CSIT9000F/MSBD9000A: 2017 Fall Semester Final Exam

Name	**Model Solution**
Student Number	

Instructions

1. The exam time is 180 minutes.
2. There are total 11 questions. The full mark is 100 points.
3. Write your answers in the exam paper only.

Problem	Marks
1 (6 pts)	
2 (6 pts)	
3 (6 pts)	
4 (15 pts)	
5 (10 pts)	
6 (10 pts)	
7 (10 pts)	
8 (7 pts)	
9 (10 pts)	
10 (12 pts)	
11 (8 pts)	
Total	

Problem 1 (6 pts) (*Boundary Following Robot and Production System*) Recall that our boundary-following robot has eight sensors s_1 - s_8 that detect if the eight surrounding cells are free for it to occupy: clockwisely, s_1 returns 1 iff the surrounding cell in the north-west direction is not free for it to occupy, s_2 returns 1 iff the surrounding cell in the north direction is not free for it to occupy, and so on. The robot has four actions: *north*, *east*, *south*, and *west*. We introduce four features x_i (for $s_{2i}+s_{2i+1}$, assuming s_9 is s_1), $i = 1, 2, 3, 4$, and under the assumption of no tight space, come up with the following production system to control the robot to follow the inside boundary of the room clockwisely and outside boundary of an object counterclockwisely:

$$
\begin{aligned}
r_1: &\quad x_4\overline{x_1} \rightarrow north,\\
r_2: &\quad x_3\overline{x_4} \rightarrow west,\\
r_3: &\quad x_2\overline{x_3} \rightarrow south,\\
r_4: &\quad x_1\overline{x_2} \rightarrow east,\\
r_5: &\quad 1 \rightarrow north.
\end{aligned}
$$

Recall that the order of the rules may matter. Now if we swap r_1 and r_2, thus making r_2 the first rule, and r_1 the second, will the new system still achieve the goal? Explain your answer: give an informal proof if you think it still works but give a counterexample if you think it will not work anymore.

Suggested Solution:
The conditions of r_1 and r_2 are exclusive with each other, so their order does not matter.

Problem 2 (6 pts) (*Stimulus-Response Agents)* An artificial ant lives in a two-dimensional grid world and is supposed to follow a continuous pheromone trail (one cell wide) of marked cells. The ant occupies a single cell and faces either up, left, down, or right. It is capable of three actions, namely, move one cell ahead (m), turn to the left while remaining in the same cell (l), turn to the right while remaining in the same cell (r). The ant can sense whether or not there is a pheromone trace in the cell immediately ahead of it (in the direction it is facing).

Show that it is impossible to design a stimulus-response system to control the ant to follow the trail even under the assumption that it starts in a cell where it can sense a pheromone trace, there is no loop, and once it reaches the end of the trail, it has achieved the goal.

Suggested Solution:
Show two states that have the same sensory state but need to take two different actions.

Problem 3 (6 pts) (*Perceptrons*) Again consider our robot example. The condition for the robot moving *north* is $\overline{x_1}\,\overline{x_2}\,\overline{x_3}\,\overline{x_4} + x_4\overline{x_1}$. Can this condition be represented by a perceptron (with inputs x_1, x_2, x_3, x_4)? If yes, give a perceptron for it. If not, explain why.

Suggested Solution:

It can be represented by the following perceptron: $\theta = 0$, $w_1 = -3$ (for x_1), $w_2 = w_3 = -1$, $w_4 = 2$.

If $x_4 = 0$, then the condition is true iff $\overline{x_1}\,\overline{x_2}\,\overline{x_3}$ is true, iff $x_1 = x_2 = x_3 = 0$. For the perceptron, it outputs 1 iff $-3x_1 - x_2 - x_3 \geq 0$ iff $x_1 = x_2 = x_3 = 0$.

If $x_4 = 1$, then the condition is true iff $x_1 = 0$. For the perceptron, it outputs 1 iff $-3x_1 - x_2 - x_3 + 2 \geq 0$ iff $x_1 = 0$.

Problem 4 (15 pts) (*Perceptron Learning and GSCA*) Consider the following samples:

ID	x_1	x_2	x_3	OK
1	0	0	0	No
2	0	0	1	Yes
3	1	0	0	No
4	1	1	0	Yes

where x_1, x_2, and x_3 are some features that should not concern us here.

1. (6 pts) Use these four instances to train a single perceptron using the error-correction procedure. Use the learning rate = 1, and the initial weights all equal to 0. Recall that the threshold is considered to be a new input that always have value "1". Please give your answer by filling in the following table, where weight vector (w_1, w_2, w_3, t) means that w_i is the weight of input x_i, and t is the weight for the new input corresponding to the threshold. You can stop when the weight vector converges.

