Surname:	Personal name:	ID #:
Circle the time/room of your tutorial:		8:30, MC 2035 (TUT 101)
11:30, RCH 204 (TUT 107)	12:30, MC 2034 (TUT 104)	12:30, RCH 305 (TUT 106)
1:30, RCH 308 (TUT 105)	4:30, MC 4060 (TUT 102)	4:30, MC 2034 (TUT 103)

Due Wednesday, Oct. 14, by noon, to the appropriate box near MC 4065.
Attach this sheet as a cover sheet on your submission.

Question 1 (12 marks).

Give proofs in Natural Dection showing the following. For full credit, use ONLY the basic rules of the Natural Deduction system (Huth and Ryan, top of p. 27, or the lecture notes) on the first three. You may receive partial credit if you use one of the derived rules MT (*modus tollens*), PBC (proof by contradicton), LEM (law of excluded middle), or ¬¬i.

(a) $\{p \to q\} \vdash_{ND} \neg q \to \neg p$

(b) $\{\neg(r \vee q),\ s \to (p \vee q),\ p \to r\} \vdash_{ND} \neg s$.

(c) $\{q \vee r\} \vdash_{ND} (q \wedge s) \vee (s \to r)$.

(d) $\vdash_{ND} \big((p \to q) \to p\big) \to p$. For part (d), you may use the derived rules listed above without loss of credit.

Question 2 (12 marks). For each of the following, determine whether or not the indicated proof exists. If one exists, give one. If none exists, carefully explain why not.

(a) $\{p \to r,\ r \to p,\ (p \wedge q) \to r\} \vdash_{ND} (p \vee q) \to r$.

(b) $\{p \to r,\ r \vee p,\ (p \wedge q) \to r\} \vdash_{ND} (p \vee q) \to r$.

(c) $\{\neg p \to \neg r,\ \neg(s \vee p),\ q \to (r \vee s)\} \vdash_{ND} \neg q$.

Marks:	1		2		Total:	

Study Exercises

To assist you with thinking about the topics of the course and about how to solve the problems, we provide the following suggestions for exercises. You may work on them on your own or with classmates (or both) as you choose. When you get stuck, please feel free to ask an instructor or IA.

How many exercises you work on is entirely up to you. We do suggest, however, that knowing the answers is NOT the important part. The important part is to practice finding such answers yourself. You will need to do this on exams.

SE 1. Give proofs in Natural Deduction of the following. If you find proofs that use one or more derived rules, use them as hints to find a proof using only basic rules. (Note: the equivalence rules are **not** proof rules. Do **not** use them for these.)

(a) $\{\varphi \vee \beta\} \vdash_{ND} (\neg(\neg\varphi \wedge \neg\beta))$.

(b) $\{\neg(\varphi \wedge \beta)\} \vdash_{ND} \neg\varphi \vee \neg\beta$.

(c) $\{\varphi \vee (\beta \wedge \gamma)\} \vdash_{ND} (\varphi \vee \beta) \wedge (\varphi \vee \gamma)$.

(d) $\{(\varphi \vee \beta) \wedge (\varphi \vee \gamma)\} \vdash_{ND} (\varphi \vee (\beta \wedge \gamma))$.

(e) $\{p \to q\} \vdash_{ND} (r \to p) \to (r \to q)$.
(This one and the next appeared on the Spring '15 midterm.)

(f) $\{(p \wedge q) \vee (\neg p \wedge \neg q)\} \vdash_{ND} (p \to q)$.

(g) For additional practice, see any of Exercises 1.2.1–3 in Huth and Ryan, pp. 78–80. (You won't have time to do all of them!)

SE 2. The following ask you to transfrom one proof into another. As a first step, write down what the presumed proof must look like. Then describe how to modify it into the second proof. Σ represents any set of formulas and α, α_1, β, etc. are arbitrary formulas.

(a) Show that if $\Sigma \vdash_{ND} p$, then $\Sigma \vdash_{ND} \neg p \to q$.

(b) Show that if $\Sigma \cup \{\alpha_2\} \vdash \beta$, then also $\Sigma \cup \{\alpha_1\} \vdash_{ND} \beta$.

(c) Show that if there is a proof $\Sigma \cup \{\neg\alpha\} \vdash_{ND} \perp$, then there is a proof of $\Sigma \vdash_{ND} \alpha$.

SE 3. On the Island of Knights and Knaves, each of the inhabitants is either a knight or a knave (but never both). Knights always tell the truth (they only make true statements), and knaves always lie (they only make false statements).

You meet three inhabitants of the Island: A, B, and C. The first two make the following statements.

A: "Each of us is a knave."

B: "Exactly one of us is a knight."

If we use p to mean A is a knight, q to mean B is a knight, and r to mean C is a knight, then A's statement yields the formula $p \leftrightarrow (\neg p \wedge \neg q \wedge \neg r)$.

(a) Write down a formula describing what you know from B's statement.

(b) From the two formulas, infer whether each of A, B and C is a knight or a knave.