

# Binary Multiplication

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# Calculator.py

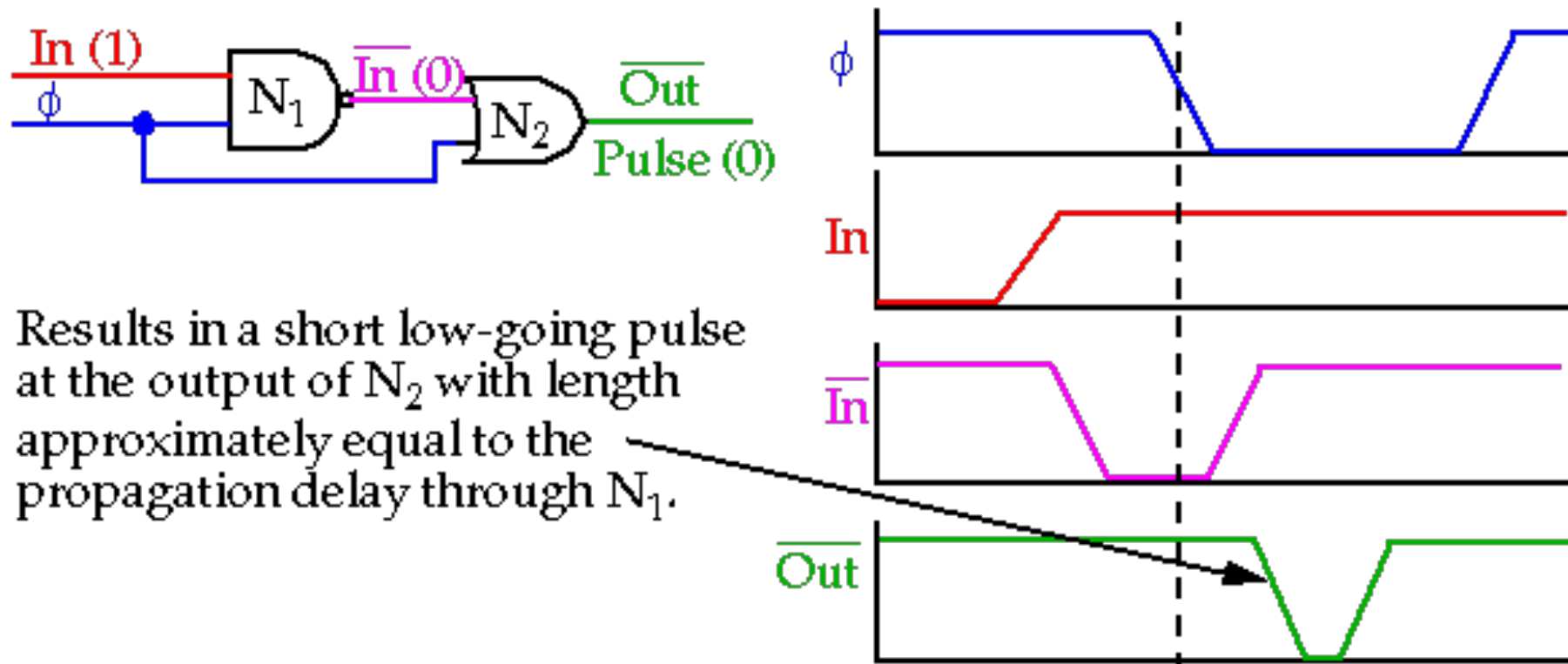
- Lets you interact with the FPGA via the keyboard
  - Translates ASCII <--> Binary for you

```
$ python3 calculator.py -s /dev/ttyUSB1
```

```
$ python3 calculator.py --help
```

```
python3 (not “python”)
```

