EECS 182 Deep Neural Networks
Fall 2025 Anant Sahai and Gireeja Ranade

Discussion 4

1. Maximal Update Parameterization During Training

In this problem, we will recover the Maximum Update Parametrication scaling during training. During training, updates to weights are very much correlated with the inputs (unlike during initialization).

Assume we are using the SignGD optimizer (which is a simplified version of Adam) with minibatch size 1. For simplicity, consider a neural network layer with input $\mathbf{x} \in \mathbb{R}^{d_{\text{in}}}$ that is sampled from an i.i.d. unit Gaussian, and weights $W \in \mathbb{R}^{d_{\text{out}} \times d_{\text{in}}}$.

(a) First, compute the gradient $\nabla_W \mathcal{L}(\mathbf{y})$ for $\mathbf{y} = W\mathbf{x} + \mathbf{b}$ and loss function $\mathcal{L}$. Your answer should be in terms of $\mathbf{x}_i$ and $\mathbf{g} = \nabla_{\mathbf{y}} \mathcal{L}(\mathbf{y})$.

(b) In SignGD, we know that W is updated as below:

$$W_{t+1} \leftarrow W_t + \eta \,\text{sign}\left(\nabla_W \mathcal{L}(\mathbf{y})\right).$$

What is the expected RMS norm squared of the change in features $\Delta\mathbf{y} = \eta\,\textbf{sign}(\nabla_W \mathcal{L}(\mathbf{y}))\mathbf{x}_i$? How does this scale with d_{out} or d_{in}? What constant should we multiply the update by to ensure that the expected RMS norm squared of $\Delta\mathbf{y}$ does not depend on either d_{out} or d_{in}?

2. Understanding Newton-Schulz

Let us consider a parameter matrix $W \in \mathbb{R}^{d_{\text{out}} \times d_{\text{in}}}$. Define the degree-3 odd polynomial p as:

$$p(W) = \frac{1}{2}\left(3I_{d_{\text{out}}} - WW^T\right)W.$$

In this problem, we will study how the iteration $W_{k+1} = p(W_k)$ affects the singular values of W_k.

(a) **Show that the iteration acts only on the singular values of W.** i.e. if $W = U\Sigma V^T$ is the SVD, then show that

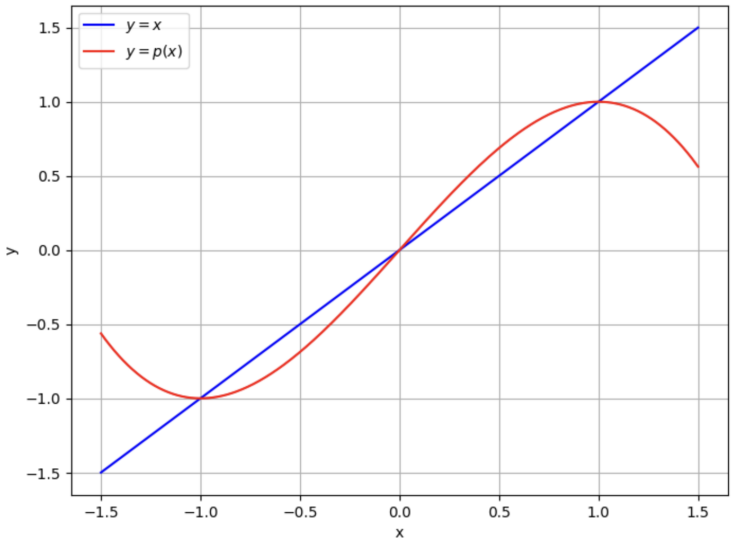
$$p(W) = Up(\Sigma)V^T.$$

Hint: First show that $WW^T = U(\Sigma\Sigma^T)U^T$.

(b) Write down the fixed point equation for $p(x) = \frac{3}{2}x - \frac{1}{2}x^3$. **Solve for all fixed points**, i.e. x^* such that $x^* = p(x^*)$.

(c) We define a fixed point x^* of $p(x)$ as *locally stable* if $|\frac{d}{dx}p(x^*)| < 1$. First, convince yourself that a stable fixed point means that the distance towards the fixed point decreases with more iterations. **Determine which fixed points of $p(x)$ are stable and which are unstable.**

(d) Below are plots of $y = p(x)$ and $y = x$. Pick different starting points for x and show graphically how iteration $x = p(x)$ eventually converges to a stable fixed point.
Use "cobweb diagram" to show how x evolves over time.



(e) Suppose the singular value starts as $+\sigma$. **For which values of σ does it converge to $+1$? What does it do for other values?**

(f) **Explain why this iteration can be viewed as an approximate way to make W closer to an orthogonal matrix** (with singular values near ± 1) and what we must ensure before using the iterations.

Contributors:

- Kevin Frans.
- Anant Sahai.
- Joey Hong.

