Investing in Climate Capital under Structural Change *

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Abstract

We investigate optimal investment into climate capital in a dynamic general equilibrium climate-economy model with endogenous structural change. First, because climate is an unpriced, nonrival and nonexcludable capital, climate capital proxied by temperature is overinvested, and represents a shrinking production base. Second, by differentiating the production of investment and consumption, we show that social cost of carbon can be perceived as a reduction in physical capital. Second, we distinguish two final sectors in terms of productivity growth and climate vulnerability. The sluggish services sector may drive down aggregate productivity growth, notoriously known as the Baumol's cost disease. We find heterogeneous climate vulnerability results in the climate Baumol's cost disease. Investing into climate capital affects the net impact combining both cost diseases. Third, our numerical results show that heterogeneous vulnerability exacerbates aggregate climate damage absent structural change, but reduces damage under the rising services sector. We conclude that investment in climate capital should not only factor in unpriced climate capital, but also be tailored to net Baumol's cost disease.

Keywords: Structural change, Climate capital, Integrated assessment model, Social cost of carbon, Baumol's cost disease.

JEL: O41, O44, H23, Q58.

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1 Introduction

Economic growth may be hampered by a stagnant services sector (Baumol, 1967) and by climate change (Nordhaus and Yang, 1996). This paper shows that climate change would have a similar effect as Baumol's cost disease. Optimal greenhouse gas emission reduction should take this into account.

The paper can be summarized with three arguments. First, climate is an *essential 'bad' capital* used in economic production[1]. Climate is an *essential* input to economic production. Growing crops requires decent climate conditions including appropriate temperature, humidity, sunshine, etc. White-collar workers in a high-tech company demand air-conditioning. A higher temperature is typically assumed to exert an adverse impact on economic production. In this sense, climate is a *'bad'* capital. Climate is a *stock*. Climate change is driven by the historical accumulation of emissions. Thus, climate embodies similar inter-temporal properties as physical or human capital (Lucas Jr, 1988).

Because climate is underpriced, carbon dioxide is overinvested. When climate is recognized as capital bad, carbon dioxide is naturally a disinvestment. In addition, climate is a public good, as climate capital is both *nonrival* and *nonexcludable*. On one hand, like ideas, climate capital is *nonrival*. Once the cost of attaining a lower level of climate capital, *i.e.* temperature, has been incurred, the climate capital can be leveraged repeatedly at no additional costs. One agent's use of temperature does not affect another agent's use. Furthermore, climate capital is *nonexcludable*, unlike (patented, trademarked or copyrighted) ideas. Nobody is capable of appropriating the property of climate, whether they are individuals, firms, or countries. No agent can stop another agent from using temperature. This implies that there are endless incentives to invest carbon dioxide emissions in climate capital. In plain language, because emitting carbon dioxide is free, profit-maximizing firms do not care whether carbon emissions are contributing to climate change.

The second argument is that heterogeneous climate vulnerability between sectors reflects the potentials to produce with climate capital, causing the climate Baumol's cost disease. Although climate capital is uniformly acquired, the ability to produce varies between sectors at any level of climate capital. For some sectors, climate capital is by its very nature a core requisite, implying a higher climate vulnerability. Other economic activities are not fastidious about climate conditions. As climate change progresses due to overinvested climate capital, the climate vulnerability gap can be expected to widen between these sectors. This division echoes Baumol (1967) that conceptually differentiated progressive with non-progressive sectors due to their technological structure. If products in sectors are

[1] There are few precedents for considering climate as a capital (Arrow et al., 2004, 2012; Barrage, 2020). See also Weitzman (2016) who viewed environmental quality as a stock of capital.

not substitutes, more and more productive factors would flow from progressive to stagnant sectors. Eventually, the overall productivity growth declines as the non-progressive sectors are expanding, notoriously known as the Baumol's cost disease[2]. Likewise, if a sector is more and more vulnerable to ongoing climate change, productive factors are increasingly absorbed into it, rendering the overall economy more vulnerable to climate change. In effect, higher climate vulnerability is analogous to slower productivity growth, giving rise to the climate Baumol's cost disease.

The third argument is that investing in climate capital should factor in structural change. The climate Baumol's cost disease recognizes the role of heterogeneous climate vulnerability, but its net effect depends on both technological structure and climate vulnerability. More specifically, climate vulnerability in each sector can be either aggravated or compensated by technological change. If a sector with high climate vulnerability is blessed with high productivity growth, climate impact can be less worrisome because the technological structure makes this sector more resilient. On the contrary, if another with high climate vulnerability is further depressed by gloomy technological prospects, the relative price of production in this sector will grow even higher. Consequently, Baumol's cost disease bites harder. In this case, cutting carbon investment can generate dual benefits including both avoided climate damages and moderated cost disease.

Although the economics literature on climate impact is burgeoning (e.g. Bakkensen and Barrage, 2018; Carleton et al., 2022; Waldinger, 2022; Cruz and Rossi-Hansberg, 2023, etc.), the conception of climate as capital, to our knowledge, has not been formalized in climate-economy models. The paper makes some attempts in this regard. We start with standard properties that naturally come along with identifying climate as a capital. For example, climate capital delivers negative returns, and the marginal return to climate capital is non-decreasing at a higher temperature. Besides, nonrival climate capital features decreasing returns to scale. Because climate capital is also nonexcludable, the nonrivalry contributes to the negative climate externality, compared to the positive externality in the knowledge capital. Nobel Prize Committee (2018) points out that externalities bridge the contributions of Romer and Nordhaus. Along its route, we emphasize that nonrivalry and nonexcluability of climate capital are the fundamental cause of climate change. Moreover, the paper interprets climate damage functions as the ability to produce with climate capital, which can differ in sectors.

To shed the light on optimal carbon investment under structural change, this paper then establishes a dynamic general equilibrium climate-economy model with endogenous

[2] Nordhaus (2008) provides empirical evidence for Baumol's cost disease. Some recent studies explore the Baumol's cost disease in dynamic growth models, and confirm that the rising price of services relative to goods will slow aggregate productivity growth (Ngai and Pissarides, 2007; Herrendorf et al., 2021).

structural change. The climate-economy linkage stands on the shoulders of Golosov et al. (2014), Nordhaus (2017) and Barrage (2020). Economic production requires energy as essential input, which will generate carbon emissions. Unabated carbon emissions will enter the atmosphere, and are investment into climate capital that is also indispensable for economic activities. On the one hand, our model differentiates *two* final production sectors–goods and services, both of which use energy and climate, among others, as input. Dividing the economy into goods and services is not uncommon in structural change literature (e.g. Moro, 2015; Leon-Ledesma and Moro, 2020; Herrendorf et al., 2021), motivated by the observed slower technological growth in services. In addition, we find some preliminary empirical evidence that services are less susceptible to climate change than goods. Thus, technological growth and climate vulnerability combine to shape the relative scarcity between goods and services. On the other hand, goods and services are leveraged for both consumption and investment (*two* final expenditures). Therefore, the model is characterized by a *two-by-two* structure. Following Herrendorf et al. (2021), García-Santana et al. (2021) and Buera et al. (2020), we induce structural transformation in both consumption and investment via the price effect[3]. For both expenditures, the substitution elasticity is less than unit between goods and services (Herrendorf et al., 2013, 2021; García-Santana et al., 2021). Hence, the comparative scarcity between goods and services determines their relative price, leading up to structural change. The results of the model are generalized as follows.

First, we provide a novel representation of the social cost of carbon based on investment. By differentiating the production of investment and consumption, we find that the trade-off between abatement costs and avoided climate damage is based on investment rather than consumption. This result theoretically supports that climate is a capital. Thus, the real cost of carbon can be perceived as a drag on physical capital. Existing studies usually define the social cost of carbon as the consumption loss due to an additional tone of CO_2 (Nordhaus, 2014; Golosov et al., 2014; Barrage, 2020). In one-sector growth model, economic production is utilized for either consumption or investment without differences. Thus, investment-equivalent social cost of carbon is precisely identical to consumption-equivalent social cost of carbon. Moreover, social cost of carbon denominated in terms of consumption has straightforward welfare implications because consumption is a direct measure of welfare, unlike investment. In comparison, Arrow et al. (2012) adopts the social cost of carbon to approximate the change in environmental capital. In their language, the social cost of carbon denominated by investment reflects the cost in wealth instead of directly in income, and hence also has welfare implications owing to its close relevance

[3]The income effect is the other important force in incurring structural change (Buera and Kaboski, 2009; Boppart, 2014; Comin et al., 2021; Alder et al., 2022).

to sustainability. It should be noted that the choice of numeraire only influences the numerical value of social cost of carbon, whereas optimal allocation is immune to either choice. Investment-equivalent social cost of carbon is linked to the consumption-equivalent social cost of carbon by the relative price of investment to consumption.

Second, we theoretically demonstrate how climate capital can influence the Baumol's cost disease. For one thing, we assume the Cobb-Douglas functions in both goods and services sectors with the same factor intensity. For another, we allow for different technological growth and climate vulnerability in both sectors. As a result, we find the relative price of services to goods is pinned down by the relative technological growth combined with relative climate vulnerability of goods to services. Absent different technological growths, the relative price of services to goods is expected to decline, because the climate impact on services is supposedly less significant than on goods. Thus, the production of goods is comparatively scarcer than of services due to climate change, exacerbating the climate impact via the climate Baumol's cost disease. By comparison, after accounting for different technological growth, relative price of services to goods will increase, but to a extent less than absent climate change. Although it is more susceptible to climate change, the goods sector is also experiencing faster technological growth, building up resilience to climate impact. In other words, the net Baumol's cost disease is ameliorated.

Third, we quantify the model. In line with the literature, we assume that the productivity growth in the goods sector is three times that in the services sector. The capital stock in 2100 is identical to that in the DICE model for comparability. We adopt the damage function of DICE, assuming that the relative damage level in the goods sector is two times higher than services. Using the data from World Bank, we pin down the factor shares used in each sector in the initial period.

Fourth, our numerical results validate that the Baumol's cost disease is an important consideration for investment into climate capital. When two sectors are only different in climate vulnerability, capital stock reduces by 12.34% in 2100 and 19.37% lower in 2150, compared to a decrease of 11.25% and 17.47% under homogeneous climate vulnerability. Thus, the climate Baumol's cost disease aggravates the aggregate climate impact. As a consequence, a more stringent climate policy is required to achieve the optimal allocation, which achieves the net-zero carbon emission in 2095, earlier than the 2100 for homogeneous climate vulnerability. When accounting for differentiated productivity growth, both capital stock and consumption damages are lower under heterogeneous climate vulnerability in this century, compared to under homogeneous vulnerability. This is because the expanding services sector is less vulnerable to climate change, or alternatively, the goods sector that is more vulnerable enjoys a higher productivity growth, showing a stronger resilience to climate change. A less strict abatement policy is required for optimal allocation, putting off 10 years for achieving the net-zero emission. In addition, we quantify two definitions of social cost of carbon. When climate capital is optimally invested, the social cost of carbon stands for a investment loss of \$254 per tone of CO_2, and a consumption loss of \$182 per tone of CO_2. Compared to investment, consumption is composed of a higher ratio of services. As the relative price of services is climbing over time, so will the price of consumption relative to investment. Given a numeraire with a higher price, consumption-equivalent social cost of carbon is lower than investment-equivalent one. Moreover, we find that a lower social time preference rate will generate a larger gap between the values of two definitions.

Although we only consider goods and services in this paper, our results imply that the stringency of climate policy should be tailored to the net Baumol's cost disease. Suppose there is a sector in the economy that has low productivity growth and high climate vulnerability, then we have strong incentives to reduce the carbon investment into climate capital. Disinvesting into climate capital helps ameliorate the Baumol's cost disease. Otherwise, the long-run economy growth would be plagued by both climate change and exacerbated Baumol's cost disease. In addition, recall climate vulnerability reflects the ability to produce with given climate capital. Apart from disinvesting into climate capital, adaptation investment into the sector also enhances its ability to produce. Thus, our paper has policy implications for both abatement and adaptation under the background of structural change.

This paper relates to the current literature in the following ways. First, existing literature typically denominates the social cost of carbon in terms of consumption (Nordhaus, 2014; Golosov et al., 2014), we propose an alternative definition in term of investment. In so doing, we show that social cost of carbon is a drag on productive capital rather than directly on consumption. We also demonstrate two definitions of social cost of carbon can be bridged by the relative price of investment to consumption. Second, the paper is relevant to studies on climate damages. Casey et al. (2021) assume that the climate system evolves exogenously, and analyzes the climate damages on consumption and investment with the focus on heterogeneous damages between sectors. Our study is consistent with theirs in finding that heterogeneous climate vulnerability will exacerbate aggregate damage level, which could be explained by the less than unity substitution elasticity between each product in producing investment and consumption. This paper, however, differs from theirs in that we focus on the background of structural change. The net Baumol's cost disease is a critical factor in determining the realized impact of climate change. Third, the paper falls within the broad category of structural change economics(Herrendorf et al., 2014), and we add that climate vulnerability is also complementary to incurring the price effect.

The remainder of the paper is organized as follows. Section 2 formalize climate as a capital. Section 3 establishes a climate-economy model and theoretically analyzes the incentives to disinvest into climate capital. Section 4 introduces the calibration process. Section 5 discusses the quantitative results. Some sensitivity analyses and extensions are included in Section 6, and Section 7 concludes the paper.

2 An overview of climate capital

This section discusses several fundamental properties associated with climate capital, which have not been formalized previously. Where necessary, we compare climate capital with other common factors in production. In so doing, we are able to interpret the cause of climate change from the perspective of investment. In addition, we note the link between climate capital and climate damage function that is commonly used in climate economics literature.

Temperature change T_t is considered as an appropriate proxy for climate capital, consistent with extant studies (Barrage and Nordhaus, 2023; Golosov et al., 2014; Cruz and Rossi-Hansberg, 2023). In addition to climate capital, economic production at period t also requires physical capital K_t, labor L_t, ideas A_t, and energy E_t.

$$Y_t = F(A_t, T_t, K_t, L_t, E_t) \tag{1}$$

where F represents some technology to utilize these factors in production. The last argument energy E_t can be either fossil fuels that will emit carbon dioxide or renewable energy with no carbon emissions.

