Efficient frictional contacts for soft body dynamics via ADMM

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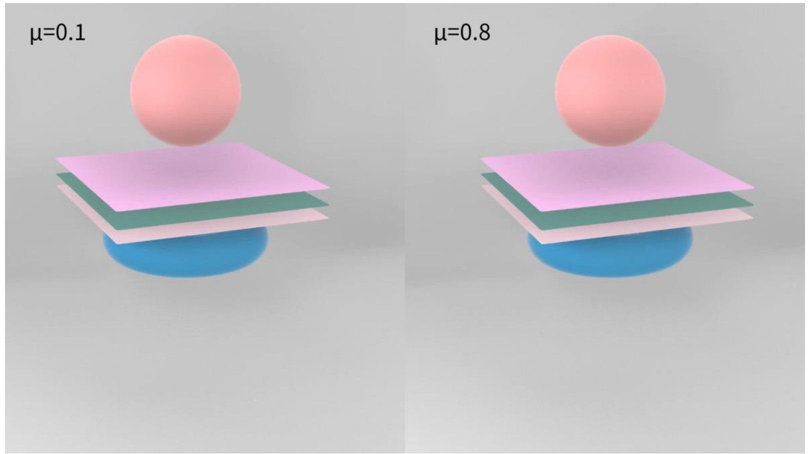

