

LECTURE 8: NETWORK EFFECTS AND CASCADING BEHAVIOR

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CSIT 6000K: Social Networks and Social Computing: A Data Science Perspective

Thursdays 07:30 PM - 10:20 PM

Spreading Through Networks

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□ Spreading through networks:

- Cascading behavior
- Diffusion of innovations
- Network effects
- Epidemics

□ Behaviors that cascade from node to node like an epidemic

□ Examples:

□ Biological:

- Diseases via contagion

□ Technological:

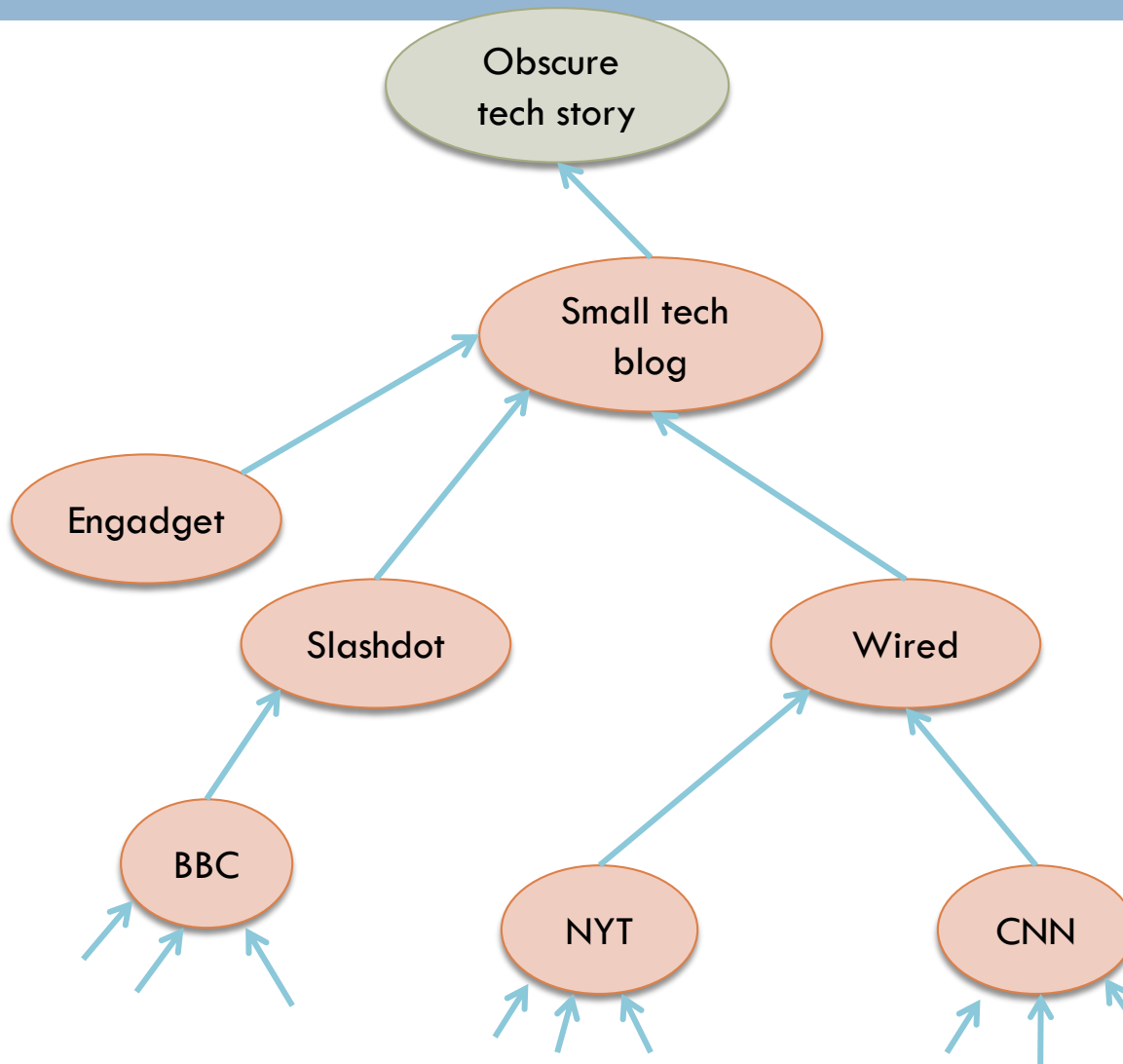
- Cascading failures
- Spread of information

□ Social:

- Rumors, news, new technology
- Viral marketing

Information Diffusion

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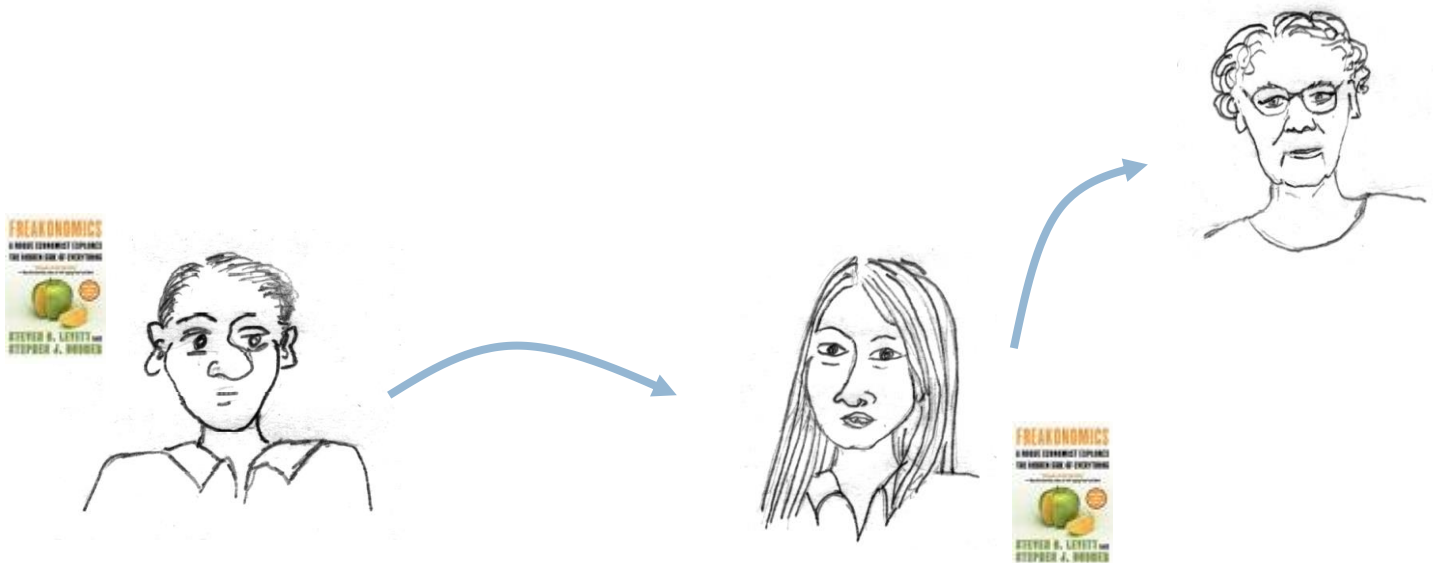


Diffusion in Viral Marketing

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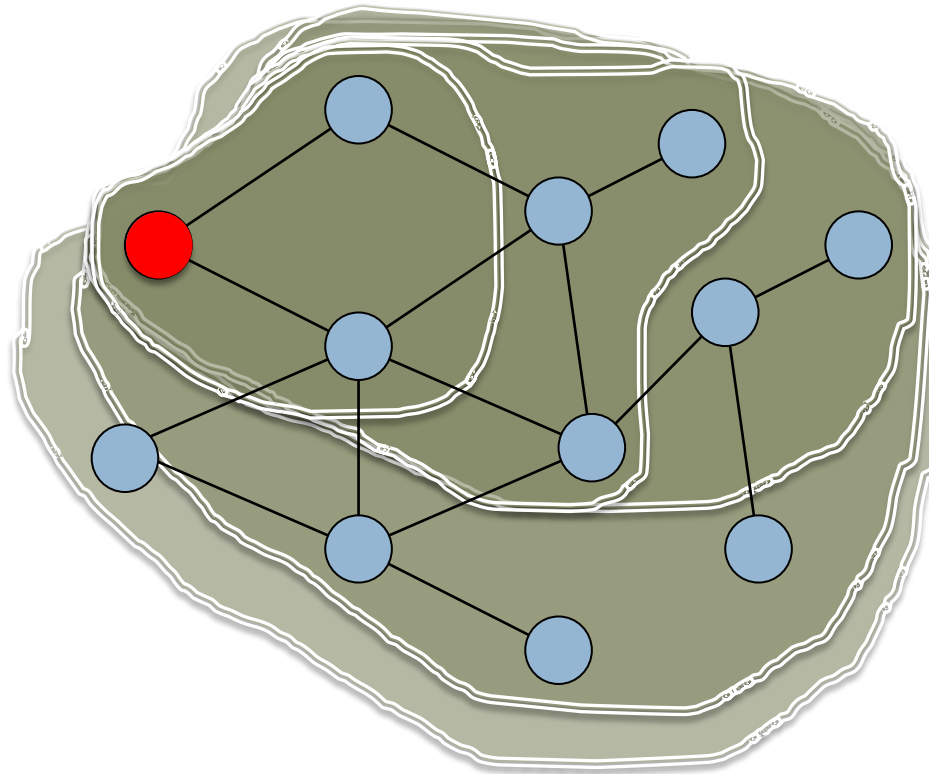
□ Product adoption:

