PS4

Weimin Zhou

Due: 17/Oct/2018

Question 2. Business Cycle Fluctuations [PS4Q2.m]: contains all codes of this part.

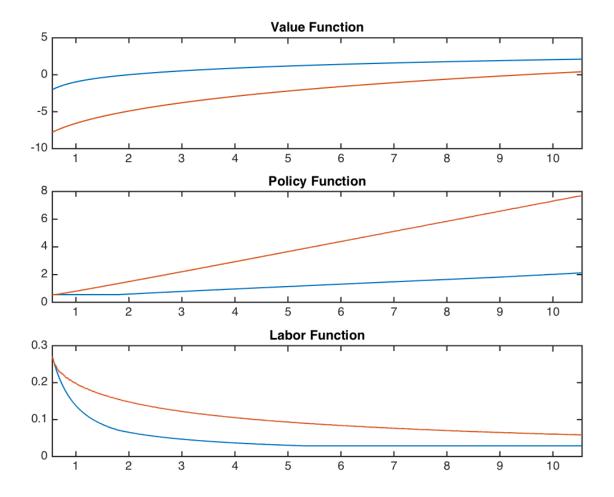
1. Add productivity shocks which follow a two-states Markov chain where the two values of the shock are 1.01 and 1/1.01. Set the unconditional mean of zt to one.

Value Function Iterations for the deterministic growth model are completed.

Elapsed time is 8.321668 seconds.

Number of iterations:

69



For plotting above figures, I combined Monotonicity, Concavity conditions and Howard's Improvements all together, expecting a higher speed.

I used the trick that setting transition function more diverse, and we could see the mean is still close to one, but the graph can give difference between two different shocks by above figures. A larger productivity shock achieved a high value function and higher policy function g(k) and higher working hours.

logged variance of some variables in the model for good shock

output capital hours consumption investment 0.0549 0.1884 0.2518 0.5505 0.9281

logged variance of some variables in the model for bad shock

output capital hours consumption investment 0.0012 0.4551 0.1332 0.2703 0.7347

HP filter variance of some variables in the model for good shock

output capital hours consumption investment 1.0e-05 *

0.0086 0.8036 0.0069 0.7780 0.8414

HP filter variance of some variables in the model for bad shock

output capital hours consumption investment 1.0e-04~* $0.0096 \quad 0.2547 \quad 0.0035 \quad 0.1955 \quad 0.2639$

ARIMA(1,0,0) Model for computing Persistence of Output

Conditional Probability Distribution: Gaussian

| | Standard | t | |
|-----------|-------------|-------------|-----------|
| Parameter | Value | Error | Statistic |
| | | | |
| Constant | 0.000565529 | 1.09595e-05 | 51.6018 |
| AR{1} | 0.982764 | 7.0642e-05 | 13911.9 |
| Variance | 5.14532e-08 | 2.55632e-08 | 2.01278 |

Persistence of some variables in the model for good shock

output capital hours consumption investment [0.9834] [1.0000] [0.9828] [1.0000] [0.9998]

Persistence of some variables in the model for bad shock

| output | capital | hours | consumption | investment |
|----------|----------|--------|-------------|------------|
| [0.9861] | [1.0000] | [0.992 | [0.9996] | [0.9996] |

Correlation Coeffcients Table for Good Shocks

| output | capital | hours | consumption | investment |
|---------|----------------|---------|-------------|------------|
| 1.0000 | <u>-0.4931</u> | 0.9887 | -0.5484 | 0.5599 |
| -0.4931 | 1.0000 | -0.6075 | 0.9972 | -0.9962 |
| 0.9887 | -0.6075 | 1.0000 | -0.6581 | 0.6684 |
| -0.5484 | 0.9972 | -0.6581 | 1.0000 | -0.9999 |
| 0.5599 | -0.9962 | 0.6684 | -0.9999 | 1.0000 |

