Surname:	Personal name:	ID #:
Circle the time/room of your tutorial:		8:30, MC 2035 (TUT 101)
11:30, RCH 204 (TUT 107)	12:30, MC 2034 (TUT 104)	12:30, RCH 305 (TUT 106)
1:30, RCH 308 (TUT 105)	4:30, MC 4060 (TUT 102)	4:30, MC 2034 (TUT 103)

Due Wednesday, Sept. 30, by noon, to the dropbox near MC 4065.

Attach this page as a cover page on your submission.

Question 1 (12 marks).
For this question, use the precise definition of a well-formed formula (slide 21 of the overheads).

Let φ be a well-formed formula and let $\varphi = a_1 a_2 \ldots a_n$, where each a_i denotes a single symbol: either a propositional variable, a connective symbol, or a parenthesis.

Prove by structural induction that every two propositional variables in φ have a connective between them. In other words, for every $i < j$, if a_i and a_j are propositional variables, then there is some $i < k < j$ such than the symbol a_k is a connective symbol.

Be careful to lay our your induction precisely.

Question 2 (12 marks).
For each of the propositional formulas given below, answer all three of the following questions: Is the formula satisfiable? Is the formula a tautology? Is the formula a contradiction? Use truth tables and/or valuation trees to justify your answers.

(a) $(q \vee r) \to p$

(b) $(\neg(p \to q) \to (p \wedge \neg q)) \wedge ((p \wedge \neg q) \to \neg(p \to q))$

(c) $(p \to q) \leftrightarrow \neg(\neg p \vee q)$

Question 3 (12 marks).
Prove or disprove each of the following semantic entailment statements. Use truth tables and/or valuation trees to justify your answers.

(a) $\{\neg p \to \neg q, p \to r\} \models \neg(q \wedge \neg r)$

(b) $\{p \to \neg q, r \to q\} \models r \to p$

(c) $\{p \to \neg q, r \to q\} \models p \to \neg r$

Marks:	1		2		3		Total:	

Study Exercises

To assist you with thinking about the topics of the course and about how to solve the problems, we provide the following suggestions for exercises. You may work on them on your own or with classmates (or both) as you choose. When you get stuck, please feel free to ask an instructor or IA.

How many exercises you work on is entirely up to you. We do suggest, however, that knowing the answers is NOT the important part. The important part is to practice finding such answers yourself. You will need to do this on exams.

SE 1. Huth & Ryan, Exercise 1.4.6, p. 85.

SE 2. Huth & Ryan, Exercises 1.4.7, 1.4.8, and 1.4.10, pp. 85–86.

SE 3. For each of the propositional formulas given below, answer all three of the following questions: Is the formula satisfiable? Is the formula a tautology? Is the formula a contradiction? Use truth tables and/or valuation trees to justify your answers.

- $((p \land q) \to r) \to ((p \to r) \lor (q \to r))$
- $\neg(p \to q) \to (\neg p \to \neg q)$
- $(p \land \neg r) \land \neg\neg(r \land q) \land (r \land \neg q)$

SE 4. Prove or disprove each of the following equivalence statements. Use proofs using logical equivalences (identities) to justify your answers. Some of these were done in class.

- $(p \land q) \lor (q \land r) \equiv q \land (p \lor r)$
- $p \lor (p \land q) \equiv p$
- $\neg(\neg p \lor \neg(r \lor s)) \equiv (p \land r) \lor (p \land s)$
- $\neg(\neg(p \land q) \lor p) \equiv \mathrm{F}$
- $p \equiv p \land (q \to p)$
- $p \equiv p \land (\neg(\neg q \land \neg p) \lor p)$
- $p \land (\neg(\neg q \land \neg p) \lor p) \equiv q$

SE 5. Prove or disprove each of the following semantic entailment statements. Justify your answers.

- $\{\neg p \to \neg r, \neg(p \land q)\} \models r \to q$
- $\{p, p \to q, q \to r\} \models r$
- $\{p \to q, q \to r, \neg r\} \models \neg p$
- $\{\neg p \to (q \lor r), p \to q, \neg q\} \models r$
- $\{(p \lor r) \lor (p \to q), \neg(p \to q), r \to (p \to q)\} \models p$
- $\{p \to (q \lor r), p \to \neg r, p\} \models q$
- $\{p \to (q \lor r), p \to \neg r, p\} \models \neg q$