2. (3 pts) What is the Boolean function corresponding to your perceptron?
3. (6 pts) From the same training set, use the GSCA algorithm to learn a set of rules.

Suggested Solution:

1.

ID	Weight vector
Initial	(0, 0, 0, 0)
1	(0,0,0,-1)
2	(0,0,1,0)
3	(-1, 0 , 1,-1)
4	(0,1,1,0)
1	(0,1,1,-1)
2	(0,1,1,-1)
3	(0,1,1,-1)
4	(0,1,1,-1)
1	(0,1,1,-1)

2. Boolean function: $x_2 \vee x_3$.

3.
i. We begin with : $true \supset OK$.
Choose the feature that yields the largest value of r_α:
$r_{x_1} = 0.5, r_{x_2} = 1, r_{x_3} = 1$
Suppose we choose x_2, this will generate : $x_2 \supset OK$
ii. Eliminate instance 4 from the training set. For the remaining positive instances in Σ:
$r_{x_1} = 0, r_{x_2} = 0, r_{x_3} = 1$
So we choose x_3 and generate: $x_3 \supset OK$
So far these two rules covers all the positive instances in the original training set.

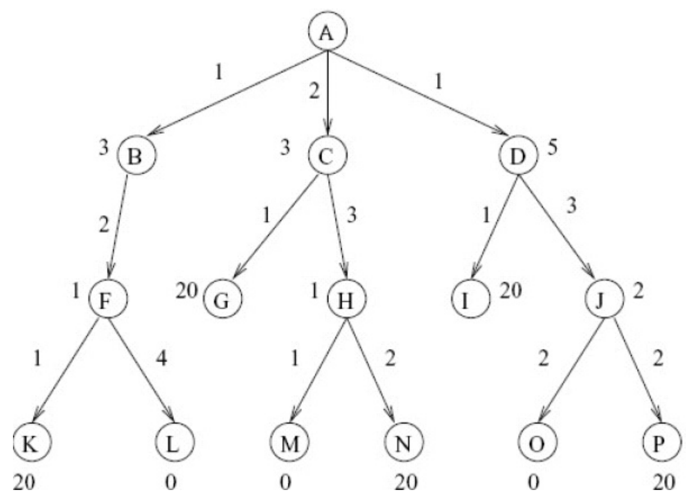
Problem 5 (10 pts) (*Game theory and auction*). Consider the single item first-price auction with three bidders B_i, $i = 1, 2, 3$, and assume that when there is a tie, the order to break the tie is $B_1 > B_2 > B_3$ (so for example, if B_1 and B_3 are tied, then B_1 wins). Suppose B_i values the item x_i, $i = 1, 2, 3$, and the information is common knowledge. Suppose further that each can bid with any value in the interval $[0, 1]$.

- Make this auction into a game in normal form $(\{B_1, B_2, B_3\}, R_1, R_2, R_3, u_1, u_2, u_3)$ by defining R_i (the set of pure strategies for player B_i) and u_i (player B_i's utility function). You can assume that all players are risk neutral.
- Suppose $0 \le x_1 = x_2 = x_3 \le 1$. Are there Nash equilibria? If yes, give all of them. If no, give a proof.

Suggested Solution:
$R_i = [0, 1]$. $u_i(b_1, b_2, b_3) = x_i - b_i$ if B_i wins and 0 otherwise. B_1 wins iff $b_1 \ge b_2$ and $b_1 \ge b_3$. B_2 wins iff $b_2 > b_1$ and $b_2 \ge b_3$. B_3 wins iff $b_3 > b_1$ and $b_3 > b_2$.

Nash equilibria: two of them bid x_1 the third one bid no larger than x_1: all (b_1, x_2, x_3) such that $b_1 \le x_1$, (x_1, b_2, x_3) such that $b_2 \le x_2$, and (x_1, x_2, b_3) such that $b_3 \le x_3$.

Problem 6 (10 pts) Consider the following state space. The number next to a state is the value of the heuristic function on the state, and the number next to an arc from state α to state β is the cost of the corresponding operator $\Pi_{\alpha\beta}$. If the number next to a state is 0, that means that the state is a goal state.



