Project #5: Predicting the temporal dynamics of a turbulent shear flow

In this project we will consider the low-order model of the near-wall cycle in a wall-bounded turbulent flow provide here. This model consists of a system of 9 ODEs to predict the evolution of 9 temporal coefficients, associated with 9 spatial modes representing the mean flow, the streaks, the stramwise vortices, the spanwise and normal vortices, as well as their interactions. We want to model the temporal dynamics of this system using data-driven methods, and to this end you need to perform the following tasks:

1. By modifying the initial condition as indicated in the repository, create a total of 1,000 different time series with a span of 4,000 time units each. Discard any time series that relaminarize within this time horizon.

2. Pick a few time series and plot the temporal evolution of the 9 coefficients. Discuss whether this dynamical system is chaotic. Visualize the various velocity components in several time steps.

3. Calculate the turbulence statistics of the flow by averaging in time and over 200 time series. Plot the mean flow and the Reynolds stresses as a function of the wall-normal direction.

4. Use an LSTM to predict the temporal evolution of the 9 coefficients. Compare the predicted turbulence statistics with those of the original system. Pick one time series, and compare the original and predicted temporal evolution of the coefficients. Discuss the results.

5. Assess whether you can learn the temporal dynamics of the system by employing Poincaré maps and Lyapunov exponents. You can read more about these here.

6. Use the SINDy framework to identify the dynamics of the system and compare with the results above.

7. *Optional task* (not required for full grade, but interesting to learn more about the problem): Use the Koopman-based framework proposed here to perform the same predictions and compare with the results above.