Quantitative Macroeconomics

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Question 1. Computing Transitions in a Representative Agent Economy

Consider the following closed optimal growth economy populated by a large number of identical infinitely lived households that maximize:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\}, \tag{1}$$

over consumption and leisure $u(c_t) = \ln c_t$, subject to:

$$c_t + i_t = y_t \tag{2}$$

$$y_t = k_t^{1-\theta} \ (z \ h_t)^{\theta} \tag{3}$$

$$i_t = k_{t+1} - (1-\delta) \ k_t \tag{4}$$

Set labor share to θ=.67. Also, to start with, set h_t=.31 for all t. Population does not grow.

(a) Compute the steady-state. Choose z to match an annual capital-output ratio of 4, and an investment-output ratio of .25.

(b) Double permanently the productiviy parameter z and solve for the new steady state.

(c) Compute the transition from the first to the second steady state and report the time-path for savings, consumption, labor and output.

(d) Unexpected shocks. Let the agents believe productivity z_t doubles once and for all periods. However, after 10 periods, surprise the economy by cutting the productivity z_t back to its original value. Compute the transition for savings, consumption, labor and output.

(e) Bonus Question: Can taxes explain differences in the speed of transition to steady-state?

- Add a permanent consumption tax. Recompute the new steady state, and the transitions.
- Add a permanent capital tax. Recompute the new steady state, and the transitions.

(f) Bonus Question: Boldrin, Christiano and Fisher (AER, 2001) and Christiano (Minn QR, 1989)

- What if preferences take the form of Boldrin, Christiano and Fisher (AER, 2001)? That is, abstracting from labor choice,

$$u(c) = \ln(c_t - bc_{t-1}). \tag{5}$$

Recompute the transition as posed in Question 1.

- What if preferences take the form of Christiano (Minn QR, 1989)? That is, abstracting from growth,

$$u(c) = \ln(c_t - \bar{c}) \tag{6}$$

Recompute the transition as posed in Question 1. Plot the differences in the time path of savings.

- Now, allow for growth, i.e., $z_t = z_0(1+\lambda_z)^t$, and replicate Christiano's Chart 1-4 for Japan, and extend the exercise to as many countries as you can (e.g. China, Taiwan, Korea, South Africa and Zambia). Get historical data for the U.K. (as long time series as you can), and replicate those Charts.

(g) Bonus Question: Labor Choice Allow for elastic labor supply. That is, let preferences be

$$u(c_t, 1-h_t) = \ln c_t - \kappa \frac{h_t^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} \tag{7}$$

and recompute the transition as posed in Question 1.

Question 2. General Equilibrium with Labor Supply, Uncertainty, and Progressive Labor Income Tax

Consider a 2-period economy with 400 agents heterogeneous in their their initial wealth, y_0 and their permanent productivity η_y. Agents save (or borrow) at the risk-free interest rate r and consume in the first period. In the second period, agents receive income from their return on savings (or pay the debt) and consume. Labor income is endogenous because agents choose how much to work for given wages in both periods. Labor is elastic. Today's wages depend on a common component (w) and on individual permanent productivity η_y, and tomorrow's wages depend on a common component (w), on individual permanent component η_y, and on individual transitory shock ε_y with $E(\varepsilon_y) = 0$. Agents receive a positive shock ε_y with probability one half, and a negative shock $-\varepsilon_y$ with the complementary probability. That is, there are three sources of heterogeneity in this model: agents can be different because they are born different (initial wealth y_0 and permanent productivity η_y) and/or because what happens to them over the life cycle (income shocks ε_y).

This way, agents solve the following problem:

$$V(y_0, \eta; \tau) = \max_{c\geq 0, c'\geq 0, a, h, h'} \quad u(c,h) + \beta E u(c', h')$$

subject to

$$\begin{aligned} c + a &= (1-\tau)wh + y_0 + T_1 \\ c' &= (1-\tau)w'h' + (1+r)a + T_2 \end{aligned}$$

with wage income process given by:

$$\begin{aligned} w &= \eta_y \\ w' &= \eta_y + \varepsilon_y \end{aligned}$$

Table 1: Parameter values

Parameter	Value	Description
β	0.99	Discount factor
η_y	[1, 1.5, 2.5, 3]	Permanent worker productivity
ε_y	0.05	Workers' productivity shock

and borrowing constraint:

$$a \geq -\frac{1}{1+r}\left(y_1 - \varepsilon_y\right).$$

The last inequality is the natural borrowing constraint. Note that today's wages are given by permanent productivity $w = \eta_y$ and wages tomorrow are given by a permanent component and a transitory component $w' = \eta_y + \varepsilon_y$. Taxes τ are proportional labor income taxes. All households receive a lump-sum transfer in the first and in the second period T_1 and T_2 that exactly exhausts the government revenues from the collected labor income tax.