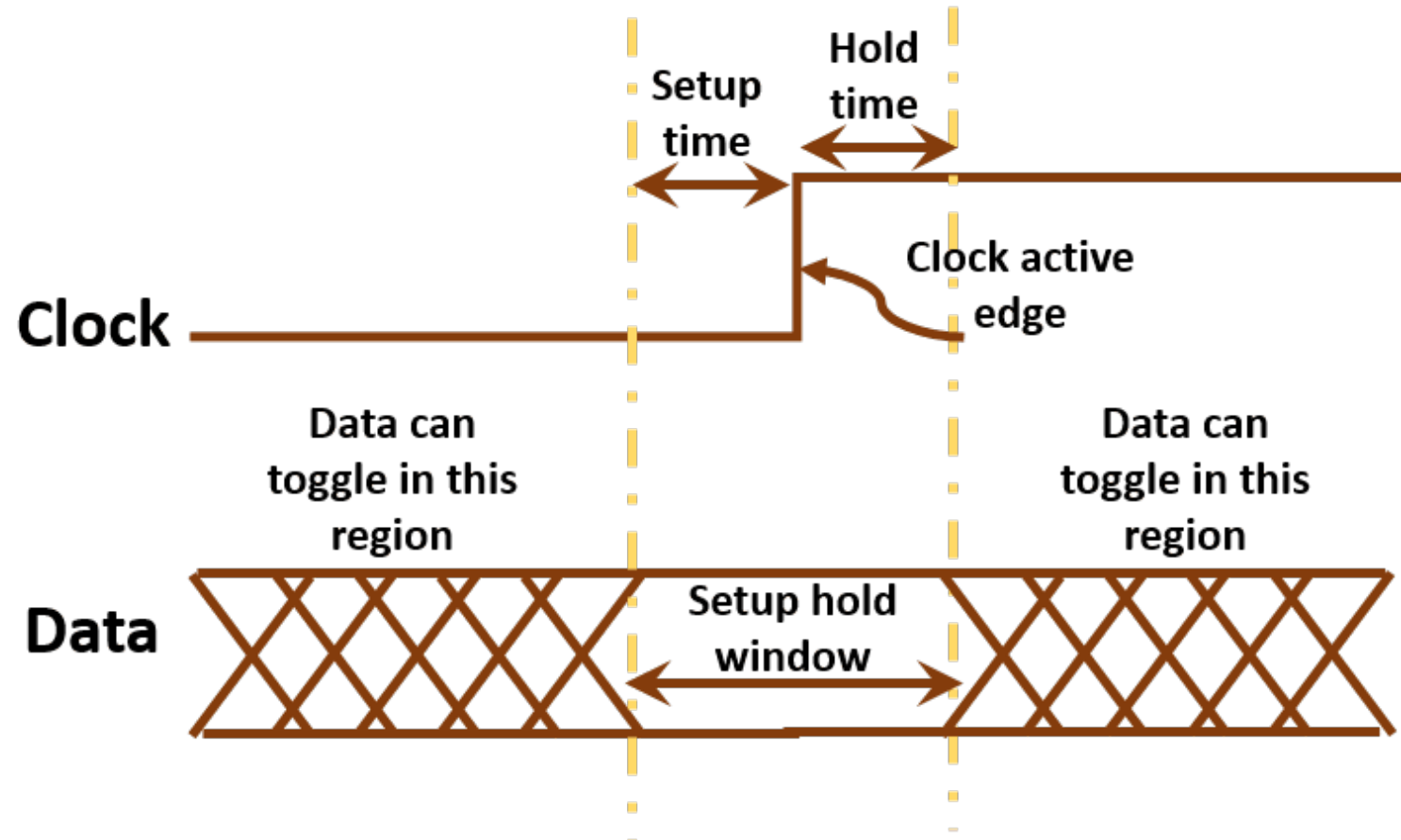
# Review: Glitch



# Last Time: Setup and Hold Time

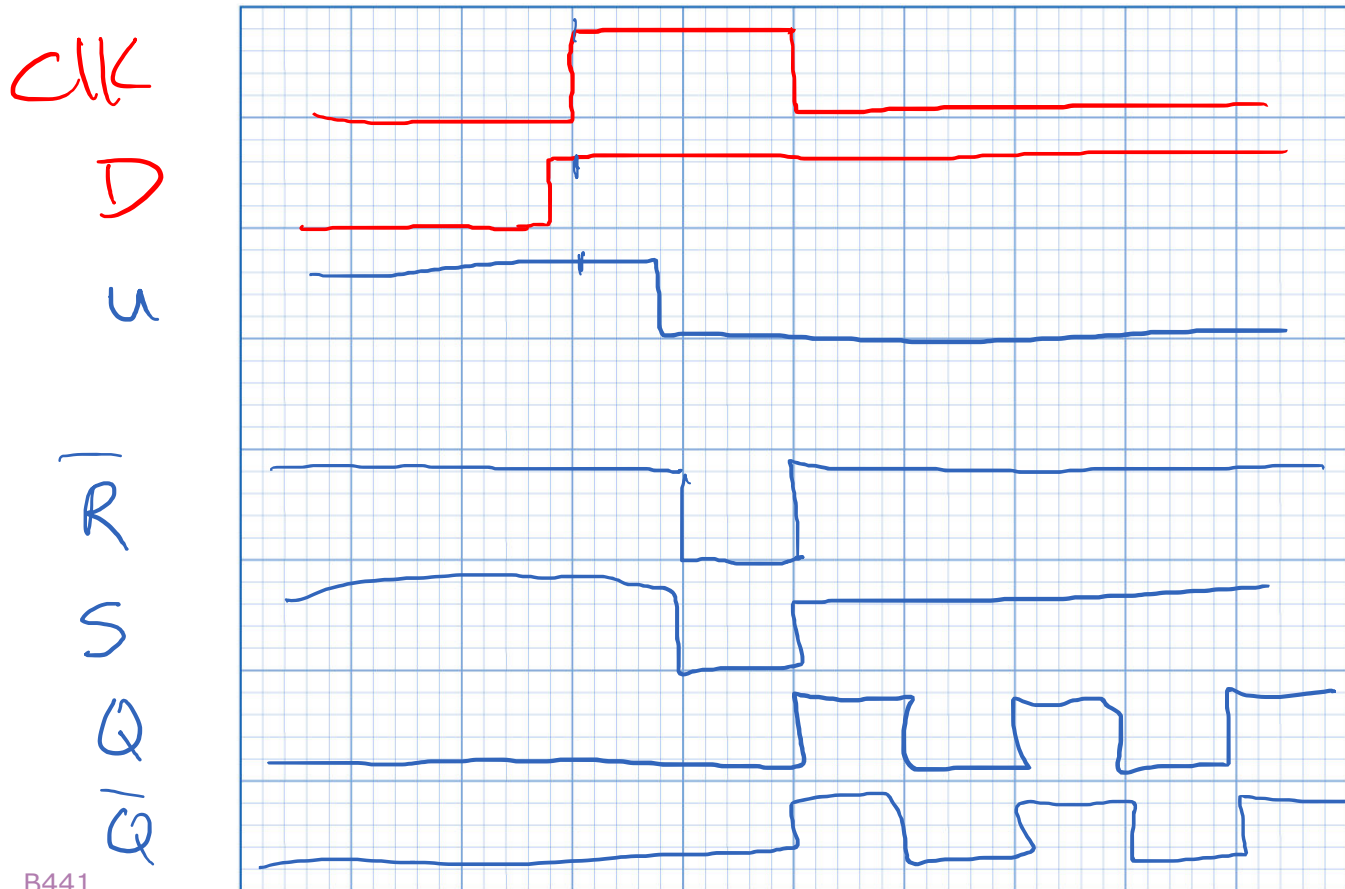
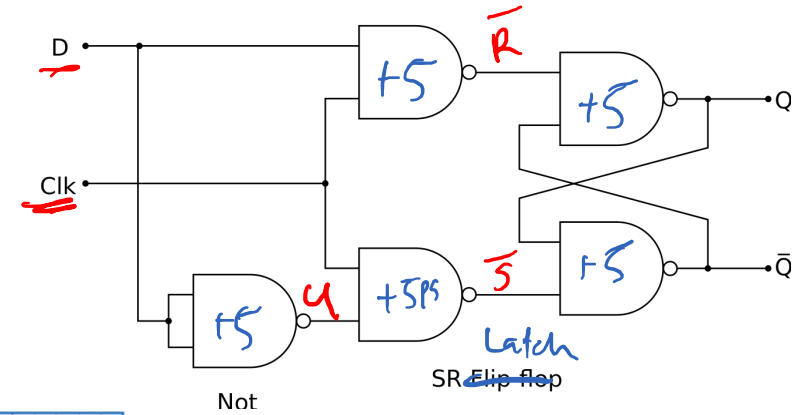
- **Setup Time:** minimum time the inputs to a flip-flop must be stable before the clock edge
- **Hold Time:** minimum time the inputs to a flip-flop must be stable after the clock edge

