

## PS 3 Q2 - Quant Macro

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### Question 2. General Equilibrium with Labor Supply, Uncertainty, and Progressive Labor Income Tax

In this section, we are required to solve the economy numerically, I use the following system of nonlinear equations to pin down policy function: [in [root1.m](#)]

$$C^{-\sigma} = R\beta\{.5 * (cph)^{-\sigma} + .5 * ((cpl)^{-\sigma})\}$$

$$\eta_y c^{-\sigma} = \kappa h^{1/\nu}$$

$$c = \eta_y h + y_0 - a$$

$$cph = (\eta_y + \epsilon)h' + R * a$$

$$cpl = (\eta_y - \epsilon)h' + R * a$$

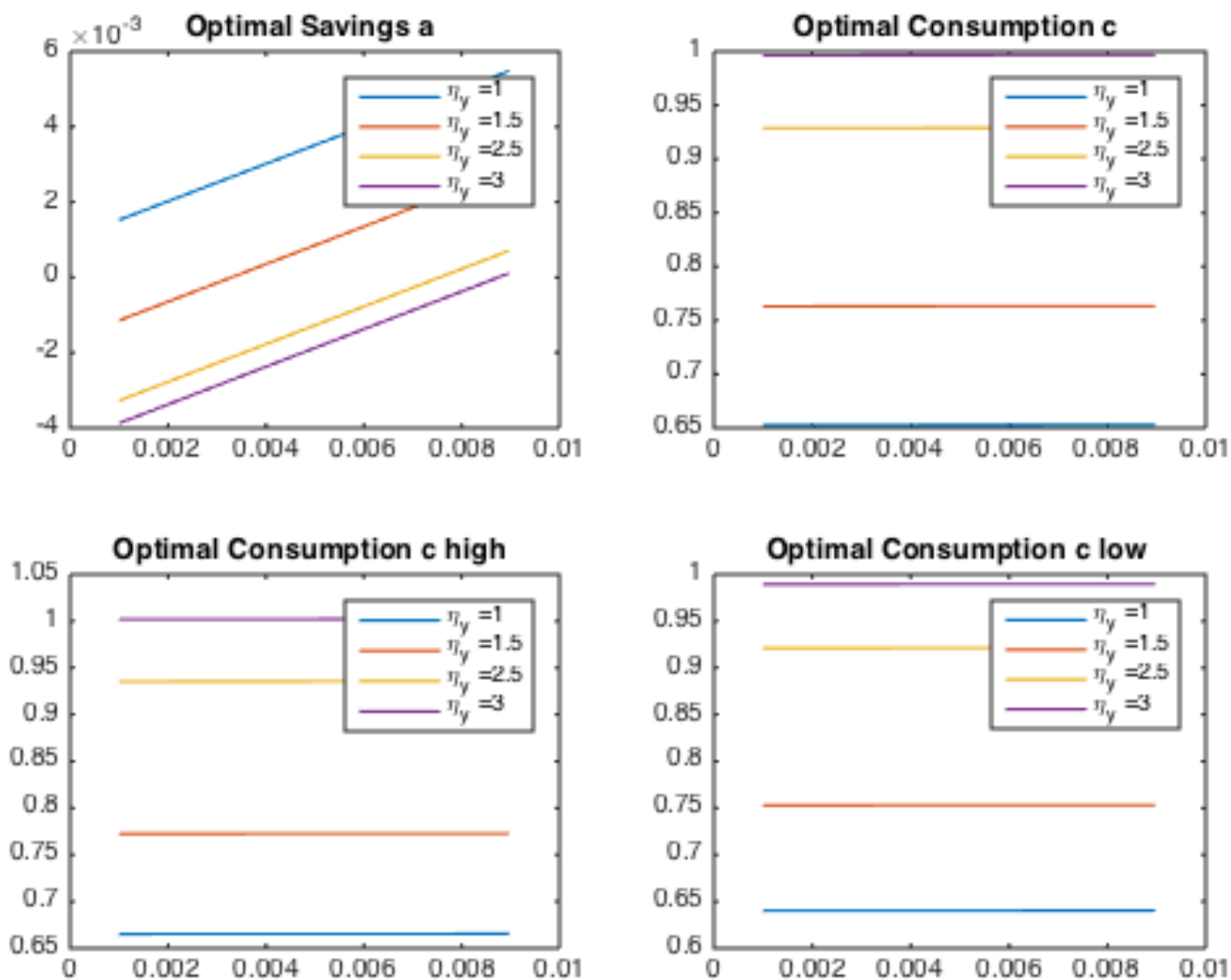
$$.5 * \{(\kappa h^{1/\nu})/(\eta_y + \epsilon) - ((\eta_y + \epsilon)h' + R * a)^{-\sigma}\} + .5 * \{(\kappa h^{1/\nu})/(\eta_y - \epsilon) - ((\eta_y - \epsilon)h' + R * a)^{-\sigma}\} = 0$$
$$h - ((\eta/\kappa) * R * \beta * (.5 * (\kappa * hph^{1/\nu})/(\eta + \epsilon)) + .5 * (\kappa * hpl^{1/\nu})/(\eta - \epsilon)))^{(\nu)}$$

