



Outlier

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Outlier

Clustering:

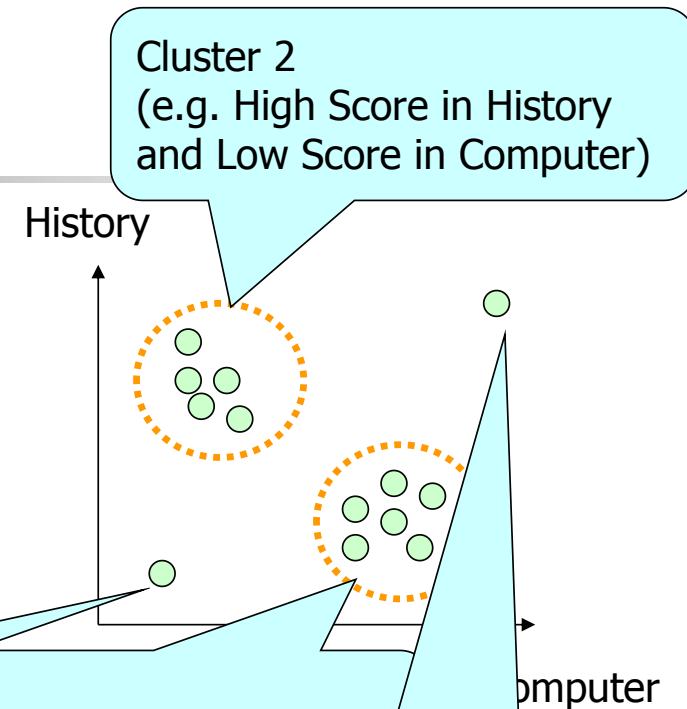
	Computer	History
Raymond	100	40
Louis	90	45
Wyman	20	95
...

Outlier
(e.g. Low Score in Computer
and Low Score in History)

Cluster 1
(e.g. High Score in Computer
and Low Score in History)

Outlier
(e.g. High Score in Computer
and High Score in History)