▣ Senders and followers of recommendations



Spread of Diseases (e.g. Covid-19)

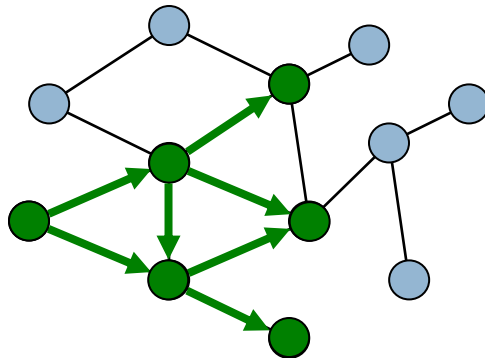
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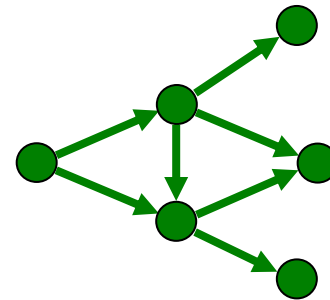
Network Cascades

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- Contagion that spreads over the edges of the network
- It creates a propagation tree, i.e., **cascade**



Network



Cascade

(propagation graph)

Terminology:

- Stuff that spreads: Contagion
- “Infection” event: Adoption, infection, activation
- We have: Infected/active nodes, adoptors

How to Model Diffusion?

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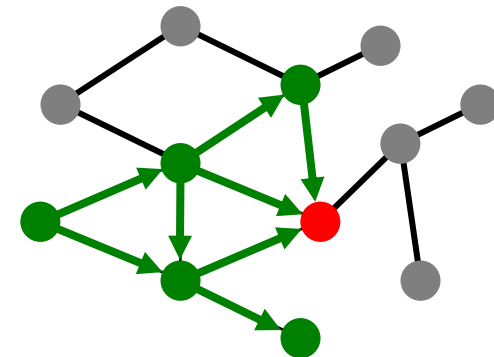
□ Probabilistic models:

□ Models of influence or disease spreading

- An infected node tries to “push” the contagion to an uninfected node

□ Example:

- You “catch” a disease with some prob. from each active neighbor in the network



□ Decision based models (today!):

□ Models of product adoption, decision making

- A node observes decisions of its neighbors and makes its own decision

□ Example:

- You join demonstrations if k of your friends do so too

DECISION BASED MODEL OF DIFFUSION



Decision Based Models

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□ Two ingredients:

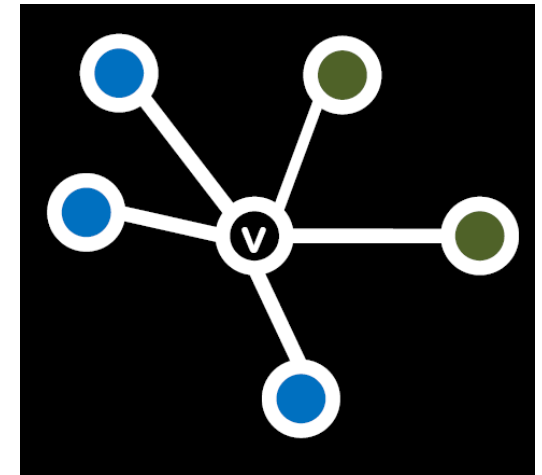
▣ Payoffs:

- Utility of making a particular choice

▣ Signals:

- Public information:
 - What your network neighbors have done
- (Sometimes also) Private information:
 - Something you know
 - Your belief

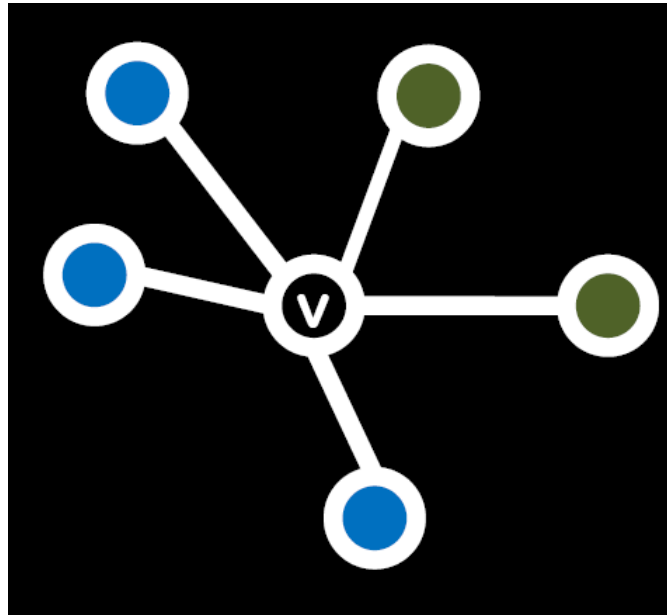
□ Now you want to make the optimal decision



Game Theoretic Model of Cascades

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- **Based on 2 player coordination game**
 - 2 players – each chooses technology A or B
 - Each person can only adopt **one** “behavior”, **A** or **B**
 - You gain more payoff if your friend has adopted the **same** behavior as you



Local view of the network of node v

Example: BlueRay vs. HD DVD

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The Model for Two Nodes

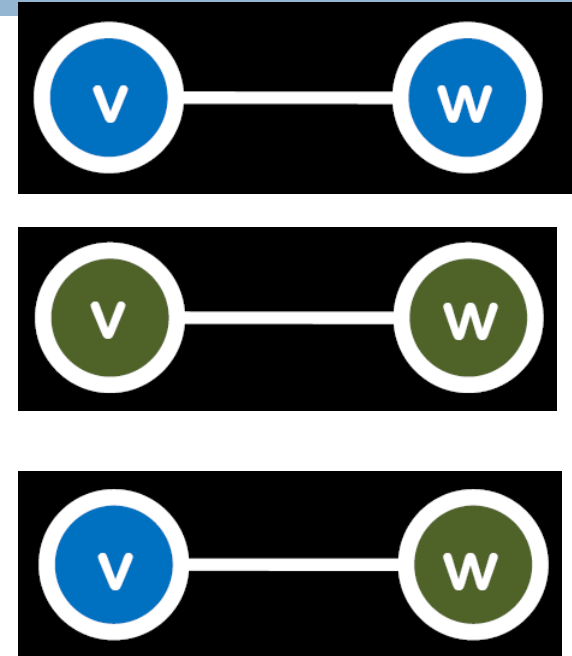
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□ *Payoff matrix:*

- If both v and w adopt behavior A , they each get payoff $a > 0$
- If v and w adopt behavior B , they each get payoff $b > 0$
- If v and w adopt the opposite behaviors, they each get 0

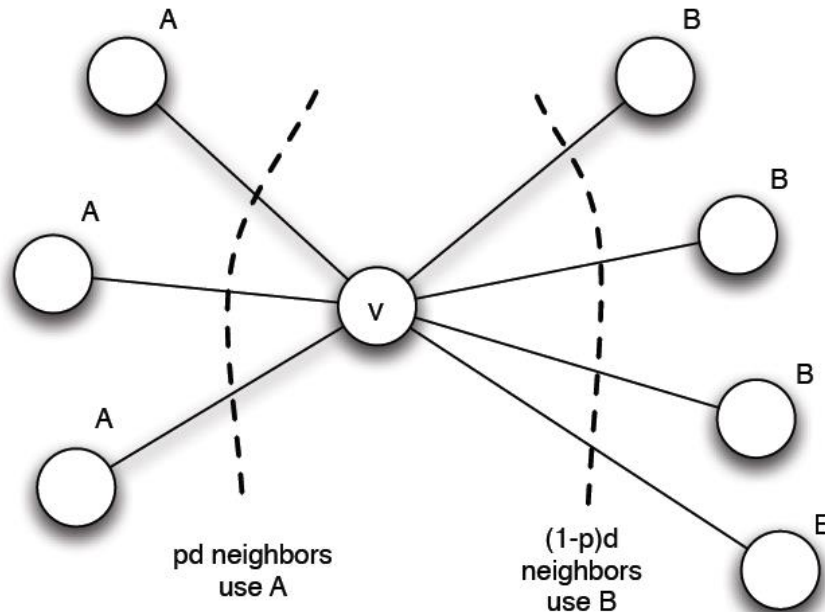
□ *In some large network:*

- Each node v is playing a copy of the game with each of its neighbors
- **Payoff:** sum of node payoffs per game



Calculation of Node v

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Threshold:

v chooses A if

$$p > q = \frac{b}{a+b}$$

- Let v have d neighbors
- Assume fraction p of v 's neighbors adopt A
 - $Payoff_v = a \cdot p \cdot d$ if v chooses A
 - $= b \cdot (1-p) \cdot d$ if v chooses B
- **Thus: v chooses A if: $a \cdot p \cdot d > b \cdot (1-p) \cdot d$**

Example Scenario

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□ Scenario:

Graph where everyone starts with B .