# Setup and Hold Time



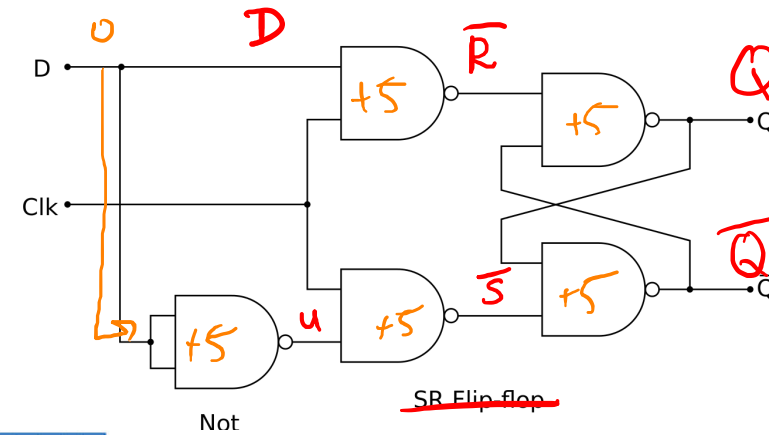
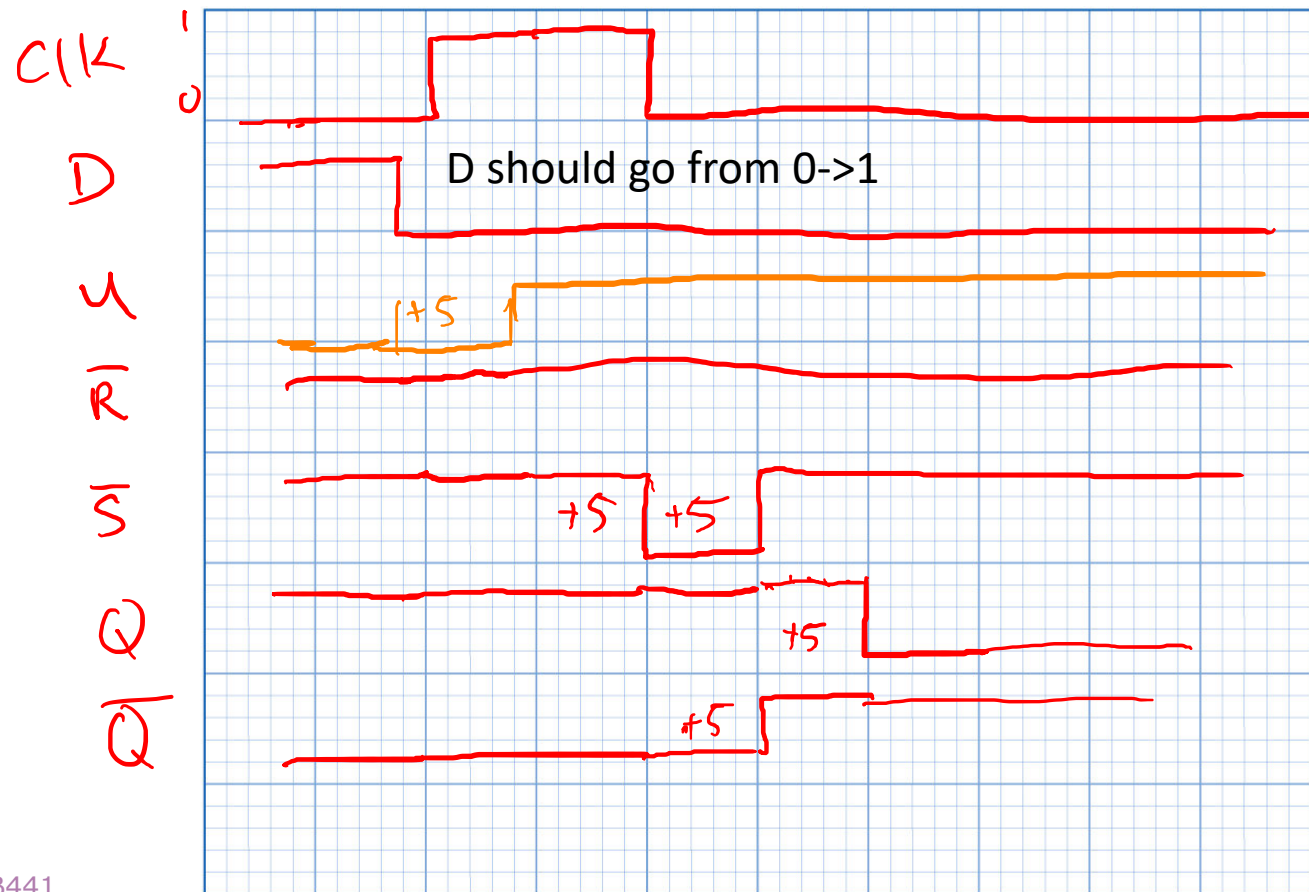
# Setup Time Problems