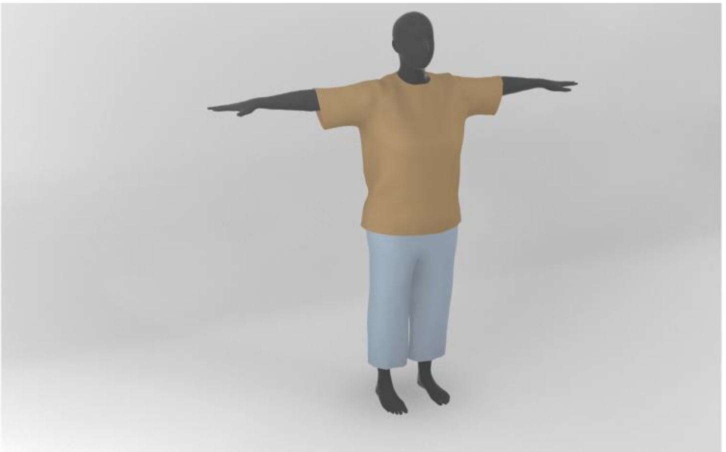

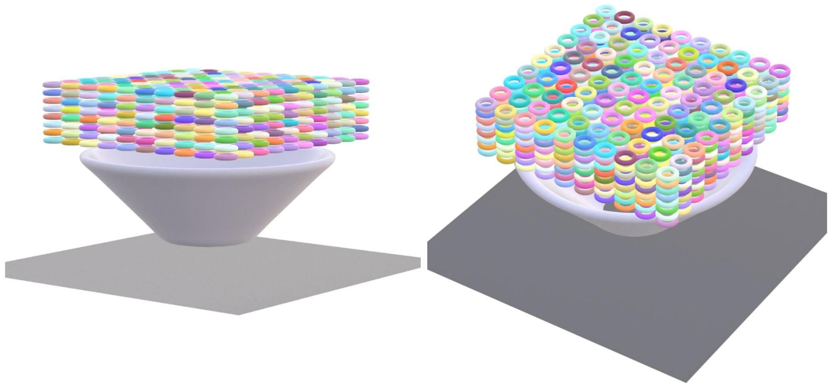

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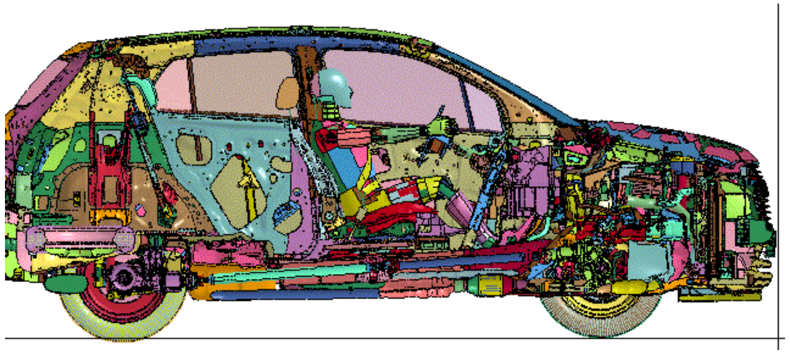
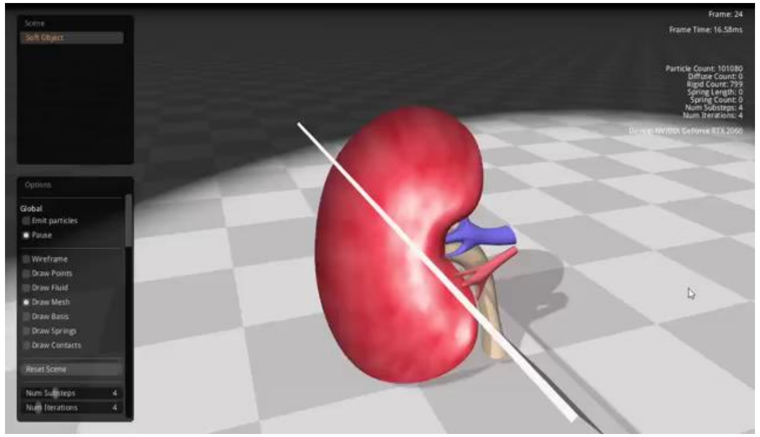
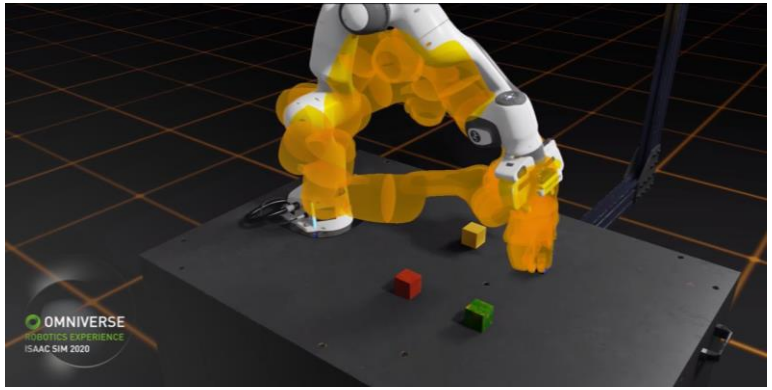
Contact Simulation Applications



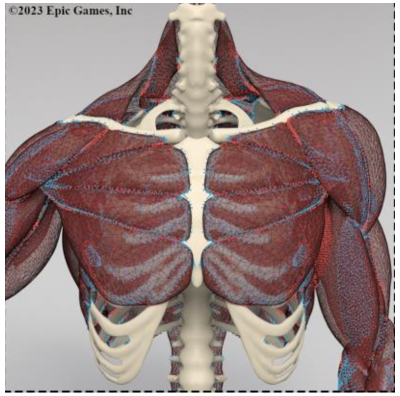
Industry Design



Virtual Surgery



Robotics



Biomechanics

Movies

Autopilot Training Platform

Backgrounds

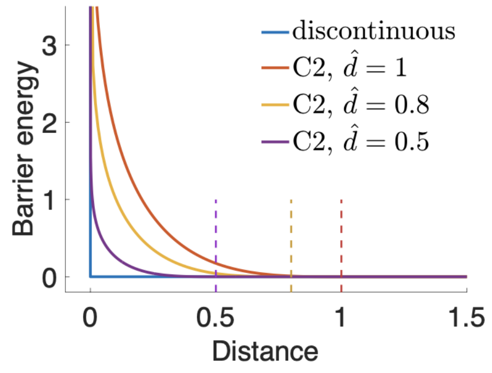
- Backward Euler Time Integration

$$\underbrace{\boldsymbol{x}^{t+1}}_{\text{New Pos}} = argmin_{\boldsymbol{x}} \underbrace{E(\boldsymbol{x})}_{\text{Incremental Potential}} = \underbrace{\frac{1}{2h^2}(\boldsymbol{x}-\widetilde{\boldsymbol{x}})^T \overset{\text{Mass Matrix}}{\mathbf{M}}(\boldsymbol{x}-\overset{\text{Predicted Position}}{\widetilde{\boldsymbol{x}}})}_{\text{Inertia Term}} + \underbrace{U(\mathbf{x})}_{\text{Elasticity Energy}}$$

