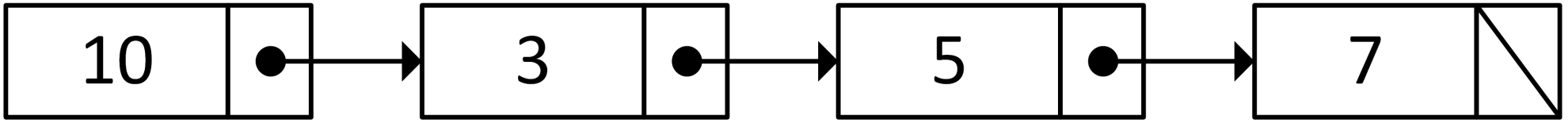
Linked Data Structures

Readings: CP:AMA 17.5

Linked lists

Racket's list type is more commonly known as a ***linked list***.



Each ***node*** contains an ***item*** **(first)** and a *link* to the ***next*** node **(rest)**.

There is no "official" way of implementing a linked list in C.

In this unit we present a typical linked list structure and several strategies for working with this structure.

In general, a linked list is a pointer to a *linked list node* (`llnode`).

```
struct llnode {
  int item;
  struct llnode *next;
};
```

A C structure can contain a *pointer* to its own structure type. This is the first **recursive data structure** we have seen in C.

If a linked list is a pointer to a node, then an empty list is represented by a `NULL` pointer.

In the following slides we present both a **functional** (Racket-like) approach and an **imperative** (C-like) approach to working with linked lists. They are not mutually exclusive and can be combined (they both use `llnode`s).

The two approaches are not formal distinctions, but they help illustrate the respective programming paradigms.

We also present a **wrapper** strategy and an **augmentation** strategy that can be combined and used with either approach.

Functional approach

Helper functions to create a *"functional"* atmosphere:

```
int first(struct llnode *lst) {
  assert (lst != NULL);
  return lst->item;
}

struct llnode *rest(struct llnode *lst) {
  assert (lst != NULL);
  return lst->next;
}

struct llnode *empty(void) {
  return NULL;
}

bool is_empty(struct llnode *lst) {
  return lst == empty();
}
```

At the heart of the functional approach is the `cons` function.

In Racket, `cons` acquires dynamic memory (similar to C's `malloc`).

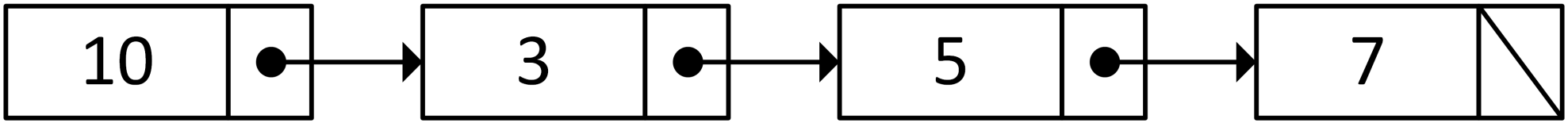
Our C `cons` `returns` a **new** node that *links* to the rest.

```
struct llnode *cons(int f, struct llnode *r) {
  struct llnode *new = malloc(sizeof(struct llnode));
  new->item = f;
  new->next = r;
  return new;
}
```

This is very similar to how Racket's `cons` is implemented.

We can use our new C `cons` function the same way we use the Racket `cons` function.

```
struct llnode *my_list = cons(10, cons(3, cons(5,
                                  cons(7, empty()))));
```



We can also use `cons` in different ways (*e.g.,* with mutation).

```
struct llnode *my_list  = empty();
my_list = cons(7, my_list);
my_list = cons(5, my_list);
my_list = cons(3, my_list);
my_list = cons(10, my_list);
```

Using the functional approach, we can write recursive functions that closely mirror their Racket equivalents.

```
(define (length lst)
  (if (empty? lst) 0
      (+ 1 (length (rest lst)))))
```

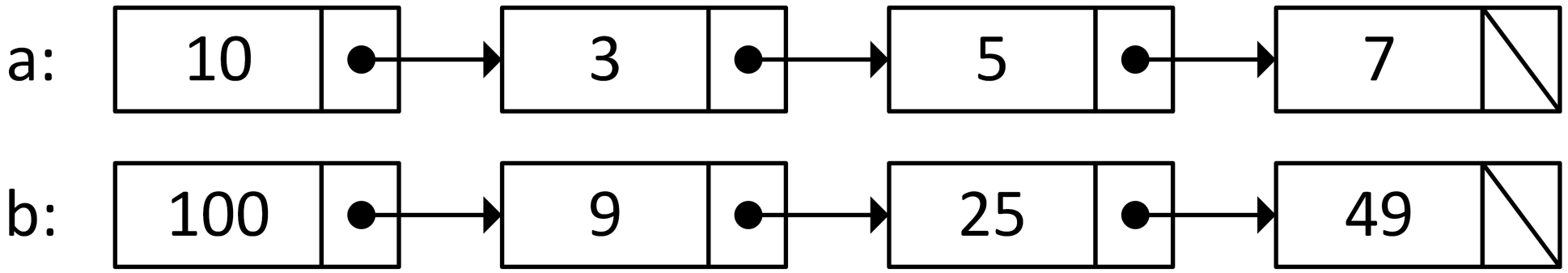
```
int length(struct llnode *lst) {
  if (is_empty(lst)) {
    return 0;
  }
  return 1 + length(rest(lst));
}
```