Problem: to find all outliers





Outlier

- Applications

- Fraud Detection

- Detect unusual usage of credit cards or telecommunication services

- Medical Analysis

- Finding unusual response to various medical treatment

- Customized Marketing

- Customers with extremely low or extremely high incomes

- Network

- A potential network attack

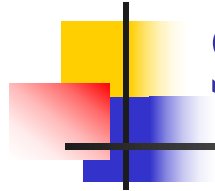
- Software

- A potential bug



Outlier

- Statistical Model
- Distance-based Model
- Density-Based Model

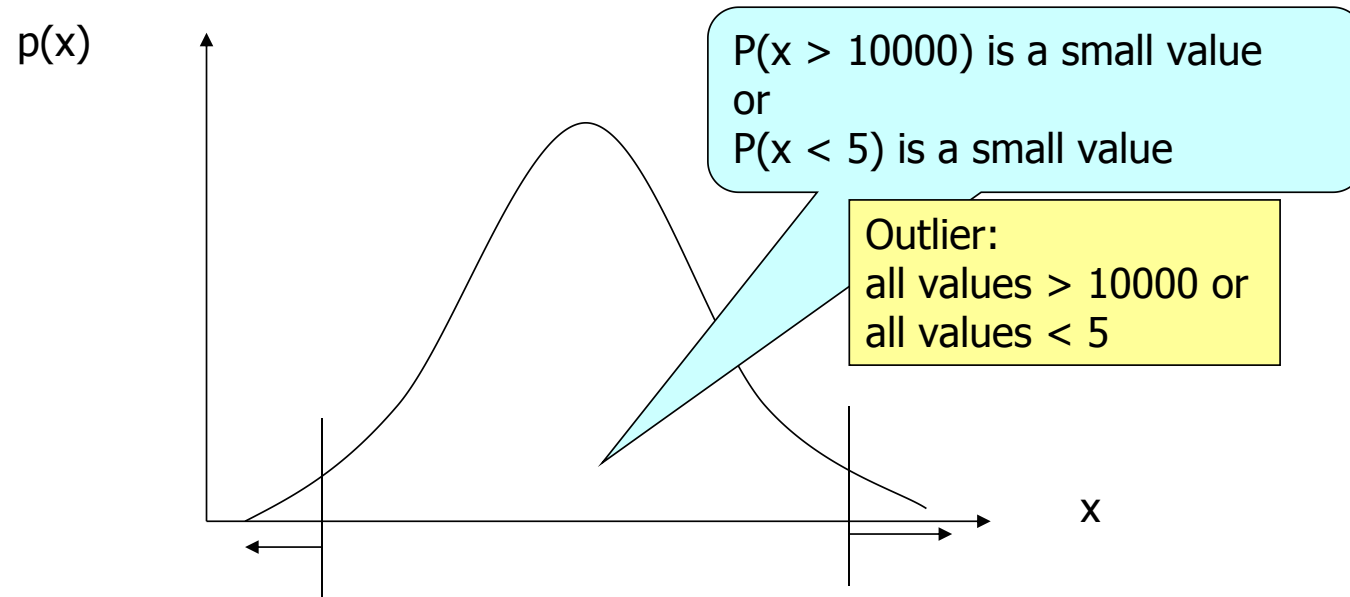


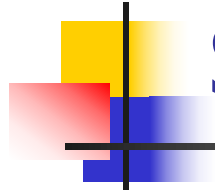
Statistical Model

- An outlier is an observation that is numerically distant from the rest of the data
- E.g.,
 - Consider 1-dimensional data
 - How is a data point considered as an outlier?

Statistical Model

- Assume the 1-dimensional data follows the normal distribution





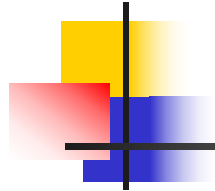
Statistical Model

- Disadvantage
 - Assume that the data follows a particular distribution



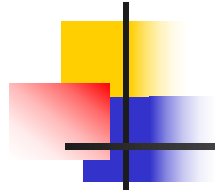
Outlier

- Statistical Model
- Distance-based Model
- Density-Based Model



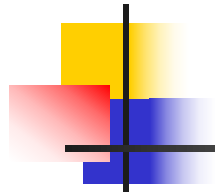
Distance-based Model

- Advantage
 - This model does not assume any distribution
- Idea
 - A point p is considered as an outlier if there are too few data points which are close to p