Correlation Coeffcients Table for Bad Shocks

| output | capital | hours | consumption | investment |
|---------|---------|---------|-------------|------------|
| 1.0000 | -0.9753 | 0.8453 | -0.9726 | 0.9734 |
| -0.9753 | 1.0000 | -0.9002 | 0.9993 | -0.9994 |
| 0.8453 | -0.9002 | 1.0000 | -0.9151 | 0.9144 |
| -0.9726 | 0.9993 | -0.9151 | 1.0000 | -1.0000 |
| 0.9734 | -0.9994 | 0.9144 | -1.0000 | 1.0000 |

Variance Decomposition of output (Check Comovement) for Good Shocks

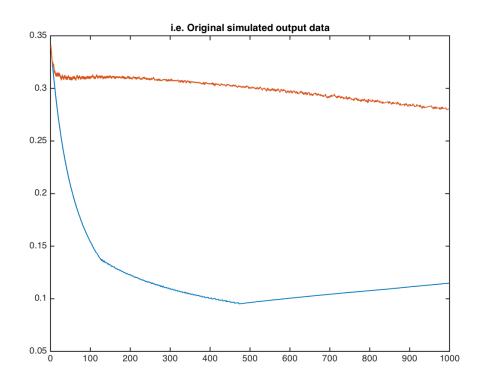
| Var(capital) | 0.0174 | | |
|------------------|---------------------|--|--|
| Var(labor-hours) | 0.0004 | | |
| Var(consumption) | <mark>0.4763</mark> | | |
| Var(investment) | <mark>0.5059</mark> | | |

Variance Decomposition of output (Check Comovement) for Bad Shocks

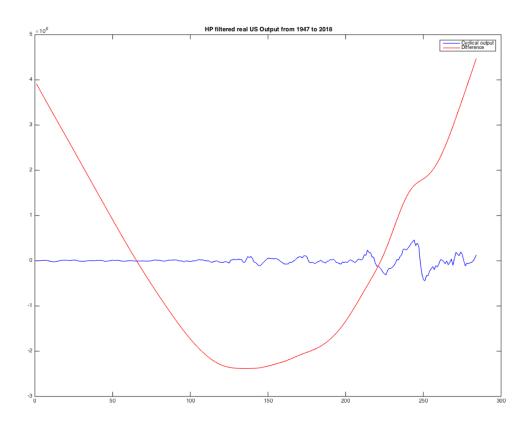
| Var(capital) | 0. <mark>8168</mark> |
|------------------|----------------------|
| Var(labor-hours) | 0.0003 |
| Var(consumption) | 0. 0901 |

I downloaded the US.data and just for a rough hint due to time limit, I checked output and investment, consumption:

| logged | logged variance of some variables of US data | | | | | | |
|--------|--|------------|--|--|--|--|--|
| output | consumption | investment | | | | | |
| 1.9337 | 2.0406 | 2.0105 | | | | | |



HP filtered output data of US.



| HP filter v | HP filter variance of some variables in US data | | | | | |
|-------------|---|-------------|------------|--|--|--|
| output | | consumption | investment | | | |
| | | 1.0e+10 * | | | | |
| | 1.1355 | 0.3760 | 0.7477 | | | |

| Persisten | Persistence of some variables in US data | | | | |
|-----------|--|-----------|------------|--|--|
| output | co | nsumption | investment | | |
| [1.000 | [00 | [1.0000] | [1.0000] | | |

Since I include the 1947 to 2018 quarterly data, persistence will be of course very large, since I didn't make any manipulation for such a long time-series data.