Algorithm:

- Given a range of interest rates to guess, e.g. while loop in Matlab;
- for every element in  $\eta_y$  (4 elements), and for every element in  $y_0$ , solve above nonlinear equations (7 equations 7 unknowns: a, c, h, hph, hpl, cph, cpl), during this step, I update the initial guess expecting to have a higher speed.
- Assets Market Clearing condition to pin down interest rate: if sum of all individual's assets in  $[\eta_y \times y_0]$  is positive: agents are willing to lend than borrow, hence, we need to decrease interest rate; otherwise, if negative total assets, we should increase interest rate, and then continue to while loop. {this step in Matlab will take a very long time, hence, Numba packages in Python which could speed up the computation is a very great source.}

(a) Without Tax Case

(Figure 1) Optimal savings  $a$  (panel (a)), consumption  $c$  (panel (b)), and consumption  $c_0$  (panel (c)) as functions of the initial wealth and the permanent productivity.



(Figure 2, 3)

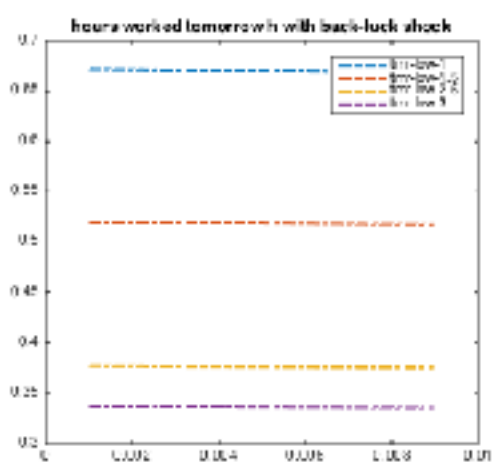
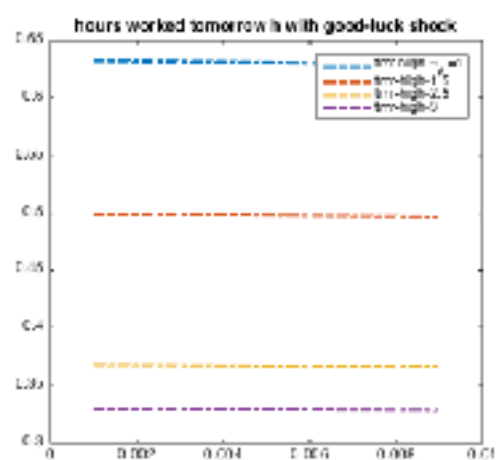
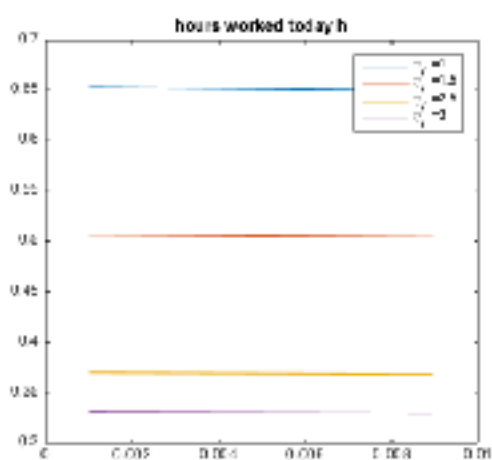
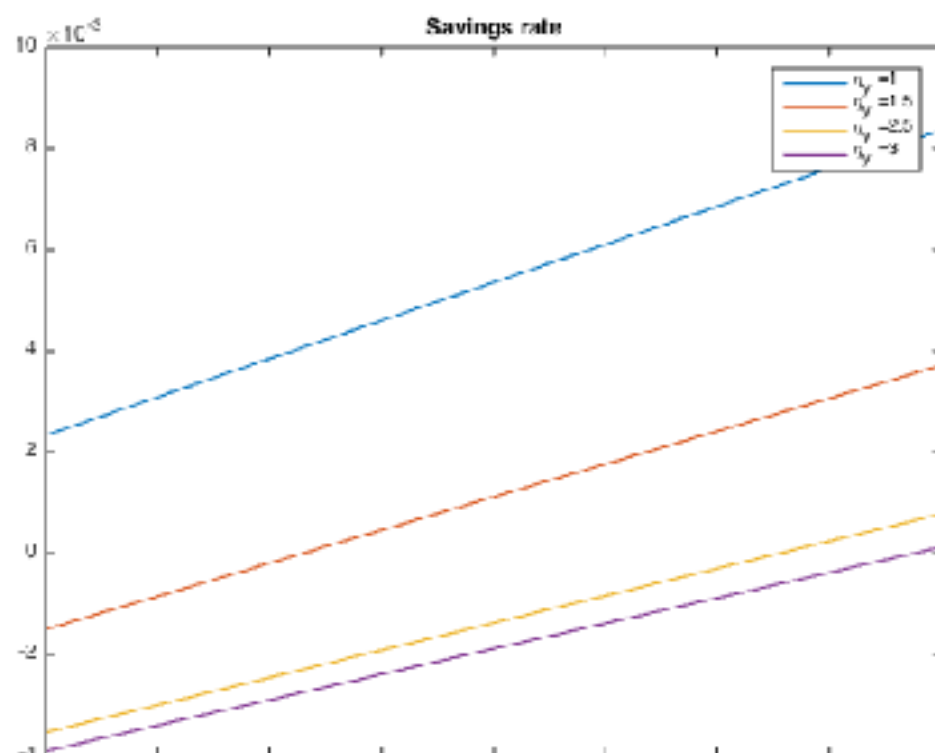