2.1 Negative and increasing returns

Unlike other factors, climate represented by global mean temperature change is a bad capital. More input of other factors (*i.e.* physical capital, human capital, energy and ideas) generate more revenues, whereas an increase in temperature causes economic losses:

$$\frac{\partial F}{\partial T_t} < 0 \tag{2}$$

Moreover, as the temperature increases further, the marginal returns to climate capital is increasing:

$$\frac{\partial^2 F}{\partial^2 T_t} > 0 \tag{3}$$

Increasing returns to climate capital captures both the beliefs (Barrage and Nordhaus, 2023; Weitzman, 2010; Pindyck, 2021) and some empirical evidence (Burke et al., 2015;

Newell et al., 2021) that the climate impact is exacerbated at a higher temperature.

2.2 Decreasing returns to scale

In nature, climate capital is a *public bad*, because it is both nonrival and nonexcludable. Consequently, climate capital is overinvested, giving rise to global warming.

Climate capital is nonrival. An individual's use of climate capital does not preclude others from using climate capital. Once climate capital is produced (represented by global temperature change), all firms use it for economic production without paying for additional costs. When global mean temperature rises, an enduring heat wave may ensue in an African village, devastating the harvests of all peasants. One peasant's adversity cannot lower the possibility or extent of another peasant in the same village. In Singapore, a coastal high-tech company is simultaneously plagued by the sea-level rise that floods the working office due to the same climate capital, *i.e.* global mean temperature change. While the sea level is rising in Singapore, miserable peasants find no reasons to believe that flooding makes global temperature increase or decrease, nor will the company think the heat wave in Africa will cool down the global suddenly. Climate capital, once produced, can be enjoyed by everyone and hence is not scare. However, good climate capital is scare in that, for example, there is some unknown level of global temperature change leading up to maximum global economic production, *ceteris paribus*.

Nonrivalry of climate capital implies that production is characterized by decreasing returns to scale. The standard replication argument is valid for physical capital, human capital and energy, but not for ideas and climate capital. Romer (1986, 1990) has illuminated that ideas are nonrival and consequently that economic production features increasing returns to scale. Because climate is a bad capital, climate capital is characterized by decreasing returns to scale. In other words, for any $\lambda > 1$, we have:

$$
\begin{aligned}
F(A_t, T_t, \lambda K_t, \lambda L_t, \lambda E_t) &= \lambda F(A_t, T_t, K_t, L_t, E_t) \\
F(\lambda A_t, T_t, \lambda K_t, \lambda L_t, \lambda E_t) &> \lambda F(A_t, T_t, K_t, L_t, E_t) \\
F(A_t, \lambda T_t, \lambda K_t, \lambda L_t, \lambda E_t) &< \lambda F(A_t, T_t, K_t, L_t, E_t) \qquad (4)
\end{aligned}
$$

Climate capital is also nonexcludable, leading up to overinvestment. Compared to knowledge capital that can be partially excludable in the presence of patents, the property of climate capital cannot be appropriated in reality. Suppose that a firm reduces one unit of carbon investment into climate capital and shoulders the associated abatement costs. Because climate capital is not excludable, all firms in the economy can benefit from this one unit of reduced carbon that lowers the level of climate capital. Thus, the abatement costs for the reduced carbon investment cannot be compensated by private revenues. In the market, when the social price of carbon is not defined, no firm is motivated to disinvest carbon into climate capital.

2.3 Climate capital and damage function

Although climate capital proxied by global mean temperature is uniform to everybody, the ability to produce with climate capital can be quite different. In other words, climate impact are heterogeneous. This can be explained by geographic endowments, adaptation technology, industry structure, etc. In existing literature (e.g.: Barrage and Nordhaus, 2023; Golosov et al., 2014; Barrage, 2020; Cruz and Rossi-Hansberg, 2023), the damage function pioneered by (Nordhaus, 1992) is commonly leveraged to reflect the productivity of climate capital. Admittedly, the damage function is among the most uncertain parts in climate-economy models in terms of both forms and parameters (Pindyck, 2021). The paper makes no efforts to determine any well-suited damage function. Instead, as we will see below, the analysis only requires that climate damage functions should be sector-specific so as to reflect the differentiated productivity of climate capital between sectors.

3 Model and theory

This section establishes a dynamic climate-economy model with endogenous structural change. The climate-economy structure is borrowed from Nordhaus (2017), Golosov et al. (2014) and Barrage (2020). The economic module is augmented with a two-by-two structure. On the one hand, we allow for two final production sectors with heterogeneous productivity growth rate and climate vulnerability, both of which can affect the relative price of products in two sectors[4]. On the other hand, following (Greenwood et al., 1997; Herrendorf et al., 2014; Foerster et al., 2022), we differentiate two final expenditures of economic production–consumption and investment in terms of their compositions of final products. Thus, structural change can take place within both consumption and investment when the price in one sector is changing relative to the other. We start from the competitive market, and shows its equivalence to social planner problem by introduce a carbon tax.

Given the established model, we first theoretically show how to achieve the optimal allocation in the presence of climate externality. This result constitutes the fundamental incentive for addressing climate change–unpriced climate capital and overinvestment. Then, we show the relative price between two products, which can be perceived as the acuteness of the Baumol's cost disease. Investing in climate capital changes the relative price, and

[4]Our model can be easily extended to a multi-sector case.

hence becomes another important consideration in optimal carbon investment decision.

3.1 Households

The economy accommodates an infinitely-lived, representative household whose lifetime utility is determined by:

$$U_0 \equiv \sum_{t=0}^{\infty} \beta^t U(C_t) \tag{5}$$

where β denotes the social time of preference and C_t market consumption at period t. In fact, the household's utility can also be affected directly by climate change. For example, it can influence amenities, biodiversity or health conditions households value[5]. Since the paper focuses on economic dynamics, we abstract from these possibilities.

The consumption bundle is composed of two different products:

$$C_t = F_C(C_{1t}, C_{2t}) \tag{6}$$

where C_{it} stands for consumption of product $i \in \{1, 2\}$ respectively. F_C captures the household's preference for each product, and can be regarded as a costless technology aggregating both consumption.

The representative household should satisfy its budget constraint in each period:

$$p_{1t}C_{1t} + p_{2t}C_{2t} + K_{t+1} \leq w_t L_t + (1 + r_t - \delta)K_t + \Pi_t + Tran_t \tag{7}$$

where K_{t+1} is the capital holdings at time $t+1$, w_t the wage rate, L_t the labor supply, r_t the rental rate of capital, δ the capital depreciation rate, Π_t the dividends from the energy sector, $Tran_t$ the lump-sum transfer of carbon tax levied by the government. We set investment as the numeraire, normalizing its price to one in each period, and define p_{it} as the price of product i.

Thus, the household's first-order condition requires that saving decisions and allocations of goods and services in consumption must satisfy:

$$\frac{U_{cgt}/p_{gt}}{U_{cg,t+1}/p_{g,t+1}} = \frac{U_{cst}/p_{st}}{U_{cg,t+1}/p_{s,t+1}} = \beta(1 + r_t - \delta) \tag{8}$$

where U_{cit} represents the partial derivative of instantaneous utility function with respect to

[5] This consideration can be represented by either introducing a climate variable such as temperature rise into the utility function (Barrage, 2020) or considering the climate impact on non-market goods (Tol, 1994; Sterner and Persson, 2008; Drupp and Hänsel, 2021).

consumption i at time t. This equation demonstrates that, between two subsequent periods, the representative household will equate the price-adjusted marginal rates of substitution in each consumption to the return rate on saving.

3.2 Two final production sectors

There are two final sectors whose production functions follow from Eq.(1). Further, each sector $i \in \{1, 2\}$ adopts a constant-returns-to-scale technology $\tilde{F}$ to combine physical capital, labor and energy, and satisfies the Inada conditions. By comparison, both climate capital and knowledge capital feature non-constant returns to scale, represented by $\hat{F}$. Thus, we have:

$$\begin{aligned} Y_{1t} &= F_1(A_{1t}, T_t, K_{1t}, L_{1t}, E_{1t}) \\ &= \hat{F}_1(A_{1t}, T_t)\tilde{F}_1(K_{1t}, L_{1t}, E_{1t}) \quad (9) \\ Y_{2t} &= F_2(A_{2t}, T_t, K_{2t}, L_{2t}, E_{2t}) \\ &= \hat{F}_2(A_{2t}, T_t)\tilde{F}_2(K_{2t}, L_{2t}, E_{2t}) \quad (10) \end{aligned}$$

Note that climate capital is uniformly utilized in both sectors, whereas other factors and production technologies can be sector-specific.

In a competitive market, profit-maximizing firms in both sectors should equate their marginal products to their prices:

$$\begin{aligned} p_{1t}F_{1lt} &= p_{2t}F_{2lt} = w_t \quad (11) \\ p_{1t}F_{1kt} &= p_{2t}F_{2kt} = r_t \\ p_{1t}F_{1Et} &= p_{2t}F_{2Et} = p_{Et} \end{aligned}$$

where F_{ijt} represents the partial derivative of sector i production function with respect to rival input $j \in \{K, L, E\}$, and p_{Et} denotes the energy price.

In addition, production in both sectors can be utilized for either consumption or investment such that:

$$\begin{aligned} Y_{1t} &= C_{1t} + I_{1t} \quad (12) \\ Y_{2t} &= C_{2t} + I_{2t} \end{aligned}$$

where I_{it} is the output from sector i used for investment.

Total nominal final output can thus be defined as:

$$Y_t = p_{1t}Y_{1t} + p_{2t}Y_{2t} \tag{13}$$

3.3 Investment production sector

In the economy, there is also an intermediate investment sector that adopts a constant-to-scale technology and combines the production from two final sectors to produce final investment:

$$I_t = F_I(A_{It}, I_{1t}, I_{2t}) \tag{14}$$

where A_{It} denotes an exogenous investment-specific technical change to produce aggregate investment. We do not allow for a direct climate impact on investment production in Eq.(14). However, temperature change indirectly influences investment because as it reduces goods and services supplied to produce investment[6].

3.4 Energy sector

Using both capital K_{Et} and labor L_{Et}, the intermediate energy sector produces with a constant-return-to-scale technology. Hence, energy production is given by:

$$E_t = A_{Et}\tilde{F}_E(K_{Et}, L_{Et}) \tag{15}$$

Energy production is allocated to two final sectors as input. Following Barrage (2020), we assume that carbon-based energy can be unlimitedly supplied and therefore incurs zero Hotelling rents. Climate capital is in reality also used for energy production, but we make simplifications here considering that the energy sector accounts for a relatively small proportion in the economy.

Energy firms can choose to produce a fraction μ_t of energy with some zero-emission technologies at an additional abatement investment $\Theta_t(\mu_t E_t)$. For each unit of emission,

[6] Allowing for the climate impact on investment explicitly will exacerbate the climate impact on growth rate. Fankhauser and Tol (2005) and Dietz and Stern (2015) show that such growth effect can be embodied in the climate impact on capital depreciation explicitly, analogous to modelling a climate impact on investment production. A climate impact on economic growth will acutely exacerbate economic losses compared to its frontier, whereas the level effect is more modest (Pindyck, 2021; Cai et al., 2023). However, there still exist some intellectual gaps between modelling practices and empirical evidence to support the growth effect of climate change (Dell et al., 2012; Burke et al., 2015; Newell et al., 2021). Further, Herrendorf et al. (2021) and García-Santana et al. (2021) find the exogenous investment-specific technical change plays a very limited role in driving long-run growth. Given all these, we only implicitly account for the climate impact on investment.

firms are obliged to pay the carbon tax. Thus, the profits of energy producers are:

$$\Pi_t = p_{Et}E_t - [(1-\mu_t)E_t]\tau_{Et} - w_t L_{Et} - r_t K_{Et} - \Theta_t(\mu_t E_t) \tag{16}$$

where τ_{Et} denotes the carbon tax on uncontrolled carbon emissions from two final sectors $\{E_i^{unc}\}_{i=0}^t = \{(1-\mu_t)(E_{1i}+E_{2i})\}_{i=0}^t$. We only consider an aggregate carbon control rate μ_t, and we discuss the implications of differentiated control rates in Section 6.

Again, in the competitive market, energy producers equate the marginal products of each input to their corresponding prices:

$$\begin{aligned} (p_{Et} - \tau_{Et})F_{Elt} &= w_t \\ (p_{Et} - \tau_{Et})F_{Ekt} &= r_t \end{aligned} \tag{17}$$

Moreover, the formulation in Eq.(16) implies that profit-maximizing energy producers are incentivised to equate the marginal benefit of avoided tax payment per unit of uncontrolled carbon emissions to marginal abatement costs:

$$\tau_{Et} = \Theta_t'(\mu_t E_t) \tag{18}$$

In each period, the productive factors are freely mobile across sectors:

$$\begin{aligned} L_t &= L_{1t} + L_{2t} + L_{Et} \\ K_t &= K_{1t} + K_{2t} + K_{Et} \\ E_t &= E_{1t} + E_{2t} \end{aligned} \tag{19}$$

where aggregate labor force L_t is exogenously given at each period.

3.5 Carbon cycle and climate

We follow the convention of climate economics literature in viewing temperature as a sufficient proxy for climate change. Specifically, atmospheric temperature change T_t at period t is determined by the historical path of carbon emissions after control $\{E_i^{unc}\}_{i=0}^t = \{(1-\mu_i)(E_{1i}+E_{2i})\}_{i=0}^t$, initial climate conditions $\boldsymbol{M}_0$ including atmospheric carbon concentrations, deep ocean temperatures, etc., and exogenous shifters $\{\boldsymbol{\eta}_i\}_{i=0}^t$ such as land-based emissions:

$$T_t = \Phi(\boldsymbol{M}_0, E_0^{unc}, E_1^{unc}, ..., E_i^{unc}, \boldsymbol{\eta}_0, ..., \boldsymbol{\eta}_t) \tag{20}$$

where $\frac{\partial T_{t+j}}{\partial E_t^{unc}}$ holds for $\forall t, j \geq 0$. The very nature that climate change, proxied by T_t, hinges on the *stock* of carbon emissions in the atmosphere hints that climate is a capital. Note that temperature is not exclusively dictated by current-period carbon emission, which is a *flow* variable.

3.6 Competitive equilibrium

Now we can present the standard definition of competitive equilibrium in the economy, augmented with the climate system.