It is also possible to write some functions iteratively.

```
int length_iterative(struct llnode *lst) {
  int length = 0;
  while (!is_empty(lst)) {
    length++;
    lst = rest(lst);
  }
  return length;
}
```

In Racket, *and the functional programming paradigm*, most functions that **produce** a list construct a **new list**.

```
(define (sqr-list lst)
  (cond [(empty? lst) empty]
        [else (cons (* (first lst) (first lst))
                    (sqr-list (rest lst)))]))

(define a '(10 3 5 7))
(define b (sqr-list a))
```



A C function written with a functional approach also `returns` a **new list**.

```
struct llnode *sqr_list(struct llnode *lst) {
  if (is_empty(lst))  return empty();
  return cons(first(lst) * first(lst),
              sqr_list(rest(lst)));
}
```

As an exercise, try writing a non-recursive (iterative) version of `sqr_list` that returns a new list (it's tricky).

Since we have been working with C imperatively, it might now seem more "natural" that a `sqr_list` function would square (or *mutate*) each item in the list.

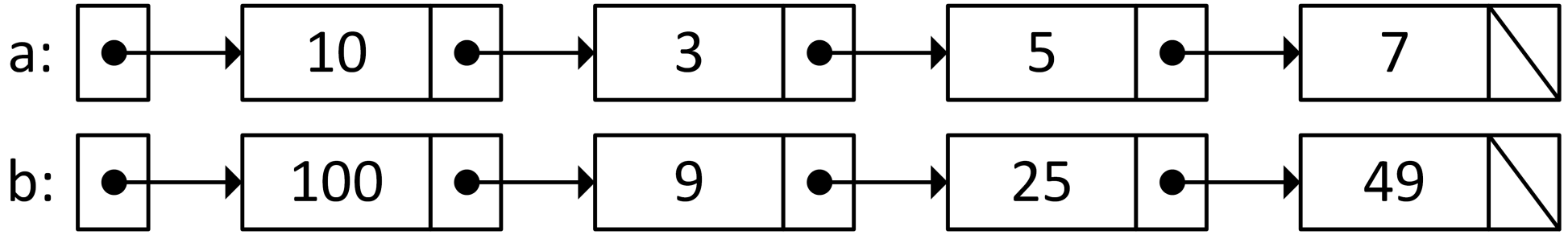
However, this is not a functional approach. We introduce an imperative version of `sqr_list` shortly.

In Racket, lists are immutable, and there is a special `mcons` function to generate a mutable list.

In the Scheme language, lists are mutable. This is one of the significant differences between Racket and Scheme.

To correctly use the `sqr_list` function, the result should be stored in a separate variable.

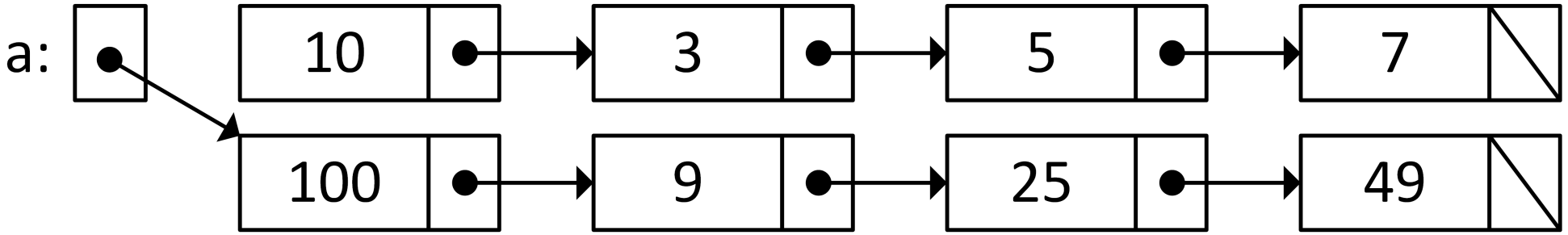
```
struct llnode *a = cons(10, cons(3, cons(5, cons(7, empty()))));
struct llnode *b = sqr_list(a);
```



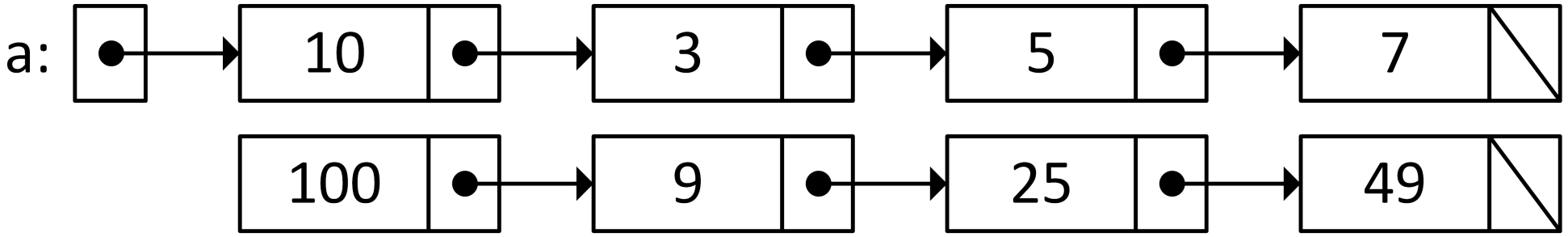
Unfortunately, if the function is misunderstood or used **incorrectly**, it can create a **memory leak**.

