Productivity and Efficiency Analysis

6) Multiple outputs and bad outputs

6c) Bad outputs

Timo Kuosmanen

Aalto University School of Business

https://people.aalto.fi/timo.kuosmanen

Undesirable outputs (=bads)

- In addition to desirable outputs, production often involves side products that are unintended and undesirable
- Classic example: pollution
- Other examples
  - Electricity distribution application: interruptions
  - Banks: non-performing loans

Two approaches in DEA

1) Model bads **b** as inputs

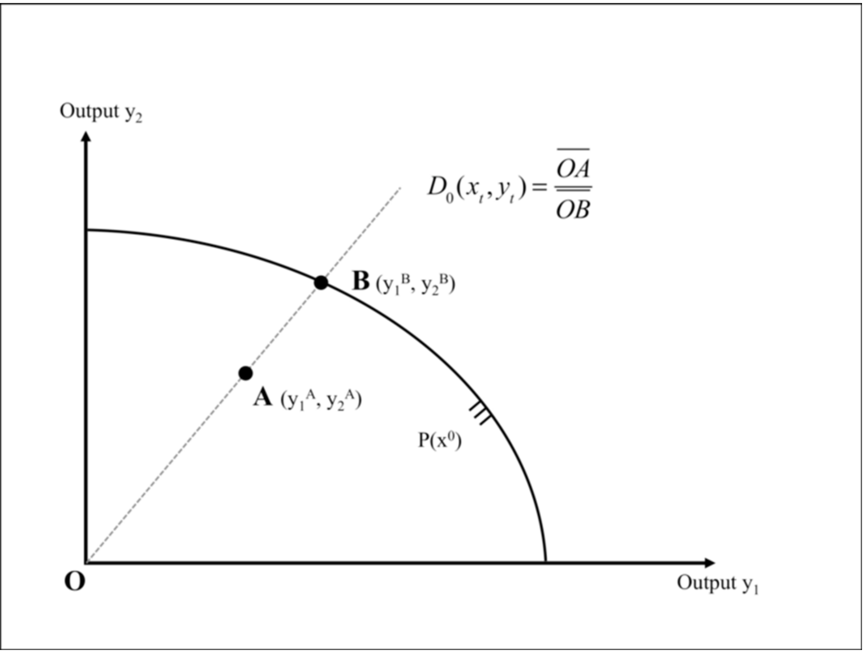
$T^{DEA} = \{(\mathbf{x},\mathbf{y},\mathbf{b}) \mid \mathbf{x} \geq \mathbf{X\lambda};\ \mathbf{b} \geq \mathbf{B\lambda};\ \mathbf{y} \leq \mathbf{Y\lambda};\ \mathbf{1}'\boldsymbol{\lambda}=1;\ \boldsymbol{\lambda} \geq \mathbf{0}\}$

2) Model bads **b** as weakly disposable outputs (Shephard, 1970)

$T^{DEA} = \{(\mathbf{x},\mathbf{y},\mathbf{b}) \mid \mathbf{x} \geq \mathbf{X\lambda};\ \mathbf{b}/\theta = \mathbf{B\lambda};\ \mathbf{y}/\theta \leq \mathbf{Y\lambda};\ 0 \leq \theta \leq 1;\ \mathbf{1}'\boldsymbol{\lambda}=1;\ \boldsymbol{\lambda} \geq \mathbf{0}\}$

where θ is an abatement factor.

