LECTURE 5:COMMUNITY STRUCTURE IN NETWORKS

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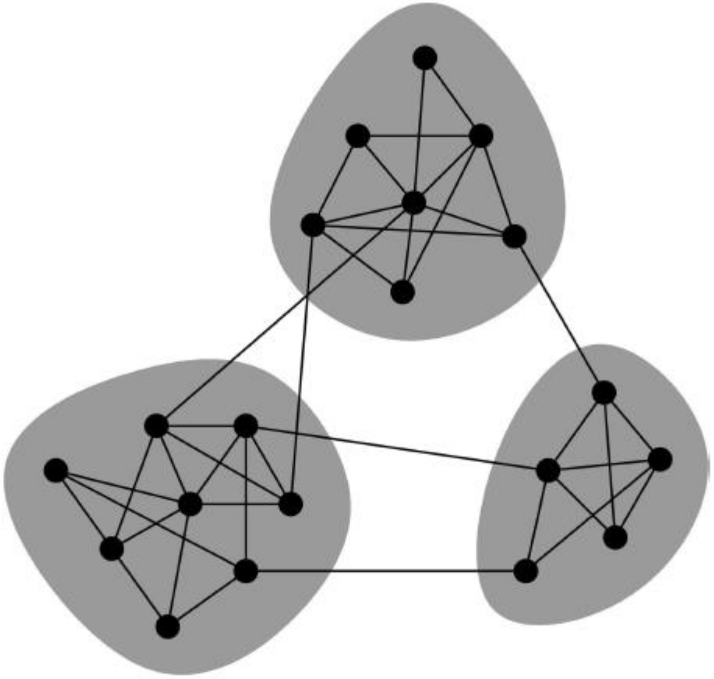
CSIT 6000K: Social Networks and Social Computing: A Data Science Perspective

Thursdays 07:30 PM - 10:20 PM

Networks & Communities

- We often think of networks "looking" like this:



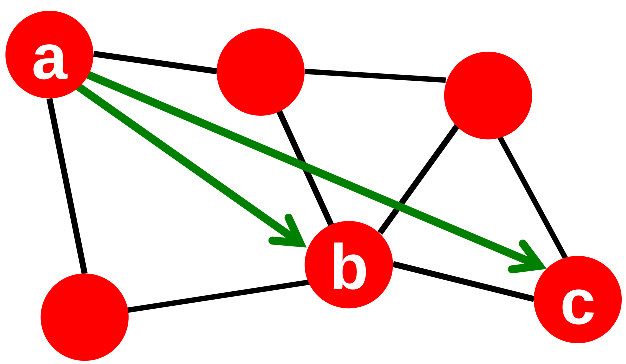
- What lead to such conceptual picture?

Networks: Flow of Information

- **How information flows through the network?**
  - **What structurally distinct roles do nodes play?**
  - **What roles do different links (short vs. long) play?**
- **How people find out about new jobs?**
  - Mark Granovetter, part of his PhD in 1960s
  - People find the information through personal contacts
- **But:** Contacts were often **acquaintances** rather than close friends
  - **This is surprising:** One would expect your friends to help you out more than casual acquaintances
- **Why is it that acquaintances are most helpful?**

Granovetter's Answer

- Two perspectives on **friendships**:
  - **Structural:** Friendships span different parts of the network
  - **Interpersonal:** Friendship between two people is either **strong** or **weak**
- **Structural role: Triadic Closure**

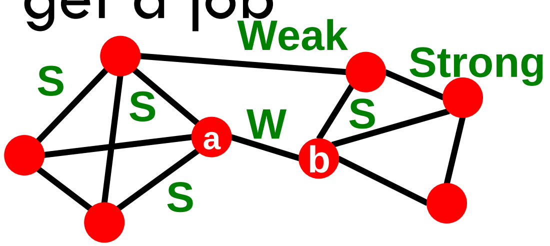


Which edge is more likely a-b or a-c?

If two people in a network have a friend in common there is an increased likelihood they will become friends themselves

Granovetter's Explanation

- **Granovetter makes a connection between social and structural role of an edge**
- **First point:**
  - Structurally embedded edges are also socially strong
  - Edges spanning different parts of the network are socially weak
- **Second point:**
  - The long range edges allow you to gather information from different parts of the network and get a job
  - Structurally embedded edges are heavily redundant in terms of information access

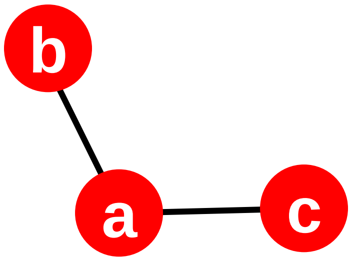


Triadic Closure

- **Triadic closure == High clustering coefficient**

Reasons for triadic closure:

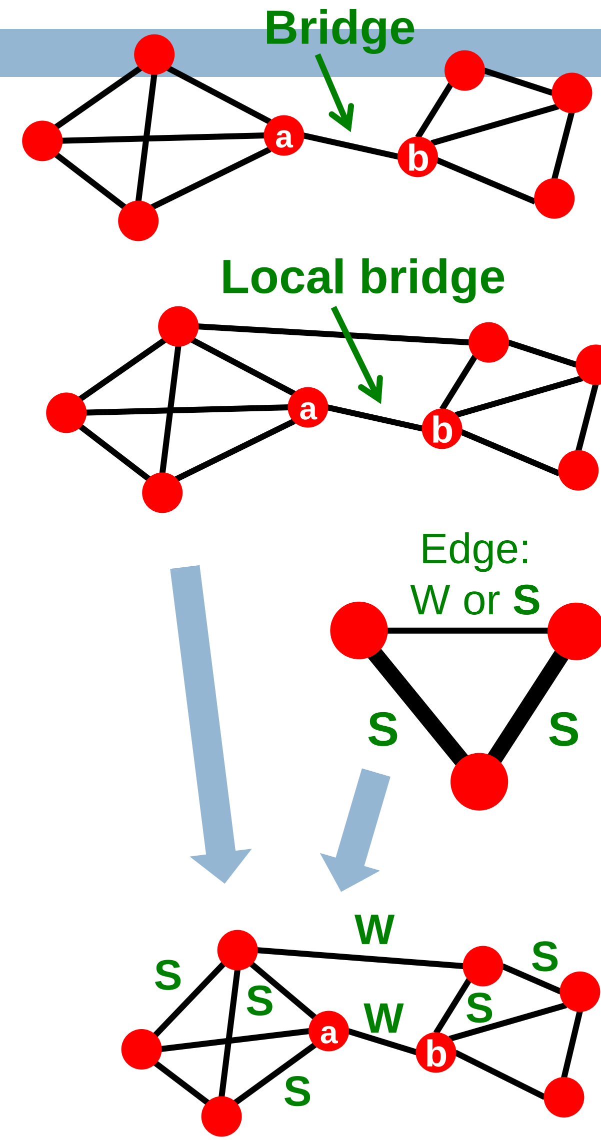
- If B and C have a friend A in common, then:
  - B is more likely to meet C
    - (since they both spend time with A)
  - B and C trust each other
    - (since they have a friend in common)
  - A has incentive to bring B and C together
    - (as it is hard for A to maintain two disjoint relationships)



- **Empirical study by Bearman and Moody:**
  - Teenage girls with low clustering coefficient are more likely to contemplate suicide

Granovetter's Explanation

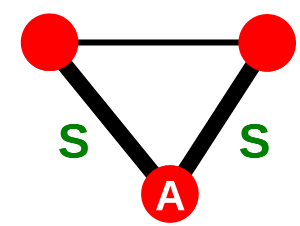
- Define: **Bridge edge**
  - If removed, it disconnects the graph
- Define: **Local bridge**
  - Edge of Span > 2
    (Span of an edge is the distance of the edge endpoints if the edge is deleted. Local bridges with long span are like real bridges)
- Define: Two types of edges:
  - **Strong** (friend), **Weak** (acquaintance)
- Define: **Strong triadic closure:**
  - Two strong ties imply a third edge
- **Fact:** If strong triadic closure is satisfied then **local bridges are weak ties!**

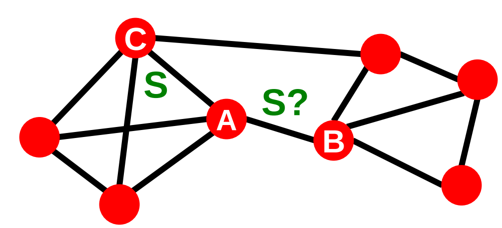


Local Bridges and Weak ties

- **Claim:** If node satisfies **Strong Triadic Closure** and is involved in at least **two strong ties**, then any local bridge adjacent to must be a weak **tie.**

- **Proof by contradiction:**
  - satisfies **Strong Triadic Closure**
  - Let be local bridge and a strong tie
  - Then must exist because of **Strong Triadic Closure**
  - But then is not a bridge!





Tie strength in real data

- **For many years the Granovetter's theory was not tested**
- But, today we have large who-talks-to-whom graphs:
  - Email, Messenger, Cell phones, Facebook
- **Onnela et al. 2007:**
  - Cell-phone network of 20% of country's population
  - **Edge strength:** # phone calls