The following two statements each create a memory leak.

```
a = sqr_list(a);  // original list is lost
```




```
sqr_list(a);      // new list is lost
```



The best strategy to avoid a memory leak is to provide a clear contract.

The `cons` function can cause a similar problem:

```
struct llnode *a = cons(7, empty());
cons(5, a);  // memory leak
```

This is not a concern in Racket because it uses **garbage collection** and *automatically* frees memory.

In Racket, we are not concerned about `free`ing a list. However, this is necessary in C.

```
void free_list(struct llnode *lst) {
  if (lst != NULL) {
    free_list(rest(lst));
    free(lst);
  }
}
```

Note that it is important to `free` the rest of the list before `free`ing the first node.

After `free(lst)`, any use of `lst` is invalid.

Both of these *iterative* `free_list` functions are equivalent.

```
void free_list_iterative_1(struct llnode *lst) {
  while (lst != NULL) {
    struct llnode *backup = lst;
    lst = rest(lst);
    free(backup);
  }
}

void free_list_iterative_2(struct llnode *lst) {
  while (lst != NULL) {
    struct llnode *backup = rest(lst);
    free(lst);
    lst = backup;
  }
}
```

This “backup” technique (using temporary pointers) is used more frequently when we introduce the imperative approach.

To further illustrate how `cons` is used, consider the `insert` from *insertion sort* (insert into a *sorted* list of numbers).

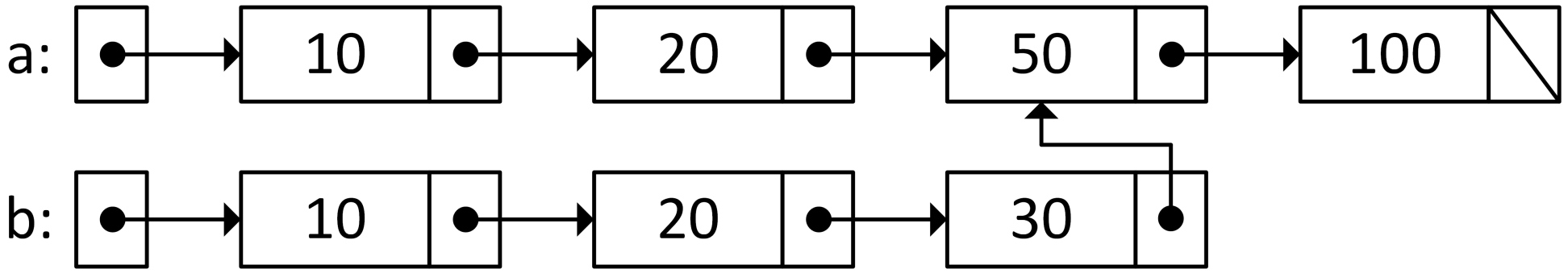
```
(define (insert n slst)
  (cond
    [(empty? slst) (cons n empty)]
    [(<= n (first slst)) (cons n slst)]
    [ else (cons (first slst) (insert n (rest slst)))]))
```

```
struct llnode *insert(int n, struct llnode *slst) {
  if (is_empty(slst)) {
    return cons(n, empty());
  } else if (n <= first(slst)) {
    return cons(n, slst);
  } else {
    return cons(first(slst), insert(n, rest(slst)));
  }
}
```

Consider this example:

```
struct llnode *a = cons(10,cons(20,cons(50,cons(100,empty()))));
struct llnode *b = insert(30, a);
```

The lists **share the last two nodes**.



Freeing either list causes an invalid memory access when the other is accessed.

```
free_list(a);
free_list(b);   // INVALID
```

This problem can occur with just the `cons` function.

```
struct llnode *a = cons(7, empty());
struct llnode *b = cons(5, a);
a = cons(3, a);

free_list(a);
free_list(b);  // INVALID
```

Once again, Racket's **garbage collector** automatically `free`s memory, so this is not a concern in Racket.

The functional approach works well for some applications, but is not well suited for environments where there are specific memory management issues and no garbage collection.

Imperative approach

In an "**imperative**" approach to linked lists, individual nodes are manipulated and changed (mutated) directly.

An imperative list function may *change* a list instead of generating a new list.

For example, the following `sqr_list_m` (for mutate) function *changes* the items in the list.

```
void sqr_list_m(struct llnode *lst) {
  while (lst != NULL) {
    lst->item *= lst->item;
    lst = lst->next;
  }
}
```

If a new list is desired, a *copy* of the original list can be made.

```
struct llnode *copy_list(struct llnode *lst) {
  if (lst == NULL) return NULL;
  return cons(lst->item, copy_list(lst->next));
}
```

The copy can then be changed.

```
struct llnode *orig = cons(10, cons(3, cons(5,
                                cons(7, empty()))));

struct llnode *dup = copy_list(orig);
sqr_list_m(dup);
```

As an alternative to the `cons` function, we write a function that inserts a new node at the start (front) of an existing list.

For example, we would like to have the following code sequence.