- Assume 5ps/ MOSFET



# Setup Time Problems

- Assume 5ps/ MOSFET



Screwed  
this  
up  
redo  
next  
time...

# Decimal Multiplication

$$\begin{array}{r} 6 \\ \times 3 \\ \hline 18 \end{array}$$

$$\begin{array}{r} 62 \\ \times 3 \\ \hline 186 \end{array}$$

$$\begin{array}{r} 62 \\ \times 31 \\ \hline 62 \\ + 1860 \\ \hline 1922 \end{array}$$

$$\begin{array}{r} 624 \\ \times 312 \\ \hline 1248 \\ 6240 \\ + 187200 \\ \hline 194688 \end{array}$$

Multiplicand

Multiplier

Partial Products

Product



# Binary Multiplication

$$\begin{array}{r} 6 \\ \times 3 \\ \hline 18 \end{array}$$

$$\begin{array}{r} \phantom{00}0110 \\ \times \phantom{00}0011 \\ \hline 0010110 \\ 0110000 \\ \hline 00000000 \\ + 00000000 \\ \hline 001000100 \\ \phantom{00}\uparrow \\ 16 + 2 = 18 \end{array}$$

$$\begin{array}{r} a_3 \ a_2 \ a_1 \ a_0 \\ \times 8 \phantom{0000} b \\ \hline \end{array}$$

$$\begin{array}{r} a_3 \ a_2 \ a_1 \ a_0 \\ \times \phantom{0000} 1 \\ \hline a_3 \ a_2 \ a_1 \ a_0 \end{array}$$

$$\begin{array}{r} a_3 \ a_2 \ a_1 \ a_0 \\ \times \phantom{0000} 0 \\ \hline 0 \ 0 \ 0 \ 0 \end{array}$$

