

# LECTURE 11: NETWORKS WITH SIGNED EDGES

Prof. Pan Hui

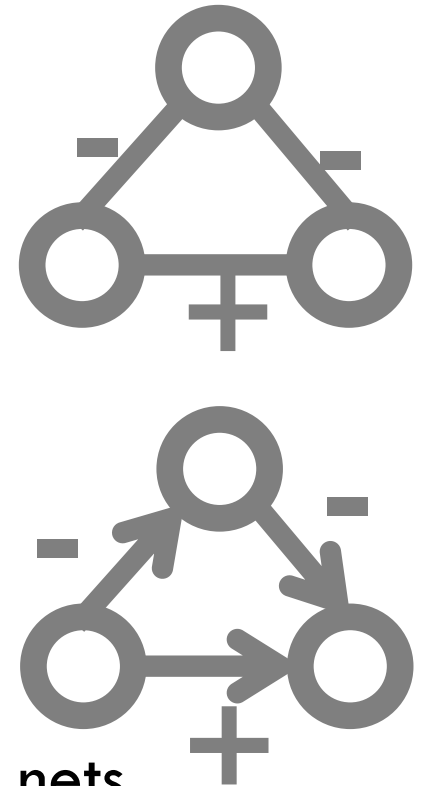
CSIT 6000K: Social Networks and Social Computing: A Data Science Perspective

Thursdays 07:30 PM - 10:20 PM

# Signed Networks

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- Networks with **positive** and **negative** relationships
- Our basic unit of investigation will be **signed triangles**
- First we talk about **undirected** networks then **directed**
- **Plan for this lecture:**
  - **Model:** Consider two soc. theories of signed nets
  - **Data:** Reason about them in large online networks
- **Application:** Predict if A and B are linked with + or -



# Signed Networks

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- Networks with **positive** and **negative** relationships
- Consider an **undirected complete graph**
- Label each edge as either:
  - **Positive**: friendship, trust, positive sentiment, ...
  - **Negative**: enemy, distrust, negative sentiment, ...
- Examine triples of connected nodes A, B, C

# Theory of Structural Balance

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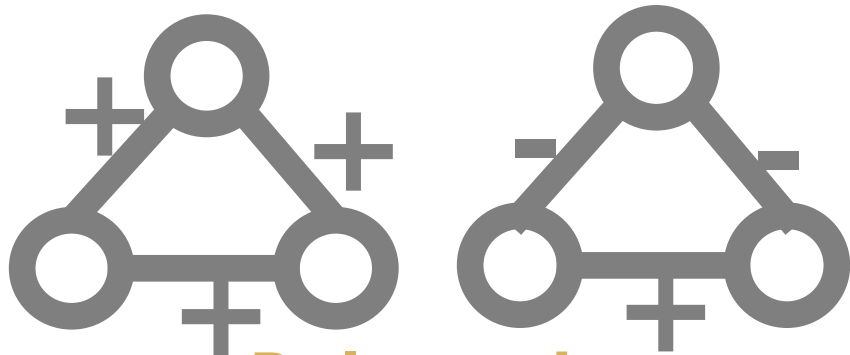
## □ Start with the intuition [Heider '46]:

□ Friend of my friend is my friend

□ Enemy of enemy is my friend

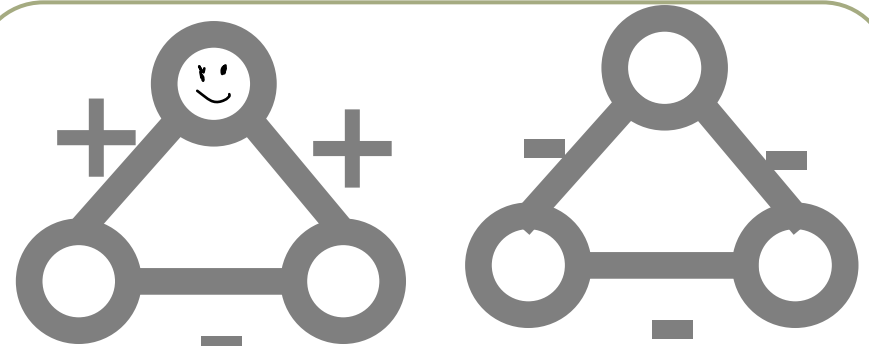
□ Enemy of friend is my enemy

## □ Look at connected triples of nodes:



**Balanced**

**Consistent** with “friend of a friend” or  
“enemy of the enemy” intuition



**Unbalanced**

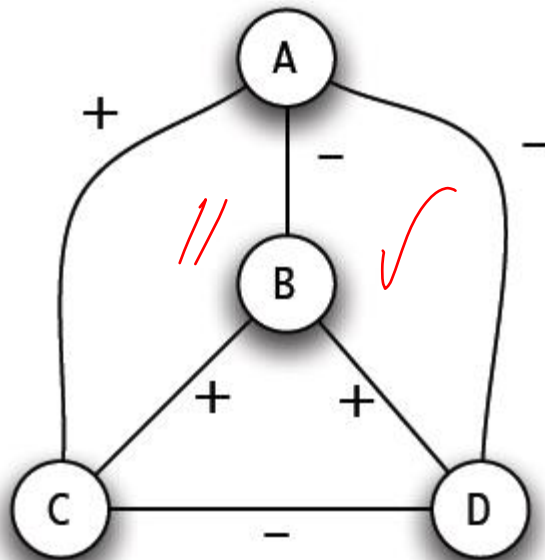
**Inconsistent** with the “friend of a friend” or  
“enemy of the enemy” intuition

# Balanced/Unbalanced Networks

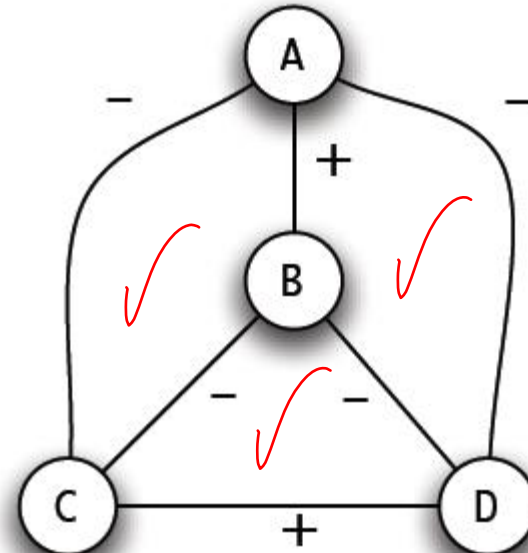
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□ Graph is **balanced** if every connected triple of nodes has:

