Problem 2

a) by technique showed in class

$$L = \lim_{n\to\infty} \frac{f(n)}{g(n)}$$

$$= \lim_{n\to\infty} \frac{n^2 + 27n\log n + 2016}{n^2\log n + 2016}$$

$$= \lim_{n\to\infty} \frac{2n + 27\log n + \frac{27n}{n\cdot\ln 2}}{2n\log n + \frac{n^2}{n\cdot\ln 2}}$$ by l'Hôpital's Rule

$$= \lim_{n\to\infty} \frac{2n + 27\log n + \frac{27}{\ln 2}}{2n\log n + \frac{n}{\ln 2}}$$

$$= \lim_{n\to\infty} \frac{2 + \frac{27}{n\cdot\ln 2}}{2\log n + \frac{2n}{n\cdot\ln 2} + \frac{1}{\ln 2}}$$ by l'Hôpital's Rule

$$= \lim_{n\to\infty} \frac{27\cdot -\frac{1}{n^2\cdot\ln 2}}{\frac{2}{n\cdot\ln 2}}$$ by l'Hôpital's Rule

$$= \lim_{n\to\infty} -\frac{27}{2\cdot n}$$

$$\approx 0$$

Thus, we can show that $f(n) \in O(g(n))$

b) by technique showed in class

$$L = \lim_{n\to\infty} \frac{f(n)}{g(n)}$$

$$= \lim_{n\to\infty} \frac{10^n + 99n^{10}}{75^n + 25n^{27}}$$

$$= \frac{\lim_{n\to\infty}(10^n + 99n^{10})}{\lim_{n\to\infty}(75^n + 25n^{27})} = \frac{\lim_{n\to\infty} 10^n + \lim_{n\to\infty} 99n^{10}}{\lim_{n\to\infty} 75^n + \lim_{n\to\infty} 25n^{27}} = 0$$

Thus we can show that $f(n) \in O(g(n))$

c). by tech. showed in class

$$L = \lim_{n \to \infty} \frac{f(n)}{g(n)}$$

$$= \lim_{n \to \infty} \frac{\sqrt{n}}{(\log n)^7}$$

$$= \lim_{n \to \infty} \frac{\frac{1}{2} \cdot n^{-\frac{1}{2}}}{\frac{7 \cdot (\log n)^6}{n \ln 2}} = \lim_{n \to \infty} \frac{n^{\frac{1}{2}} \cdot \ln 2}{14 (\log n)^6}$$ by L'Hospital Rule

$$= \lim_{n \to \infty} \frac{\frac{1}{2} n^{-\frac{1}{2}} \cdot \ln 2}{\frac{14 \cdot 6 (\log n)^5}{n \ln 2}} = \lim_{n \to \infty} \frac{n^{\frac{1}{2}} (\ln 2)^2}{168 (\log n)^5}$$ by L'Hospital Rule

...... keep using L'Hospital Rule

~~finally~~ we can get

$L \approx 0$

Thus $f(x) \in O(g(n))$