- Primal based method

$$\boldsymbol{x}^{t+1} = argmin_{\boldsymbol{x}} E(\boldsymbol{x}) + P(\boldsymbol{x})$$

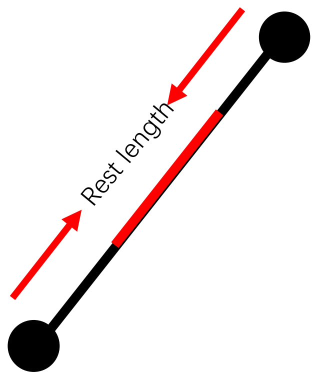
  - Penalty method
  - Barrier method (IPC, 2020)
    - Penetration free
    - Continuous collision detection (time consuming)



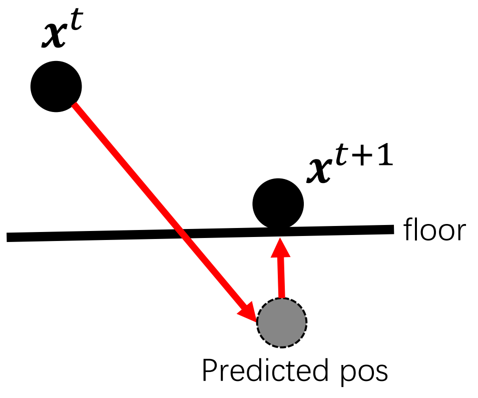
Backgrounds

- Dual based method

$$x^{t+1} = argmin_{\boldsymbol{x}} \frac{1}{2}(\boldsymbol{x} - \tilde{\boldsymbol{x}})^T \mathbf{M}(\boldsymbol{x} - \tilde{\boldsymbol{x}})$$
$$s.t. \quad C_1(x) \le 0,$$
$$C_2(x) = 0$$



Distance constraints



Collision constraints

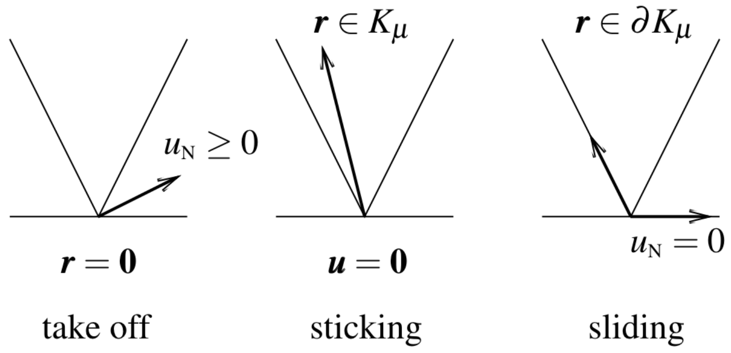
PBD[2006], XPBD[2016] (for small scenes)

- Hybrid method

$$\boldsymbol{x}^{t+1} = argmin_{\boldsymbol{x}} E(\boldsymbol{x})$$
$$s.t. \quad C_1(x) \le 0,$$
$$C_2(x) = 0$$

$$\longrightarrow \begin{cases} M\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \mathbf{f}(t, \boldsymbol{x}, \boldsymbol{v}) + \underbrace{\mathrm{J}^{\mathrm{T}}\boldsymbol{r}}_{\mathbf{f}^c} \\ \boldsymbol{u} = \mathrm{J}\boldsymbol{v} + \boldsymbol{u}_{\mathrm{f}} \\ \forall i = 1 \dots n, (\boldsymbol{r}^i, \boldsymbol{u}^i) \in K_\mu. \end{cases}$$

Gilles Daviet [2021, 2023]