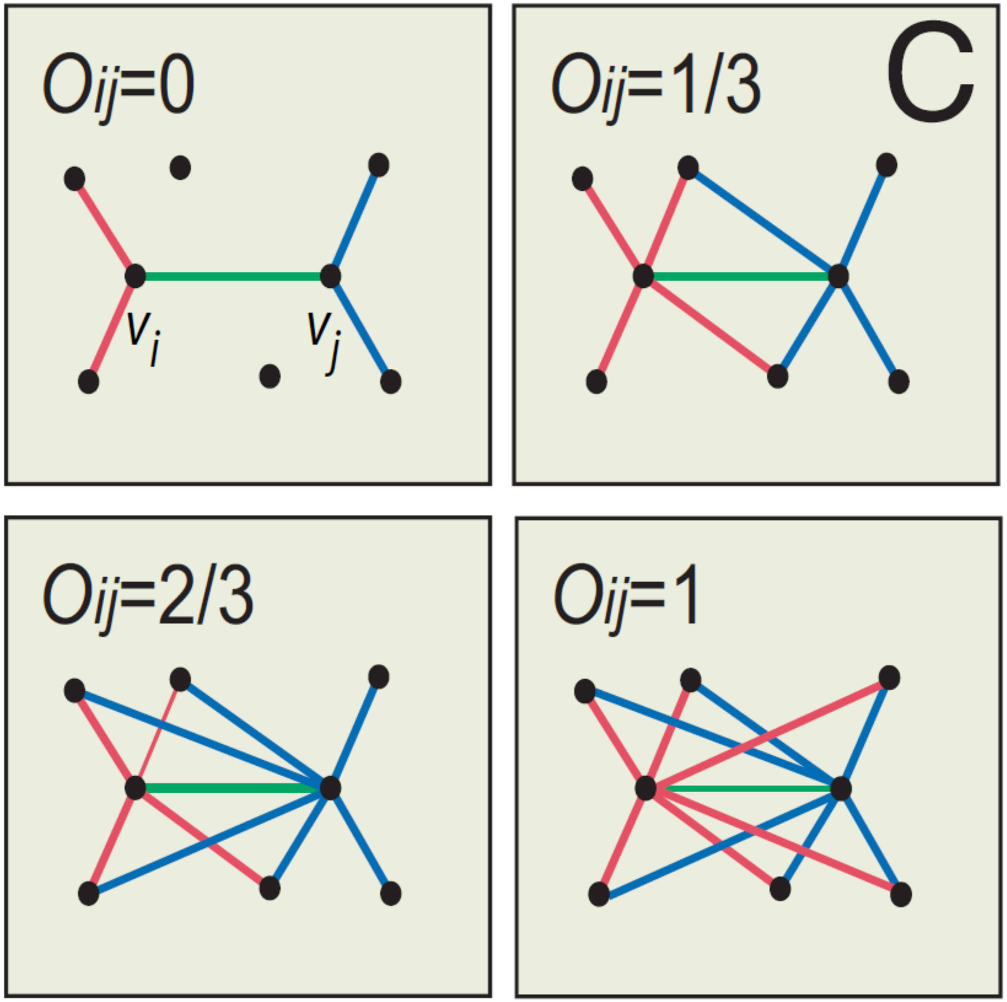
Neighborhood Overlap

- **Edge overlap:**

$$O_{ij} = \frac{N(i) \cap N(j)}{N(i) \cup N(j)}$$

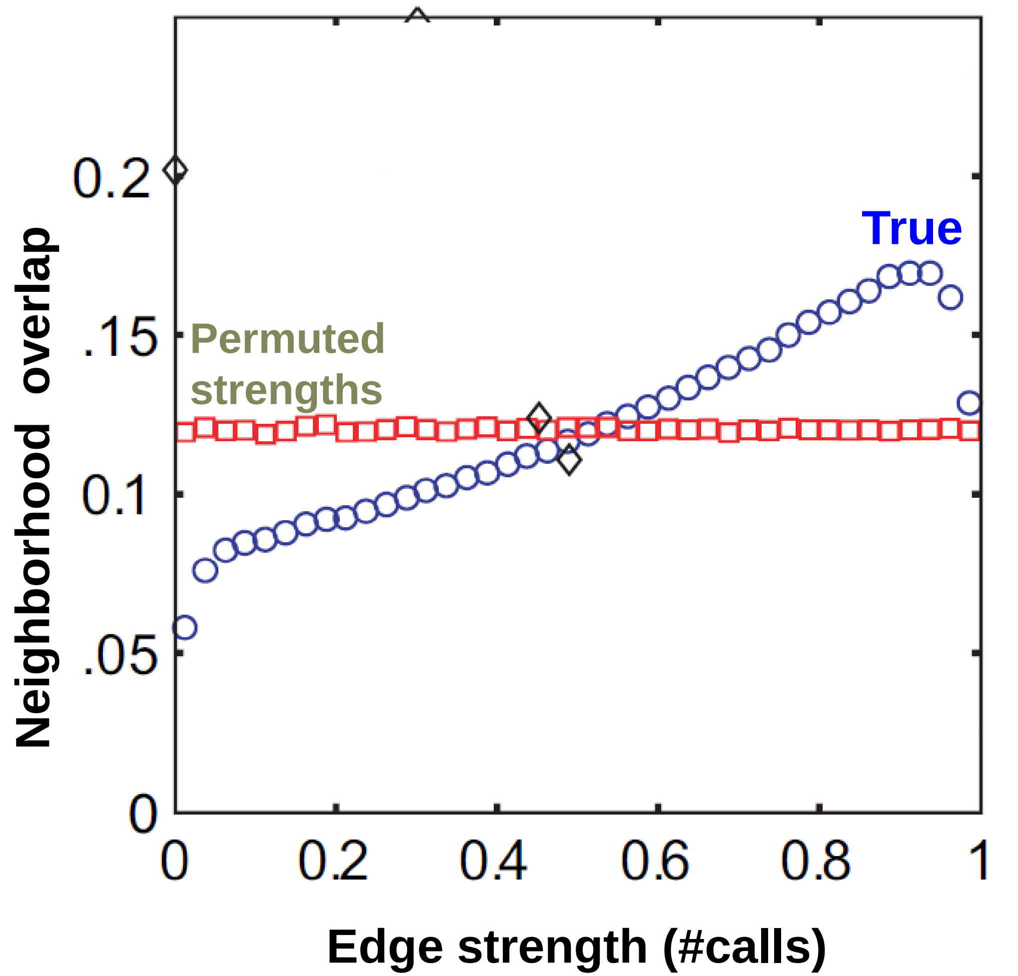
  - $N(i)$ … a set of neighbors of node i

- Overlap = 0 when an edge is a **local bridge**



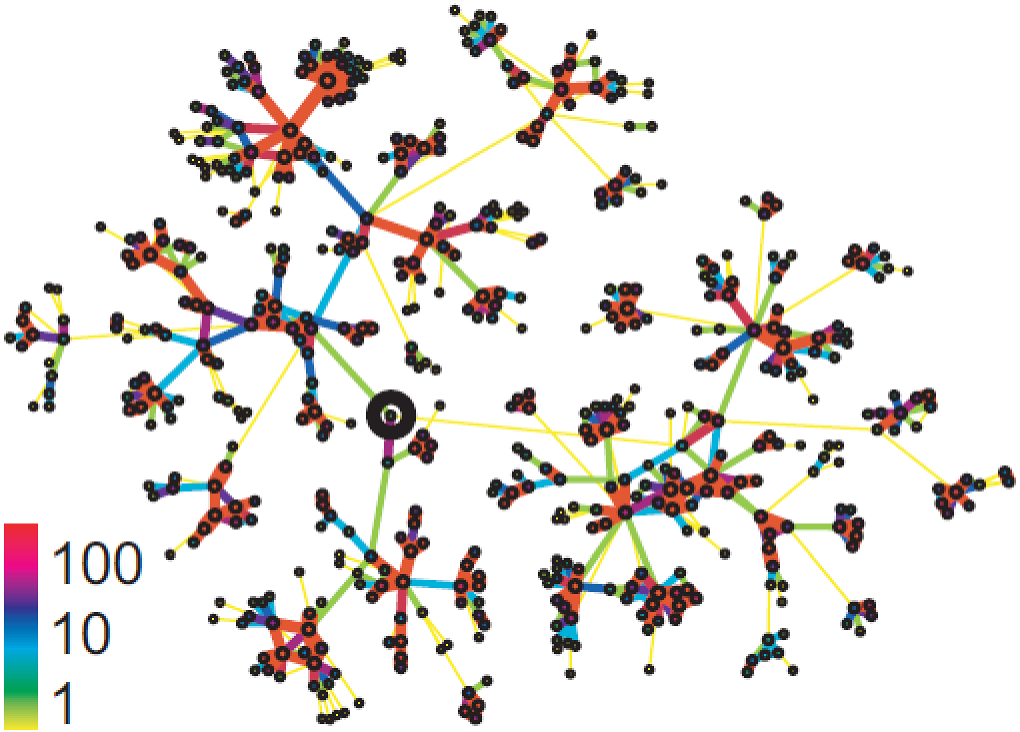
Phones: Edge Overlap vs. Strength

- **Cell-phone network**
- **Observation:**
  - Highly used links have high overlap!



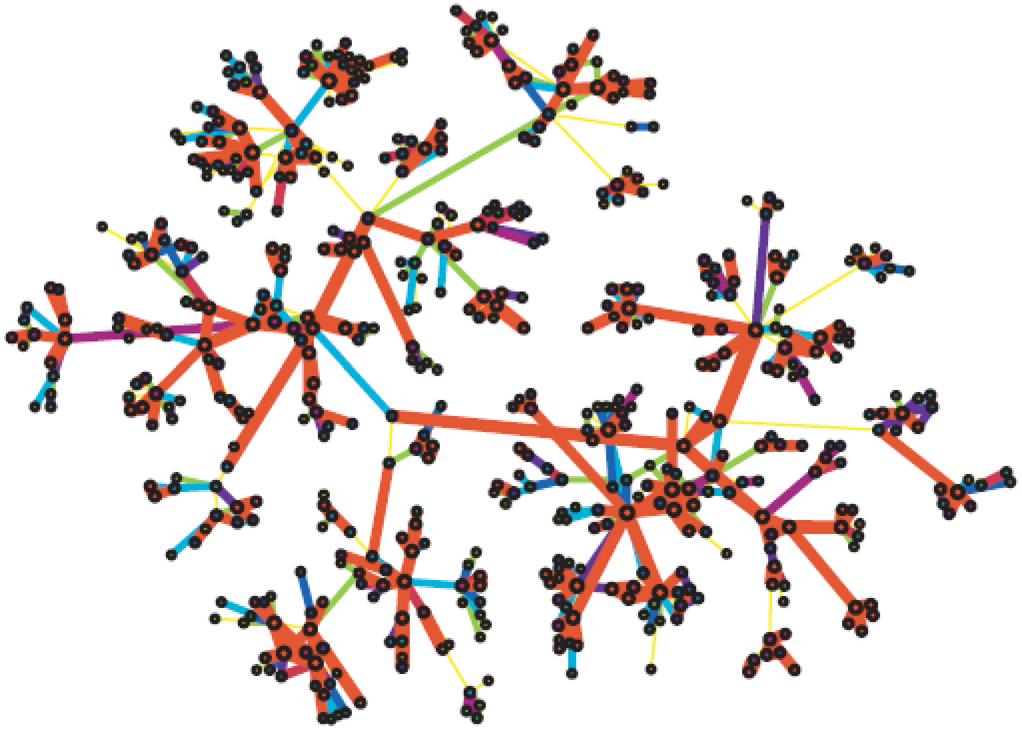
- **Legend:**
  - **True:** The data
  - **Permuted strengths:** Keep the network structure but randomly reassign edge strengths

Real Network, Real Tie Strengths



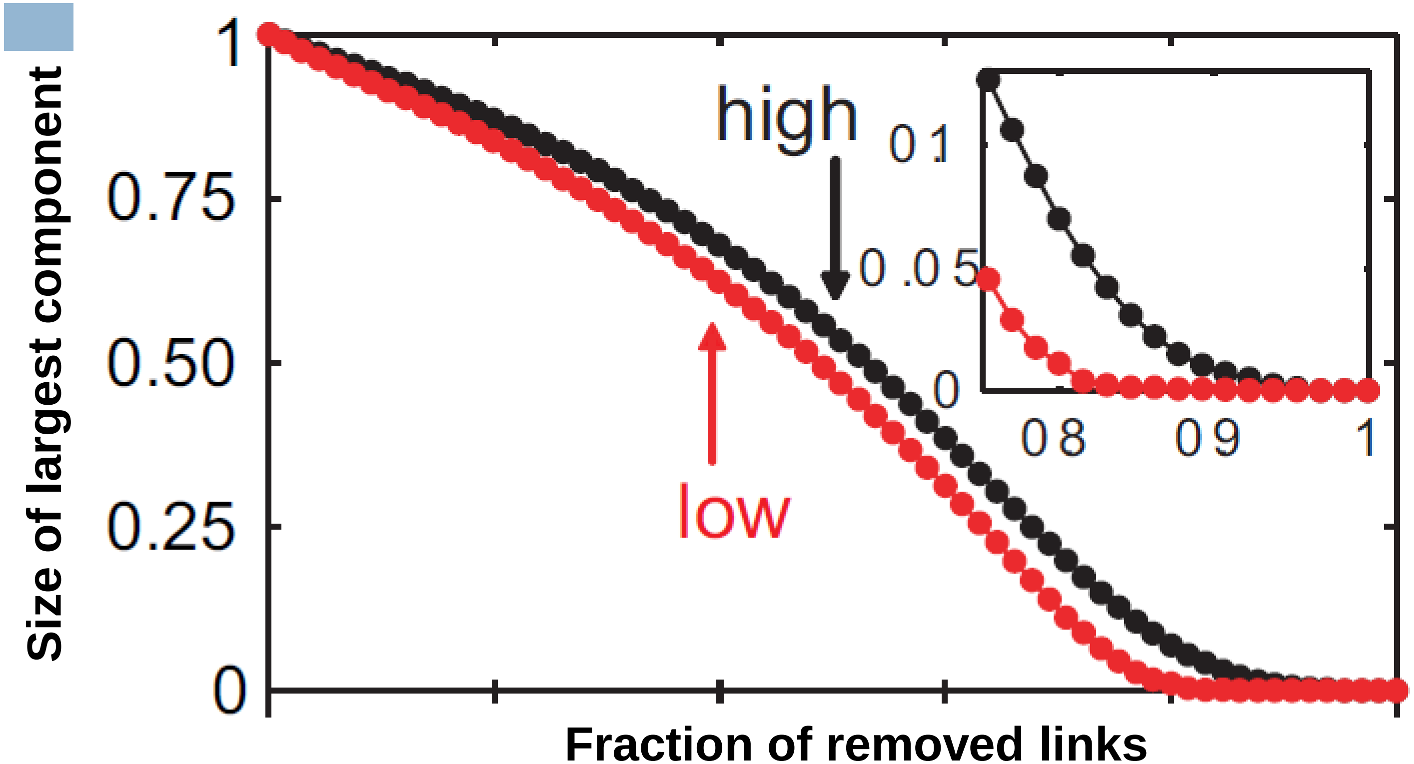
- **Real edge strengths in mobile call graph**
  - Strong ties are more embedded (have higher overlap)

Real Net, Permuted Tie Strengths



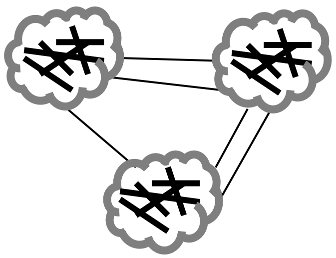
- **Same network, same set of edge strengths** but now **strengths are randomly shuffled**

