PS #2

November 16, 2018

In this PS you will have to modify the setup you used in PS#1 by including an elastic labor supply. The instantanous utility of the individual is now given by

$$U(c,n) = \log(c) - \Gamma \frac{n^{1+\gamma}}{1+\gamma}$$

so there is now endogenous labor supply. All the other elements of the model remain the same.

1 Solution of the model without aggregate risk.

Abstract from aggregate risk and fix the TFP $z_t = \bar{z}$ for all t .

1.1 Household problem

taking prices as given (r, w) solve the following:

1. Write down the problem of the household in a recursive form and find the optimal intratemporal and intertemporal conditions.
2. Use the intratemporal condition and the budget constraint to write the problem of the household in terms of a single decision variable (savings a').
   - Note that you might have to use an implicit function as an analytical expression to solve for consumption out of the budget constraint might not exist.
3. For a given value of wages and the interest rate, write the code to solve for the policy function $a'(e,a)$
   (a) Compare the policy functions you obtain by varying the transition matrix of the employment state e.
      i. Use the transition matrix implied by good times in PS#1 $\pi_{gge'e}$ and
      ii. The transition matrix implied by bad times in PS#1 $\pi_{bbe'e}$
   (b) Recover the optimal labor decision $n(e,a)$ and compare the compare this policy function for the cases:
      i. Use the transition matrix implied by good times in PS#1 $\pi_{gge'e}$ and
      ii. The transition matrix implied by bad times in PS#1 $\pi_{bbe'e}$

1.2 Equilibrium

1. Write down the equilibrium conditions in the capital market, the labor markets and the goods market.
2. Solve for the equilibrium
   (a) Make a guess on the level of aggregate capital and aggregate labor.
   (b) Find the corresponding equilibrium prices
   (c) Solve for the problem of the household
   (d) Aggregate and find the invariant distribution $\mu(e,a)$
   (e) Compute the aggregate savings and labor using the policy functions $a'(e,a)$ and $n(e,a)$ using the measure $\mu(e,a)$

(f) **Update your guess** until convergence

  i. You have to design the updating rule and the convergence criteria

  ii. Explain your choices.

3. Compare the two equilibria that are implied if you set $\bar{z} = z_l$ and $\bar{z} = z_H$.

   - Note here that you have to change also the transition probabilities of the employment state.

4. Calibrate γ and Γ such that if you average the total labor in the previous two scenarios you get an average of $\frac{N_{z_l}+N_{z_h}}{2} = \bar{N} \approx 0.97$.

2 The solution with aggregate risk

Now include the process for z presented in PS#1

1. Write down the recursive problem of the household.

   (a) Be explicit about how the expected value is computed (use the transition matrix $\pi_{z'ze'e}$.

   (b) Describe the state vector and explain why the distribution $\mu(e, a)$ is a state.

2. Simplify the problem of the household by replacing $\mu(e, a)$ in the state vector for K. Include the following two mappings in the problem:

   (a) Use the β's you found in PS#1 to construct the mapping $K' = H(K, z)$

   (b) Construct the mapping $N = G(K, z)$ by

$$G(K, z) = \begin{cases} N_{z_l} & \text{if } z = z_l \\ N_{z_H} & \text{otherwise} \end{cases}$$

3. Solve the problem of the household by VFI and find the policy function $a'(e, a, K, z)$

   - Note that H is used to form expectations about future capital.
   - Function G is only used to find the prices r, w for a given level of capital and z.

   (a) Plot the policy function and explain.

4. Simulate the model and store the following:

   (a) The sequence of prices w_t, r_t you found using functions H, G.

   (b) The sequence of asset distributions of the simulation.

5. Compare the sequence of prices w_t, r_t with the sequence of prices implied by the sequence of assets distribution.

   - Here you have to aggregate over asset holdings to find aggregate capital and also aggregate over the labor decision (using $n(.)$).
   - Once you recover the two aggregates you can recover the implied prices using the firms FOC.

6. Propose a way to update H, G to improve the approximation of your solution. So the differences you found in 5 become smaller.