- All 3 edges labeled +, or
- Exactly 1 edge labeled +



Unbalanced

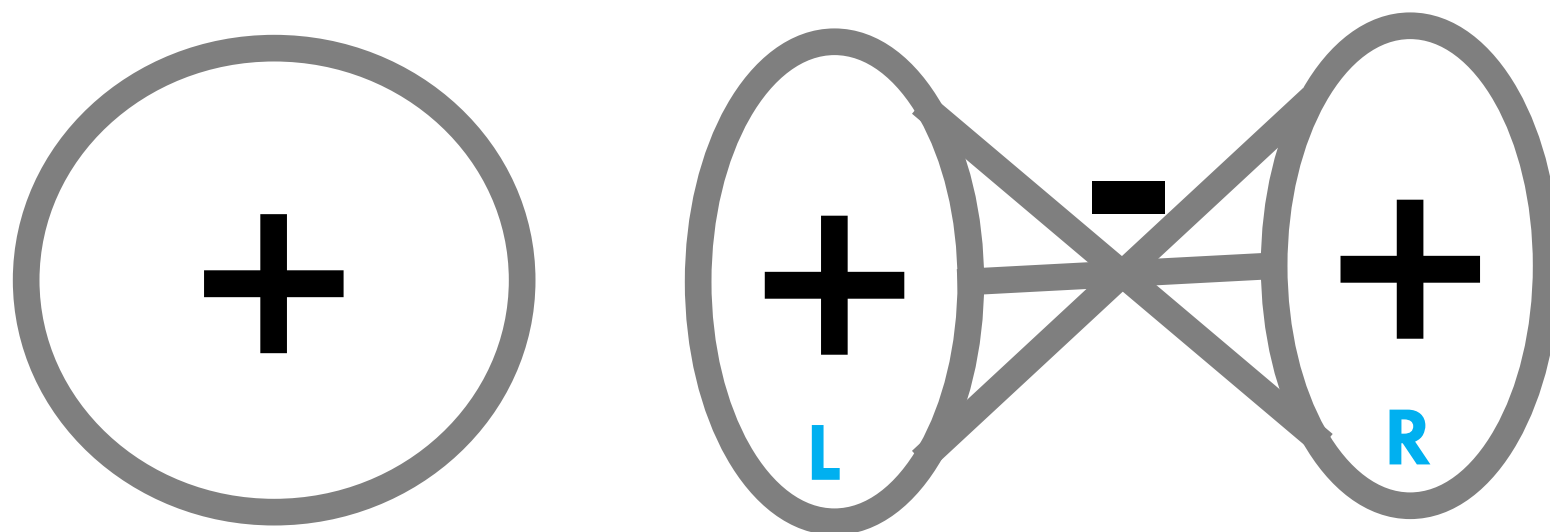


Balanced

# Local Balance $\rightarrow$ Global Factions

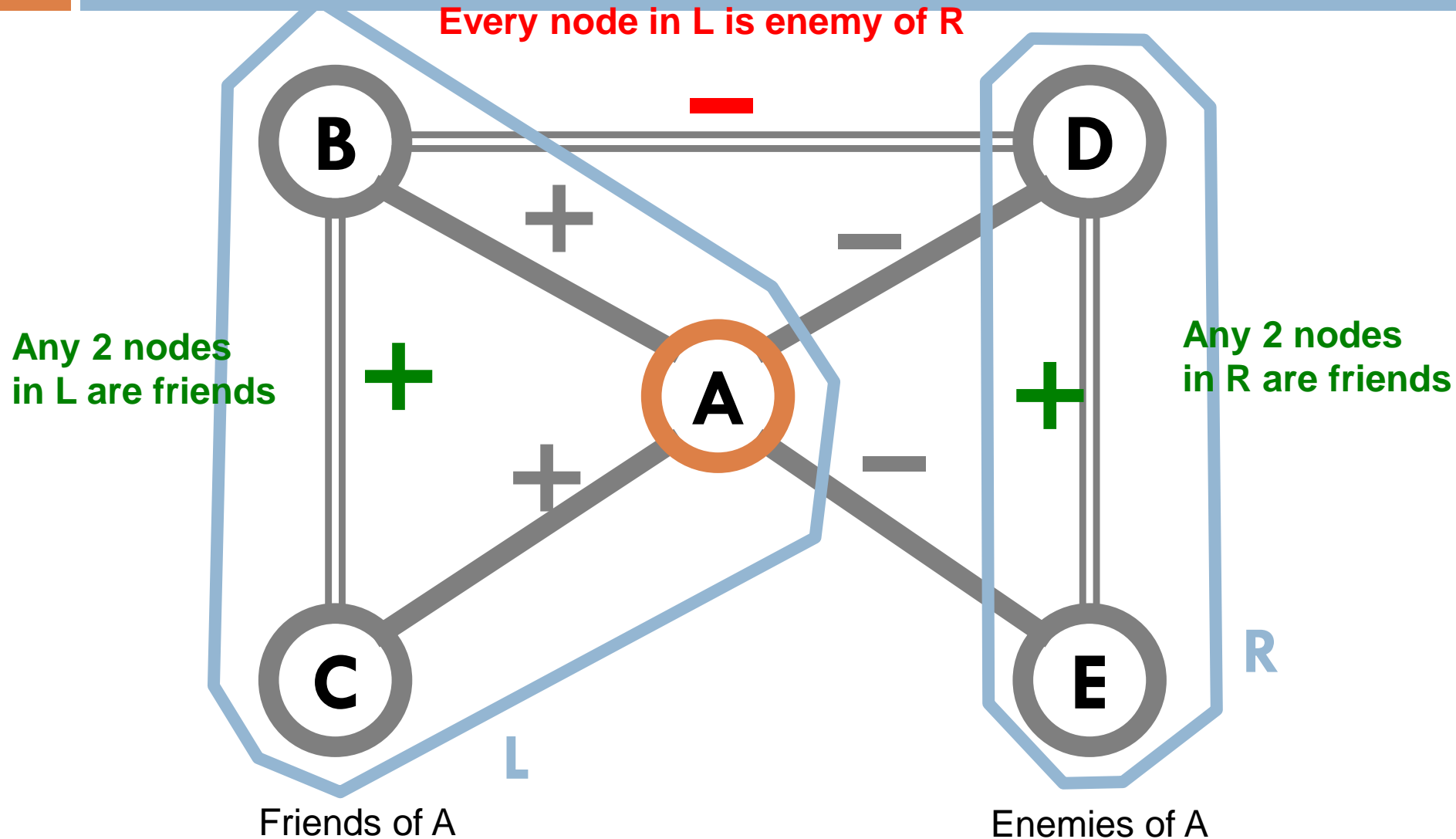
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- **Balance implies global coalitions** [Cartwright-Harary]
- If **all triangles are balanced**, then either:
  - The network contains only positive edges, or
  - Nodes can be split into 2 sets where negative edges only point between the sets



# Analysis of Balance

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# Example: International Relations

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## □ International relations:

