CS245 Tutorial

Sept. 28, 2015

1. Consider the set of propositional formulas: $\{\ b \to h,\ h \to (d \vee \neg b),\ b,\ d\ \}$

   Is this set of formulas satisfiable? I.e., is there a truth valuation such that each formula in the set is satisfied.

   Yes, the set of formulas is satisfiable.
   Consider the truth valuation: $t(b) = \mathrm{T}, t(d) = \mathrm{T}, t(h) = \mathrm{T}$.

2. Do the premises semantically entail (logically imply) the conclusion? Answer this question using a truth table. Explain.

   $\{p \vee \neg q, p \to r, \neg q \to r\} \models r$

   The definition of semantically entails states that the premises semantically entail the conclusion iff every truth valuation which satisfies the premises also satisfies the conclusion. We can construct a truth table which lists every possible truth valuation (assignment of true or false to each proposition) and check. We see below that whenever all of the premises are true, the conclusion r is also true (see rows marked with $\lhd$). Therefore the premises semantically entail r.

p	q	r	$p \vee \neg q$	$p \to r$	$\neg q \to r$	r	
T	T	T	T	T	T	T	$\lhd$
T	T	F	T	F	T	F	
T	F	T	T	T	T	T	$\lhd$
T	F	F	T	F	F	F	
F	T	T	F	T	T	T	
F	T	F	F	T	T	F	
F	F	T	T	T	T	T	$\lhd$
F	F	F	T	T	F	F	

3. Do the premises semantically entail (logically imply) the conclusion? Answer this question using a truth table. Explain.

$$\{\neg p \to (\neg q \vee r), p \to r, \neg r\} \models q$$

Again, we can construct a truth table which lists every possible truth valuation and check. We see below that there are cases where all of the premises are true, but the conclusion q is not true. In particular, the row marked $\lhd$ is the counter example. All of the premises are true, but the conclusion is false. Therefore the premises do not semantically entail q.

p	q	r	$\neg p \to (\neg q \vee r)$	$p \to r$	$\neg r$	q	
T	T	T	T	T	F	T	
T	T	F	T	F	T	T	
T	F	T	T	T	F	F	
T	F	F	T	F	T	F	
F	T	T	T	T	F	T	
F	T	F	F	T	T	T	
F	F	T	T	T	F	F	
F	F	F	T	T	T	F	$\lhd$

4. Prove the equivalence: $\neg(p \vee q) \vee (\neg p \wedge q) \equiv \neg p$

$\neg(p \vee q) \vee (\neg p \wedge q)$

$\equiv (\neg p \wedge \neg q) \vee (\neg p \wedge q)$	De Morgan
$\equiv \neg p \wedge (\neg q \vee q)$	Distributive
$\equiv \neg p \wedge \texttt{T}$	Excluded Middle
$\equiv \neg p$	Simplification I

5. Prove the equivalence: $p \wedge (p \to q) \equiv (p \wedge q)$

$p \wedge (p \to q)$

$\equiv p \wedge (\neg p \vee q)$	Implication
$\equiv (p \wedge \neg p) \vee (p \wedge q)$	Distributive
$\equiv \texttt{F} \vee (p \wedge q)$	Contradiction
$\equiv p \wedge q$	Simplification I

6. Prove the equivalence: $(p \leftrightarrow q) \equiv ((p \wedge q) \leftrightarrow (p \vee q))$

$((p \wedge q) \leftrightarrow (p \vee q))$	
$\equiv ((p \wedge q) \rightarrow (p \vee q)) \wedge ((p \vee q) \rightarrow (p \wedge q))$	Equivalence
$\equiv (\neg(p \wedge q) \vee (p \vee q)) \wedge (\neg(p \vee q) \vee (p \wedge q))$	Implication (2×)
$\equiv (\neg p \vee \neg q \vee p \vee q) \wedge (\neg(p \vee q) \vee (p \wedge q))$	De Morgan (*)
$\equiv (\neg p \vee p \vee \neg q \vee q) \wedge (\neg(p \vee q) \vee (p \wedge q))$	Commutative
$\equiv (\mathtt{T} \vee \mathtt{T}) \wedge (\neg(p \vee q) \vee (p \wedge q))$	Excluded Middle (2×)
$\equiv \mathtt{T} \wedge (\neg(p \vee q) \vee (p \wedge q)$	Idempotence
$\equiv \neg(p \vee q) \vee (p \wedge q)$	Simplification I
$\equiv (\neg p \wedge \neg q) \vee (p \wedge q)$	De Morgan
$\equiv ((\neg p \wedge \neg q) \vee p) \wedge ((\neg p \wedge \neg q) \vee q)$	Distributive
$\equiv ((\neg p \vee p) \wedge (\neg q \vee p)) \wedge ((\neg p \vee q) \wedge (\neg q \vee q))$	Distributive (2×)
$\equiv (\mathtt{T} \wedge (\neg q \vee p)) \wedge ((\neg p \vee q) \wedge \mathtt{T})$	Excluded Middle (2×)
$\equiv (\neg q \vee p) \wedge (\neg p \vee q)$	Simplification I (2×)
$\equiv (q \rightarrow p) \wedge (p \rightarrow q)$	Implication (2×)
$\equiv p \leftrightarrow q$	Equivalence

(*) Note: it's okay to drop the brackets and implicitly use the associativity rule.