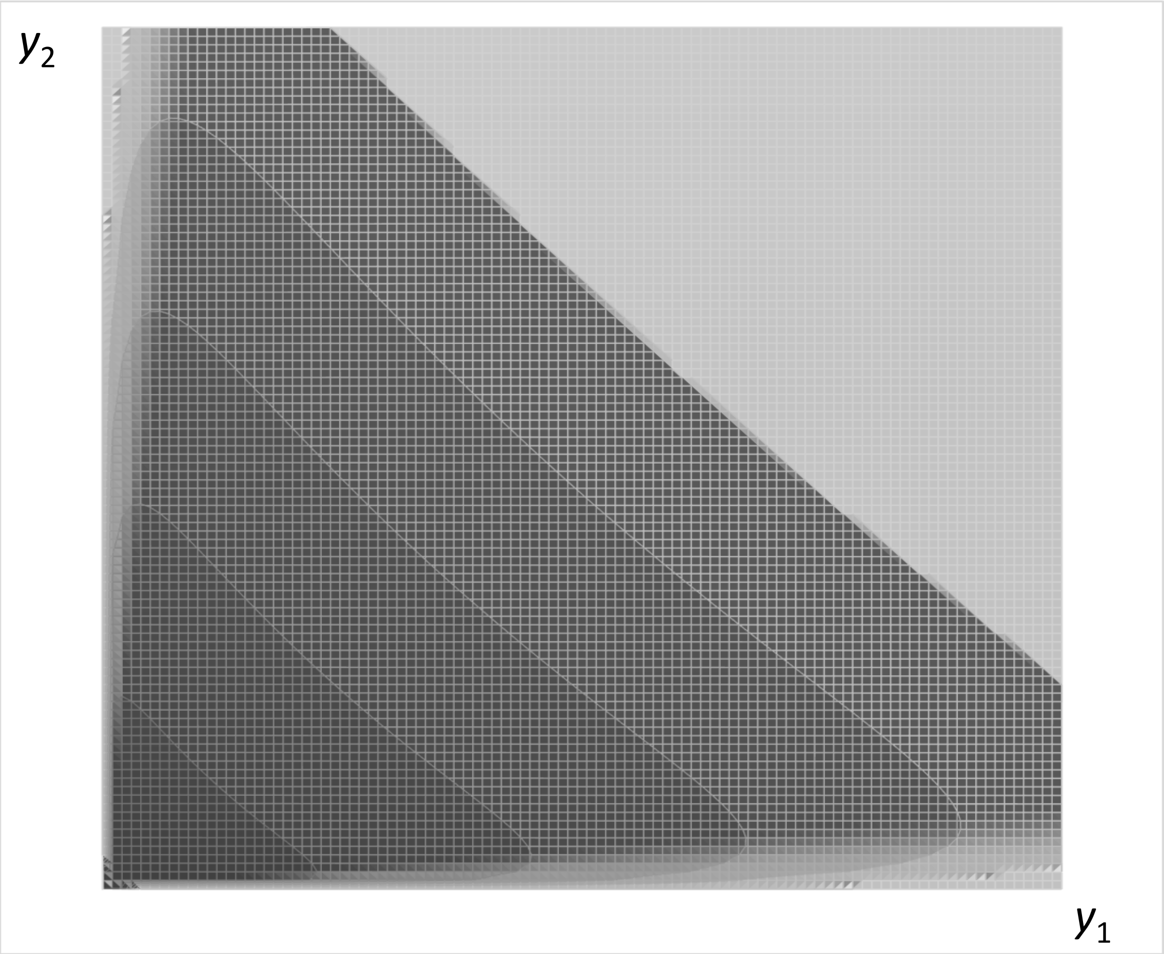


$\alpha_{11} = 0.1$

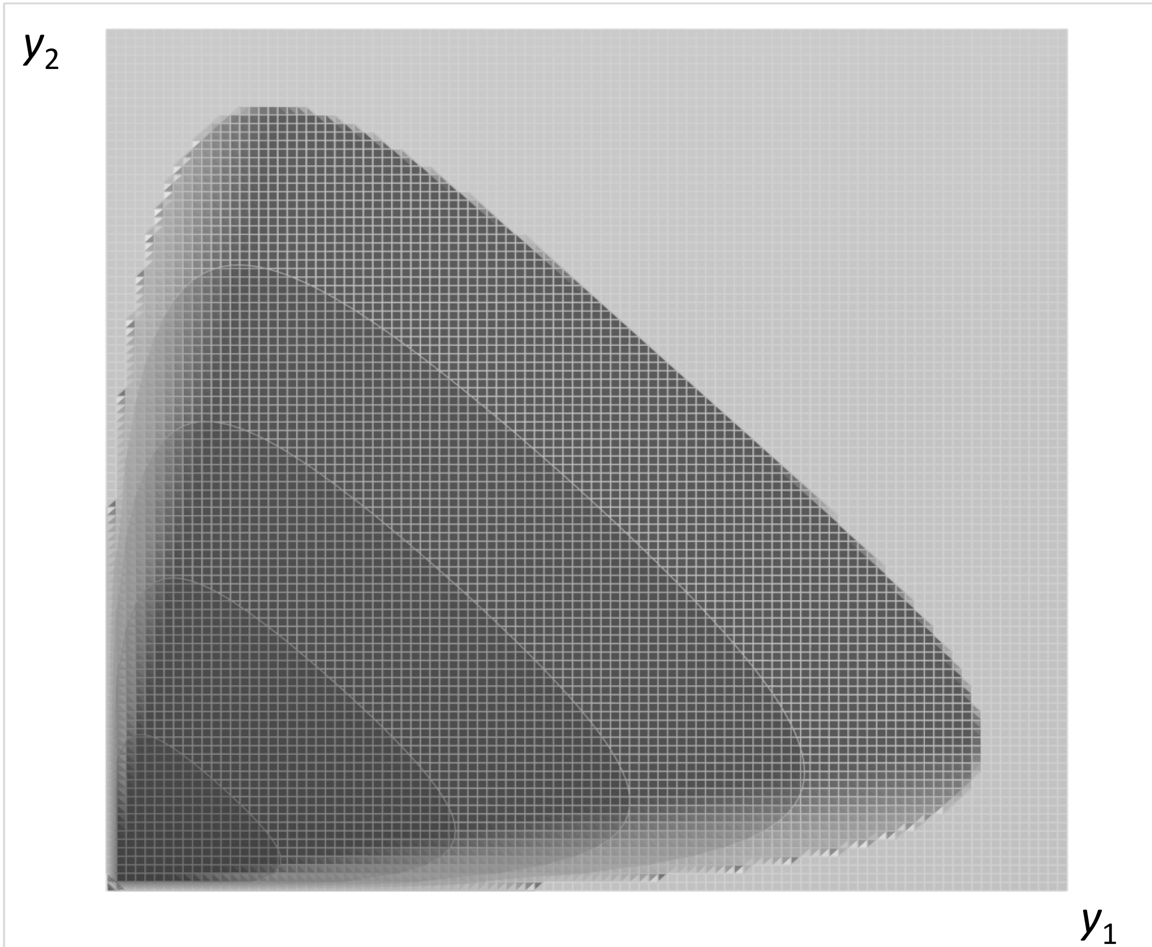
Note: homog. and symmetry imply $\alpha_{12} = -0.1 = \alpha_{21}$ and $\alpha_{22} = 0.1$.