- **Positive** edge: alliance

- **Negative** edge: animosity

## □ Separation of Bangladesh from Pakistan in 1971: US supports Pakistan. Why?

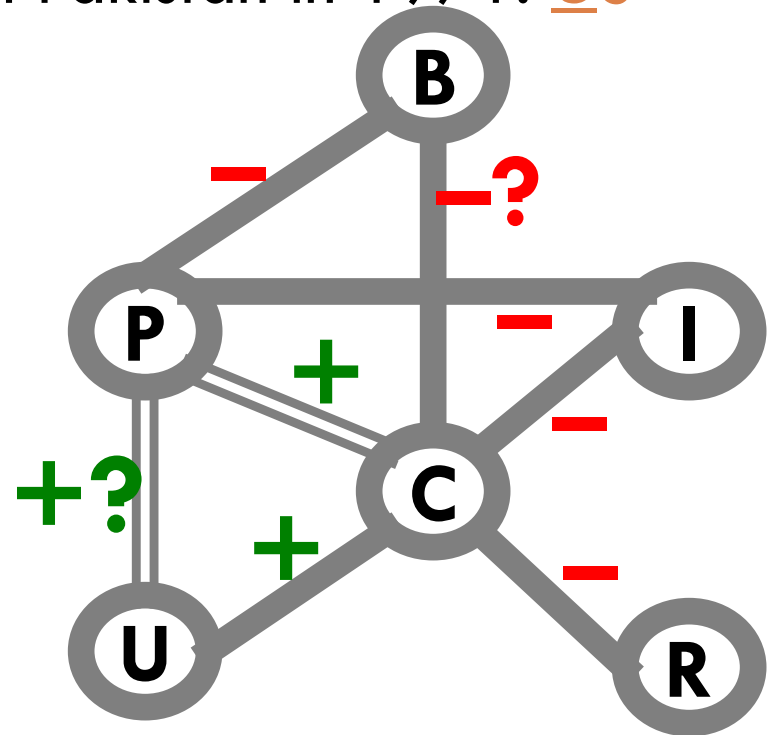
- USSR was enemy of China

- China was enemy of India

- India was enemy of Pakistan

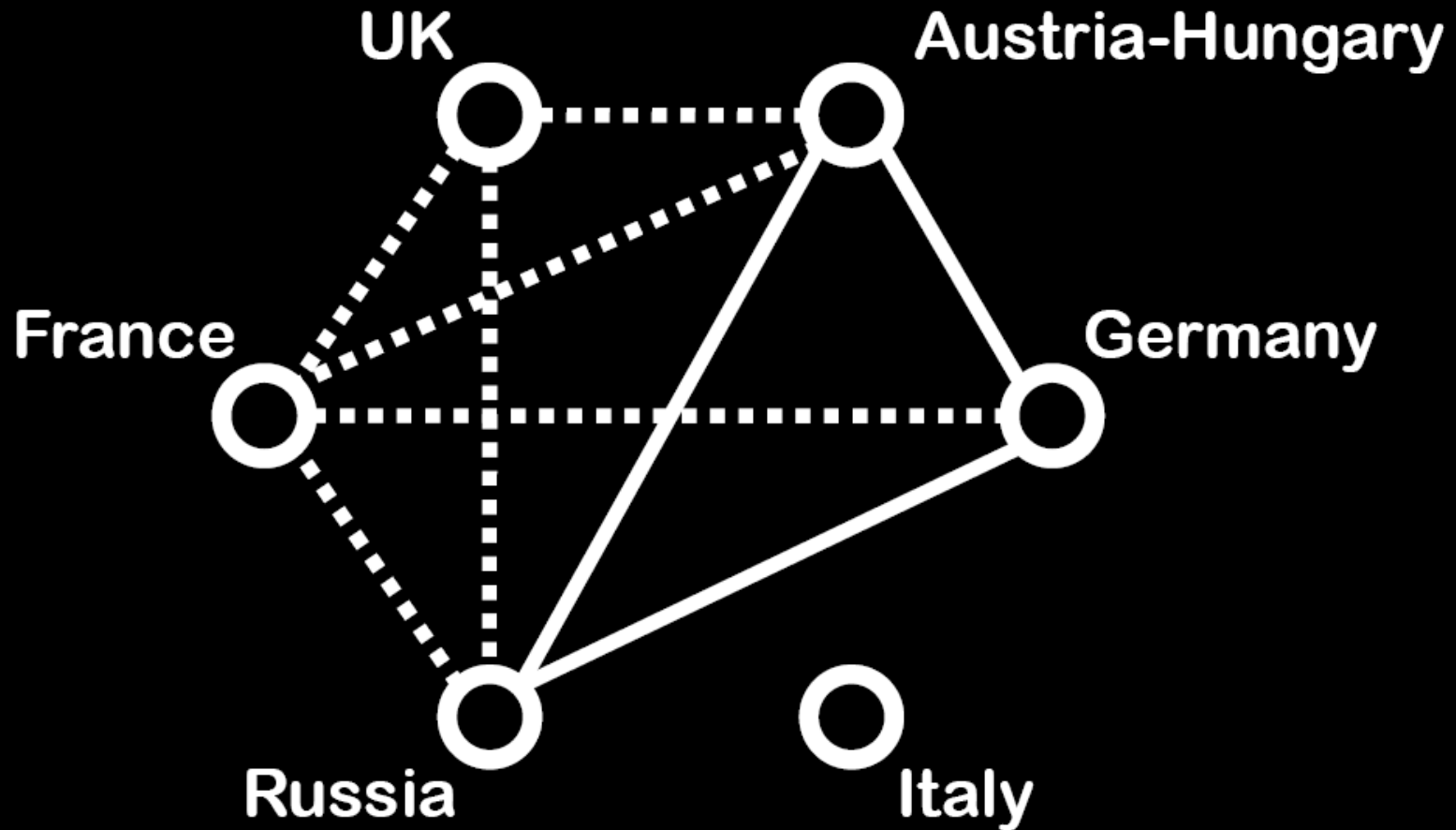
- US was friendly with China

- China vetoed Bangladesh from U.N.

