MECA0029–1 Theory of vibration

Analysis of the dynamic behaviour of a stadium

Academic year 2024 – 2025

Some small stadiums, for attending events such as football, are built as truss structures instead of concrete. One example is depicted in Fig. 1. In this project, a design inspired from such stadiums will be studied, along with the influence of supporters on the structure.

Figure 1 – Example of a truss stadium (source: Southern Bleacher Company).

Work instructions

1. The work will be done **by groups of two students maximum**. A report will be submitted in which all the **descriptions**, **results** and **discussions** have to figure out. More information on the form of the report and evaluation criteria can be found on eCampus under "Objectives of the project".
   Reminder: a report must be written as a standalone work.

   - The report and the implementation (MATLAB/Python codes, NX files) have to represent your own work. **Any plagiarism will be severely punished**!
   - **40 pages** maximum are tolerated. You will be penalized if the report is too long (or extremely short).

2. The deadline for the submission of your work is fixed to **December 5, 2024** at **11:59 PM**. No additional delay will be tolerated.

   Your work must be uploaded on eCampus. The report (.pdf) and an archive (.zip) containing the following must be provided:

   - The MATLAB/Python codes
   - The Siemens NX files (.prt, .fem and .sim only)

   For sorting purposes, name your main files as follows:

   - Report: `MECA0029_Group_XX.pdf`.
   - Siemens NX file (.sim): `MECA0029_Group_XX.sim`.
   - MATLAB/Python files: the main files must be named `MECA0029_Group_XX_Y.m`, where XX is the number of your group and Y is the file number. You can use and name other files and subfunctions at your own convenience.

1 Modeling of the structure

Recommended deadline: October 24, 2024

The structure to study is a section of a simplified model of a truss stadium. It is built according to Fig. 2. It is composed of three main support structures of a height of 5, 3 and 1 [m]. The support structures are spaced from 4 [m]. The upper transverse bars, colored in red in Fig. 2a, are spaced from 1 [m] in the x-direction. The whole structure has a width of 2 [m].

All the beams of the truss are made of steel with the following material properties:

- Density: $\rho = 7800$ [kg/m^3].
- Poisson's ratio: $\nu = 0.3$ [-].
- Young's modulus: $E = 210$ [GPa].

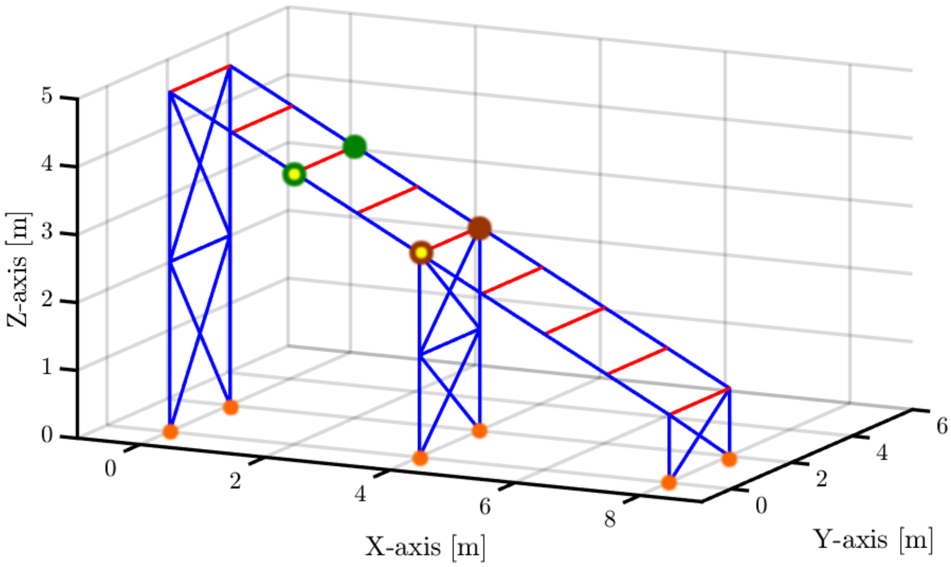
They are assumed to have a **circular hollow cross-section** with an outer diameter of 150 [mm] and a wall thickness of 5 [mm].

The main support structures are **clamped** at the intersection with the ground. The corresponding nodes are colored in orange in Fig. 2a.