Illustration: b as input



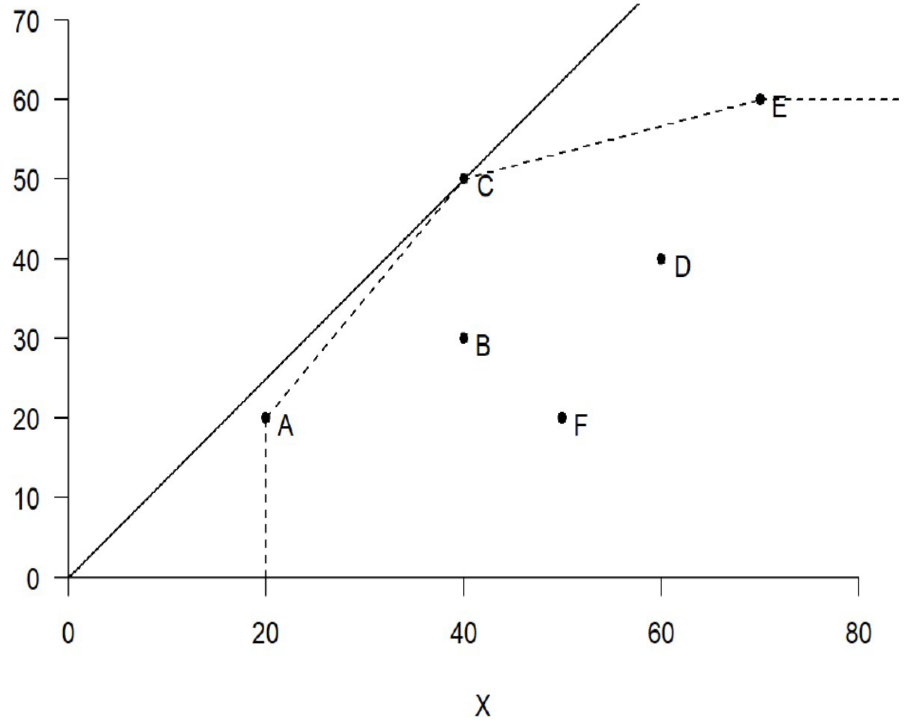


Illustration: no disposability of b (b = Bλ)

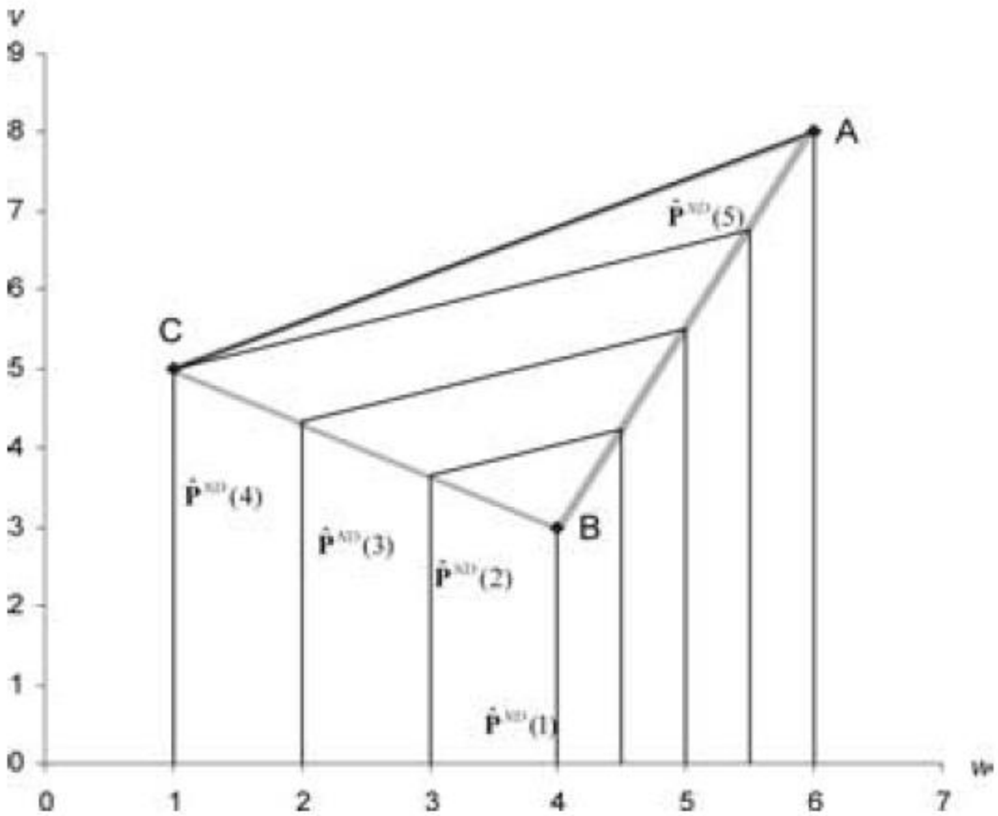


Figure 1. Output sets with nondisposability (ND) of undesirable output

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Source: Kuosmanen (2005) *AJAE*