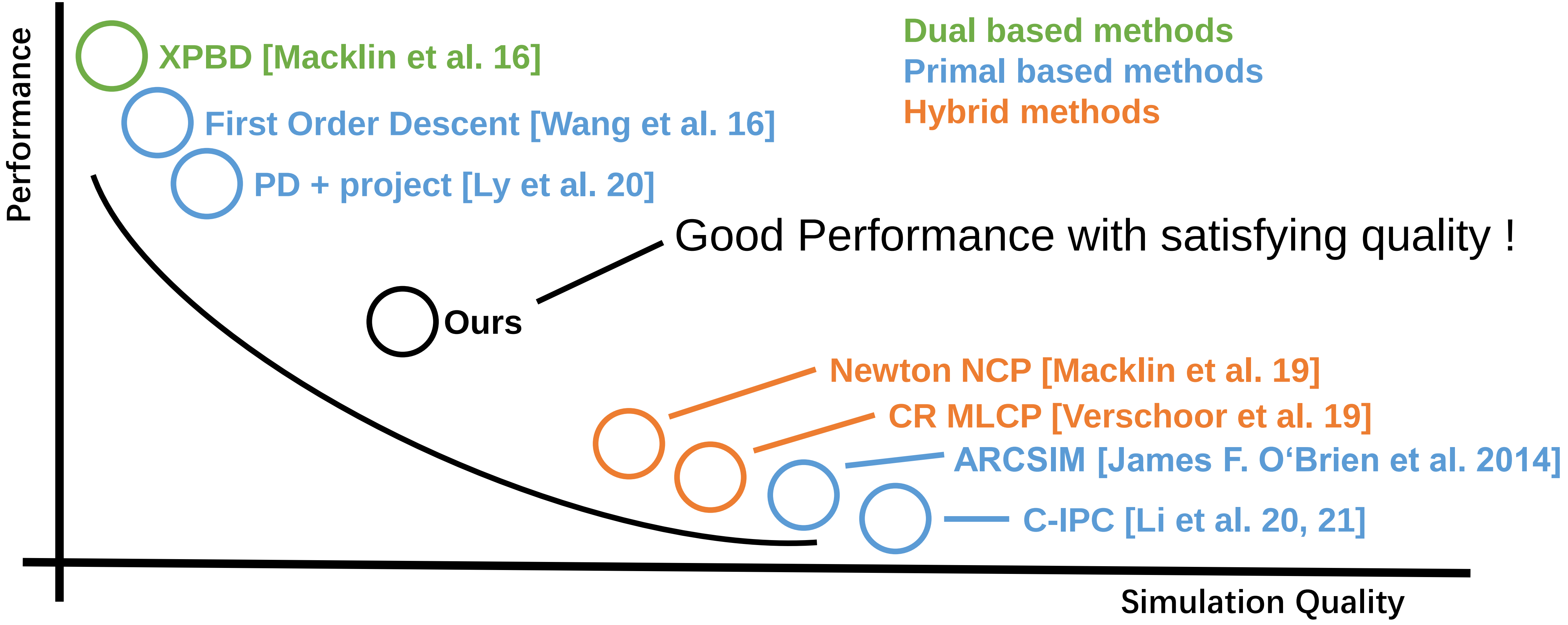


Coulomb friction model

- Delassus Operator $\mathbf{J}\mathbf{A}^{-1}\mathbf{J}^{\mathrm{T}}$ (Large Sparse Matrix Inversion, time consuming)

Motivation

- Habitat of our method



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ADMM (Alternating Direction Method of Multipliers)

- ADMM: solving convex optimization problem

$$\min_{\mathbf{x},\mathbf{z}} \; f(\mathbf{x}) + g(\mathbf{z}), \;\; s.t. \; \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{z} + \mathbf{c} = 0,$$

$$\Downarrow$$

$$L(\mathbf{x},\mathbf{z},\boldsymbol{\lambda}) = f(\mathbf{x}) + g(\mathbf{z}) + \frac{\rho}{2}\|\mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{z} + \mathbf{c} + \boldsymbol{\lambda}\|^2 + \mathbf{k}.$$

- ADMM use a Gauss-Seidel type iterative algorithm:

$$\mathbf{x}^{l+1} := \arg\min_{\mathbf{x}} L(\mathbf{x}, \mathbf{z}^l, \boldsymbol{\lambda}^l),$$
$$\mathbf{z}^{l+1} := \arg\min_{\mathbf{z}} L(\mathbf{x}^{l+1}, \mathbf{z}, \boldsymbol{\lambda}^l),$$
$$\boldsymbol{\lambda}^{l+1} := \boldsymbol{\lambda}^l + \rho(\mathbf{A}\mathbf{x}^{l+1} + \mathbf{B}\mathbf{z}^{l+1} + \mathbf{c}).$$