Link Removal by Strength



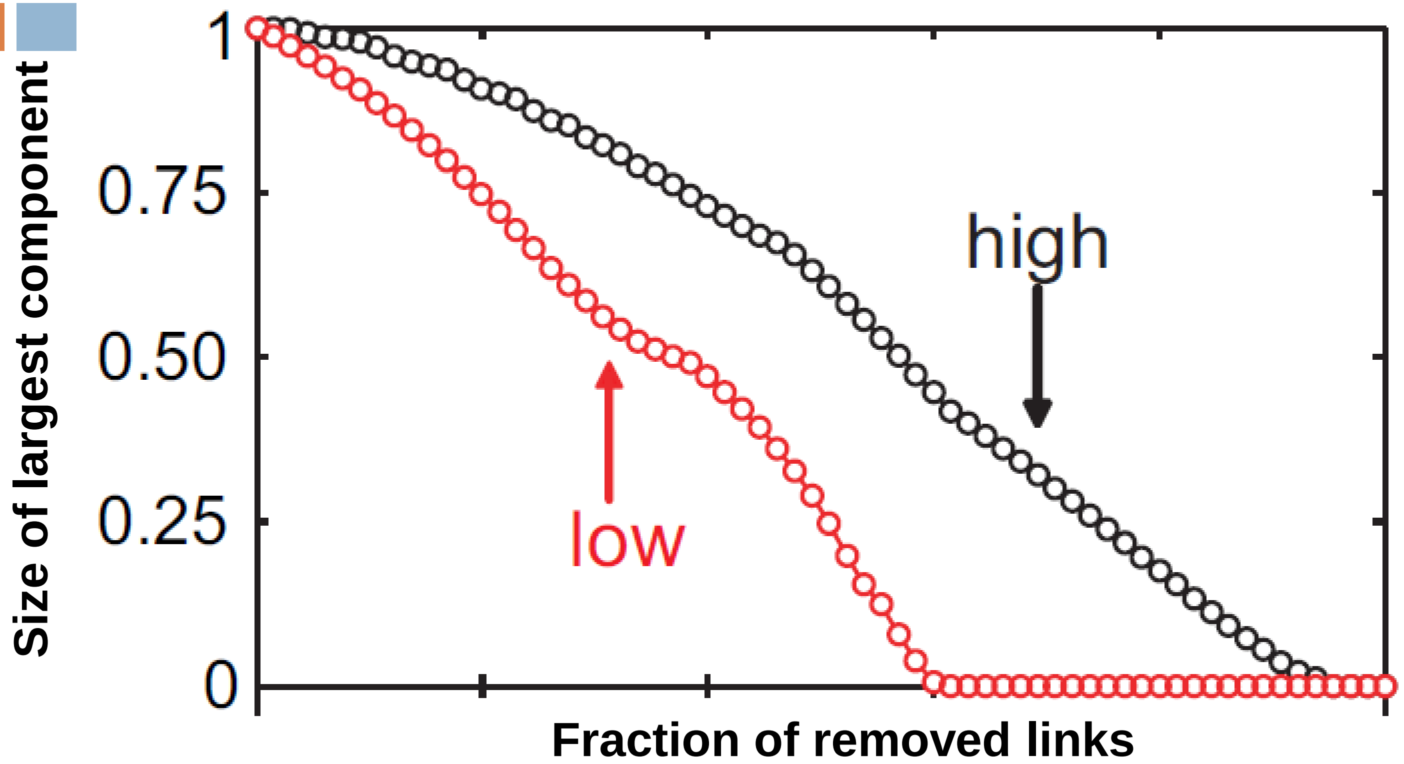
Low disconnects the network sooner

- Removing links by **strength (#calls)**
  - Low to high
  - High to low



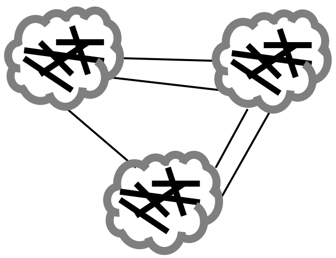
Conceptual picture of network structure

Link Removal by Overlap



Low disconnects the network sooner

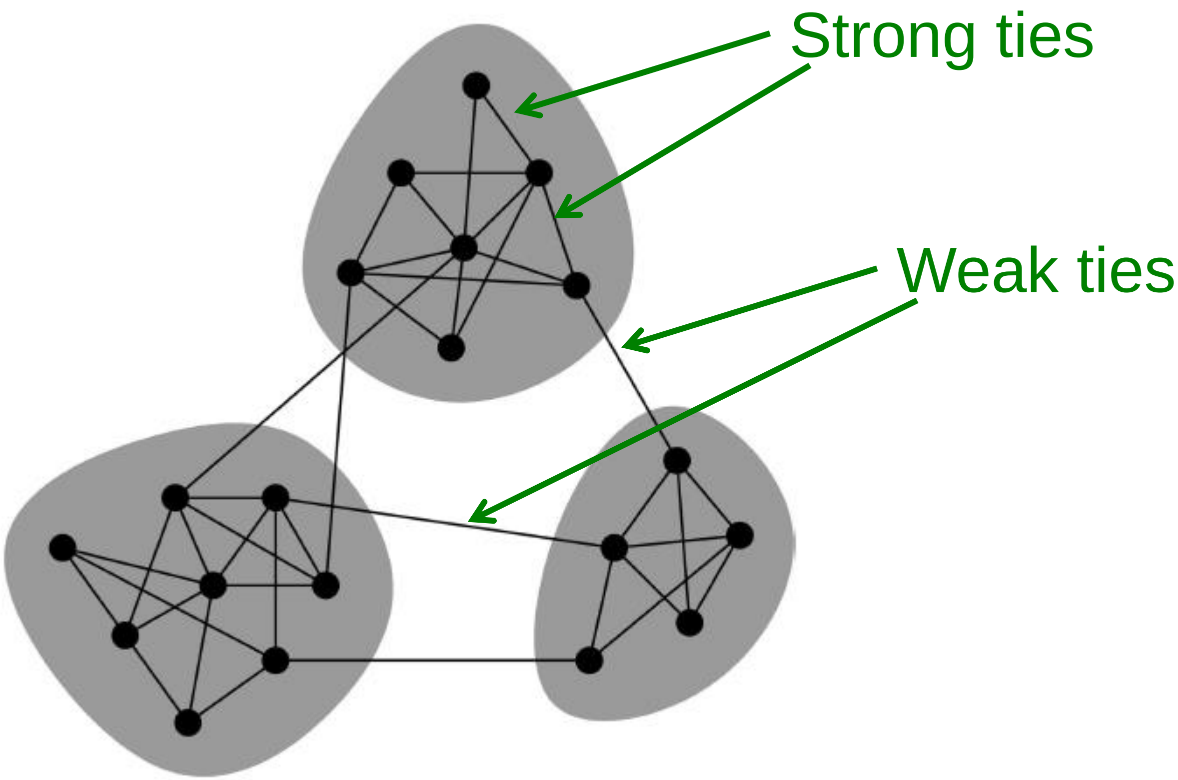
- Removing links based on **overlap**
  - Low to high
  - High to low



