Homework #1

1 Initial values and exogenous process

1. Give initial values to the value functions

$$v(k,1;\bar{k},z_g)$$
$$v(k,0;\bar{k},z_g)$$
$$v(k,1;\bar{k},z_b)$$
$$v(k,0;\bar{k},z_b)$$

   - Build your guess by assuming the agent expects that the aggregate and individual states $(\bar{k},z,\epsilon)$ will not change in the future and his policy is to save exactly the same level of capital he has initially k.

2. Compute the transition probabilities $\pi_{zz'\epsilon\epsilon'}$ such that

   (a) "The process for (z,ϵ) is chosen so that the average duration of both good and bad times is eight quarters and so that the average duration of an unemployment spell is 1.5 quarters in good times and 2.5 quarters in bad times. We also impose $\pi_{gb00}\pi_{gb}^{-1} = 1.25\pi_{bb00}\pi_{bb}^{-1}$ and $\pi_{bg00}\pi_{bg}^{-1} = 0.75\pi_{gg00}\pi_{gg}^{-1}$."

   (b) $u_g = 0.04$ and $u_b = 0.1$

   (c) $\pi_{gg10} = 0.005$ and $\pi_{bb10} = 0.02$

2 Workers problem and simulation

- To solve the exercises use the following
  1. The following parameter values
     (a) $\beta = 0.95$
     (b) $(\alpha,\delta,\sigma,z_g,z_b,u_g,u_b)$ as presented in K&S.
  2. The initial values of the value function constructed in Exercise 1.
  3. The transition matrix for (z,ϵ) computed in Exercise 1.
  4. The following grid for individual capital $\{0,0.1,0.2,\ldots,4.9,5\}\cup\{5.3,5.6,5.9,\ldots,50\}$
  5. A grid for aggregate capital given by $\{16,\cancel{15.04,15.08,15.12},\ldots,18.5\}$

1. Solve the model by VFI with the following parametrized expectations:

$$\log\left(\bar{k}'\right) = \begin{cases}\beta_{0g}+\beta_{1g}\log(\bar{k}) & \text{if } z=z_g\\ \beta_{0b}+\beta_{1b}\log(\bar{k}) & \text{if } z=z_b\end{cases}$$
$$\text{where} \quad \beta_{0g}=\beta_{0b}=0$$
$$\beta_{1g}=\beta_{1b}=1$$

   (a) Recover the policy function for savings $a'(k,K,\epsilon,z,a)$

   (b) plot $a'(k,K,\epsilon,z,a)$ and discuss

2. Simulate the model for 2000 periods and 1000 individuals.

   (a) Simulate a sequence of z_t

   (b) Set the initial distribution of wealth at the same level of capital for all workers. $k=17$. Aggregate capital is the average capital along the population.

   (c) Adjust the initial level of unemployment accordingly with the value of z_1.

   (d) Store the time series for aggregate capital K_t and TFP z_t

3 Solution of the model

1. Find the parameter values for the approximate equilibrium with parametrized expectations

   (a) Estimate the following model with the simulated data

   $$\log\left(\bar{k}'\right) = \begin{cases} \beta_{0g} + \beta_{1g}\log(\bar{k}) & \text{if } z = z_g \\ \beta_{0b} + \beta_{1b}\log(\bar{k}) & \text{if } z = z_b \end{cases}$$

   (b) Update the guess on the parameters β

   (c) Simulate the model with the new expectations function.

   (d) Iterate until convergence on $\beta's$

   (e) Evaluate the fit of the expectations model in equilibrium (R^2)

   (f) Plot the asset distribution in the economy and discuss.

   (g) Compare the asset distribution in equilibrium after 7 periods of being in a bad state (low z) as opposed to being in the high state. Discuss.

2. Solve again the model following the steps in 3 with the following change:

   (a) Half of the agents always have the expectations given by the case described in exercise 3.1

   $$\beta_{0g} = \beta_{0b} = 0$$
   $$\beta_{1g} = \beta_{1b} = 1$$

   do not update these values for these agents in each iteration. Only for the other 50% of agents.

   (b) Compare the welfare of the agents that have the model with the best fit and the agents with the expectations that are never updated. Discuss.

The code KS_sol.m is a guide for the solution.