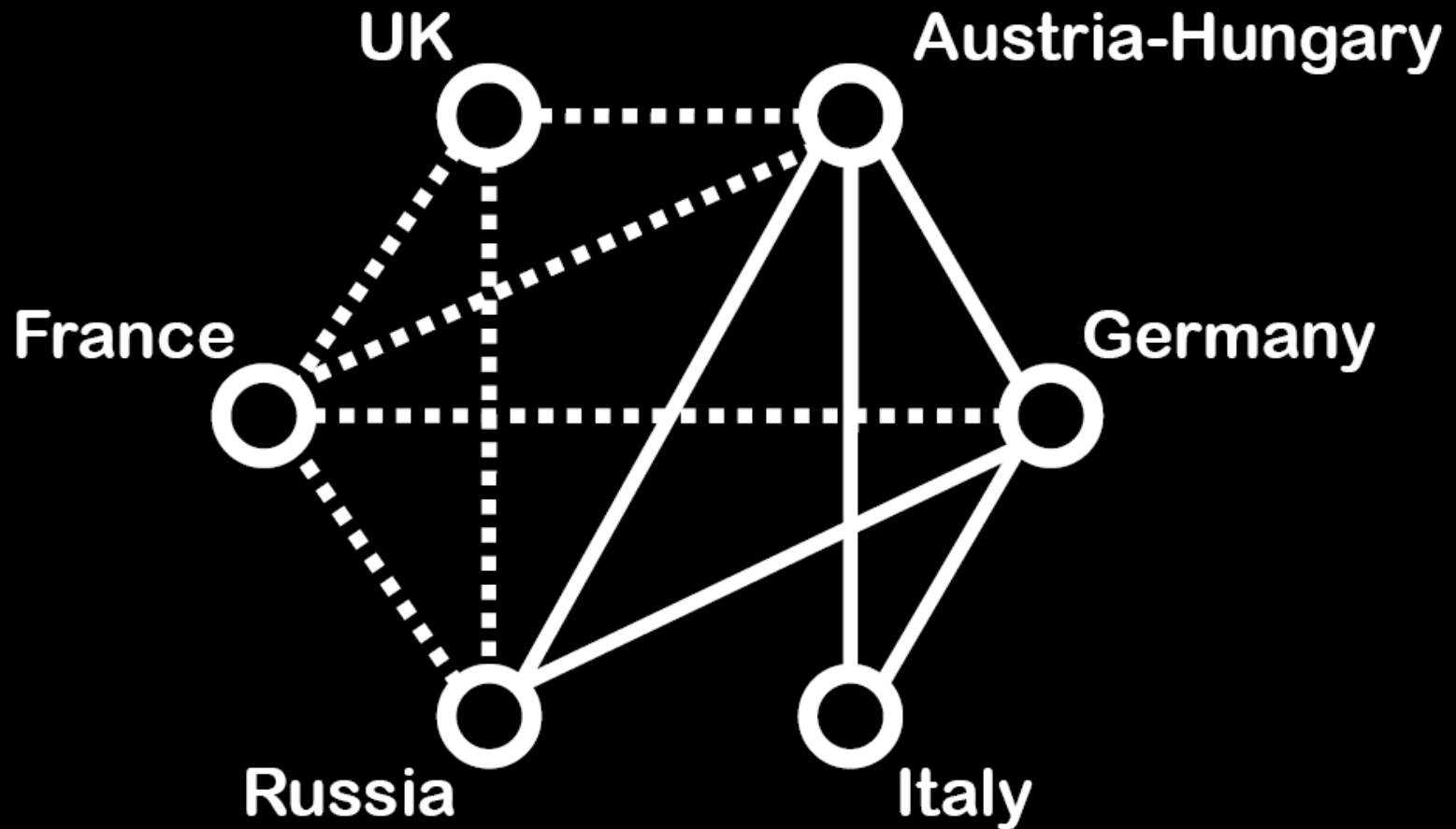




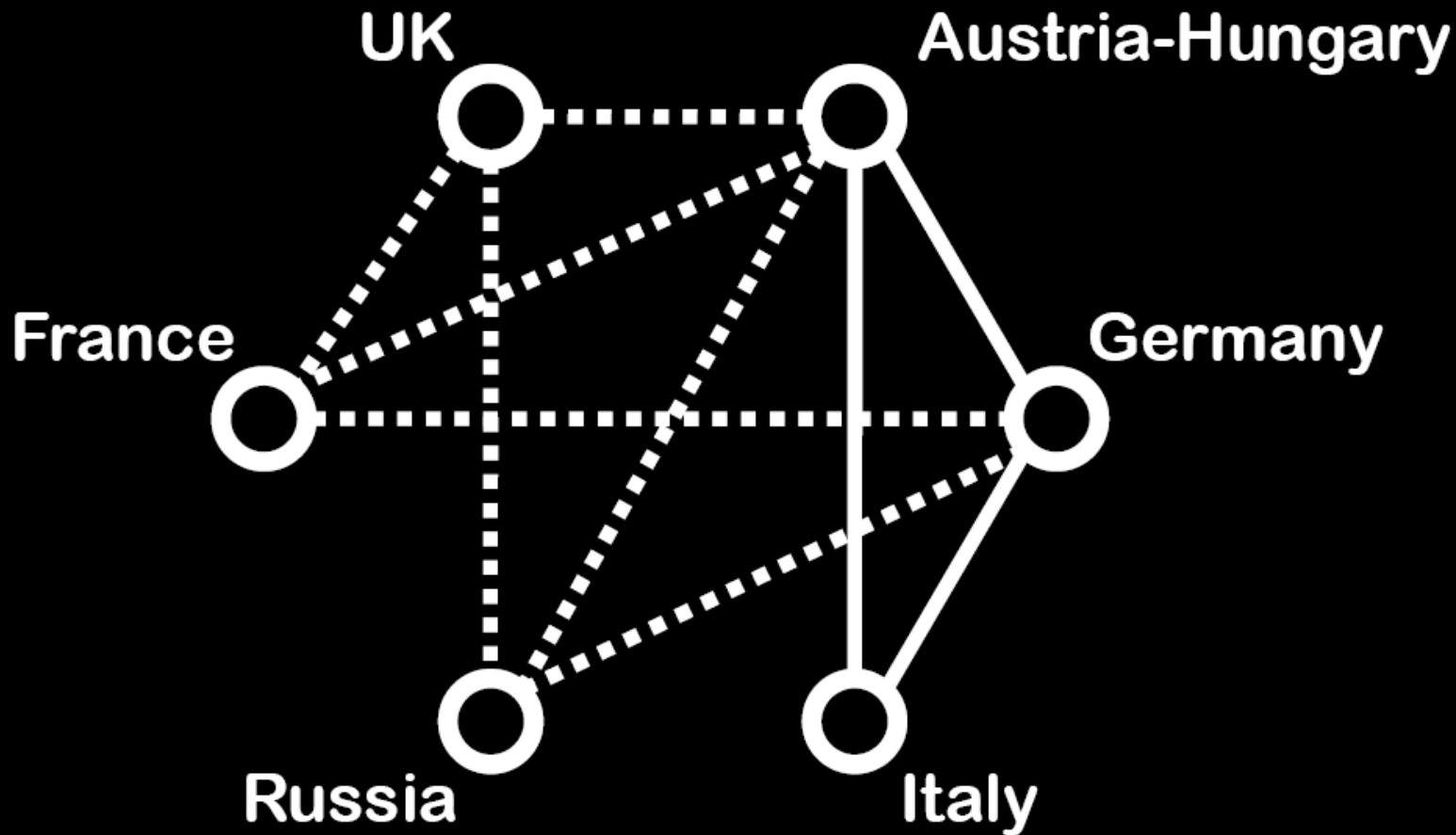
# 1872-1881



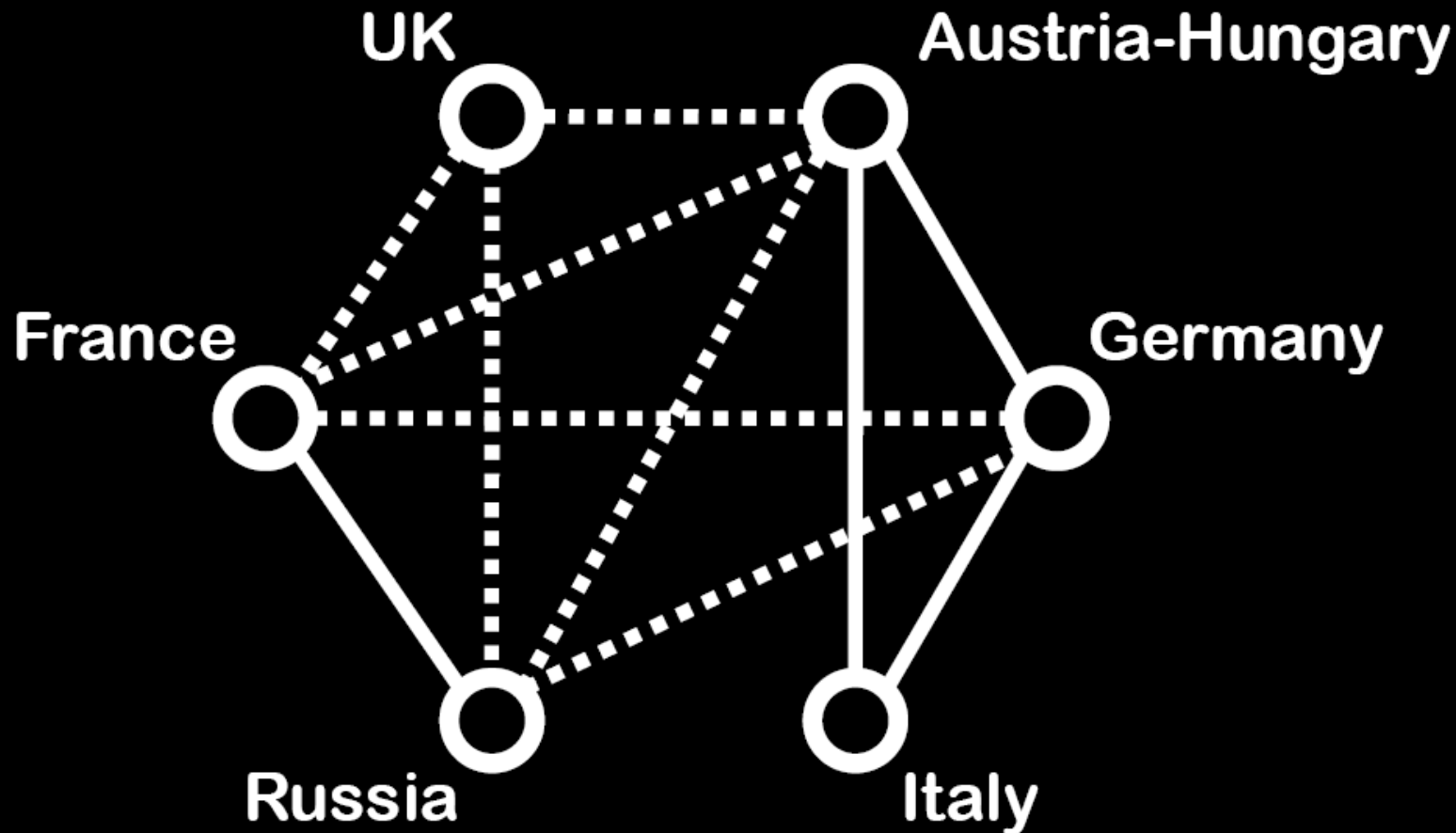
# 1882



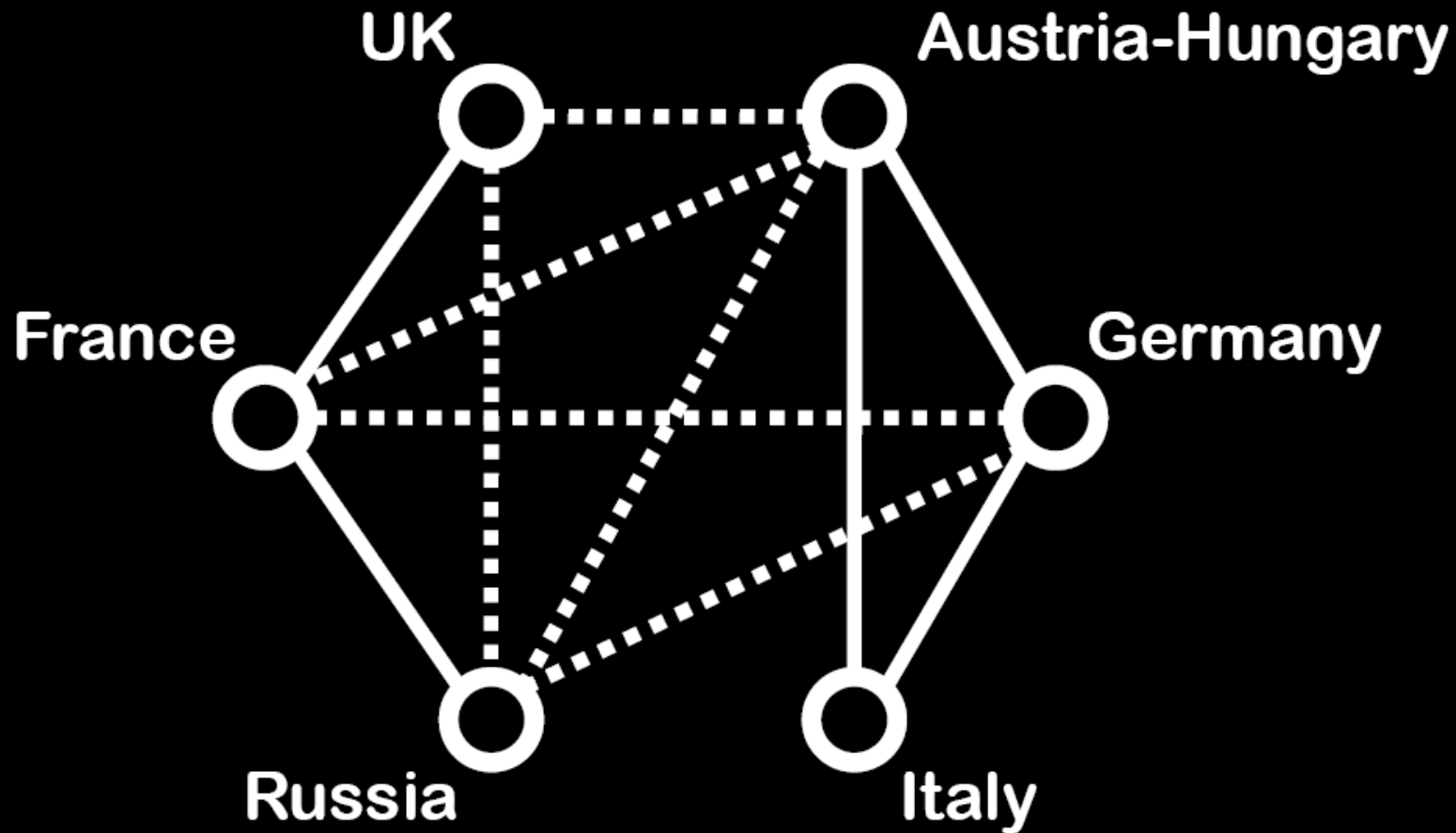
# 1890



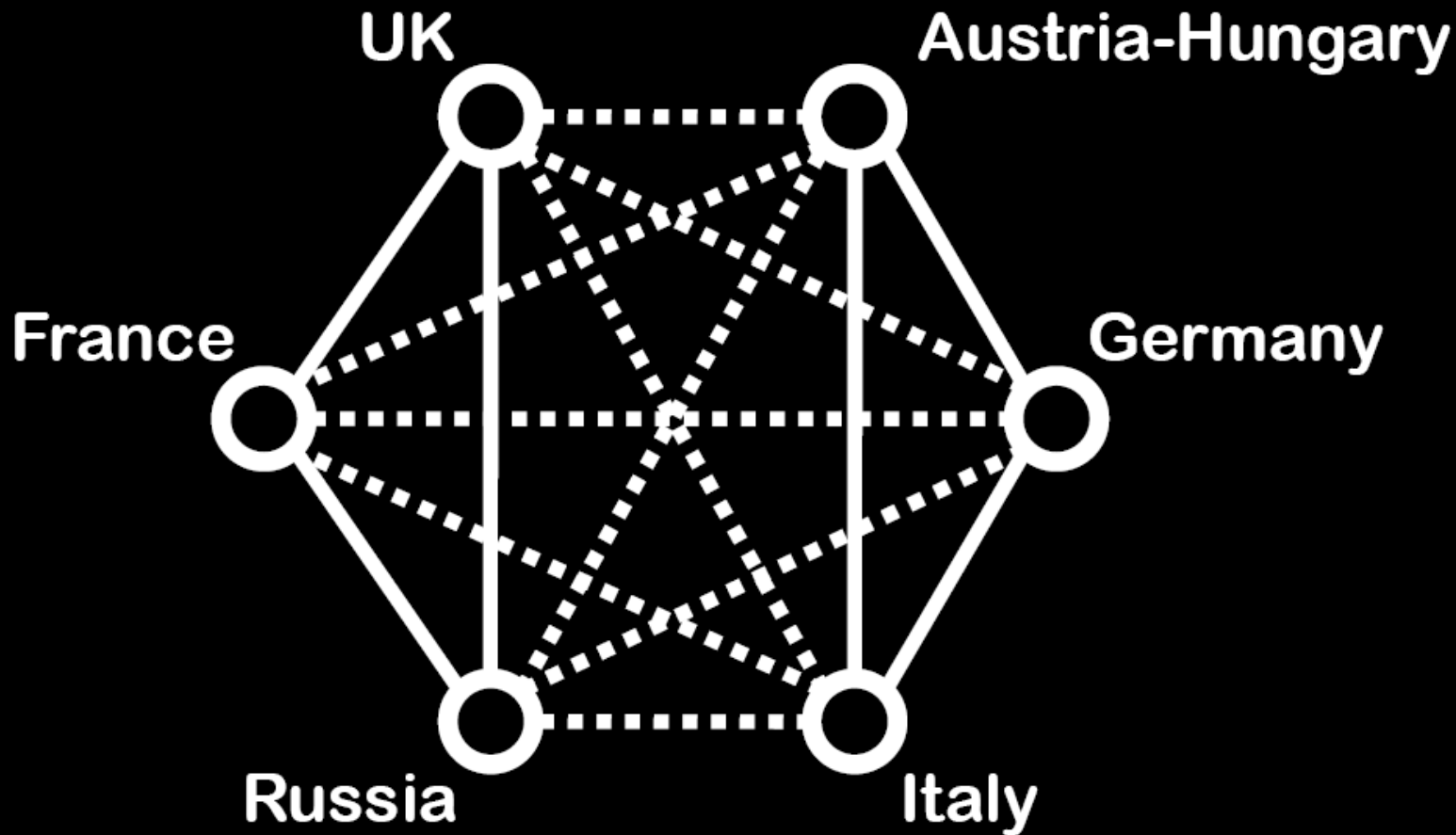
# 1891-1894



# 1904



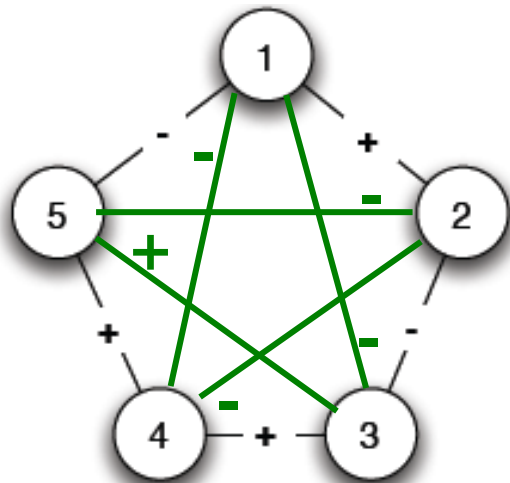
# 1907



# Balance in General Networks

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□ So far we talked about complete graphs



