Inside out: Including Self-Perception In Peer Effect Modeling

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October 19, 2023

Abstract

Theoretical work on peer effects builds on the premise of positive peer effects. Yet, this assumption does not always support empirical findings that contradict it or document heterogeneous peer effects. I address this gap by proposing a novel model of social interactions that formalizes the mechanism of social comparison. The proposed model is a modified version of the local-average model and integrates the notion of self-perception. The core of this model is the outcome production function, which is a concavely increasing function of one's own productivity and effort and the product of self-perceived ability and peer effort. Five main results are established in this study. *First*, a unique (but not alway interior) Nash equilibrium exists and is a weighted average of productivities, where the weights are a nonlinear function of the network structure, the degree of social comparison, and the perceived abilities. *Second*, peer productivity does not always positively affect individual equilibrium effort, and the direction of this effect strictly depends on one's own and peer self-perception. *Third*, the overall distribution of self-perception in the network affects everyone's effort, and both the network density and the degree of social comparison can exacerbate this effect, regardless of its direction. Furthermore, the presence of individuals with low self-concepts is not unconditionally harmful (only above a certain threshold), but the larger their proportion in the network, the lower the average effort in the network. This study offers the first formalization of social comparison through a microeconomic model. Its conceptual differences from the existing models of social interactions has also significant implications for policy interventions.

JEL classification: D85, J24, Z13
Keywords: Social interactions; Social comparison; Self-perception; Peer effects in education

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I would like to particularly thank Beatrix Eugster and Bryan Graham for extensive discussions and their mentorship, as well as Caroline Chuard, Andrea Burro, and Samuel Kang for early stage exchanges.

1 Introduction

People have a drive to compare themselves with others (Festinger, 1954). This process is known as *social comparison*—a psychological process whereby individuals establish the value of their own attributes and ideas—and is key in the formation of the *self-concept*—the set of beliefs and self-perceptions one holds about oneself (Myers and Twenge, 2015). Social comparison is also deemed to be a potential mechanism underlying *peer effects*—the influence that peers have on one's own behaviors and outcomes.[1] However, little has been done in providing a theoretical framework that formalizes this particular mechanism.

Theoretical work on peer effects has mostly focused on formalizing the mechanism of social learning through two main models: the local-average model and the local-aggregate model.[2] The local-average model emphasizes the role of social norms (e.g., peer pressure) on one's outcomes (Blume et al., 2015; Boucher, 2016; Boucher et al., 2022; Liu, Patacchini, and Zenou, 2014; Patacchini and Zenou, 2008; Ushchev and Zenou, 2020), while the local-aggregate model highlights the role of knowledge spillovers on one's outcomes (Ballester, Calvó–Armengol, and Zenou, 2006; Bramoullé, Djebbari, and Fortin, 2009; Bramoullé, Kraton, and D'Amours, 2014; Liu, Patacchini, and Zenou, 2014).[3] Since social learning presumes a positive relationship between one's own and peer outcomes, both models build on the premise of positive peer effects.

However, starting from this assumption hinders providing a theoretical foundation for empirical findings that contradict it.[4] Additionally, it impedes the recognition of heterogeneous patterns typically observed in the empirical literature (see Cools and Patacchini, 2021; Sacerdote, 2011, 2014, for an overview).

I address this gap by proposing a novel model of social interactions that formalizes the mechanism of social comparison. By integrating and analyzing the interplay between self-perception and peer influence on one's own outcomes, the model manages to capture positive, negative, and heterogeneous peer effects.

The proposed model is a modified version of the local-average model, which is a static, full-information game that builds upon the following premises. First, agents differ from one another with respect to two predetermined characteristics: productivity and self-perception of their abilities.[5] Second, agents care only about maximizing their expected outcome and simultaneously choose the optimal amount of effort that maximizes their target outcome. Third, the outcome is determined by two components: a concavely increasing function of one's own productivity and effort and the product of self-perceived ability and peer effort.

The outcome production function is the core of this model and embodies the following rationale.

[1] The *invidious comparison model* is an example where social comparison is at play. The model posits that exposure to better achieving peers harms the academic outcomes of other students (Sacerdote, 2011).

[2] Social learning describes the process of adopting beliefs and behaviors through observation, imitation, or interaction with others (Bandura, 1977). It is a broader concept that involves both learning from others and their experiences (i.e., *knowledge spillovers*) while recognizing rewards and punishments associated with given behaviors (e.g., social pressure and social norms).

[3] See Jackson and Zenou (2015)

[4] Findings from four studies in particular strongly question the positive peer effects hypothesis. Antecol, Eren, and Ozbeklik (2016) and Carrell, Sacerdote, and West (2013), for instance, show that exposure to high-ability students can adversely affect other students' achievement. On the other hand, Duflo, Dupas, and Kremer (2011) and Booij, Leuven, and Oosterbeek (2017) show that low-ability students do not necessarily have a negative impact on other students' academic performance.

[5] Productivity is thought of as a function of one's abilities, demographics, and family background, while self-perception of one's own abilities is regarded as a function of one's past performance relative to others (e.g., past ranks).

If agents harbor doubts about their own abilities, then the presence of peers with whom they compete or compare imposes extra psychological costs. As a result, outcomes are lower in the presence of social interaction than when realized individually. On the other hand, if agents perceive themselves as capable, the presence of peers serves as an extrinsic motivator, spurring them to exert even greater effort than they would in isolation. This ultimately leads to higher outcomes in the presence of social interactions than in their absence. These dynamics are, furthermore, regulated by the degree of social comparison—the intensity with which agents in a given network compare themselves to others.

Five main results are established in this study. *First*, a unique and—under given circumstances—interior Nash equilibrium exists and is a weighted average of productivities, where the weights are a nonlinear function of the network structure, the degree of social comparison, and the perceived abilities. In contrast to the original local-average model, weights can also be negative.[6]

Second, a marginal change in peer productivity on individual equilibrium effort (as well as outcome and utility) can be both positive and negative. The direction of the effect is primarily determined by one's self-perception and, if two agents are indirectly connected, by the self-perception of the intermediary individuals as well. In terms of magnitude, the marginal effect of peer productivity never exceeds 1 and is always smaller on outcome and utility than that on effort.

Third, the marginal effect of one's own self-perception on one's own effort is always positive, while the marginal effect of peer self-perception can be both positive and negative. The simulation results suggest that the distribution of individual self-perception in the network affects everyone's equilibrium effort. In general, the average effort in the network decreases as the fraction of individuals with negative self-perception increases, but their presence becomes detrimental only beyond a certain threshold. Furthermore, individuals with different self-perceptions of their own abilities react differently to higher proportions of low self-perception individuals. While agents with positive beliefs about their abilities exert less effort because of smaller gains, those with negative beliefs exert higher effort because of lower psychological costs.

Fourth, marginal increases in the degree of social comparison can affect both positively and negatively the equilibrium effort; the primary factor is one's self-perception. However, the main role of social comparison is to "magnify" the peer effect, regardless of its sign. The more people compare to each other, the larger the psychological cost for those who suffer social comparison, and the larger the benefit for the more competitive agents who benefit from social comparison.

Fifth, a denser network features higher average equilibrium efforts when the fraction of individuals with negative beliefs about their abilities does not exceed a certain threshold; above that threshold, the average equilibrium effort decreases as network density increases. In general, just as the degree of social comparison exhibits, network density also acts as an amplifier of peer effects.

Finally, a comparison with the local-average model reveals that the conceptual differences between the two frameworks yield opposite results when the same groupwise policy intervention is enacted. For example, by increasing everybody's productivity, the local-average model converges to a norm, while the social comparison model distances even further outcomes of individuals with high and low self-concepts. These same conceptual differences also call for different policy interventions to achieve similar outcomes, with options ranging from groupwise interventions in

[6]This is the consequence of introducing self-perception, which can be either positive or negative.

the local-average setting to targeted measures on individuals with low self-concepts in the social comparison model.

This paper contributes to the study of the effects of social interactions and network structure on individual behavior and outcomes in three major ways. *First*, it formalizes an alternative mechanism that is deemed to drive peer effects, namely, social comparison. In an attempt to bring the theoretical and the empirical research on peer effects somewhat closer, the most recent theoretical work on peer effects has mainly focused on studying the microfoundation of the linear-in-means model (see e.g., Blume et al., 2011, 2015; Boucher and Fortin, 2016; Manski and Mayshar, 2003) while overlooking potential mechanisms other than social learning.

Second, this research departs from the common assumption of exclusively positive peer effects and allows for both positive and negative peer effects. While some models have proposed negative peer effects in the context of public goods provision (Allouch, 2015; Bramoullé, Kraton, and D'Amours, 2014; Bramoullé and Kraton, 2007), the majority of models, particularly in the education context, assume positive peer effects (e.g., Ballester, Calvó–Armengol, and Zenou, 2006; Calvó–Armengol, Patacchini, and Zenou, 2009; Liu, Patacchini, and Zenou, 2014; Ushchev and Zenou, 2020).

Finally, the present study also provides a theoretical foundation for the growing empirical literature on the effects of ordinal rank (Bertoni and Nisticò, 2023; Buechel, Mechtenberg, and Petersen, 2018; Carneiro et al., 2022; Delaney and Devereux, 2021; Elsner and Isphording, 2017; Elsner, Isphording, and Zölitz, 2021; Murphy and Weinhardt, 2020). Although some theoretical models account for ordinal rank (Tincani, 2014, 2017) or one's comparative advantage in the group (Cicala, Spenkuch, and Fryer, 2018) to provide a theoretical foundation for the empirical literature on ordinal ranks, little has been done in the realm of game theory on networks.

To the best of the author's knowledge, this is the first economics paper in which a model of social comparison has been proposed that formalizes the notion of self-perception within a game theory framework. However, there are some limitations that warrant further research, such as allowing for incomplete information.[7] integrating social norms and self-perception in the same model to understand the interplay between social learning and social comparison, and reformulating the optimization problem in order to make it apt for structural estimation, as in Calvó–Armengol, Patacchini, and Zenou (2009) and Boucher et al. (2022).

The remainder of the paper is structured as follows. Section 2 motivates the importance of including self-perception in the peer effect models and presents the model, including the equilibrium solution. Section 3 presents the comparative statics with respect to productivity, self-perception, degree of social comparison, and network density. In Section 4, the presented model is compared with the original local-average model, and the policy implications of these differences are discussed. Section 5 concludes and outlines potential future research avenues.

2 The model

The following section serves three main purposes. First, it clarifies why it is important and reasonable to include the notion of self-perception in the context of peer effect studies. Second, this section illustrates how an abstract notion such as self-perception can be conceptualized in

[7] As shown by Jackson and Yariv (2007) and de Martí and Zenou (2015), modeling with incomplete information can substantially change the equilibrium outcome.

an economic model by presenting a modified version of the local-average model. Finally, this section explores the advantages of the social comparison model. To this end, I show how the latter contributes to the understanding of alternative mechanisms underlying peer effects other than social norms, and I illustrate how it can provide a theoretical foundation for different (even contrasting) results observed in empirical studies.

2.1 The role of self-perception in understanding peer effects

Self-perception (or self-concept) refers to an individual's conception of self. It describes the set of beliefs one has about one's views and characteristics, such as abilities, personality, and appearance, just to mention a few. The formation of these beliefs often relies on evaluations that are informed by comparisons with others (Myers and Twenge, 2015).

As such, self-perception is a byproduct of a psychological process known as social comparison. Social comparison is a notion put forward by the psychologist Leon Festinger back in 1954. He posited that people have a natural drive for self-evaluation and that in absence of objective evaluation means, they engage in social comparison to gain information about themselves and to determine their relative position in the social hierarchy (or simply rank) within a particular group or society.[8]

Peer influence comes into play precisely when we compare ourselves with others and determine our own rank. For instance, in assessing our position on the social ladder of intelligence, we form beliefs about both ourselves and others.[9] Recent research reveals that these beliefs and expectations about others' behaviors significantly influence our own behaviors (Bursztyn and Yang, 2022).

Social comparison can occur in various domains, including appearance (Arduini, Iorio, and Patacchini, 2019), wealth (Bursztyn et al., 2014), social status (Bursztyn et al., 2018), and most importantly intelligence and academic achievements (Bursztyn, Egorov, and Jensen, 2019). Considering the nature of the educational environment, with its emphasis on rankings, classroom standings, and competitive structures, it is straightforward to see how the notion of social comparison holds a particular relevance in the context of education. Students, in fact, often gauge their own progress and success by comparing their grades, test scores, and academic achievements to those of their classmates.

These evaluations can affect not only one's perception of one's academic competence but also other dimensions, such as motivation (Ames, 1981; Buechel, Mechtenberg, and Petersen, 2018), confidence and assessment of one's abilities Murphy and Weinhardt (2020), future aspirations (Elsner and Isphording, 2017; Elsner, Isphording, and Zölitz, 2021), educational choices (Delaney and Devereux, 2021) and risky habits (Elsner and Isphording, 2018).

Furthermore, students seem to have particular preferences for upward comparison, especially when experiencing improvement (Dijkstra et al., 2008; Gruder, 1977; Suls and Tesch, 1978). This upward comparison, however, can have twofold consequences. On the one hand, believing to be smarter than one actually is can help achieve one's goals by creating a self-fulfilling prophecy that promotes motivation even during more difficult times (Willard and Gramzow, 2009). On the other

[8]Social hierarchy refers to the organization and ranking of individuals within a group or society based on perceived or actual differences in various domains, including wealth, power, status, as well as personal attributes, such as intelligence, beauty, and physical abilities (Lee, Pratto, and Johnson, 2011).

[9] In other words, perceiving others as more intelligent leads us to believe that we have inferior cognitive abilities.

hand, it can breed dissatisfaction and elicit more negative evaluations of one's abilities, thereby leading to a lower self-concept (Dijkstra et al., 2008).

The above considerations highlight the importance of incorporating the concepts of self-perception and social comparison into models of peer effects. The dual nature of potential effects that social comparison can have suggests that if peer effects are driven by social comparison, the assumption that the marginal effect of peer ability is consistently positive may no longer be fully justified. Furthermore, recent studies on the consequences of ordinal ranks in academic achievement suggest that peer effects in education are at least partly driven by relative performance (Cicala, Spenkuch, and Fryer, 2018; Puljic, 2023; Tincani, 2014, 2017). Therefore, to enhance our understanding of the underlying mechanisms of peer effects, both in education and more broadly, it is necessary to deviate slightly from the current standard and introduce new notions, such as self-perception.

Before moving on to the presentation of the model, two remarks are in order. First, the model approximates four specific clues that were presented in this section and can be summarized as follows: (i) Social comparison implies the existence of a social rank, (ii) self-perception is a function of this rank, (iii) students compare to each other, especially upward, and (iv) this comparison can lead to both positive and negative peer effects.

Second, the model formalizes the specific notion of *academic self-concept*, which must not be confused with concepts such as self-esteem. *Academic self-concept* specifically focuses on one's self-perception and beliefs about one's *academic abilities* and performance.[10] Self-esteem, on the other hand, pertains to one's overall evaluation of their worth and value as a person, encompassing both academic and nonacademic aspects of the self (Myers and Twenge, 2015). Not surprisingly, academic self-concept—whether one thinks they are good in school—strongly predicts academic performance while self-esteem—whether one thinks they are great—does not (Marsh and O'Mara, 2008).

Throughout the text, I use the terms "academic self-concept," "self-perception of one's abilities," and "beliefs about one's abilities" interchangeably. Importantly, academic self-perception is closely linked to one's level of self-confidence, which is a broader concept encompassing academic self-concept as well. Self-confidence refers to an individual's subjective assessment of their own capabilities and their belief in their ability to successfully complete tasks and achieve desired outcomes. Therefore, it is appropriate to consider students with high academic self-concepts as highly self-confident individuals.

2.2 Setup and notation

Example. To better understand the setup of the theoretical framework, it is helpful to envision a specific thought experiment. Imagine a scenario where students undergo a transition from one educational environment to another, such as moving from high school to college. As part of this transition, students are required to take an entrance exam, which is evaluated based on an absolute scale rather than a curve. Prior to the exam, students have the opportunity to meet and interact with their new peers and form social connections (e.g., through tutorials).

