CS 245

Predicate Logic

Summary of Additional Inference Rules for Natural Deduction

∀-introduction: ∀i

$$\frac{\left[\begin{array}{l} u \text{ fresh} \\ \vdots \\ \alpha[u/x] \end{array}\right]}{\forall x \cdot \alpha}$$

∀-elimination: ∀e

$$\frac{\forall x \cdot \alpha}{\alpha[t/x]}$$

∃-introduction: ∃i

$$\frac{\alpha[t/x]}{\exists x \cdot \alpha}$$

∃-elimination: ∃e

$$\frac{\exists x \cdot \alpha \qquad \left[\begin{array}{l} \alpha[u/x] \quad u \text{ fresh} \quad \text{assumption} \\ \quad \vdots \\ \quad \beta \end{array}\right]}{\beta}$$

u should not occur free in α or β

Reflexivity: =i

$$\frac{}{t = t}$$

Substitution: =e

$$\frac{t_1 = t_2 \qquad \alpha[t_2/x]}{\alpha[t_1/x]} \qquad \frac{t_1 = t_2 \qquad \alpha[t_1/x]}{\alpha[t_2/x]}$$