Balanced?

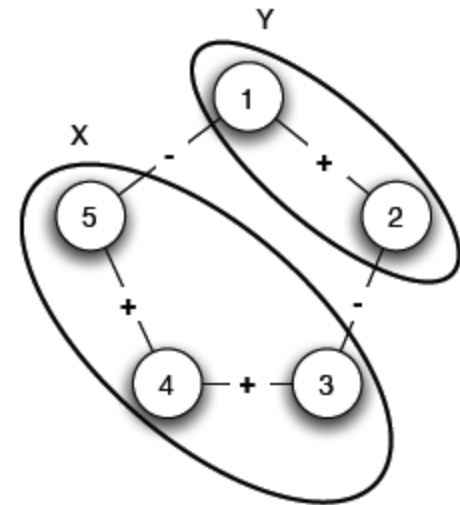
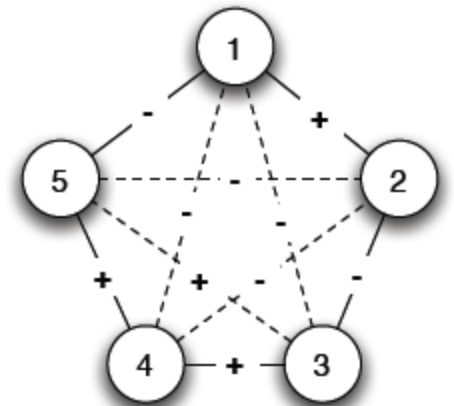
## Def 1: Local view

Fill in the missing edges to achieve balance

## Def 2: Global view

Divide the graph into two coalitions

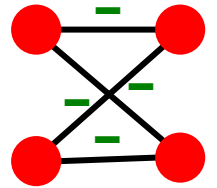
The 2 definitions are **equivalent!**



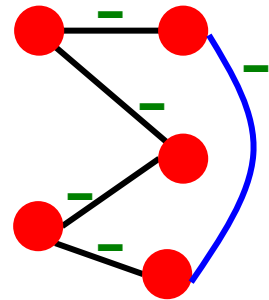
# Is a Signed Network Balanced?