Illustration: Shephard technology (b/θ= Bλ)

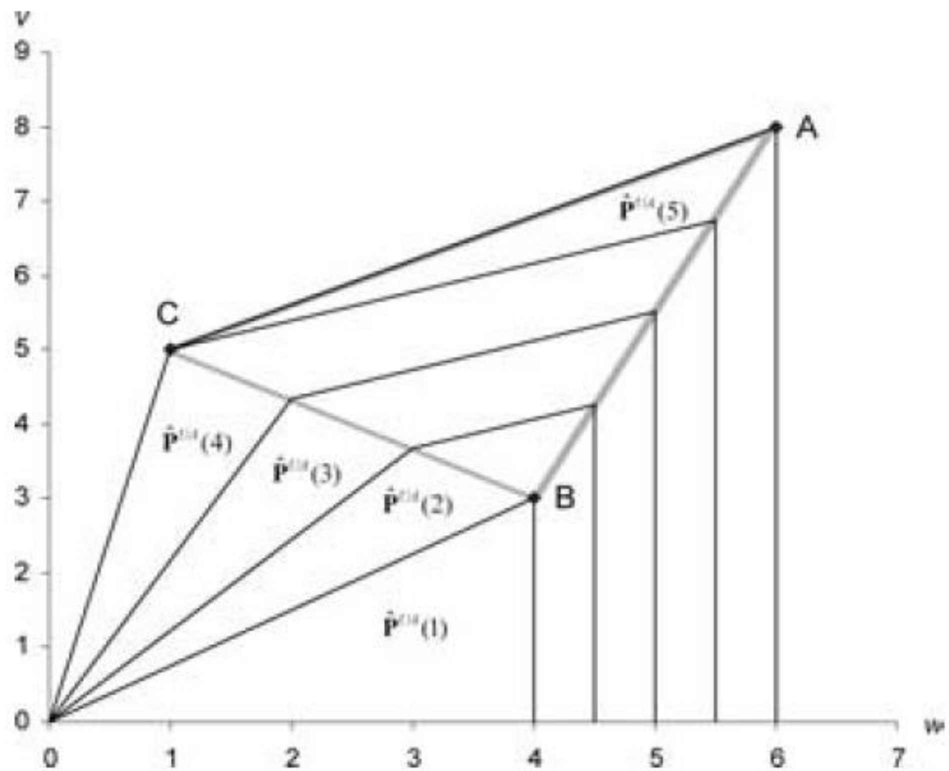

Figure 2. Output sets with conventional specification of weak disposability that implicitly assumes uniform abatement (UA) across firms

Source: Kuosmanen (2005) *AJAE*

Illustration: Shephard technology ($b/\theta = B\lambda$)

