CS245 Tutorial

Oct. 19, 2015

1. Express the following English sentences as predicate logic formulas.

   (a) All second-year students are hardworking.

   (b) No one can be clever without being hardworking.

   (c) Clever students work hard.

   (d) Those who do not work hard are lazy.

   (e) The lazy students are exactly those who do not work hard.

   (f) Not being lazy is equivalent to being hardworking.

   (a) $\forall x \cdot (T(x) \wedge S(x)) \rightarrow H(x)$

   (b) $\neg(\exists x \cdot C(x) \wedge \neg H(x))$

   (c) $\forall x \cdot (C(x) \wedge S(x)) \rightarrow H(x)$

   (d) $\forall x \cdot \neg H(x) \rightarrow L(x)$

   (e) $\forall x \cdot (L(x) \wedge S(x)) \leftrightarrow \neg H(x)$

   (f) $\forall x \cdot \neg L(x) \leftrightarrow H(x)$

   where

$T(x)$	is true iff x is in second year
$S(x)$	is true iff x is a student
$C(x)$	is true iff x is clever
$H(x)$	is true iff x is hardworking
$L(x)$	is true iff x is lazy

2. Express the following English sentences as predicate logic formulas.

   (a) London stores only supply stores outside of London.
   (b) No store supplies itself.
   (c) There are no stores in Grimsby but there are some in Halifax.
   (d) Stores do not supply stores that are supplied by stores which they supply.
   (e) There is no more than one store in any one city.
   (f) Stores which supply each other are always in the same city.

   (a) $\forall x \cdot \forall y \cdot (I(x, \text{London}) \wedge S(x, y)) \rightarrow \neg I(y, \text{London})$
   (b) $\neg\, \exists x \cdot\, S(x, x)$
   (c) $(\neg\, \exists x \cdot\, I(x, \text{Grimsby})) \wedge (\exists y \cdot\, I(y, \text{Halifax}))$
   (d) $\forall x\, \forall y \cdot \forall z \cdot (S(x, y) \wedge S(y, z)) \rightarrow \neg S(x, z)$
   (e) $\neg(\exists x \cdot, y, p I(x, p) \wedge I(y, p) \wedge \neg(x = y))$
   (f) $\forall x \cdot \forall y \cdot\, S(x, y) \rightarrow \exists p \cdot\, I(x, p) \wedge I(y, p)$

   where

$S(x, y)$	is true iff store x supplies store y
$I(x, p)$	is true iff store x is in city p
$x = y$	is true iff x is equal to y

3. Let φ be the first-order logic formula,

$$\exists x \cdot C(x, \text{fred}) \wedge R(\text{wilma}, x),$$

where fred and wilma are constant symbols and $C^{(2)}$ and $R^{(2)}$ are binary predicate symbols.

(a) Give an interpretation (a model) $\mathcal{M}_1$ such that $\mathcal{M}_1 \models \varphi$.
Explain why your choice of $\mathcal{M}_1$ is correct.

Let $\mathcal{M}_1$ be the interpretation that consists of

- Domain: Let the domain (or universe) $dom(\mathcal{M}_1)$ be {Fred, Wilma, Barney, Betty}.
- Constants: let fred map to Fred and wilma map to Wilma.
- Functions: there are no function symbols.
- Predicates: let C map to the relation $\{\langle \text{Wilma}, \text{Fred}\rangle\}$ and let R map to the relation $\{\langle \text{Wilma}, \text{Fred}\rangle, \langle \text{Wilma}, \text{Wilma}\rangle\}$.

There is a value for x for which the formula is true; i.e., $x = \text{wilma}$.

(b) Give an interpretation (a model) $\mathcal{M}_2$ such that $\mathcal{M}_2 \not\models \varphi$.
Explain why your choice of $\mathcal{M}_2$ is correct.

Let $\mathcal{M}_2$ be the interpretation that consists of

- Domain: Let the domain (or universe) $dom(\mathcal{M}_2)$ be {Fred, Wilma, Barney, Betty}.
- Constants: let fred map to Fred and wilma map to Wilma.
- Functions: there are no function symbols.
- Predicates: let C map to the relation $\{\langle \text{Wilma}, \text{Fred}\rangle\}$ and let R map to the relation $\{\langle \text{Wilma}, \text{Fred}\rangle, \langle \text{Fred}, \text{Fred}\rangle, \langle \text{Fred}, \text{Wilma}\rangle\}$.

There is no value for x for which the formula is true.

4. Let φ be the first-order logic formula,

$$\forall x \cdot \forall y \cdot \mathit{Loves}(x, y) \to \mathit{Loves}(y, x)$$

$\mathit{Loves}^{(2)}$ is a binary predicate symbol.

(a) Give an interpretation (a model) $\mathcal{M}_1$ such that $\mathcal{M}_1 \models \varphi$.
Explain why your choice of $\mathcal{M}_1$ is correct.

Let $\mathcal{M}_1$ be the interpretation that consists of

- Domain: Let the domain (or universe) $dom(\mathcal{M}_1)$ be {Fred, Wilma}.
- Constants: there are no constant symbols.
- Functions: there are no function symbols.
- Predicates: let *Loves* map to the relation $\{\langle \text{Wilma}, \text{Fred} \rangle, \langle \text{Wilma}, \text{Wilma} \rangle, \langle \text{Fred}, \text{Wilma} \rangle\}$.

For all values of x and y the formula is true.

(b) Give an interpretation (a model) $\mathcal{M}_2$ such that $\mathcal{M}_2 \not\models \varphi$.
Explain why your choice of $\mathcal{M}_2$ is correct.

Let $\mathcal{M}_2$ be the interpretation that consists of

- Domain: Let the domain (or universe) $dom(\mathcal{M}_2)$ be {Fred, Wilma}.
- Constants: there are no constant symbols.
- Functions: there are no function symbols.
- Predicates: let *Loves* map to the relation $\{\langle \text{Wilma}, \text{Wilma} \rangle, \langle \text{Fred}, \text{Fred} \rangle, \langle \text{Fred}, \text{Wilma} \rangle\}$.

The formula is not true for all values of x and y; i.e., it is false for $x =$ Fred, $y =$ Wilma.

5. Show that $\forall x \cdot P(x) \wedge M(x), \forall x \cdot H(x) \rightarrow P(x) \quad \not\models \quad \exists x \cdot M(x) \wedge H(x)$. In other words, show that $\exists x \cdot M(x) \wedge H(x)$ does not logically follow from the premises.

To show that the conclusion does not logically follow from the premises, we must find an interpretation (a model) $\mathcal{M}$ in which the premises are true but the conclusion is false. I.e., we must find a counter example.

It is a good idea when looking for a counter example to start with small domains of discourse.

Let $\mathcal{M}$ be the interpretation that consists of:

- Domain: Let the domain (or universe) $dom(\mathcal{M})$ be {A}.
- Constants: there are no constant symbols.
- Functions: there are no function symbols.
- Predicates: let M map to the relation $\{A\}$, let H map to the relation $\{\}$, and let P map to the relation $\{A\}$. In other words, the predicates M and P are always true in this interpretation and the predicate H is always false in this interpretation.

We can verify that the premises are true in this interpretation but the conclusion is false.