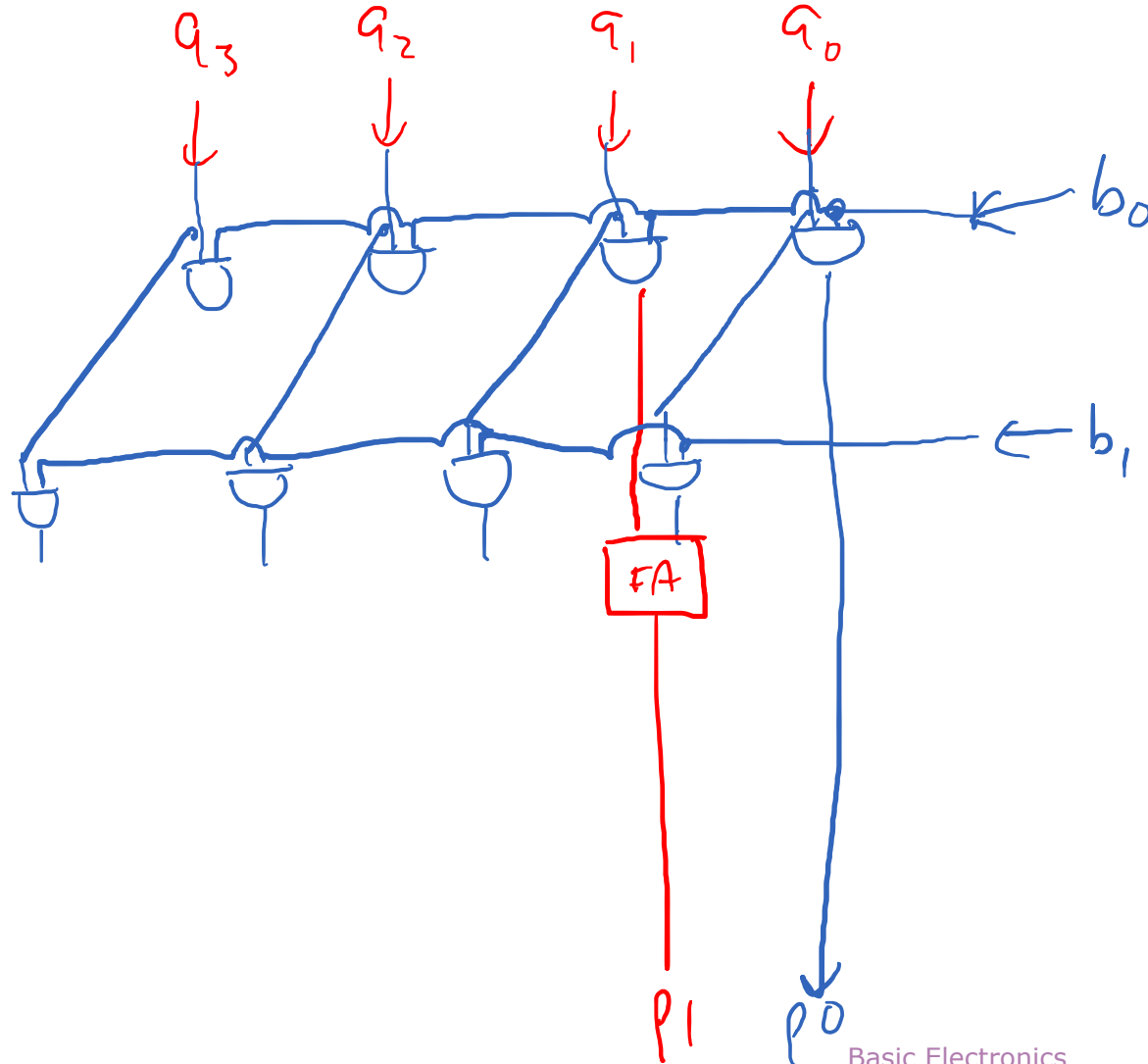
# Binary Multiplication

$$\begin{array}{r} 6 \\ \times 3 \\ \hline \end{array}$$

$$\begin{array}{r} 0110 \\ \times 0011 \\ \hline \end{array}$$

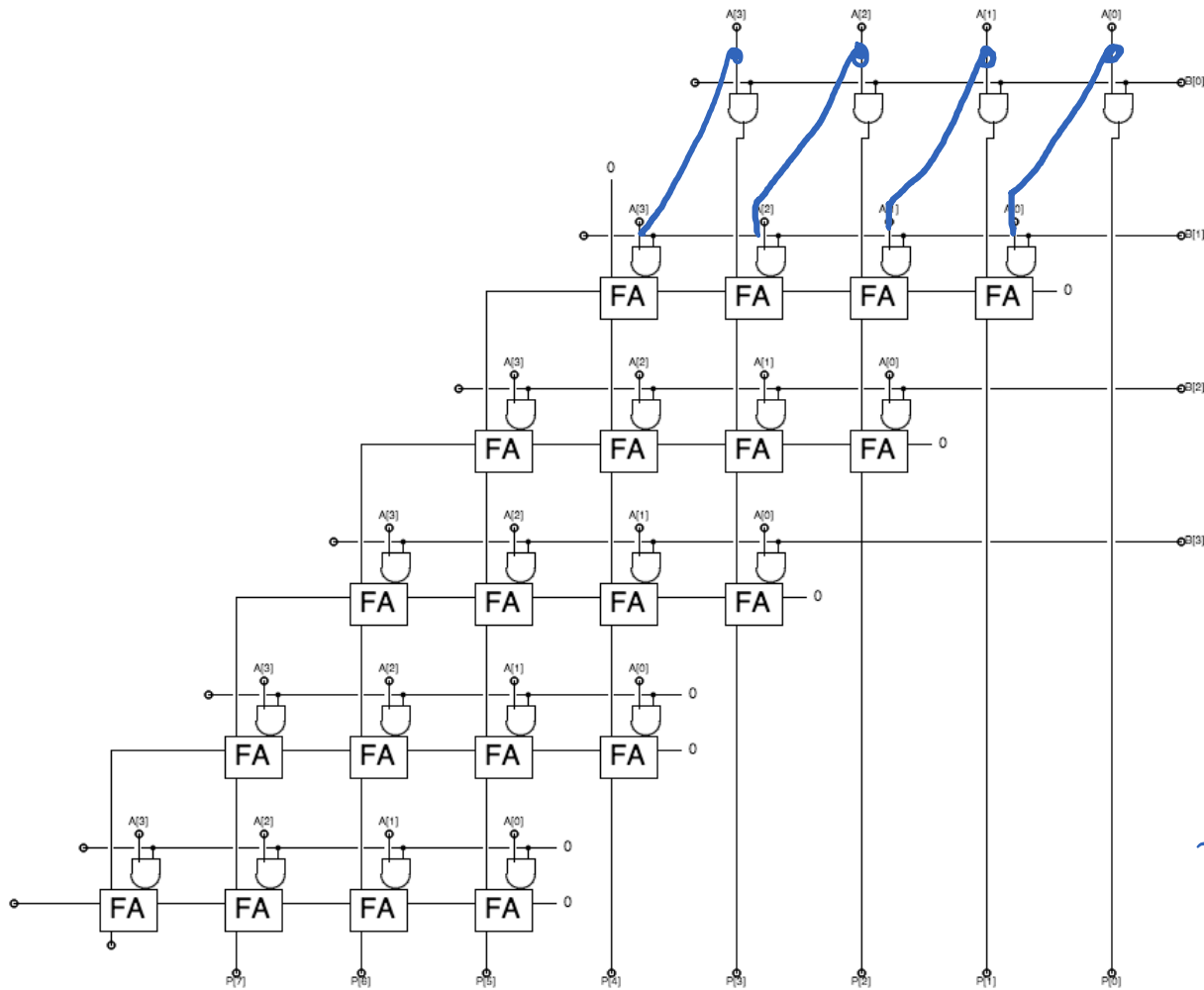
						a3	a2	a1	a0
						b3	b2	b1	b0
						<hr/>			
						b0a3	b0a2	b0a1	b0a0
+				b1a3	b1a2	b1a1	b1a0		0
+		b2a3	b2a2	b2a1	b2a0		0		0
+	b3a3	b3a2	b3a1	b3a0		0	0		0
	<hr/>								
p7	p6	p5	p4	p3	p2	p1	p0		