Definition 1. *A competitive equilibrium consists of a sequence of exogenously-given productivity $\{A_{1t}, A_{2t}, A_{It}, A_{Et}\}_{t=0}^{\infty}$, a series of allocations $\{C_{1t}, C_{2t}, I_{1t}, I_{2t}, L_{1t}, L_{2t}, L_{Et}, K_{1,t+1}, K_{2,t+1}, K_{E,t+1}, E_{1t}, E_{2t}, \mu_t, T_t\}_{t=0}^{\infty}$, a set of prices $\{r_t, w_t, p_{1t}, p_{2t}, p_{Et},\}_{t=0}^{\infty}$ and a series of policies $\{\tau_{Et}\}_{t=0}^{\infty}$ such that in each period, given prices and policies:*
(i) the household solves the utility-maximizing problem subject to the budget constraint,
(ii) firms in two final production sectors and two intermediate sectors (energy and investment) maximize profits,
(iii) temperature changes in line with the carbon cycle constraint, and
(iv) markets clear.

By virtue of the above definition, we now demonstrate what a first-best carbon tax is needed to decentralize the optimal allocation in the competitive equilibrium:

Proposition 1. *The allocations $\{C_{1t}, C_{2t}, I_{1t}, I_{2t}, L_{1t}, L_{2t}, L_{Et}, K_{1,t+1}, K_{2,t+1}, K_{E,t+1}, E_{1t}, E_{2t}, \mu_t, T_t\}_{t=0}^{\infty}$, along with initial capital stock K_0, initial carbon concentrations $\boldsymbol{M}_0$, and climate shifters $\{\boldsymbol{\eta}_i\}_{i=0}^{t}$ in a competitive equilibrium satisfy:*

$$F_1(A_{1t}, T_t, L_{1t}, K_{1t}, E_{1t}) \geq C_{1t} + I_{1t} \tag{21}$$

$$F_2(A_{2t}, T_t, L_{2t}, K_{2t}, E_{2t}) \geq C_{2t} + I_{2t} \tag{22}$$

$$F_C(C_{1t}, C_{2t}) \geq C_t \tag{23}$$

$$F_I(A_{It}, I_{1t}, I_{2t}) + (1-\delta)K_t \geq K_{t+1} + \Theta_t(\mu_t E_t) \tag{24}$$

$$E_t \leq A_{Et} F_E(K_{Et}, L_{Et}) \tag{25}$$

$$T_t \geq \Phi(\boldsymbol{M}_0, (1-\mu_0)(E_{10}+E_{20}), ..., (1-\mu_t)(E_{1t}+E_{2t}), \boldsymbol{\eta}_0, ..., \boldsymbol{\eta}_t) \tag{26}$$

$$L_t \geq L_{1t} + L_{2t} + L_{Et} \tag{27}$$

$$K_t \geq K_{1t} + K_{2t} + K_{Et} \tag{28}$$

$$E_t \geq E_{1t} + E_{2t} \tag{29}$$

Therefore, given an allocation that maximizes the household's net present utility Eq.(5) and simultaneously satisfies constraints Eq.(21)-Eq.(29), letting λ_{It} the Lagrange multiplier on *the capital accumulation constraint Eq.(24), one can formulate a carbon tax equal to:*

$$\underbrace{\Theta'_t}_{Marginal\quad abatement\quad cost} = \underbrace{F_{I1t}F_{1Et} - \frac{F_{I1t}F_{1lt}}{F_{Elt}} = F_{I2t}F_{2Et} - \frac{F_{I2t}F_{2lt}}{F_{Elt}}}_{Marginal\quad product\quad of\quad energy}$$
$$= (-1)\sum_{j=0}^{\infty}\beta^j \overbrace{\left(\underbrace{\frac{U_{C,t+j}}{\lambda_{It}}\frac{\partial C_{t+j}}{\partial C_{1,t+j}}\frac{\partial Y_{1,t+j}}{\partial T_{t+j}}\frac{\partial T_{t+j}}{\partial E_t^{unc}}}_{Impacts\quad on\quad sector\quad 1} + \underbrace{\frac{U_{C,t+j}}{\lambda_{It}}\frac{\partial C_{t+j}}{\partial C_{2,t+j}}\frac{\partial Y_{2,t+j}}{\partial T_{t+j}}\frac{\partial T_{t+j}}{\partial E_t^{unc}}}_{Impacts\quad on\quad sector\quad 2}\right)}^{Marginal\quad product\quad of\quad carbon\quad investment} \tag{30}$$

such that the competitive market can achieve the optimal allocation as in the social planner problem, and the discounting of output damages in period t is governed by:

$$\frac{\lambda_{It}}{\beta\lambda_{I,t+1}} = F_{Ig,t+1}F_{gk,t+1} + (1-\delta) = F_{Is,t+1}F_{sk,t+1} + (1-\delta) \tag{31}$$

Proof: See Appendix. Eq.(30) describes the classic wisdom (Baumol, 1972) to address climate (environmental) externality. The optimal allocation requires **equating** marginal product of carbon-based energy **to** marginal abatement costs, **and also to** current-value marginal product of carbon investment, namely climate impact due to an additional tone of carbon emission. To decentralize such optimal allocation, the global planner needs to levy a Pigouvian tax equal to Θ'_t.

However, the theoretical finding deviates from previous studies primarily in establishing investment, rather than consumption, as the core metric in the trade-off. Marginal abatement cost is by construction denominated by investment as previously introduced. The marginal product of energy is represented by how much final investment per unit of energy can produce net of the opportunity cost in energy production. The marginal impact of carbon emission is also denominated in terms of investment. Under optimal allocation, Pigouvian tax is numerically equal to the social cost of carbon. Social cost of carbon is conventionally defined as the shadow price of carbon measured by the utility of per unit of consumption (Nordhaus, 2014; Golosov et al., 2014; Barrage, 2020)[7]. Thus, the common definition reflects the amount of consumption one would like to sacrifice today in order to reduce an additional unit of welfare-reducing emissions.

Motivated by this result, we provide an alternative definition for social cost of carbon, *i.e.* investment loss due to an additional tone of carbon emission. Recall in Section 2 that

[7] Consumption-equivalent social cost of carbon can be generalized as $SCC_{CE} = \frac{\partial U}{\partial E_t}/\frac{\partial U}{\partial C_t}$, while investment-equivalent social cost of carbon is formulated as $SCC_{IE} = \frac{\partial U}{\partial E_t}/\frac{\partial U}{\partial I_t}$.

climate is a capital and carbon is an investment. If economic production is not utilized for the investment in abatement technology, it can be leveraged to accumulate physical capital. However, if no abatement technology is adopted, climate capital will be overinvested. Eventually, the global mean temperature rises, generating an impact equivalent to a loss in physical capital. The pigouvian tax in Eq.(30) seeks an optimal investment bundle that guarantees the equalized marginal product of each kind of investment. Therefore, the real cost of carbon is on investment rather than consumption, and climate change is due to disinvestment instead of malconsumption.

In fact, two definitions of social cost of carbon deviate only when investment and consumption are produced in different ways. Denoting the marginal value of investment λ_{It} and that of consumption λ_{Ct}, one can establish the following identity:

$$\frac{\lambda_{It}}{\lambda_{Ct}} = \frac{F_{C1t}}{F_{I1t}} = \frac{F_{C2t}}{F_{I2t}} \tag{32}$$

The marginal value of investment relative to that of consumption is governed by how each final product is transformed into between consumption and investment. In one-sector climate-economy models without any other distortions like Nordhaus (2014) and Golosov et al. (2014), a unit of final production can be either consumed or invested without any differences. Thus, both F_{C1t}/F_{I1t} and F_{C2t}/F_{I2t} are cancelled out, and two definitions of social cost of carbon are numerically identical.

Although social cost of carbon as a price is different, optimal allocation is fixed regardless of any numeraire. In addition, both definitions have their strengths and weaknesses. Consumption-equivalent social cost of carbon does not reflect the very nature that climate change is a drag on investment, but has convenient welfare implications because it demonstrates the consumption loss due to an additional tone of carbon emission. In comparison, investment-equivalent social cost of carbon cannot provide direct welfare interpretations. After all, people care consumption rather than investment[8].

3.7 Drivers of structural change

We induce structural change within both consumption and investment through the price effect[9]. The price effect reflects that the value-added share of a certain product can also increase when its relative price goes up. We assume two final production sectors are only different in technological growth rate and climate vulnerability. Following Herrendorf et

[8]Having said that, investment-equivalent social cost of carbon can be utilized for indirect welfare analysis.

[9]Existing studies generalize two broad forces behind structural change—the income effect and the price effect. The income effect captures that as income increases, so will the value-added share of products with a higher income elasticity.

al. (2014), the production functions in both sectors adopt the Cobb-Douglas form with identical factor intensity[10]:

$$
\begin{aligned}
Y_{1t} &= \hat{F}_1(A_{1t}, T_t) K_{1t}^{\alpha} L_{1t}^{1-\alpha-\nu} E_{1t}^{\nu} \\
Y_{2t} &= \hat{F}_2(A_{2t}, T_t) K_{2t}^{\alpha} L_{2t}^{1-\alpha-\nu} E_{2t}^{\nu}
\end{aligned} \tag{33}
$$

where how climate capital interacts with knowledge capital in the production functions remains undefined without loss of generality.

In addition, both the consumption and investment functions are assumed to be the CES form:

$$
C_t = \left(\omega_c^{\frac{1}{\epsilon_c}} C_{1t}^{\frac{\epsilon_c - 1}{\epsilon_c}} + (1-\omega_c)^{\frac{1}{\epsilon_c}} C_{2t}^{\frac{\epsilon_c - 1}{\epsilon_c}} \right)^{\frac{\epsilon_c}{\epsilon_c - 1}} \tag{34}
$$

$$
I_t = A_{It} \left(\omega_I^{\frac{1}{\epsilon_I}} I_{1t}^{\frac{\epsilon_I - 1}{\epsilon_I}} + (1-\omega_I)^{\frac{1}{\epsilon_I}} I_{2t}^{\frac{\epsilon_I - 1}{\epsilon_I}} \right)^{\frac{\epsilon_I}{\epsilon_I - 1}} \tag{35}
$$

where ω_c and ω_I are the weights of product 1 in consumption and investment. ϵ_c and ϵ_I are the substitution elasticities between two products in producing final consumption and investment, both of which are less than unit. Note that consumption and investment can be different in terms of weight of each product, substitution elasticity, and investment-specific technological progress.

Proposition 2. *Given the production function in Eq.(33), it is straightforward to show that the relative price between two products in competitive market is pinned down by:*

$$
\frac{p_{2t}}{p_{1t}} = \frac{\hat{F}_1(A_{1t}, T_t)}{\hat{F}_2(A_{2t}, T_t)} \tag{36}
$$

such that given the consumption and investment production functions as Eq.(34) and Eq.(35), the share of product i will increase in tandem with it relative price. Thus, structural change takes place via the price effect. Moreover, two definitions of social cost of carbon are linked

[10]The price effect can also occur when sectors differ in capital intensity or the substitution elasticity between reproducible factors (Acemoglu and Guerrieri, 2008; Alvarez-Cuadrado et al., 2017). We abstract from these two possibilities for two reasons. First, Herrendorf et al. (2015) showed that sectors with different productivity growths alone can fit well the post-war structural change in the United States. Second, the Baumol's cost disease, which is the focus of this paper, is concerned with the technological structure behind each sector.

by the transformation ratio:

$$\frac{SCC_{CE}}{SCC_{IE}} = \underbrace{\frac{1}{A_{It}}}_{ISTC} \times \underbrace{\frac{\left[\omega_c \hat{F}_1(A_{1t}, T_t)^{\epsilon_c - 1} + (1-\omega_c)\hat{F}_2(A_{2t}, T_t)^{\epsilon_c - 1}\right]^{\frac{1}{\epsilon_c - 1}}}{\left[\omega_I \hat{F}_1(A_{1t}, T_t)^{\epsilon_I - 1} + (1-\omega_I)\hat{F}_2(A_{2t}, T_t)^{\epsilon_I - 1}\right]^{\frac{1}{\epsilon_I - 1}}}}_{SC \quad interacted \quad with \quad climate} \tag{37}$$

Proof: See Appendix. Proposition 2 presents two relative prices that are related to the the Baumol's cost disease and social cost of carbon respectively.

The first is the relative price between two final products in Eq.(84), jointly determined by technological productivity growth and climate vulnerability. We discuss the relative price under three different cases, and show their relevance to the Baumol's cost disease. Note that the substitution elasticity between two final products are less than unit in both Eq.(34) and Eq.(35).

Case 1: Only technological productivity affects the relative price. In structural change literature, climate vulnerability is normally not included. Thus, following the common assumption that technology enters the production function in multiplicative form, Eq.(84) boils down to:

$$\frac{p_{2t}}{p_{1t}} = \frac{A_{1t}}{A_{2t}} \tag{38}$$

Assume that sector 1 has a robust productivity growth, while sector 2 is stagnant in growth (like services). Thus, the relative price of product 2 to product 1 will increase over time. As products in both sectors are necessary in producing consumption and investment, more and more productive resources will flow into the non-progressive sector because of its increasing relative price, and the share of this sector is expanding in the economy. In the long run, the aggregate productivity growth slows down due to an growing sector with slow productivity growth. Thus, *the Baumol's cost disease* occurs, as studied in Ngai and Pissarides (2007) and Herrendorf et al. (2021).

Case 2: Only climate vulnerability affects the relative price. Climate vulnerability between two sectors, determined by climate capital, can also influence the relative price, leading up to *the climate Baumol's cost disease*. Assume that two final sectors have fixed knowledge capital so that technological productivity is constant in both sectors. Thus, Eq.(84) can be rewritten to:

$$\frac{p_{2t}}{p_{1t}} = \frac{\hat{F}_1(T_t)}{\hat{F}_2(T_t)} \tag{39}$$

where $\hat{F}_i(T_t)$ reflects the ability of sector i to produce to uniform climate capital T_t. As explained in Section 2, it can be perceived as the climate damage function, and governs heterogeneous climate vulnerability. Thus, a higher level of climate capital T_t implies a high damage level, and hence a lower level of $\hat{F}_i(T_t)$. As climate capital is nonrival and nonexcludable, climate capital is overestimated and a higher level of global mean temperature is attained. Assume that economic production in sector 2 is more reliant on climate capital. Thus, given a higher temperature uniform to both sectors, $\hat{F}_2(T_t)$ is increasingly lower than $\hat{F}_1(T_t)$. Thus, the relative price of product 2 is climbing gradually. Again, because both products are necessary and cannot be well substituted, the expenditure on product 2 will increase in accordance. Thus, given an expanding sector in the economy more vulnerable to climate change, the aggregate economy will become increasingly vulnerable to climate change. In effect, a sector that is more vulnerable to climate change is technically a sector that has slower productivity growth.