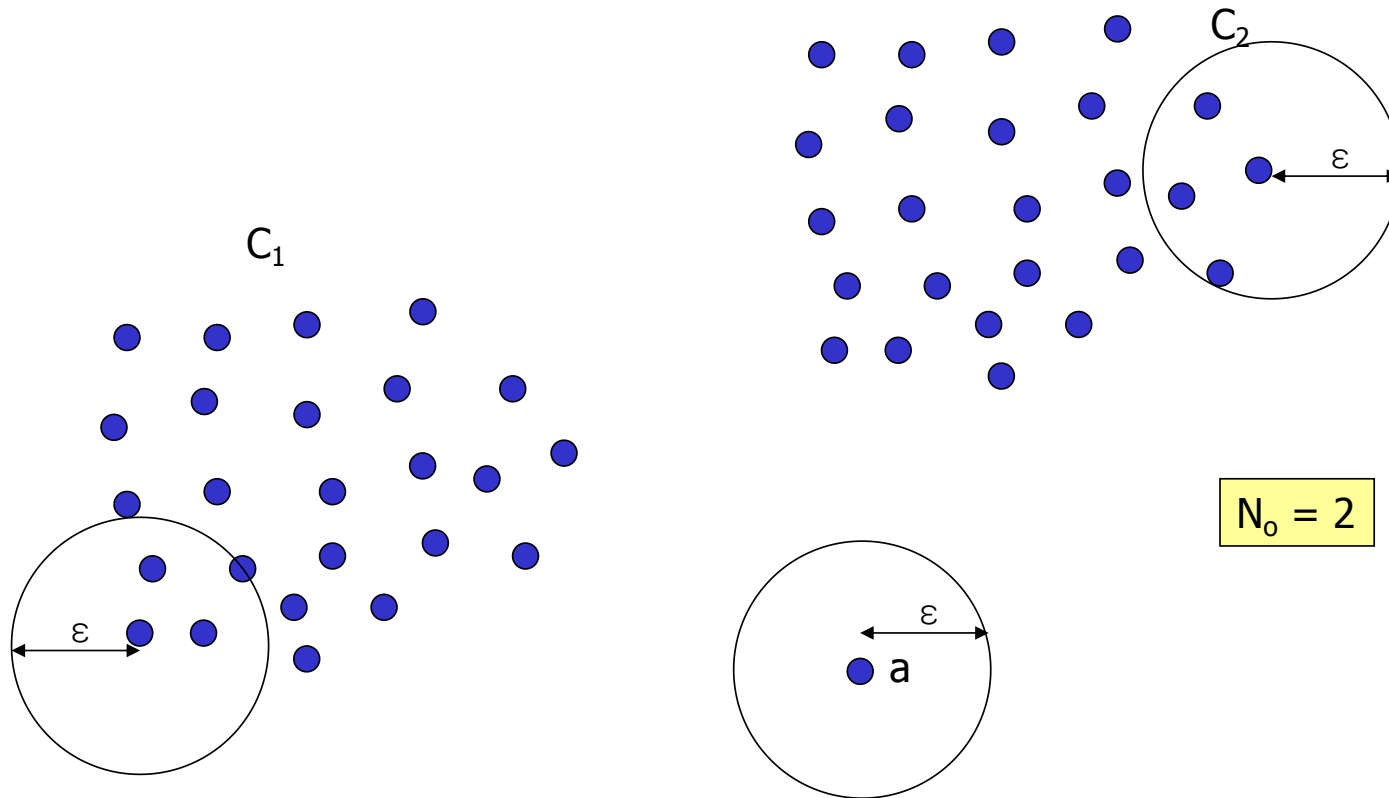


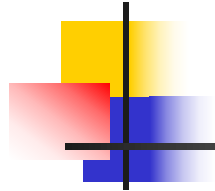
Distance-based Model

- Given a point p and a non-negative real number ε ,
 - the *ε -neighborhood* of point p , denoted by $N(p)$, is the set of points q (including point p itself) such that the distance between p and q is within ε .
- Given a non-negative integer N_0 and a non-negative real number ε
 - A point p is said to be an outlier if
 - $N(p) \leq N_0$