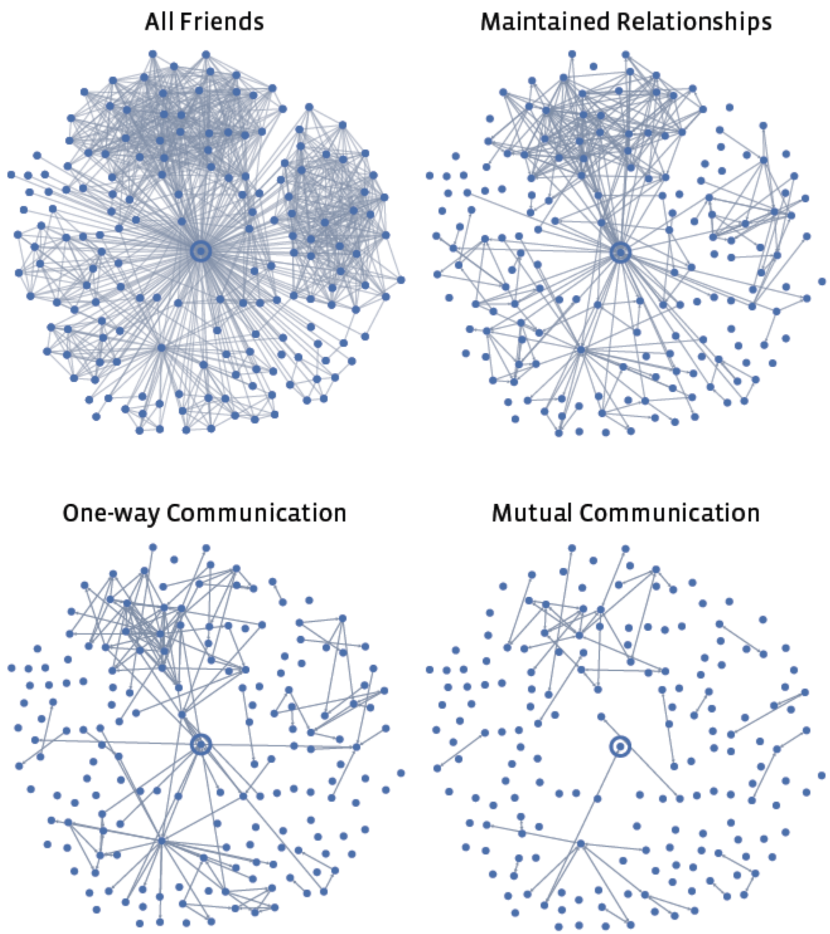
Conceptual picture of network structure

Conceptual Picture of Networks

- **Granovetter's theory leads to the following conceptual picture of networks**



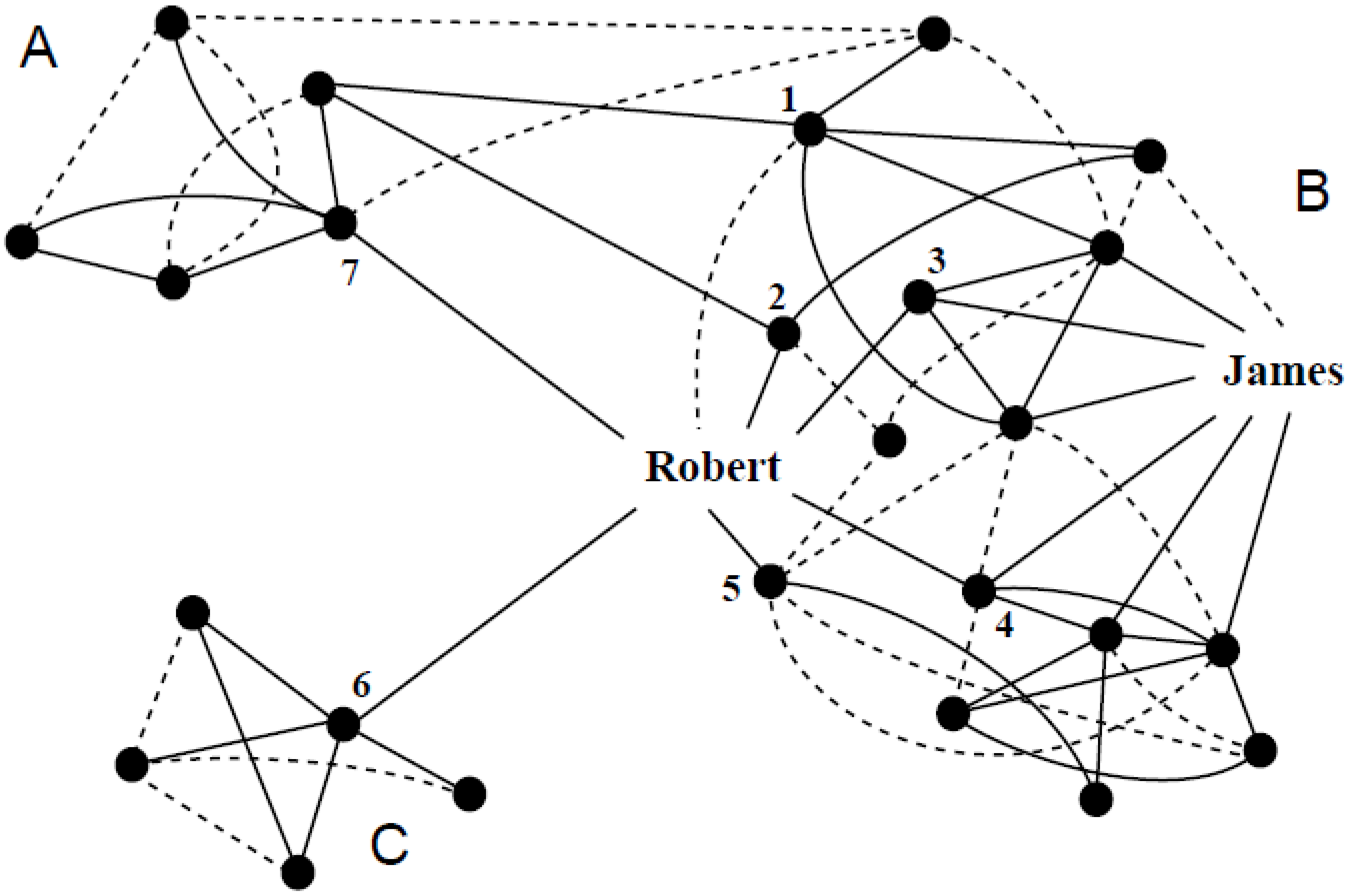
Facebook User's Tie Strength



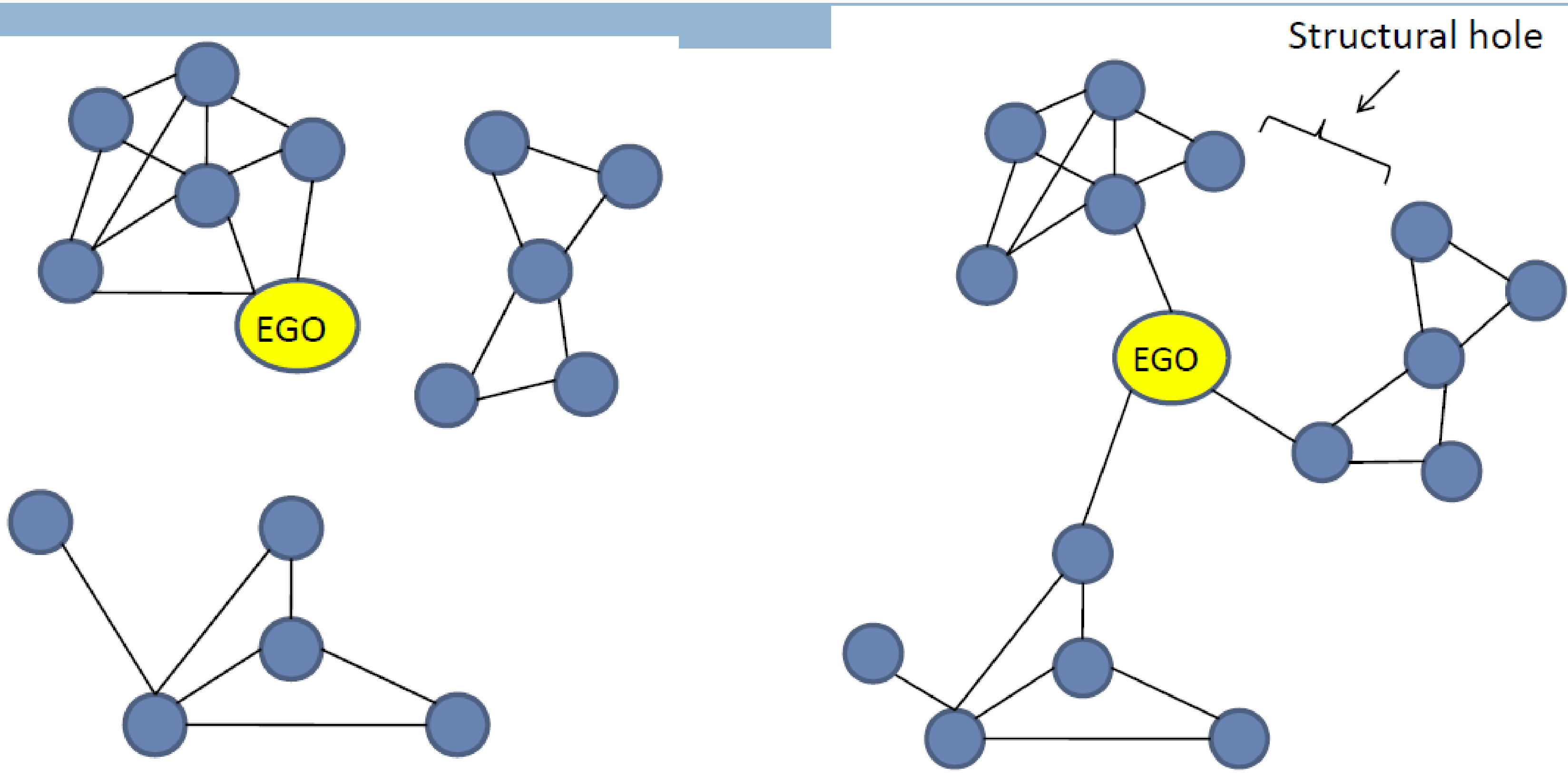
Four different views of a Facebook User's network neighborhood, show the structure of links corresponding respectively to all declared friendships, maintained relationships, one-way communication, and reciprocal communication

SMALL DETOUR: STRUCTURAL HOLES

Small Detour: Structural Holes



Structural Holes



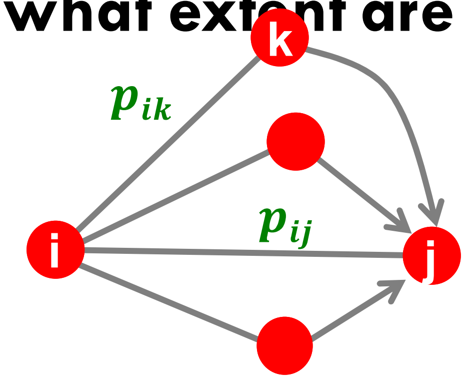
Few structural holes

Many structural holes

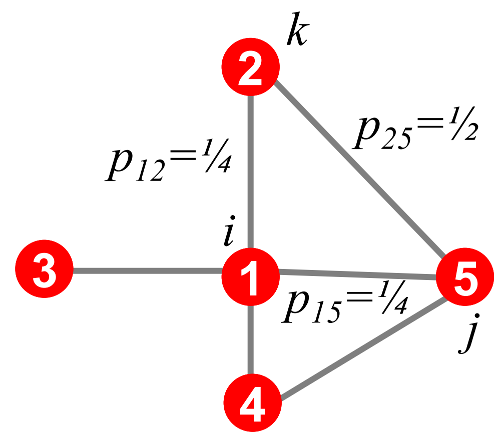
Structural Holes provide ego with access to novel information, power, freedom

Structural Holes: Network Constraint

- **The "network constraint" measure** [Burt]:
  - **To what extent are person's contacts redundant**



$$p_{uv} = 1/du$$



  - **Low**: disconnected contacts
  - **High**: contacts that are close or strongly tied

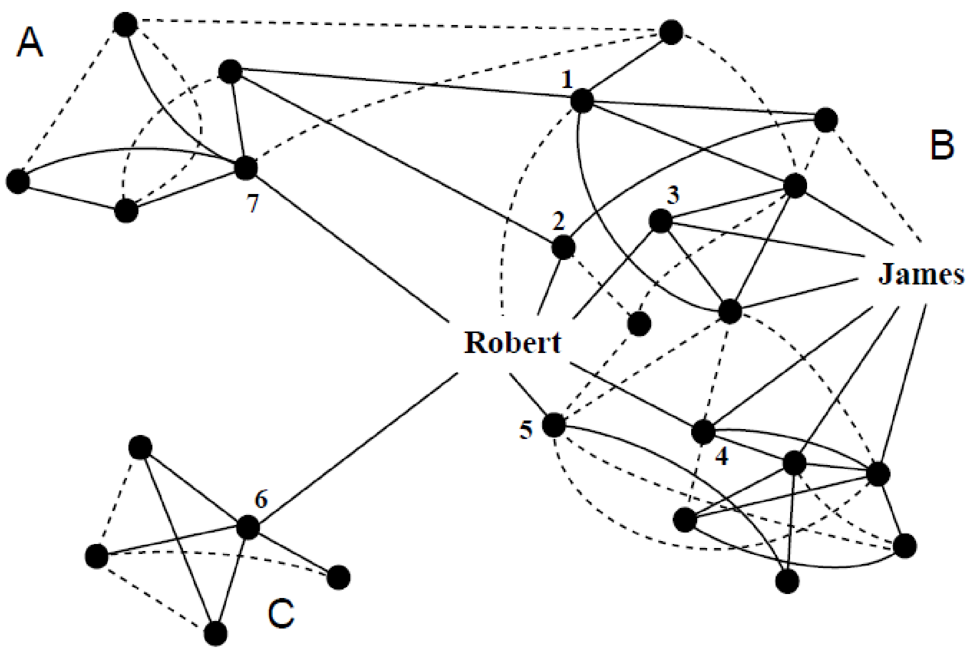
$$c_i = \sum_j c_{ij} = \sum_j \left[p_{ij} + \sum_k \left(p_{ik} p_{kj} \right) \right]^2$$

p_{uv} … prop. of u's "energy" invested in relationship with v

p_{uv}

	1	2	3	4	5
1	.00	.25	.25	.25	.25
2	.50	.00	.00	.00	.50
3	1.0	.00	.00	.00	.00
4	.50	.00	.00	.00	.50
5	.33	.33	.00	.33	.00

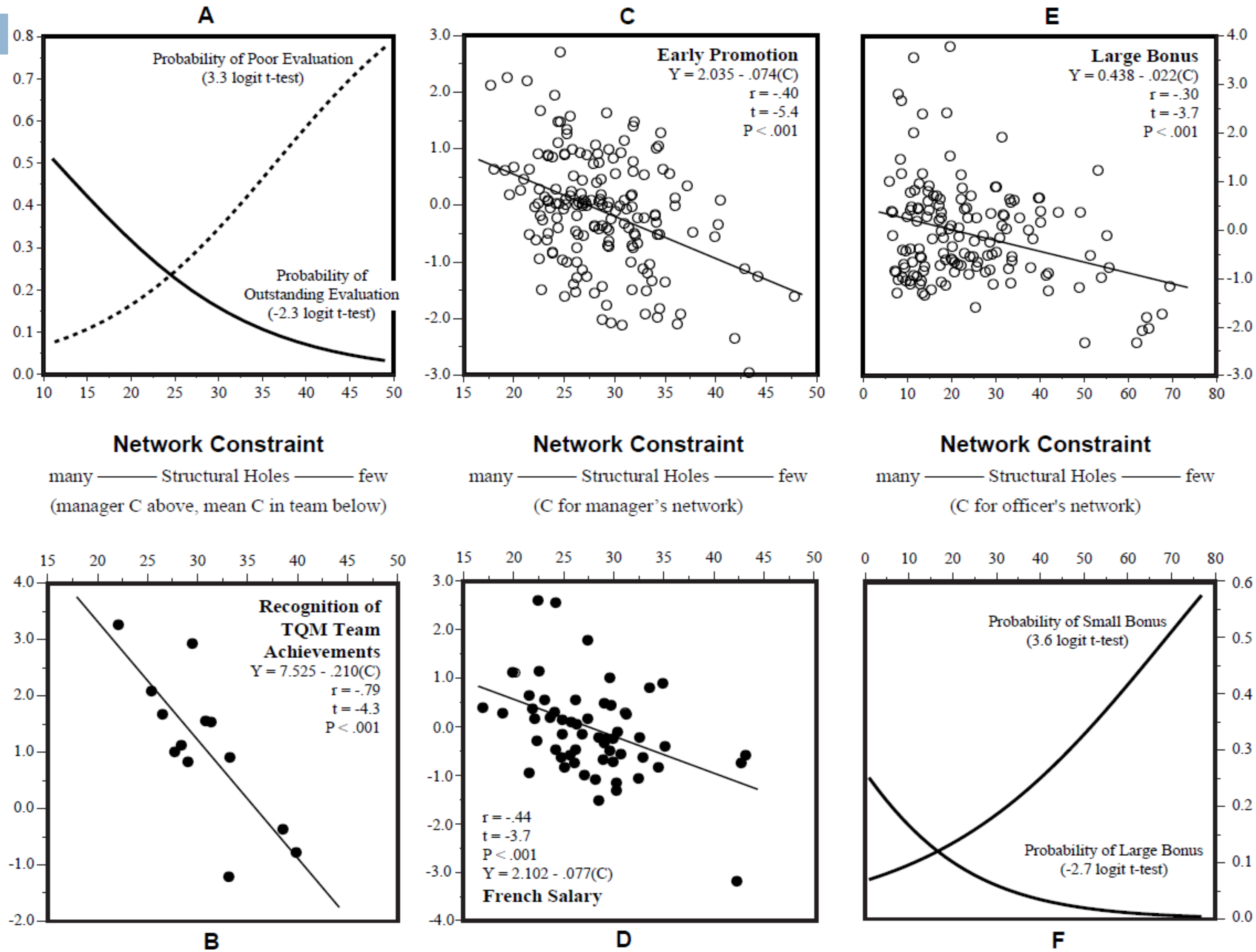
Example: Robert vs. James



- **Constraint:** To what extent are person's contacts redundant
  - Low: disconnected contacts
  - High: contacts that are close or strongly tied