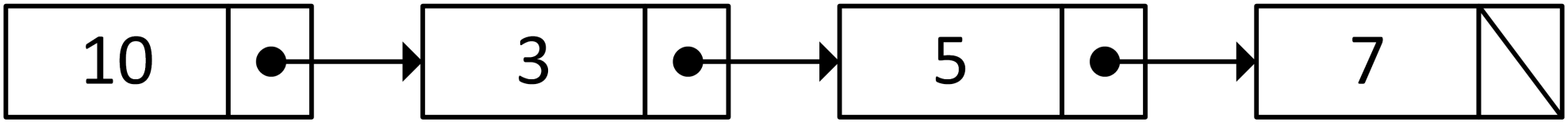
```
struct llnode *lst = NULL;

add_to_front(7, lst);   // won't work
add_to_front(5, lst);   // won't work
```

Remember, to have a function change a variable (*i.e.,* `lst`), we need to pass a *pointer* to the variable.

```
add_to_front(7, &lst);
add_to_front(5, &lst);
```

Since `lst` is already a pointer, we need to pass a **pointer to a pointer**.

```
void add_to_front(int n, struct llnode **ptr_front) {
  struct llnode *new = malloc(sizeof(struct llnode));
  new->item = n;
  new->next = *ptr_front;
  *ptr_front = new;
}
```




```
struct llnode *lst = NULL;
add_to_front( 7, &lst);
add_to_front( 5, &lst);
add_to_front( 3, &lst);
add_to_front(10, &lst);
```

With the imperative approach, nodes can also be *removed*.

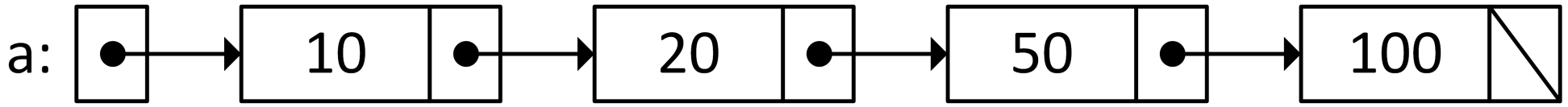
```
int remove_from_front(struct llnode **ptr_front) {
  struct llnode *front = *ptr_front;
  int retval = front->item;
  *ptr_front = front->next;
  free(front);
  return retval;
}
```

Instead of `returning` nothing (`void`), it is more useful to `return` the value of the item being removed.

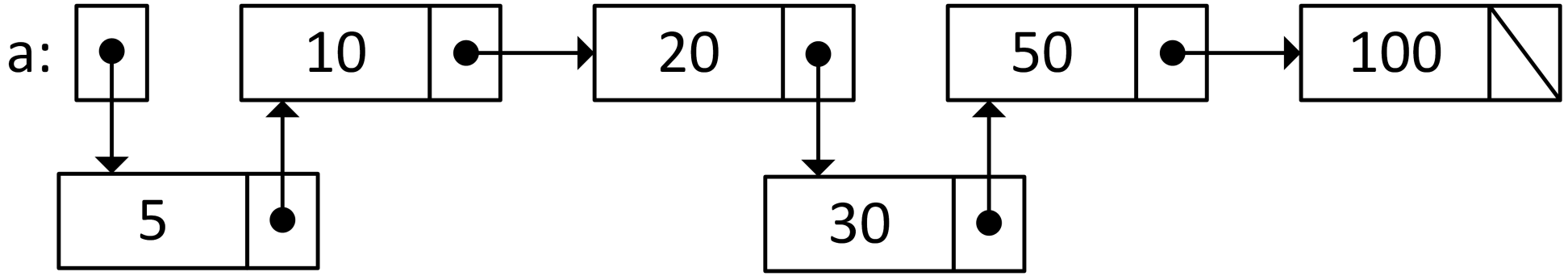
This function `inserts` a node into an *existing* sorted list.

```
void insert(int n, struct llnode **ptr_front) {
    struct llnode *cur = *ptr_front;
    struct llnode *prev = NULL;
    while ((cur != NULL) && (n > cur->item)) {
      prev = cur;
      cur = cur->next;
    }
    struct llnode *new = malloc(sizeof(struct llnode));
    new->item = n;
    new->next = cur;
    if (prev == NULL) { // new first node
      *ptr_front = new;
    } else {
      prev->next = new;
    }
  }
}
```

Revisiting the previous (functional) insert example,




```
insert( 5, &a);
insert(30, &a);
```



An alternative *"destructive"* approach uses a Racket-like programming interface (functions produce *new* lists), but each list passed to a function may be destroyed (`free`d).

```
struct llnode *insert(int n, struct llnode *slst) {
  if (is_empty(slst)) {
    return cons(n, empty());
  } else if (n <= first(slst)) {
    return cons(n, slst);
  } else {
    int f_backup = first(slst);
    struct llnode *r_backup = rest(slst);
    free(slst);
    return cons(f_backup, insert(n, r_backup));
  }
}
```

This approach has been taught in previous offerings of CS 136.

The imperative approach is still susceptible to misuse if we have pointers to the middle of the the list.

```
struct llnode *a = NULL;
add_to_front(5, &a);
struct llnode *b = a;
add_to_front(7, &a);
add_to_front(9, &b);
```

Freeing either list `a` or `b` makes the other list invalid.

