Quant Macro (Part 2) PS3 - Quantitative Model of Sovereign Debt

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Abstract

ref: Luis's lecture slides and "Sovereign Debt" by Mark Aguiar and Manuel Amador.

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1 Model Set up

Assumptions:

- Small open economy with a stochastic endowment yt, which follows a Markov process.
- The government preferences are given by $E_0 \sum \beta^t u(c_t)$
- the govt. trades one-period bonds with risk neutral investors.
- timing within a period t is:

1. Outstanding debt b_t and the gdp realization y_t.
2. Decision to repay of default.
3. If not in default, new issuance decision b_{t+1} at price:

$$q(b_{t+1}, y_t)$$

4. If there is default the country is excluded from financial markets. With probability λ can return
5. The economy suffers a loss of τ fraction of the endowment during default

1.1 Problem of government

$$V(b,y) = \max_{D\in[0,1]} \left[(1-D)V^{ND}(b,y) + DV^{D}(y)\right] \tag{1}$$

where (2)

$$V^{ND}(b,y) = \max_{b'} \left[u(y + q(b',y)b' - b) + \beta E[V(b',y')|y]\right] \tag{3}$$

$$V^{D}(b,y) = u((1-\tau)y + (1-\lambda)\beta E[V^{D}(y')|y] + \lambda\beta E[V^{D}(0,y')|y] \tag{4}$$

1.2 Creditors

break-even condition is (uncertainty only comes from y'):

$$q(b',y) = \frac{E[1 - D(b',y')|y]}{R}$$

1.3 Equilibrium Computation

Since in Equilibrium: $q(b',y) = \frac{E[1-I[V^{ND}(b',y';q)\leq V^{D}(y',q)|y]}{R}$

1. guess q^0, no default guess $q^0(b',y) = 1/R$
2. find policy functions of gov't
3. For every combination of (b',y) compute price:

$$\hat{q}(b',y) = \frac{E[1 - D(b',y';q)|y]}{R}$$

4. update q with:

$$q^1(b',y) = a\hat{q}(b',y) + (1-a)q^0(b',y)$$

5. Iterate until convergence.

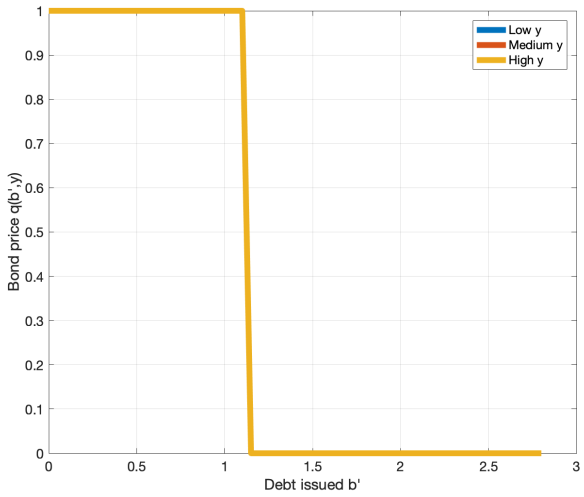
2 Q1: compute the bond-price

See first few lines in matlab file: Sov_debt_model_new_m.m

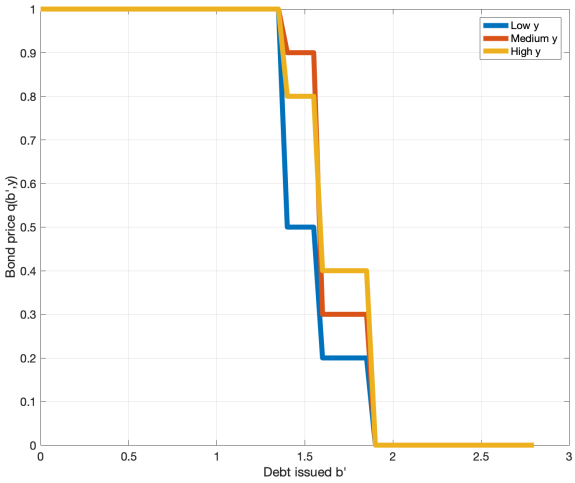
3 Q2: Is there default along the equilibrium path?

Figure 1: Bond-price menu

(a) For different y fix a same $\tau = 0.2$



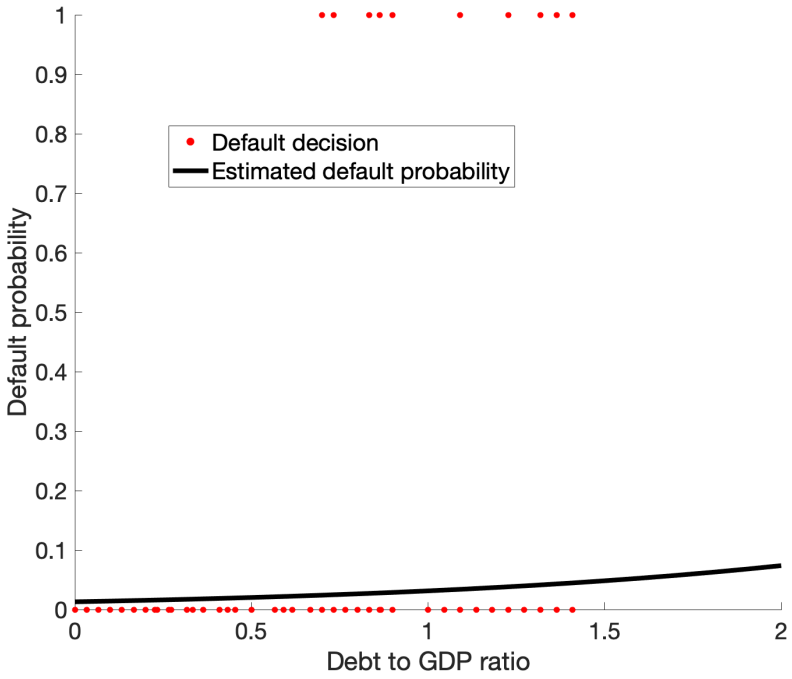
(b) For different y, $\tau = [0.1, 0.2, 0.5]$



(a) By adjusting a little bit the parameter values, I obtain that the country is 3.04% of the time in default. I didn't use simulation since all parameters including the 3 by 3 transition need to vary, and this is too large, so I use guess and try. (relative parameters values are kept in the main file).

(b)

Figure 2: the probability of default at $t+1$ conditional on the debt-to-GDP ratio b_{t+1}/y_t.



4 Q3. Using the data in default_IDEA.dta (Stata file)

Table 1: Logit Regression Table

	(1) default
default	
debt_to_gdp	1.724***
	(9.98)
Constant	−2.200***
	(−17.75)
Observations	2223

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

To calibrate our parameters to make the coefficients of Logit regression as close as possible to the real Data, I tried with different combination of $\beta \in [0.6, 0.96]$ and $\tau \in [[0.01, 0.2], [0.3, 0.4], [0.5, 0.6]]$. The closest results I've simulated are the following:

$$\beta = 0.9052, \tau = [0.0362, 0.2276, 0.4276],$$

with true data regression coefficients: $[-2.199848^{***} 1.72411^{***}]$, and simulated coefficients are $[-4.4083, 1.5853]$. The correspond matlab file is Q3.m