In this thought experiment, students are characterized by two attributes: *actual knowledge* and

[10] Note that *self-concept*, in general, is a broader term that encompasses one's perception of the self over *multiple dimensions*, including the academic sphere.

perceived ability rank. They are assumed to possess complete information about the two traits. That is, they know both their own as well as their peers' actual knowledge and perceived abilities. They impute their actual knowledge from past achievement tests, while their self-perceived ability rank is informed by their academic performance history, that is, their grades and rankings in *past* grades (e.g., high school, primary school).

All students share a common objective: maximizing the test score on the entrance exam. This test score depends on several factors, including students' knowledge, their study effort (which is both limited and costly), their academic self-concept, and the expected average effort among their peers. The rationale is as follows.

If students harbor doubts about their own academic abilities, the competitive nature of the exam, coupled with the presence of hard-working peers, imposes a threat to their own success and additional psychological costs. Consequently, students with low academic self-concepts are expected to achieve lower test scores when taking exams in the presence of peers compared to when they take the exam individually.

On the other hand, if students perceive themselves as capable, the presence of peers serves as an extrinsic motivator, spurring them to exert even greater effort than they would in the absence of peers. As a result, students with high academic self-concepts are expected to achieve higher test scores than if they were taking the exam individually. In essence, the model formalizes a self-fulfilling prophecy mechanism that entirely depends on one's own beliefs about their academic abilities.

The network. Let us consider a network $\mathcal{G}$ of $N \geq 2$ students (or agents) described by the $N \times N$ *adjacency matrix* $\mathbf{G} = [g_{ij}]$, with $g_{ij} = \{0, 1\}$ for all pairs of students (i, j), where 1 indicates that i and j are directly connected while 0 indicates that they are not. The tie between a pair of students can be thought of as either a friendship or as membership in the same social group (e.g., class).

With these examples in mind, it is natural to let $\mathbf{g}$ be a *undirected* and *nonregular* graph. That is, relationships are reciprocal, and the degree (i.e., a student's number of direct connections) differs across students. In other words, if i is j's friend, then j must also be i's friend.

Formally, this implies that the adjacency matrix $\mathbf{G}$ and its row-normalized version, $\widehat{\mathbf{G}} = [\widehat{g}_{ij}]$, are symmetric. Each element of $\widehat{\mathbf{G}}$ is defined as $\widehat{g}_{ij} := \frac{g_{ij}}{d_i} \in [0, 1]$, where $d_i := \sum_{j=1}^{N} g_{ij}$ denotes student i's degree.

Agents and their attributes. Each agent $i = 1, 2, ..., N$ is characterized by a certain *productivity* level, $\alpha_i \in \mathbb{R}_+$, and the centered *self-perceived ability rank* $r_i \in \left[-\frac{1}{2}, \frac{1}{2}\right]$, where $r_i < 0$ ($r_i > 0$) indicates that the student believes to have below (above) average abilities.

Both characteristics are considered exogenous and independent of each other. Productivity is to be thought of as a function of $q > 0$ individual exogenous characteristics, $\mathbf{x}_i \in \mathbb{R}^q$, including knowledge, cognitive and socioemotional abilities, and parental investment. Self-perceived ability rank, instead, is to be thought of as the level to which an individual believes in his or her own abilities rather than the actual performance rank. Although it is reasonable to consider the perceived rank as a function of actual, *past* performance ranks, it is treated as independent of any current ability rank as well as one's productivity.

Preferences, outcome, and choice variable. Students are assumed to care only about their *academic performance*, $y_i \in [y_{min}, y_{max}]$, which is assumed to be bounded just as real-life academic performance measures are (e.g., test scores).

Students' choice variable is *study effort*, $e_i \in \mathbb{R}_+$. More precisely, students aim high by choosing the level of effort that minimizes the distance from the maximal possible value of academic performance:[11]

$$e_i^* = \arg\max_{e_i} \mathbb{E}\left[U_i(y_i) \middle| \mathcal{I}_i\right] = \arg\max_{e_i} \mathbb{E}\left[y_i(\alpha_i, e_i, \mathbf{e}_{-i}, \mathbf{g}_i) \middle| \mathcal{I}_i\right] - y_{max}, \tag{1}$$

where $\mathcal{I}_i$ denotes the set of information available to student i to predict their future performance.[12]

The *technology of academic performance* is assumed to depend on one's productivity, academic self-concept, and one's own and peers' level of effort in the following way:

$$y_i(\alpha_i, r_i, e_i, \mathbf{e}_{-i}, \mathbf{g}_i) = \alpha_i e_i - \frac{1}{2}e_i^2 + \phi \ln(r_i + 1)\, e_i \bar{e}_{-i} + \epsilon_i, \tag{2}$$

where $0 \leq \phi \leq 1$ denotes the degree of *social comparison* in the network, $\bar{e}_{-i} := \sum_{j \neq i} \widehat{g}_{ij} e_j$ denotes the average effort among i's direct peers, and ϵ_i is a random event that may affect students' performance (e.g., illness).[13] Students are assumed to know nothing about ϵ_i. Their best guess based on their information set, $\mathcal{I}_i$, is to expect no random event for anybody in the network (including themselves). That is, $\mathbb{E}[\epsilon_j | \mathcal{I}_i] = 0$ for any $i, j = 1, 2, \ldots, N$.

The first part of the technology is standard (see Boucher and Fortin, 2016; Liu, Patacchini, and Zenou, 2014; Ushchev and Zenou, 2020). This reflects the idea that academic performance increases in both productivity and effort but not indefinitely—resulting in a concave relationship with both arguments. Furthermore, the product between the two arguments implies that a unit of effort exerted by a highly productive student generates a higher performance than that of a unit of effort exerted by a low-productivity student. Or put differently, everything else being equal, a high-productivity student must exert fewer units of effort to achieve the same outcome as a low-productivity student.

The third term of the specification is what distinguishes this social interaction model from the leading ones, such as the local-average (Ushchev and Zenou, 2020) or the local-aggregate model (Liu, Patacchini, and Zenou, 2014). More precisely, the novelty lies in the logarithmic function of the self-perceived ability rank. In light of the previous discussion, this function is to be interpreted as the mathematical formalization of the notion of self-concept, which in general can be defined as a function of a social rank.

For the structure of the third term, the product between e_i and $\ln(r_i + 1)$ captures the idea that one's academic self-concept regulates one's level of effort (Honicke and Broadbent, 2016). By further multiplying this product with peer average effort, $\bar{e}_{-i}$, the model formalizes the notion that peer effects are not unconditionally beneficial to everybody but that their benefits are rather contingent on one's self-perception of own abilities relative to others, as suggested by the previously

[11] In the case of a grade point average, $y_{max} = 4$ (in the US), or $y_{max} = 1600$ for the SAT.

[12] Note that $\mathbb{E}\left[y_i(\alpha_i, e_i, \mathbf{e}_{-i}, \mathbf{g}_i) \middle| \mathcal{I}_i\right] \leq y_{max}$. This means that $\mathbb{E}\left[U_i(y_i) \middle| \mathcal{I}_i\right] \leq 0$. This is the reason why maximizing $\mathbb{E}\left[U_i(y_i) \middle| \mathcal{I}_i\right]$ minimizes the distance between $\mathbb{E}\left[y_i(\alpha_i, e_i, \mathbf{e}_{-i}, \mathbf{g}_i) \middle| \mathcal{I}_i\right]$ and y_{max}.

[13] To better understand what the parameter ϕ captures, let us consider a network of adolescents versus a network of retirees. Since adolescents are more susceptible to social comparison, ϕ takes on a higher value in this network than in that of retirees, who are also likely to compare themselves but to a much lesser extent than adolescents.

reviewed recent research. Depending on this perception, peer externalities can be either positive or negative.

In fact, if i and j are friends and $\{\alpha_i, \boldsymbol{\alpha}_{-i}, e_i, \boldsymbol{e}_{-i}, \boldsymbol{g}_i\} \subseteq \mathcal{I}_i$ (i.e., everybody's productivity and effort are known), then

$$\frac{\partial \mathbb{E}\left[y_i \middle| \mathcal{I}_i\right]}{\partial e_j} \lesseqgtr 0, \quad \frac{\partial \mathbb{E}\left[\partial U_i(y_i) \middle| \mathcal{I}_i\right]}{\partial e_j} \lesseqgtr 0 \qquad \Leftrightarrow \qquad r_i \lesseqgtr 0. \tag{3}$$

That is, if student i believes in their own academic abilities, then a marginal increase in j's effort results in a positive externality. However, if i doubts their own academic abilities, then a marginal increase in j's effort results in a negative externality. This is the formal expression of the fact that social comparison leads to both positive and negative peer effects.

The choice of the logarithmic function to express academic self-perception and map it to academic performance is motivated by three specific features of the function itself.

First, $\ln(r_i + 1)$ is monotonically increasing in r_i. From a purely technical point of view, this is a conventional property for differentiation with respect to r_i. From a more conceptual point of view, this reflects the empirical pattern shown by numerous studies, whereby students with higher confidence in their academic abilities tend to perform better academically, as well as to experience higher satisfaction than those with lower levels (see Doménech-Betoret, Abellán-Roselló, and Gómez-Artiga, 2017; Honicke and Broadbent, 2016). That is,

$$\frac{\partial y_i(\cdot)}{\partial r_i} > 0 \qquad \text{and} \qquad \frac{\partial U_i\left(y_i(\cdot)\right)}{\partial r_i} > 0. \tag{4}$$

Second, $\ln(r_i + 1)$ takes positive values for $r_i > 0$ and negative values for $r_i < 0$. This formalizes Bandura's (1977) idea that low beliefs in one's abilities to perform a given task tend to hijack people's perseverance in that task, while high beliefs tend to increase it. Formally, if $r_i^H > 0$ and $r_i^L < 0$, then

$$y_i\left(\cdot, r_i^L\right) < y_i\left(\cdot, r_i^H\right) \qquad \text{and} \qquad U_i\left(y\left(\cdot, r_i^L\right)\right) < U_i\left(y\left(\cdot, r_i^H\right)\right). \tag{5}$$

Third, the slope of the function is steeper for lower values of r_i, implying that the marginal benefits of an increase in one's beliefs about one's academic abilities are greater for students with lower self-confidence than those for students who already believe in their abilities. Formally,

$$\left.\frac{\partial y_i(\cdot)}{\partial r_i}\right|_{r_i = r_i^L} > \left.\frac{\partial y_i(\cdot)}{\partial r_i}\right|_{r_i = r_i^H}. \tag{6}$$

Finally, before discussing the equilibrium and comparative statics, it is important to highlight two other noteworthy features that this model specification entails.

First, in contrast to the local-average model studied by Ushchev and Zenou (2020), efforts are not unconditionally strategic complements since

$$\frac{\partial^2 U_i\left(y_i(\cdot)\right)}{\partial e_i e_j} \lesseqgtr 0 \qquad \Leftrightarrow \qquad r_i \lesseqgtr 0. \tag{7}$$

Rather, efforts are strategic complements only if i believes they have above-average abilities, strategic substitutes if i does not believe in their abilities, and independent if i believes they are average.

Second, the cross-effect of individual i's effort and self-perception is positive on both the academic outcome and the individual utility:

$$\frac{\partial^2 y_i(\cdot)}{\partial e_i r_i} \geq 0 \qquad \text{and} \qquad \frac{\partial^2 U_i\left(y_i(\cdot)\right)}{\partial e_i \partial r_i} \geq 0. \tag{8}$$

That is, increases in one's effort improve both the outcome and the utility as beliefs in one's abilities also improve.

2.3 Nash Equilibrium

When maximizing the utility function, individuals are assumed to choose e_i simultaneously with other players. Hence, the effort choice of the other agents is taken for given, just as it is the network structure, $\mathbf{G}$.

By plugging (2) in (1) and computing the first-order condition (FOC) with respect to e_i, it follows that any i's best-response function yields:

$$e_i = \alpha_i + \phi \ln(r_i + 1)\bar{e}_{-i}. \tag{9}$$

Let us define $w_i := \ln(r_i + 1)$ and let $\mathbf{W} := \ln(\mathbf{R} + \mathbf{I})$ be the logarithm matrix of the $N \times N$ diagonal matrix $(\mathbf{R} + \mathbf{I})$, where $\mathbf{I}$ is the identity, and $\mathbf{R}$ is a diagonal matrix, with $diag(\mathbf{R}) = (r_1, r_2, \ldots, r_N)^T$ and off-diagonal elements equal to zero.[14] Considering that $\bar{e}_{-i} := \sum_{j \neq i}^{N} \widehat{g}_{ij} e_j = \widehat{g}_i \mathbf{e}$, expression (9) can be re-expressed in matrix form as follows:

$$\mathbf{e} = \boldsymbol{\alpha} + \phi \mathbf{W} \widehat{\mathbf{G}} \mathbf{e}, \tag{10}$$

where $\boldsymbol{\alpha} := (\alpha_1, \alpha_2, \ldots, \alpha_N)^T$ is the productivity vector and $\mathbf{e} := (e_1, e_2, \ldots, e_N)^T$ is the effort vector. By reshuffling and re-expressing (10) in terms of effort, one can find the following result.

Proposition 1. *(Equilibrium efforts, performance, and utilities) Consider a network described by the row-normalized $N \times N$ adjacency matrix $\widehat{\mathbf{G}}$, where $N \geq 2$ denotes the number of agents. Let $\boldsymbol{\alpha}$ be a $N \times 1$ vector describing agents' productivity, and $\mathbf{W}$ be a diagonal matrix that describes agents' self-perception, with $w_{ii} = \ln(r_i + 1)$, for all $i = 1, 2, ..., N$, and $r_i \in \left[-\frac{1}{2}, \frac{1}{2}\right]$. Assume that agents solve (1), then it can be shown that, for $0 \geq \phi \geq 1$, $\left(\mathbf{I} - \phi \mathbf{W} \widehat{\mathbf{G}}\right)^{-1}$ exists and that the following results hold true.*

(i) A unique (but not always interior) Nash equilibrium $\mathbf{e}^*$ of the following form exists:

$$\mathbf{e}^* = \widehat{\mathbf{M}} \boldsymbol{\alpha}, \tag{11}$$

where

$$\widehat{\mathbf{M}} := \left(\mathbf{I} - \phi \mathbf{W} \widehat{\mathbf{G}}\right)^{-1} = \sum_{k=0}^{\infty} \phi^k \left(\mathbf{W} \widehat{\mathbf{G}}\right)^k. \tag{12}$$

[14] Note that sum $(\mathbf{R} + \mathbf{I})$ is a diagonal matrix, precisely because both $\mathbf{R}$ and $\mathbf{I}$ are diagonal matrices. Therefore, to compute the matrix logarithm $\mathbf{W}$, the sum $(\mathbf{R} + \mathbf{I})$ does not need to be diagonalized first.

(ii) Each student's equilibrium outcome is given by

$$y_i^*(\alpha_i, r_i, e_i^*, \overline{e}_{-i}^*, \mathbf{g}_i) = \frac{1}{2}\widehat{m}_{ii}\alpha_i + \frac{1}{2}\sum_{j\neq i}^{N}\widehat{m}_{ij}\alpha_j. \tag{13}$$

(iii) Each student's equilibrium utility is given by

$$U_i^*(\alpha_i, r_i, e_i^*, \overline{e}_{-i}^*, \mathbf{g}_i) = \frac{1}{2}\widehat{m}_{ii}\alpha_i + \frac{1}{2}\sum_{j\neq i}^{N}\widehat{m}_{ij}\alpha_j - y_{max}. \tag{14}$$

Before moving on directly to comparative statics, it is important to briefly discuss the above result to ensure its full understanding. For this reason, a few remarks are in order.

First, thanks to the Neumann series expansion, it follows that the single elements of matrix $\widehat{\mathbf{M}}$ take the following form:[15]

$$\widehat{m}_{ij} = \begin{cases} 1 + w_i\left(\phi^2 \sum_{k\neq i} w_k \widehat{g}_{ik}^2 + \phi^3 \sum_{k\neq i}\sum_{\ell\neq\{i,k\}} w_k w_\ell \widehat{g}_{ik}\widehat{g}_{k\ell}\widehat{g}_{\ell i} + \ldots\right), & \text{if } j = i \\ w_i\left(\phi\widehat{g}_{ij} + \phi^2 \sum_{k\neq\{i,j\}} w_k \widehat{g}_{ik}\widehat{g}_{kj} + \phi^3 \sum_{k\neq\{i,j\}}\sum_{\ell\neq\{i,j,k\}} w_k w_\ell \widehat{g}_{jk}\widehat{g}_{k\ell}\widehat{g}_{\ell i} + \ldots\right), & \text{if } j \neq i\,. \end{cases} \tag{15}$$

The diagonal elements (i.e., when $j = i$) are the marginal effects of one's productivity α_i, whereas the off-diagonal elements (i.e., when $j \neq i$) are the marginal effects of other (directly or indirectly linked) agents' productivity on their own equilibrium effort.

