CS 245 / Fall 2015
Propositional Logic
Summary of Logical Identities

Commutativity

$$\alpha \wedge \beta \equiv \beta \wedge \alpha$$
$$\alpha \vee \beta \equiv \beta \vee \alpha$$
$$\alpha \leftrightarrow \beta \equiv \beta \leftrightarrow \alpha$$

Associativity

$$\alpha \wedge (\beta \wedge \gamma) \equiv (\alpha \wedge \beta) \wedge \gamma$$
$$\alpha \vee (\beta \vee \gamma) \equiv (\alpha \vee \beta) \vee \gamma$$

Distributivity

$$\alpha \vee (\beta \wedge \gamma) \equiv (\alpha \vee \beta) \wedge (\alpha \vee \gamma)$$
$$\alpha \wedge (\beta \vee \gamma) \equiv (\alpha \wedge \beta) \vee (\alpha \wedge \gamma)$$

De Morgan's Laws

$$\neg(\alpha \wedge \beta) \equiv \neg\alpha \vee \neg\beta$$
$$\neg(\alpha \vee \beta) \equiv \neg\alpha \wedge \neg\beta$$

Double Negation

$$\neg(\neg\alpha) \equiv \alpha$$

Excluded Middle

$$\alpha \vee \neg\alpha \equiv \mathtt{T}$$

Contradiction

$$\alpha \wedge \neg\alpha \equiv \mathtt{F}$$

Implication

$$\alpha \rightarrow \beta \equiv \neg\alpha \vee \beta$$

Contrapositive

$$\alpha \rightarrow \beta \equiv \neg\beta \rightarrow \neg\alpha$$

Equivalence

$$\alpha \leftrightarrow \beta \equiv (\alpha \rightarrow \beta) \wedge (\beta \rightarrow \alpha)$$

Idempotence

$$\alpha \vee \alpha \equiv \alpha$$
$$\alpha \wedge \alpha \equiv \alpha$$

Simplification I (a.k.a Absorbtion)

$$\alpha \wedge \mathtt{T} \equiv \alpha$$
$$\alpha \vee \mathtt{T} \equiv \mathtt{T}$$
$$\alpha \wedge \mathtt{F} \equiv \mathtt{F}$$
$$\alpha \vee \mathtt{F} \equiv \alpha$$

Simplification II

$$\alpha \vee (\alpha \wedge \beta) \equiv \alpha$$
$$\alpha \wedge (\alpha \vee \beta) \equiv \alpha$$