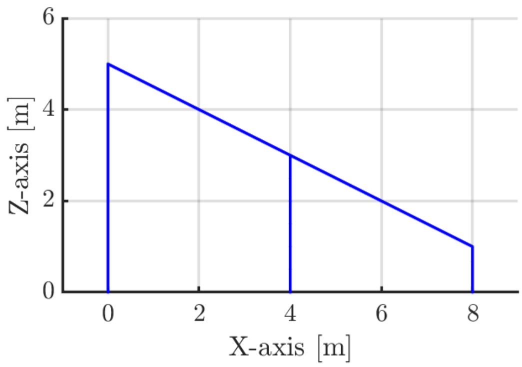
In order to simulate the presence of supporters sitting in the stadium, lumped masses are added to **each extremity of the transverse bars**. We will assume that **51 supporters** are sitting on the structure and that **each supporter weighs 80 [kg]**. The total mass of the supporters is **uniformly distributed over the 18 extremity nodes**. The rotational inertias of the supporters are neglected. The considered nodes are highlighted in pink in Fig. 3.

It is asked to

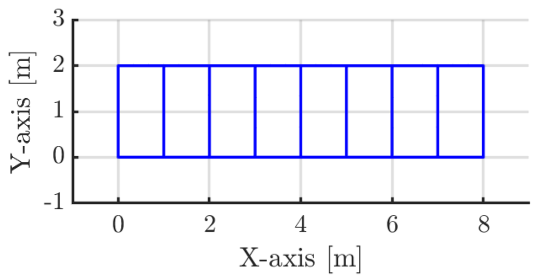
- Build the stadium model in **NX** based on 3D beam elements and extract the first **six natural frequencies** along with their corresponding **mode shapes**.
- Build the stadium model in **MATLAB/Python** using the 3D beam model seen in the theory, extract the first **six natural frequencies** and draw the corresponding **mode shapes** of the structure.
- Make a **convergence study** (as a function of the number of elements per beam) in **both software**.
- Compute the **total mass** of the system composed of the stadium and the supporters using a rigid-body mode in translation.



(a) 3D view. The clamped nodes are the orange dots, the impact locations are the green and brown dots, and the observation nodes are the yellow dots.



(b) XZ view.



(c) XY view.

Figure 2 – Stadium section model.

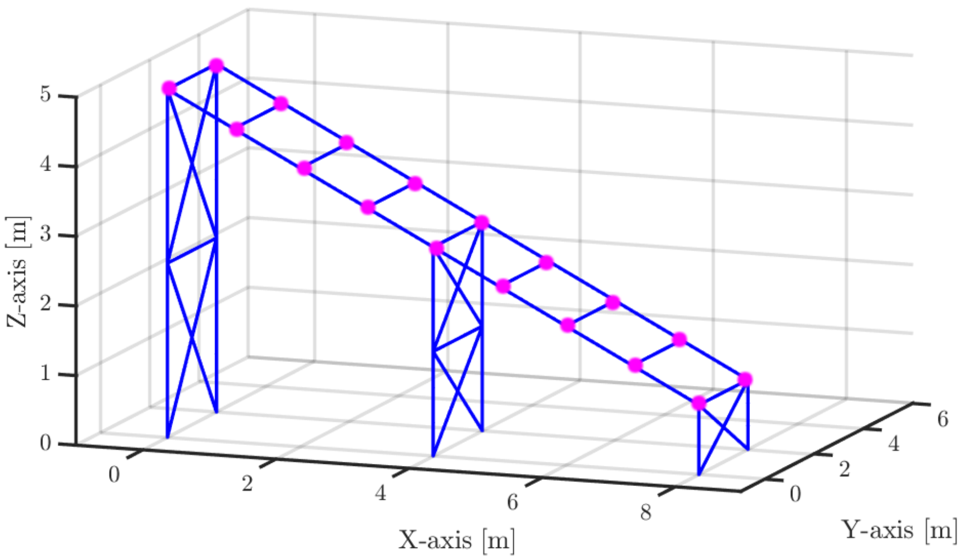


Figure 3 – Nodes where the concentrated masses are applied.

2 Transient response

Recommended deadline: November 21, 2024

In a common effort to make the structure move below their feet, some supporters begin to jump off their seats. In the present case, this represents **9 people**. The resulting force on the stadium is assumed to be a **sine** wave with a frequency of **2 [Hz]**. The supporters are jumping up to a height of **20 [cm]** above their floor level. We will consider that the impact with the floor lasts **0.1 [s]**. In order to approximate the amplitude of the sine wave, we make the assumption that all the potential energy has been converted into kinetic energy right before the impact and that **80 %** of the momentum of the supporters is transferred to the stadium. The direction of the force is **purely vertical** along the z-axis. The resulting force is **uniformly distributed over the two nodes colored in green** in Fig. 2a.