Small set S of early adopters of A

□ Hard-wire S – they keep using A no matter what payoffs tell them to do

□ Assume payoffs are set in such a way that nodes say:

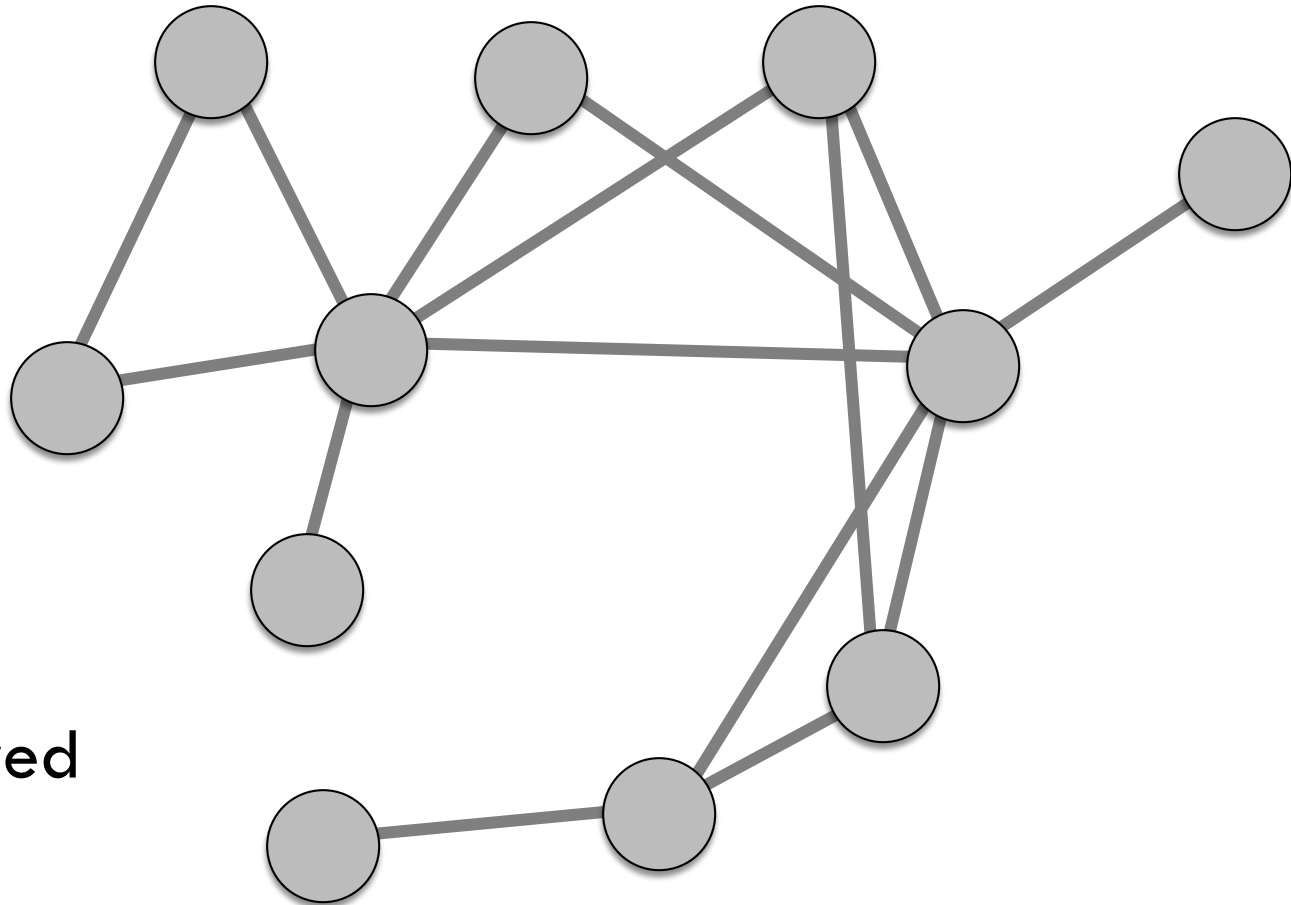
**If more than 50% of my friends take A
I'll also take A**

(this means: $a = b - \epsilon$ and $q > 1/2$)