Existing literature (Barrage and Nordhaus, 2023; Golosov et al., 2014; Barrage, 2020, e.g.:) usually treats the economy as a single sector, and studies the incentives to address climate change. In this paper, we show that because climate capital is nonrival, nonexcludable and unpriced, it is overestimated. This is the first incentive for reducing investment into climate capital in Proposition 1. We add that the climate Baumol's cost disease is another important incentive to cope with climate change seriously after accounting for heterogeneous climate vulnerability. Long-run economic growth is hampered by climate change due to unpriced climate capital, and this adverse impact can be potentially aggravated by the climate Baumol's cost disease.

Case 3: Both affect the relative price. In reality, technological productivity and climate vulnerability jointly determine the relative price as in Eq.(84). Suppose that sector 2 has lower productivity growth and higher climate vulnerability, its relative price will be higher than both Case 1 and Case 2. Put differently, *the net Baumol's cost disease* is even more acute. Thus, failure to pricing climate capital can drag down aggregate economic growth severely, and there is a stronger incentive to disinvest into climate capital. Suppose that sector 2 has high productivity growth and also high climate vulnerability. Under the plausible assumption that the impact of productivity growth outweighs climate vulnerability, the relative price of product 2 to product 1 may also increase, but to an extent less than absent climate change. The net Baumol's cost disease is ameliorated. Thus, the urgency to address climate change is reduced. This reasoning may be surprising at first sight, but can be valid. A sector with higher productivity growth implies a higher resilience to climate change. Although the sector may be more vulnerable to climate change, it can soon recover from climate damage by rapidly accumulating knowledge capital.

The second price bridges two definitions of social cost of carbon, and in fact reflects the relative price of investment to consumption. As shown by Eq.(37), the ratios depends on investment-specific technical change A_{It}, how climate capital interacts with knowledge capital in each final sector $F_i(A_{it}, T_t)$, the weight of product 1 in producing investment and consumption ω_I and ω_c, and the substitution elasticity between two products in investment and consumption ϵ_I and ϵ_c. In climate-economy models with one final production sectors, because the productions of investment and consumption are not specified, all items are cancelled out[11]. The consumption-equivalent social cost of carbon is the same as the investment-equivalent one. Our specifications of investment and consumption production are admittedly not comprehensive. But they serve as good examples for illustrating that only under very strict condition that one would expect two definitions of social cost of carbon are the same. The real cost of carbon is on investment, and but can be denominated in terms of consumption after transformation.

4 Calibration

4.1 Sector division

We divide the whole economy into goods and services for two reasons.

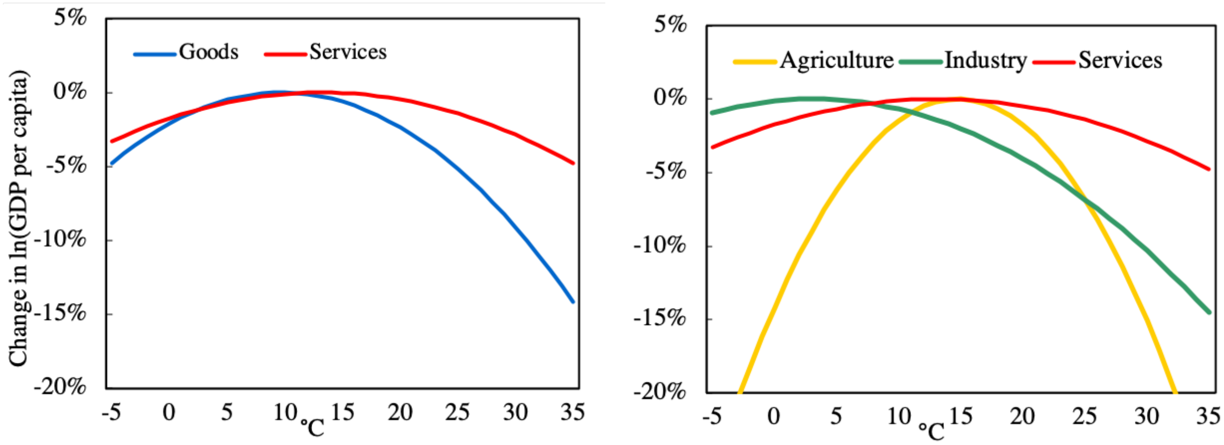
Reason 1: On the climate-to-economy side, the services sector appears to be less vulnerable to weather shocks than the goods sector. Following Burke et al. (2015) but focusing on sectoral impact, we investigate

$$\Delta Y_{it}^{j} = h(T_{ij}) + g(P_{ij}) + \mu_i + \zeta_t + \theta_i t + \theta_{i2} t^2 + \epsilon_{it} \tag{40}$$

where ΔY_{it}^{j} is the output growth rate of sector j (either goods or services) in country i at year t, T annual average temperature, P precipitation, μ_i country-specific constant terms, ζ_t year fixed effects and $\theta_i t + \theta_{i2} t^2$ flexible country-specific time trends.

Figure 1 shows when the global annual average temperature rises above around 13 degrees Celsius, the impact on the goods sector will soon be larger than on the services, and the gap is widened at a higher temperature (Panel a). Even when subtracting the agriculture sector, that is among the most vulnerable to temperature, weather shocks take a heavier toll on the left industry production than services (Panel b). Moreover, the adverse impact on industry can start at a temperature much lower than services. These results are consistent with Casey et al. (2021) and Rudik et al. (2021) that labor productivity can suffer a heavier loss under heat stress in outdoor activities which are more concentrated in the goods sector.

[11] Section 6 examines a simplified case where there are two final sectors producing investment and consumption respectively.



(a) Impact on goods and services

(b) Impact on three sectors

Notes: The figure shows how sectoral GDP growth can be affected by temperature shocks using the data from Burke et al. (2015).

Figure 1. Sectoral damages

Reason 2: The sluggish services sector may drive down aggregate productivity growth.

The services sector affects aggregate productivity growth. One-sector climate-economy models do not explicitly include this prospect. Evolving growth path in each sector combines to determine aggregate growth path (Foerster et al., 2022), and hence an expanding sector with a lower-than-average growth rate is capable of slowing down aggregate growth, which is roughly the core of Baumol's cost disease (Baumol, 1967; Nordhaus, 2008). In the past decades, a strand of literature has been actively engaged in leveraging the growth models to analyze the implications of rising services on long-run economic growth (e.g.: Duarte and Restuccia, 2010; Ngai and Pissarides, 2007; Leon-Ledesma and Moro, 2020; Duernecker et al., 2017). However, the rise of services receives surprisingly deficient attention in the climate economics literature.

Given such division, we calibrate the model.

4.2 Households

The representative household maximize the population-weighted lifetime utility:

$$\sum_{t=0}^{\infty} \beta^t N_t U(c_{1t}, c_{2t}) \tag{41}$$

where N_t is aggregate population projections following the DICE model (Nordhaus, 2017). c_{1t} represents consumption of goods, and c_{1t} consumption of services.

We use the literature to calibrate the consumption production function Eq.(34) (see Table 1). The gap in substitution elasticity between earlier and recent literature can be reconciled by two different accounting approaches matching the data, final expenditure and value added (Herrendorf et al., 2013). We take a benchmark value of 0.2 for parameter ϵ_C governing the substitution elasticity between goods and services in consumption. In addition, the services expenditure share in consumption is 0.8. Overall, our chosen values are consistent with existing studies that observe the complementary relationship between goods and services in consumption as well as a relatively larger proportion of services.

Table 1. Parameters of the utility function

Source	Calibrated to	$1-\omega_c$	ϵ_c
Buera and Kaboski (2009)	USA	NA	0.5
Duarte and Restuccia (2010)	USA	0.96	0.4
Herrendorf et al. (2013)	USA	0.81	0.002
Moro (2015)	USA	0.95	0.4
Alder et al. (2022)	AUS, CAN, GBR & USA	0.60-0.77	0.00-0.17

4.3 Production sectors

We assume the Cobb-Douglas production technology for both final production sectors, with capital, labor and energy as inputs. Thus, the production functions are given as:

$$\begin{aligned}\tilde{F}_1(L_{1t}, K_{1t}, E_{1t}) &= K_{1t}^{\alpha} L_{1t}^{1-\alpha-\beta} E_{1t}^{\beta} \\ \tilde{F}_2(L_{2t}, K_{2t}, E_{2t}) &= K_{2t}^{\alpha} L_{2t}^{1-\alpha-\beta} E_{2t}^{\beta}\end{aligned} \tag{42}$$

The identical factor intensity in two sectors are assumed to simplify the model. If capital share differs in each sector, this would generate another force for structural change via the price effect, as discussed above[12]. Thus, following Nordhaus (2017) and Golosov et al. (2014), we adopt a capital share of $\alpha = 0.3$ and an energy share of $\beta = 0.03$ for both sectors.

For the sectoral productivity growth and climate vulnerability, we assume:

$$\begin{aligned}\hat{F}_1(A_{1t}, T_t) &= A_{1t}(1 - D_1(T_t)) \\ \hat{F}_2(A_{2t}, T_t) &= A_{2t}(1 - D_2(T_t))\end{aligned} \tag{43}$$

where $D_i(T_t)$ represents the fraction of output loss due to temperature change T_t in sector i at time t. We follow both the DICE model (Nordhaus, 1992, 2017) and Golosov et al.

[12]One could argue that energy share is higher in the goods sector as it is more energy-intensive. Given our main intention is to understand the interactions between climate change and structural change dynamics, we assume energy intensity is also equivalent in both sectors.

(2014) where climate damage enters the production function in multiplicative form. The departure from previous works is that we allow for differentiated impact of temperature rise on producing both goods and services. To pin down the productivity growth in each sector, we first abstract from climate damages. Then, we determine the relative ratio of the goods sector to the services from existing studies (see Table 2), and, taking this ratio as given, mimic the capital stock in the DICE model at 2100. This strategy guarantees the closest comparability to DICE at an identical wealth level. Taking a benchmark value of 3 for γ_1/γ_2, we have $\gamma_1 = 10.86\%$ and $\gamma_2 = 3.62\%$.

Table 2. Parameters of sectoral productivity growth

Source	Calibrated to	γ_1/γ_2	γ_a/γ_2	γ_m/γ_2
Leon-Ledesma and Moro (2020)	USA	5.43	NA	NA
Comin et al. (2021)	USA	NA	2.64	1.20
Buera et al. (2020)	USA	NA	4.17	1.75

Now we move on to investment production function Eq.(35). Recently studies delving into the structural change within investment arrive at consistent findings that exogenous investment-specific technical change has a minor or even negligible effect in long-run growth (Herrendorf et al., 2021; García-Santana et al., 2021; Buera et al., 2020). Herrendorf et al. (2021) finds that the productivity growth in goods can be three times larger than investment-specific technical change. Therefore, in addition to assuming no climate impact, we further assume it remains constant for simplicity, namely, $\gamma_{I,2015} = 0\%$. The investment-specific technology level in the initial period (in 2015) is normalized to one. Moreover, we adopt $\omega_I = 0.57$ and $\epsilon_I = 0.5$, both of which are consistent with existing studies (Table 3).

Table 3. Parameters of the final investment production function

Source	Calibrated to	ω_I	ϵ_I
Herrendorf et al. (2021)	USA	0.65	0.00
García-Santana et al. (2021)	Countries in PWT & WIOD	0.58	0.51
Buera et al. (2020)	USA	0.52	0.01

For the energy sector, we also assume a Cobb-Douglas production function:

$$E_t = A_{Et} K_{Et}^{\alpha_E} L_{Et}^{1-\alpha_E} \tag{44}$$

where the labor share is 0.403 adopted from Barrage (2020). In addition, the energy sector is assigned to the same labor-augmented productivity growth as the goods sector.

For value added shares of goods and services in initial period, we refer to the World Development Indicators. Goods account for 33.57% of aggregate value added, and services is 66.43%. We further combine with first order conditions to determine the factor share in each sector in the initial period.

4.4 Carbon cycle and climate models

The current paper borrows the carbon cycle and climate model from the DICE model (Nordhaus, 2017), which generates a warming of 3.1°C for a doubling of carbon concentrations in equilibrium and a transient climate sensitivity of 1.7°C.

First, the equations of the carbon cycle include three reservoirs (the atmosphere M_t^{At}, the upper oceans and the biosphere M_t^{Up}, and the deep oceans M_t^{Lo}):

$$\begin{pmatrix} M_t^{At} \\ M_t^{Up} \\ M_t^{Lo} \end{pmatrix} = \begin{pmatrix} \phi_{11} & \phi_{21} & 0 \\ \phi_{12} & \phi_{22} & \phi_{32} \\ 0 & \phi_{23} & \phi_{33} \end{pmatrix} \begin{pmatrix} M_{t-1}^{At} \\ M_{t-1}^{Up} \\ M_{t-1}^{Lo} \end{pmatrix} + \begin{pmatrix} E_t^M + E_t^{Land} \\ 0 \\ 0 \end{pmatrix} \tag{45}$$

where ϕ_{ij} measures the carbon flow between reservoirs and E_t^{Land} represents exogenous land carbon emissions.

Second, the increased carbon concentrations in the atmosphere elevate radiative forcing:

$$\chi_t = \kappa \left[\ln \left(M_t^{At} / M_{1750}^{At} \right) / \ln(2) \right] + \chi_t^{Ex} \tag{46}$$

where χ_t^{Ex} is the exogenous forcing from other greenhouse gases.

Last, higher radiative forcing raises the atmospheric temperature T_t and, indirectly, the deep ocean temperature T_t^{Lo}:

$$\begin{pmatrix} T_t \\ T_t^{Lo} \end{pmatrix} = \begin{pmatrix} 1 - \zeta_1\zeta_2 - \zeta_1\zeta_3 & \zeta_1\zeta_3 \\ 1 - \zeta_4 & \zeta_4 \end{pmatrix} \begin{pmatrix} T_{t-1} \\ T_{t-1}^{Lo} \end{pmatrix} + \begin{pmatrix} \zeta_1 \chi_t \\ 0 \end{pmatrix} \tag{47}$$

where $\{\zeta_i\}_{i=1}^4$ are parameters governing heat exchange between the atmosphere and the ocean.