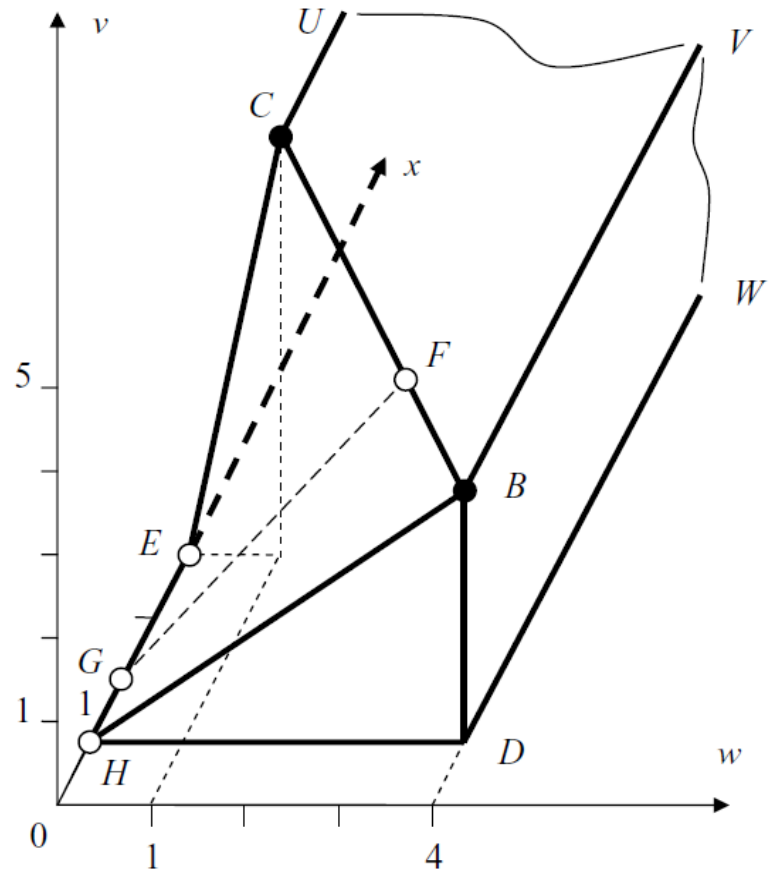

Figure 1. The Shephard technology induced by activities B and C

Source: Kuosmanen and Podinovski (2009) *AJAE*

Correct formulation of weakly disposable DEA technology (Kuosmanen, 2005)

$T^{DEA} = \{(\mathbf{x},\mathbf{y},\mathbf{b}) \mid \mathbf{x} \geq \mathbf{X\lambda};\ \mathbf{b} = \boldsymbol{\theta}'\mathbf{B\lambda};\ \mathbf{y} \leq \boldsymbol{\theta}'\mathbf{Y\lambda};\ \mathbf{0} \leq \boldsymbol{\theta} \leq \mathbf{1};\ \mathbf{1}'\boldsymbol{\lambda} = 1;\ \boldsymbol{\lambda} \geq \mathbf{0}\}$

where $\boldsymbol{\theta}$ is a vector of *n* abatement factors.

Linearization:

$T^{DEA} = \{(\mathbf{x},\mathbf{y},\mathbf{b}) \mid \mathbf{x} \geq \mathbf{X}(\boldsymbol{\lambda}+\boldsymbol{\mu});\ \mathbf{b} = \mathbf{B\lambda};\ \mathbf{y} \leq \mathbf{Y\lambda};\ \mathbf{1}'(\boldsymbol{\lambda}+\boldsymbol{\mu}) = 1;\ \boldsymbol{\lambda} \geq \mathbf{0};\ \boldsymbol{\mu} \geq \mathbf{0}\}$

Correct formulation of weakly disposable DEA technology (Kuosmanen, 2005)

