Résultat Gaussian-GARCH-HN:

Estimation	Gaussian-GARCH-HN						
Parameters	M^{Ess} Ret	M ^{Ess} Ret-opt	M ^{Ess} Ret-VIX	M ^{Qua} Rett-opt	M ^{Qua} Ret-VIX		
a_0	$2.176 e^{-11}$	$1.874 e^{-04}$	$3.285 e^{-05}$	$4.454 e^{-04}$	$3.779 e^{-08}$		
	$(0.003 e^{-05})$	$(0.045 e^{-07})$	$(0.156 e^{-09})$	$(0.078 e^{-06})$	$(0.014 e^{-04})$		
a_1	$3.285 e^{-04}$	$3.345 e^{-04}$	$3.600 e^{-04}$	$3.257 e^{-05}$	$1.279 e^{-0.5}$		
	$(0.782 e^{-07})$	$(0.891 e^{-07})$	$(1.264 e^{-07})$	$(2.451 e^{-07})$	$(1.564 e^{-06})$		
a_2	_	_	_				
	_	_	_	_	_		
b_1	$6.255 e^{-01}$	$1.124 e^{-03}$	$2.510 e^{-01}$	$1.123 e^{-03}$	$9.221~e^{-01}$		
	$(0.213 e^{-03})$	$(0.739 e^{-03})$	$(0.049 e^{-03})$	$(0.077 e^{-03})$	$(0.037 e^{-03})$		
γ	$8.622 e^{+00}$	$1.142 e^{+00}$	$9.258 e^{+00}$	$8.424 e^{+00}$	$9.750 e^{+00}$		
	$(0.048 e^{-08})$	$(0.785 e^{-08})$	$(0.028 e^{-08})$	$(0.001 e^{-07})$	$(0.001 e^{-06})$		
λ_0	$2.300 \mathrm{e}^{-03}$	$6.573 e^{-01}$	$1.353 e^{-06}$	$6.573 e^{-01}$	$8.882 e^{-07}$		
	$(0.007 e^{-02})$	$(0.004 e^{-02})$	$(0.002 e^{-02})$	$(0.001 e^{-03})$	$(0.008 e^{-02})$		
γ^*	_	_	_	_			
	_	_	_	_	_		
π	_	_	_	$1.382 e^{+00}$	$1.356 e^{+00}$		
	_	_	_	(0.00009)	(0.00204)		
ϱ	_	_	0.9718	_	0.843		
	_	_	(0.00001)	_	(0.00001)		
Times (h)	0.0034	17.453	0.00191	27.802	0.0016		
Model Properties :			_	_	_		
Persistence	_	_	_	_	_		
Log-likelihood	3601.888	6157.784	5391.126	6375.122	4508.567		
Pricing performances :			_	_	_		
in-IVRMSE	0.06380	0.05609	0.05737	0.05231	0.05397		
out-IVRMSE	0.08184	0.07343	0.07463	0.06875	0.07031		
Predictibility of VIX:			_	_	_		
MPE_{VIX}	0.98350	0.87433	0.81234	0.81472	0.77446		
MAE_{VIX}	0.99918	0.90143	0.83630	0.82288	0.74113		
$RMSE_{VIX}$	1.19059	1.22255	1.00021	1.12682	0.96737		

 $Table \ 1: Summary \ fits \ of \ Joint \ MLE-estimation \ procedure \ with \ Returns \ data, Option-Returns \ data \ or \ VIX-Returns \ data \ in \ of \ sample \ 2009-2010.$

Résultat Gaussian-GARCH-GJR:

Estimation	Gaussian-GARCH-GJR			
Parameters	M^{Ess} Ret	M^{Ess} Ret-VIX		
a_0	$1.445 e^{-05}$	$4.966 e^{-06}$		
	(0.00075)	(0.00008)		
$ a_1 $	$3.107e^{-01}$	$1.240 \ e^{-02}$		
	(0.00450)	(0.00120)		
a_2	$1.055 e^{-01}$	$2.314 e^{-02}$		
	(0.00001)	(0.00001)		
b_1	$6.311 e^{-01}$	$8.504 e^{-01}$		
	(0.00001)	(0.00001)		
γ	_	_		
	_	_		
λ_0	$4.208 e^{-03}$	$1.989 e^{-01}$		
	(0.00481)	(0.00261)		
γ^*	_	_		
	_	_		
ϱ	_	0.8924		
	_	(0.01179)		
Times (h)	0.0018	0.0066		
Model Properties :				
Persistence	_	_		
Annualized volatility	_	_		
Leverage coefficient	_	_		
Log-likelihood	5284.952	6075.361		
Pricing performances :				
IVRMSE in sample	0.061832	0.052961		
IVRMSE out of sample	0.069847	0.064482		
Predictibility of VIX::				
MPE_{VIX}	0.860545	0.654981		
MAE_{VIX}	0.980997	0.639153		
$RMSE_{VIX}$	1.001299	0.803589		

 $Table\ 2:\ Summary\ fits\ of\ Joint\ MLE-estimation\ procedure\ of\ historical\ parameters\ with\ Option-Returns\ data\ or\ VIX-Returns\ data\ in\ of\ sample\ 2009-2010.$

Résultat IG-GARCH:

Table 3: Estimated parameters for the IG model and the two stochastic discount factors.

Joint-Estimation	Returns	Returns-Option		Returns-VIX	
Model		M_t^{ess}	M_t^{Ushp}	M_t^{ess}	M_t^{Ushp}
Parameters :				•	
W	$9.8322~e^{-06}$	$9.9095~e^{-06}$	$9.9109~e^{-06}$	$1.0166 e^{-06}$	$9.8762~e^{-06}$
Stand.Dev	(0.00485)	(0.00452)	(0.00301)	(0.00248)	(0.00752)
b	$1.2158 \ e^{-03}$	$1.4019 \; e^{-03}$	$2.0123 e^{-03}$	$2.0418 e^{-03}$	$8.6267 e^{-03}$
Stand.Dev	(0.00001)	(0.00001)	(0.00015)	(0.00045)	(0.00003)
c	$4.5438 e^{-0.5}$	$5.1407 e^{-0.5}$	$4.7859 e^{-05}$	$4.5018 e^{-05}$	$4.4913 e^{-05}$
Stand.Dev	(0.00428)	(0.00035)	(0.00025)	(0.00002)	(0.00001)
a	$3.3174 e^{+03}$	$3.0174 e^{+03}$	$3.3174 e^{+02}$	$3.3175 e^{+03}$	$3.3174 e^{+03}$
Stand.Dev	(0.00004)	(0.01520)	(0.03471)	(0.01450)	(0.00001)
η	$-7.5313 e^{-03}$	$-9.0121 e^{-03}$	$-8.176 e^{-03}$	$-7.465 e^{-03}$	$-7.516 e^{-03}$
Stand.Dev	(0.00150)	(0.00457)	(0.00145)	(0.00365)	(0.00096)
ν	$1.2596 \ e^{+02}$	$1.2592 e^{+02}$	$1.2584~e^{+02}$	$1.2588 \ e^{+02}$	$1.2583 \ e^{+02}$
Stand.Dev	(0.00001)	(0.00001)	(0.00001)	(0.00001)	(0.00001)
π	_	_	1.1305	_	1.2971
Stand.Dev	_	_	(0.00072)	_	(0.00108)
ϱ		_	_	$9.9455 e^{-01}$	$9.9625 e^{-01}$
Stand.Dev	_	_	_	(0.00150)	(0.00206)
Model Properties:					
Times (h)	0.0097	19.5574	20.4178	0.0028	0.0041
Persistence	_	_	_	_	_
Annualized volatility	_	_	_	_	_
Leverage coefficient	_	_	_	_	_
Log-likelihood	3873.447	8919.914	8930.341	4451.978	4625.001
Pricing performances:					
IVRMSE in sample	0.054358	0.046160	0.043546	0.046483	0.043872
IVRMSE out of sample	0.067427	0.061058	0.056641	0.061834	0.057568
Predictibility of VIX:					
MPE_{VIX}	0.9851	0.7248	0.7187	0.6709	0.6392
MAE_{VIX}	1.0967	0.7408	0.7293	0.6823	0.6505
$RMSE_{VIX}$	1.2023	0.8682	0.8593	0.8210	0.7846