Wrapper strategy

In the ***wrapper strategy***, we **"wrap"** each list so it is inside of another structure. The definition of a linked list changes slightly to be a pointer to this new structure, and not just a single node.

```
struct llist {
  struct llnode *front;
};
```

This may seem insignificant, but it has 3 advantages:

- a cleaner interface (avoids double pointers)
- less susceptible to misuse
- it can store **additional information**

A wrapper approach would typically have a “create” and a “destroy” function.

```
struct llist *create_list(void) {
  struct llist *lst = malloc(sizeof(struct llist));
  lst->front = NULL;
  return lst;
}

void destroy_list(struct llist *lst) {
  free_list(lst->front);
  free(lst);
}
```

Many of the previous functions introduced can be “wrapped” inside of another function.

```
void add_to_front(int n, struct llist *lst) {
  previous_add_to_front(n, &lst->front);
}
```

Overall, this provides a cleaner interface.

```
struct llist *lst = create_list();
add_to_front(5, lst);
add_to_front(7, lst);
destroy_list(lst);
```

A significant advantage of the wrapper approach is that **additional information** can be stored in the list structure.

The run time of the previous `length` functions are $O(n)$.

With the wrapper, we can store (or "cache") the length of *in the wrapper structure*, so the length can be retrieved in $O(1)$ time.

```
struct llist {
  struct llnode *front;
  int length;
};

// TIME: O(1)
int length(struct llst *lst) {
  return lst->length;
}
```

Naturally, other list functions would have to update the `length` when necessary.

```
struct llist *create_list(void) {
  struct llist *lst = malloc(sizeof(struct llist));
  lst->front = NULL;
  lst->length = 0;  // *****NEW
  return lst;
}

void add_to_front(int n, struct llist *lst) {
  previous_add_to_front(n, &lst->front);
  lst->length++;    // *****NEW
}
```

A common addition to a wrapper structure is a `back` pointer that points to the *last node* in the list.

This allows for an $O(1)$ `add_to_back` function.

```
void add_to_back(int n, struct llist *lst) {
  struct llnode *new = malloc(sizeof(struct llnode));
  new->item = n;
  new->next = NULL;
  if (lst->length == 0) { // empty list
    lst->front = new;
    lst->back = new;
  } else {
    lst->back->next = new;
    lst->back = new;
  }
  lst->length++;
}
```

Other functions would also have to be changed to update `back`.

By working with a *list* structure instead of directly working with *node* structures, there is less chance for misuse (*e.g.,* pointing to nodes in the middle).

However, the wrapper approach introduces entirely new ways that the information can be corrupted. For example, what if the `length` field does not accurately reflect the true length?

Whenever the same information is stored in more than one way, it is susceptible to *integrity* (consistency) issues.

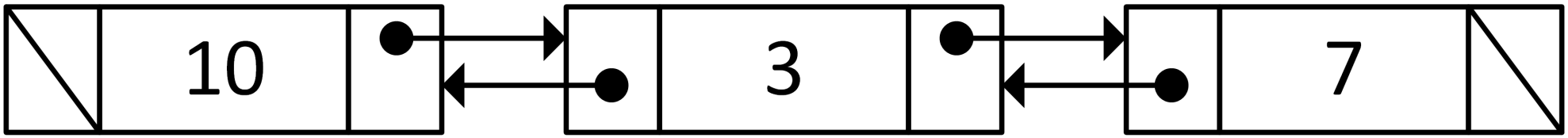
For example, a naïve user may think that the following statement removes all of the nodes from the list.

```
lst->length = 0;
```

Augmentation strategy

In an **augmentation strategy**, each *node* is *augmented* to include additional information about the node or the structure.

The most common augmentation for a liked list is to create a ***doubly linked list***, where each node also contains a pointer to the *previous* node. When combined with a `back` pointer in a wrapper, a doubly linked list can add or remove from the front **and back** in $O(1)$ time.



- The **functional** approach is appropriate when there is no mutation and extensive memory management is not required (*i.e.,* when there is a garbage collector),
- the **imperative** approach is appropriate when there is mutation or memory management is necessary,
- the **wrapper** strategy provides a cleaner interface and can be used to store additional information in a *wrapper structure*, and
- the **augmentation** strategy can be used to store additional information in *each node*.

These approaches and strategies also apply to other linked data structures (*e.g.,***trees**).