From (15), it is straightforward to see that the marginal effect of one's productivity on one's equilibrium effort—in addition to the direct effect of one's productivity—also includes the "reflected effect" of one's productivity on direct peers' effort (see second term) and circular friendships such as triads (see third term).[16] The reflected effects are, however, attenuated compared to one's direct effect, and this attenuation intensifies as the friendship circle grows.[17]

On the other hand, the marginal return to peer productivity on one's equilibrium effort includes both the effects of direct as well as indirect ties' productivity. Further details on the marginal effects of own and peer productivity are foreseen in the next section on comparative statics.

The second point worth noting is that for the solution to be interior, the following result must hold.[18]

Corollary 1. *(Lower bound on r_i for low-productivity students). For each $i = 1, 2, ..., N$, to guarantee $e_i^* > 0$, the following inequality must hold true:*

$$r_i > \exp\left(-\frac{\alpha_i}{\phi\overline{e}_{-i}^*}\right) - 1, \tag{16}$$

where $\overline{e}_{-i}^* = \widehat{\mathbf{g}}_i\mathbf{e}^* = \widehat{\mathbf{g}}_i\left(\widehat{\mathbf{M}}\boldsymbol{\alpha}\right)$, and $\widehat{\mathbf{g}}_i$ is the i-th row of the row-normalized matrix $\widehat{\mathbf{G}}$.

Corollary 1 suggests that for the equilibrium solution to be interior, individuals with very low productivities relative to their peers cannot have too low self-perceptions. To see why, one needs

[15]See Appendix A for more details.
[16]Circular friendships are friendships that start and end at the same node.
[17]This happens for two reasons. On the one hand, since $0 \leq \phi \leq 1$, it means that $\lim_{k\to\infty}\phi^k = 0$. On the other hand, because $0 \leq \widehat{g}_{ij} \leq 1$, the more $\widehat{g}_{ij}$s are multiplied with each other, the smaller their product becomes.
[18]As shown in Appendix A the lower bound on r_i can be derived by calculating $e_i^* > 0$ from the equation (9).

to recognize that $\overline{e}^*_{-i}$ is a weighted average of the productivity among i's direct peers and that the main argument of the exponential function is a ratio of one's productivity relative to the peers' weighted productivity.[19]

Now, if one's productivity is larger than the peer average productivity—that is the argument within the exponential function exceeds 1 in absolute terms—, then r_i can take any value within its interval of definition $[-0.5, 0.5]$.

If, however, $\frac{\alpha_i}{\phi \overline{e}_{-i}} < 0.7$—that is, i is (much) less productive than its peers—then r_i can no longer take any value on the interval $[-0.5, 0.5]$, but it has to be smaller in absolute terms than -0.5 in order for e^*_i to be strictly positive. In other words, individuals with much lower productivities than those of their peers cannot have too low self-perceptions; otherwise, they are bound to exert zero effort.[20]

Another noteworthy aspect to consider is that there are precisely three cases in which the equilibrium effort exactly corresponds to one's productivity, i.e., $e^*_i = \alpha_i$, for all $i = 1, 2, ..., N$. First: Students do not compare to each other, that is, if $\phi = 0$. Second: There are no social interactions, and all students are isolated, that is, if $\widehat{g}_{ij} = 0$ for all $i, j = 1, 2, ..., N$. Third: All students perceive themselves as average, that is, if $r_i = 0$, for all $i = 1, 2, ..., N$.

While the first and second cases may appear obvious, the implications of the third case are more intriguing. This suggests that when individuals believe themselves to be average, with no significant differences from others, there is no motivation or ambition for further improvement.

To conclude, the list of remarks, it is worth discussing how this model contributes to the understanding of alternative mechanisms underlying peer effects other than social norms. To this end, let us consider equation (9) and the following result derived therefrom.

Proposition 2. *(Effect of peer effort). Given a best-response function defined as in (9), then it follows that:*

(i) For all $i, j = 1, 2, ..., N$ and $i \neq j$, a marginal change in individual j's effort leads to a change in individual i's equilibrium effort, e^*_i, that does not exceed 1 in absolute terms,

$$\left| \frac{\partial e^*_i}{\partial e_j} \right| < 1.$$

(ii) The sign of the effect is uniquely determined by individual i's self-perceived ability rank, r_i. In fact,

$$\frac{\partial e^*_i}{\partial e_j} = \phi \ln(r_i + 1) \lesseqgtr 0 \quad \Leftrightarrow \quad r_i \lesseqgtr 0 \,.$$

The first part of Proposition 2 states that the marginal effect of an increase in a peer's effort on one's own effort never exceeds one in absolute value. In simpler terms, if a friend studies one extra hour per week, their own effort does not change by exactly one hour but by somewhat less.

The second part of Proposition 2 further suggests that the direction in which one's effort changes only depends on one's perception of one's academic abilities. If one believes that one has above-average abilities (i.e., $r_i > 0$), then a marginal increase in direct peers' effort results in an increase in one's own effort. In other words, one's and others' efforts are *complements*.

[19] Appendix B.1 presents the same bound for the dyadic case, which may ease the interpretation of (16).

[20] Notice that this bound can be seen as more than just a simple theoretical artifact and rather as an expression of the so-called Dunning-Kruger effect, suggesting that people with low cognitive abilities tend to be more overconfident and lack awareness of their cognitive limits.

If, however, one lacks confidence in one's abilities and believes that one has below-average abilities (i.e., $r_i < 0$),[21] then a marginal increase in direct peers' effort leads to a decrease in their own equilibrium effort. Hence, one's and others' efforts are *substitutes*.

Finally, in the case where students perceive themselves to be precisely average ($r_i = 0$), any changes in peers' effort have no impact on their equilibrium effort.

This threefold nature is the core peculiarity of this model that also distinguishes it from the local-average and the local-aggregate models. While the latter models allow for positive peer effects only, here, the relationships between one's own and peer effort can potentially take three turns: (i) *complementarity at the top*, (ii) *discouragement at the bottom*, and (iii) *indifference in the middle*.

The main advantage of the present model lies in its versatility, as it provides a theoretical foundation for explaining a range of empirical findings, even in cases where one's effort and peer effort may not necessarily exhibit strategic complementarity.

For instance, these empirical findings include the negative impact of high performers on low-ability students, (Antecol, Eren, and Ozbeklik, 2016; Burke and Sass, 2013; Carman and Zhang, 2012; Carrell, Sacerdote, and West, 2013; Feld and Zölitz, 2017; Gibbons and Telhaj, 2016; Imberman, Kugler, and Sacerdote, 2012), the absence of an effect on middle-performance students (Antecol, Eren, and Ozbeklik, 2016; Imberman, Kugler, and Sacerdote, 2012), the negative impact of better peers on high-ability students (Antecol, Eren, and Ozbeklik, 2016) or the positive effect of better-performing peers on low-ability students (Carrell, Fullerton, and West, 2009).

According to the logic of the model, the negative effect of high-performers on low-ability students may be driven by a discouragement effect experienced by the low-ability students.[22] If exposure to better-performing peers led to lower evaluations of their own abilities, low-ability students may exhibit lower engagement and effort levels from the beginning.

Likewise, the absence of an effect on middle-ability students may be attributed to their lack of interest in competition and in achieving top positions because of their modest beliefs about their academic abilities.

On the other hand, there may also be cases when students misjudge their abilities. Highly productive students may perceive themselves as not smart enough and exert less effort when exposed to more competitive peers. Conversely, low-ability students with high confidence in their ability to learn exert more effort when exposed to more hard-working peers. Either case provides a theoretical explanation for why high-ability students suffer from the presence of other good students or why low-ability students may benefit from the presence of high-performing peers.

Together, all these examples demonstrate how the present model can effectively capture and can provide theoretical explanations for contrasting empirical patterns.

3 Comparative statics

To better understand the model and its implications, I now present the comparative statics with respect to the key parameters defining the Nash equilibrium in the following order: one's and

[21] Note that although in real life self-efficacy and actual abilities are positively related (i.e., low-ability students tend to have low self-efficacy, high-ability students tend to have high self-efficacy), this model also allows above-average-ability students to have low self-efficacy.

[22] Notice that this does not preclude the possibility that the effect is also driven by assortative matching, as suggested by Carrell, Sacerdote, and West (2013).

peer productivity, one's and peer self-perceived ability rank, the degree of social comparison in the network, and the network density.

3.1 Effect of productivity

Using (9) and (11), one can find the following results.

Proposition 3. *(Effect of productivity). Given a Nash equilibrium as in (11), it follows that:*

(i) For all $i, j = 1, 2, ..., N$, a marginal change in individual j's productivity, α_j, leads to a change in individual i's equilibrium effort, e_i^*, that can be either positive or negative but does not exceed 1 in absolute terms. That is,

$$\left|\frac{\partial e_i^*}{\partial \alpha_j}\right| = |\widehat{m}_{ij}| < 1.$$

*(ii) For all $i, j = 1, 2, ..., N$, a marginal change in individual j's productivity, α_j, leads to a change in individual i's equilibrium performance, $y_i^*(\boldsymbol{\alpha}, \mathbf{r}, \mathbf{e}^*, \mathbf{G})$, and equilibrium utility, $U_i^*(\boldsymbol{\alpha}, \mathbf{r}, \mathbf{e}^*, \mathbf{G})$, that is equivalent to exactly **half** of the change in equilibrium effort, e_i^*, which can be either positive or negative but does not exceed 1 in absolute terms. That is,*

$$\left|\frac{\partial y_i^*(\boldsymbol{\alpha}, \mathbf{r}, \mathbf{e}^*, \mathbf{g})}{\partial \alpha_j}\right| = \left|\frac{\partial U_i^*(\boldsymbol{\alpha}, \mathbf{r}, \mathbf{e}^*, \mathbf{g})}{\partial \alpha_j}\right| = \frac{1}{2}|\widehat{m}_{ij}| < 1.$$

*(iii) If i and j are **directly linked**, then the direction of the effect of j's productivity on i's effort, performance, and utility is **solely** determined by the **i's beliefs** in their abilities, r_i. If i believes in their own abilities (i.e., $r_i > 0$), then the effect is positive; otherwise, it is negative.*

*(iv) If i and j are **indirectly linked**, then the direction of the effect of j's productivity on i's effort, performance, and utility is determined by **i's beliefs and** all the **intermediary agents' beliefs** (i.e., r_k, with $k \neq i, j$).*

(v) In equilibrium, effort, e_i^*, academic performance, $y_i^*(\boldsymbol{\alpha}, \mathbf{r}, \mathbf{e}^*, \mathbf{g})$, and utility, $U_i^*(\boldsymbol{\alpha}, \mathbf{r}, \mathbf{e}^*, \mathbf{g})$ increase in one's own productivity, α_i. That is,

$$\widehat{m}_{ii} > 0\,, \qquad \forall\; i = 1, 2, ..., N.$$

The first two points are obtained from (11) and (13)-(14). The first interesting aspect that emerges from the comparison of points (i) and (ii) is that the productivity effect is always smaller on the outcome and utility than that on one's effort in equilibrium. This result is the byproduct of the explicit distinction of action versus outcome and aligns with a commonly observed empirical pattern whereby peer effects observed on behaviors are larger than those on academic outcomes (Sacerdote, 2014).

The points (iv) and (iv) of Proposition 3 are less straightforward. To see why the direction of productivity effects from direct and indirect ties can differ, it is helpful to examine equation (15).

Let us consider the second expression of equation (15). The first term represents the marginal return of direct friends' productivity. The second term captures the return to productivity of indirect friends who are one node apart from individual i, while the third term captures that of agents who are two nodes apart, and so on. While all terms are multiplied by w_i, the function of individual i's beliefs about their abilities, the second and third terms additionally depend on the intermediary nodes' beliefs, denoted as w_k and w_ℓ.[23]

Finally, Point (v) of Proposition 3 suggests that marginal increases in one's productivity always translate into positive changes in one's equilibrium effort. This result is a direct consequence of the spectral radius of $\widehat{\mathbf{WG}}$ being strictly smaller than 1 in absolute terms.[24] Considering the first expression of (15), even if w_i is negative, its product with other w_j's and g_{ij}'s never exceeds 1. Hence, the expression of $\widehat{m}_{ii}$ must always be positive.

Before moving on to the next determinant of the equilibrium effort it is worth briefly comparing these results with the ones found by Ushchev and Zenou (2020) and Liu, Patacchini, and Zenou (2014) for local-average and the local-aggregate models, respectively. This serves for a better understanding of the peculiarities and novelties of the social comparison model.

The first main difference lies in the sign of the marginal change in equilibrium effort as peer productivity changes. While in both the local-average and the local-aggregate model the effect is positive, here it can be both positive and negative. In all three models, however, the effect is strictly smaller than 1 in absolute terms.

The second main difference lies in the marginal change in equilibrium utility given a marginal change in peer productivity. While both here as well as in the original model the marginal effect of productivity on equilibrium utility can be both positive and negative (although for different reasons), in the original local-average model this marginal effect is not *always* smaller and linearly proportional to the marginal effect on effort as it is the case in the social comparison model.

3.2 Effect of self-perceived ability rank

By differentiating the best-response function in (9) with respect to r_i and r_j, one can find the following results.[25]

Proposition 4. *(Effect of self-perceived rank). Given a best-response function as in (9), it follows that:*

(i) A marginal increase in individual perception of one's abilities increases one's optimal effort. Formally,

$$\frac{\partial e_i^*}{\partial r_i} > 0\,. \tag{17}$$

(ii) A marginal increase in peer perception of their abilities can both increase and decrease their own optimal effort, as

$$de_i^* = \phi \ln(r_i + 1)\frac{d\bar{e}_{-i}}{dr_j}\,. \tag{18}$$

[23]In Appendix B.2 I also present the example of a three-agent star network—where one agent (the central node) is linked to everybody else—where the dependence on the intermediary node's (in the example the central node) beliefs is even more salient.

[24]See Appendix A for the proof.

[25]Proofs can be found in Appendix A.

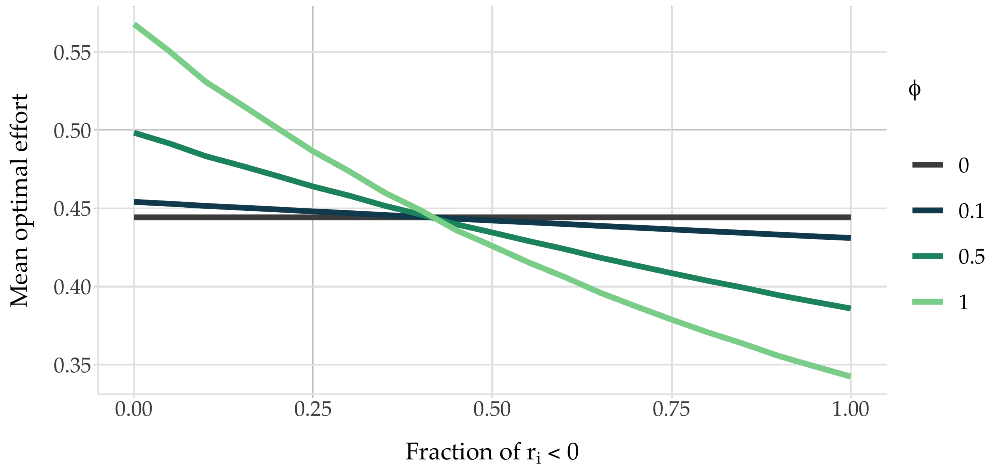

Figure 1: Mean effort given the fraction of L-type individuals.

Since the derivative of (9) with respect to r_i depends on only positive variables—including ϕ and peer average effort—it is straightforward to see that as one improves the self-assessment of one's abilities, their optimal effort also increases.

Less straightforward is the direction that a marginal change in peer self-assessment can have on one's equilibrium effort. From (18) it is clear that the sign depends both on their own and the peer self-perception of their abilities. How peer self-perception influences the impact of peer self-perception on one's equilibrium effort is somewhat clearer from equation (15), which suggests that the direction of this impact depends on the individual's self-assessment and the assessments of intermediate nodes.

