Report 1: Martingale

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1 QUESTION 1

Q: In Experiment 1, based on the experiment results calculate and provide the estimated probability of winning \$80 within 1000 sequential bets. Thoroughly explain your reasoning for the answer using the experiment output. Your explanation should NOT be based on estimates from visually inspecting your plots.

Answer:
Theoretically the probability of winning \$80 in 1000 trails is almost 100%. The winning chance is 18/38 for each bet. In our strategy (bet amount double for every lost), to win \$80 is to win 80 bets. The game results follow binomial distribution. Since the sample size n is 1000, which is large enough (where nP(1-P) > 5, P is the probability of winning 18/38 = 9/19). The distribution of the total number of winnings in the sample, X, is approximately Normal, where: mean = nP and standard deviation = $\sqrt{nP(1-P)}$. Here is the calculation:

$$P = 9/19, 1 - P = 10/19, n = 1000$$

$$\mu = nP = 9000/19$$

$$\sigma = \sqrt{nP(1-P)} = 300/19$$

$$z = (80 - \mu)/\sigma = -24.93$$

$$P(X \geqslant 80) = 1$$

2 QUESTION 2

Q: In Experiment 1, what is the estimated expected value of winnings after 1000 sequential bets? Thoroughly explain your reasoning for the answer.

Answer:
The estimated expected winning for 1000 sequential bets is equal to the time of expected winning bets, which according to Question 1 is nP = 9000/19 = \$473.68.

3 QUESTION 3

Q: In Experiment 1, do the mean upper standard deviation line and mean lower standard deviation lines reach a maximum (or minimum) value and then stabilize? Do the standard deviation lines converge as the number of sequential bets increases? Thoroughly explain why it does or does not.

Answer:
Yes, both mean upper and lower standard deviation lines converged to the mean line once the simulator winning reaches $80. This is because we fill in the data with 80 once we reach the $80 goal, so no more runs occur thus standard deviation is 0. But the standard deviation did not stabilize during simulator runs.

4 QUESTION 4

Q: In Experiment 2, based on the experiment results calculate and provide the estimated probability of winning $80 within 1000 sequential bets. Thoroughly explain your reasoning for the answer using the experiment output. Your explanation should NOT be based on estimates from visually inspecting your plots.

Answer:
To lose the game, one needs to lost all money which is any amount from $256 to $256 + $79 = $335. It takes 9 sequential lost bets lose all money. The probability for lose the game is $(10/19)^9 = 0.003$.
From my experiment, there are 627 out of 1000 episodes reached $80 and 373 out of 1000 episodes reached -$256 at the end of 1000 trials. So the estimated probability of winning $80 within 1000 sequential bets is 0.627.

5 QUESTION 5

Q: In Experiment 2, what is the estimated expected value of our winnings after 1000 sequential bets? Thoroughly explain your reasoning for the answer.

Answer:
Take the result from Question 4, we know that 0.627 has got $80 and 0.373 has lost -$256.

$$0.627 * \$80 + 0.373 * (-\$256) = -\$45.328$$

So the expected winnings for 1000 sequential bets is to lose $45.

6 QUESTION 6

Q: In Experiment 2, do the mean upper standard deviation line and mean lower standard deviation lines reach a maximum (or minimum) value and then stabilize? Do the standard deviation lines converge as the number of sequential bets increases? Thoroughly explain why it does or does not.

Answer:
Yes, the standard deviation lines stabilized.
No, both standard deviation lines did not converge.
The reason for stabilize is that there is a limit of $256 to lose. And reason for not converge is that there are 2 results for the experiment – win $80 or lose $256, the difference between the 2 results is quite big, thus standard deviation is large.

7 QUESTION 7

Q: What are some of the benefits of using expected values when conducting experiments instead of simply using the result of one specific random episode?

Answer:
There are randomness on each episodes, but with a large number of sample sizes (lots of episodes) the effect of randomness has been reduced to the minimum. Thus the expected value is more deterministic.