- **Network constraint:**
  - James:
  - Robert:

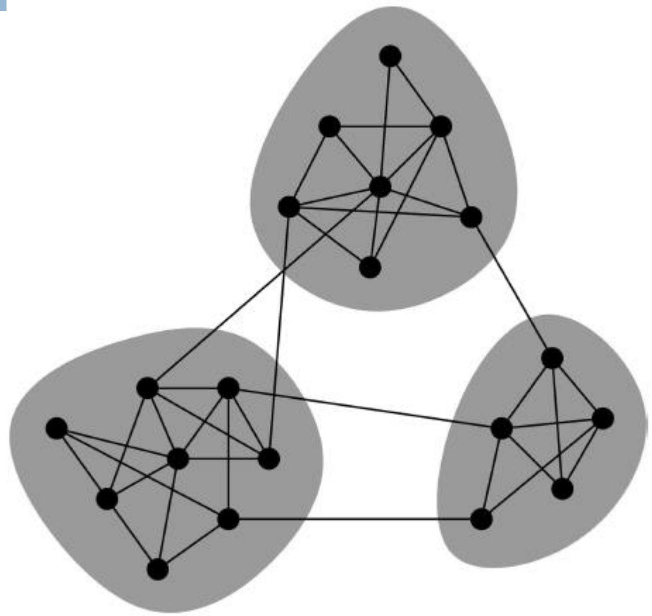
Spanning the Holes Matters



NETWORK COMMUNITIES

Network Communities

- Granovetter's theory (and common sense) suggest that networks are composed of **tightly connected sets of nodes**

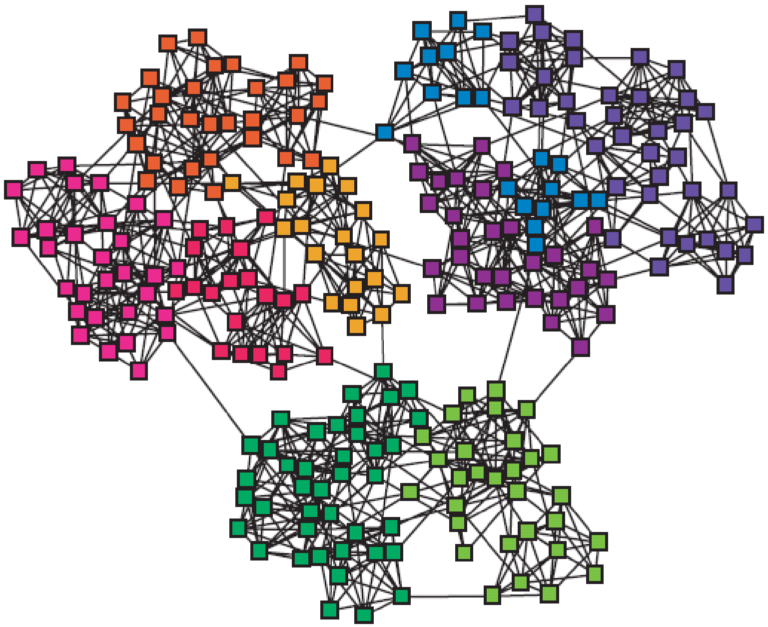


Communities, clusters, groups, modules

- **Network communities:**
  - Sets of nodes with **lots** of connections **inside** and **few** to **outside** (the rest of the network)

Finding Network Communities

- **How to automatically find such densely connected groups of nodes?**



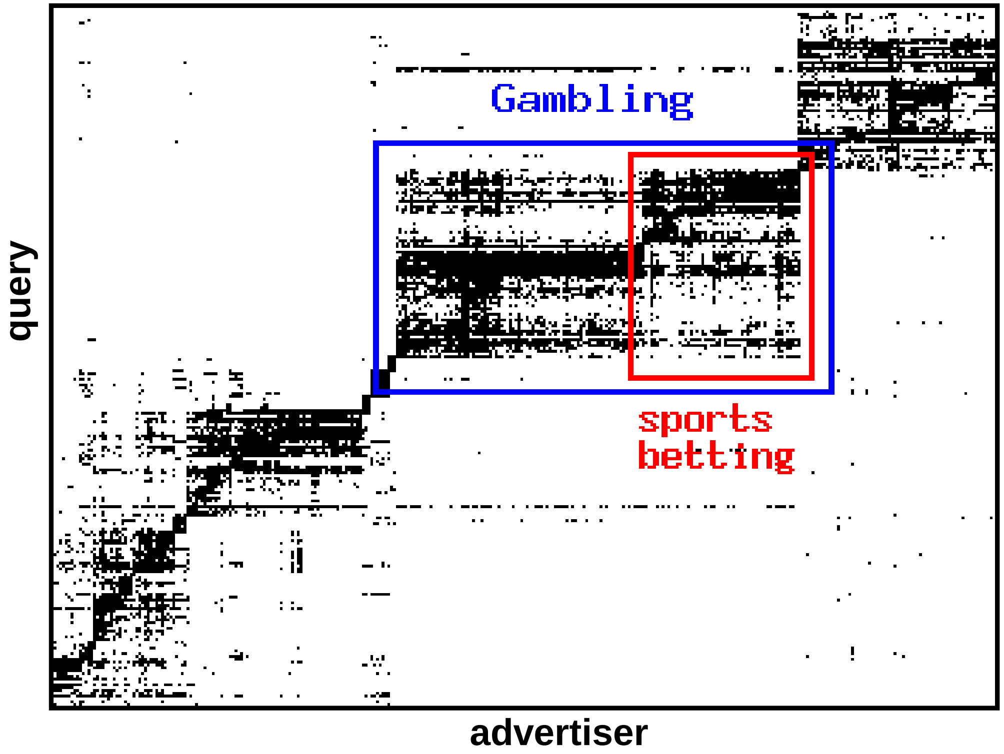
- Ideally such automatically detected clusters would then correspond to real groups

- For example:

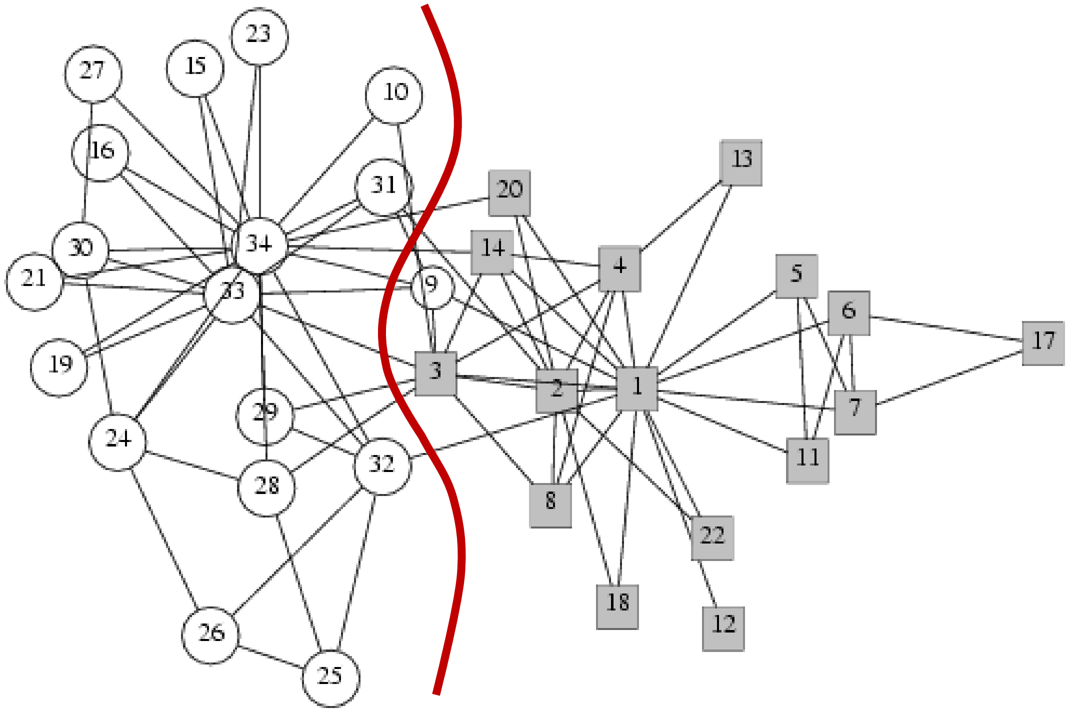
Communities, clusters, groups, modules

Micro-Markets in Sponsored Search

Find micro-markets by partitioning the "query x advertiser" graph:



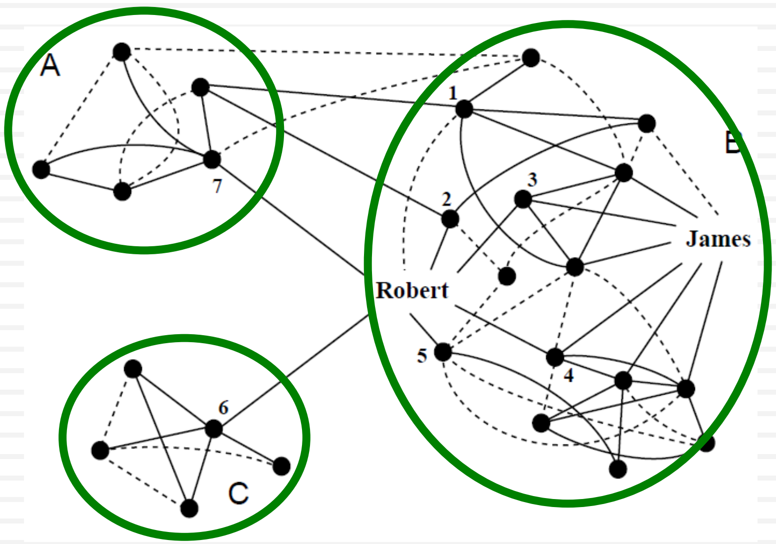
Social Network Data

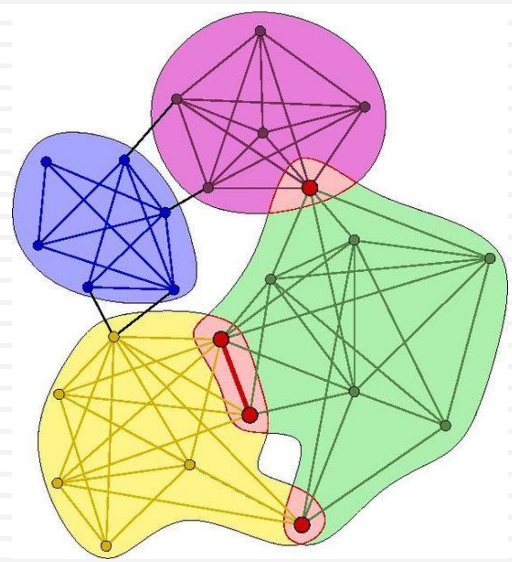


- **Zachary's Karate club network:**
  - Observe social ties and rivalries in a university karate club
  - During his observation, conflicts led the group to split
  - Split could be explained by a minimum cut in the network

Community Detection

How to find communities?