Assume the following parameter values:

Note that to solve for the GE we need a distribution of initial endowment/wealth and permanent productivity $\Psi(y_0, \eta_y)$. We assume y_0 is uniformly distributed conditional on permanent productivity η_y. Specifically, for each η we assume that initial wealth is computed with the following two steps: First, compute a uniformly distributed initial wealth across the population, $y_0 \sim U[0.001, 0.009]$ with 100 points (which implies a total of 400 households for this economy). Second, replace the initial wealth of agents with $y_0 \in [0.0055, 0.0087]$ by $y_0 = 0.001$. This implies that the correlation between initial wealth and permanent productivity is zero. This will artificially create a bulk of more initially poor households and a small elite of initially rich households for each η_y.

Preferences can take this shapes:

$$u(c, h) = \frac{c^{1-\sigma}}{1-\sigma} - \kappa \frac{h^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}$$

with $\sigma = 3.00$, $\kappa = 4.00$, and $\nu = 4.00$.

(a) The case without taxes, $\tau = T_1 = T_2 = 0$. Let's start solving this economy without taxes. That is, solve for the GE economy assuming that $\tau = T_1 = T_2 = 0$. Use both CRRA and quadratic preferences defined above.

- Define the general equilibrium (GE) of this economy and provide a solution algorithm to compute it.
- Plot the optimal solutions as functions of initial wealth and discuss your results. To ease the exposition, plot the following Figures (note that each subfigure may have different panels or subfigures):
  - (Figure 1) Optimal savings a (panel (a)), consumption c (panel (b)), and consumption c' (panel (c)) as functions of the initial wealth and the permanent productivity.

To do this plot the endogenous variable of interest in the vertical axis and y_0 in the horizontal axis. For a and c there should be one line per η_y and for c' there should be one line for each pair (η_y, ε_y).

· (Figure 2) Savings rate (i.e., $a/(y_0 + wh(1-\tau))$. Do this separately for each value of η_y.

· (Figure 3) Plot optimal hours worked today h (panel (a)) as a function of initial wealth y_0 and η_y in the same way that you plotted a or c. In the same graph, plot optimal hours worked tomorrow h' (panel(b)) as function of initial wealth y_0, η_y and ε_y as you did for c'.

· (Figure 4) Plot before-tax labor income today wh (panel (a)) as a function of initial wealth y_0 and η_y in the same way that you plotted a or c. In the same fashion, plot today's after-tax labor share as $(1-\tau)wh/((1-\tau)wh + y_0 + T_1)$, panel (b). In the same graph, plot optimal labor income tomorrow h' (panel(c)) as function of initial wealth y_0, η_y and ε_y as you did for c'. In the same fashion, plot tomorrow's after-tax labor share as $(1-\tau)w'h'/((1-\tau)w'h' + (1+r)a + T_2)$, panel (d).

· (Figure 5) Consumption growth (panel (a)) and income growth (panel (b)). Note that consumption is $g_c = g_c = \frac{c'-c}{c}$, which depends on η_y and on the value of transitory shocks. Plot in the same panel the expected value of consumption growth $E[g_c]$ which is a function of η. Note that income growth is $g_{wh} = \frac{w'h'-wh}{wh}$, where wh is labor income for workers. Note that this growth rate depends on η_y and on the value of transitory shocks. Plot in the same panel the expected value of income growth $E[g_{wh}]$. Plot also the elasticity of expected consumption growth with respect to the expected income growth $E(g_c)/E(g_{wh})$ (panel (c)). There is one such ratio per η_y. Finally, plot the ratio between actual elasticity against the expected one, that is, plot $(g_c/g_y)/(E(g_c)/E(g_y))$ (panel (d)).

· (Figure 6) Capital market clearing. In the vertical axis show aggregate supply, $A = \sum_i a$, aggregate demand, K, and aggregate excess of demand $K - A$, and in the horizontal axis show values of the interest rate in an interval that contains the equilibrium interest rate.

· (Figure 7) Welfare V as a function of initial wealth.

- Plot aggregate savings, $A = \sum_i a$ if $a \geq 0$, and aggregate debt, $D = \sum_i a$ if $a < 0$, for values of the interest rate in an interval that contains the equilibrium interest rate. Discuss.
- How does the policy function a differ across this model and previous models without labor supply? (ii) Did aggregate savings increase with labor supply? (iii) Is the interest higher or lower with labor supply? Discuss your results.

(b) The case with flat-rate taxes, $\tau = 0.115$ and equally redistributed transfers $T \neq 0$ This implies that you need to solve the associated GE economy (i.e., find the interest rate and transfers that clear the markets).

- Redo items (a)-(c) above, now with flat-rate labor income taxes.

(c) The case with progressive taxes with a Heathcote-Storesletten-Violante tax function with $\theta = 0.18$ and $\lambda = 0.15$ equally redistributed transfers $T \neq 0$. This implies that you need to solve the associated GE economy (i.e., find the interest rate and transfers that clear the markets).

- Redo items (a)-(c) above, now with progressive labor income taxes.

(d) Re-do Question 3 with (a) homogeneous initial wealth and (b) pareto initial wealth distribution.