Correlation Coeffcients Table of US data

| Output | Consumption | Investment |
|--------|-------------|------------|
| 1.0000 | 0.9997 | 0.9915 |
| 0.9997 | 1.0000 | 0.9899 |
| 0.9915 | 0.9899 | 1.0000 |

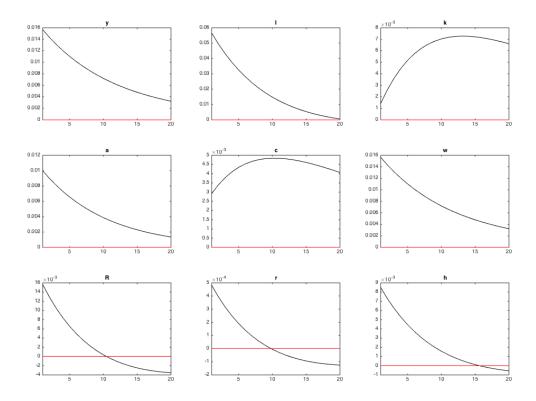
Variance Decomposition of output (Check Comovement) in US data

| Var(Consumption) | 0.9409 |
|------------------|--------|
| Var(Investment) | 0.0591 |

Remarks:

The basic benchmark model of RBC cannot capture as many features in the US data. Since in the RBC basic model, agents are perfect forward-looking, and hence smooth their consumption and investment till a steady states, which could be proved there's a unique equilibrium for rational agents. However, in the US data, fluctuations are happened as always. Hence, only persistence matches since it's a long time series data.

3. Impulse Response Function of output, consumption, investment, capital, hourse to a productivity shock.



What we can tell from the IMF is just the results of benchmark RBC with endogenous labor supply (elastically), which denotes a sensitive behavior of investment and interest rate and hours of working, hence, the capital stock is increasing on the other side.

For a deeper understand of why we are required to do logged and HP-filtered data and look at the Impulse Response Function of output, consumption, labor w.r.t Productivity shocks. By browsing: Dongya Koh & Raül Santaeulàlia-Llopis, 2017. "Countercyclical Elasticity of Substitution," Working Papers 946, Barcelona Graduate School of Economics.

Since my simulated data from benchmark RBC cannot exhibit such great results, hence, to look at the labor market countercyclical phenomenon, I attached a few results from this paper just to compare:

Table 5: Cyclical Behavior of NCES(SR)-CD(LR) Business Cycle Model and U.S.1954.I-2012.III

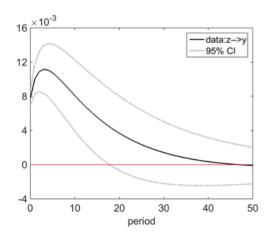
| | U.S. Data | | | NCES(SR)-CD(LR) | | |
|------------------------------------|---------------|--------------|--------------|-----------------|---------------|--------------|
| | v_x | ho(y,x) | ho(x,x') | v_x | $\rho(y,x)$ | $\rho(x,x')$ |
| Outroots | | | | | | |
| Output: | 0.01 | 1.00 | 0.07 | 0.00 | 1.00 | 0.70 |
| y | 3.31 | 1.00 | 0.87 | 3.28 | 1.00 | 0.72 |
| Labor Market: | | | | | | |
| eh | 3.64 | 0.88 | 0.91 | 3.32 | 0.91 | 0.72 |
| e | 2.51 | 0.82 | 0.93 | 3.04 | 0.91 | 0.72 |
| h | 0.28 | 0.73 | 0.82 | 0.03 | 0.51 | 0.73 |
| w | 0.84 | -0.13 | 0.75 | 0.63 | -0.14 | 0.75 |
| lp | 0.81 | 0.15 | 0.73 | 0.58 | 0.19 | 0.70 |
| ls | 0.55 | -0.34 | 0.75 | 0.52 | -0.35 | 0.80 |
| | | | | | | |
| Elasticity: | | | | | | |
| w/r | 10.70 | -0.75 | 0.82 | 9.32 | -0.77 | 0.73 |
| eh/k | 3.39 | 0.90 | 0.91 | 3.50 | 0.88 | 0.72 |
| σ | - | - | - | 2.21 | -0.48 | 0.90 |
| Cons./Inv.: | | | | | | |
| | 0.15 | 0.03 | 0.00 | 0.70 | 0.06 | 0.72 |
| $egin{array}{c} c \ i \end{array}$ | 2.15 32.29 | 0.93 0.95 | 0.88 0.86 | 0.72 18.24 | -0.26 0.92 | 0.73 0.72 |
| $\stackrel{\imath}{R}$ | 0.01 | 0.95 | 0.82 | 0.00 | 0.92 | 0.72 |
| \boldsymbol{n} | 0.01 | 0.60 | 0.02 | 0.00 | 0.04 | 0.72 |
| Shocks: | | | | | | |
| a^* | _ | _ | _ | 1.42 | 0.79 | 0.71 |
| v^* | 1.00 | 0.00 | 0.92 | 0.98 | 0.24 | 0.92 |
| g^* | | - | - | 0.84 | 0.34 | 0.71 |
| b^* | _ | _ | _ | 0.18 | -0.12 | 0.75 |
| | | | | | | |
| TFP Residual: | | | | | | |
| z^* | 0.91 | 0.70 | 0.75 | 0.74 | 0.83 | 0.71 |
| | | | | | | |