4.5 Sectoral climate damages

In the DICE model, the damage function is calibrated to match a damage of 2.1% damage of income at 3°C. We adopt the damage function as in DICE-2016, and suppose that the curvature of the damage function is identical in both sectors. Furthermore, our calibration matches that the relative climate damage in the goods sector is three times that of services and also that at a temperature rise of 3°C, the combined damage from two sectors amount to that of DICE.The damage function in the IAMs has always been one of the most uncertain components, and this concern also applies here. However, although the damage function is highly uncertain, our analysis only requires that sector is different in the productivity of climate capital. Therefore, the benchmark model adopts a damage function for output in each sector as follows:

$$1 - D_1(T_t) = \frac{1}{1 + 0.004352 * T_t^2} \tag{48}$$

$$1 - D_2(T_t) = \frac{1}{1 + 0.001414 * T_t^2} \tag{49}$$

4.6 Abatement costs

In the DICE model, the abatement cost function is directly governed by the control rate, whereas the abatement costs in this study are related to the unit of abated carbon emission. Thus, following Barrage (2020), we recalibrate the abatement cost function through a logistic approximation to the abatement cost curve implied by Nordhaus (2017):

$$\Theta_t(\mu_t E_t) = \frac{\overline{a} P_t^{backstop}}{1 + a_t \exp(b_{0t} - b_{1t}(\mu_t E_t)^{b2})} \cdot (\mu_t E_t) \tag{50}$$

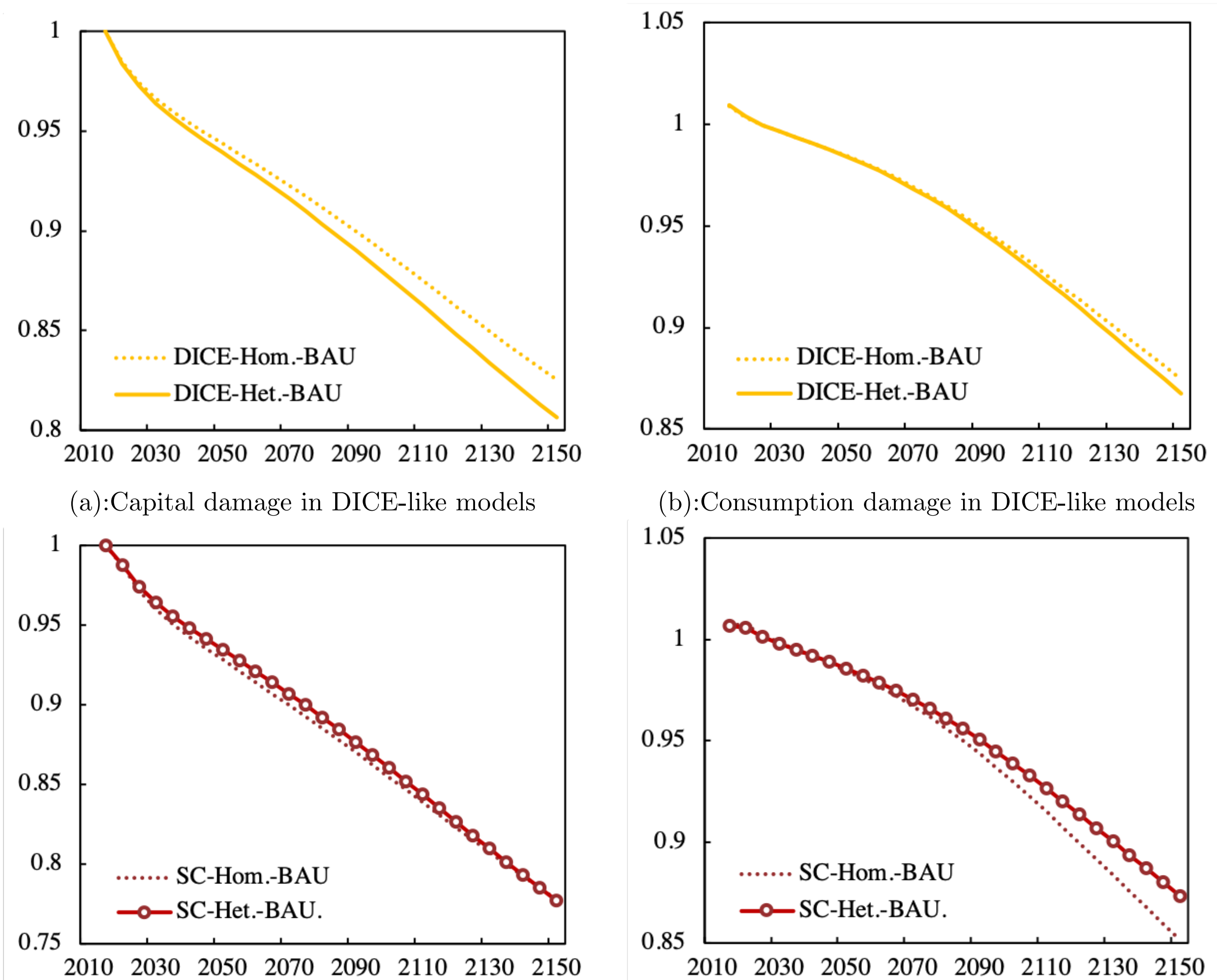
where $P_t^{backstop}$ is the price for backstop technology.

Finally, we have $\overline{a} = 0.7464$, $a_t = 0.6561 + 0.8881t$, $b_{0t} = 7.864 - 1.4858t$, $b_{1t} = 1.6791 - 0.3157t$ and $b_2 = 0.4207$. It should be noted that the abatement costs is denominated by investment product whereas the abatement costs in DICE adopt the final output as numeraire.

5 Quantitative results

In this section, we present the numerical results of the constructed climate-economy model. Two standard scenarios are considered throughout, a business-as-usual (BAU) scenario where the climate externality is completely ignored so that no abatement policy is adopted, and an optimal scenario where a carbon tax is adopted equating marginal abatement cost to marginal climate damage. We also assume, where necessary and for ease of exposition, an additional scenario with no climate (pure economic growth model) to demonstrate the economic frontier. We first present the damage level to explore whether the climate Baumol's cost disease is consistent with the theoretical results in Section 3. Then, we quantify the social cost of carbon with two different definitions, namely, investment-equivalent and consumption-equivalent, and explain their implications.

5.1 Climate damage and abatement incentive



(a):Capital damage in DICE-like models

(b):Consumption damage in DICE-like models

(c):Capital damage under structural change

(d):Consumption damage under structural change

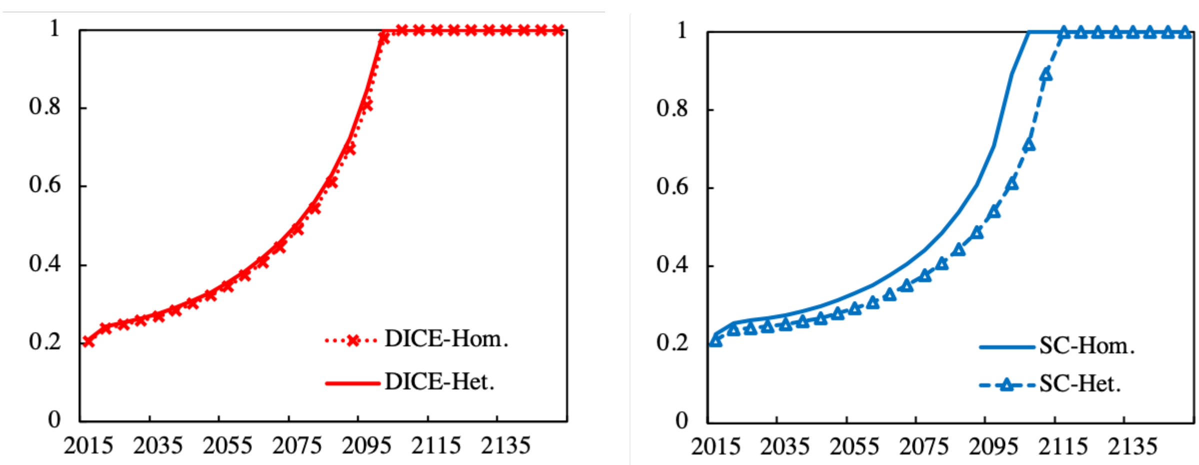
Notes: Panels (a)-(d) show that after considering both homogeneous(Hom.) and heterogeneous(Het.) climate vulnerability between sectors, the relative level of each economic variable in the business-as-usual scenarios compared to the their frontiers under no climate externality in each period. DICE means two identical sectors, SC means structural change.

Figure 2. Capital and consumption damage

Figure 2 displays the relative level of capital stock and consumption in the BAU scenario compared to their economic frontier without climate damage. DICE-like models denotes that two sectors have identical productivity growth, while under structural change, the productivity growth rate in goods is three times that in services.

When two sectors have the same productivity growth, only the climate Baumol's cost disease is operating, as the Case 2 in Section 3. Panels (a) and (b) show that climate damage is indeed higher under heterogeneous climate vulnerability, validating the impact of the climate Baumol's cost disease. When no abatement policy is adopted, capital stock is 5.62% (6.07%) lower in 2050, 11.25% (12.34%) lower in 2100, and 17.47% (19.37%) lower in 2150 than its frontier under homogeneous (heterogeneous) damage (Panel a). Aggregate consumption under heterogeneous damage is also in general lower than homogeneous one (Panel b), but the gap is narrower than capital damage. For DICE-like models, in 2050, the consumption level is only 0.03 percentage points higher when sectors are equally vulnerable, and this difference increases to 0.71 percentage points in 2150.

If we assume the goods sector has a productivity growth two times faster than services, a different pattern is obtained. Under heterogeneous climate vulnerability, both capital and consumption damages are lower before the end of this century, compared to under homogeneous climate vulnerability. Because both investment consists of an increasing proportion of services, relative capital damage curve under heterogeneous damage overlaps with or is even slightly above that under homogeneous damage in initial periods (Panel c). Put differently, the trend that services is gaining its importance in investment makes economic growth more 'resilient'. It should be cautioned, however, that in the long-run heterogeneous climate damages remain a non-negligible concern, because capital stock under heterogeneous climate vulnerability starts to be increasingly lower than under homogeneous vulnerability. By comparison, aggregate consumption is elevated in the displayed period (Panel d), due to a higher dependence on services in consumption than investment, and also to that capital damage only begins to be higher under heterogeneous damages in later periods.



(a):DICE (b):Structural change

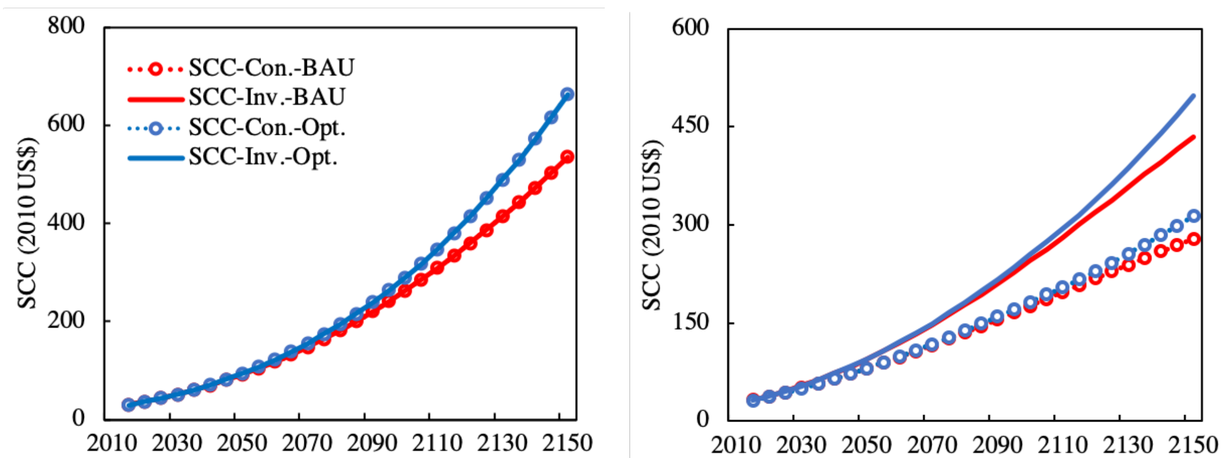
Notes: Panels (a)-(b) show the optimal abatement rate under both homogeneous(Hom.) and heterogeneous(Het.) climate vulnerability.

Figure 3. Optimal abatement rate

Figure 3 displaying the optimal abatement rate confirms that the incentives to reduce investment into climate capital is influenced by the Baumol's cost disease. When only the climate Baumol's cost disease is operating, the abatement policy is more stringent under heterogeneous climate vulnerability, although the its gap with under homogeneous climate vulnerability is not obvious. In comparison, if one divides the economy into goods and services, the abatement policy is less strict under heterogeneous climate vulnerability. The reason is that although the goods sector is more vulnerable to climate change, its productivity growth rate is also higher than services. Thus, the net Baumol's cost disease is 'soothed'.

5.2 Social cost of carbon: consumption equivalent v.s. investment equivalent

In Section 4, we theoretically show that the actual trade-off between abatement cost and climate damage resides in how the final investment is affected, and proposes investment-equivalent social cost of carbon. In this part, we quantify the social cost of carbon of both definitions (Figure 4). For better illustration, we compare DICE-like models with models under structural change.



(a) DICE-like models

(b) Structual change

Notes: Two definitions of social cost of carbon are presented—consumption equivalent social cost of carbon (SCC-Con.) and investment equivalent one (SCC-Inv.). BAU denotes the business-as-usual scenario (no abatement), and Opt. denotes the optimal scenario.

Figure 4. Social cost of carbon

It can be readily seen in Panel a that two definitions of social cost of carbon generate identical numerical values in the DICE-like models. In a symmetric world, a unit of final production can be used to produce the same amount of investment or consumption, and therefore one can arbitrarily choose either definition. In contrast, under structual change (Panel b), there is an obvious divergence between the consumption-equivalent social cost of carbon and the investment-equivalent one. In 2100, the social cost of carbon comes to a investment loss of \$254 per tone of CO_2, or a consumption loss of \$182 per tone of CO_2 under the optimal scenario. In 2150, this gap further expands, with investment-equivalent social cost of carbon climbing to $497, 59% high than consumption-equivalent one. Because two final production sectors differ in productivity growth rate, climate vulnerability and also their compositions in consumption and investment, one unit of final production can no longer be converted to a equalized amount between investment and consumption. More concretely, since the production of consumption is assumed to be more heavily reliant on services, which has a lower productivity growth, its price relative to investment will rise, eventually leading to a lower value of social cost of carbon denominated by consumption than investment equivalent.