Softbody Contacts Formulation

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Softbody with contacts

$$\boldsymbol{x}_{t+1} = \arg\min_{\boldsymbol{x}} \underbrace{\frac{1}{2h^2}(\boldsymbol{x}-\tilde{\boldsymbol{x}})^T\mathbf{M}(\boldsymbol{x}-\tilde{\boldsymbol{x}})}_{\text{Inertia Term}} + \underbrace{U(\boldsymbol{x})}_{\text{Elasticity}} + \underbrace{F(\boldsymbol{x},\boldsymbol{v})}_{\text{Contacts Potential}}$$

Introduce two auxiliary variables $\boldsymbol{z}_i = \boldsymbol{D}_i\boldsymbol{x}, \boldsymbol{p} = \boldsymbol{v}$

$$\boldsymbol{x}_{t+1} = \arg\min_{\boldsymbol{x},\boldsymbol{z},\boldsymbol{p}} \frac{1}{2h^2}(\boldsymbol{x}-\tilde{\boldsymbol{x}})^T\mathbf{M}(\boldsymbol{x}-\tilde{\boldsymbol{x}}) + \sum_i U(\boldsymbol{z}_i) + F(\boldsymbol{p})$$

$$s.t.\ \boldsymbol{Dx} = \boldsymbol{z}, \boldsymbol{x} - \boldsymbol{x}_t - h\boldsymbol{p} = \boldsymbol{0} \text{ (equation constraints)}$$

Reform equation constraint

$$\begin{bmatrix} \boldsymbol{W}_e\boldsymbol{D} \\ \frac{1}{h}\boldsymbol{W}_c \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} -\boldsymbol{W}_e & 0 \\ 0 & -\boldsymbol{W}_c \end{bmatrix} \begin{bmatrix} \boldsymbol{z} \\ \boldsymbol{p} \end{bmatrix} + \begin{bmatrix} 0 \\ -\boldsymbol{W}_c\frac{1}{h}\boldsymbol{x}_t \end{bmatrix} = 0$$

$$\Rightarrow$$

$$\min_{\boldsymbol{x},\boldsymbol{z}} f(\boldsymbol{x}) + g(\boldsymbol{z})$$

$$s.t.\ \boldsymbol{Ax} + \boldsymbol{Bz} + \boldsymbol{c} = \boldsymbol{0}$$

ADMM Formulation

ADMM Solver

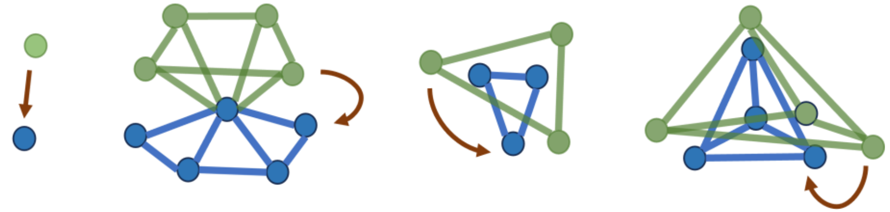
1. Global strain propagation $\boldsymbol{x}^{l+1} \leftarrow \arg\min_{x} L_1$

$$L_1 = \frac{1}{2h^2}(\boldsymbol{x}-\widetilde{\boldsymbol{x}})^T\mathbf{M}(\boldsymbol{x}-\widetilde{\boldsymbol{x}}) + \frac{1}{2}\left|\boldsymbol{W}_e^{\frac{1}{2}}(\boldsymbol{Dx}-\boldsymbol{z}^l+\boldsymbol{\lambda}_e^l)\right|^2 + \frac{1}{2}\left|\boldsymbol{W}_c^{\frac{1}{2}}(\boldsymbol{x}-\boldsymbol{hp}^l-\boldsymbol{x}_t+\boldsymbol{\lambda}_c^l)\right|^2$$