To obtain a complete picture of how variations in self-perception by other agents affect individual optimal effort, one would need to consider several specific cases (e.g., direct, or indirect friends who are 1, 2, or k nodes apart), but the problem becomes readily overly complex and intractable by proceeding in this way.

To overcome the complexity of the problem, I rely on a simulation exercise in which I hold both the productivity distribution, the size, and structure of the network constant, varying only the distribution of r_i among agents.

The simulation consists of the following steps. First, I draw a random Erdos-Renyi network of size $N = 20$ (i.e., approximately the size of a classroom) with medium density ($P(\mathbf{G}) = 0.30$) and a random vector of productivities $\boldsymbol{\alpha}$.[26]

Let L-types (H-types) denote agents with negative (positive) self-perceptions, that is, $r_i < 0$ ($r_i > 0$). The fraction of L-types is then defined as $\pi = P(r_i < 0)$. For all possible fractions $\pi \in [0, 1]$, I randomly draw 1,000 times $N \times 1$ random vectors of $\mathbf{r} = (r_1, r_2, ..., r_N)$ from the interval $[-0.5, 0.5]$. Since there are only 21 possible fractions of L-types in a network of 20 agents (with $\pi = 0$ included), I end up with a total of 21,000 different random vectors of $\mathbf{r}$.

To also explore the impact of social comparison, I consider three different levels of $\phi \in \{0.1, 0.5, 1\}$. Starting with one of these levels, I compute the vector of equilibrium efforts for each of the 21,000 random vectors $\mathbf{r}$ by using expression (11). The average equilibrium effort is then calculated as the arithmetic mean of the resulting effort vector.

[26] The random vector of productivities is drawn from a normal distribution centered at 0.5 and with standard deviation of 0.15. Potential negative values are substituted with 0.05.

The result of the simulation exercise is presented in Figure 1, which depicts the relationship between the mean equilibrium effort and the fraction of L-types in the network.

Figure 1 reveals an interesting relationship. As the fraction of agents with pessimistic beliefs about their abilities increases, the average optimal effort in the network decreases. This negative effect appears to be amplified by higher values of social comparison (ϕ). This indicates that when individuals heavily rely on others' as their reference point to evaluate their own abilities, the overall effort level in the network declines more pronouncedly as the fraction of L-types increases.

In Figure 1, the amplification effect caused by stronger degrees of social comparison can be observed by comparing the steepness of the lines. Indeed, the light-green line representing the situation with the strongest social comparison ($\phi = 1$) also has the steepest slope, indicating a more significant decrease in effort as the fraction of pessimistic agents increases.

Another noteworthy peculiarity of Figure 1 is that all lines intersect at a specific point. This intersection point is where the average optimal effort equals the average productivity. This can be inferred from the fact that all lines intersect with the dark-gray horizontal line where $\phi = 0$. In fact, when $\phi = 0$, the equilibrium solution is simply $\mathbf{e}^* = \boldsymbol{\alpha}$.

Now, this implies the existence of a threshold (of approximately 40% in Figure 1) in the fraction of individuals with negative self-perceptions for any $\phi > 0$. Below this threshold, the average optimal effort exceeds what is expected in the absence of social comparison (or social interactions in general), while above this threshold, the mean optimal effort falls below the level that is observed without social comparison (or social interactions).

From a policy perspective this is a very interesting result for two reasons. First, this result suggests that everybody's self-perception matters for the overall effort choice in the network. Second, Figure 1 also indicates that the presence of individuals with negative self-perceptions is not unconditionally harmful to the entire network. While it is true that when the fraction of such individuals increases, the average effort in the equilibrium decreases, their presence becomes detrimental to the entire network only when their fraction exceeds a given threshold. The latter is reached when the average optimal effort is equal to the effort level that can be observed in the absence of social interactions.

At this point, in light of Proposition 4, it is natural to ask the following question. How do agents with different self-perceptions of their abilities react in equilibrium when exposed to a more optimistic or pessimistic learning environment?

To answer this question, I compute the averages by agent type (i.e., L vs. H) by using the optimal efforts computed in the previous step. Figure 2 presents the results from these computations. Each panel presents the results for a different value of ϕ. The light-green line represents the mean optimal effort of H-type agents, while the dark-blue line represents the mean optimal effort of L-type agents.

The top left panel represents the simplest case scenario where individuals do not compare themselves with others (i.e., when $\phi = 0$). In the latter case, the equilibrium effort of both types is exactly equal to their average productivity. Since productivity is randomly assigned,[27] the average effort of both types is essentially identical and equal to the average productivity.

As soon as social comparison is considered, the average optimal effort for the two agent types

[27] Recall that r_i and α_i are considered independent in this model, and the simulation exercise respects this abstraction from reality.

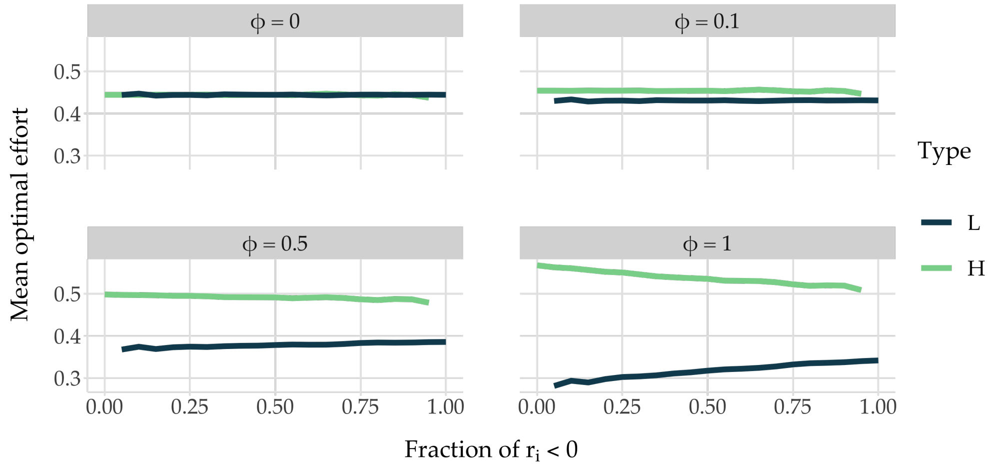

Figure 2: Effect of r_i on optimal effort for individuals with high (H) and low (L) self-assessment.

begins to diverge, with the effort of type H being consistently higher than that of type L for all $\phi > 0$—despite their average productivity being similar.

Moreover, they seem to follow more or less parallel patterns when ϕ is low, but when ϕ increases and reaches its maximum (i.e., $\phi = 1$), it appears that the two groups are affected differently by an increasing number of L-type individuals. While the average equilibrium effort of H-type agents decreases, the average optimal effort of L-type agents increases.

The main reason for this interesting result is attributed to the way the outcome production function is specified. The specification of the outcome technology implies that one's self-perception determines whether peer effort is beneficial or detrimental to individual academic performance. While H-type individuals benefit from more hard-working peers, L-types experience a decline in academic performance when their peers exert greater effort. Referring to equation (9), this indicates that for any value of $\phi > 0$, H-types consistently exert effort levels that exceed their own productivity, while L-types consistently exert effort levels below their productivity.

As a result, when the fraction of L-types increases, there is a rise in the proportion of individuals whose effort falls below their productivity. This implies that H-types have less to gain from further increasing their effort, as the average peer effort decreases. On the other hand, L-types have less to lose, as the impact of diligent peers becomes progressively smaller due to their decreasing presence in the network.

The above analysis reveal three noteworthy results that can be summarize in the following conjecture.

Conjecture 1. *(Effect of peer self-perceived ability rank).*
Let us consider a network of N agents characterized by either a high or low self-perception of their abilities, which is denoted by $r_i > 0$ and $r_i < 0$, respectively. Let $\pi := P(r_i < 0)$ be the fraction of agents with a negative self-perception of their abilities and let $\boldsymbol{\iota}$ be a $N \times 1$ vector of ones. Individuals with negative self-perception of their abilities are defined as L-type agents, and all others are defined as H-type agents. Finally, let $0 < t < 1$ denote a certain threshold. Then,

(i) As $\pi \to 1$, the average equilibrium effort, $\mathbf{e}^{*T}\boldsymbol{\iota}$, decreases.

(ii) If $\pi \leq t$, then the average effort in the network is greater than the average productivity, that is, $\frac{1}{N}\mathbf{e}^{*T}\boldsymbol{\iota} \geq \frac{1}{N}\boldsymbol{\alpha}^T\boldsymbol{\iota}$.

(iii) If $\pi > t$, then the average effort in the network is lower than the average productivity, that is, $\frac{1}{N}\mathbf{e}^{*T}\boldsymbol{\iota} < \frac{1}{N}\boldsymbol{\alpha}^T\boldsymbol{\iota}$.

(iv) As $\phi \rightarrow 1$, the negative impact of L-type agents' presence on others' equilibrium effort becomes stronger.

(v) As $\pi \rightarrow 1$, the average equilibrium effort among L-type agents increases, while the average effort among H-type agents decreases.

3.3 Effect of social comparison

I now shift the focus to examining the comparative statics of the degree of social comparison, ϕ. Starting from the best-response function, one finds the following results.

Proposition 5. *(Effect of social comparison). Given a best-response function specified as in (9), it follows that:*

(i) The total effect of a marginal change in the degree of social comparison on individual equilibrium effort is given by:

$$de_i^* = \ln(r_i + 1)\bar{e}_{-i}\, d\phi + \phi \ln(r_i + 1)\frac{d\bar{e}_{-i}}{d\phi}. \tag{19}$$

*(ii) For all $i = 1, 2, ..., N$, a marginal change in the degree of social comparison, ϕ, increases (decreases) the **individual effort** in the equilibrium, if individual i has positive (negative) beliefs about his or her own abilities. Formally,*

$$\frac{\partial e_i^*}{\partial \phi} \gtreqless 0 \qquad \Leftrightarrow \qquad r_i \gtreqless 0. \tag{20}$$

The first point suggests that changes in the degree of social comparison affect the equilibrium effort through two different channels, a direct and an indirect one. The direct effects are steered by one's self-perception, while the indirect effects originate from the effect the degree of social comparison has on one's peers, and its sign is determined by the product of one's and others' self-perceptions.

The second point further suggests that the direction in which equilibrium effort changes given a marginal increase in the degree of social comparison, ϕ, primarily depends on one's self-perception. Hence, it can be both positive and negative. Since the first term of equation (19) is always larger in absolute terms than the second, and the sign of the first term is determined by the individual self-perception, r_i, it follows that the sign of the total derivative is determined by one's beliefs about one's abilities.

If an individual holds positive beliefs about his or her own abilities, a further increase in social comparison benefits his or her own equilibrium effort. If, however, one does not have a positive perception of the self, stronger comparison reinforces only personal discouragement and even further decreases equilibrium effort.

Furthermore, as mentioned earlier, the degree of social comparison acts as an amplifier of peer influence on one's effort. Specifically, as social comparison increases, both peer effort and peer self-perception have a greater impact on one's equilibrium effort, regardless of the direction of this influence.

From a practical point of view, this suggests that the differences in the magnitude of peer effects observed in the empirical literature may also be the result of cultural differences, such as different degrees of social comparison, and not only by different outcomes and peer characteristics considered.

Additionally, worth noting is that the degree of social comparison and the taste for conformism from the original local-average model (denoted as λ in Ushchev and Zenou, 2020) lead to similar results but for conceptually different reasons.

First, if different from zero, both parameters can lead the equilibrium effort to be higher or lower than one's productivity. In the original local-average model the individual equilibrium effort is larger (smaller) than the one's productivity if one's productivity level is larger (smaller) than that of their peers. In the social comparison model, instead, the equilibrium effort exceeds one's productivity if one has positive beliefs about one's abilities, and it falls short of one's productivity if one has negative beliefs instead. Hence, while in the original case effort exceeds or falls short of one's productivity because of differences between individual and peer *actual* abilities, here, it purely depends on one's *perception* of that difference.

Second, in individualistic societies where there is neither social comparison (i.e., when $\phi = 0$) nor social norms, the local-average model and the social comparison model yield the same result in equilibrium. Namely, the equilibrium efforts simply correspond to the individual productivities.

Third, both in perfectly conformist societies ($\lambda \to 1$ in Ushchev and Zenou, 2020) and in societies with extreme social comparison ($\phi \to 1$), equilibrium efforts depend on weighted productivity, although weights are defined differently. While in the local-average model weights are always positive and depend on the network structure only, here, they can also be negative and depend on the individual self-perceptions of the entire network.

3.4 Effect of network density

Provided that network structure is one of the determinants of the equilibrium effort in this model, I conclude the analysis on comparative statics by studying how equilibrium effort changes as network density (i.e., connectedness among agents) increases.

The more elegant approach for studying the impact of network density on equilibrium effort consists in comparing networks with and without a given connection, as done by Ushchev and Zenou (2020). However, in my model, this exercise may easily become overly complex and intractable because of agents' twofold nature. Namely, adding a new tie between two agents with positive beliefs may have totally different consequences than those of adding a tie between two agents with opposite beliefs about their abilities. These results may further change depending on the distribution of beliefs among existing ties.

Therefore, to overcome the complexity of the problem, I rely once again on simulations. I begin with an initially empty graph consisting of $N = 20$ agents. I gradually increase the density by adding one nonexistent tie at a time in a random order until all agents are connected with each other and the resulting network density is 1. This process generates 190 different versions of the graph (i.e., $N(N-1)/2$ possible connections). Next, I generate a random vector of productivities from a normal distribution with a mean of 0.5 and a standard deviation of 0.15 and define eleven different values of social comparison, ϕ, ranging from 0 to 1 with increments of 0.1.

To examine whether the impact differs for different distributions of self-perceived ranks, I

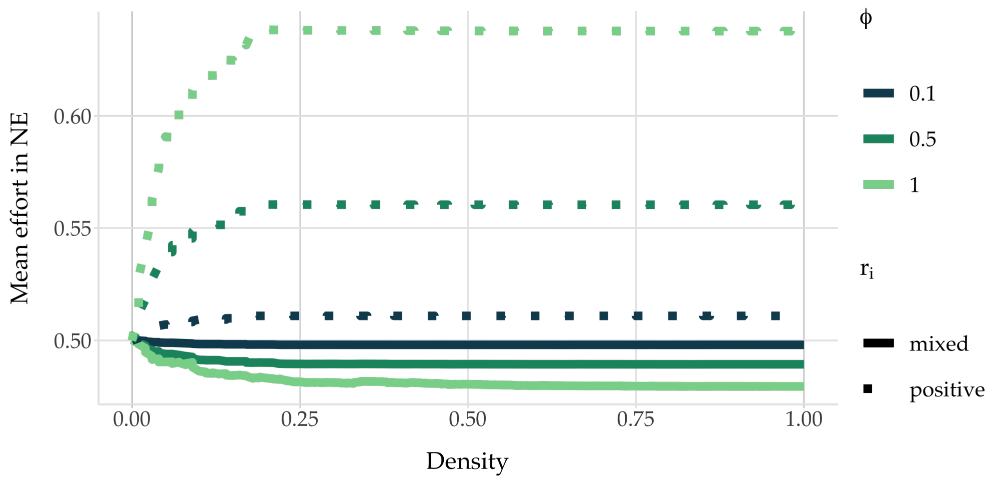

Figure 3: MEAN EFFORT IN EQUILIBRIUM GIVEN DIFFERENT DEGREES OF NETWORK DENSITY.

consider two different distributions for the vector $\mathbf{r}$. A *mixed distribution* is randomly drawn from the interval $[-0.5, 0.5]$, and a *positive distribution* is randomly drawn from the interval $[0, 0.5]$. Each of these random vectors is generated 10,000 times.

For each degree of social comparison, network structure, and self-perception distribution, I compute the vector of equilibrium efforts and its arithmetic mean.[28] This results in a total 41,800,000 ($= 11 \times 190 \times 10,000 \times 2$) simulations, or 10,000 mean equilibrium efforts for each combination of ϕ, network density, and self-perception distribution.

Figure 3 presents the result from this exercise. Each point of the line is the average of the 10,000 mean equilibrium efforts by network density, degree of social comparison, and self-perception distribution.

The line colors represent different degrees of social comparison, where dark blue indicates low ($\phi = 0.1$), dark green indicates medium ($\phi = 0.5$), and light green indicates high social comparison ($\phi = 1$). The line types distinguish between two types of self-perception distributions: the dotted lines denote cases where all agents hold positive beliefs, whereas the solid lines indicate cases where agents have mixed beliefs about their abilities.