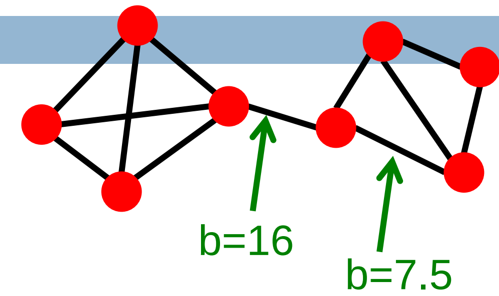




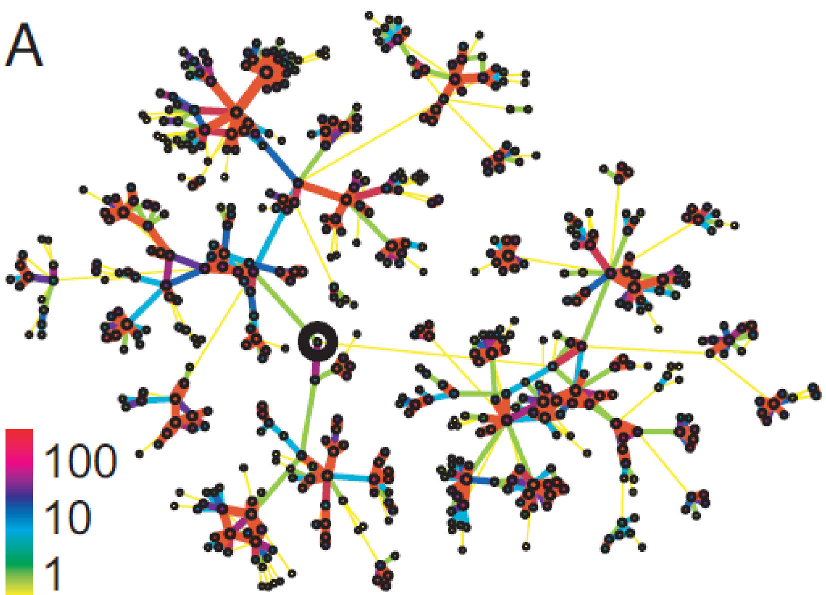
We will work with **undirected** (unweighted) networks

Method 1: Strength of Weak Ties

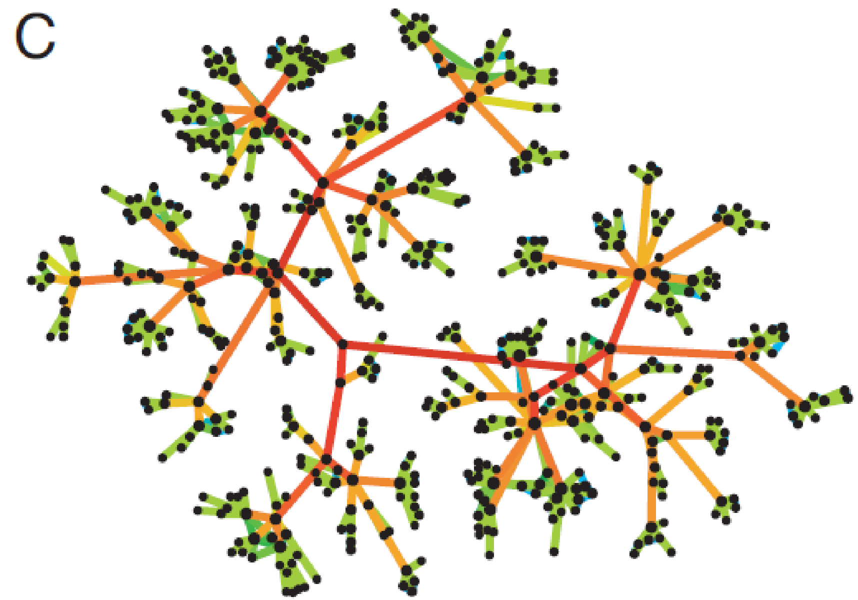
- **Edge betweenness:** Number of shortest paths passing over the edge



- **Intuition:**



Edge strengths (call volume) in real network



Edge betweenness in real network

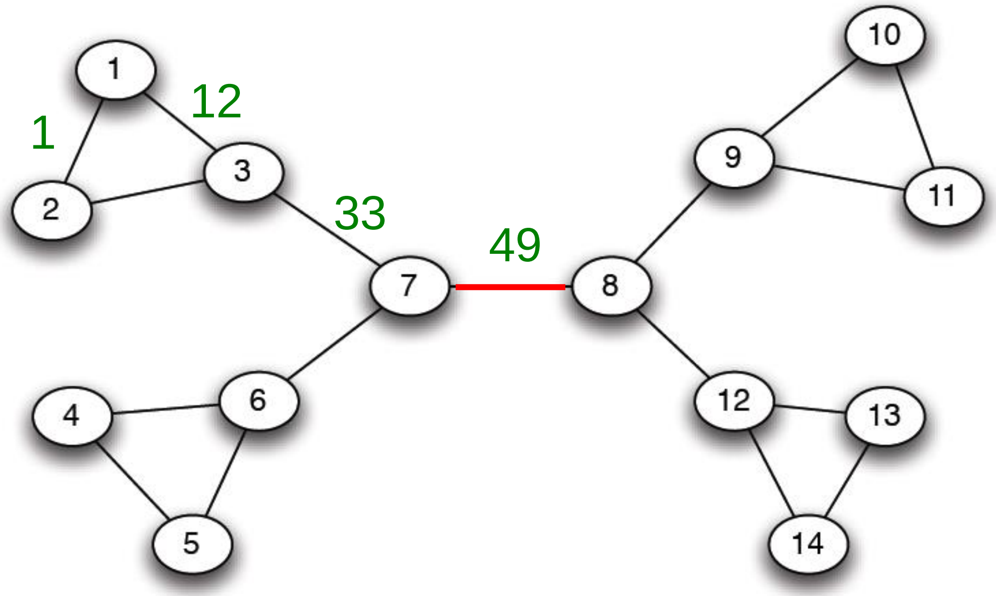
Method 1: Girvan-Newman

- Divisive hierarchical clustering based on the notion of edge **betweenness:**

  Number of shortest paths passing through the edge

- **Girvan-Newman Algorithm:**
  - **Undirected unweighted networks**
  - Repeat until no edges are left:
    - Calculate betweenness of edges
    - Remove edges with highest betweenness
  - Connected components are communities
  - Gives a hierarchical decomposition of the network

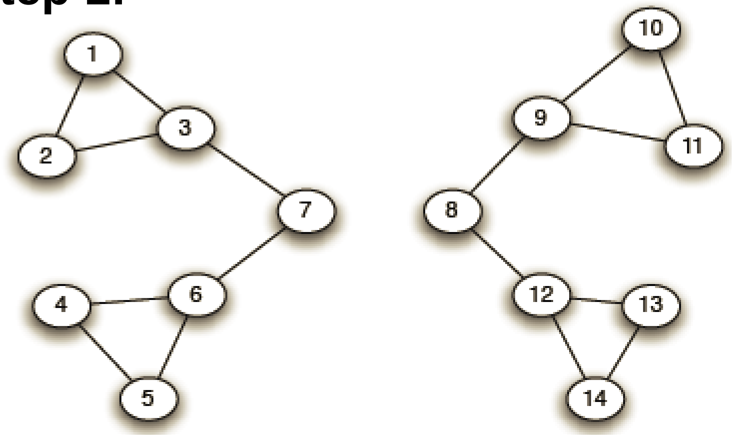
Girvan-Newman: Example



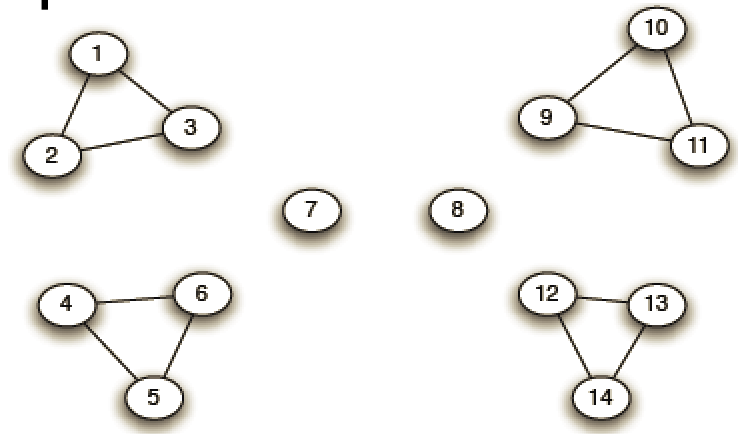
Need to re-compute betweenness at every step

Girvan-Newman: Example

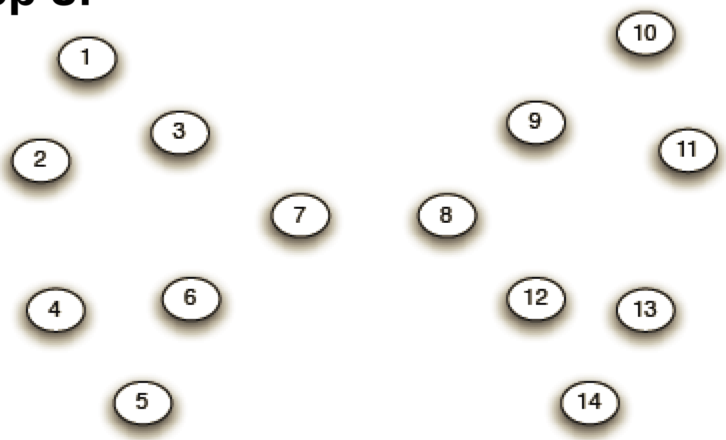
Step 1:



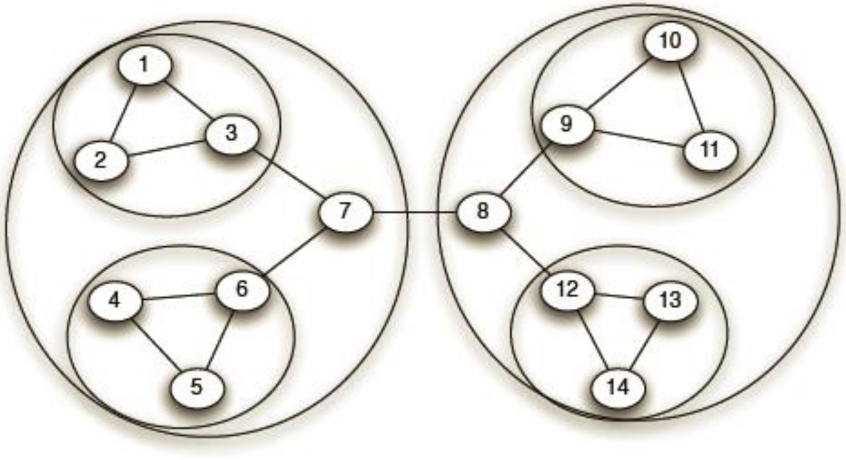
Step 2:



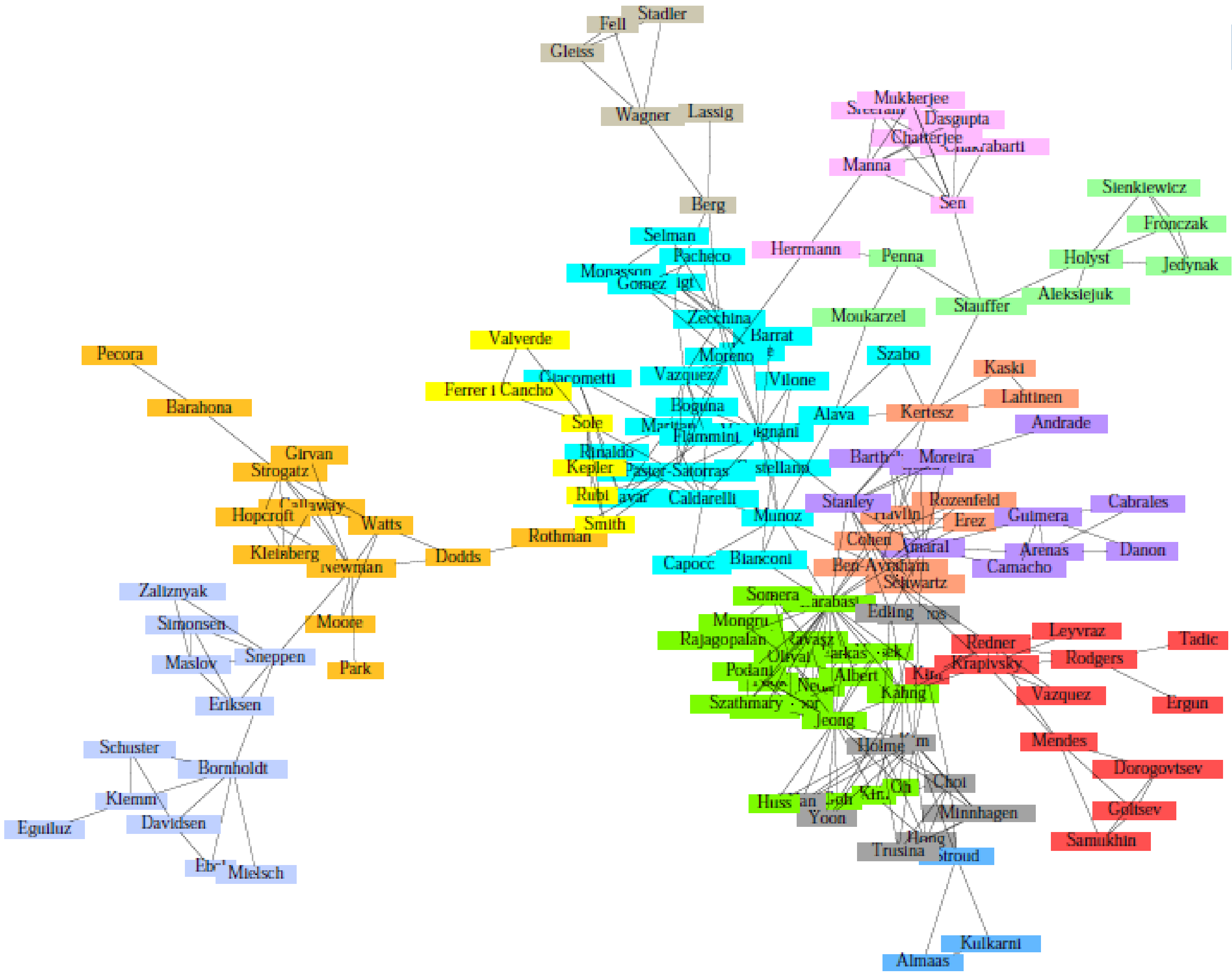
Step 3:



Hierarchical network decomposition:



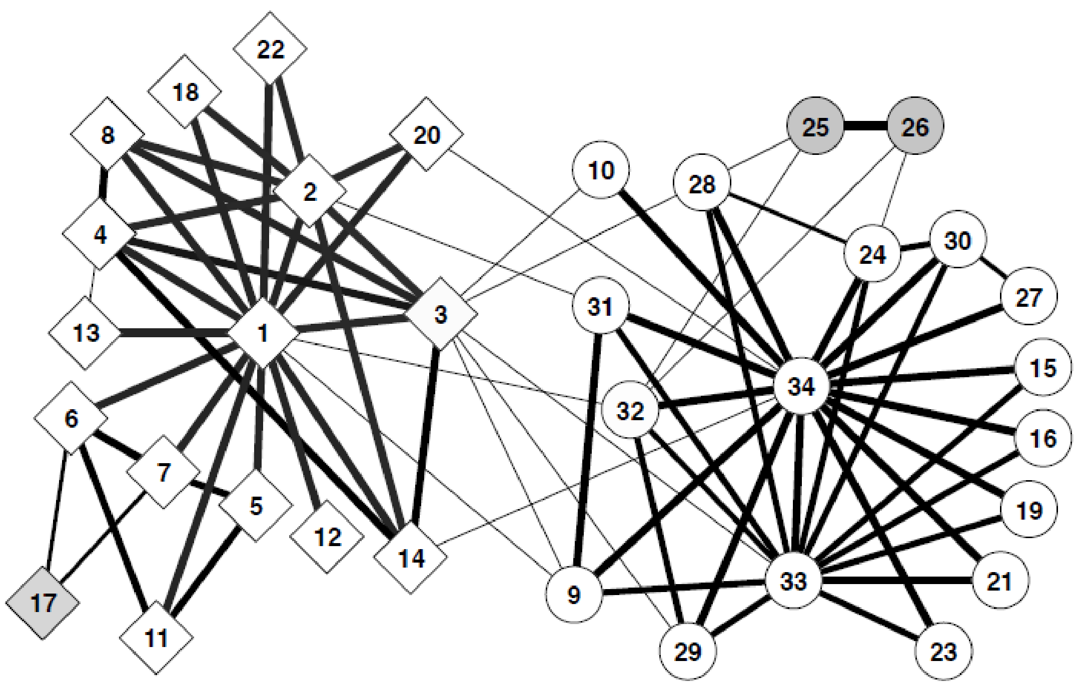
Girvan-Newman: Results

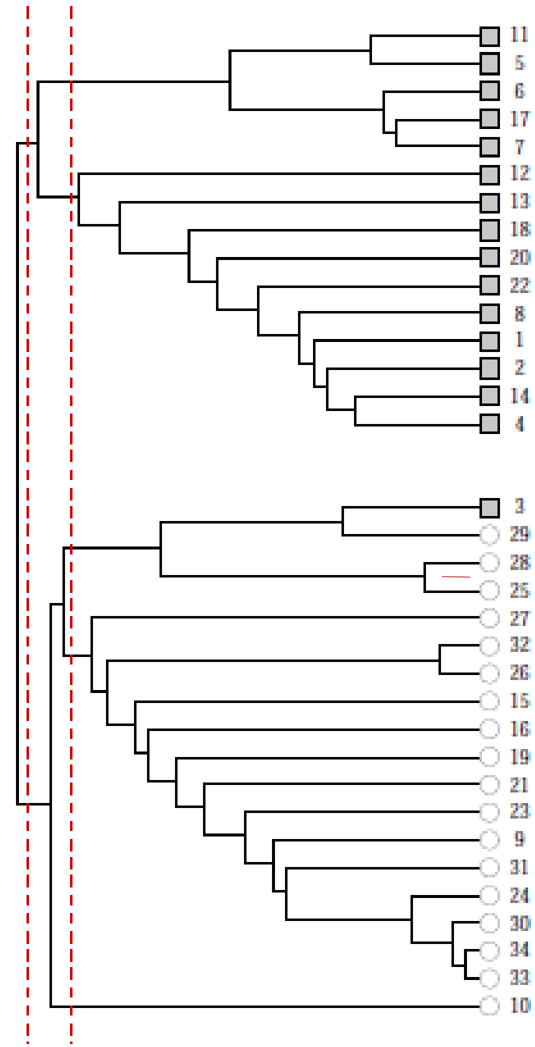


Communities in physics collaborations

Girvan-Newman: Results

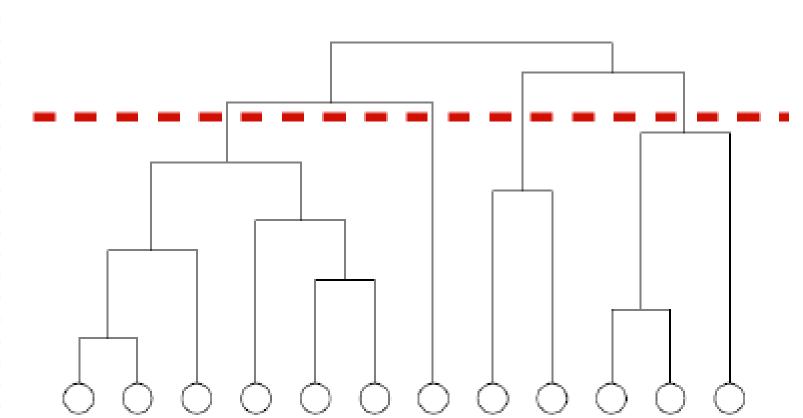
- **Zachary's Karate club:** Hierarchical decomposition





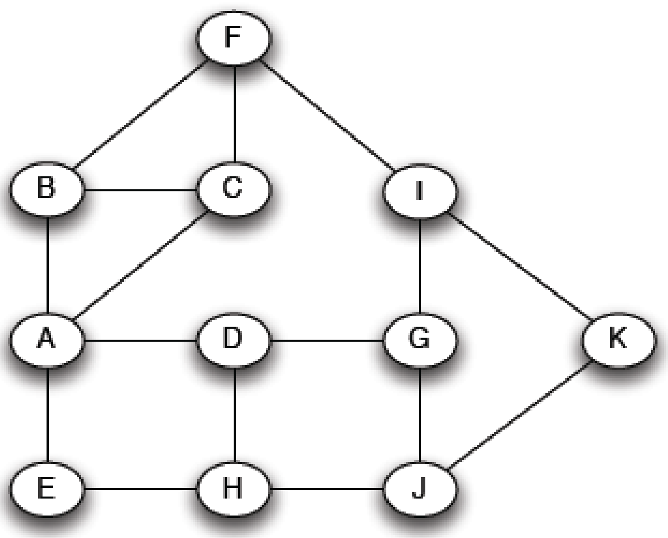
We need to resolve 2 questions

1. **How to compute betweenness?**
2. **How to select the number of clusters?**

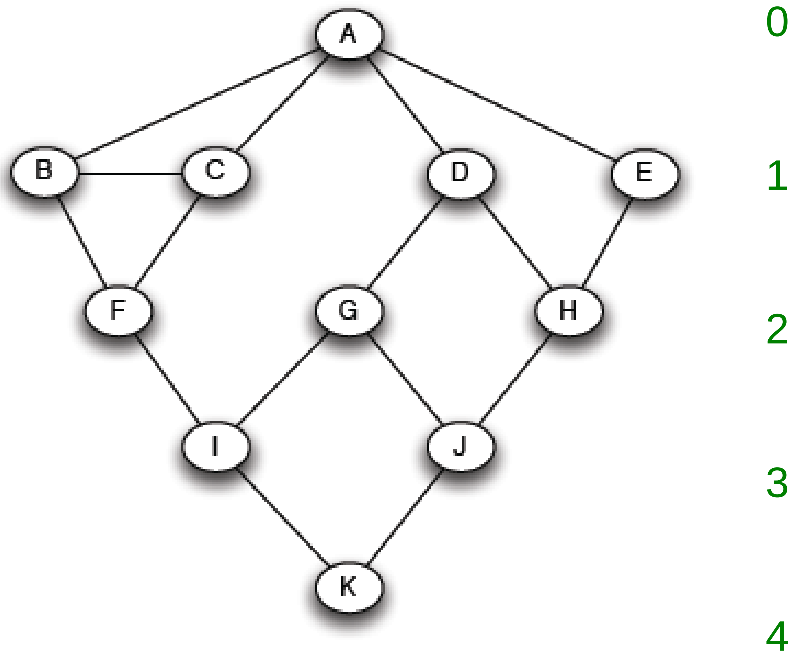


How to Compute Betweenness?

- **Want to compute betweenness of paths starting at node A**

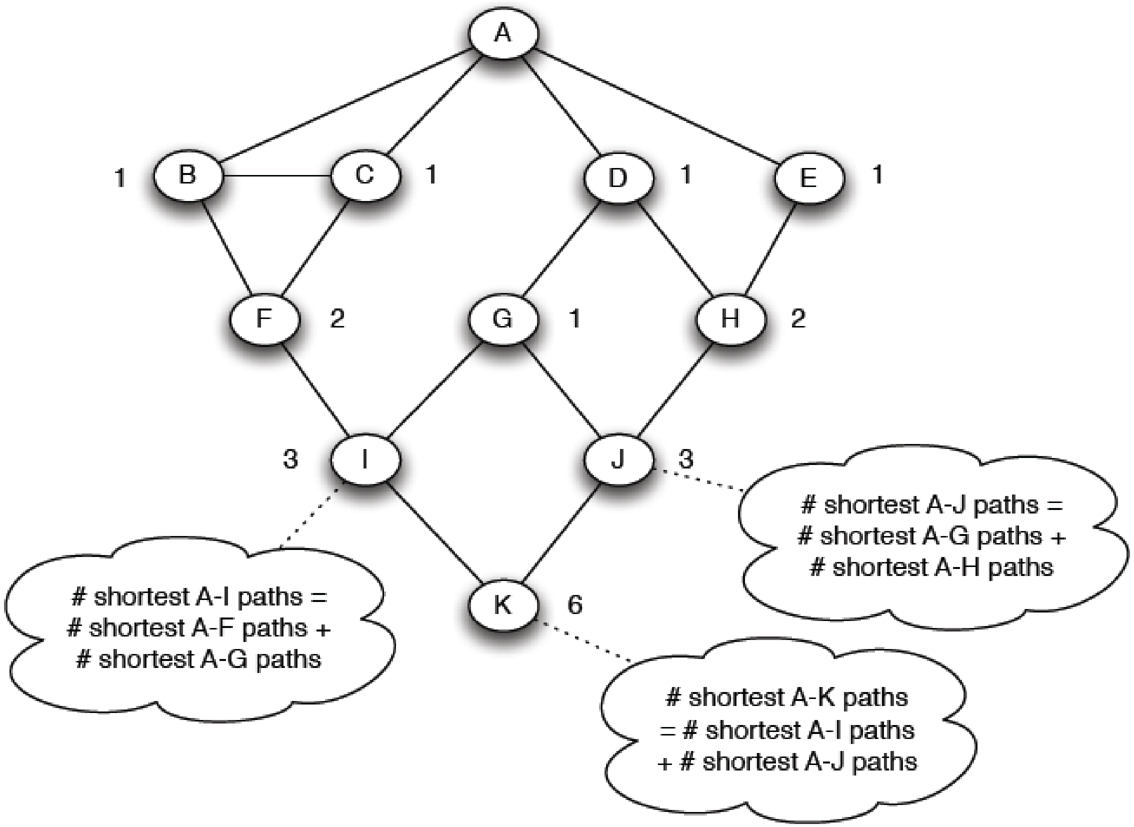


- **Breath first search starting from A:**



How to Compute Betweenness?