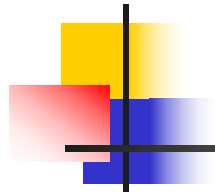
Distance-based Model



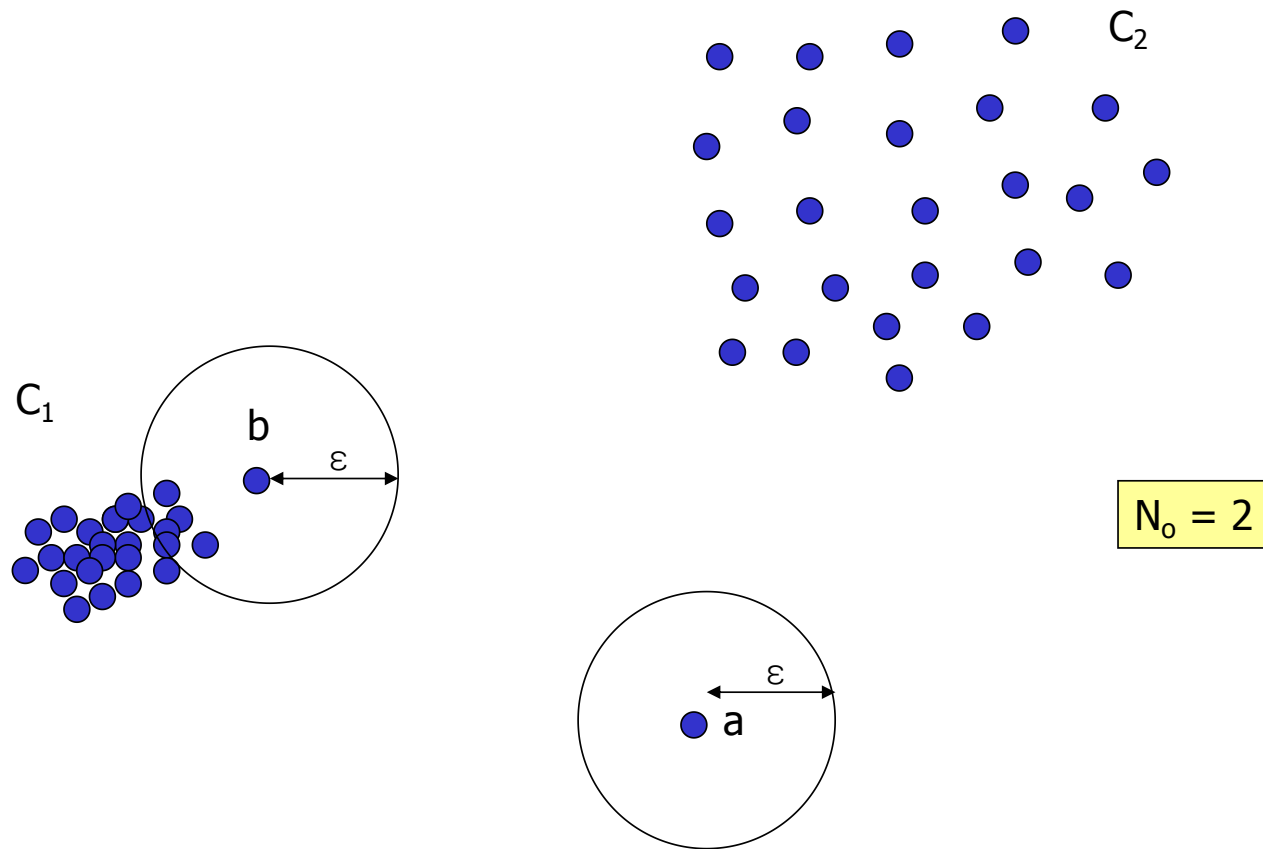


Distance-based Model

- Is the distance-based model “perfect” to find the outliers?



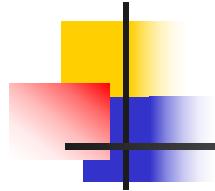
Distance-based Model





Outlier

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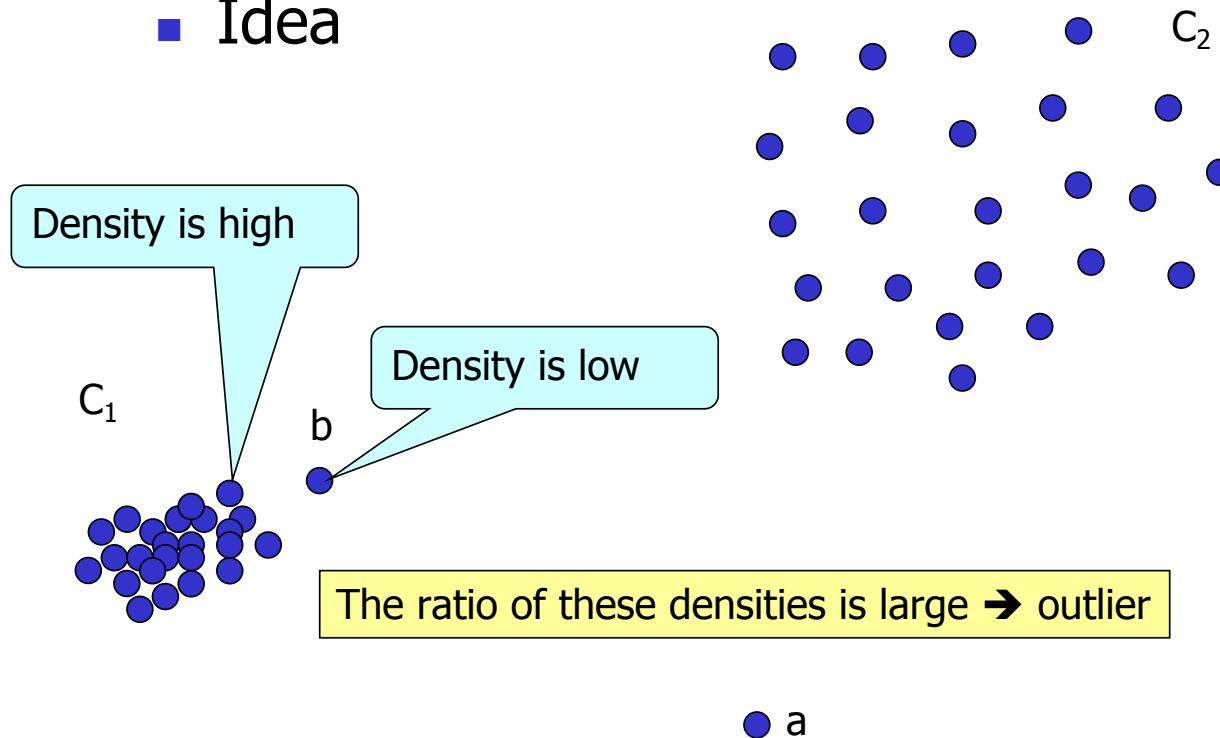