# Combinational Multiplication



					a3	a2	a1	a0
			x		b3	b2	b1	b0
				b0a3	b0a2	b0a1	b0a0	
+			b1a3	b1a2	b1a1	b1a0		0
+		b2a3	b2a2	b2a1	b2a0		0	0
+	b3a3	b3a2	b3a1	b3a0		0	0	0
p7	p6	p5	p4	p3	p2	p1	p0	

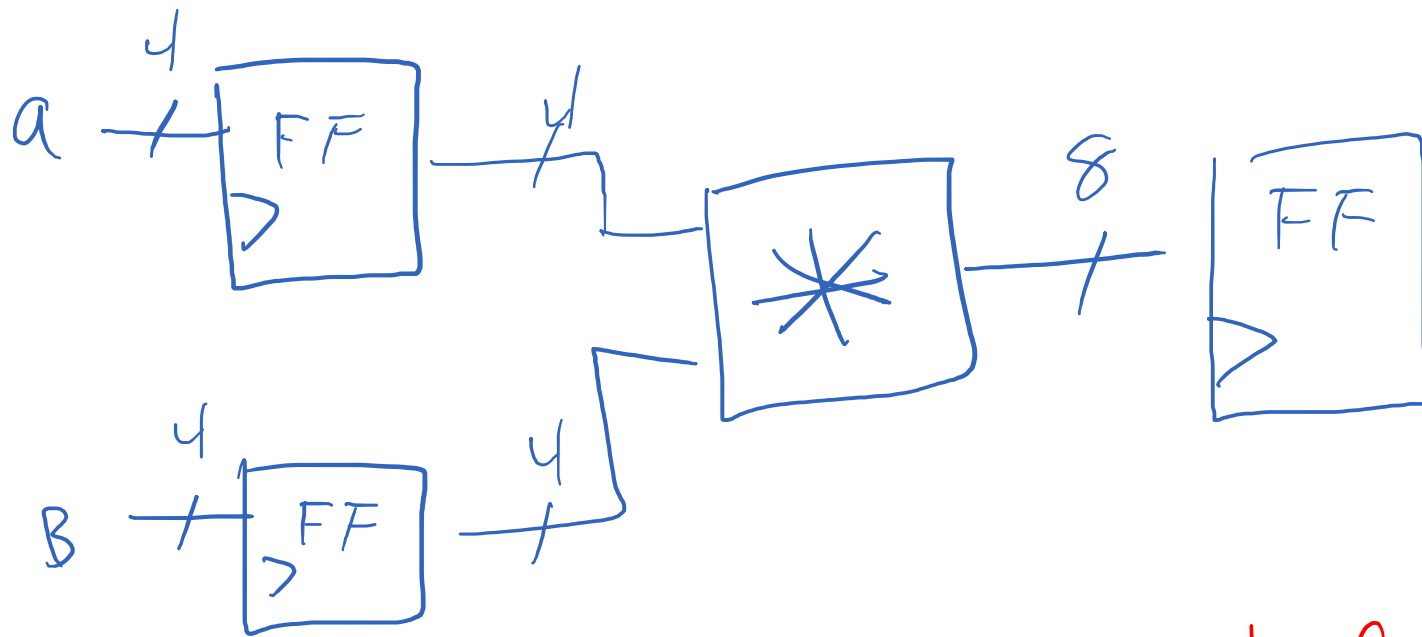
# Combinational Multiplication



					a3	a2	a1	a0
			x		b3	b2	b1	b0
				b0a3	b0a2	b0a1	b0a0	
+			b1a3	b1a2	b1a1	b1a0		0
+		b2a3	b2a2	b2a1	b2a0		0	0
+	b3a3	b3a2	b3a1	b3a0		0	0	0
p7	p6	p5	p4	p3	p2	p1	p0	

$$\begin{array}{rcccccc}
 & & b_0 a_3 & b_0 a_2 & b_0 a_1 & b_0 a_0 \\
 + & b_1 a_3 & b_1 a_2 & b_1 a_1 & b_1 a_0 & 0 \\
 \hline
 & q_4 & q_3 & q_2 & q_1 & q_0 \\
 + & b_2 a_3 & b_2 a_2 & b_2 a_1 & b_2 a_0 & \\
 \hline
 r_5 & r_4 & r_3 & r_2 & r_1 & r_0
 \end{array}$$