BSTs

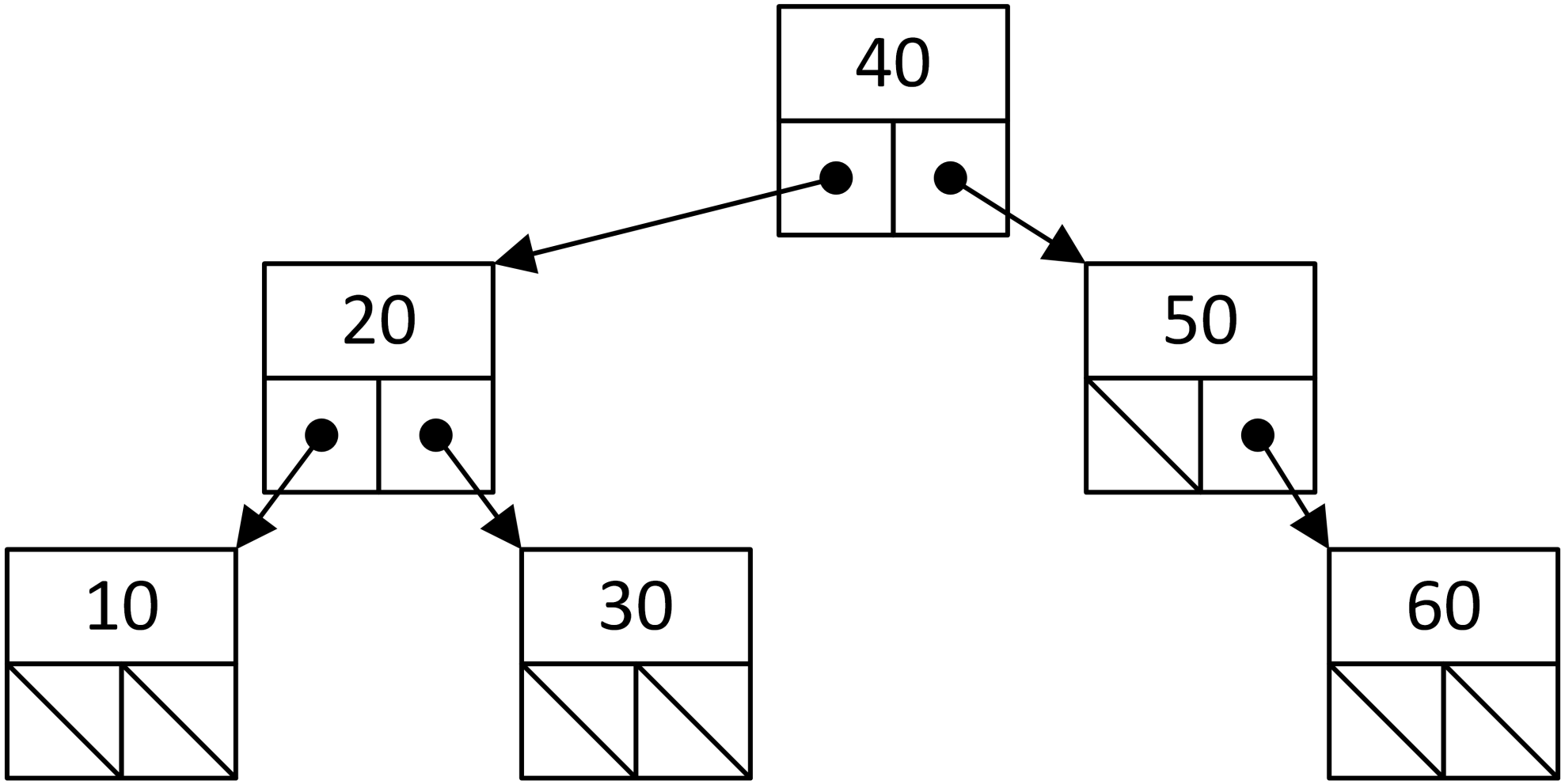
In previous courses, we used key/value pairs with *Binary Search Trees (BSTs)*

In this course, we only store **single items** (keys) in BSTs. **Values are considered an *augmentation***.

Similar to linked lists, a BST is pointer to *a BST node* (`bstnode`), and an empty tree is `NULL`.

```
struct bstnode {
  int item;
  struct bstnode *left;
  struct bstnode *right;
};
```

The definition of a BST includes the **ordering property**.



Additional tree definitions

The ***depth*** of a node is the number of nodes from the root to the node, including the root. The *depth* of the root node is 1.

The ***height*** of a tree is the maximum *depth* in the tree. The *height* of an empty tree is 0.

For the tree on the previous slide, the *depth* of the node with 50 is 2, and *height* of the tree is 3.

Other courses may use slightly different definitions of *depth* and *height*.

Making new nodes

In Racket, dynamic memory is used to *"make"* a structure.

For example, `(posn 3 4)` (previously `(make-posn 3 4)`) obtains space from the Racket heap for the structure.

