Surname:	Personal name:	ID #:
Circle the time/room of your tutorial:		8:30, MC 2035 (TUT 101)
11:30, RCH 204 (TUT 107)	12:30, MC 2034 (TUT 104)	12:30, RCH 305 (TUT 106)
1:30, RCH 308 (TUT 105)	4:30, MC 4060 (TUT 102)	4:30, MC 2034 (TUT 103)

Due Wednesday, Oct. 7, by noon, to the dropbox near MC 4065.

Attach this page as a cover page on your submission.

Question 1 (12 marks).
Consider the two fragments of code given below, where P_1, P_2, P_3, and P_4 are blocks of code.

Fragment #1

```
if ( a ∨ ¬b ) {
    if ( a ∧ b ) { P1 }
    else if ( ¬b ) { P2 }
    else { P3 }
}
else {
    P4
}
```

Fragment #2

```
if ( a ∧ b ) {
    P1
}
else if ( b ) {
    P4
}
else {
    P2
}
```

(a) For Fragment #1, express in propositional logic the conditions under which each of the blocks of code P_1, P_2, P_3, and P_4 will be executed. Do not simplify for this part.

(b) For Fragment #2, express in propositional logic the conditions under which each of the blocks of code P_1, P_2, and P_4 will be executed. Do not simplify for this part.

(c) Give equivalence proofs to show that Fragment #1 and Fragment #2 have the same behavior. For any unreachable (dead) code, give an equivalence proof that the condition under which the code would be executed are a contradiction (equivalent to false). For any reachable code, give an equivalence proof that the conditions under which the code would be executed are equivalent in both fragments.

Question 2 (8 marks).
Consider formulas that may have $\perp$ as a constant symbol (a "nullary connective"), such that the formula $\perp$ has value F under any valuation. ("$\perp$" may also occur as a sub-formula, e.g. "$(\perp \vee p)$", etc.)

Show that the set $\{\rightarrow, \perp\}$ is an adequate set of connectives, by showing that it can define each of the connectives $\wedge$, $\vee$ and $\neg$.

Marks:	1		2		3		Total:	

Question 3 (15 marks).
Prove or disprove each of the following semantic entailment statements. To prove that a semantic entailment statement holds, give a resolution refutation proof. If a semantic entailment statement does not hold, explain why.

(a) $\{(q \vee r) \to s\} \models q \to s$

(b) $\{q \to r,\ \neg s \to \neg r\} \models q \leftrightarrow s$

(c) $\{\neg r \to s,\ q \vee \neg r,\ s \to \neg p,\ \neg\neg p\} \models q$

(d) $\{\neg r \vee \neg p,\ \neg q \to r,\ \neg s \to p\} \models q \vee s$

(e) $\{\neg r \vee \neg p,\ \neg q \to r,\ \neg s \to p\} \models \neg(q \vee s)$

Study Exercises

To assist you with thinking about the topics of the course and about how to solve the problems, we provide the following suggestions for exercises. You may work on them on your own or with classmates (or both) as you choose. When you get stuck, please feel free to ask an instructor or IA.

How many exercises you work on is entirely up to you. We do suggest, however, that knowing the answers is NOT the important part. The important part is to practice finding such answers yourself. You will need to do this on exams.

SE 1. Huth & Ryan, Exercises 1.4.12, 1.4.13, and 1.4.14, pp. 86.

SE 2. Huth & Ryan, Exercises 1.5.2, 1.5.3, and 1.5.5, pp. 87–88.

SE 3. As an exercise in formal inductive proof, give a complete proof that a definable connective is unnecessary. That is, prove the following, where $\circ$ is any binary connective symbol.

> Suppose that there is a formula α such that
>
> - α does not contain the connective $\circ$, and
> - The formula $p \circ q$ is equivalent to α.
>
> Then for any formula, which may contain $\circ$, there is an equivalent formula which does not contain $\circ$.

You may think this obvious (or you may not). But do a formal induction, just for practice.

SE 4. Give a resolution proof of the entailment $\{q \to s,\ r \to p\} \models (q \vee r) \to (s \vee p)$. Remember to negate the conclusion and then convert to CNF.

SE 5. Convert the formula $(p \to q) \to (\neg r \to (p \wedge q))$ to an equivalent formula in conjunctive normal form (CNF). Simplify when possible, by using the appropriate logical identities—Simplification I & II, Contradiction, Excluded Middle, and/or Idempotence. (Note: There are many possible orders in which to do the conversion. Although any order will ultimately work, you can simplify your conversion, and do less, by doing the "right" ones first.)