Example Scenario

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$$S = \{u, v\}$$

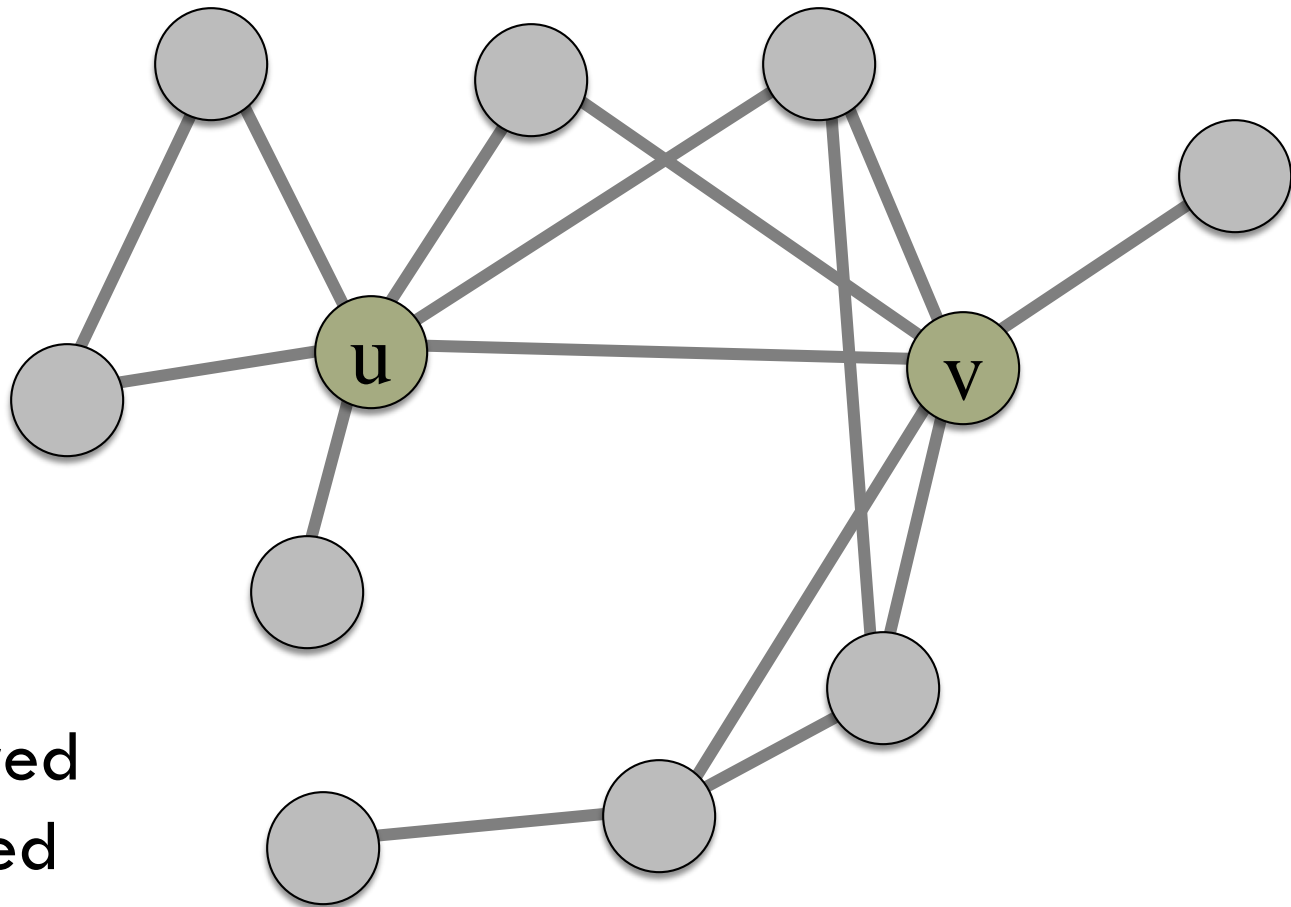


If **more** than
50% of my
friends are red
I'll be red

Example Scenario

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$$S = \{u, v\}$$

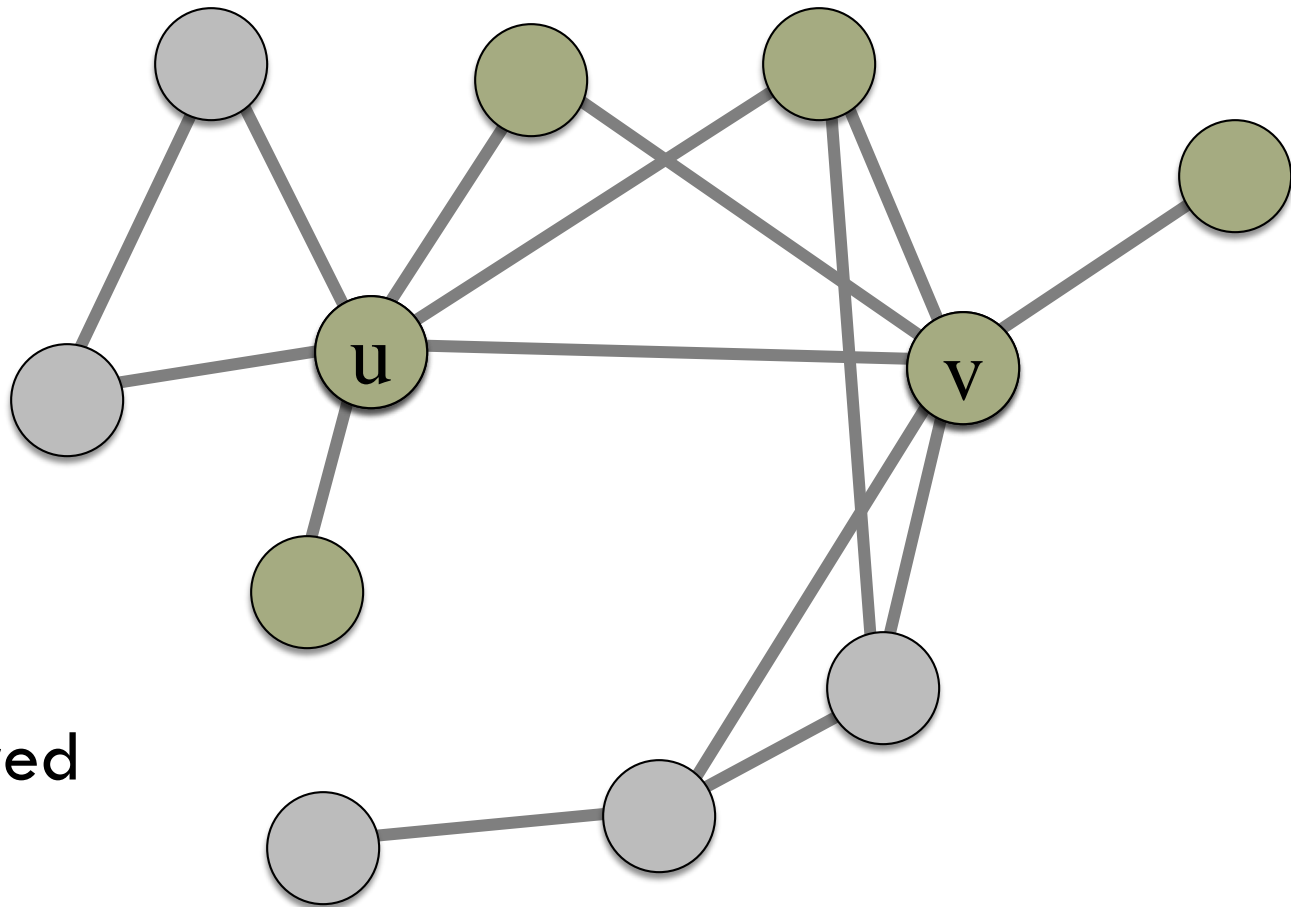


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Example Scenario

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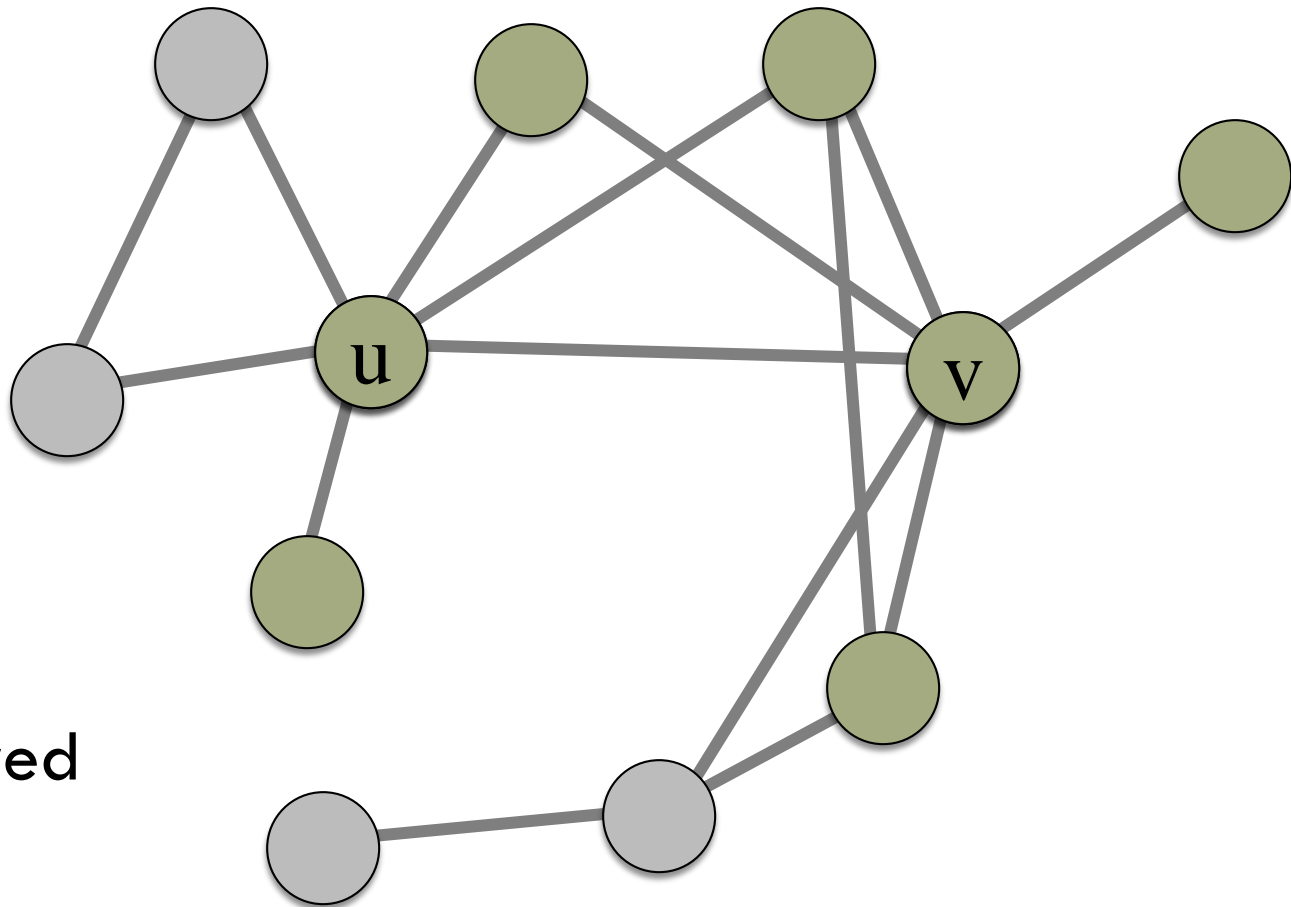