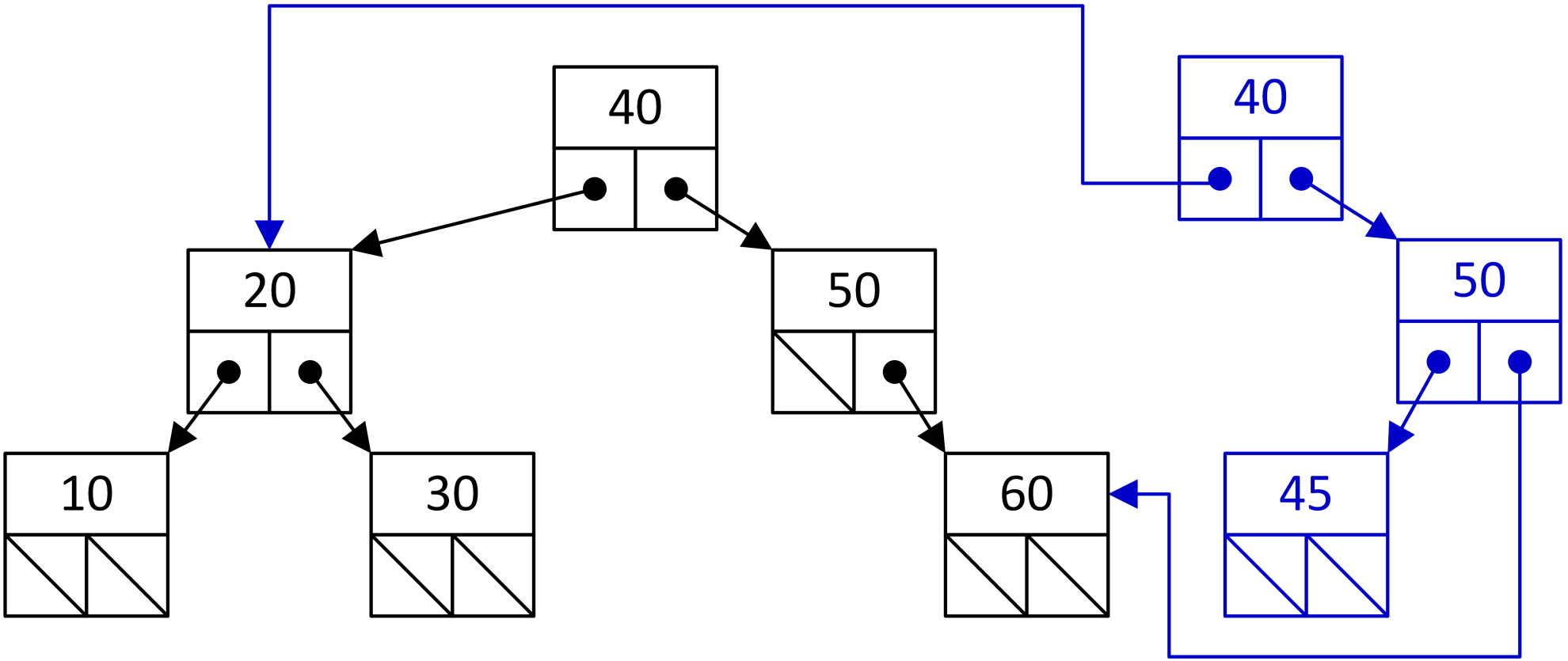
In a **functional** approach, we can similarly *"make"* a bstnode.

```
struct bstnode *make_bstnode(int i, struct bstnode *l,
                                    struct bstnode *r) {

  struct bstnode *new = malloc(sizeof(struct bstnode));
  new->item = i;
  new->left = l;
  new->right = r;
  return new;
}
```

Using a functional approach with BSTs may cause memory management problems similar to the problem shown with the linked list `insert` function.

```
// functional approach
struct bstnode *bst_insert(int i, struct bstnode *t) {
  if (t == NULL) {
    return make_bstnode(i, NULL, NULL);
  } else if (i == t->item) {
    return t;
  } else if (i < t->item) {
    return make_bstnode(t->item,
                        bst_insert(i, t->left), t->right);
  } else {
    return make_bstnode(t->item, t->left,
                        bst_insert(i, t->right);
  }
}
```

```
struct bstnode my_tree = make_bstnode(40,
  make_bstnode(20, make_bstnode(10,NULL,NULL),
                   make_bstnode(30,NULL,NULL)),
  make_bstnode(50,NULL, make_bstnode(60,NULL,NULL)));

struct bstnode new_tree = bst_insert(45, my_tree);
```



An imperative approach can be used to *change* an *existing* tree.

Like with linked lists, we must pass a *pointer to a pointer* to the root BST node.

```
// imperative approach (recursive)
void bst_insert(int i, struct bstnode **ptr_root) {
  struct bstnode *t = *ptr_root;
  if (t == NULL) { // tree is empty
    *ptr_root = make_bstnode(i, NULL, NULL);
  } else if (t->item == i) {
    return;
  } else if (i < t->item) {
    bst_insert(i, &(t->left));
  } else {
    bst_insert(i, &(t->right));
  }
}
```

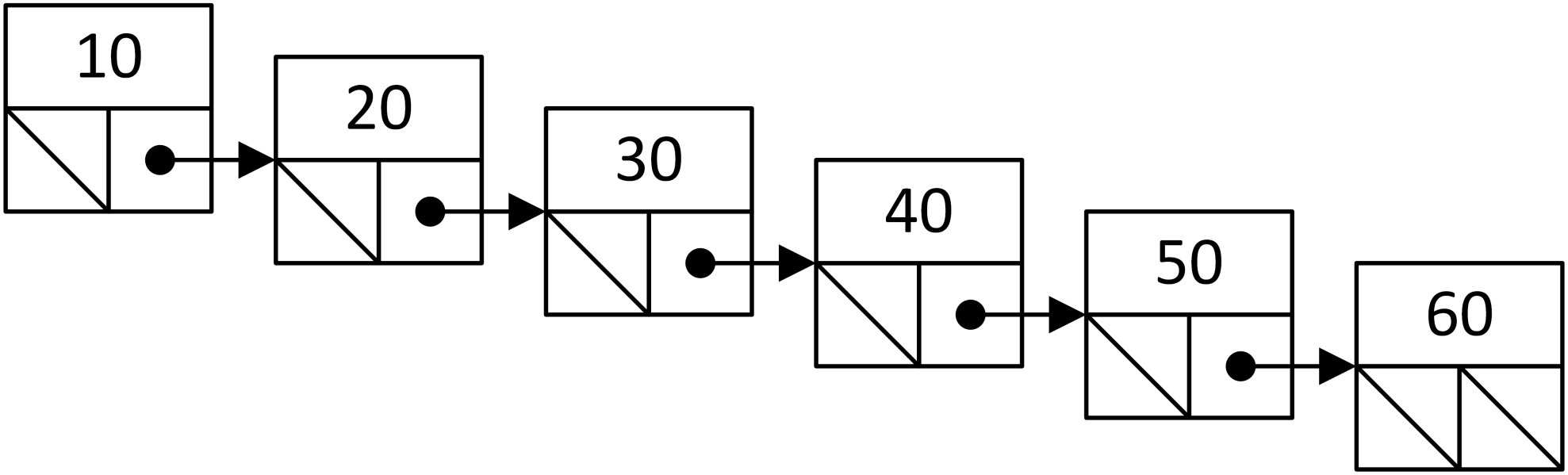
```c
// imperative approach (iterative)
void bst_insert(int i, struct bstnode **ptr_root) {
  struct bstnode *prev = NULL;
  struct bstnode *cur = *ptr_root;
  while (cur != NULL) {
    if (i == cur->item) return;
    prev = cur;
    if (i < cur->item) cur = cur->left;
    else cur = cur->right;
  }
  struct bstnode *new = make_bstnode(i, NULL, NULL);
  if (prev == NULL) { // empty tree
    *ptr_root = new;
  } else if (i < prev->item) {
    prev->left = new;
  } else {
    prev->right = new;
  }
}
```