2. Local elasticity projection $z^{l+1} \leftarrow \arg\min_{z} L_2$

$$L_2 = \sum_{i=1}^{n_e} U(\boldsymbol{z}_i) + \frac{1}{2}\left|\boldsymbol{W}_e^{\frac{1}{2}}(\boldsymbol{Dx}-\boldsymbol{z}^l+\boldsymbol{\lambda}_e^l)\right|^2$$

#m independent problems
Solved in parallel



3. Frictional contact projection $\boldsymbol{p}^{l+1} \leftarrow \arg\min_{\boldsymbol{p}} L_3$

$$L_3 = I_{K_\mu}(\boldsymbol{p}) + \frac{1}{2}\left|\boldsymbol{W}_c^{\frac{1}{2}}(\boldsymbol{x}-\boldsymbol{hp}^l-\boldsymbol{x}_t+\boldsymbol{\lambda}_c^l)\right|^2$$

4. Dual update

$$\boldsymbol{\lambda}_e^{l+1} = \boldsymbol{\lambda}_e^l + \boldsymbol{Dx}^{l+1} - \boldsymbol{z}^{l+1}$$
$$\boldsymbol{\lambda}_c^{l+1} = \boldsymbol{\lambda}_e^l + \boldsymbol{x}^{l+1} - h\boldsymbol{p}^{l+1} - \boldsymbol{x}_t$$

Simple vector addition

ADMM Solver

1. Global strain propagation $\boldsymbol{x}^{l+1} \leftarrow \arg\min_{\boldsymbol{x}} L_1$

$$L_1 = \frac{1}{2h^2}(\boldsymbol{x}-\widetilde{\boldsymbol{x}})^T \mathbf{M}(\boldsymbol{x}-\widetilde{\boldsymbol{x}}) + \frac{1}{2}\left|\boldsymbol{W}_e^{\frac{1}{2}}(\boldsymbol{D}\boldsymbol{x}-\boldsymbol{z}^l+\boldsymbol{\lambda}_e^l)\right|^2 + \frac{1}{2}\left|\boldsymbol{W}_c^{\frac{1}{2}}(\boldsymbol{x}-\boldsymbol{h}\boldsymbol{p}^l-\boldsymbol{x}_t+\boldsymbol{\lambda}_c^l)\right|^2$$

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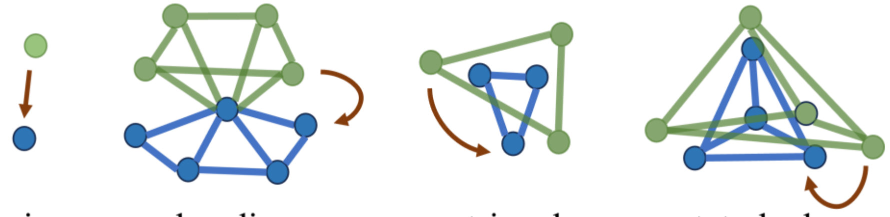
Solved in parallel



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$$\boldsymbol{\lambda}_e^{l+1} = \boldsymbol{\lambda}_e^l + \boldsymbol{D}\boldsymbol{x}^{l+1} - \boldsymbol{z}^{l+1}$$
$$\boldsymbol{\lambda}_c^{l+1} = \boldsymbol{\lambda}_e^l + \boldsymbol{x}^{l+1} - h\boldsymbol{p}^{l+1} - \boldsymbol{x}_t$$

Simple vector addition

ADMM Solver

- Step 1: Global Strain Propagation: Solving a linear system

$$\boldsymbol{x}^{l+1} \leftarrow \arg\min_x L_1 = \arg\min_x \frac{1}{2h^2}(\boldsymbol{x}-\widetilde{\boldsymbol{x}})^T\mathbf{M}(\boldsymbol{x}-\widetilde{\boldsymbol{x}}) + \frac{1}{2}\left|\boldsymbol{W}_e^{\frac{1}{2}}(\boldsymbol{D}\boldsymbol{x}-\boldsymbol{z}^l+\boldsymbol{\lambda}_e^l)\right|^2 + \frac{1}{2}\left|\boldsymbol{W}_c^{\frac{1}{2}}(\boldsymbol{x}-\boldsymbol{h}\boldsymbol{p}^l-\boldsymbol{x}_t+\boldsymbol{\lambda}_c^l)\right|^2$$

