Surname:	Personal name:	ID #:

Circle the time/room of your tutorial:		8:30, MC 2035 (TUT 101)
11:30, RCH 204 (TUT 107)	12:30, MC 2034 (TUT 104)	12:30, RCH 305 (TUT 106)
1:30, RCH 308 (TUT 105)	4:30, MC 4060 (TUT 102)	4:30, MC 2034 (TUT 103)

Due Wednesday, Sept. 23 by noon, to the dropbox near MC 4065.

Attach this page as a cover page on your submission.

Question 1.
Translate the following sentences into propositional logic. For each atom that you use, explicitly define the English statement which it represents. Choosing which English statements to represent by atoms is part of the question.

(a) I will go to school, although I feel like sleeping in.

(b) My exam will be easy to mark if and only if my writing is easy to read.

(c) Whether or not there is something interesting on Netflix, I will do my assignment.

(d) If there is a thunderstorm I won't go to the party, but I will do my assignment.

(e) If it is Spring, then both Waterloo and Toronto are rainy.

(f) If it is not raining or it is sunny, then I do not use an umbrella.

(g) I will bring my umbrella, unless it is sunny.

(h) If it rains, Alex will stay at home; otherwise Alex will go to the market or to school.

(i) Alex will go to the party, unless Sam does not go to the party.

Question 2.
Give the truth table for each of the following formulas.

(a) $\big(p \to (q \to r)\big)$.

(b) $\big((p \wedge q) \to r\big)$.

(c) $\big(\neg(p \to q) \to \neg p\big)$.

Marks:	1		2		Total:	

Study Exercises

To assist you with thinking about the topics of this assignment and about how to solve the problems, we provide the following suggestions for exercises. You may work on them on your own or with classmates (or both) as you choose. When you get stuck, please feel free to ask an instructor or IA.

How many exercises you work on is entirely up to you. We do suggest, however, that knowing the answers is NOT the important part. The important part is to practice finding such answers yourself. You will need to do this on exams.

SE 1. Huth & Ryan, Exercise 1.1.1, p. 78. Any or all of the parts.

SE 2. Briefly explain why each of the following **cannot** be translated into propositional logic.

(a) Hand your solutions in to the drop box.

(b) Can this sentence be translated into propositional logic?

(c) Fruit flies like a banana.

SE 3. Huth & Ryan, Exercises 1.3, pp. 81–2. Any or all of the questions.

SE 4. Huth & Ryan, Exercises 1.4.1–2, p. 82, 84. Any or all of the parts.

SE 5. The "Polish notation" for formulas is defined as follows.

- Any propositional variable is a formula (an atom).
- If α is a formula, then $\neg\alpha$ is a formula.
- If α and β are formulas, then $\wedge\alpha\beta$ is a formula.
- Nothing else is a formula.

As you will prove in this exercise, formulas in Polish notation are unambiguous, even though they have no parentheses.

(a) For each of the following formulas in Polish notation, give the steps to construct it. Also give the corresponding formula in standard notation.

   i. $\wedge p \wedge q \neg p$.

   ii. $\wedge \neg \wedge p q p$.

(b) Prove, using induction on the length of a formula, that no formula is a proper prefix of another formula. That is, if φ is a formula, and η is any expression of length 1 or more, then their contatenation $\varphi\eta$ is not a formula.

(c) Using the previous result, prove that every formula in Polish notation is a formula in only one way.

SE 6. The following claim is false. Thus its “proof” must contain at least one error.

Claim: Every non-negative integer is even.

Proof by mathematical induction.

Base case: 0 is even.

Inductive step: Suppose that the claim holds up to k.

To prove that $k+1$ is even, note that $k+1=(k-1)+2$.

Since $k-1\leq k$, the hypothesis of the inductive step applies: $k-1$ is even.

By the definition of even, $k-1=2m$ for some $m\in\mathbb{Z}$.

Thus $k+1=(k-1)+2=2m+2=2(m+1)$, where $m+1\in\mathbb{Z}$.

Therefore, by the definition of even, $k+1$ is even.

Thus, by the principle of induction, every non-negative integer is even.

(a) Specify the exact property being used in the induction. Give both the precise assumption in the inductive step and the precise conclusion of the inductive step.

(b) Explain precisely where and how the “proof” makes its mistake(s). (Do not simply say it can’t be right; identify exactly what is wrong.)