Although the upper-row supporters seem very content with their performance, their colleagues from the middle rows are not impressed and decide to copy the same move to show them how it should be done. They jump harder in order to teach them a lesson, up to **30 [cm]**. The impact points are now **colored in brown** in Fig. 2a. Despite all their efforts, this move ends up in huge burst of laughter directed towards the supporters of the middle rows.

The responses will be analyzed at **two specific locations, colored in yellow** in Fig. 2a. These points will be referred to as "observation nodes".

It is asked to (in MATLAB/Python)

- Compute the **damping matrix** using the **proportional damping** assumption such that the damping ratio of the **first two** modes is equal to **0.5 %**. List the damping ratios for modes 1 to 6.
- For **both load cases**, compute an approximate solution using the **mode displacement** method. Plot the **time** evolutions **in the z-direction** at both observation nodes.
- For **both load cases**, compute an approximate solution using the **mode acceleration** method. Plot the **time** evolutions **in the z-direction** at both observation nodes.
- **Compare and discuss** the results of **both** approximation methods and discuss the **convergence** in terms of the number of modes included in the superposition.
- For **both load cases**, compute the solution by time integration using the **Newmark** algorithm. **Justify** the choice of the time step and integration parameters. Plot the **time** evolutions **in the z-direction** and the amplitude of their corresponding **discrete Fourier transforms**, at both observation nodes.
- **Discuss** the results. Why were the mid-rows supporters not impressed by the move of their colleagues? Why did everybody laugh at them when they tried it themselves?

3 Reduction methods

Recommended deadline: December 5, 2024

To decrease the number of degrees of freedom, a part of the structure is condensed and **only the translation degrees of freedom** of the **5** nodes highlighted in Fig. 4 are retained.

It is asked to (in MATLAB/Python)

- Use **Guyan-Irons** method and **compare** the natural frequencies with the **initial** finite element model.
- Use **Craig-Bampton** method and choose an appropriate number of modes such that the relative error on the six previously identified natural frequencies is **less than 1 %**. **Compare** the results with the **initial** model and with **Guyan-Irons** method.
- Compute again the response to the excitation of the **upper-row supporters** using **Newmark** algorithm and the reduced order model. Show the interest of reduction methods by comparing the responses at **both observation points** in terms of **accuracy**, **size of the problem** and **computation time**.

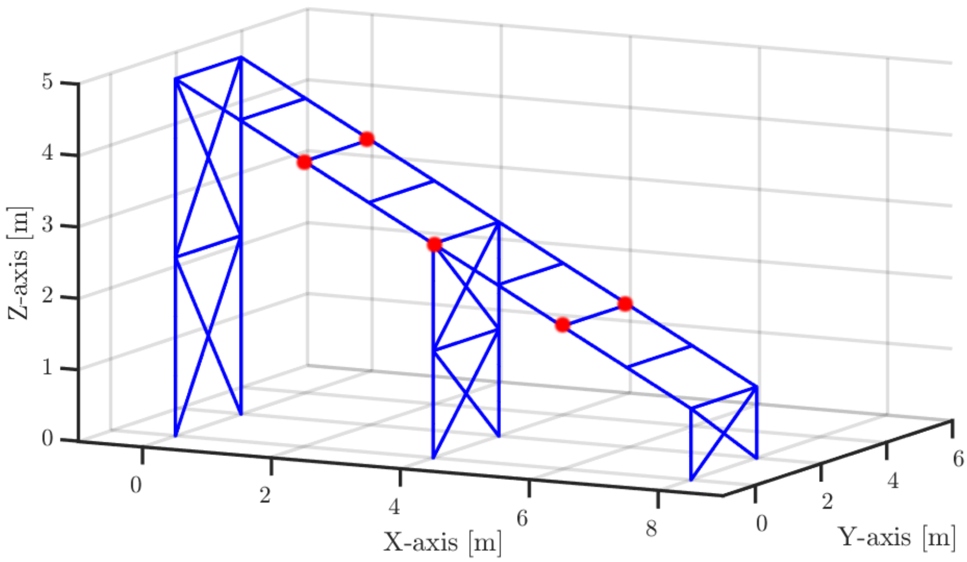


Figure 4 – Nodes retained for the reduction.