8 FIGURES

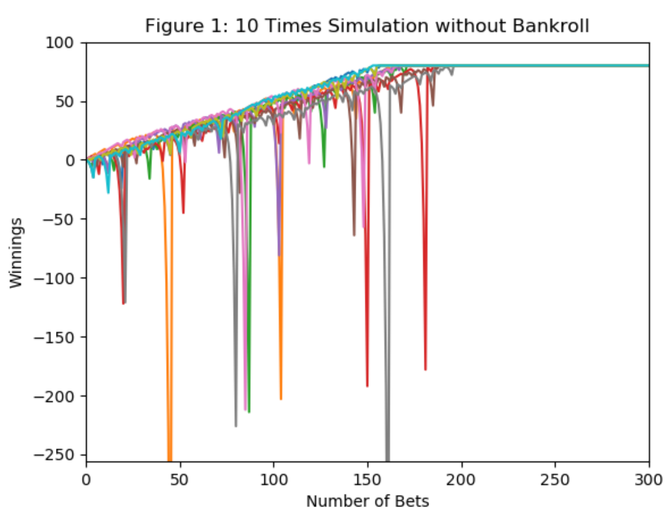


Figure 2: Mean & SD of 1000 Times Simulation without Bankroll

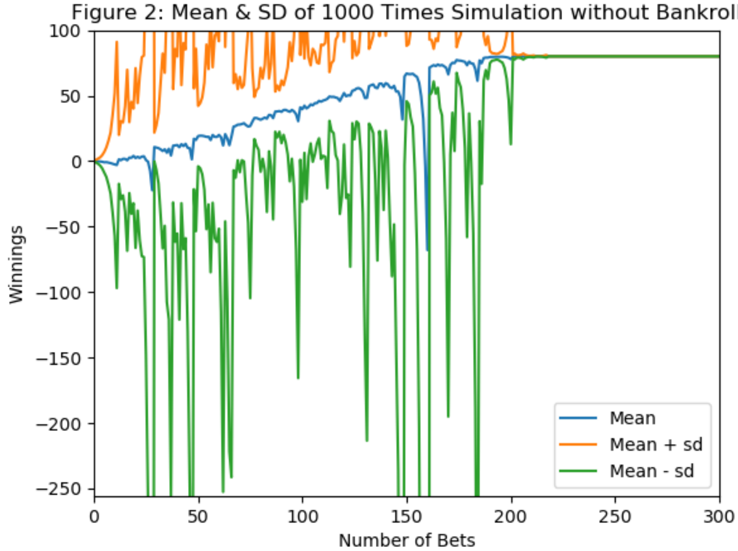
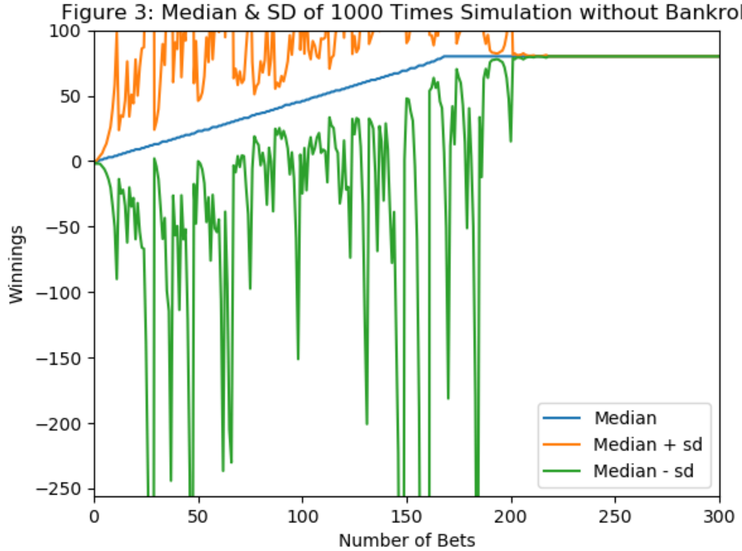


Figure 3: Median & SD of 1000 Times Simulation without Bankroll



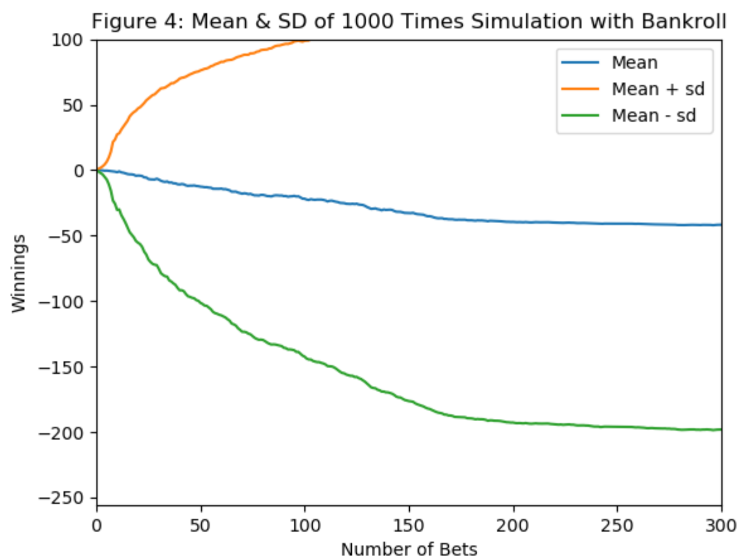


Figure 4: Mean & SD of 1000 Times Simulation with Bankroll
Mean
Mean + sd
Mean - sd
Winnings
Number of Bets
100
50
0
−50
−100
−150
−200
−250
0
50
100
150
200
250
300

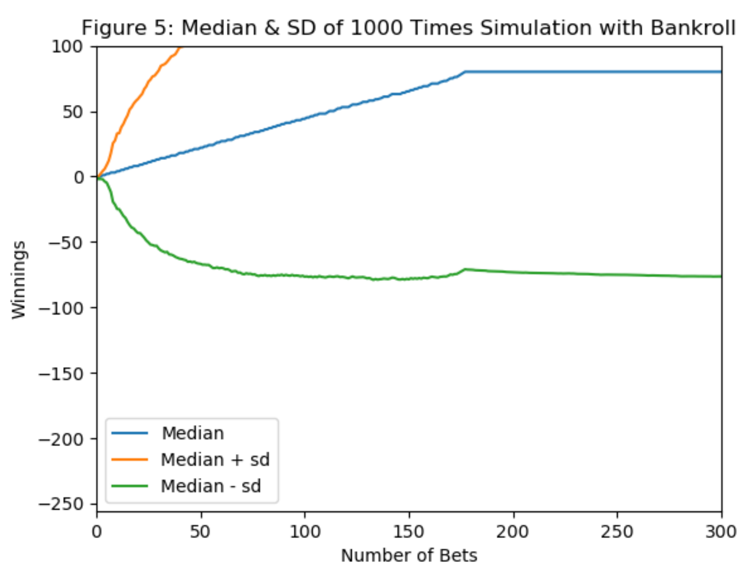


Figure 5: Median & SD of 1000 Times Simulation with Bankroll
Median
Median + sd
Median - sd
Winnings
Number of Bets
100
50
0
−50
−100
−150
−200
−250
0
50
100
150
200
250
300