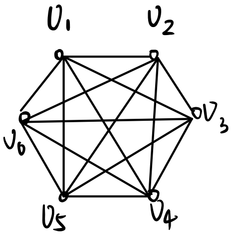
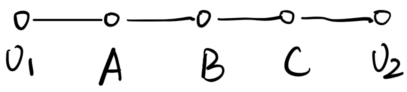
$Q_{1\text{-}1}$

12, There are 5 points in the path of length 4, v_1 must be the first point, v_2 must be the last point, Two of A, B, C are v_3 and v_4. the last remaining point maybe v_5 or v_6.



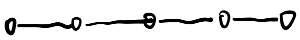


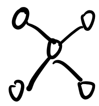
So the answer is $C_2^1 \cdot A_3^3 = 2\times3\times2\times1 = 12$

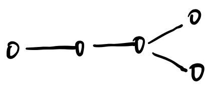
$Q_{1\text{-}2}$

We can categorize the discussion, we can easily know that there are at least 4 and at most 10 friendships, totally 21 kinds

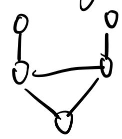
① Four friendships

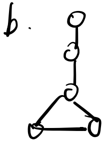
a 

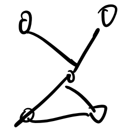
c 

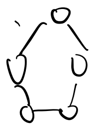
b. 

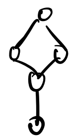
② Five friendships

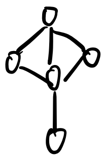
a. 

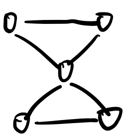
b. 

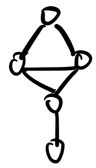
c. 

d. 

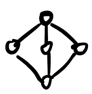
e 

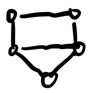
③ a. 

b. 

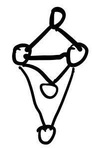
c. 

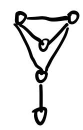
6 friendships

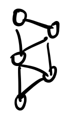


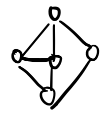


④ 7 friendships

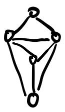


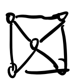




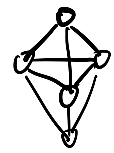


⑤

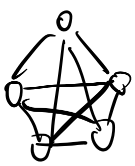




⑥ 9 friendships

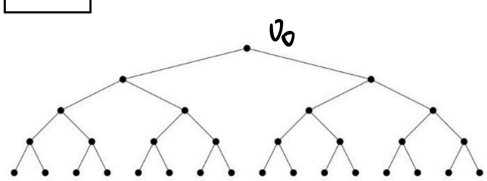


⑦



10 friendships

$Q_{1\text{-}3}$



① first level amount = 0

② Second level amount = 2×8 = 16 Total = 64

③ Third level amount = 4×4 = 16

④ Forth level amount = 4×8 = 32

Q1-4 We need to find the villages that were the longest distance apart.

According to the figure

A B C D E F G H I J K L M N O

A 0
B 5
C 9
D 14
E 2
F 17
G 24
H 16
I 18
J 4
K 21
L 25
M 24
N 21
O 24

It's too hard to compute the distance, so I change my mind. As far as possible, the hospital should be located in a well-connected area, which means that there are direct links to other villages.

① First hospital: C which can cover A B C, D, E, F, H, I, J.

Villages that are uncovered: G, K, L, M, N, O

② Second hospital could be located at L, M, N

So the answer are {C, L}, {C, M}, {C, N}

Q2

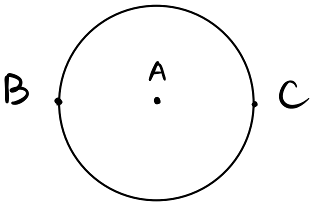
2-1 Yes, because there are at most 20 nodes whose distance from v_1 is exactly two. When there is no node that overlap, there are exactly 20 nodes that are at distance of 2 from v_1.

2-2 No, If the five friends of each of the five people in $v_2 \sim v_6$ are not the same, then there are at most $5 \times 4 = 20$ people with a distance of 2 from v_1.

2-3 Yes, If the five friends of each of the five people in $v_2 \sim v_6$ are the same, then there are at least 4 people with a distance of 2 from v_1.

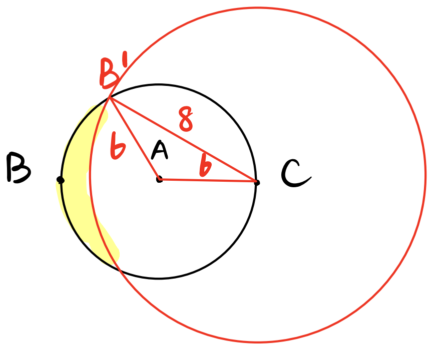
Q3

Strong Triadic Closure Property is that if a node has strong ties to two neighbors, then these neighbors must have at least a weak tie between them. Consider the extreme case where the river is 30 miles long and 30 farmers are spread evenly over those 30 miles. If AC is 6 miles away, AB is also 6 miles away and BC is 12 miles away, which is the longest distance before BC, and the title shows that BC are connected by a weak tie, so the Strong Triadic Closure property is satisfied.



Q4

As shown in the figure below, the nodes in the yellow part of the network are not able to satisfy the Strong Triadic Closure property



Q5

From the Handshaking Theorem we know the sum of degree of all the vertices $= 2 \times E$ (E means the number of edges of G)

Let N means the number of vertices, so the sum of degree of all vertices is $N * k$, too.

$$N \cdot k = 2 \cdot E \Rightarrow E = \frac{N}{2} \cdot k$$

Assume that N is odd, we already known the k is odd, which means that the result of $N * k$ is odd, too. However the result of $2 \cdot E$ is even. So N must be even, which means that $\frac{N}{2}$ is an integer so the conclusion is right.