Most tree functions are more easily written and understood with a recursive approach.

Trees and efficiency

What is the efficiency of `bst_insert`?

The *worst case* is when the tree is ***unbalanced***, and *every* node in the tree must be visited.



In this example, the running time of `bst_insert` is $O(n)$, where n is the number of nodes in the tree.

The running time of `bst_insert` is $O(h)$: it depends more on the *height* of the tree (h) than the *size* of the tree (n).

The definition of a ***balanced tree*** is a tree where the height (h) is $O(\log n)$.

Conversely, an **un**balanced tree is a tree with a height that is **not** $O(\log n)$. The height of an unbalanced tree is $O(n)$.

Using the `bst_insert` function we provided, inserting the nodes in *sorted order* creates an *unbalanced* tree.

With a **balanced** tree, the running time of the `insert`, `remove` and `search` functions are all $O(\log n)$.

With an **unbalanced** tree, the running time of each function is $O(h)$.

A ***self-balancing tree*** "re-arranges" the nodes to ensure that tree is always balanced. With a good self-balancing implementation, the `insert` and `remove` functions *preserve the balance of the tree* **and** have an $O(\log n)$ running time.

In CS 240 and CS 341 you will see *self-balancing trees*.

Self-balancing trees often use an augmentation strategy.

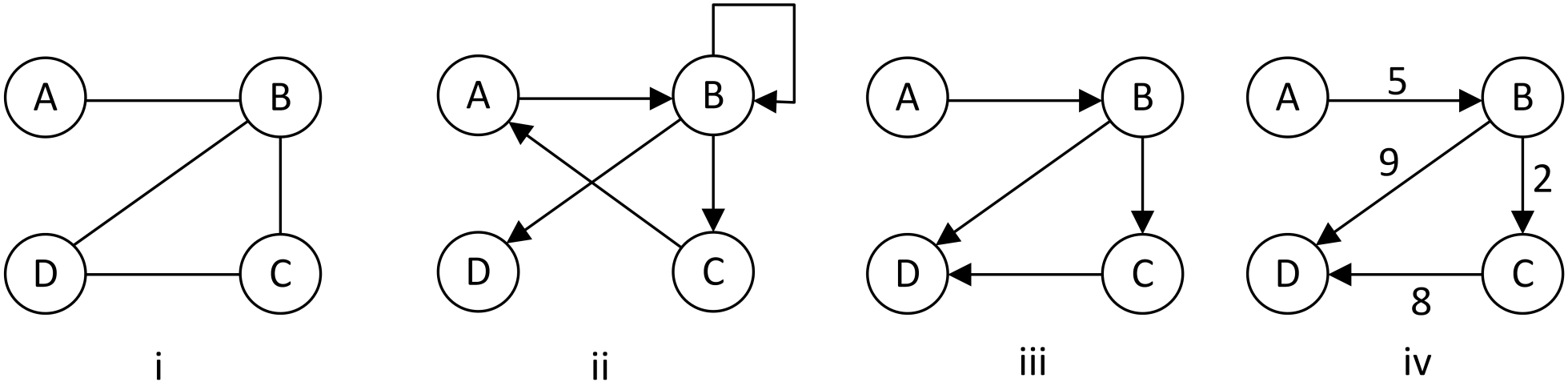
A popular tree **augmentation** is to store in *each node* the **size** of its subtree.

```
struct bstnode {
  int item;
  struct bstnode *left;
  struct bstnode *right;
  int size;                  // *****NEW
};
```

This augmentation allows us to retrieve the size of the tree in $O(1)$ time. It also allows us to implement a `select` function in $O(h)$ time. `select(k)` finds the `k`-th smallest item in the tree.

Graphs

Linked lists and trees can be thought of as *"special cases"* of a ***graph*** data structure. Graphs are the only core data structure we are **not** working with in this course.



Graphs link **nodes** with **edges**. Graphs may be undirected (i) or directed (ii), allow cycles (ii) or be acyclic (iii), and have labeled edges (iv) or unlabeled edges (iii).

Goals of this Section

At the end of this section, you should be able to:

- use the new linked list and tree terminology introduced
- use linked lists and trees with a functional or imperative approach and with a wrapper or augmented strategy
- explain the memory management issues related to working with linked lists and trees

- explain why an unbalanced tree can affect the efficiency of tree functions
- explain how a wrapper or augmentation strategy can be used to improve efficiency