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- Graph is **balanced** if and only if it contains **no cycle with an odd number of negative edges**
- **How to compute this?**
  - Find connected components on  $+$  edges
    - If we find a component of nodes on  $+$  edges that contains a  $-$  edge  $\Rightarrow$  **Unbalanced**
  - For each component create a super-node
  - Connect components A and B if there is a negative edge between the members
  - Assign super-nodes to sides using BFS



Even length cycle

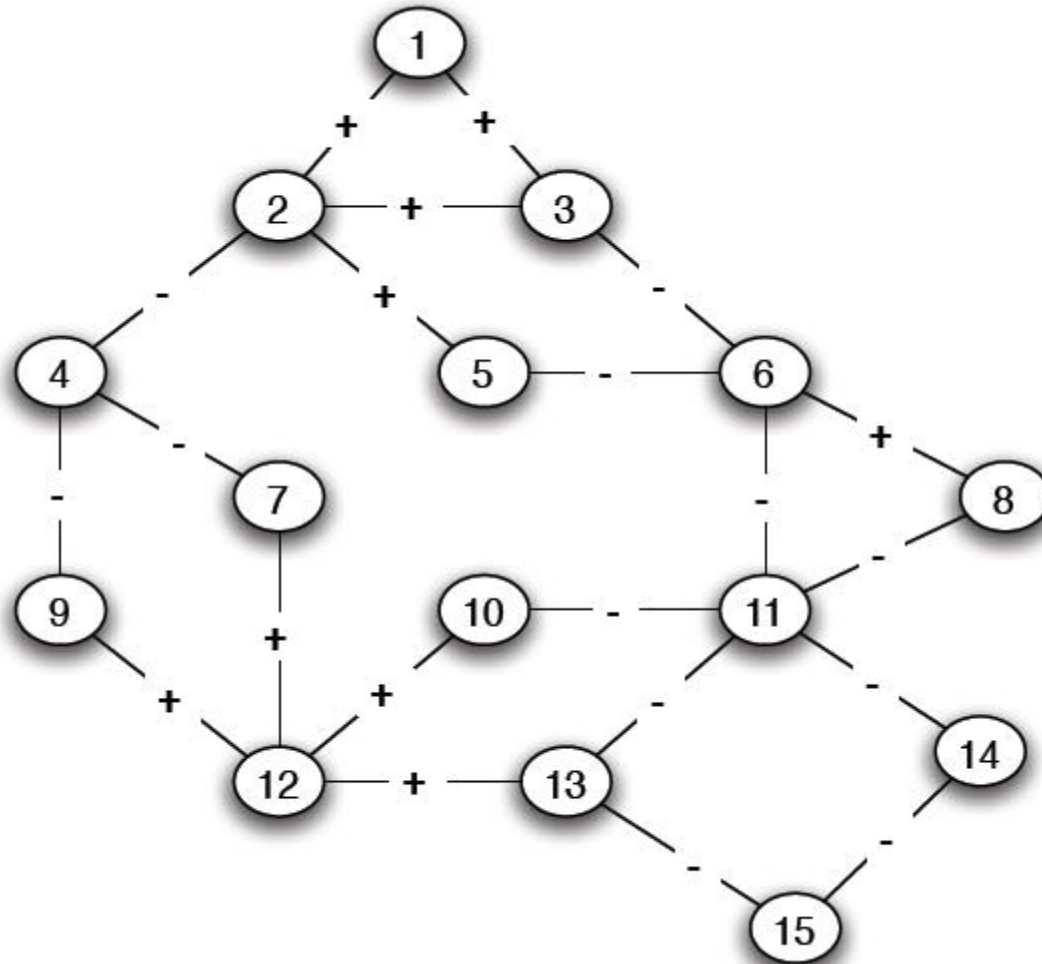


Odd length cycle



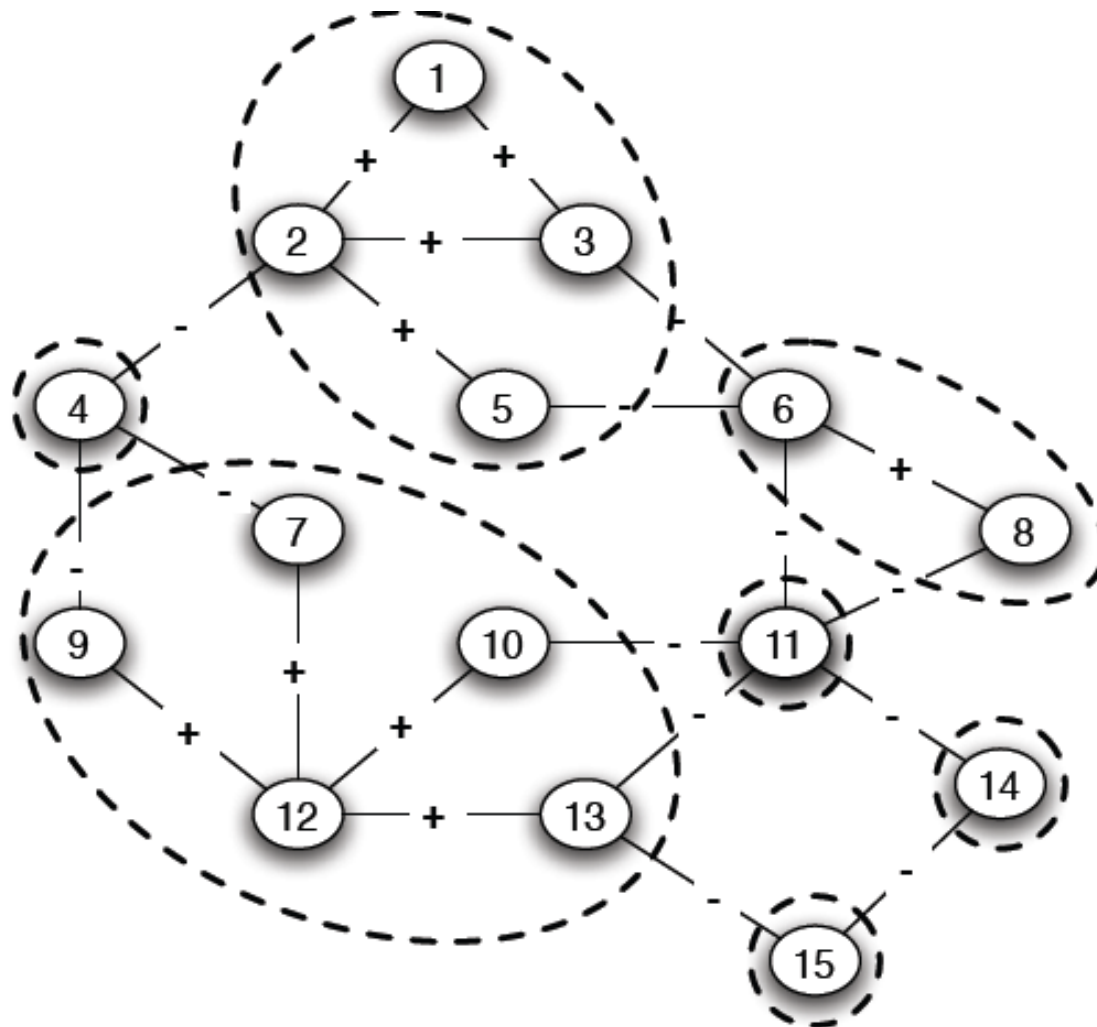
# Signed Graph: Is it Balanced?