# Problems with Combinational Multiplication

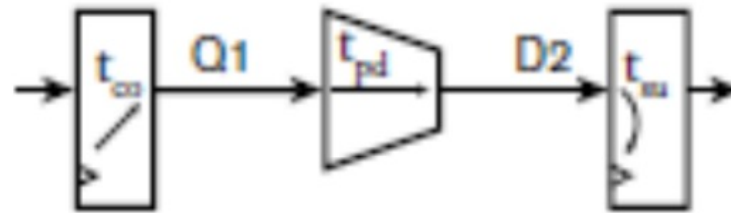


not ready before clock edge

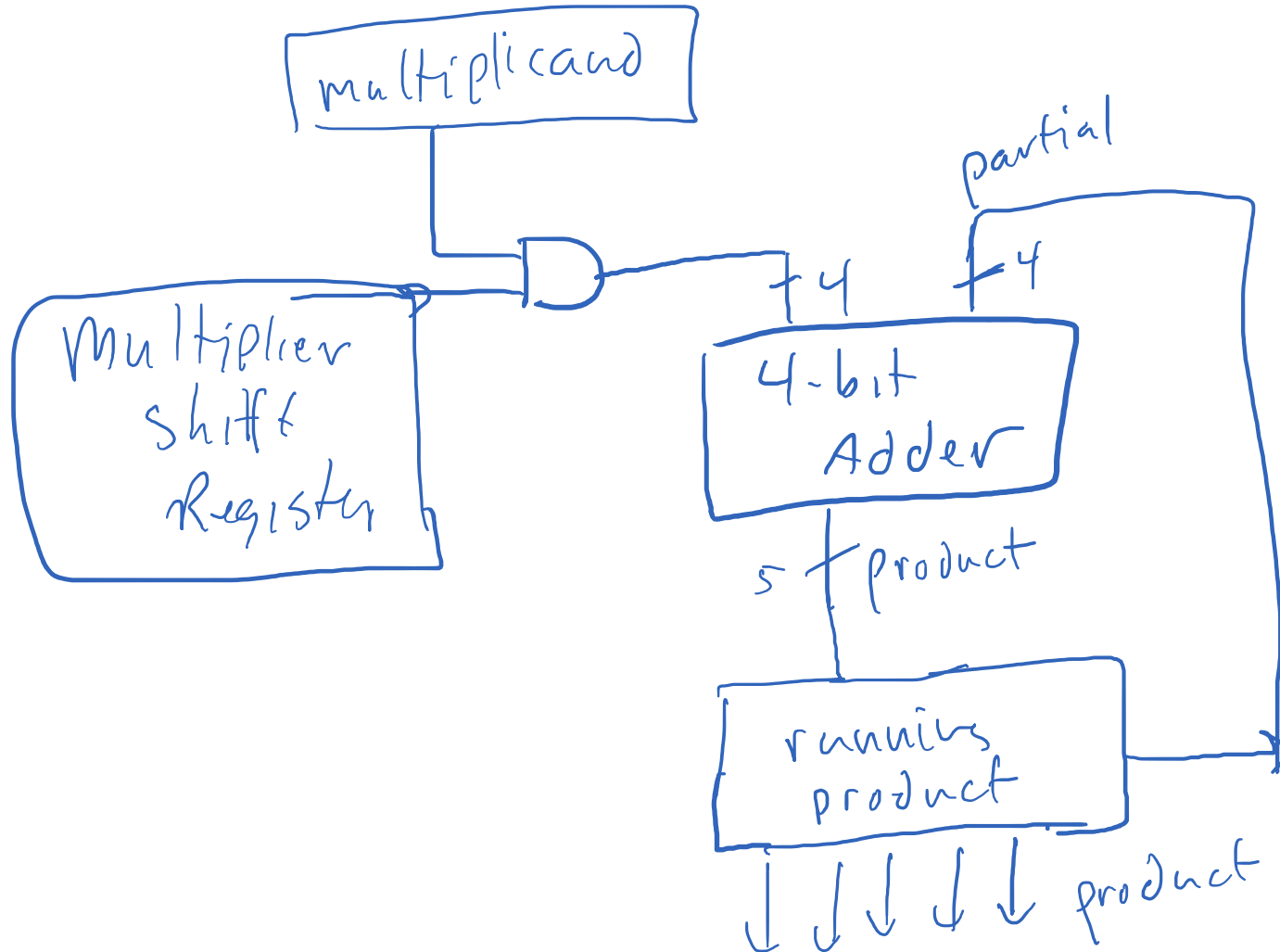
$\Rightarrow$  Setup time Violation

# Problems with Combinational Multiplication

- Setup Time Violations!