Figure 4.

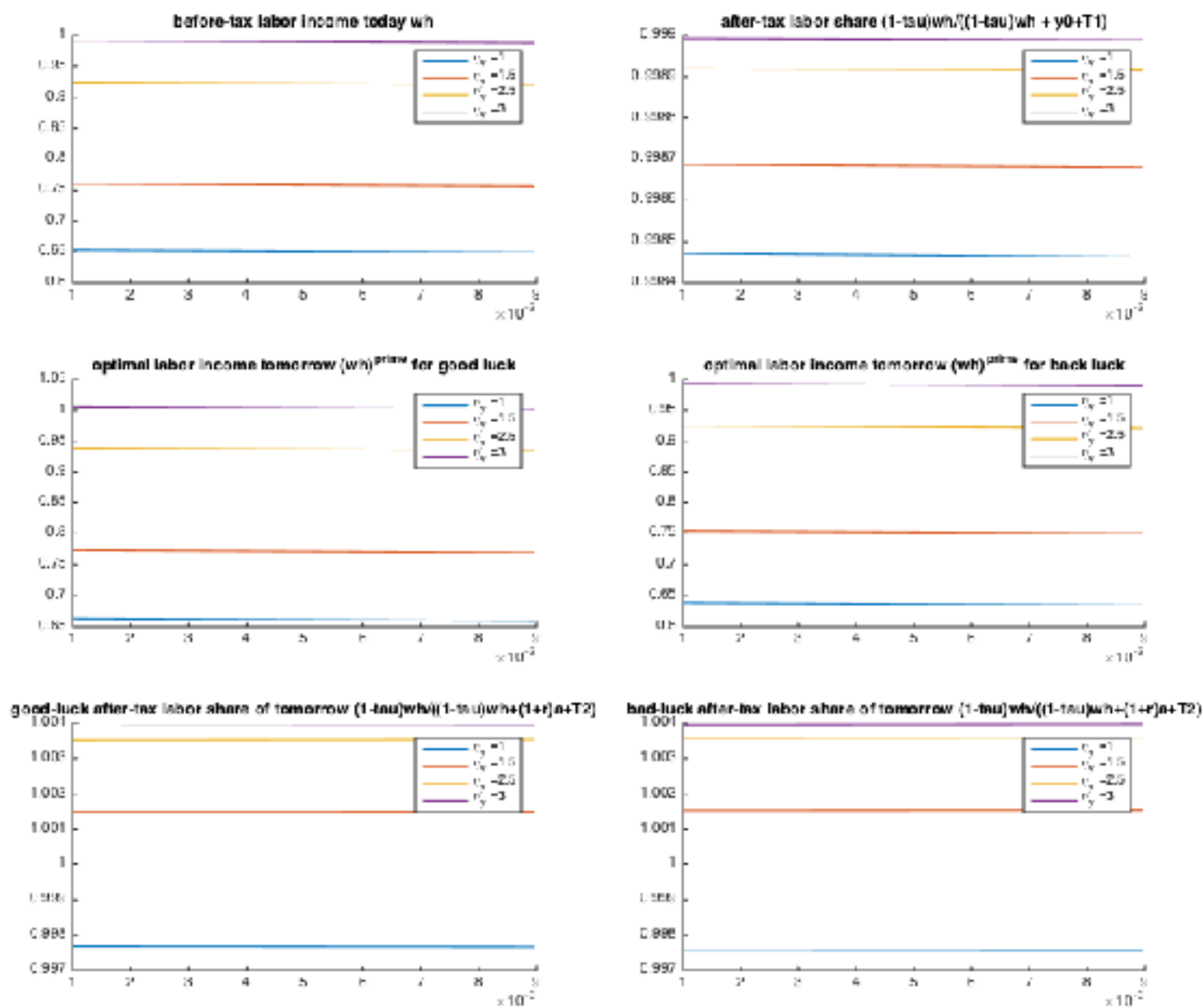


Fig 5.