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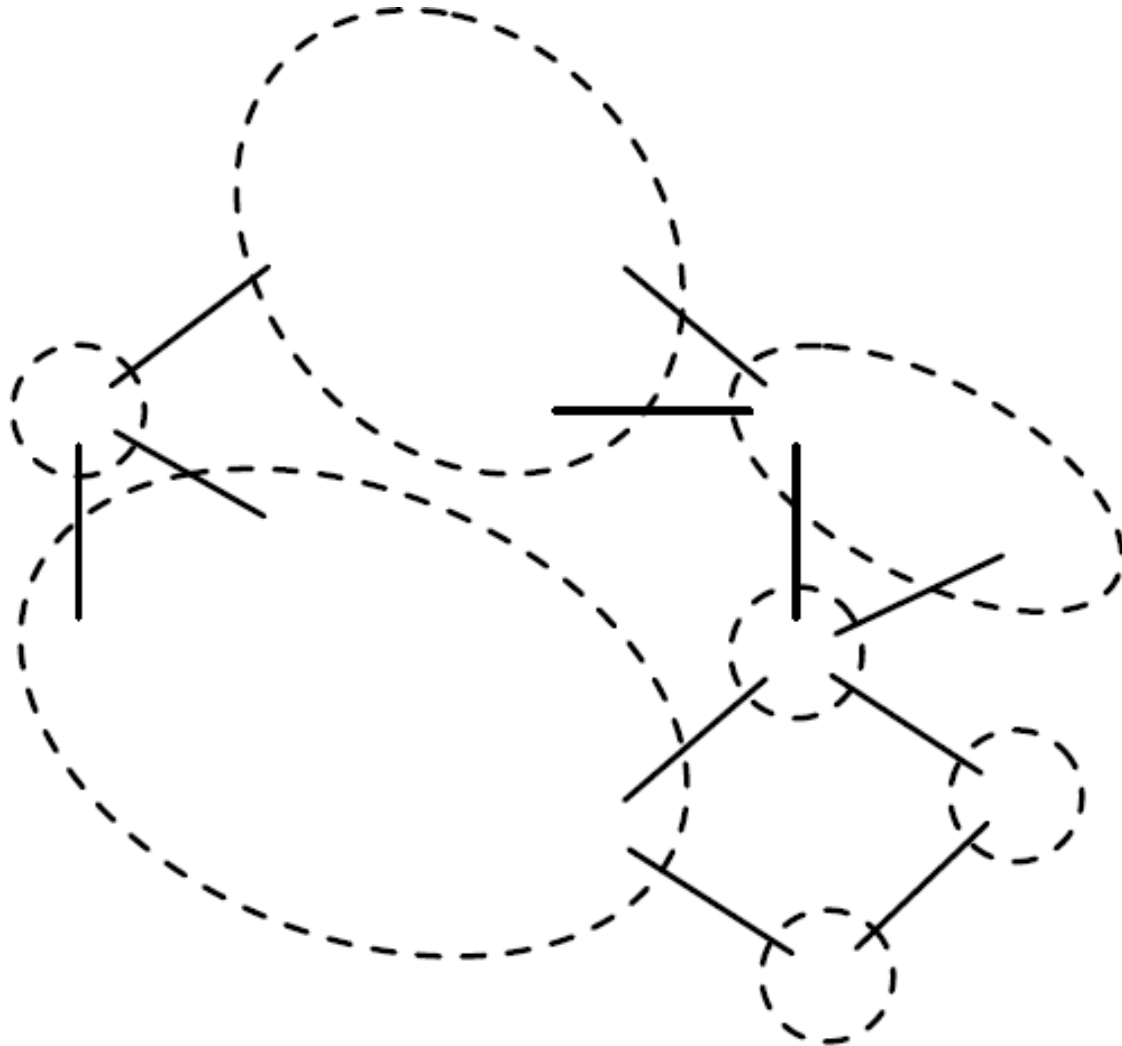
# Positive Connected Components

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# Reduced Graph on Super-Nodes

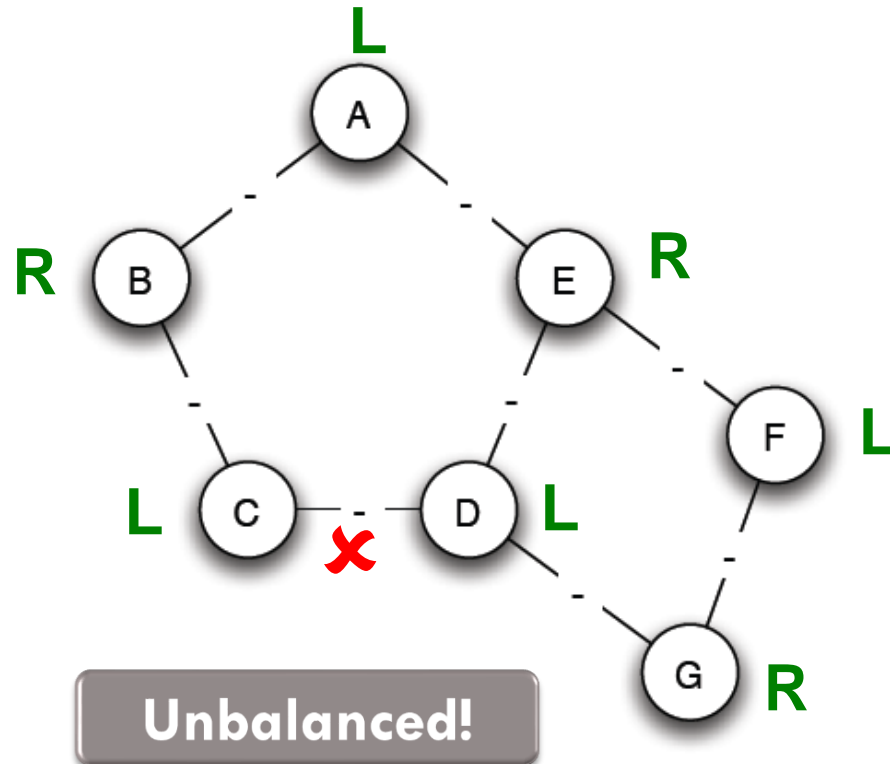
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# BFS on Reduced Graph

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- Using BFS assign each node a **side**
- Graph is **unbalanced** if any two super-nodes are assigned the **same side**



# EXPLORING REAL DATA

4/7/2022

# Real Large Signed Networks

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□ Each link  $A \rightarrow B$  is **explicitly** tagged with a sign:

□ **Epinions:** Trust/Distrust

- Does A trust B's product reviews?  
(only positive links are visible)

□ **Wikipedia:** Support/Oppose

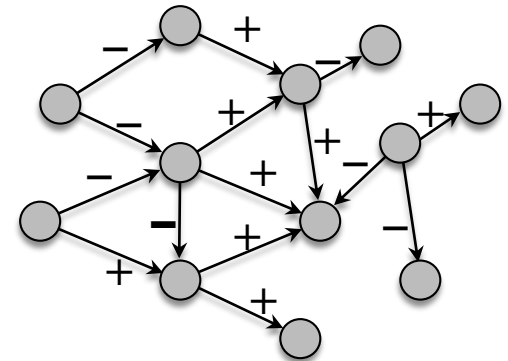
- Does A support B to become Wikipedia administrator?

□ **Slashdot:** Friend/Foe

- Does A like B's comments?

□ **Other examples:**

- Online multiplayer games



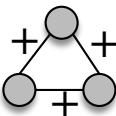
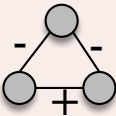
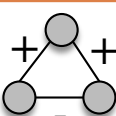
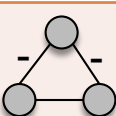
	Epinions	Slashdot	Wikipedia
Nodes	119,217	82,144	7,118
Edges	841,200	549,202	103,747
+ edges	85.0%	77.4%	78.7%
- edges	15.0%	22.6%	21.2%

# Balance in Network Data

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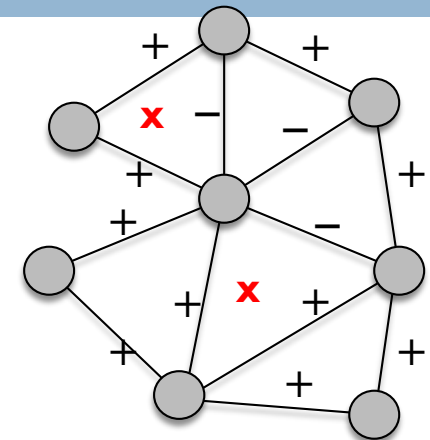
## □ Does structural balance hold?