6 Sensitivity and extensions

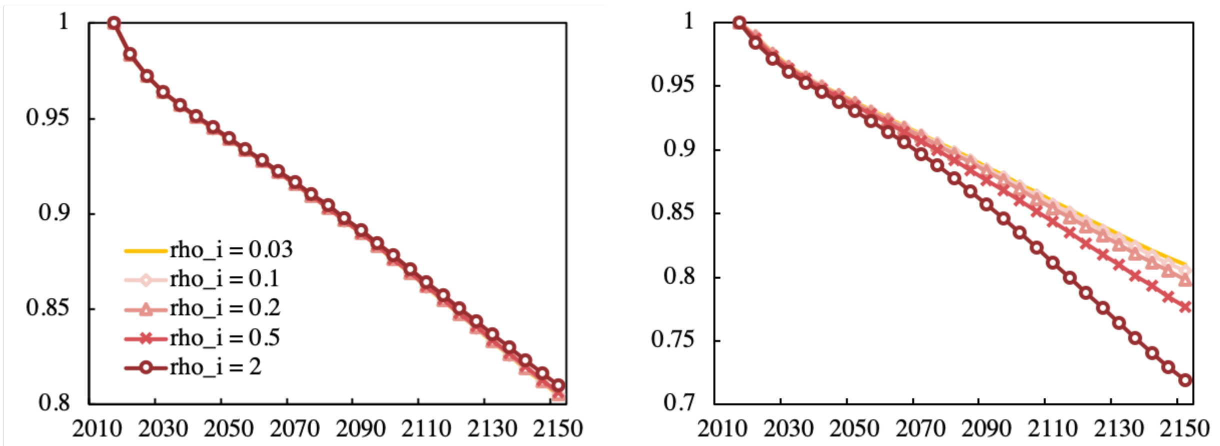
In this section, we initially test the sensitivity of our results by changing the values of some key parameters. Then, we provide two alternative models as extension.

6.1 Sensitivity analyses

First, the substitution elasticity between two sectors can affect the net impact of the Baumol's cost disease. We focus on the elasticity in investment production. In general, extant studies are consistent that the substitution elasticity between the two is low. In our baseline model, we choose a value of 0.5. Some studies argue that this value could be even lower and close to zero. Thus, we test the robustness of our results by choosing a value of 0.03, 0.1 and 0.5. Also, we perform the model with a value of 2 as a comparison, where two product can easily substitute each other in formulating final investment. Results are displayed in Figure 5. One should note that changing the substitution elasticity can work through altering both the price effect and the potential to transforming to a less vulnerable sector.

When sectors are only different in climate vulnerability, a lower-than-unit substitution elasticity yields a lower level of capital stock, although changes are negligible (Panel a). When goods and services are assumed to be substitutes (rho_i as 2), we observe that relative climate damages are slightly alleviated in the DICE-like models. When accounting for the rising services, we find relative climate damages also vary under the lower-than-unit elasticity. A closer inspection reveals that a lower elasticity will bring about a lower relative damage. Moreover, when two products are considered as substitutes, the Baumol's cost disease is no longer operating, and the relative damage level is considerably aggravated.

Second, the social time preference rate is a key factor in driving the results of the social cost of carbon and debates around it abound. We are specifically interested in whether such choices will also matter to the gap between two definitions of social cost of carbon. We address this problem by altering the benchmark value of the social time preference

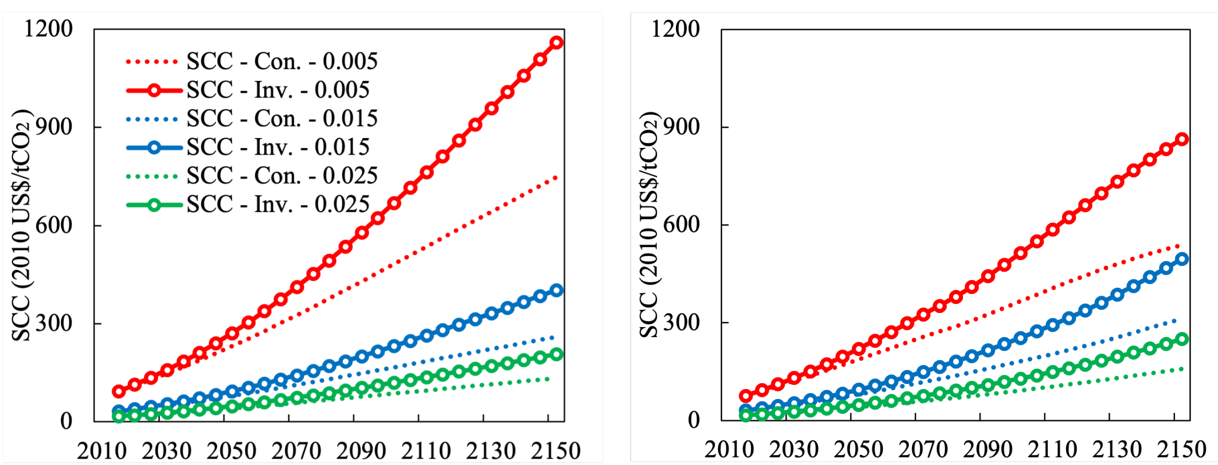


(a) DICE-like models

(b) Structural change

Notes: The figure shows the relative level of capital stock under heterogeneous damage to its economic frontier (without climate damage). rho_i is the substitution elasticity between goods and services in producing final investment.

Figure 5. Capital damage under different substitution elasticities



(a) Business-as-usual scenario

(b) Optimal scenario

Notes: Two definitions of social cost of carbon are presented in the figure, which are consumption equivalent social cost of carbon (SCC-Con.) and investment equivalent one (SCC-Inv.). Results are reported choosing different values of social time preference (0.005, 0.015 or 0.025).

Figure 6. Social cost of carbon and the discount rate

from 0.015 (which is used by the DICE model) to a percentage point higher and lower. Figure 15 shows that when a lower social time preference is chosen, the implied social cost of carbon under both scenarios will climb dramatically. Moreover, the gap is amplified between the consumption equivalent social cost of carbon and the investment equivalent one. For example, the gap in 2100 between two definitions is enlarged from \$34/tCO$_t$ (0.025) to \$52/tCO$_t$ (0.015) and further to \$187/tCO$_t$ (0.005) under the business-as-usual scenario. A similar picture is observed in the optimal scenario.

6.2 Alternative models

We present two alternative models and discuss their implications. All proofs are included in the appendix.

Case 1: Consumption and investment. The model can be simplified to a two-sector version that produces consumption and investment respectively. The two-sector growth model resembles in structure that in Greenwood et al. (1997) and Foerster et al. (2022), and enables us to demonstrate that the deviation between the consumption-equivalent and investment-equivalent social costs of carbon originates from the detailed consideration of the distinct production procedures for consumption and investment. It can be straightforward to show that the optimal carbon tax in this case is pinned down by:

$$\Theta^{'} = F_{IEt} - \frac{F_{IIt}}{F_{EIt}} = (-1)\sum_{j=0}^{\infty}\beta^j(\underbrace{\frac{\lambda_{C,t+j}}{\lambda_{It}}\frac{\partial Y_{C,t+j}}{\partial T_{t+j}}}_{Impact\quad on\quad consumption} + \underbrace{\frac{\lambda_{I,t+j}}{\lambda_{It}}\frac{\partial Y_{I,t+j}}{\partial T_{t+j}}}_{Impact\quad on\quad investment})\frac{\partial T_{t+j}}{\partial E_t^{unc}} \quad (51)$$

This identity also establishes the shadow price of investment as denomination, but the climate damage aggregates both consumption damage and investment damage. In the baseline model, energy input should be utilized to produce two final products first, and then two products are combined to produce the aggregate investment. By comparison, the benefits of one additional tone of carbon emission is now represented by how much aggregate investment it can straightly produce. To sum up, the trade-off still centers on investment instead of consumption.

Case 2: Differential abatement costs between sectors. In the model, we do not allow for the more realistic case that each production sector can be characterized by different abatement costs (Gillingham and Stock, 2018). This can be easily included in the model by introducing two individual abatement cost functions. In so doing, one can establish that the marginal abatement cost in each sector should be equalized and also equal to marginal climate damages:

$$\Theta_{gt}^{'} = \Theta_{st}^{'} = MD \tag{52}$$

where Θ_{it} denotes the abatement cost function in sector i and MD the marginal climate damages aggregating both sectors. We have shown that our numerical results are generally still robust under different abatement costs given a single abatement technology, but it is of practical interests to include differentiated abatement functions in each sector[13].

7 Conclusion

This paper establishes a dynamic general equilibrium model with endogenous structural change. Using the model, we investigates optimal carbon investment into climate capital under structural change.

Two reasons stand out for reducing carbon investment into climate capital. First, climate is an essential capital used in economic production, but it is unpriced, nonrival, and nonexcludable. Thus, climate capital to overestimated, and too much carbon emissions imply deteriorating productive base, which can be perceived as reduction in physical capital. Second, overinvested climate capital has the potential to amplify the climate vulnerability between sectors, leading up to the climate Baumol's cost disease. Eventually, climate vulnerability and productivity growth combines to determine the net impact of Baumol's cost disease, which can drag down long-run economic growth. Whenever there is a sector with higher climate vulnerability and lower productivity growth, disinvesting into climate capital is urgent and highly demanded; when vulnerable sectors have high productivity growth, aggregate climate impact may be less worrisome.

[13] Vogt-Schilb et al. (2018) shows that after accounting for the value of abatement capital in the future, optimal marginal abatement cost can differ in each sector. We do not include this concern in the current paper, but it can be an interesting extension.

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A Appendix One: Proofs

Proof of Proposition 1

We first state the representative household's problem and associated first order conditions. We assume throughout the solution to the household's problem is interior. The representative household seeks to maximize lifetime utility according to:

$$U_0 \equiv \sum_{t=0}^{\infty} \beta^t U(C_t) \tag{53}$$

The household faces the following two constraints Eq.(6) and Eq.(7).

Eq.(6) describes how each kind of product is tranformed into final aggregate consumption, and Equation Eq.(7) is the budget constraint faced by the household. Letting ζ_t and γ_t be the Lagrange multiplier on Eq.(6) and Eq.(7) at time t respectively, the first order conditions are given by:

$[C_t]$:

$$\beta^t U_{Ct} = \zeta_t \tag{54}$$

$[C_{1t}]$:

$$\zeta_t F_{C1t} = \gamma_t p_{1t} \tag{55}$$

$[C_{2t}]$:

$$\zeta_t F_{C2t} = \gamma_t p_{2t} \tag{56}$$

$[K_{t+1}]$:

$$\gamma_{t+1}(1 + r_{t+1} - \delta) = \gamma_t \tag{57}$$

For the producers of goods and services, their problems to maximize profits are:

$$\max_{K_{it}, L_{it}, E_{it}} p_{it} Y_{it} - r_t K_{it} - w_t L_{it} - p_{et} E_{it}, \quad where \quad i \in \{1, 2\} \tag{58}$$

Thus, the first order conditions to each input factor satisfy:

$$\begin{aligned} p_{1t}F_{1lt} &= p_{2t}F_{2lt} = w_t \\ p_{1t}F_{1kt} &= p_{2t}F_{2kt} = r_t \\ p_{1t}F_{1Et} &= p_{2t}F_{2Et} = p_{Et} \end{aligned} \tag{59}$$

The investment producer solves:

$$\max_{I_{1t}, I_{2t}} p_{It}F_I(A_{It}, I_{1t}, I_{2t}) - p_{1t}I_{1t} - p_{2t}I_{2t} \tag{60}$$

where the price of final investment product in each period is normalized to one. The corresponding first order conditions are:

$$\begin{aligned} F_{I1t} &= p_{1t} \\ F_{I2t} &= p_{2t} \end{aligned} \tag{61}$$

For the energy sector, the representative firm solves:

$$\max_{K_{Et}, L_{Et}, \mu} \Pi_t = p_{Et}A_{Et}F_E(K_{Et}, L_{Et}) - [(1-\mu_t)E_t]\tau_{Et} - w_tL_{Et} - r_tK_{Et} - \Theta_t(\mu_tE_t) \tag{62}$$

The associated first-order conditions are:

$$\begin{aligned} (p_{Et} - \tau_{Et})F_{Elt} &= w_t \\ (p_{Et} - \tau_{Et})F_{Ekt} &= r_t \\ \tau_{Et} &= \Theta_t^{'}(\mu_tE_t) \end{aligned} \tag{63}$$

Assume that the government only levies carbon tax and makes a lump-sum transfer to the household:

$$Tran_t = [(1-\mu_t)E_t]\tau_{Et} \tag{64}$$

Direction: If initial conditions and the allocations satisfy a competitive equilibrium, constraints Eq.(21)-Eq.(29) are hence satisfied. First, adding up the household's budget constraint (7), the definition of energy profits (62), and the government transfer (64), we have:

$$p_{gt}C_{gt} + p_{st}C_{st} + K_{t+1} \leq w_t(L_t - L_{Et}) + (1-\delta)K_t + p_{Et}E_t + r_t(K_t - K_{Et}) - \Theta_t(\mu_tE_t) \tag{65}$$

Substituting into the market clearing conditions as per capital, labor and energy Eq.(19) gives:

$$p_{gt}C_{gt} + p_{st}C_{st} + K_{t+1} \leq w_t(L_{gt} + L_{st}) + (1-\delta)K_t + p_{Et}(E_{gt} + E_{st}) + r_t(K_{gt} + K_{st}) - \Theta_t(\mu_t E_t) \tag{66}$$

Invoking the factor prices based on first order conditions (59) yields:

$$\begin{aligned} p_{gt}C_{gt} + p_{st}C_{st} + K_{t+1} \leq p_{gt}F_{glt}L_{gt} + p_{st}F_{slt}L_{st} + (1-\delta)K_t + p_{gt}F_{gEt}E_{gt} + p_{st}F_{sEt}E_{st} \\ + p_{gt}F_{gkt}K_{gt} + p_{st}F_{skt}K_{st} - \Theta_t(\mu_t E_t) \end{aligned} \tag{67}$$

Substituting the Euler's theorem based on the assumption of constant returns to scale in two final production sectors gives:

$$p_{gt}C_{gt} + p_{st}C_{st} + K_{t+1} \leq p_{gt}Y_{gt} + p_{st}Y_{st} + (1-\delta)K_t + \Theta_t(\mu_t E_t) \tag{68}$$

Recalling the utilization rule of final production (12), the above equation can be rewritten to:

$$K_{t+1} \leq p_{gt}I_{gt} + p_{st}I_{st} + (1-\delta)K_t + \Theta_t(\mu_t E_t) \tag{69}$$

Revoking the Euler's theorem based on the assumption of constant returns to scale in the investment production sector:

$$K_{t+1} \leq I_t + (1-\delta)K_t + \Theta_t(\mu_t E_t) \tag{70}$$

This gives the capital accumulation constraint as in (24). The carbon cycle constraint, the consumption aggregation constraint, the investment producer's budget constraint and the energy producer's budget constraint all hold by definition in competitive equilibrium.

