Resolution Example — Blocks world

Suppose we have the following predicates.

$On(x, y)$:	x is directly on y,
$Above(x, y)$:	x is above y (but not necessarily directly on it)
$Clear(x)$:	x is clear (has nothing on it),
$Green(x)$:	x is colored green,
$OnTable(x)$:	x is on the table.

and suppose that Σ consists of the following statements.

$On(A, B)$,
$On(B, C)$,
$On(C, D)$,
$On(D, E)$,
$Green(D)$,
$\neg Green(B)$

$\forall x \cdot \forall y \cdot \big(On(x, y) \rightarrow Above(x, y)\big)$,
$\forall x \cdot \forall y \cdot \forall z \cdot \big(\big(Above(x, y) \wedge Above(y, z)\big) \rightarrow Above(x, z)\big)$,
$\forall x \cdot \forall y \cdot \big(Above(x, y) \rightarrow \neg Clear(y)\big)$,
$\forall x \cdot \forall y \cdot \big(Above(x, y) \rightarrow \neg OnTable(x)\big)$,

or, in clause form,

1. $On(A, B)$
2. $On(B, C)$
3. $On(C, D)$
4. $On(D, E)$
5. $Green(D)$
6. $\neg Green(B)$
7. $\neg On(x, y) \vee Above(x, y)$
8. $\neg Above(x, y) \vee \neg Above(y, z) \vee Above(x, z)$
9. $\neg Above(x, y) \vee \neg Clear(y)$
10. $\neg Above(x, y) \vee \neg OnTable(x)$

For each of the following queries φ, determine whether the conclusion is a logical consequence of Σ (i.e., does $\Sigma \models \varphi$ hold?). Use resolution to determine whether $\Sigma \cup \{\neg\varphi\} \vdash_{Res} \bot$. (This works because Resolution is a sound and complete refutation system.)

(a) Is there a block on the table that has a green block directly on it?

$$\exists x \cdot \exists y \cdot (OnTable(x) \wedge On(y, x) \wedge Green(y))$$

(b) Is there a green block that has a block above it that is not green?

$$\exists x \cdot \exists y \cdot (Green(x) \wedge Above(y, x) \wedge \neg Green(y))$$

(c) Is there a green block that has a block directly on it that is not Green?

$$\exists x \cdot \exists y \cdot (Green(x) \wedge On(y, x) \wedge \neg Green(y))$$