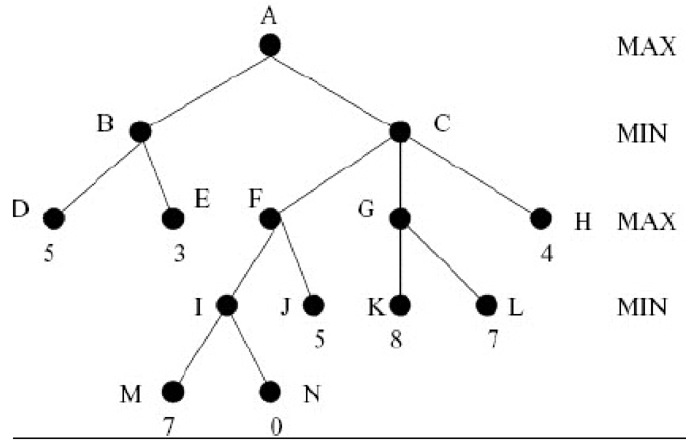
(2.1) Give the sequence of the states expanded by A^* algorithm, starting from the root A and terminating at a goal state. Notice that whenever there is a tie, we prefer nodes at deeper levels, and on the same level, left to right.

(2.2) Can you adjust the heuristic function so that the goal state O is returned? Notice that your heuristic function still needs to be admissible, and you don't need to do this question if O is already the terminating goal state in your solution to (2.1).

Suggested Solution:
(2.1) state sequence: A, B, F, C, H, M.

(2.2) Many possible solutions. An easy one is to change D's value to 1 and J's value to 1. You can even change them to 0.

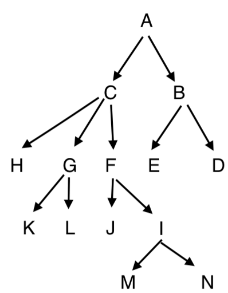
Problem 7 (10 pts) Consider the following minimax search tree.



- Perform a left to right apha-beta pruning on the tree, and list the nodes that are pruned. Notice that when a node is pruned, all of its descendants are pruned as well.
- Re-order the children of some of the nodes, so that left-to-right alpha-beta pruning will prune the most number of nodes.

Suggested Solution:
(a). L.

(b). See below (K and L can be swapped. G and F can be swapped. M and N do not matter. Pruned: L, I, D)



Problem 8 (7 pts) Let x_{ij} be propositional symbols (variables), and A the following sentence:

$$(x_{11} \wedge x_{12}) \vee \cdots \vee (x_{n1} \wedge x_{n2}).$$

By introducing new variables if necessary, convert A into a set of clauses so that the number of clauses is polynomial in n.

Suggested Solution:
Let $y_i \equiv x_{i1} \wedge x_{i2}$. Then A can be converted into the following clauses:

$$
\begin{aligned}
&y_1 \vee \cdots \vee y_n, \\
&\neg y_1 \vee x_{11}, \\
&\neg y_1 \vee x_{12}, \\
&y_1 \vee \neg x_{11} \vee \neg x_{12}, \\
&\ldots \\
&\neg y_n \vee x_{n1}, \\
&\neg y_n \vee x_{n2}, \\
&y_n \vee \neg x_{n1} \vee x_{n2}.
\end{aligned}
$$

A few people let

$$B \equiv \neg[(x_{11} \wedge x_{12}) \vee \cdots \vee (x_{n1} \wedge x_{n2})]. \tag{1}$$

Then B yields the following clauses:

$$\neg x_{11} \vee \neg x_{12}, \cdots, \neg x_{n1} \vee \neg x_{n2}$$

and A is $\neg B$. This is not correct as you have to convert (1) to clauses which would beg the same question.

Problem 9 (10 pts) Let $Fools(x, y, t)$ stands for "x fools y at time t". Represent the following statements in first-order logic:

1. Everyone can fool every other person sometimes.
2. There is exactly one time when someone fools everyone else.
3. At no time one can fool all other people.
4. One cannot fool another person all the time.
5. For any two people x and y, if x can fool y some time, then y can also fool x some other time.

Suggested Solution:

1. $\forall x, y.x \neq y \supset \exists t.Fools(x, y, t)$,
2. $\exists t[\exists x \forall y(x \neq y \supset Fools(x, y, t)) \wedge \forall u(\exists x \forall y(x \neq y \supset Fools(x, y, u)) \supset u = t)]$,
3. $\forall t, x \exists y.y \neq x \wedge \neg Fools(x, y, t)$ Or
   $\neg \exists t \exists x \forall y(y \neq x \supset Fools(x, y, t))$,
4. $\forall x, y \exists t.\neg Fools(x, y, t)$,
5. $\forall x, y, t.Fools(x, y, t) \supset \exists u(u \neq t \wedge Fools(y, x, u))$.

Problem 10 (12 pts) We are given the following facts:

> Tony, Mike, and John are members of the Alpine Club. Every member of the Alpine Club is either a skier or a mountain climber or both. No mountain climber likes rain, and all skiers like snow. Mike dislikes whatever Tony likes and likes whatever Tony dislikes. Tony likes rain and snow.