Direction: If constraints Eq.(21)-Eq.(29) are satisfied, one can construct competitive equilibrium.

Thus, the social planner problem can be established as:

$$
\begin{aligned}
&\max \sum_{t=0}^{\infty} \beta^t U(C_t) \\
&+ \sum_{t=0}^{\infty} \beta^t \lambda_{1t} \left[F_1(A_{1t}, T_t, L_{1t}, K_{1t}, E_{1t}) - C_{1t} - I_{1t}\right] \\
&+ \sum_{t=0}^{\infty} \beta^t \lambda_{2t} \left[F_2(A_{2t}, T_t, L_{2t}, K_{2t}, E_{2t}) - C_{2t} - I_{2t}\right] \\
&+ \sum_{t=0}^{\infty} \beta^t \lambda_{Ct} \left[F_C(C_{1t}, C_{2t}) - C_t\right] \\
&+ \sum_{t=0}^{\infty} \beta^t \lambda_{It} \left[F_I(A_{It}, I_{1t}, I_{2t}) + (1-\delta)K_t - K_{t+1} - \Theta_t(\mu_t E_t)\right] \\
&+ \sum_{t=0}^{\infty} \beta^t \xi_t \left\{T_t - \Phi[\boldsymbol{M}_0, (1-\mu_0)(E_{10}+E_{20}), ..., (1-\mu_t)(E_{1t}+E_{2t}), \boldsymbol{\eta}_0, ..., \boldsymbol{\eta}_t]\right\} \\
&+ \sum_{t=0}^{\infty} \beta^t \chi_{lt} \left[L_t - L_{1t} - L_{2t} - L_{Et}\right] \\
&+ \sum_{t=0}^{\infty} \beta^t \chi_{kt} \left[K_t - K_{1t} - K_{2t} - K_{Et}\right] \\
&+ \sum_{t=0}^{\infty} \beta^t \chi_{Et} \left[A_{Et} F_E(K_{Et}, L_{Et}) - E_{1t} - E_{2t}\right]
\end{aligned} \tag{71}
$$

The F.O.C w.r.t. C_{1t} is:

$$\lambda_{Ct} \frac{\partial C_t}{\partial C_{1t}} = \lambda_{1t}$$

The F.O.C w.r.t. C_{2t} is:

$$\lambda_{Ct} \frac{\partial C_t}{\partial C_{2t}} = \lambda_{2t}$$

The F.O.C w.r.t. C_t is:

$$U_{Ct} = \lambda_{Ct}$$

The F.O.C w.r.t. K_{t+1} is:

$$\beta \lambda_{I,t+1}(1-\delta) + \beta \chi_{k,t+1} = \lambda_{It}$$

The F.O.C w.r.t. K_{1t} is:

$$\lambda_{1t}F_{1kt} = \chi_{kt}$$

The F.O.C w.r.t. K_{2t} is:

$$\lambda_{2t}F_{2kt} = \chi_{kt}$$

The F.O.C w.r.t. K_{Et} is:

$$\chi_{Et}F_{Ekt} = \chi_{kt}$$

The F.O.C w.r.t. I_{1t} is:

$$\lambda_{It}F_{I1t} = \lambda_{1t}$$

The F.O.C w.r.t. I_{2t} is:

$$\lambda_{It}F_{I2t} = \lambda_{2t}$$

The F.O.C w.r.t. L_{1t} is:

$$\lambda_{1t}F_{1lt} = \chi_{lt}$$

The F.O.C w.r.t. L_{2t} is:

$$\lambda_{2t}F_{2lt} = \chi_{lt}$$

The F.O.C w.r.t. L_{Et} is:

$$\chi_{Et}F_{Elt} = \chi_{lt}$$

The F.O.C w.r.t. E_{1t} is:

$$\lambda_{1t}F_{1Et} - \lambda_{It}\mu_t\Theta' - \chi_{Et} = \sum_{j=0}^{\infty}\beta^j\xi_{t+j}(1-\mu_t)\frac{\partial T_{t+j}}{\partial E_t^{unc}}$$

The F.O.C w.r.t. E_{2t} is:

$$\lambda_{2t} F_{2Et} - \lambda_{It} \mu_t \Theta^{'} - \chi_{Et} = \sum_{j=0}^{\infty} \beta^j \xi_{t+j} (1 - \mu_t) \frac{\partial T_{t+j}}{\partial E_t^{unc}}$$

The F.O.C w.r.t. T_t is:

$$\lambda_{1t} \frac{\partial Y_{1t}}{\partial T_t} + \lambda_{2t} \frac{\partial Y_{2t}}{\partial T_t} + \xi_t = 0$$

The F.O.C w.r.t. μ_t is:

$$\lambda_{It} \Theta^{'} = \sum_{j=0}^{\infty} \beta^j \xi_{t+j} \frac{\partial T_{t+j}}{\partial E_t^{unc}}$$

Rearranging above equations yields:

$$\begin{aligned} \Theta^{'} &= F_{I1t} F_{1Et} - \frac{F_{I1t} F_{1lt}}{F_{Elt}} = F_{I2t} F_{2Et} - \frac{F_{I2t} F_{2lt}}{F_{Elt}} \\ &= (-1) \sum_{j=0}^{\infty} \beta^j \left\{ \frac{\lambda_{1,t+j}}{\lambda_{It}} \frac{\partial Y_{1,t+j}}{\partial T_{t+j}} + \frac{\lambda_{2,t+j}}{\lambda_{It}} \frac{\partial Y_{2,t+j}}{\partial T_{t+j}} \right\} \\ &= (-1) \sum_{j=0}^{\infty} \beta^j \left(\underbrace{\frac{U_{C,t+j}}{\lambda_{It}} \frac{\partial C_{t+j}}{\partial C_{1,t+j}} \frac{\partial Y_{1,t+j}}{\partial T_{t+j}} \frac{\partial T_{t+j}}{\partial E_t^{unc}}}_{Impacts \quad on \quad sector \quad 1} + \underbrace{\frac{U_{C,t+j}}{\lambda_{It}} \frac{\partial C_{t+j}}{\partial C_{2,t+j}} \frac{\partial Y_{2,t+j}}{\partial T_{t+j}} \frac{\partial T_{t+j}}{\partial E_t^{unc}}}_{Impacts \quad on \quad sector \quad 2} \right) \end{aligned} \tag{72}$$

where the government's discounting of output damages in period t is governed by:

$$\frac{\lambda_{It}}{\beta \lambda_{I,t+1}} = F_{I1,t+1} F_{1k,t+1} + (1 - \delta) = F_{I2,t+1} F_{s2,t+1} + (1 - \delta) \tag{73}$$

Q.E.D.

Proof of Proposition 2

There are two final sectors producing goods and services respectively in the economy. Each sector will employ rival factors including physical capital labor, and energy for production. Production in both sectors requires nonrival factors including knowledge capital and climate capital. Thus, the production functions are given by:

$$\begin{aligned} Y_{1t} &= \hat{F}_1(A_{1t}, T_t) K_{1t}^{\alpha} L_{1t}^{1-\alpha-\nu} E_{1t}^{\nu} \\ Y_{2t} &= \hat{F}_2(A_{2t}, T_t) K_{2t}^{\alpha} L_{2t}^{1-\alpha-\nu} E_{2t}^{\nu} \end{aligned} \tag{74}$$

In addition, there is an intermediate energy production using capital and labor:

$$E_t = A_{Et} K_{Et}^{\alpha_E} L_{Et}^{1-\alpha_E} \tag{75}$$

Note that we assume the energy sector is immune to climate change.

Then, market clearing conditions will be given by:

$$\begin{aligned} L_t &= L_{1t} + L_{2t} + L_{et} \\ K_t &= K_{1t} + K_{2t} + K_{et} \\ E_t &= E_{1t} + E_{2t} \end{aligned} \tag{76}$$

Firms in two final sectors shall decide their production plans to maximize profits:

$$\max_{K_{it}, L_{it}, E_{it}} p_{it} Y_{it} - r_t K_{it} - w_t L_{it} - p_{Et} E_{it} \tag{77}$$

Firms in energy sector will decide their production plans according to:

$$\max_{K_{Et}, L_{Et}} p_{Et} E_t - r_t K_{Et} - w_t L_{Et} \tag{78}$$

Thus, one can obtain the identity across sectors using the interest rate:

$$\begin{aligned} r_t &= \alpha p_{1t} \hat{F}_1(A_{1t}, T_t) \left(\frac{K_{1t}}{L_{1t}}\right)^{\alpha-1} \left(\frac{E_{1t}}{L_{1t}}\right)^{\nu} \\ &= \alpha p_{2t} \hat{F}_2(A_{2t}, T_t) \left(\frac{K_{2t}}{L_{2t}}\right)^{\alpha-1} \left(\frac{E_{2t}}{L_{2t}}\right)^{\nu} \\ &= \alpha_E p_{Et} A_{Et} \left(\frac{K_{Et}}{L_{Et}}\right)^{\alpha_E-1} \end{aligned} \tag{79}$$

In a similar vein, the identity using the wage will be:

$$
\begin{aligned}
w_t &= (1-\alpha-\nu)p_{1t}\hat{F}_1(A_{1t},T_t)\left(\frac{K_{1t}}{L_{1t}}\right)^{\alpha}\left(\frac{E_{1t}}{L_{gt}}\right)^{\nu} \\
&= (1-\alpha-\nu)p_{2t}\hat{F}_2(A_{2t},T_t)\left(\frac{K_{2t}}{L_{2t}}\right)^{\alpha}\left(\frac{E_{2t}}{L_{2t}}\right)^{\nu} \\
&= (1-\alpha_E)p_{Et}A_{Et}\left(\frac{K_{Et}}{L_{Et}}\right)^{-\alpha_E}
\end{aligned}
\tag{80}
$$

Finally, the identity using the energy price is given by:

$$
\begin{aligned}
p_{Et} &= \nu p_{1t}\hat{F}_1(A_{1t},T_t)\left(\frac{K_{1t}}{L_{1t}}\right)^{\alpha}\left(\frac{E_{1t}}{L_{1t}}\right)^{\nu-1} \\
&= \nu p_{2t}\hat{F}_2(A_{2t},T_t)\left(\frac{K_{2t}}{L_{2t}}\right)^{\alpha}\left(\frac{E_{2t}}{L_{2t}}\right)^{\nu-1}
\end{aligned}
\tag{81}
$$

Combining the above three identities, the capital-labor ratios between three sectors should satisfy:

$$
\frac{K_{1t}}{L_{1t}} = \frac{K_{2t}}{L_{2t}} = \frac{K_{et}}{L_{et}} \tag{82}
$$

Likewise, the energy-labor ratios between two final sectors should satisfy:

$$
\frac{E_{1t}}{L_{1t}} = \frac{E_{2t}}{L_{2t}} \tag{83}
$$

Substituting Eq.(82)-Eq.(83) into Eq.(79) to Eq.(81), one can obtain the relative price of services to goods is given by:

$$
\frac{p_{2t}}{p_{1t}} = \frac{\hat{F}_1(A_{1t},T_t)}{\hat{F}_2(A_{2t},T_t)} \tag{84}
$$

Given the production productions of consumption and investment Eq.(34) and Eq.(35), it is straightforward to show that the cost minimization problem for producing both yields:

$$
\frac{p_{1t}C_{1t}}{p_{2t}C_{2t}} = \frac{\omega_c}{1-\omega_c}\left(\frac{p_{1t}}{p_{2t}}\right)^{1-\epsilon_c} \tag{85}
$$

$$
\frac{p_{1t}I_{1t}}{p_{2t}I_{2t}} = \frac{\omega_I}{1-\omega_I}\left(\frac{p_{1t}}{p_{2t}}\right)^{1-\epsilon_I} \tag{86}
$$

Substituting Eq.(84) into above gives:

$$\frac{C_{1t}}{C_{2t}} = \frac{\omega_c}{1-\omega_c}\left(\frac{\hat{F}_1(A_{1t},T_t)}{\hat{F}_2(A_{2t},T_t)}\right)^{\epsilon_c} \tag{87}$$

$$\frac{I_{1t}}{I_{2t}} = \frac{\omega_I}{1-\omega_I}\left(\frac{\hat{F}_1(A_{1t},T_t)}{\hat{F}_2(A_{2t},T_t)}\right)^{\epsilon_I} \tag{88}$$

The consumption production function Eq.(34) can be rewritten in two different ways:

$$C_t = \left(\omega_c^{\frac{1}{\epsilon_c}} + (1-\omega_c)^{\frac{1}{\epsilon_c}}\left(\frac{C_{2t}}{C_{1t}}\right)^{\frac{\epsilon_c-1}{\epsilon_c}}\right)^{\epsilon_c} C_{1t} \tag{89}$$

$$C_t = \left(\omega_c^{\frac{1}{\epsilon_c}}\left(\frac{C_{1t}}{C_{2t}}\right)^{\frac{\epsilon_c-1}{\epsilon_c}} + (1-\omega_c)^{\frac{1}{\epsilon_c}}\right)^{\frac{\epsilon_c}{\epsilon_c-1}} C_{2t} \tag{90}$$