Density-Based Model

- Advantage:
 - This model can find some “local” outliers

Density-Based Model

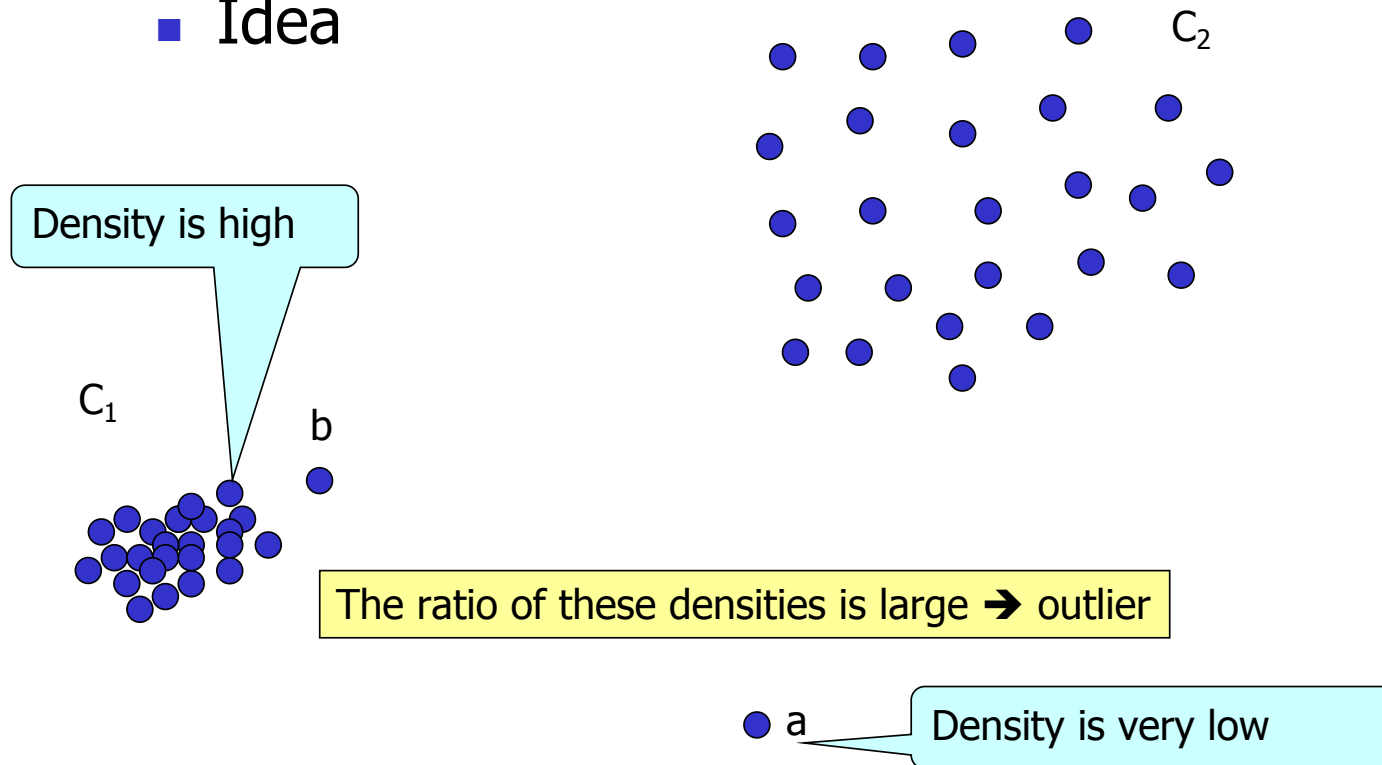
■ Idea





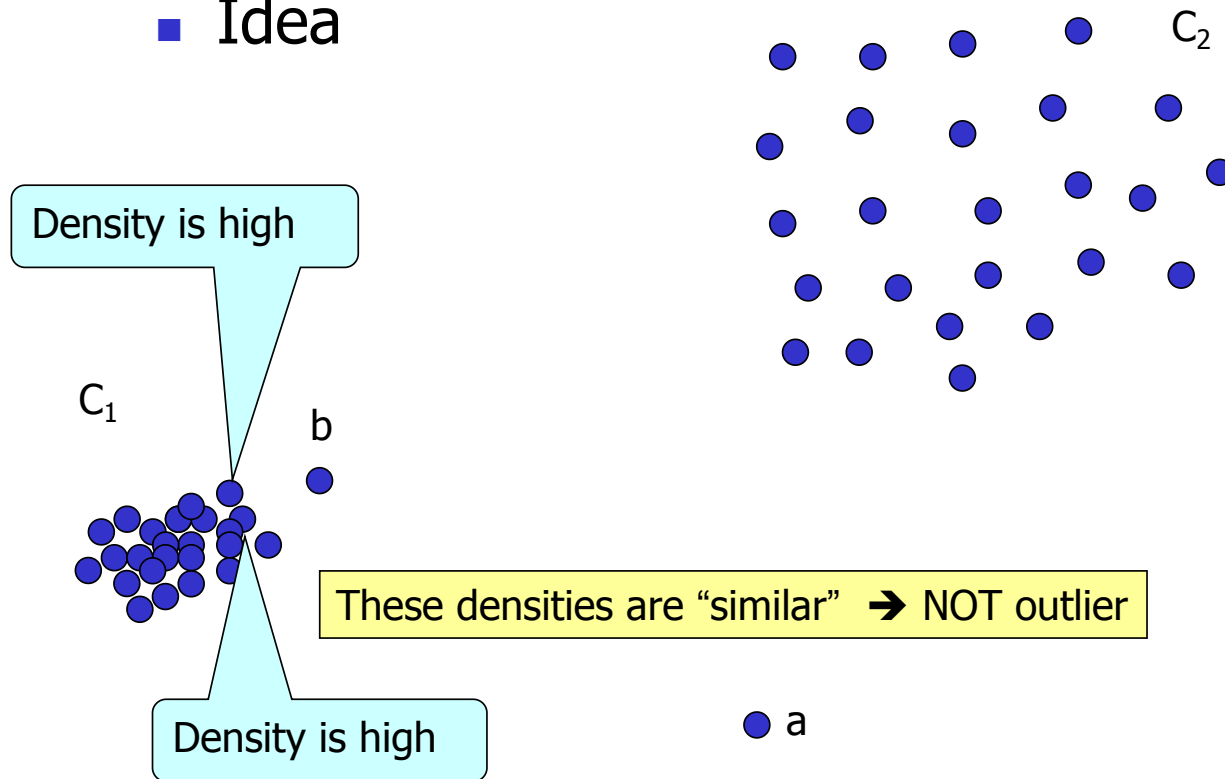
Density-Based Model

- Idea



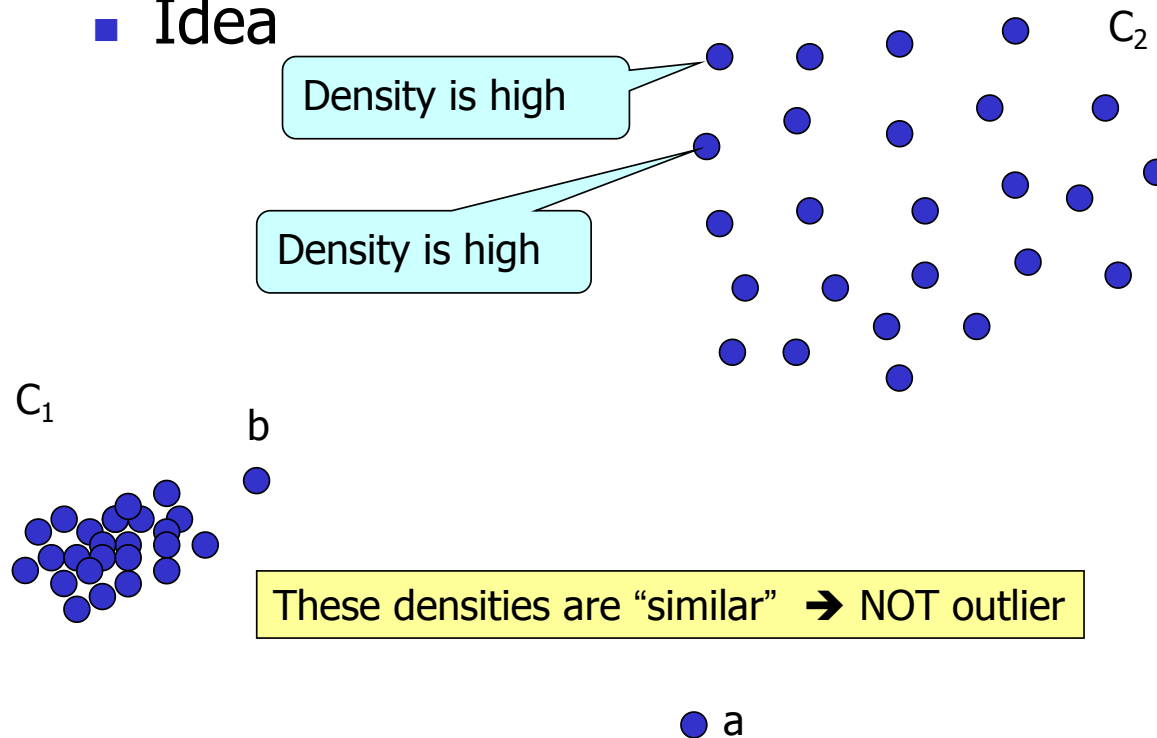
Density-Based Model

- Idea



Density-Based Model

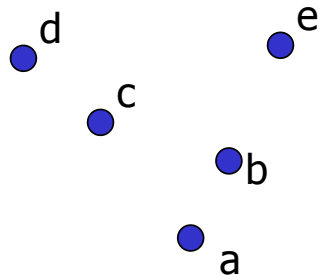
■ Idea





Density-Based Model

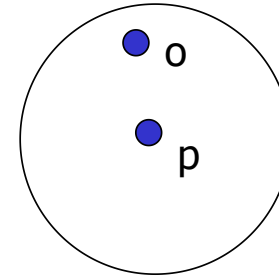
- Formal definition
 - Given an integer k and a point p ,
 - $N_k(p)$ is defined to be the ε -neighborhood of p (excluding point p)