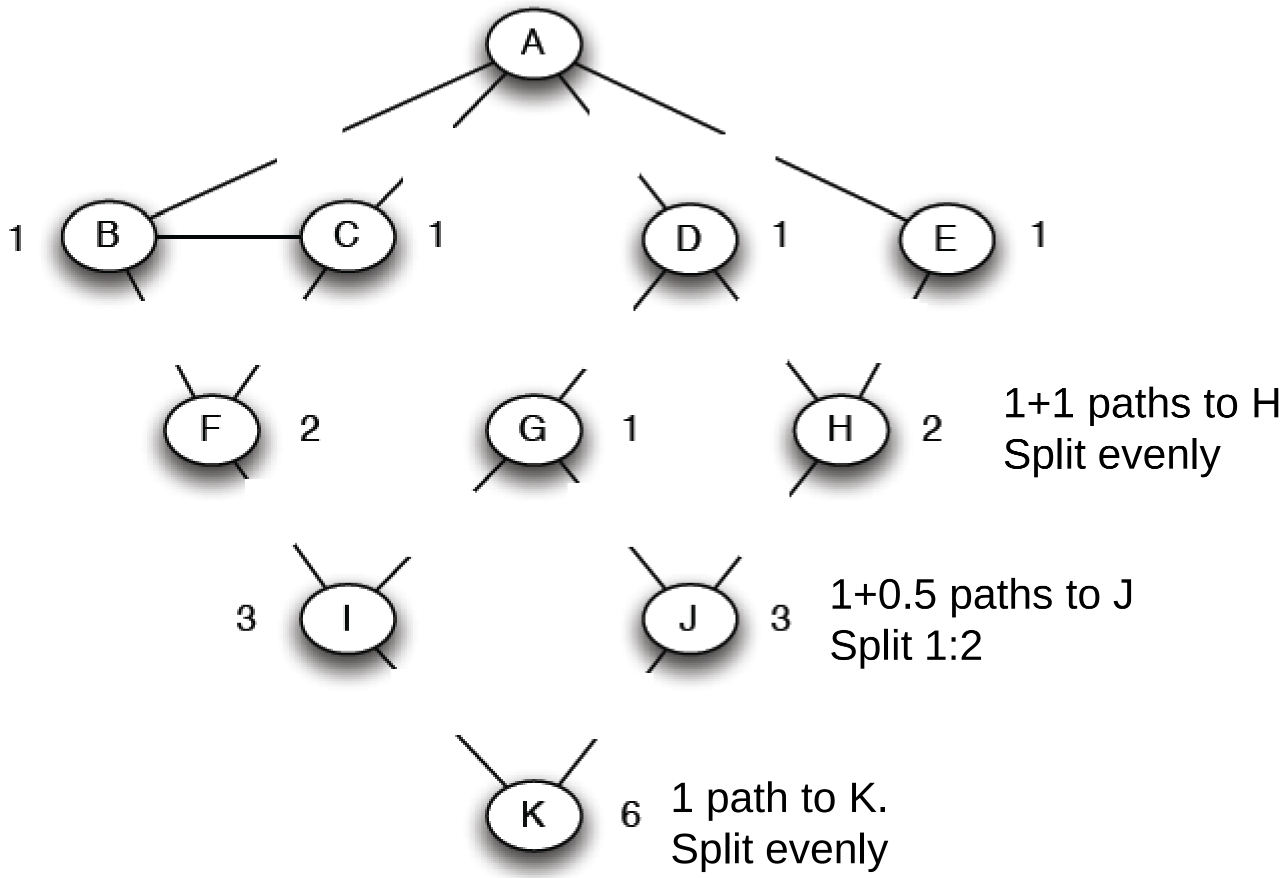
- **Count the number of shortest paths from *A* to all other nodes of the network:**



How to Compute Betweenness?

- **Compute betweenness by working up the tree:** If there are multiple paths count them fractionally

The algorithm:
•Add edge **flows**:
-- node flow = 1+∑child edges
-- split the flow up based on the parent value
• Repeat the BFS procedure for each starting node *U*



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1. **How to compute betweenness?**
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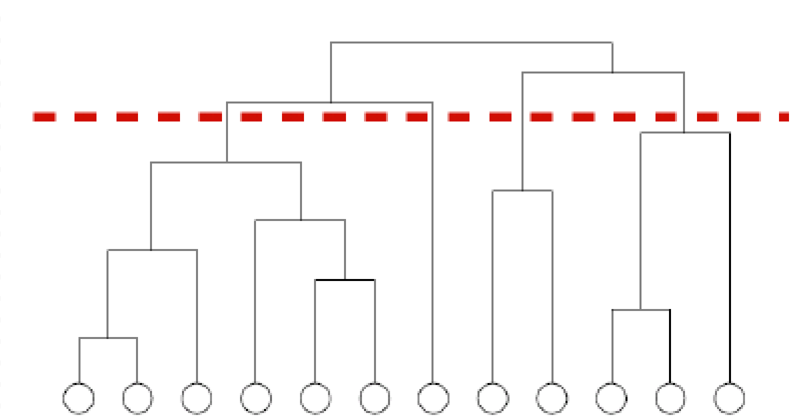
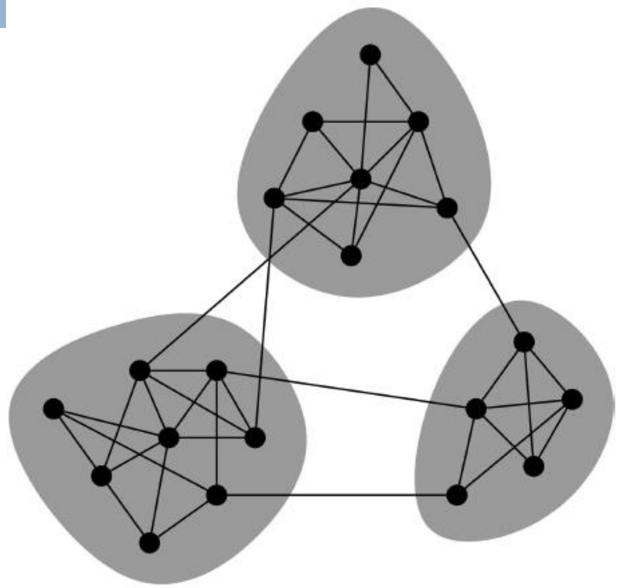


Network Communities

- **Communities:** **sets of tightly connected nodes**
- Define: **Modularity Q**
  - A measure of how well a network is partitioned into communities
  - Given a partitioning of the network into groups $s \in S$:

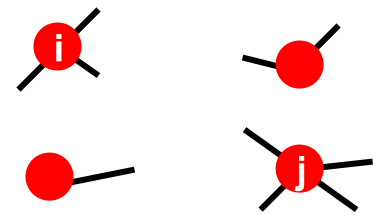
$$Q \propto \sum_{s \in S} [\ (\#\text{ edges within group } s) - (\text{expected }\#\text{ edges within group } s)\ ]$$



Need a null model!

Null Model: Configuration Model

- **Given real on nodes and edges, construct rewired network**
  - Same degree distribution but random connections
  - Consider as a multigraph
  - **The expected number of edge between nodes *i* and *j*** of degrees k_i and k_j equals to: $k_i \cdot \frac{k_j}{2m} = \boldsymbol{\frac{k_i k_j}{2m}}$
    - The expected number of edges in (multigraph) G':
      - $= \frac{1}{2}\sum_{i \in N}\sum_{j \in N}\frac{k_i k_j}{2m} = \frac{1}{2} \cdot \frac{1}{2m}\sum_{i \in N} k_i \left(\sum_{j \in N} k_j\right) =$
      - $= \frac{1}{4m} 2m \cdot 2m = m$



Note:
$\sum_{u \in N} k_u = 2m$

Modularity

- **Modularity of partitioning S of graph G:**
  - Q ∝ $\sum_{s \in S}$ [(# edges within group *s*) – (expected # edges within group *s*)]
  - $Q(G,S) = \underbrace{\frac{1}{2m}}\sum_{s\in S}\sum_{i\in s}\sum_{j\in s}\left(A_{ij} - \frac{k_i k_j}{2m}\right)$

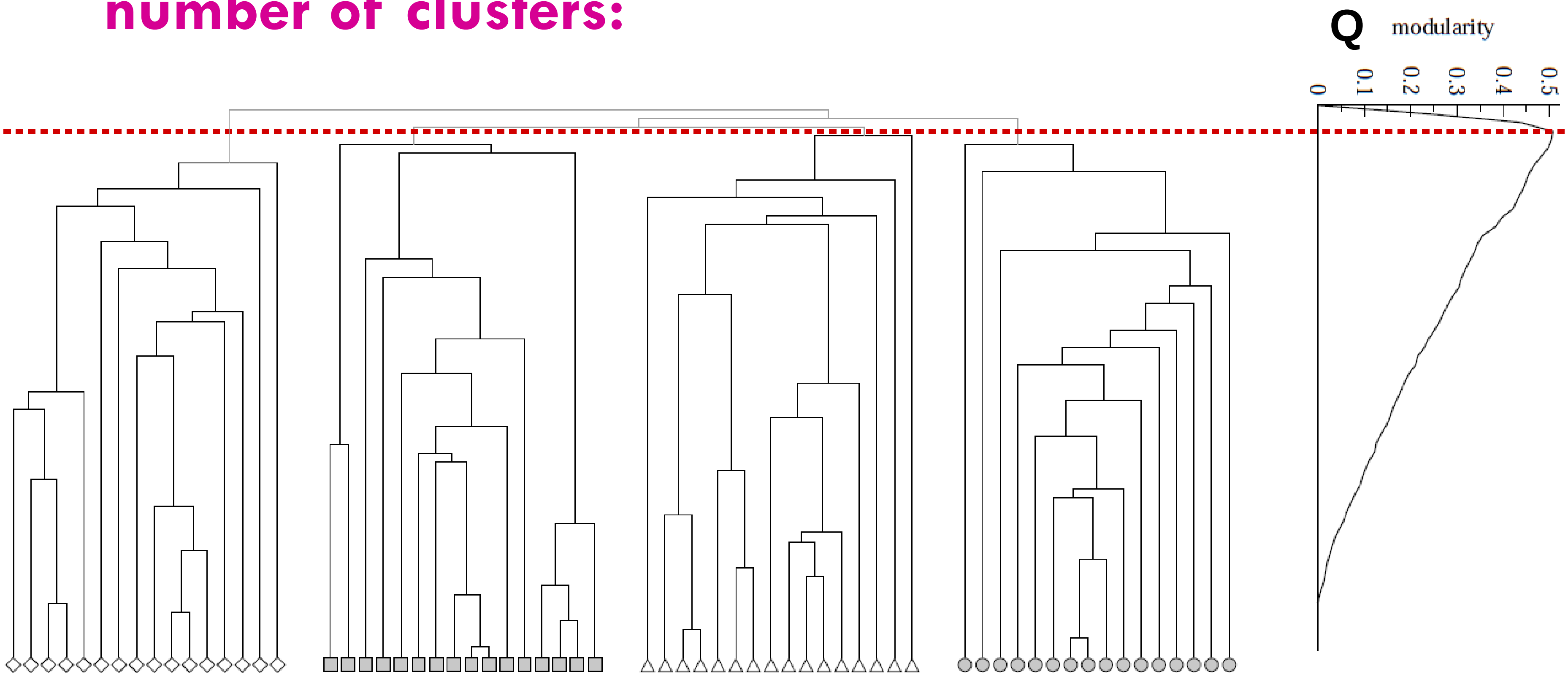
Normalizing cost.: -1<Q<1

A_{ij} = 1 if i→j,
0 else

- **Modularity values take range [−1,1]**
  - It is positive if the number of edges within groups exceeds the expected number
  - 0.3<Q<0.7 means significant community structure

Modularity: Number of clusters

- **Modularity is useful for selecting the number of clusters:**



Why not optimize modularity directly?