Combining Eq.(87) and above two equations, we obtain:

$$C_t = \left(\omega_c^{\frac{1}{\epsilon_c}} + \frac{(1-\omega_c)}{\omega_c^{\frac{\epsilon_c-1}{\epsilon_c}}}\left(\frac{\hat{F}_2(A_{2t},T_t)}{\hat{F}_1(A_{1t},T_t)}\right)^{\frac{\epsilon_c-1}{\epsilon_c}}\right)^{\frac{\epsilon_c}{\epsilon_c-1}} C_{1t} \tag{91}$$

$$C_t = \left(\frac{\omega_c}{(1-\omega_c)^{\frac{\epsilon_c-1}{\epsilon_c}}}\left(\frac{\hat{F}_1(A_{1t},T_t)}{\hat{F}_2(A_{2t},T_t)}\right)^{\frac{\epsilon_c-1}{\epsilon_c}} + (1-\omega_c)^{\frac{1}{\epsilon_c}}\right)^{\frac{\epsilon_c}{\epsilon_c-1}} C_{2t} \tag{92}$$

Rearranging both equations and adding together, we have:

$$C_t = \left(\omega_c\hat{F}_1(A_{1t},T_t)^{\epsilon_c-1} + (1-\omega_c)\hat{F}_2(A_{2t},T_t)^{\epsilon_c-1}\right)^{\frac{1}{\epsilon_c-1}}\left(\frac{C_{1t}}{\hat{F}_1(A_{1t},T_t)} + \frac{C_{2t}}{\hat{F}_2(A_{2t},T_t)}\right) \tag{93}$$

In a similiar vein, for investment we have:

$$I_t = A_{It}\left(\omega_I\hat{F}_1(A_{1t},T_t)^{\epsilon_I-1} + (1-\omega_I)\hat{F}_2(A_{2t},T_t)^{\epsilon_I-1}\right)^{\frac{1}{\epsilon_I-1}}\left(\frac{I_{1t}}{\hat{F}_1(A_{1t},T_t)} + \frac{I_{2t}}{\hat{F}_2(A_{2t},T_t)}\right) \tag{94}$$

Thus, the transformation ratio between two definitions of social cost of carbon is given by:

$$\begin{aligned}\frac{SCC_{CE}}{SCC_{IE}} &= \frac{\lambda_I}{\lambda_c} \\ &= \frac{F_{C1t}}{F_{I1t}} = \frac{F_{C2t}}{F_{I2t}} \\ &= \frac{\left[\omega_c \hat{F}_1(A_{1t}, T_t)^{\epsilon_c - 1} + (1-\omega_c)\hat{F}_2(A_{2t}, T_t)^{\epsilon_c - 1}\right]^{\frac{1}{\epsilon_c - 1}}}{A_{It}\left[\omega_I \hat{F}_1(A_{1t}, T_t)^{\epsilon_I - 1} + (1-\omega_I)\hat{F}_2(A_{2t}, T_t)^{\epsilon_I - 1}\right]^{\frac{1}{\epsilon_I - 1}}}\end{aligned} \tag{95}$$

Q.E.D.

Proof of Equation (57)

We directly present the social planner problem in a two sector model with consumption and investment as follows:

$$
\begin{aligned}
&\max \sum_{t=0}^{\infty} \beta^t U(C_t) \\
&+ \sum_{t=0}^{\infty} \beta^t \lambda_{Ct} \left[F_C(A_{Ct}, T_t, L_{Ct}, K_{Ct}, E_{Ct}) - C_t \right] \\
&+ \sum_{t=0}^{\infty} \beta^t \lambda_{It} \left[F_I(A_{It}, T_t, L_{It}, K_{It}, E_{It}) + (1-\delta)K_t - K_{t+1} - \Theta_t(\mu_t E_t) \right] \\
&+ \sum_{t=0}^{\infty} \beta^t \xi_t \left\{ T_t - \Phi[\boldsymbol{M}_0, (1-\mu_0)(E_{C0}+E_{I0}), ..., (1-\mu_t)(E_{Ct}+E_{It}), \boldsymbol{\eta}_0, ..., \boldsymbol{\eta}_t] \right\} \\
&+ \sum_{t=0}^{\infty} \beta^t \chi_{lt} \left[L_t - L_{Ct} - L_{It} - L_{Et} \right] \\
&+ \sum_{t=0}^{\infty} \beta^t \chi_{kt} \left[K_t - K_{Ct} - K_{It} - K_{Et} \right] \\
&+ \sum_{t=0}^{\infty} \beta^t \chi_{Et} \left[A_{Et} F_E(K_{Et}, L_{Et}) - E_{Ct} - E_{It} \right]
\end{aligned}
\tag{96}
$$

The F.O.C w.r.t. C_t is:

$$U_{Ct} = \lambda_{Ct}$$

The F.O.C w.r.t. K_{t+1} is:

$$\beta \lambda_{I,t+1}(1-\delta) + \beta \chi_{k,t+1} = \lambda_{It}$$

The F.O.C w.r.t. K_{Ct} is:

$$\lambda_{Ct} F_{Ckt} = \chi_{kt}$$

The F.O.C w.r.t. K_{It} is:

$$\lambda_{It} F_{Ikt} = \chi_{kt}$$

The F.O.C w.r.t. K_{Et} is:

$$\chi_{Et} F_{Ekt} = \chi_{kt}$$

The F.O.C w.r.t. L_{Ct} is:

$$\lambda_{Ct} F_{Clt} = \chi_{lt}$$

The F.O.C w.r.t. L_{It} is:

$$\lambda_{It} F_{Ilt} = \chi_{lt}$$

The F.O.C w.r.t. L_{Et} is:

$$\chi_{Et} F_{Elt} = \chi_{lt}$$

The F.O.C w.r.t. E_{Ct} is:

$$\lambda_{Ct} F_{CEt} - \chi_{Et} = \sum_{j=0}^{\infty} \beta^j \xi_{t+j} (1-\mu_t) \frac{\partial T_{t+j}}{\partial E_t^{unc}}$$

The F.O.C w.r.t. E_{It} is:

$$\lambda_{It} F_{IEt} - \lambda_{It} \mu_t \Theta^{'} - \chi_{Et} = \sum_{j=0}^{\infty} \beta^j \xi_{t+j} (1-\mu_t) \frac{\partial T_{t+j}}{\partial E_t^{unc}}$$

The F.O.C w.r.t. T_t is:

$$\lambda_{Ct} \frac{\partial Y_{Ct}}{\partial T_t} + \lambda_{It} \frac{\partial Y_{It}}{\partial T_t} + \xi_t = 0$$

The F.O.C w.r.t. μ_t is:

$$\lambda_{It} \Theta^{'} = \sum_{j=0}^{\infty} \beta^j \xi_{t+j} \frac{\partial T_{t+j}}{\partial E_t^{unc}}$$

Reformulating above equations yields:

$$\Theta^{'} = F_{IEt} - \frac{F_{Ilt}}{F_{Elt}} = (-1) \sum_{j=0}^{\infty} \beta^j \left(\frac{\lambda_{C,t+j}}{\lambda_{It}} \frac{\partial Y_{C,t+j}}{\partial T_{t+j}} + \frac{\lambda_{I,t+j}}{\lambda_{It}} \frac{\partial Y_{I,t+j}}{\partial T_{t+j}} \right) \frac{\partial T_{t+j}}{\partial E_t^{unc}} \tag{97}$$

where the government's discounting of output damages in period t is governed by:

$$\frac{\lambda_{It}}{\rho \lambda_{I,t+1}} = F_{Ik,t+1} + (1-\delta) \tag{98}$$

Q.E.D.

Proof of Equation (60)

The social planner problem with different abatement costs can be established as:

$$
\begin{aligned}
&\max \sum_{t=0}^{\infty} \beta^t U(C_t) \\
&+ \sum_{t=0}^{\infty} \beta^t \lambda_{1t} \left[F_1(A_{1t}, T_t, L_{1t}, K_{1t}, E_{1t}) - C_{1t} - I_{1t}\right] \\
&+ \sum_{t=0}^{\infty} \beta^t \lambda_{2t} \left[F_2(A_{2t}, T_t, L_{2t}, K_{2t}, E_{2t}) - C_{2t} - I_{2t}\right] \\
&+ \sum_{t=0}^{\infty} \beta^t \lambda_{Ct} \left[F_C(C_{1t}, C_{2t}) - C_t\right] \\
&+ \sum_{t=0}^{\infty} \beta^t \lambda_{It} \left[F_I(A_{It}, I_{1t}, I_{2t}) + (1-\delta)K_t - K_{t+1} - \Theta_{1t}(\mu_{1t}E_{1t}) - \Theta_{2t}(\mu_{2t}E_{2t})\right] \\
&+ \sum_{t=0}^{\infty} \beta^t \xi_t \left\{T_t - \Phi[\boldsymbol{M}_0, (1-\mu_{10})E_{10} + (1-\mu_{20})E_{20}, ..., (1-\mu_{1t})E_{1t} + (1-\mu_{2t})E_{2t}, \boldsymbol{\eta}_0, ..., \boldsymbol{\eta}_t]\right\} \\
&+ \sum_{t=0}^{\infty} \beta^t \chi_{lt} \left[L_t - L_{1t} - L_{2t} - L_{Et}\right] \\
&+ \sum_{t=0}^{\infty} \beta^t \chi_{kt} \left[K_t - K_{1t} - K_{2t} - K_{Et}\right] \\
&+ \sum_{t=0}^{\infty} \beta^t \chi_{Et} \left[A_{Et} F_E(K_{Et}, L_{Et}) - E_{1t} - E_{2t}\right]
\end{aligned}
\tag{99}
$$

The F.O.C w.r.t. C_{1t} is:

$$\lambda_{Ct} \frac{\partial C_t}{\partial C_{1t}} = \lambda_{1t}$$

The F.O.C w.r.t. C_{2t} is:

$$\lambda_{Ct} \frac{\partial C_t}{\partial C_{2t}} = \lambda_{2t}$$

The F.O.C w.r.t. C_t is:

$$U_{Ct} = \lambda_{Ct}$$

The F.O.C w.r.t. K_{t+1} is:

$$\beta\lambda_{I,t+1}(1-\delta)+\beta\chi_{k,t+1}=\lambda_{It}$$

The F.O.C w.r.t. K_{1t} is:

$$\lambda_{1t}F_{1kt}=\chi_{kt}$$

The F.O.C w.r.t. K_{2t} is:

$$\lambda_{2t}F_{2kt}=\chi_{kt}$$

The F.O.C w.r.t. K_{Et} is:

$$\chi_{Et}F_{Ekt}=\chi_{kt}$$

The F.O.C w.r.t. I_{1t} is:

$$\lambda_{It}F_{I1t}=\lambda_{1t}$$

The F.O.C w.r.t. I_{2t} is:

$$\lambda_{It}F_{I2t}=\lambda_{2t}$$

The F.O.C w.r.t. L_{1t} is:

$$\lambda_{1t}F_{1lt}=\chi_{lt}$$

The F.O.C w.r.t. L_{2t} is:

$$\lambda_{2t}F_{2lt}=\chi_{lt}$$

The F.O.C w.r.t. L_{Et} is:

$$\chi_{Et}F_{Elt}=\chi_{lt}$$

The F.O.C w.r.t. E_{1t} is:

$$\lambda_{1t}F_{1Et}-\lambda_{It}\mu_{1t}\Theta_{1t}^{'}-\chi_{Et}=\sum_{j=0}^{\infty}\beta^{j}\xi_{t+j}(1-\mu_{1t})\frac{\partial T_{t+j}}{\partial E_{t}^{unc}}$$

The F.O.C w.r.t. E_{2t} is:

$$\lambda_{2t}F_{2Et} - \lambda_{It}\mu_{2t}\Theta_{2t}^{'} - \chi_{Et} = \sum_{j=0}^{\infty}\beta^j\xi_{t+j}(1-\mu_{2t})\frac{\partial T_{t+j}}{\partial E_t^{unc}}$$

The F.O.C w.r.t. T_t is:

$$\lambda_{1t}\frac{\partial Y_{1t}}{\partial T_t} + \lambda_{2t}\frac{\partial Y_{2t}}{\partial T_t} + \xi_t = 0$$

The F.O.C w.r.t. μ_{1t} is:

$$\lambda_{It}\Theta_{1t}^{'} = \sum_{j=0}^{\infty}\beta^j\xi_{t+j}\frac{\partial T_{t+j}}{\partial E_t^{unc}}$$

The F.O.C w.r.t. μ_{2t} is:

$$\lambda_{It}\Theta_{2t}^{'} = \sum_{j=0}^{\infty}\beta^j\xi_{t+j}\frac{\partial T_{t+j}}{\partial E_t^{unc}}$$

Rearranging above equations yields:

$$\begin{aligned}
\Theta_{1t}^{'} = \Theta_{2t}^{'} &= F_{I1t}F_{1Et} - \frac{F_{I1t}F_{1lt}}{F_{Elt}} = F_{I2t}F_{2Et} - \frac{F_{I2t}F_{2lt}}{F_{Elt}} \\
&= (-1)\sum_{j=0}^{\infty}\beta^j\left\{\frac{\lambda_{1,t+j}}{\lambda_{It}}\frac{\partial Y_{1,t+j}}{\partial T_{t+j}} + \frac{\lambda_{2,t+j}}{\lambda_{It}}\frac{\partial Y_{2,t+j}}{\partial T_{t+j}}\right\} \\
&= (-1)\sum_{j=0}^{\infty}\beta^j\left(\underbrace{\frac{U_{C,t+j}}{\lambda_{It}}\frac{\partial C_{t+j}}{\partial C_{1,t+j}}\frac{\partial Y_{1,t+j}}{\partial T_{t+j}}\frac{\partial T_{t+j}}{\partial E_t^{unc}}}_{Impacts\quad on\quad sector\quad 1} + \underbrace{\frac{U_{C,t+j}}{\lambda_{It}}\frac{\partial C_{t+j}}{\partial C_{2,t+j}}\frac{\partial Y_{2,t+j}}{\partial T_{t+j}}\frac{\partial T_{t+j}}{\partial E_t^{unc}}}_{Impacts\quad on\quad sector\quad 2}\right)
\end{aligned} \tag{100}$$

where the government's discounting of output damages in period t is governed by:

$$\frac{\lambda_{It}}{\beta\lambda_{I,t+1}} = F_{I1,t+1}F_{1k,t+1} + (1-\delta) = F_{I2,t+1}F_{2k,t+1} + (1-\delta) \tag{101}$$

Q.E.D.