Notes: y denotes output per capita, c indicates consumption per capita, i indicates quality-adjusted investment per capita, R denotes the rate of return, eh=H/N is hours per capita, e is employment per capita, h is average hours per worker, w is real wage, lp indicates labor productivity, ls is labor share, eh/k is the factor labor-capital input ratio, w/r is a factor price ratio. The data series of factor prices is constructed as $w=ls_t\frac{y_t}{eh_t}$ and $r=(1-ls_t)\frac{y_t}{k_t}$. See Online Appendix A for the data definitions and variable construction. The statistic v_x refers to the variance of the time series x, $\rho(x,y)$ refers to the correlation of x with output per capita, and $\rho(x,x')$ refers to the autocorrelation of x. For the computations of these statistics all time series have been logged (except the rate of return) and HP-filtered. The data moments of z^* and v^* are computed under the assumption of full utilization (see Online Appendix A).

Table 6: Labor Market Comovements

| | 21 | eh | | h | 211 | lp | ls | | | | | |
|----------------|---------------------------|------|----------|------|-------|-------|-------|--|--|--|--|--|
| | y | еп | <u>e</u> | 11 | w | ιp | | | | | | |
| | U.S. Data 1954.I-2012.III | | | | | | | | | | | |
| \overline{y} | 1.00 | 0.88 | 0.82 | 0.73 | -0.13 | 0.15 | -0.34 | | | | | |
| eh | | 1.00 | 0.97 | 0.70 | -0.44 | -0.33 | -0.14 | | | | | |
| e | | | 1.00 | 0.51 | -0.41 | -0.39 | -0.03 | | | | | |
| h | | | | 1.00 | -0.36 | -0.02 | -0.42 | | | | | |
| w | | | | | 1.00 | 0.67 | 0.42 | | | | | |
| lp | | | | | | 1.00 | -0.39 | | | | | |
| ls | | | | | | | 1.00 | | | | | |

| | NCES(SR)-CD(LR) Model | | | | | | | | | | | |
|----------------|-----------------------|------|------|------|-------|-------|-------|--|--|--|--|--|
| \overline{y} | 1.00 | 0.91 | 0.91 | 0.51 | -0.14 | 0.19 | -0.35 | | | | | |
| eh | | 1.00 | 1.00 | 0.54 | -0.37 | -0.23 | -0.17 | | | | | |
| e | | | 1.00 | 0.47 | -0.38 | -0.23 | -0.17 | | | | | |
| h | | | | 1.00 | -0.12 | -0.07 | -0.06 | | | | | |
| w | | | | | 1.00 | 0.57 | 0.50 | | | | | |
| lp | | | | | | 1.00 | -0.43 | | | | | |
| ls | | | | | | | 1.00 | | | | | |



Notes: Data IRFs of TFP (top panel), labor share (center panel) and output (bottom panel) in logs in response to one standard deviation TFP shocks. See the identification assumptions used to generate these IRFs in Online Appendix C and a discussion of these results in Section $\frac{2.2}{38}$.