Output isoquants of translog technology

$\alpha_{11} = 0.2$



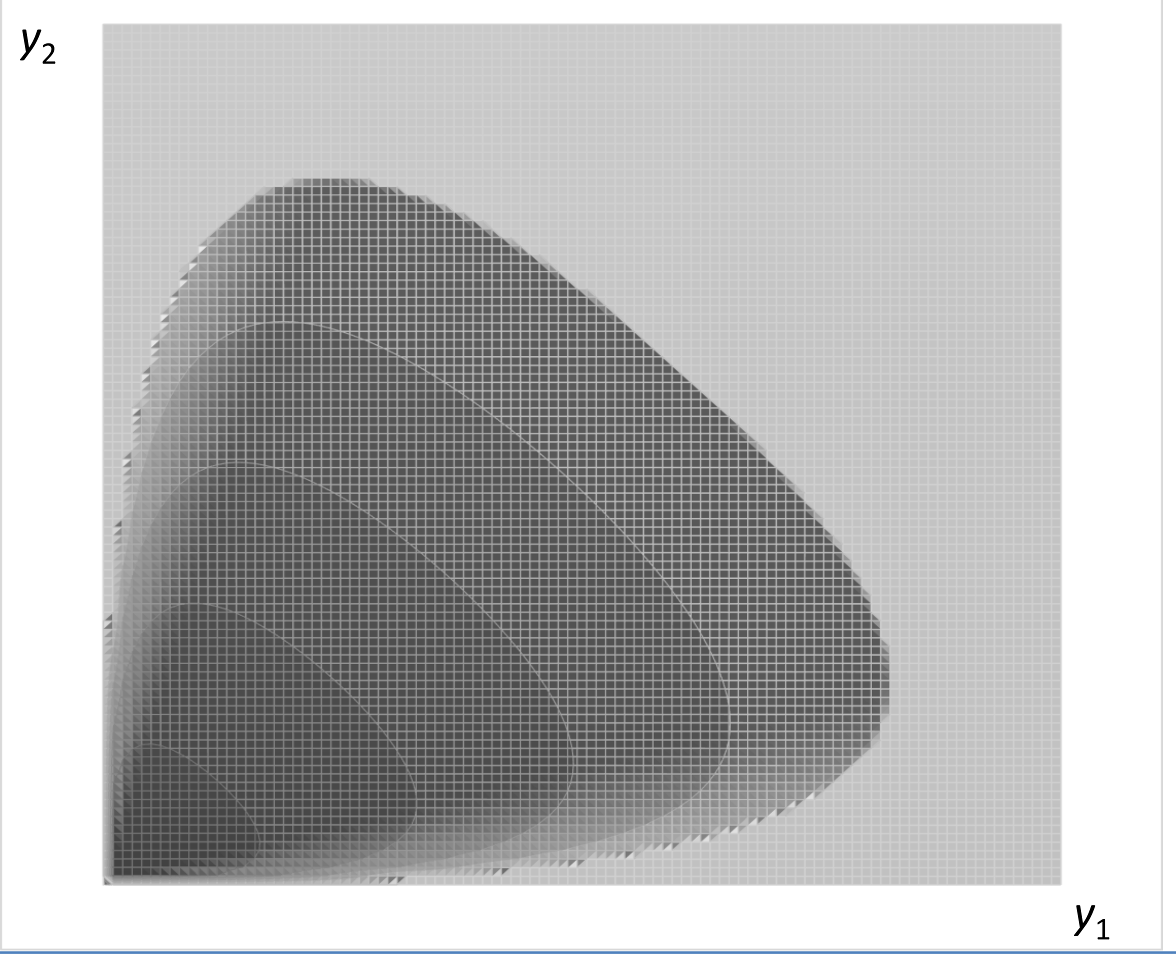
Output isoquants of translog technology



$\alpha_{11} = 0.3$

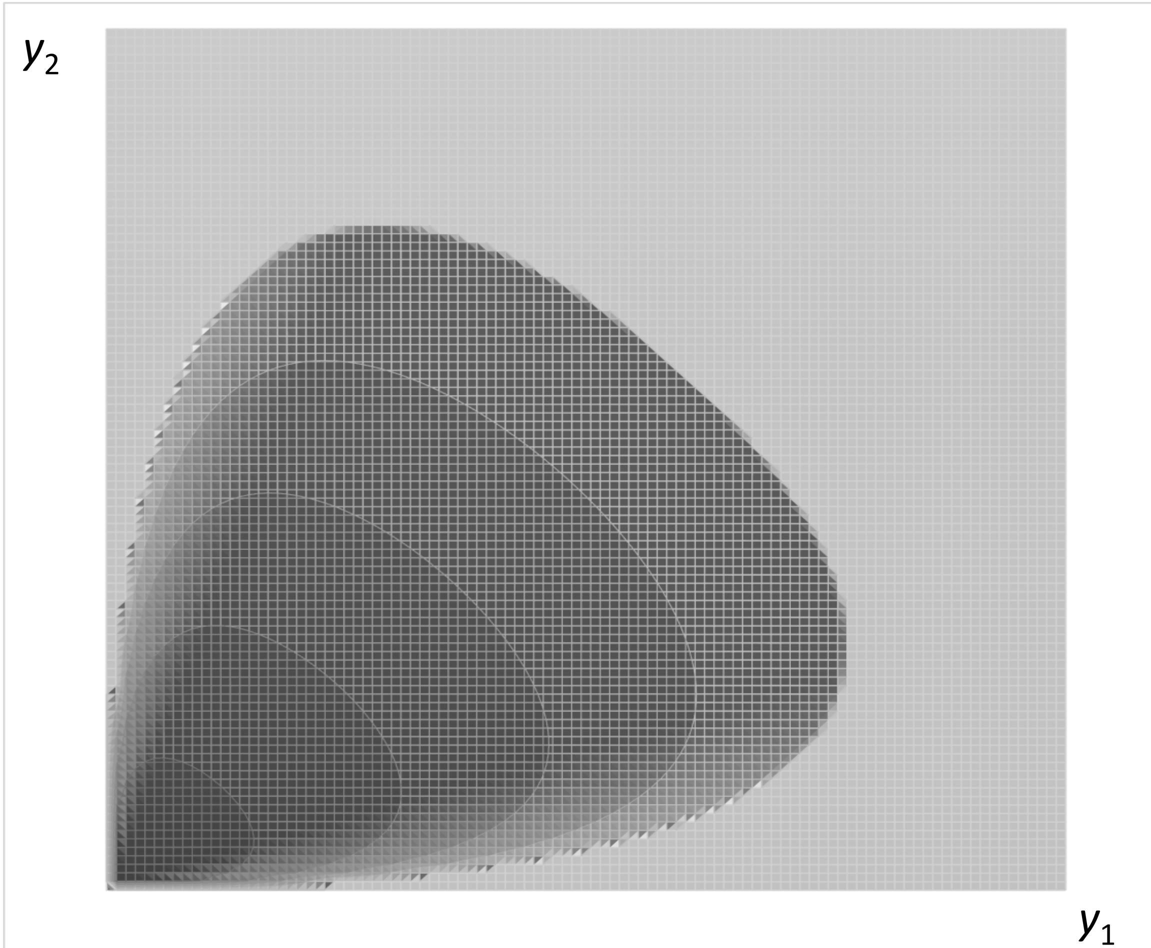
Output isoquants of translog technology

$\alpha_{11} = 0.4$



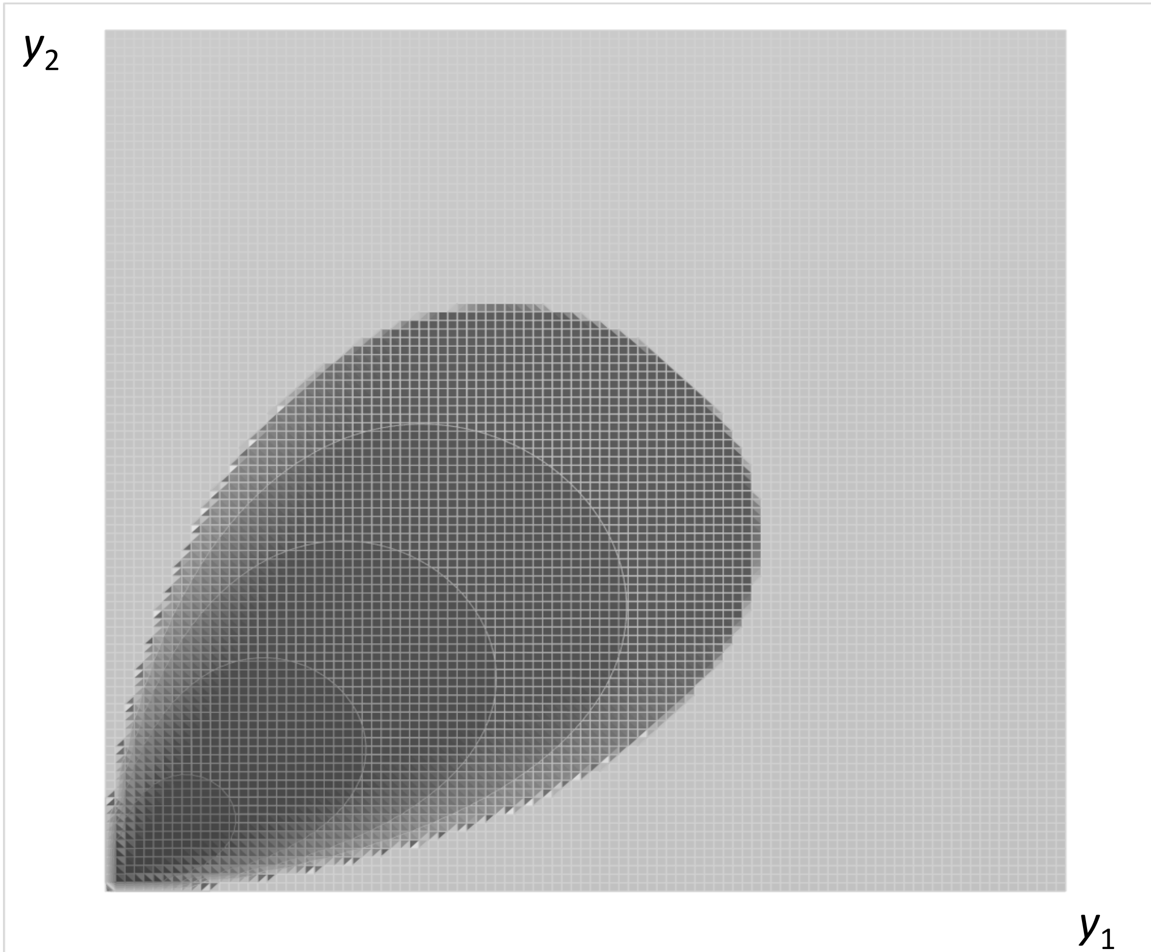
Output isoquants of translog technology

$\alpha_{11} = 0.5$



Output isoquants of translog technology

$\alpha_{11} = 1$



Cobb-Douglas output sets