 - where ε is the distance between p and the k -th nearest neighbor



$N_1(a) = ?$

$N_2(a) = ?$

Density-Based Model



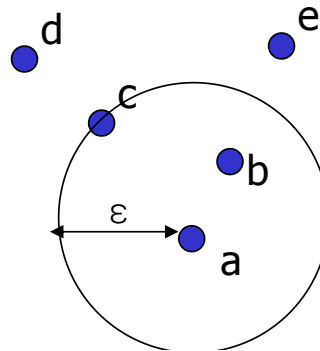
- Reachability Distance of p with respect to o
 - Given two points p and o and an integer k,
 - $\text{Reach_dist}_k(p, o)$ is defined to be $\max\{\text{dist}(p, o), \varepsilon\}$
 - where ε is the distance between p and the k-th nearest neighbor

$\text{Reach_dist}_2(a, b) = ?$

$\text{Reach_dist}_2(a, c) = ?$

$\text{Reach_dist}_2(a, d) = ?$

$\text{Reach_dist}_2(a, e) = ?$

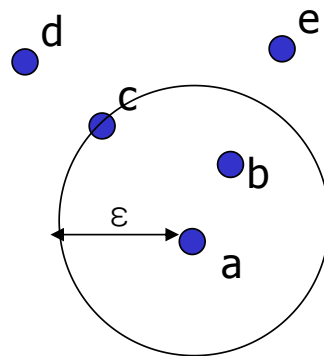


$k = 2$

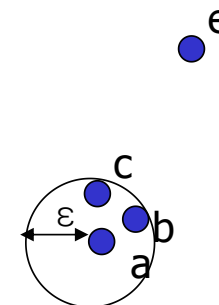
Density-Based Model

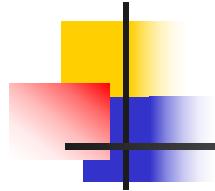
- The **average reachability distance** of p among all k nearest neighbors is equal to ε
 - where ε is the distance between p and the k -th nearest neighbor
- The **local reachability density** of p (denoted by $\text{lrd}_k(p)$) is defined to be $1/\varepsilon$

Why?



$k = 2$





Density-Based Model

- The **local outlier factor (LOF)** of a point p is equal to

$$\frac{\sum_{o \in N_k(p)} \frac{lrd_k(o)}{lrd_k(p)}}{k}$$

Density-Based Model

■ Idea