On one hand, Figure 3 confirms some of the previous results presented above. First, the fact that all solid lines lie below the dotted lines suggests that the mean equilibrium effort is higher when everyone has positive beliefs than when some agents have negative beliefs about their abilities. Second, the fact that the two light-green-shaded lines (whereby $\phi = 1$) lie on the outer edges highlights that the degree of social comparison acts as an amplifier of the impact that self-perception has on the average equilibrium effort.

On the other hand, Figure 3 reveals that network density also acts as an additional amplifier of this very same impact. In particular, when adding new ties to sparser networks, the mean equilibrium effort tends to be larger is all agents hold positive beliefs about their abilities, and even smaller when agents hold mixed beliefs.

However, the amplification caused by the increase in network density is milder than that caused by the degree of social comparison. As shown in Figure 3, all curves follow a concave pattern, so beyond a certain density (e.g., approximately 0.25 in Figure 3), further increases in density do not lead to any further significant increase or decrease in the mean equilibrium effort.

[28]For all combinations, I use that same productivity vector.

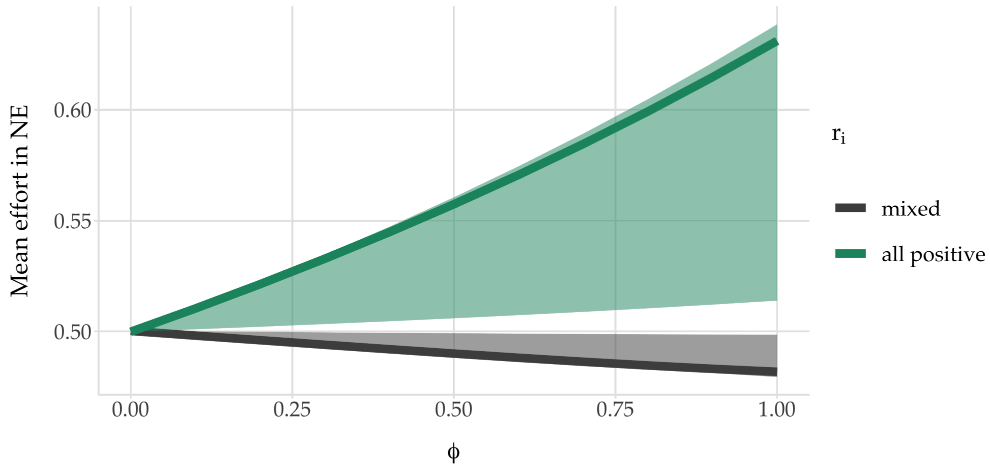

Figure 4: Mean effort for given ϕ and network densities.

To demonstrate that the amplification effect resulting from network density is less pronounced compared to that caused by social comparison, I present the findings of the same simulation exercise from a slightly different perspective, specifically emphasizing the role of social comparison.

Figure 4 shows how the mean equilibrium effort evolves as the degree of social comparison increases under the two considered scenarios: Everyone holds positive beliefs about their abilities (green line) versus some agents have negative beliefs (gray line). The shaded areas surrounding the lines represent all possible lines under the various network densities. The outer bounds correspond to the case where all agents are connected, while the inner bounds represent the case where only two agents are connected.

From Figure 4, the following mainly emerges. Given a certain network structure, a certain productivity distribution, and a *random* self-perception distribution, the mean equilibrium effort *on average* increases (decreases) as the degree of social comparison grows, if and only if all (*some*) agents hold positive beliefs about their abilities.[29]

The above results on the effects of network density can summarize through the following conjecture.

Conjecture 2. *(Effect of network density).*
Let us define network density as $P(\mathbf{G}) = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{j=1}^{N} g_{ij}$*. Let* $\boldsymbol{\iota}$ *be a* $N \times 1$ *vector of ones and let* $0 < t < 1$ *denote a certain threshold. Then,*

(i) As $P(\mathbf{G}) \to 1$*, the average equilibrium effort* $\frac{1}{N}\mathbf{e}^{*T}\boldsymbol{\iota}$ *increases if and only if* $P(r_i < 0) \leq t$ *and decreases otherwise.*

(ii) If $\boldsymbol{\alpha}_1 = \boldsymbol{\alpha}_2$*,* $N_1 = N_2$*, and* $\mathbf{r}_1 = \mathbf{r}_2$*, but* $\phi_1 < \phi_2$ *and* $P(\mathbf{G_1}) < P(\mathbf{G}_2)$*, then* $\frac{1}{N}\left|\mathbf{e}_{\mathbf{1}}^{*T}\boldsymbol{\iota}\right| < \frac{1}{N}\left|\mathbf{e}_{\mathbf{2}}^{*T}\boldsymbol{\iota}\right|$.

In terms of comparison with existing models, both of these results share similarities with the findings from the two leading models, the local-aggregate and the local-average model.

Point (i), for example, aligns with Proposition 6 presented by Ushchev and Zenou (2020). In the local-average model, they demonstrate that adding an additional link can result in both an increase

[29] Please note that the reason why Figure 4 suggests that having just one agent with negative beliefs is sufficient for the mean equilibrium effort to be lower than the average ability is due to the way the picture was generated. In this case, the vector of self-perceived ability ranks, denoted as $\mathbf{r}$, is randomly sampled without keeping the fraction of negative entries fixed, unlike in Figure 1. This variation in sampling methodology accounts for the observed differences between the two figures.

and a decrease in the equilibrium effort. Specifically, if both agents have productivities above a certain threshold, the effect is positive. However, if one of the agents has a productivity below that threshold, an increase in network density leads to a decrease in the effort of all individuals. In my case, ambivalence arises due to the fraction of individuals with low self-perceptions, rather than their productivities relative to their direct peers.

Point (ii), on the other hand, aligns with the results presented by Calvó–Armengol, Patacchini, and Zenou (2009) for local-aggregate models. In the local-aggregate model, regardless of the productivities involved, all agents consistently exert greater effort in denser networks. Similarly, in my case, the *magnitude* of the effect also increases as the network density increases.

4 Discussion

To understand the implications of the social comparison model, it is helpful to compare it with the original local-average model, given that it is a modified version of the latter. While many technical differences and similarities have been previously highlighted, the following discussion primarily centers on comparing the two models in terms of their policy implications. Specifically, I show that the two models can yield similar outcomes but for distinct underlying reasons and discuss the potential consequences for policy design. I conclude with an illustrative example of three different interventions to further illustrate the points.

4.1 Comparison to the local-average and local-aggregate models

It should be clear by now that the major difference between the two models is purely conceptual. While in the original local-average model individuals strive to be similar to others, in the social comparison model, individuals care about (and are influenced by) their own position in the social hierarchy.

However, despite the different societal portrays and two very distinct mechanisms, the two models also share some peculiarities. Precisely two of these also have important implications regarding policy interventions.

The first property regards the *ambivalent effect that the parameters characterizing social interactions—taste for conformism* in the local-average model and *degree of social comparison* in the social comparison model—have on equilibrium effort.

In the local-average model, the marginal effect of taste for conformism (denoted as θ in Ushchev and Zenou, 2020) on effort can be both positive and negative. The direction of this effect depends on whether an individual's equilibrium effort is higher or lower than the average equilibrium effort of their neighbors. As *conforming* with others becomes more important, agents who exert more effort than their neighbors (i.e., $e_i^* > \overline{e}_{-i}^*$) need to reduce their effort, whereas individuals who exert below-average effort (i.e., $e_i^* > \overline{e}_{-i}^*$) need to increase their effort to meet the group standards.

Similarly, in the social comparison model, a marginal increase in the degree of social comparison can have both positive and negative effects on the equilibrium effort. When comparison with others becomes more common, agents who believe they have above-average abilities (i.e., $r_i > 0$) increase their effort due to their higher level of confidence and motivation to succeed. In contrast, agents who believe they have below-average abilities decrease their optimal effort because of the additional psychological costs involved in social comparison.

Hence, the direction of the marginal effect of the social interaction parameter is determined by a different type of comparison with one's peers in the two models. In the original local-average model, the direction is determined by the *actual relative effort*, whereas in social comparison model, it is determined by the *relative perceived ability* to succeed in a given task.

Although subtle, this conceptual difference can result in opposite outcomes under the same policy intervention. On the one hand, taste for conformism plays the role of an "*harmonizer*" by reducing the effort of high-productivity individuals while increasing the effort of low-productivity individuals and by pushing both types of agents toward a common end: the social norm. On the other hand, social comparison plays the role of a "*divider*" by widening the gap in exerted effort between individuals with low and high beliefs in their abilities.

Since these two mechanisms likely coexist in real life, when designing policies, the consequences of both conformism and social comparison should be accounted for to avoid unintended and potentially undesired outcomes. This also calls for the development of a theoretical framework that integrates both mechanisms into a single model of peer effects.

The second shared property regards the *ambivalent impact of marginal changes in peer productivity* on an individual's equilibrium utility and outcome.[30]

In the original local-average model, peers' productivity has a positive impact on an individual's equilibrium utility if their equilibrium effort exceeds the equilibrium social norm (i.e., if $e_i^* > \overline{e}_{-i}^*$), or equivalently, if their productivity surpasses (falls short of) the weighted average of (direct and indirect) peers' productivity (see Lemma 1 in Ushchev and Zenou, 2020). Conversely, if an individual's equilibrium effort falls below the social norm (i.e., if $e_i^* < \overline{e}_{-i}^*$), then the marginal effect of peers' productivity on their own equilibrium utility is negative. At the same time, individual equilibrium effort increases regardless of whether one's own effort deviates above or below the social norm. This means that some individuals experience a utility loss despite trying harder.

The local-average model with self-perception, instead, predicts that *both* equilibrium effort and equilibrium utility change in the same direction as peer productivity marginally increases. That is, both increase as long as agents hold positive beliefs about their ability to learn and decrease when they hold negative beliefs. This implies that some individuals experience a utility loss because of preexisting differences, namely, self-perception.[31]

The different underlying reasons leading to the loss of utility for some individuals invite further reflection when designing policy interventions. Not only does one need to clarify which of the two frameworks is more appropriate (if any) but also what the scope of the intervention is: Should we focus on increasing everyone's effort or increasing everyone's utility? Answering this question is particularly relevant if conformity is seen as the primary underlying mechanism, answering this question becomes crucial.

The original local-average model calls for *groupwise* policies that simultaneously affect everyone through changes in social norms, such as introducing self-discipline education in schools where it is generally low (see Ushchev and Zenou, 2020). This, as discussed above, can effectively increase everyone's effort but can also harm some agents' well-being.

[30] Note that in the original local-average model, outcome is not separately defined from utility as here. However, if one were to define utility as a function of the outcome in the local-average model too, the same results found for utility also apply to the outcome.

[31] Notably, in my model, agents with low self-concepts also exert lower effort. These very same individuals also exert below-average effort in the original local-average framework and consequently also experience a loss in utility under similar circumstances.

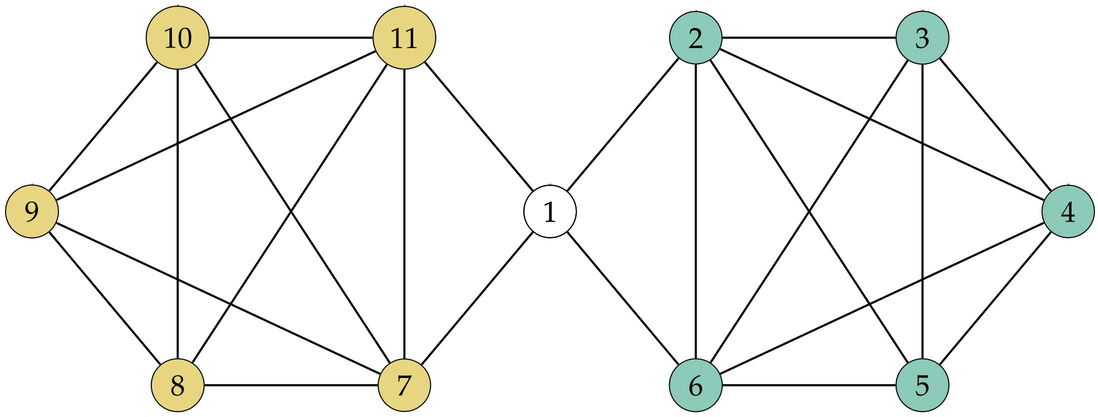

Figure 5: Bridge network.

In contrast, the social comparison model suggests targeting individuals with low self-concepts, for instance, by teaching them strategies to improve their beliefs about their abilities even when exposed to better-performing students. This could potentially lead to less competitive and stimulating learning environments for everybody and could have positive externalities on the entire network, as shown in the comparative statics.

4.2 Illustrative example

To illustrate the above points, I follow the example by Ballester, Calvó–Armengol, and Zenou (2006) and Ushchev and Zenou (2020) and consider a bridge network of 11 nodes, such as the one illustrated in Figure 5.

The equilibrium effort in the local-average model yields

$$\mathbf{e}^* = \widehat{\mathbf{M}}\boldsymbol{\alpha} = \frac{1}{1+\theta}\left(\mathbf{I} - \frac{\theta}{1+\theta}\widehat{\mathbf{G}}\right)^{-1}\boldsymbol{\alpha}, \tag{21}$$

while the equilibrium in the social comparison model yields:

$$\mathbf{e}^* = \widehat{\mathbf{M}}\boldsymbol{\alpha} = \left(\mathbf{I} - \phi\mathbf{W}\widehat{\mathbf{G}}\right)^{-1}\boldsymbol{\alpha}. \tag{22}$$

Everybody's productivity is set to 1, and both social interaction parameters, θ and ϕ, are set to 0.2. In the modified local-average model, I additionally specify the self-perception, which is chosen as follows: the central node believes to be average (i.e., $r_1 = 0$), green nodes have above-average self-perception, $r_i = 0.2$, whereas yellow nodes have below-average self-concepts, with $r_i = -0.2$. The initial total effort is very similar: 11 in the local-average model and 10.97 in the version with self-perception. Furthermore, in the latter case, the total effort among H-type agents is 5.192 and 4.781 among L-type agents.

I consider the following three types of interventions: (i) removing the key player, (ii) increasing the social norm, and (iii) improving the effort of agents with low-self concept. All three interventions are motivated by the designs of the local-aggregate, the local-average, and the social comparison model.

Key player removal. As already shown by Ushchev and Zenou (2020), removing a central node (i.e., agent 1 in Figure 5) reduces the effort of the nodes that were connected to the key player while increasing that of the other agents, so that as a result, everybody exerts the same amount of effort; exactly 1 unit.[32] However, this is not entirely true in the variant with self-perception. In this case, the total effort is reduced to 9.976, with 5.189 contributed by H-types and 4.786 by L-types. Although the removal of the central player leads to a reduction in total effort by approximately 9.09% in both cases, a structural change occurs in the modified version. Specifically, the effort of L-types slightly increases, while the effort of H-types slightly decreases. This outcome can be expected, as L-type individuals have less to lose when there is one less individual with higher effort than theirs, while H-types gain less when there are fewer participants in the competition.

Change in the norm. Let us consider a policy that uniformly increases everyone's productivity from 1 to 1.3. In both the local-average model and the version with self-perception, this policy leads to an increase of approximately 50% in total effort. In the local-average model, the total effort rises to 16.50, while in the version with self-perception, it increases to 16.460, with H-types contributing 7.788 and L-types contributing 7.172. Once again, the two policies produce very similar outcomes in terms of overall effort in both frameworks. However, there are some compositional divergences in the social comparison model. Specifically, increasing everyone's productivity does not equally affect L-type and H-type individuals. L-types experience a slightly smaller percentage increase in their total effort compared to that of the other group (i.e., 49.85% vs. 50%).

Confidence boost. Finally, let us consider a policy that increases L-types' beliefs about their abilities from below-average to precisely average ($r_i = 0$), so that their effort becomes equal to their productivity. In the modified version with self-perception, this change leads to an improvement of 2% of the total effort, which yields 11.192. This increase is completely driven by L-type agents, whose total effort jumps from 4.781 to 5 and a total change of 4.6%, while the effort of H-type individuals remains unvaried by 5.192. In the original local-average model, a similar change could be achieved by increasing the productivity of only the yellow nodes (agents with low self-perception and below-average effort) by 5%.