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Example Scenario

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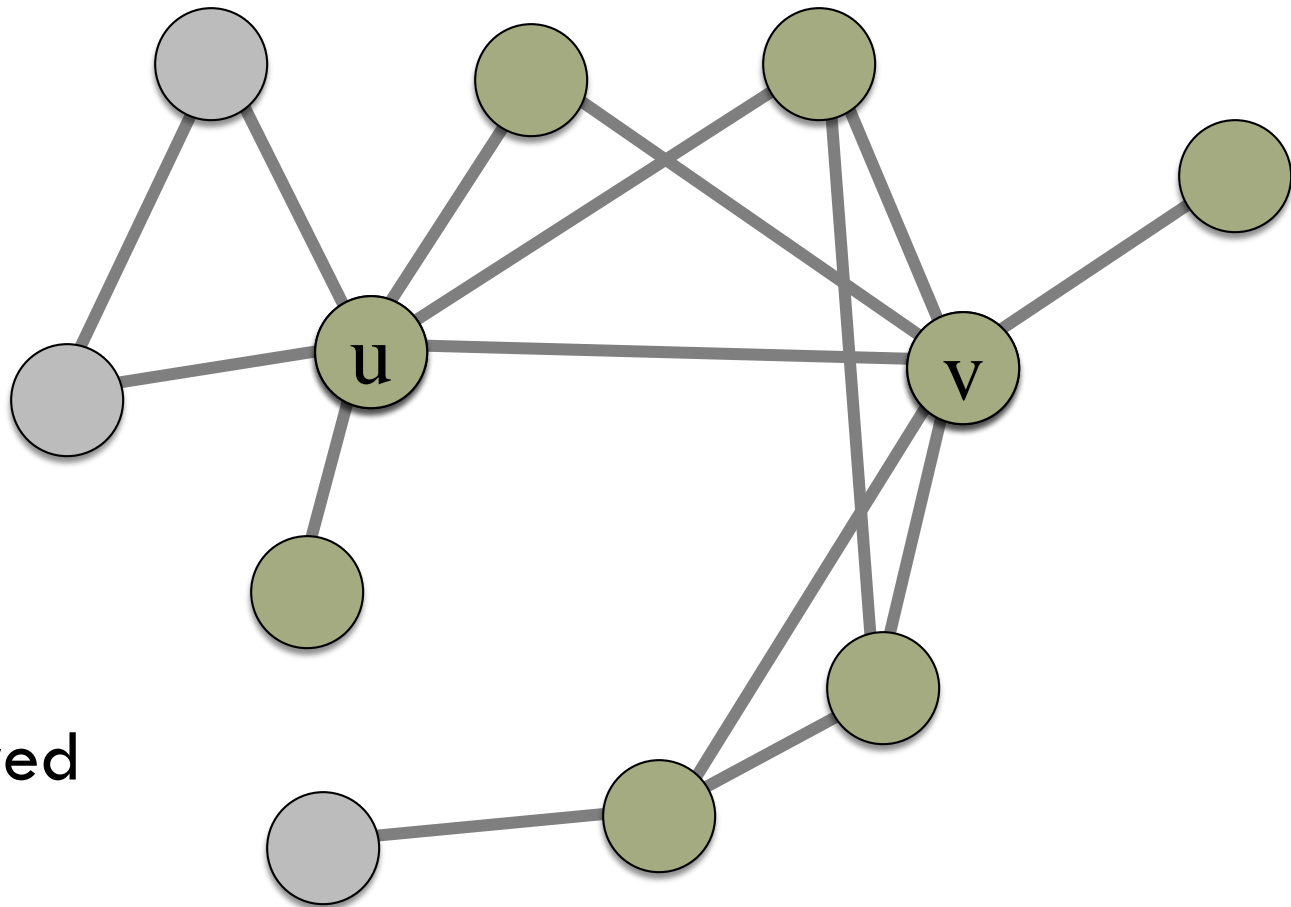


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Example Scenario

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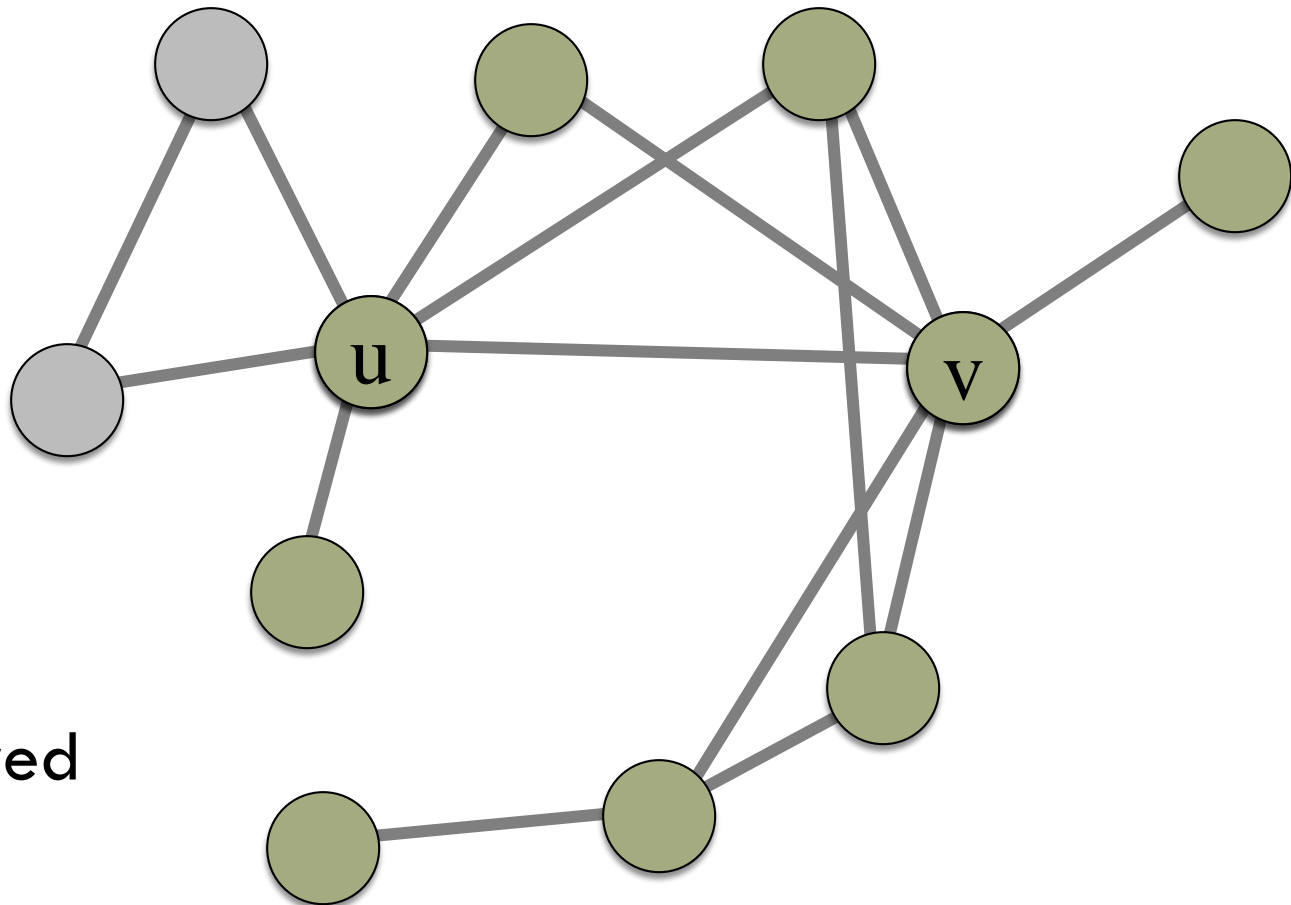


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Example Scenario

20

$$S = \{u, v\}$$



If **more** than
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I'll be red

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Infinite Graphs

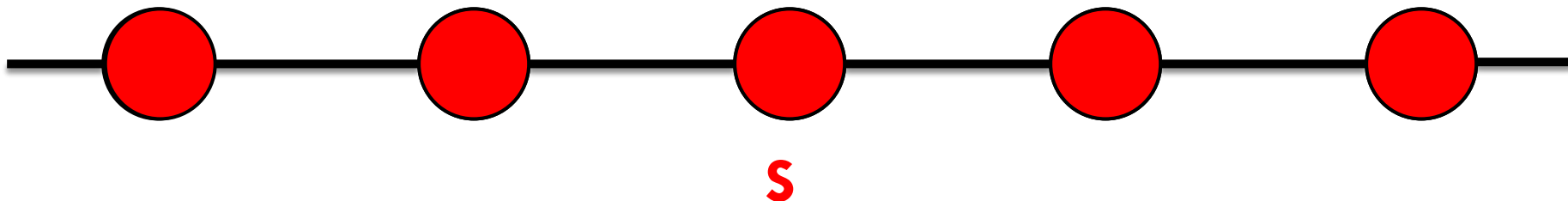
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v chooses A if $p > q$

$$q = \frac{b}{a+b}$$

- Consider infinite graph G
 - (but each node has finite number of neighbors!)
- We say that a finite set S causes a cascade in G with threshold q if, when S adopts A , eventually every node adopts A
- Example: Path

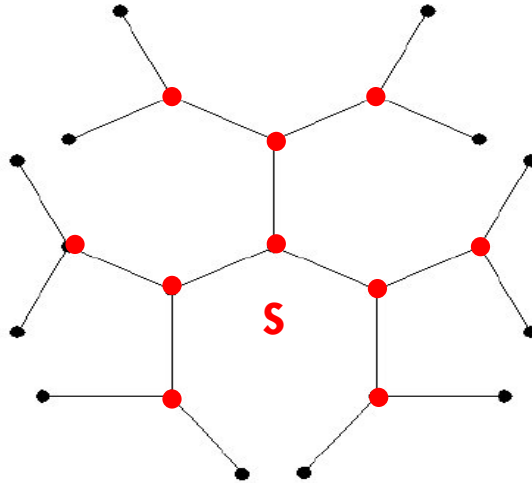
If $q < 1/2$ then cascade occurs



Infinite Graphs

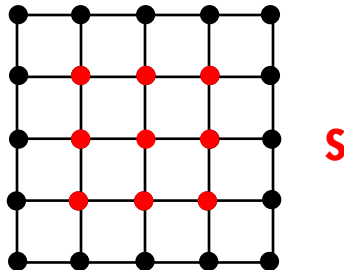
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□ Infinite Tree:



If $q < 1/3$ then
cascade occurs

□ Infinite Grid:



If $q < 1/4$ then
cascade occurs

Cascade Capacity

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□ Def:

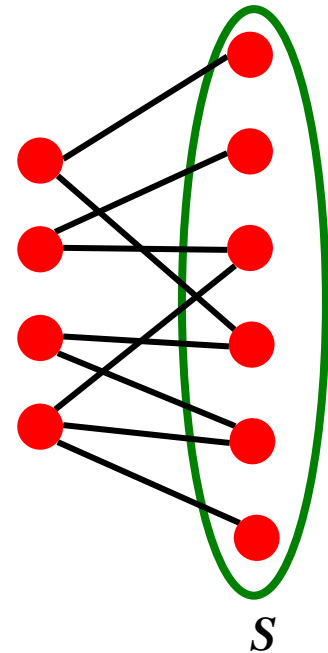
- The **cascade capacity** of a graph G is the **largest q** for which some **finite set S** can cause a **cascade**

□ Fact:

- There is no G where cascade capacity $> 1/2$

□ **Proof idea:**

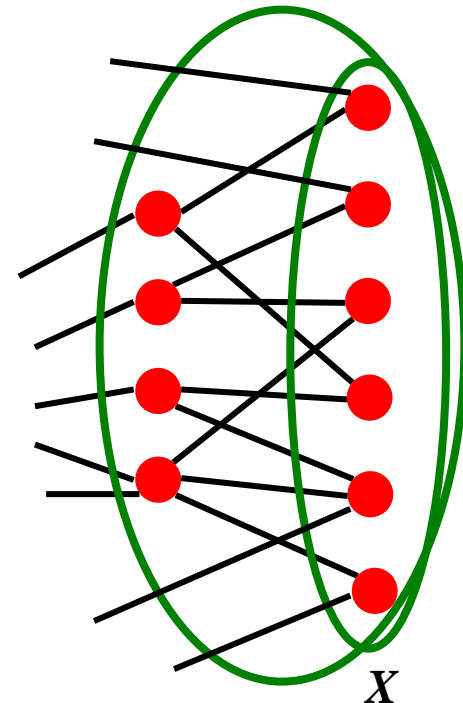
- Suppose such G exists: $q > 1/2$, finite S causes cascade
- **Show contradiction:** Argue that nodes stop switching after a finite # of steps



Cascade Capacity

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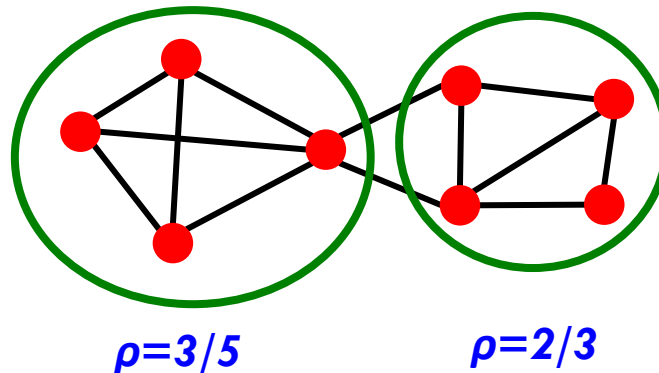
- **Fact:** There is no G where cascade capacity $> 1/2$
- **Proof sketch:**
 - Suppose such G exists: $q > 1/2$, finite S causes cascade
 - **Contradiction:** Switching stops after a finite # of steps
 - Define “potential energy”
 - Argue that it starts finite (non-negative) and strictly decreases at every step
 - “Energy”: $= |d^{\text{out}}(X)|$
 - $|d^{\text{out}}(X)| := \#$ of outgoing edges of active set X
 - The only nodes that switch have a strict majority of its neighbors in S
 - $|d^{\text{out}}(X)|$ strictly decreases
 - It can do so only a finite number of steps



Stopping Cascades

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- What prevents cascades from spreading?
- Def: **Cluster of density ρ** is a **set of nodes C** where each node in the set has at least ρ fraction of edges in C .



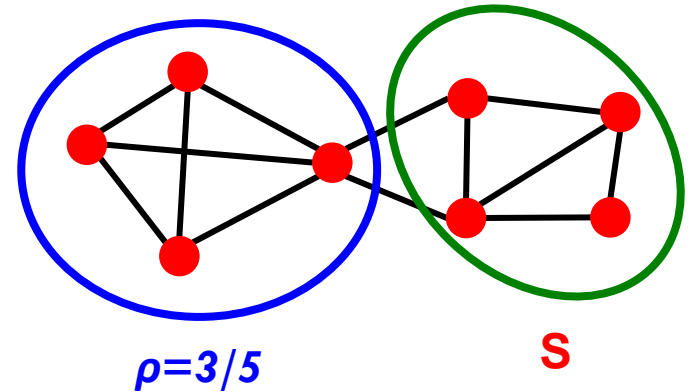
Stopping Cascades

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- Let S be an initial set of adopters of A
- All nodes apply threshold q to decide whether to switch to A

- **Two facts:**

- 1) If $G \setminus S$ contains a cluster of density $> (1-q)$ then S can not cause a cascade
- 2) If S fails to create a cascade, then there is a cluster of density $> (1-q)$ in $G \setminus S$



No cascade if $q > 2/5$

EXTENDING THE MODEL:
ALLOW PEOPLE TO ADOPT A
AND B

Cascades & Compatibility

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□ So far:

- Behaviors A and B compete
- Can only get utility from neighbors of same behavior:
 A - A get a , B - B get b , A - B get 0

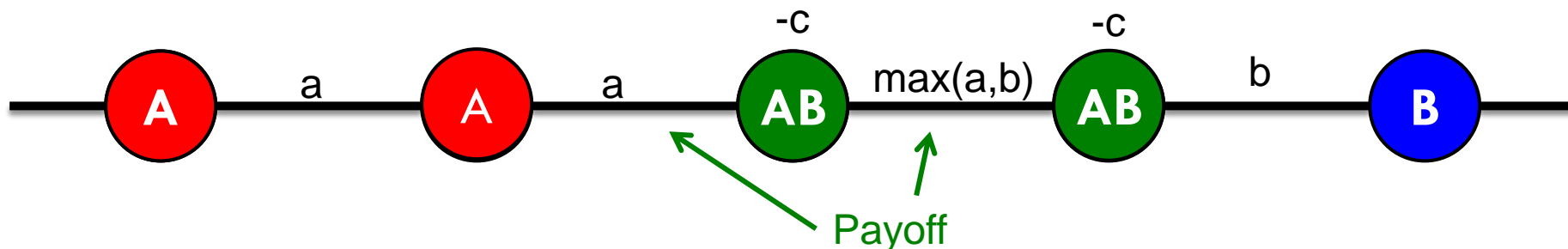
□ Let's add an extra strategy “ A - B ”

- AB - A : gets a
- AB - B : gets b
- AB - AB : gets $\max(a, b)$
- Also: Some cost c for the effort of maintaining both strategies (summed over all interactions)

Cascades & Compatibility: Model

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- Every node in an infinite network starts with B
- Then a finite set S initially adopts A
- Run the model for $t=1,2,3,\dots$
 - ▣ Each node selects behavior that will optimize payoff (given what its neighbors did in at time $t-1$)

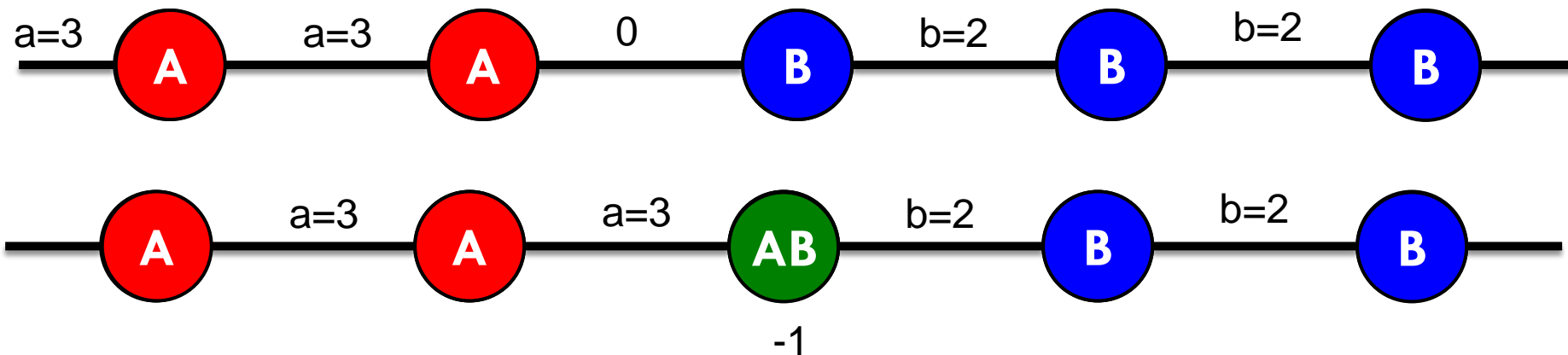


- How will nodes switch from B to A or AB ?