# Sequential Multiplication



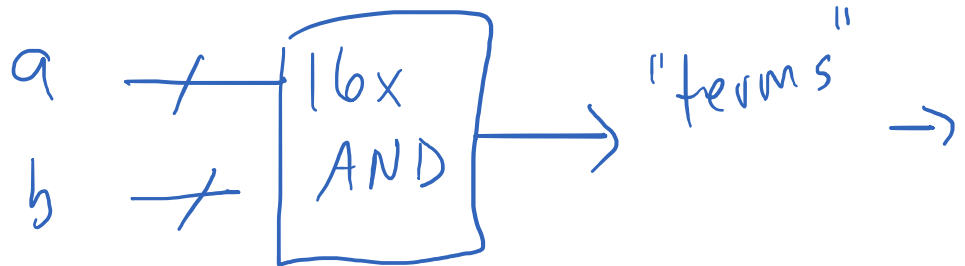
sedo Demo

					a3	a2	a1	a0
				x	b3	b2	b1	b0
					<hr/>			
					b0a3	b0a2	b0a1	b0a0
+			b1a3	b1a2	b1a1	b1a0		0
+		b2a3	b2a2	b2a1	b2a0		0	0
+	b3a3	b3a2	b3a1	b3a0		0	0	0
<hr/>								
p7	p6	p5	p4	p3	p2	p1	p0	

clk 1

+	b0a3	b0a2	b0a1	b0a0
	0	0	0	0

# Hybrid Multiplication



clock 1

# Partial products

$$\begin{array}{r} \text{clk}^2 \\ w + x = q \\ \hline y + z = r \end{array}$$

clk3

$$g+r = \text{product}$$

partial products

					a3	a2	a1	a0
				x	b3	b2	b1	b0
					<hr/>			
					b0a3	b0a2	b0a1	b0a0
+			b1a3	b1a2	b1a1	b1a0		0
+		b2a3	b2a2	b2a1	b2a0		0	0
+	b3a3	b3a2	b3a1	b3a0		0	0	0
	<hr/>							
p7	p6	p5	p4	p3	p2	p1	p0	

partial products

partial products



# Pipelined Multiplication

Addr 1

$$\begin{array}{r}
 b_0 a_3 \quad b_0 a_2 \quad b_0 a_1 \quad b_0 a_0 \\
 + b_1 a_3 \quad b_1 a_2 \quad b_1 a_1 \quad b_1 a_0 \quad 0 \\
 \hline
 q_4 \quad q_3 \quad q_2 \quad q_1 \quad q_0
 \end{array}$$

Addr 2

$$\begin{array}{r}
 b_2 a_3 \quad b_2 a_2 \quad b_2 a_1 \quad b_2 a_0 \\
 + b_3 a_3 \quad b_3 a_2 \quad b_3 a_1 \quad b_3 a_0 \quad 0 \\
 \hline
 r_4 \quad r_3 \quad r_2 \quad r_1 \quad r_0
 \end{array}$$

$a_3 \ a_2 \ a_1 \ a_0$

$\times \ b_3 \ b_2 \ b_1 \ b_0$

$$\begin{array}{r}
 b_0 a_3 \quad b_0 a_2 \quad b_0 a_1 \quad b_0 a_0 \\
 + b_1 a_3 \quad b_1 a_2 \quad b_1 a_1 \quad b_1 a_0 \quad 0
 \end{array}$$

$\leftarrow w$

$\leftarrow x$

$$\begin{array}{r}
 b_2 a_3 \quad b_2 a_2 \quad b_2 a_1 \quad b_2 a_0 \quad 0 \quad 0 \\
 b_3 a_3 \quad b_3 a_2 \quad b_3 a_1 \quad b_3 a_0 \quad 0 \quad 0
 \end{array}$$



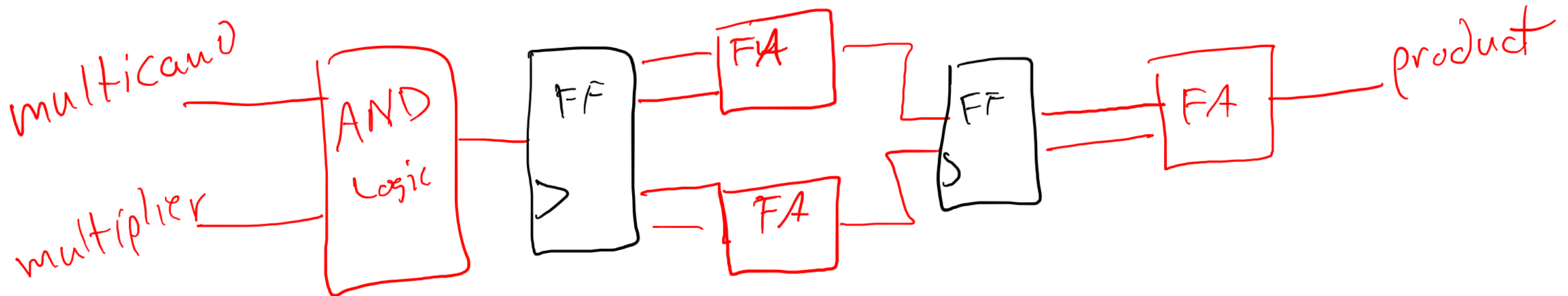
$q_4 \ q_3 \ q_2 \ q_1 \ q_0$

$+ \ r_4 \ r_3 \ r_2 \ r_1 \ r_0 \ 0 \ 0$

$p_7 \ p_6 \ p_5 \ p_4 \ p_3 \ p_2 \ p_1 \ p_0$

# Even More Multipliers

- Wallace Tree Multiplier
- Dadda Tree Multiplier
- Baugh–Wooley Tree Multiplier  $e \cdot d$



# Next Time

- Binary Division
- More Pipelining

Comb



Seq