Assessing whether it is more costly or easier to boost these individuals' self-confidence by 100% or improve their productivity by 5% is not straightforward. If we consider the percentage change in both factors, it might be tempting to conclude that targeting productivity is more effective. However, without concrete measurements, it is impossible to determine a priori whether teaching discipline is easier than teaching individuals how to properly assess what is within their power to learn and what is not. Additionally, the potential long-term effects of each intervention should not be neglected. These questions certainly present intriguing avenues for future research.

5 Concluding remarks

Social comparison—a psychological process whereby individuals establish the value of their own attributes and ideas by comparing with others—is deemed to be a potential mechanism underlying *peer effects*. Yet, little has been done in providing a theoretical framework that formalizes this

[32]This follows from the choice of equal productivity for all agents and the fact that without the key player, in each component of the network, individuals share the exact connections.

particular mechanism. Theoretical work on peer effects has primarily focused on formalizing the mechanism of social learning through two main models: the local-average model and the local-aggregate model. Both models are based on the assumption of positive peer effects. However, starting from this assumption hinders providing a theoretical foundation for empirical findings that contradict it and impedes the recognition of heterogeneous patterns typically observed in the empirical literature (see Cools and Patacchini, 2021; Sacerdote, 2011, 2014, for an overview).

I address this gap by proposing a novel model of social interactions that formalizes the mechanism of social comparison.

The proposed model is a modified version of the local-average model, that integrates the notion of self-perception. Specifically, I consider a static, full-information game where agents differ from one another with respect to two predetermined characteristics: productivity and self-perception of their abilities. Furthermore, agents are assumed to simultaneously choose the optimal amount of effort that maximizes their target outcome, which is determined by two components: a concavely increasing function of one's own productivity and effort and the product of self-perceived ability and peer effort.

The outcome production function is the core of this model and embodies the following rationale. If agents harbor doubts about their own abilities, then the presence of peers with whom they compete or compare imposes extra psychological costs. As a result, outcomes are lower in the presence of social interaction than when realized individually. On the other hand, if agents perceive themselves as capable, the presence of peers serves as an extrinsic motivator, spurring them to exert even greater effort than they would in isolation. This ultimately leads to higher outcomes in the presence of social interactions than in their absence. These dynamics are, furthermore, regulated by the degree of social comparison—the intensity with which agents in a given network compare themselves to others.

Five main results are established in this study. *First*, a unique and—under given circumstances—interior Nash equilibrium exists and is a weighted average of productivities, where the weights are a nonlinear function of the network structure, the degree of social comparison, and the perceived abilities.

Second, peer productivity does not always positively affect individual equilibrium effort, and the direction of this effect strictly depends on one's own and peer self-perception.

Third, the overall distribution of self-perception in the network affects everyone's effort, and both the network density and the degree of social comparison can exacerbate this effect, regardless of its direction. Furthermore, the presence of individuals with low self-concepts is not unconditionally harmful (only above a certain threshold), but the larger their proportion in the network, the lower the average effort in the network.

Importantly, the social comparison model presents a conceptual departure from the original local-average framework. Instead of striving for similarity, individuals in this model are driven by their position in the social hierarchy and its influence on their behavior. This conceptual difference has significant implications for policy interventions too.

The study reveals that similar policies yield different results: While one framework leads to convergences to a norm, the other leads to further divergence between different types of individuals. Moreover, different policy interventions are needed to achieve similar results, with options ranging from groupwise interventions in the local-average setting to targeted measures for the more disadvantaged in contexts with social comparison.

In this study, I provide the first formalization of social comparison (by using the notion of self-perception) in a peer effect model. However, the model has some limitations and opens up avenues for future research. Potential improvements include further exploring the relationship between productivity and self-perceived ability, extending the utility function to incorporate incomplete information or alternative outcome targets (other than y_{max}), and integrating social norms into the model. Additionally, modifying the outcome function to provide a microfoundation into a specific econometric model may be another valuable direction for further investigation.

In conclusion, this study contributes to the understanding of peer effects by introducing self-perception into the local-average model framework. The findings elucidate the complex dynamics between individuals' self-perception, peer interactions, and effort levels, offering insights for policy design and future research endeavors.

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A Proofs

A.1 Proof of Proposition 1

Proof of part (i). The equilibrium effort is given by expression (11), where $\widehat{\mathbf{M}} := \left(\mathbf{I} - \phi\mathbf{W}\widehat{\mathbf{G}}\right)^{-1}$. Hence, the equilibrium *exists* and is *unique* if $\mathbf{I} - \phi\mathbf{W}\widehat{\mathbf{G}}$ is invertible, and thus *nonsingular*. Concretely, we need to prove that

$$\det\left(\mathbf{I} - \phi\mathbf{W}\widehat{\mathbf{G}}\right) \neq 0 \,.$$

Since matrix $\mathbf{W}\widehat{\mathbf{G}}$ is not nonnegative—i.e., its elements can be negative due to $\mathbf{W}$—one cannot directly apply Theorem III of (Debreu and Herstein, 1953, p. 601) as done by Ballester, Calvó–Armengol, and Zenou (2006) to prove the existence of the inverse.

However, matrices of the form $\mathbf{I} - \mathbf{A}$ are always invertible as long as $\rho(\mathbf{A}) < 1$, where $\rho(\cdot)$ denotes the spectral radius (see Mayer, 2000, p. 618). This also ensures a stable equilibrium and asymptotic convergence in case of multiple transformations through the same matrix, which is very important for approximating $\widehat{\mathbf{M}}$ through the Neumann series as in equation (12).

In the specific case, this means that one needs to prove that

$$\phi\rho(\mathbf{W}\widehat{\mathbf{G}}) < 1 \,, \tag{A.2}$$

for any $0 \leq \phi \leq 1$ and $r_i \in \left[-\frac{1}{2}, \frac{1}{2}\right]$, for all $i = 1, 2, ..., N$. This can be accomplished by using Gerschgorin circle theorem (Mayer, 2000, p. 498).

The theorem states that all eigenvalues of a square matrix $\mathbf{A} = [a_{ij}]$ lie within the union of the so-called Gerschgorin disks. Each disk is a circle centered around a diagonal entry, a_{ii}, and radius $r_i = \sum_{j=1}^{N} |a_{ij}|$, which is equal to the sum of the absolute values of the off-diagonal entries in the corresponding row i.

Since $\mathbf{W}$ is a diagonal matrix while $diag(\widehat{\mathbf{G}}) = \mathbf{0}$, it means that the diagonal of their matrix product must also be a zero vector. Namely,

$$\mathbf{W}\widehat{\mathbf{G}} = \begin{pmatrix} w_1 & 0 & 0 & ... & 0 \\ 0 & w_2 & 0 & ... & 0 \\ 0 & 0 & \ddots & & 0 \\ 0 & 0 & \dots & & w_N \end{pmatrix} \begin{pmatrix} 0 & \widehat{g}_{12} & \widehat{g}_{13} & \cdots & \widehat{g}_{1N} \\ \widehat{g}_{21} & 0 & \widehat{g}_{23} & \dots & \widehat{g}_{2N} \\ \vdots & & 0 & \dots & \vdots \\ \widehat{g}_{N1} & \widehat{g}_{N2} & \widehat{g}_{N3} & \dots & 0 \end{pmatrix} = \begin{pmatrix} 0 & w_1 \cdot \widehat{g}_{12} & \dots & w_1 \cdot \widehat{g}_{1N} \\ w_2 \cdot \widehat{g}_{21} & 0 & \dots & w_2 \cdot \widehat{g}_{2N} \\ \vdots & \vdots & \dots & \vdots \\ w_N \cdot \widehat{g}_{N1} & w_N \cdot \widehat{g}_{N2} & \dots & 0 \end{pmatrix} .$$

Now, from Gerschgoring theorem it follows that all Gerschgorin disks are centered around zero. This means that the Gerschgorin disk with the largest radius contains all the other disks. In other words the largest Gerschgorin disk must contain all the eigenvalues, including the spectral radius.[33] Therefore, to prove (A.2) it suffices to prove that the sum of the absolute value of the

[33] Technically, the spectral radius is nothing but the largest eigenvalue of a matrix in absolute terms, that is

$$\rho(A) = \max\left\{|\lambda_1|, |\lambda_2|, ..., |\lambda_n|\right\},$$

where λ_k, with $k = 1, 2, ..., N$ denote all the eigenvalues of the generic matrix $\mathbf{A}$.

off-diagonal elements is smaller than one for all rows of the matrix $\mathbf{W}\widehat{\mathbf{G}}$. That is,

$$\sum_{j=1}^{N} |w_i \widehat{g}_{ij}| \overset{?}{<} 1, \quad \forall\, i = 1, 2, ..., N. \tag{A.3}$$

Notice that w_i is the same for the same row and that $0 \leq \widehat{g}_{ij} \leq 1$, and that $\sum_{j=1}^{N} \widehat{g}_{ij} = 1$ because $\widehat{\mathbf{G}}$ is row-normalized. This means that (A.3) can be rewritten as follows:

$$|w_i| \sum_{j=1}^{N} \widehat{g}_{ij} = |w_i| \,. \tag{A.4}$$

But since $w_i := \ln(r_i + 1)$, with $r_i \in \left[-\frac{1}{2}, \frac{1}{2}\right]$, and both $|\ln(0.5)| < 1$ and $|\ln(1.5)| < 1$, it follows that for any $r_i \in \left[-\frac{1}{2}, \frac{1}{2}\right]$ the largest Gerschgorin radius is smaller than one. Given Gerschgorin theorem this means that all eigenvalues of the matrix $\mathbf{W}\widehat{\mathbf{G}}$, including its spectral radius, $\rho\left(\mathbf{W}\widehat{\mathbf{G}}\right) < 1$.

Since ϕ is defined as a ratio (i.e., it never exceeds 1), with this I proved that $\left(\mathbf{I} - \phi\mathbf{W}\widehat{\mathbf{G}}\right)^{-1}$ exists and that the Nash equilibrium is unique and asymptotically converges to a fixed point under the specific parametrization of the model.

Even if the equilibrium is always unique, it is not guaranteed that it is also always interior. In fact, if one considers the best-response function in equation (9), it is easy to see that if $r_i < 0$ (i.e., if student i is pessimistic about the own abilities relative to others), then if might happen that $\alpha_i + \phi \ln(r_i + 1)\bar{e}_{-i} < 0$. This would cause the optimal effort to be negative, which is not an admissible solution given that I defined effort as a positive choice variable.

Players' best-response is thus given by

$$e_i = \max\left\{0, \alpha_i + \phi \ln(r_i + 1)\bar{e}_{-i}\right\}. \tag{A.5}$$

More in general, the equilibrium is not interior when at least one student is pessimistic about their abilities (i.e., $r_i < 0$ for at least one student) and $r_i < \exp\left(-\frac{\alpha_i}{\phi \bar{e}_{-i}}\right) - 1$.

The intuition behind this lower bound on r_i may become clearer through the dyadic example presented in the appendix section B, as therefrom it is evident that this lower bound strictly depends on one's productivity relative to the others. □

Proof part (ii). To prove that the *equilibrium performance* is given by (13) one needs to use equation (9) and express $\bar{e}_{-i}$ in terms of the individual equilibrium effort as follows:

$$\bar{e}_{-i} = \frac{1}{\phi \ln(r_i + 1)} \left(e_i^* - \alpha_i\right). \tag{A.6}$$

By replacing $\bar{e}_{-i}$ in (2) with (A.6) it can be found that:

$$y_i^* = \frac{1}{2} e_i^*. \tag{A.7}$$

Finally, using (11) one can re-write the optimal performance as follows

$$
\begin{aligned}
y_i^* &= \frac{1}{2}\sum_{j=1}^{N} m_{ij}\alpha_j \\
&= \frac{1}{2}\widehat{m}_{ii}\alpha_i + \frac{1}{2}\sum_{j\neq i}^{N} \widehat{m}_{ij}\alpha_j
\end{aligned}
$$

where $\widehat{m}_{ij}$ iw the ij-th element of matrix $\widehat{\mathbf{M}}$. □

Proof part (iii). To proof that the *equilibrium utility* is given by (14) we need to substitute y_i with the equilibrium performance using equation (13). In a context with *full information* where all students know both own and others' productivity and self-efficacy levels, as well as the network structure, i.e., $\{\boldsymbol{\alpha}, \mathbf{W}, \widehat{\mathbf{G}}\} \subseteq \mathcal{I}_i$, this means that

$$
\begin{aligned}
E\left[y_i^* \middle| \mathcal{I}_i\right] &= \frac{1}{2}E\left[\sum_{j=1} \widehat{m}_{ij}\alpha_j \middle| \mathcal{I}_i\right] \\
&= \frac{1}{2}\sum_{j=1} \widehat{m}_{ij}\alpha_j \\
&= \frac{1}{2}\widehat{m}_{ii}\alpha_i + \frac{1}{2}\sum_{j\neq i}^{N} \widehat{m}_{ij}\alpha_j .
\end{aligned}
$$

□

A.2 Proof of Proposition 2

Proof of part (i)-(ii). By differentiating the best-response function (9) with respect to any e_j, one finds that

$$
\frac{\partial e_i^*}{\partial e_j} = \phi \ln(r_i + 1).
$$

Considering that $0 \leq \phi \leq 1$ and that $\left|\ln(r_i + 1)\right| < 1$ because $r_i \in \left[\frac{1}{2}, \frac{1}{2}\right]$, it means that the product of the two quantities must also be smaller than 1 in absolute value, as state in part (i) of Proposition 2.

Now, considering the ϕ is always positive, it follows that the sign of the effect is uniquely determined by r_i. Namely, if $r_i < 0$, then $\ln(r_i + 1) < 0$. Similarly, if $r_i > 0$, then $\ln(r_i + 1) > 0$, and $r_i = 0$, then $\ln(r_i + 1) = 0$. This proves part (ii) of Proposition 2. □

A.3 Proof of Proposition 3

Proof of part (i). To compute the effect of anybodies' productivity on own equilibrium effort, it is useful to start from the individual formulation of optimal effort, that is,

$$
e_i^* = \sum_{j=1}^{N} \widehat{m}_{ij}\alpha_j \ ,
$$

which is obtained by multiplying the ith row of $\widehat{\mathbf{M}}$ with the productivity vector, $\boldsymbol{\alpha}$.

The first-order derivative with respect to all α_j's (i.e., with respect to the vector $\boldsymbol{\alpha}$) of the above expression gives,

$$\frac{\partial e_i^*}{\partial \boldsymbol{\alpha}} = (\widehat{m}_{i1}, \widehat{m}_{i2}, ..., \widehat{m}_{ii}, ..., \widehat{m}_{iN}) \ ,$$

which corresponds to the i-th row of matrix $\widehat{\mathbf{M}}$. This also means that the matrix of all first-order derivatives of equilibrium effort with respect to all productivities is

$$\frac{\partial \mathbf{e}^*}{\partial \boldsymbol{\alpha}} = \begin{pmatrix} \frac{\partial e_1^*}{\partial \alpha_1} & ... & \frac{\partial e_1^*}{\partial \alpha_i} & ... & \frac{\partial e_1^*}{\partial \alpha_N} \\ & \ddots & & & \\ \frac{\partial e_i^*}{\partial \alpha_1} & ... & \frac{\partial e_i^*}{\partial \alpha_i} & ... & \frac{\partial e_i^*}{\partial \alpha_N} \\ \vdots & & & \cdots & \\ \frac{\partial e_N^*}{\partial \alpha_1} & ... & \frac{\partial e_N^*}{\partial \alpha_i} & ... & \frac{\partial e_N^*}{\partial \alpha_N} \end{pmatrix} = \begin{pmatrix} \widehat{m}_{11} & ... & \widehat{m}_{1i} & ... & \widehat{m}_{1N} \\ & \ddots & & & \\ \widehat{m}_{i1} & ... & \widehat{m}_{ii} & ... & \widehat{m}_{iN} \\ \vdots & & & \cdots & \\ \widehat{m}_{N1} & ... & \widehat{m}_{Ni} & ... & \widehat{m}_{NN} \end{pmatrix} = \widehat{\mathbf{M}}.$$

This means that $\widehat{\mathbf{M}}$ is to be seen as a sort of Jacobian matrix of first-order derivatives with respect to productivities of all individuals in network $\mathbf{g}$.