Example: Path Graph

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- **Path graph:** Start with all Bs, $a > b$ (A is better)
- **One node switches to A – what happens?**
 - With just A, B: A spreads if $a > b$
 - With A, B, AB: **Does A spread?**
- **Assume $a=3, b=2, c=1$:**

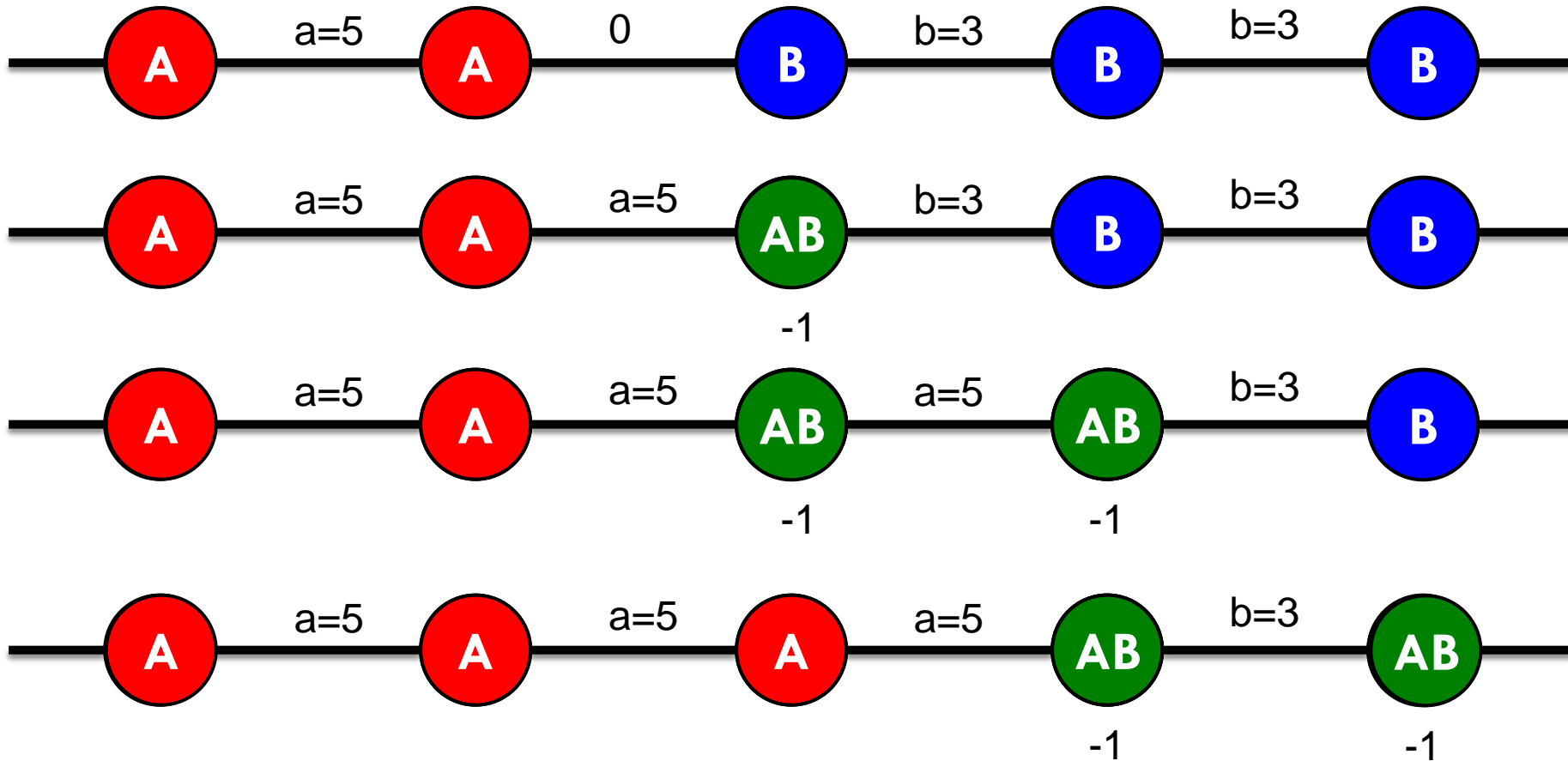


Cascade stops

Example

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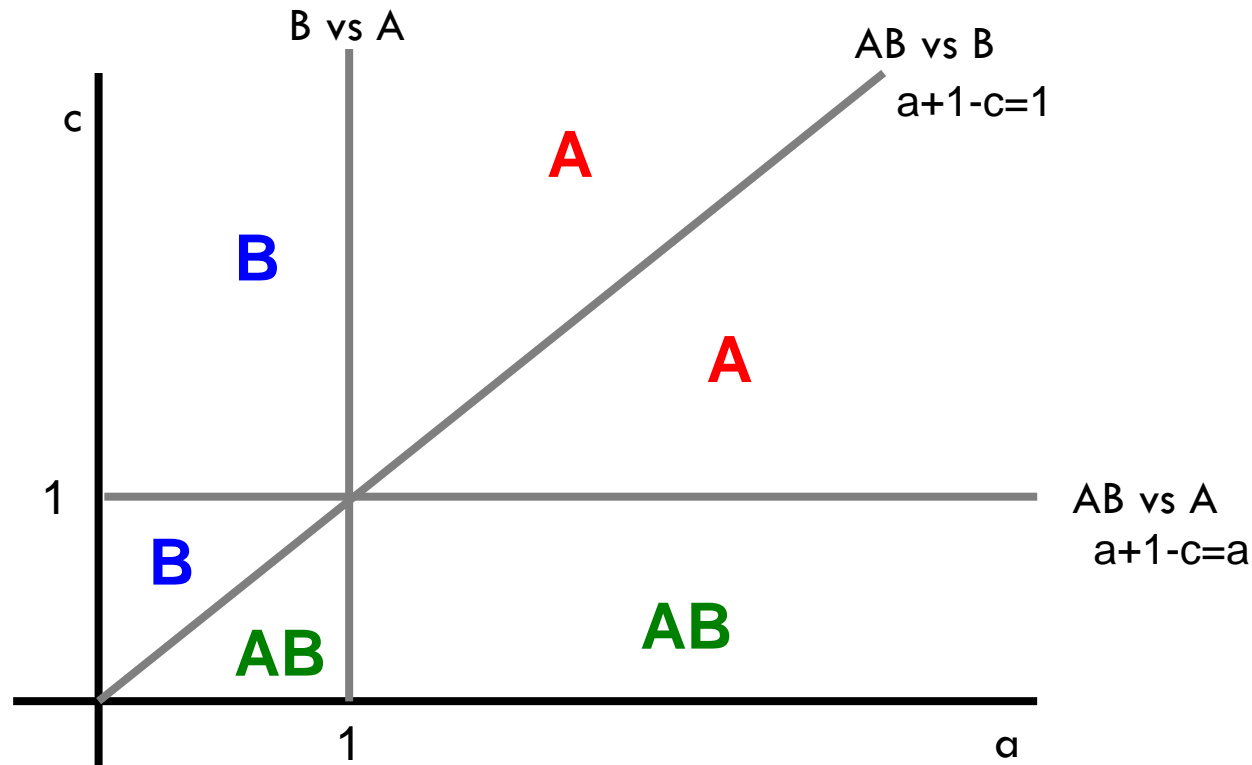
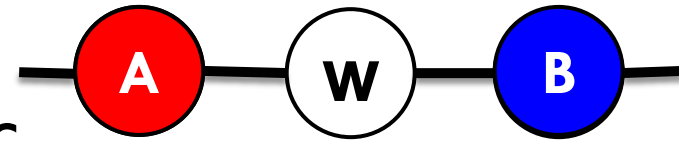
□ Let $a=5$, $b=3$, $c=1$



For what pairs (c,a) does A spread?