- ▣ Compare frequencies of signed triads in real and “shuffled” data

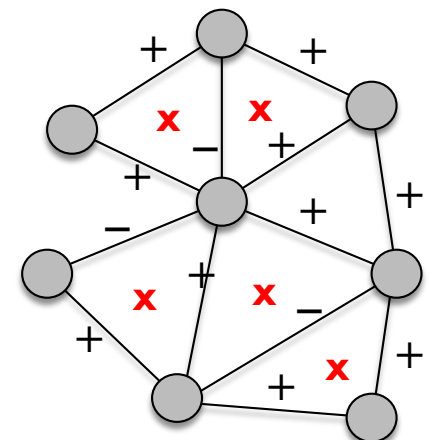
	Triad	Epinions		Wikipedia		Consistent with Balance?
		P(T)	P <sub>0</sub> (T)	P(T)	P <sub>0</sub> (T)	
Balanced		0.87	0.62	0.70	0.49	✓
		0.07	0.05	0.21	0.10	✓
Unbalanced		0.05	0.32	0.08	0.49	✓
		0.007	0.003	0.011	0.010	✗

P(T) ... fraction of a triads

P<sub>0</sub>(T)... triad fraction if the signs would be random



Real data

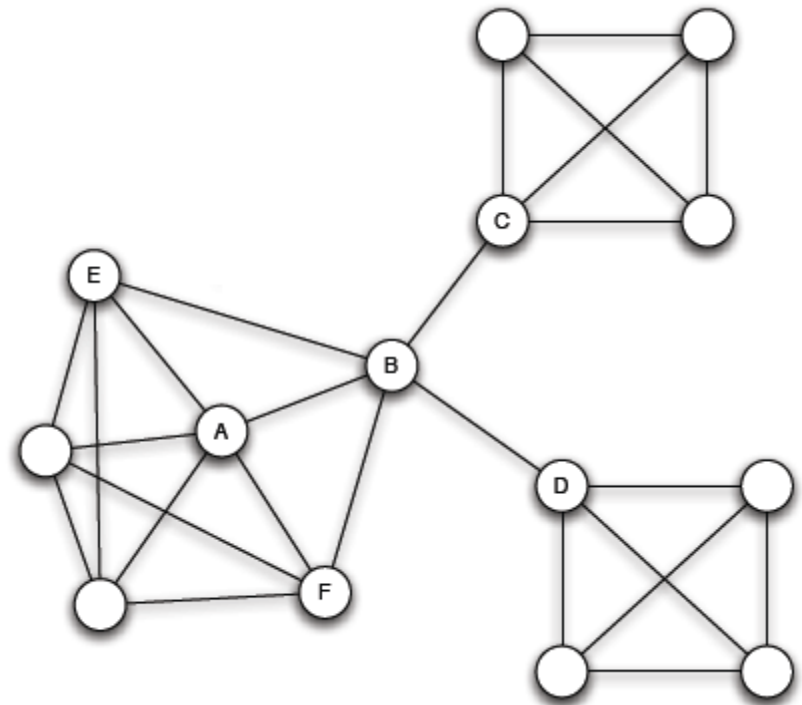


Shuffled data

# Global Structure of Signed Nets

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- Intuitive picture of social network in terms of densely linked clusters
- **How does structure interact with links?**
- **Embeddedness of link (A,B):** Number of shared neighbors

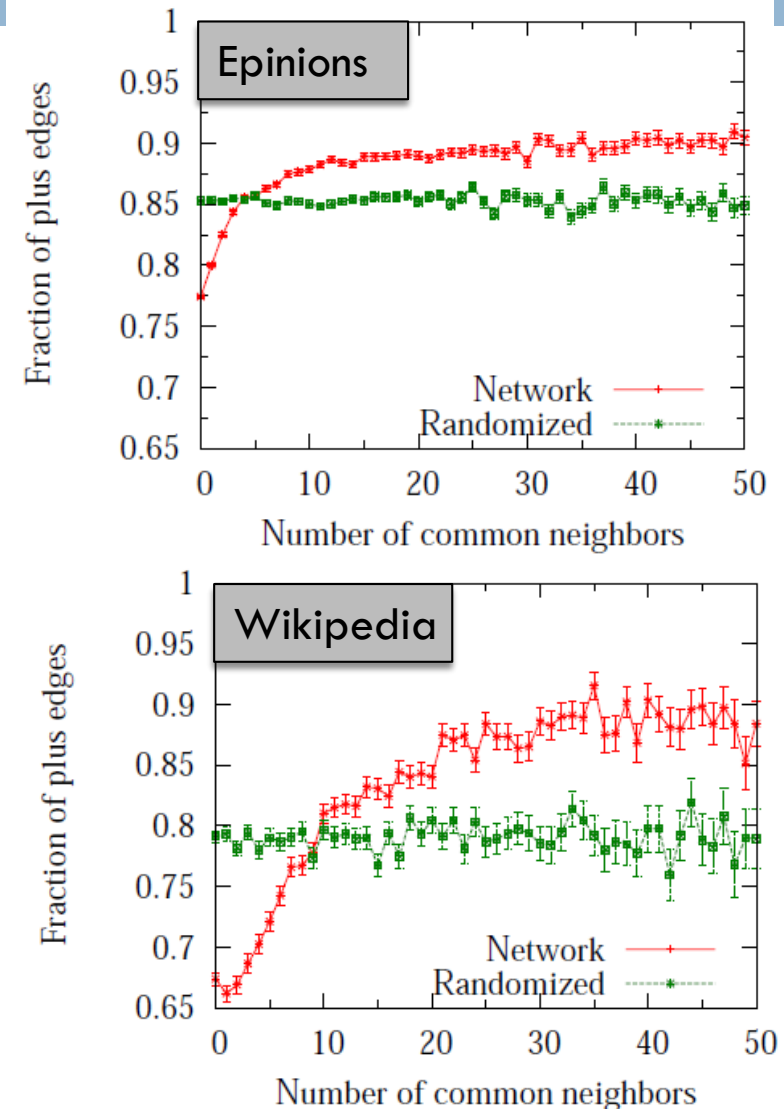




# Global Factions: Embeddedness

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- **Embeddedness of ties:**
  - ▣ Positive ties tend to be **more** embedded
- **Positive ties** tend to be more **clumped together**
  - ▣ Public display of signs (votes) in Wikipedia further attenuates this



# Global Structure of Signed Nets

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	Size		Clustering		Component	
	Nodes	Edges	Real	Rnd	Real	Rnd
Epinions: −	119,090	123,602	0.012	0.022	0.308	0.334
Epinions: +	119,090	717,027	0.093	0.077	0.815	0.870
Slashdot: −	82,144	124,130	0.005	0.010	0.423	0.524
Slashdot: +	82,144	425,072	0.025	0.022	0.906	0.909
Wikipedia: −	7,115	21,984	0.028	0.031	0.583	0.612
Wikipedia: +	7,115	81,705	0.130	0.103	0.870	0.918

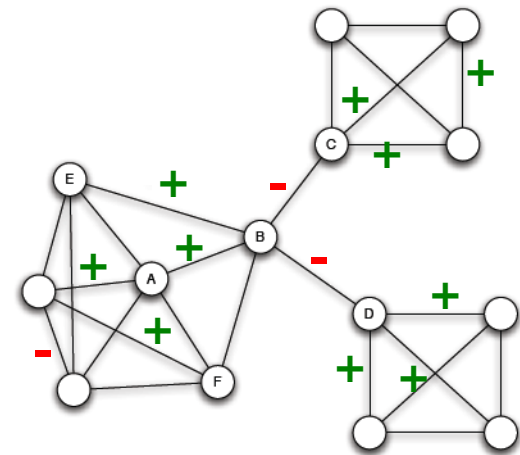
## □ Clustering:

▣ +net: More clustering than baseline

▣ −net: Less clustering than baseline

## □ Size of max. component:

▣ +/−net: Smaller than the baseline



# Directed Edges

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□ But, our networks can be really **directed**

□ How many  $\Delta$  are now explained by balance?

□ Only half (8 out of 16) **explained by Balance**

□ Can we do better?

□ Yes. **theory of status**