To see why $|\widehat{m}_{ij}| < 1$, for all $i, j = 1, ..., N$ it is useful to decompose $\widehat{\mathbf{M}}$ in its single components using the Neumann series expansion (12) as follows:

$$\phi^0 \left(\mathbf{W}\widehat{\mathbf{G}}\right)^0 = 1 \cdot \begin{pmatrix} 1 & 0 & 0 \, ... & 0 \\ \vdots & 1 & ... & \vdots \\ 0 & ... & \ddots & 0 \\ \vdots & & & 1 \end{pmatrix}$$

$$\phi \left(\mathbf{W}\widehat{\mathbf{G}}\right) = \phi \cdot \begin{pmatrix} 0 & w_1 \widehat{g}_{12} & ... & w_1 \widehat{g}_{1N} \\ w_2 \widehat{g}_{21} & 0 & ... & w_2 \widehat{g}_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ w_N \widehat{g}_{N1} & w_N \widehat{g}_{N2} & ... & 0 \end{pmatrix}$$

$$\phi^2 \left(\mathbf{W}\widehat{\mathbf{G}}\right)^2 = \phi^2 \begin{pmatrix} w_1 \sum_{k \neq 1} w_k \widehat{g}_{1k}^2 & ... & w_1 \sum_{k \neq \{1,N\}} w_k \widehat{g}_{1k} \widehat{g}_{kN} \\ w_2 \sum_{k \neq \{1,2\}} w_k \widehat{g}_{2k} \widehat{g}_{k1} & w_2 \sum_{k \neq 2} w_k \widehat{g}_{2k}^2 & \vdots \\ \vdots & \vdots \quad \ddots & \vdots \\ w_N \sum_{k \neq \{1,N\}} w_k \widehat{g}_{Nk} \widehat{g}_{k1} & ... & w_N \sum_{k \neq N} w_k \widehat{g}_{Nk}^2 \end{pmatrix}$$

$$\phi^3 \left(\mathbf{W}\widehat{\mathbf{G}}\right)^3 = \phi^3 \begin{pmatrix} w_1 \sum_{\ell \neq 1} \sum_{k \neq \{1,\ell\}} w_k w_\ell \widehat{g}_{1\ell} \widehat{g}_{\ell k} \widehat{g}_{k1} & ... & w_1 \sum_{\ell \neq N} \sum_{k \neq \{1,\ell\}} w_k w_\ell \widehat{g}_{Nk} \widehat{g}_{k\ell} \widehat{g}_{\ell 1} \\ w_2 \sum_{\ell \neq 1} \sum_{k \neq \{2,\ell\}} w_k w_\ell \widehat{g}_{1k} \widehat{g}_{k\ell} \widehat{g}_{\ell 2} & w_2 \sum_{\ell \neq 2} \sum_{k \neq \{2,\ell\}} w_k w_\ell \widehat{g}_{2\ell} \widehat{g}_{\ell k} \widehat{g}_{k2} & w_2 \sum_{\ell \neq N} \sum_{k \neq \{2,\ell\}} w_k w_\ell \widehat{g}_{Nk} \widehat{g}_{k\ell} \widehat{g}_{\ell 2} \\ \vdots & \vdots \quad \ddots & \vdots \\ w_N \sum_{\ell \neq 1} \sum_{k \neq \{N,\ell\}} w_k w_\ell \widehat{g}_{1k} \widehat{g}_{k\ell} \widehat{g}_{\ell N} & ... & w_N \sum_{\ell \neq N} \sum_{k \neq \{N,\ell\}} w_k w_\ell \widehat{g}_{N\ell} \widehat{g}_{\ell k} \widehat{g}_{kN} \end{pmatrix}$$

Since

$$\widehat{\mathbf{M}} = \phi^0 \left(\mathbf{W}\widehat{\mathbf{G}}\right)^0 + \phi^1 \left(\mathbf{W}\widehat{\mathbf{G}}\right)^1 + \phi^2 \left(\mathbf{W}\widehat{\mathbf{G}}\right)^2 + \phi^3 \left(\mathbf{W}\widehat{\mathbf{G}}\right)^3 + ... \tag{A.8}$$

this means that $\widehat{m}_{ij}$ corresponds to the sum of ij-th elements from the above matrices. That is,

$$\widehat{m}_{ij} = \begin{cases} 1 + w_i \left(\phi^2 \sum_{k \neq i} w_k \widehat{g}_{ik}^2 + \phi^3 \sum_{k \neq i} \sum_{\ell \neq \{i,k\}} w_k w_\ell \widehat{g}_{ik} \widehat{g}_{k\ell} \widehat{g}_{\ell i} + ... \right) \,, & \text{if } j = i \\ w_i \left(\phi \widehat{g}_{ij} + \phi^2 \sum_{k \neq \{i,j\}} w_k \widehat{g}_{ik} \widehat{g}_{kj} + \phi^3 \sum_{k \neq \{i,j\}} \sum_{\ell \neq \{i,j,k\}} w_k w_\ell \widehat{g}_{jk} \widehat{g}_{k\ell} \widehat{g}_{\ell i} + ... \right) \,, & \text{if } j \neq i \,. \end{cases}$$

Since $0 \leq \phi, \widehat{g}_{ij} \leq 1$, and $|w_i| < 1$, for all $i = 1, ..., N$, for all $i, j = 1, ..., N$, it means that when $i \neq j$, the productivity effect of other students on own equilibrium effort must be smaller than 1 in absolute terms. Note that this is a direct consequence of the spectral radius of $\mathbf{W}\widehat{\mathbf{G}}$ being strictly smaller than 1 in absolute terms. □

Proof of part (ii). Starting from (13) and deriving with respect to α_j, on finds that

$$\frac{\partial y_i^*}{\partial \alpha_j} = \frac{1}{2} \widehat{m}_{ij},$$

where $\widehat{m}_{ij}$ is defined as in (15) for the case when $j \neq i$.

The marginal effect of peer productivity on own utility is derived in a similar way, starting from (14). □

Proof of part (iii) and (iv). As explained in the text, the determinants influencing the direction of the marginal effect of productivity can be inferred from the second expression of equation (15) when $j \neq i$, where

$$\frac{\partial e_i^*}{\partial \alpha_j} = \begin{cases} \phi w_i \widehat{g}_{ij} & , \quad \text{if } i \text{ and } j \text{ are direct friends} \\ \phi^2 w_i \sum_{k \neq \{i,j\}} w_k \widehat{g}_{ik} \widehat{g}_{kj} & , \quad \text{if } i \text{ and } j \text{ are one node apart} \\ \phi^3 w_i \sum_{\ell \neq j} \sum_{k \neq \{i,\ell\}} w_k w_\ell \widehat{g}_{jk} \widehat{g}_{k\ell} \widehat{g}_{\ell N} & , \quad \text{if } i \text{ and } j \text{ are two nodes apart} \\ ... & , \quad \text{if } i \text{ and } j \text{ are } h \text{ nodes apart} \end{cases}$$

As it can be seen above, a derivatives depend from w_i, which represents individual beliefs about one's own abilities. Furthermore, the second and the third expressions—i.e., the cases of indirect friendships—also depend on the intermediary nodes' beliefs, denoted as w_k and w_ℓ. □

Proof of part (iv). That the return to own productivity is always positive it can be seen from the first expression of (15). Since the spectral radius of $\mathbf{W}\widehat{\mathbf{G}}$ being strictly smaller than one, one can confidently conclude that the sum of the various products between w_i's and $\widehat{g}_{ij}$'s is smaller than one absolute value. Consequently, the sum of 1 plus something the is smaller than 1 in absolute value must be strictly positive. □

Proof of Proposition (4). If one considers the best-response function (9) and computes the first-order derivative with respect to r_i one finds that:

$$\frac{\partial e_i}{\partial r_i} = \frac{\phi \bar{e}_{-i}}{r_i + 1}. \tag{A.9}$$

Considering that $0 \leq \phi \leq 1$, and that $e_i > 0$ for all $i = 1, 2, ..., N$, so that $\bar{e}_{-i} > 0$, it follows that the (A.9) must also be positive. Therefore, a marginal increase in own self-perception increases own optimal effort. □

Proof of Proposition (5). If one considers again the best-response function (9) and, this time, computes the first-order derivative with respect to ϕ, one finds that:

$$\frac{\partial e_i}{\partial \phi} = \ln(r_i + 1)\bar{e}_{-i}. \tag{A.10}$$

Since $\bar{e}_{-i} > 0$, it follows that the sign of the derivative only depends on whether r_i is greater or lesser than 0. If $r_i < 0$ the logarithm is also negative, and a marginal increase in ϕ results in a lower effort. Whereas, if $r_i > 0$ the logarithm is positive, and a marginal increase in ϕ results in a higher equilibrium effort.

□

B Examples

B.1 Dyadic case: Roommates

Consider the situation with two students characterized by the following performance production function:

$$y_i\left(\alpha_i, r_i, e_i, e_j, \mathbf{g}_i\right) = \alpha_i e_i - \frac{1}{2}e_i^2 + \ln\left((r_i+1)^{\phi}\right)e_i e_j + \epsilon_i, \tag{B.2}$$

which leads to the following best-response function:

$$e_i = \alpha_i + \phi \ln(r_i+1)e_j\,. \tag{B.3}$$

Rewriting the above expression in matrix form as in equation (10) we have

$$\boldsymbol{\alpha} = \begin{pmatrix}\alpha_1^* \\ \alpha_2^*\end{pmatrix}, \qquad \widehat{\mathbf{G}} = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \qquad \text{and} \qquad \mathbf{W} = \begin{pmatrix}w_1 & 0\\ 0 & w_2\end{pmatrix},$$

where $w_k = \ln(1+r_k)$, for $k = 1, 2$. This means that

$$\begin{aligned}\widehat{\mathbf{M}} &= \begin{pmatrix}1 & -\phi w_1\\ -\phi w_2 & 1\end{pmatrix}^{-1}\\ &= \frac{1}{1-\phi^2 w_1 w_2}\begin{pmatrix}1 & \phi w_1\\ \phi w_2 & 1\end{pmatrix}.\end{aligned}$$

Using equation (11) the effort of the two students in equilibrium yields

$$\begin{pmatrix}e_1^*\\ e_2^*\end{pmatrix} = \frac{1}{2-\phi^2 w_1 w_2}\begin{pmatrix}\alpha_1 + \phi w_1 \alpha_2\\ \alpha_2 + \phi w_2 \alpha_1\end{pmatrix},$$

whereas from equation (13) it follows that their academic performance in the optimum is

$$\begin{pmatrix}y_1^*\\ y_2^*\end{pmatrix} = \frac{1}{2-\phi^2 w_1 w_2}\begin{pmatrix}\frac{1}{2}\alpha_1 + \frac{\phi}{2} w_1 \alpha_2\\ \frac{1}{2}\alpha_2 + \frac{\phi}{2} w_2 \alpha_1\end{pmatrix}.$$

Differentiating one student's optimal effort with respect to the other student's productivity, one observes that the orientation of the marginal effect can be positive or negative, depending on the individual's beliefs in their own abilities. If a student believes they are above average, their optimal effort will increase as the other student's productivity increases, while if they believe they are below average, their optimal effort will decrease as the other student's productivity increases.

The optimal effort of one student 1 is also visually presented in Figure B.1, given the productivity of student 2 in the case where $\alpha_1 > 0$. It's worth noting that the graphs are identical for student 2's effort given student 1's productivity. The figure contains multiple panels that represent different intensities of social comparison in the network, ranging from low to high (from left to right). Additionally, different line types indicate the self-efficacy levels of student 1, with the solid line representing the lowest self-efficacy level and the dashed line representing the highest. The line colors differentiate the self-efficacy levels of student 2, with the light gray line representing the highest self-efficacy level and the black line representing the lowest.

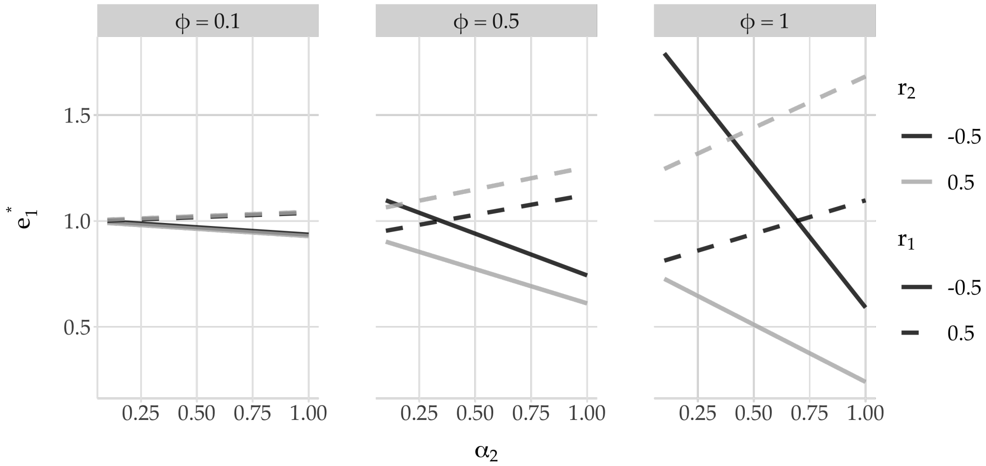


Figure B.1: Optimal efforts of one student given the roommate's productivity ($\alpha_1 \geq \alpha_2$).

In all three panels of Figure B.1, if student 1 believes they are the best, an increase in the other student's productivity results in an increase in their own effort in the optimal outcome. Conversely, if the student believes they are the worst, an increase in the other student's productivity results in a decrease in their own effort in the optimal outcome.

As ϕ increases (i.e., moving from left to right), two things become apparent. Firstly, the optimal effort becomes more responsive to changes in the other student's productivity, i.e., the curve becomes more inelastic for any value of r_1 and r_2. Secondly, as ϕ increases, the self-efficacy of the other student shifts the intercept of the curve upwards or downwards, depending on the student's own self-efficacy. If a student believes in their own abilities, higher self-efficacy levels of the other student result in higher effort levels for the student themselves, as evidenced by the fact that the gray dashed line lies above the black dashed line. Conversely, if a student lacks confidence in their own abilities, higher self-efficacy levels of the other student can be even more detrimental to their own effort, as evidenced by the fact that the gray solid line lies below the black solid line.

It is important to notice, however, that if one student is much less productive than the other (i.e., $\alpha_1 << \alpha_2$) and social comparison increases in saliency (i.e., $\phi \to 1$), then the equilibrium may not be interior anymore, or not all values of $r_1 < 0$ might be feasible. For the optimal effort to be an interior solution (i.e., $e_j^* > 0$), the following condition must hold:

$$r_j > \exp\left(-\frac{\alpha_j}{\phi\alpha_k}\right), \quad \text{with } j,k = 1,2,\, j \neq k. \tag{B.4}$$

Figure B.2 illustrates the lower bound of one student's self-efficacy given their productivity relative to the other student, α_j/α_k. The two different lines represent two different levels of social intensity in the dyad. The dashed line represents the maximal possible value (i.e., $\phi = 1$), while the solid line presents the middle value ($\phi = 0.5$).

The lower is a students productivity relative to the other—hence, the lower is the ratio of α_j to α_k—the higher is the lower bound of r_j. In other words, the lower one's productivity (hence also abilities). the less pessimist a student can be about their relative abilities.