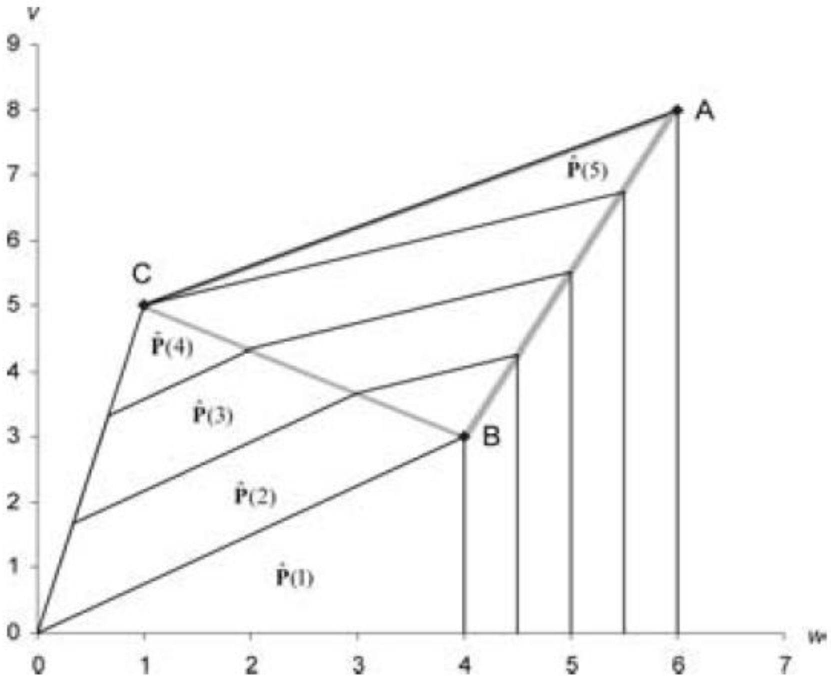

Figure 3. Output sets with correctly specified weak disposability

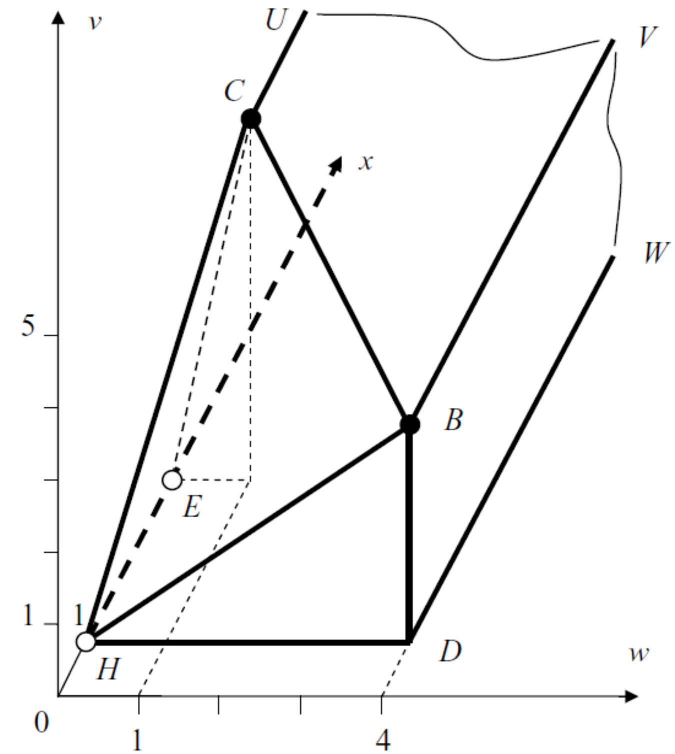

Figure 2. The Kuosmanen technology induced by activities B and C

Sources: left (Kuosmanen, 2005, *AJAE*), right (Kuosmanen and Podinovski, 2009, *AJAE*)

Weakly disposable Kuosmanen (2005) technology

Simple trick for implementation:

Include in the data set the original n observations $(\mathbf{x}_i,\mathbf{y}_i,\mathbf{b}_i)$, $i=1,\ldots,n$

plus n inactive observations $(\mathbf{x}_i,\mathbf{0},\mathbf{0})$, $i=1,\ldots,n$.

Then simply apply standard DEA to the data set of $2n$ observations.

Properties of the weakly disposable technology

THEOREM 1. *The Kuosmanen weakly disposable technology* $\hat{\mathbf{Y}}_T$ *as defined in (2) is the minimal set that contains all observed activities and satisfies the axioms of strong disposability of inputs and good outputs (A1), weak disposability of good and bad outputs (A2), and convexity (A3).*

THEOREM 2. *The Kuosmanen weakly disposable technology* $\hat{\mathbf{Y}}_T$ *as defined in (2) is the convex hull of the Shephard technology* $\hat{\mathbf{Y}}_S$ *as defined in (1).*

Source: Kuosmanen and Podinovski (2009) *AJAE*)

Unfortunately, confusion about weak disposability is persistent…

A Comment on Weak Disposability in Nonparametric Production Analysis

Rolf Färe and Shawna Grosskopf

In his 2005 paper in this journal, Kuosmanen argues that Shephard's specification of weak disposability in activity analysis (DEA) models is not correct. We show that Shephard's specification does satisfy weak disposability and is the "smallest" technology to do so.

Shadow prices and marginal abatement costs of bad outputs

- Convex quantile regression approach: Kuosmanen and Zhou (2018, WP)
- Application: Cost of Kyoto Protocol (Kuosmanen, Zhou, Dai, 2020, *Word. Dev.*)
- Forward-looking assessment of the GHG abatement cost in China (Dai, Zhou, Kuosmanen, 2020, *Energy Econ*)

Next lesson

6d) Parametric distance functions