1. Represent these facts by first-order logic sentences using the following language:

- $AC(x)$: x is a member of the Alpine Club.
- $CL(x)$: x is a mountain climber.
- $SK(x)$: x is a skier.
- $R(x)$: x likes rain.
- $SN(x)$: x likes snow.
- Constants: t (Tony), j (John), and m (Mike).

2. Convert your sentences into clauses and prove by resolution refutation the following assertion: there is a member of the Alpine Club who is a mountain climber but not a skier, i.e.

$$\exists x.AC(x) \wedge CL(x) \wedge \neg SK(x).$$

3. Who is that member?

Suggested Solution:
1.

$$\begin{aligned}
&AC(t) \wedge AC(m) \wedge AC(j),\\
&\forall x.AC(x) \supset CL(x) \vee SK(x),\\
&\neg \exists x.CL(x) \wedge R(x),\\
&\forall x.SK(x) \supset SN(x),\\
&R(t) \equiv \neg R(m),\\
&SN(t) \equiv \neg SN(t),\\
&R(t) \wedge SN(t)
\end{aligned}$$

2. The question is

$$\exists x.AC(x) \wedge CL(x) \wedge \neg SK(x)$$

To use resolution refutation, convert the KB into clauses:

1. $AC(t)$.
2. $AC(m)$.
3. $AC(j)$.
4. $\neg AC(x) \vee CL(x) \vee SK(x)$.
5. $\neg CL(x) \vee \neg SN(x)$.
6. $\neg SK(x) \vee SN(x)$.
7. $\neg R(t) \vee \neg R(m)$.
8. $R(t) \vee R(m)$.
9. $\neg SN(t) \vee \neg SN(m)$.
10. $SN(t) \vee SN(m)$.
11. $R(t)$.
12. $SN(t)$.

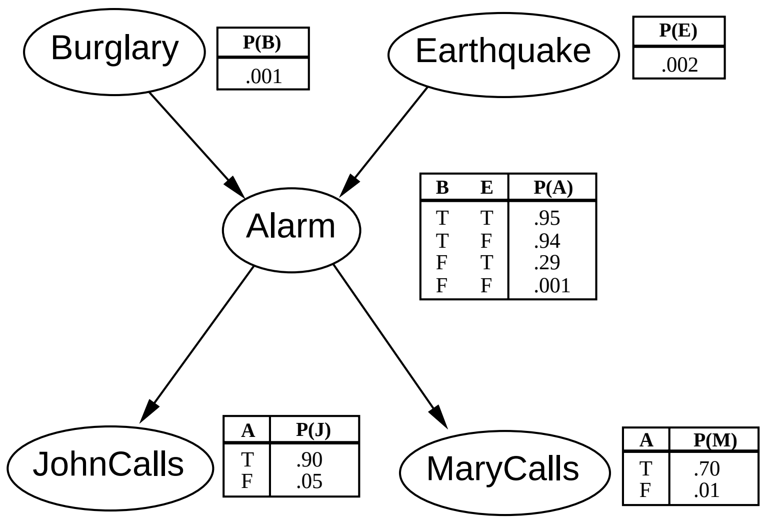
Negate the query, convert to clause, and add the answer predicate to extract the answer:

13. $\neg AC(x) \vee \neg CL(x) \vee SK(x) \vee A(x)$.

14. $\neg AC(x) \vee SK(x) \vee A(x)$. (4 and 13)
15. $SK(m) \vee A(m)$. (14 and 2)
16. $SN(m) \vee A(m)$. (15 and 6)
17. $\neg SN(t) \vee A(m)$. (16 and 9)
18. $A(m)$. (17 and 12)

3. The answer is m, i.e. Mike is the member.

Problem 11 (8 pts) Consider the following belief network:



1. Compute the joint probability of $Earthquake$ and $\neg Burglary$.
2. Compute the probability of $Alarm$.

Suggested Solution:
In the following, let E denotes $Earthquake$, B for $Burglary$, etc. You'll get fullmark as long as the formulas are correct, regardless if you have done the calculation.

1. Notice that E and B are independent, so

$$\begin{aligned} P(E \wedge \neg B) &= P(\neg B)P(E) \\ &= 0.999 * 0.002 \end{aligned}$$

2.

$$\begin{aligned} P(A) &= \sum_{x \in \{E, \neg E\}, y \in \{B, \neg B\}} P(A|x,y)P(x,y) \\ &= 0.95 * 0.001 * 0.002 + 0.94 * 0.001 * 0.998 + 0.29 * 0.999 * 0.002 + \\ &\quad 0.001 * 0.999 * 0.998 = 0.0025 \end{aligned}$$