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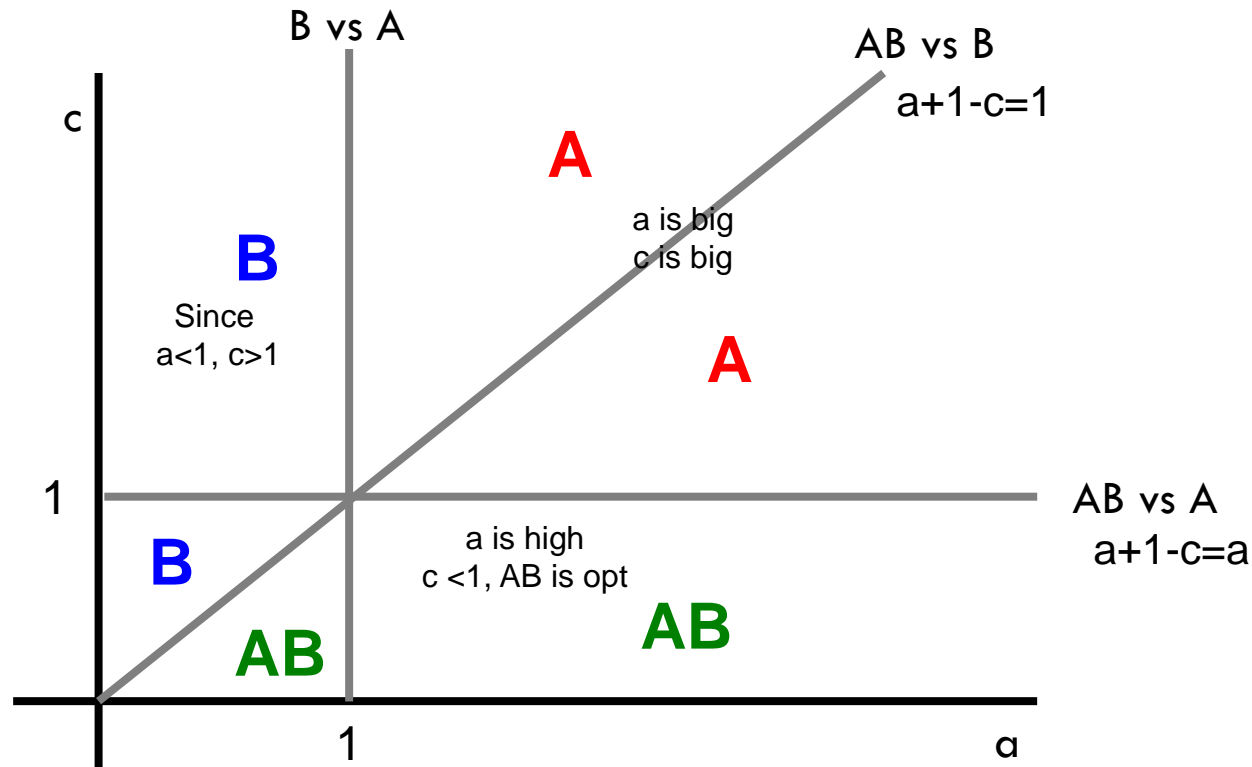
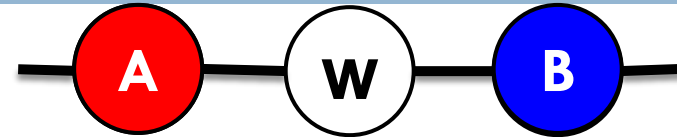
- Infinite path, start with all Bs
- Payoffs for w : A: a , B: 1 , AB: $a+1-c$
- What does node w in **A-w-B** do?



For what pairs (c,a) does A spread?

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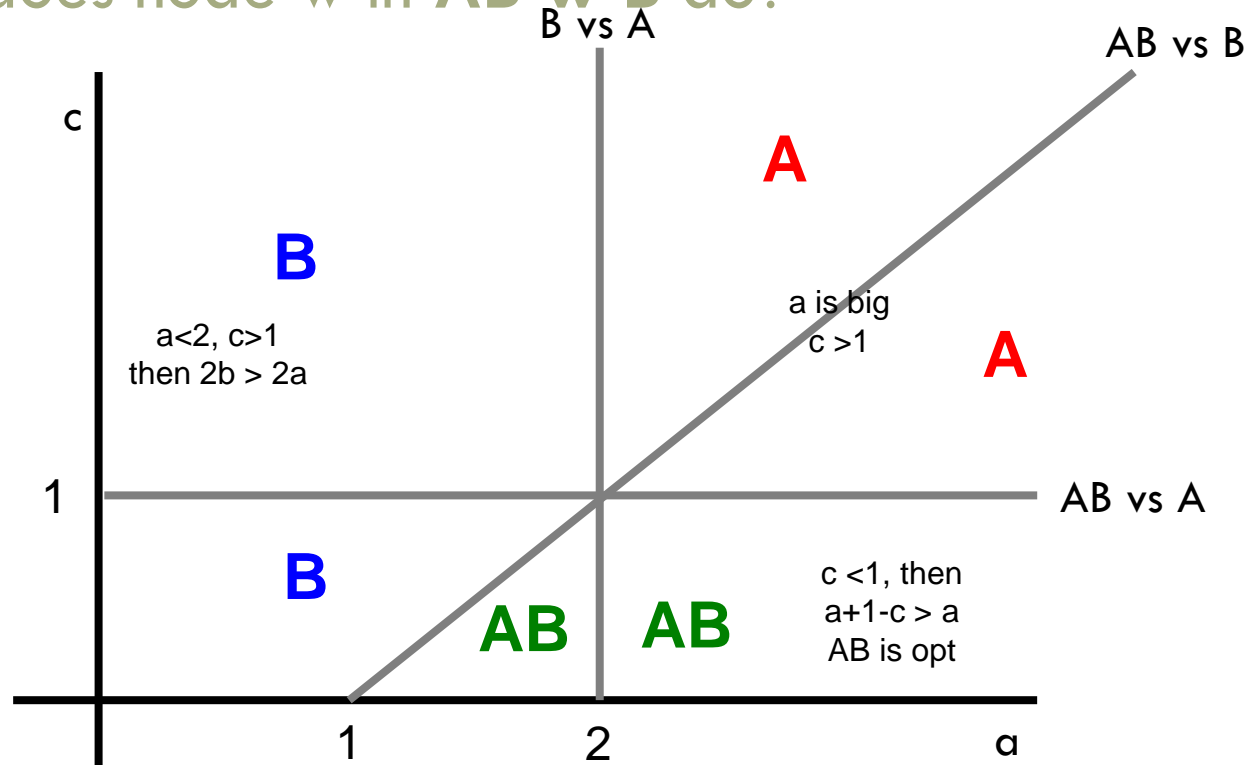
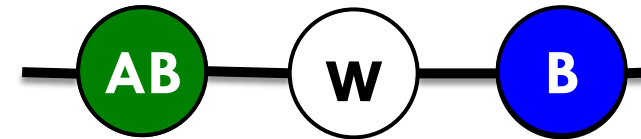
- Infinite path, start with all Bs
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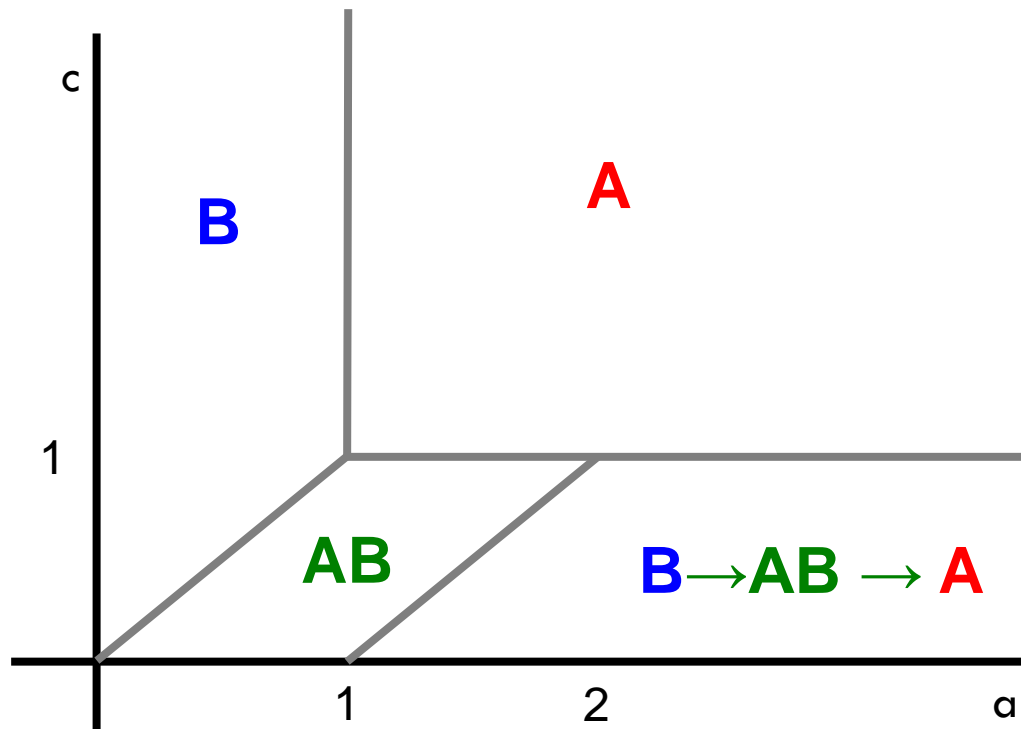
- Same reward structure as before but now payoffs for w change: A: a , B: $1+1$, AB: $a+1-c$
- Notice: Now also AB spreads
- What does node w in AB- w -B do?



For what pairs (c,a) does A spread?

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□ **Joining the two pictures:**



Lesson

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□ You manufacture default B and new/better A comes along:

- **Infiltration:** If B is too compatible then people will take on both and then drop the worse one (B)
- **Direct conquest:** If A makes itself not compatible – people on the border must choose. They pick the better one (A)
- **Buffer zone:** If you choose an optimal level then you keep a static “buffer” between A and B