- Output sets are always
  - unbounded
  - non-convex (negative returns to scope)

- Cannot handle zero outputs of specialized firms

Translog output sets

- Output sets are either
  - Unbounded and non-convex
  - Or violate free disposability

- Cannot handle zero outputs of specialized firms

- Multicollinearity

Example: Finnish electricity distribution firms

- Correlation matrix

	lnEnergy	*lnLength*	*lnCustomer*	*lnEnergy^2*	*lnLength^2*	*lnCustomer^2*	*lnEne*lnLen*	*lnEne*lnCust*	*lnLen*lnCus*	*Underground*
lnEnergy	1									
lnLength	0,76	1								
lnCustomer	0,80	0,87	1							
lnEnergy^2	0,99	0,77	0,80	1						
lnLength^2	0,78	0,99	0,84	0,80	1					
lnCustomer^2	0,87	0,88	0,98	0,88	0,87	1				
lnEne*lnLen	0,94	0,92	0,87	0,96	0,94	0,93	1			
lnEne*lnCust	0,96	0,86	0,93	0,97	0,86	0,97	0,97	1		
lnLen*lnCus	0,85	0,97	0,94	0,87	0,97	0,96	0,97	0,94	1	
Underground	0,33	-0,16	0,22	0,29	-0,17	0,24	0,10	0,28	0,02	1

Next lesson

6e) StoNED with multiple outputs