This bound can be seen as more than just a simple theoretical artifact because it also aligns with empirical observations. In fact, it can be seen as an expression of the Dunning-Kruger effect, which suggests that people with low cognitive ability may not have the skills necessary to accurately

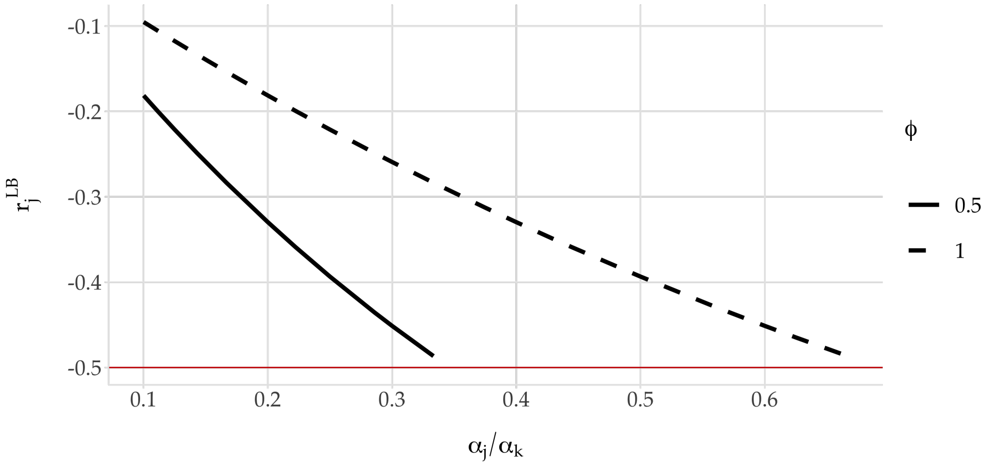
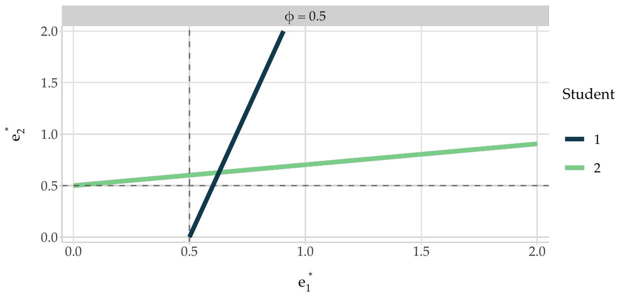


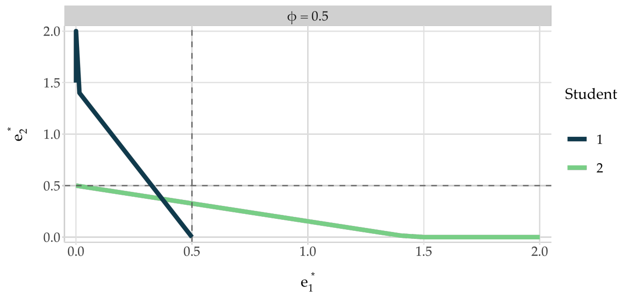
Figure B.2: r_j's LOWER BOUND.

assess their own abilities, leading to overconfidence and a lack of awareness of their cognitive limits.

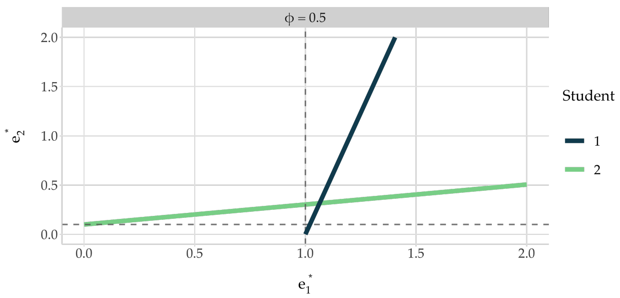
The consequence without this additional bound is that the equilibrium solution could not be interior anymore as shown in Panel (d) of Figure B.3. This not a problem though, if everybody is self-confident in own abilities.



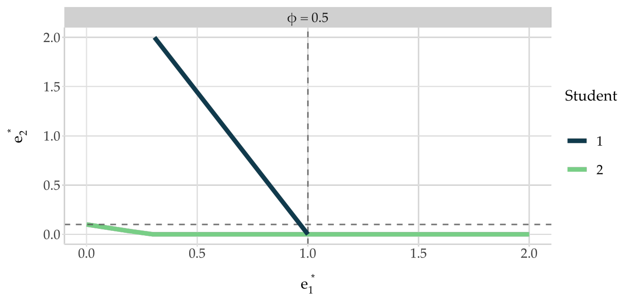
(a) $\alpha_1 = \alpha_2$, and $r_1, r_2 > 0$.



(b) $\alpha_1 = \alpha_2$, and $r_1, r_2 < 0$.



(c) $\alpha_1 >> \alpha_2$, and $r_1, r_2 > 0$.



(d) $\alpha_1 >> \alpha_2$, and $r_1, r_2 < 0$.

Figure B.3: EQUILIBRIUM REPRESENTATIONS FOR ROOMMATES EXAMPLE.

B.2 Three-agents star network

Consider a network with three students, where student 1 is connected to both other students but 2 and 3 are not directly connected, as illustrated in Figure B.4

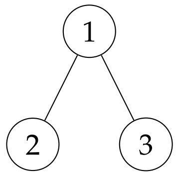


Figure B.4: Representation of cool kid example.

The network of three students is characterized by the following matrices and vectors

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_1^* \\ \alpha_2^* \\ \alpha_3^* \end{pmatrix}, \qquad \widehat{\mathbf{G}} = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad \text{and} \qquad \mathbf{W} = \begin{pmatrix} w_1 & 0 & 0 \\ 0 & w_2 & 0 \\ 0 & 0 & w_3 \end{pmatrix},$$

where $w_k = \ln(1 + r_k)$, for $k = 1, 2, 3$. This means that

$$\begin{aligned} \widehat{\mathbf{M}} &= \begin{pmatrix} 1 & -\frac{\phi}{2} w_1 & -\frac{\phi}{2} w_1 \\ -\phi w_2 & 1 & 0 \\ -\phi w_3 & 0 & 1 \end{pmatrix}^{-1} \\ &= \frac{1}{2 - \phi^2 w_1 (w_2 + w_3)} \begin{pmatrix} 2 & \phi w_1 & \phi w_1 \\ 2\phi w_2 & 2 - \phi w_1 w_3 & \phi^2 w_1 w_2 \\ 2\phi w_3 & \phi^2 w_1 w_3 & 2 - \phi w_1 w_2 \end{pmatrix} \end{aligned}$$

Following equation (11) the equilibrium efforts for the three students are then

$$\begin{pmatrix} e_1^* \\ e_2^* \\ e_3^* \end{pmatrix} = \frac{1}{2 - \phi^2 w_1 (w_2 + w_3)} \begin{pmatrix} 2\alpha_1 + \phi w_1 (\alpha_2 + \alpha_3) \\ 2\alpha_2 - \phi w_1 w_3 \alpha_2 + 2\phi w_2 \alpha_1 + \phi^2 w_1 w_2 \alpha_3 \\ 2\alpha_3 - \phi w_1 w_2 \alpha_3 + 2\phi w_3 \alpha_1 + \phi^2 w_1 w_3 \alpha_2 \end{pmatrix}. \tag{B.5}$$

Whereas, from equation (13) it follows that their equilibrium performances yield:

$$\begin{pmatrix} y_1^* \\ y_2^* \\ y_3^* \end{pmatrix} = \frac{1}{2 - \phi^2 w_1 (w_2 + w_3)} \begin{pmatrix} \alpha_1 + \frac{\phi}{2} w_1 (\alpha_2 + \alpha_3) \\ \alpha_2 - \frac{\phi}{2} w_1 w_3 \alpha_2 + \phi w_2 \alpha_1 + \frac{\phi^2}{2} w_1 w_2 \alpha_3 \\ \alpha_3 - \frac{\phi}{2} w_1 w_2 \alpha_3 + \phi w_3 \alpha_1 + \frac{\phi^2}{2} w_1 w_3 \alpha_2 \end{pmatrix}. \tag{B.6}$$

From (B.5) it is evident that the individual optimal effort, e_i^*, is influenced by: own productivity (α_i), self-assessment of own abilities relative to others (w_i), direct and indirect friends' productivity ($\alpha_{j \neq i}$), and others' beliefs about their own abilities ($w_{j \neq i}$).

To better understand how each of these determinants influences the individual optimal effort, in what follows I present the comparative statics and the relative graphical representations, that will help better understand both the specific example of a 3-agents star network, as well as the general results presented in the text.

Effect of productivity. In contrast to the dyadic example, the 3-agents star network is well suited for analyzing the productivity effects of both direct and indirect peers.

The effect of *direct friend's* productivity on own effort in the equilibrium is given by:

$$\frac{\partial e_1^*}{\partial \alpha_j} = \frac{\phi w_1}{2 - \phi^2 w_1 (w_2 + w_3)}, \quad j = 2, 3\ ; \tag{B.7}$$

$$\text{and} \quad \frac{\partial e_j^*}{\partial \alpha_1} = \frac{2\phi w_j}{2 - \phi^2 w_1 (w_2 + w_3)}, \quad j = 2, 3. \tag{B.8}$$

Just as for the general and the dyadic case, the direction of the productivity effect from direct friends exclusively depends on individuals' beliefs in own abilities. Namely, if students believe to have above average abilities (i.e., $w_i > 0$), then an increase in their direct peers' productivity results in a positive externality on their own effort. Conversely, if they believe to have below average abilities (i.e., $w_i > 0$), then an increase in their direct peers' productivity results in a negative externality on their own effort. If they, nevertheless, consider to be average (i.e., $w_i = 0$), then peers' effort has no effect on own effort.

The effect of *indirect friend's* productivity on own effort in the equilibrium is, instead, given by:

$$\frac{\partial e_j^*}{\partial \alpha_k} = \frac{\phi^2 w_1 w_j}{2 - \phi^2 w_1 (w_2 + w_3)}, \ j, k = 2, 3, j \neq k. \tag{B.9}$$

Equation (B.9) suggests that the indirect effect not only depends on the individual beliefs about own abilities, but also on the beliefs of the intermediary node (e.g., student 1 in the specific example).

This means that in a environment both where everybody is pessimistic (i.e., $w_1 < 0$ and $w_j < 0$) and where everybody is optimistic (i.e., $w_1 > 0$ and $w_j > 0$) about their own abilities, marginal changes of indirect peers' productivity can result in positive externalities. Figure B.5 visually illustrates this phenomenon.

The four panels present how optimal efforts of all three students (dotted line: student 1, solid line: student 2, dashed line: student 3) change when student 2's productivity (x-axis) changes under four different circumstances. Namely, when everybody is optimistic about their ability (panel B.5a), when everybody is pessimistic about their ability (panel B.5b), when only the central node is optimistic (panel B.5c), and when only the central node is pessimistic (panel B.5d) about their abilities.

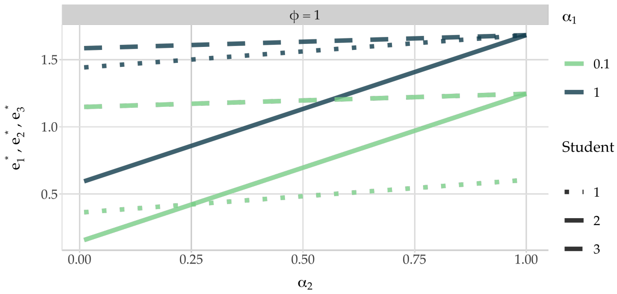
When everybody Panel (B.5a) is representative of the situation where all three students believe to have exceptionally high abilities (i.e., $r_1 = r_2 = r_3 = 0.5$).

Panel (B.5b) presents the precise opposite situation, where all three students believe to have exceptionally low abilities (i.e., $r_1 = r_2 = r_3 = -0.5$).

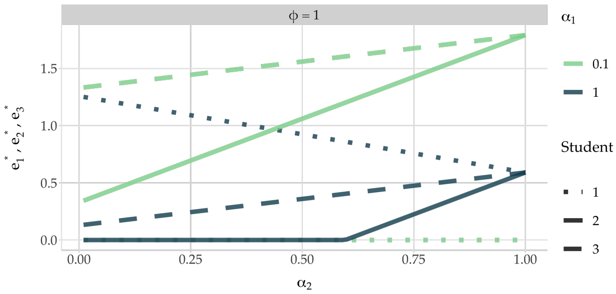
The other two panels, instead, present the results for the case when the central node and the peripheral nodes have diametrically opposed views about their own abilities.

Namely, panel (B.5c) presents the case when only the central node (student 1) is extremely confident about the own abilities and the other two are not, whereas panel (B.5d) presents the case when the central node has extremely negative beliefs about the own abilities while its neighbors are highly confident about their abilities.

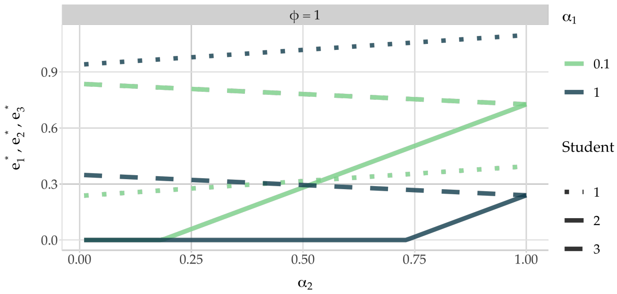
Before discussing each panel it is important to briefly comment on the structure of each panel. Each panel is divided in three sub-panels. Each of these presents results for different degrees of social comparison, ϕ, going from low to high values when proceeding from left to right. Each line type corresponds to a different student: dotted to student 1 (central node), solid to student 2, and



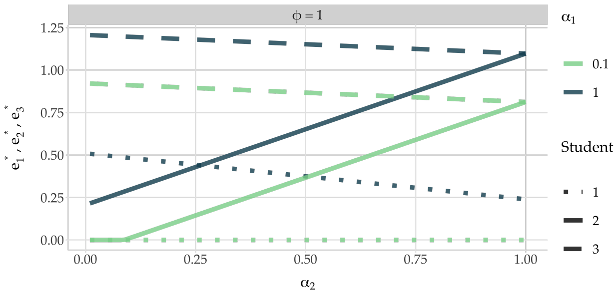
(a) All optimists ($r_1 = r_2 = r_3 = 0.5$)



(b) All pessimists ($r_1 = r_2 = r_3 = -0.5$)



(c) Central node optimist ($r_1 = 0.5, r_2 = r_3 = -0.5$)



(d) Central node pessimist ($r_1 = -0.5, r_2 = r_3 = 0.5$)

Figure B.5: Optimal efforts.

dashed to student 3. Finally, the two shades of gray denote two different e productivity levels of the central node (student 1) while the productivity of student 3 is set to be equal one (i.e., $\alpha_3 = 1$). The light gray denotes cases when student 1 has a higher productivity, whereas dark gray stands for cases when student 1 has a lower productivity.

Panel (a) of the Figure B.5 presents the scenario where all students exhibit high confidence. In this panel, several observations can be made. Firstly, the efforts of all three students increase in the productivity of student 2. This suggests that higher-performing peers have a positive influence on the efforts of their classmates when everybody believes to have equal competences. Secondly, the effect of direct and indirect peer productivity on own effort becomes stronger as the saliency of social comparison increases. This is reflected in the steeper lines in the graph, indicating a stronger relationship between peers' productivity and individual effort. Lastly, when all students have higher productivity levels, the achievable efforts for any given productivity level of student 2 also increase. This can be observed by the parallel light and dark-shaded lines, with the lighter lines being consistently positioned above the darker lines.

Moving on to Panel (b), which depicts a scenario where all students have low confidence, several interesting findings emerge. Firstly, the efforts of all students, except the central node, increase as their own productivity increases. That student 2's effort increases in the own productivity is not a surprise given Proposition 3. However, it is not immediately intuitive that while node 2's effort increases with their own productivity, node 1's effort decreases and node 3's effort increases as student 2's productivity increases. By examining the derivatives in (B.8), it becomes evident that the productivity effect of direct peers is solely dependent on the individual's beliefs in their own abilities. On the other hand, the indirect productivity effects are influenced by the product of the individual's beliefs and the beliefs of the bridging nodes. In this case, since both beliefs are negative, node 3's effort increases as student 2's effort increases.

In terms of the social comparison effect, as the saliency of social comparison increases, the effects become stronger, as observed in the steeper curves. Finally, in contrast to the scenario where all students have high confidence, the highest efforts in this panel are achieved when student 1 has lower productivity compared to the other two nodes. The dark lines consistently lie above the light-gray lines, except for student 1, where lower effort is observed when productivity is lower.

Moving to Panel (c), which represents a scenario where the majority of students have low confidence, several key observations can be made. Firstly, only node 1's effort increases as the productivity of node 2 increases, as node 1 is the only student with positive beliefs in their own ability. Secondly, the effect of indirect peer productivity on student 3's effort is negative, due to the student's low self-assessment of their own abilities. Furthermore, as the saliency of social comparison increases, the effects become stronger, leading to steeper curves. Lastly, when the central node (student 1) is highly productive, the efforts of the other two students are lower, except for student 1, whose effort level is higher when their productivity is higher.

Finally, in Panel (d), which represents a scenario where the majority of students have high confidence it emerges that efforts of both direct and indirect peers decrease as student 2's productivity increases. This suggests that in this context, higher-performing peers have a discouraging effect on the efforts of their classmates. Secondly, as in previous panels, the effects become stronger with higher saliency of social comparison, resulting in steeper curves. Lastly, when the central node (student 1) is more productive, the efforts of all three students increase.

Overall, the four panels provide a comprehensive understanding of the relationships between student confidence, peer productivity, and individual effort.