First order optimal condition $\boldsymbol{A}\boldsymbol{x}=\boldsymbol{b}$

$$(\boldsymbol{M}+h^2\boldsymbol{D}^T\boldsymbol{W}_e\boldsymbol{D}+h^2\boldsymbol{W}_c)\boldsymbol{x} = \boldsymbol{M}\widehat{\boldsymbol{x}} + h^2\boldsymbol{D}^T\boldsymbol{W}_e(\boldsymbol{z}^l-\boldsymbol{\lambda}_e^l) + h^2\boldsymbol{W}_c(\boldsymbol{p}^l-\boldsymbol{\lambda}_e^l)$$

  - Constant $\boldsymbol{A}$, **pre-factorization** can be applied

- Rayleigh damping
  - Our formulation $\boldsymbol{f}_d = -k_d\boldsymbol{D}^T\boldsymbol{W}_e\boldsymbol{D}\boldsymbol{v}_{t+1}$
  - Final system

$$(\boldsymbol{M}+h^2\boldsymbol{D}^T\boldsymbol{W}_e\boldsymbol{D}+h^2\boldsymbol{W}_c+hk_d\boldsymbol{D}^T\boldsymbol{W}_e\boldsymbol{D})\boldsymbol{x} = \boldsymbol{M}\widehat{\boldsymbol{x}} + h^2\boldsymbol{D}^T\boldsymbol{W}_e(\boldsymbol{z}^l-\boldsymbol{\lambda}_e^l) + h^2\boldsymbol{W}_c(\boldsymbol{p}^l-\boldsymbol{\lambda}_e^l) + \boldsymbol{f}_d$$

ADMM Solver

- Step3: Friction Contact Projection
  - First order optimal condition of L_3 + Schur Complement

$$L_3 = I_{K_\mu}(\boldsymbol{p}) + \frac{1}{2}\left|\boldsymbol{W}_c^{\frac{1}{2}}\left(\boldsymbol{x} - h\boldsymbol{p}^l - \boldsymbol{x}_t + \boldsymbol{\lambda}_c^l\right)\right|^2$$

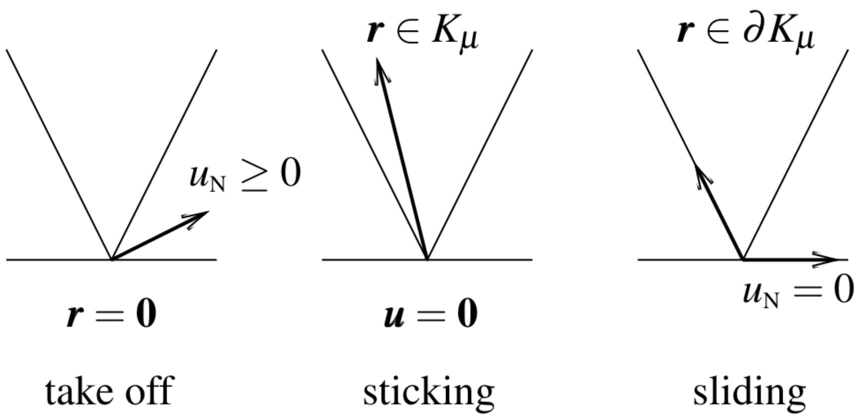
$$\Downarrow$$

Complementary Problem

$$\begin{cases} \boldsymbol{u} = \boldsymbol{Q}\boldsymbol{\gamma}^l + \frac{1}{h}\boldsymbol{J}\left(\boldsymbol{x}^{l+1} - \boldsymbol{x}_t + \boldsymbol{\lambda}_c^l\right) \\ (\boldsymbol{u}, \boldsymbol{\gamma}) \in K_\mu \end{cases}$$

Relative velocity

Contact force



Coulomb friction model

ADMM Solver

- Step3: Friction Contact Projection
  - First order optimal condition of L_3 + Schur Complement

$$L_3 = I_{K_\mu}(\boldsymbol{p}) + \frac{1}{2}\left|\boldsymbol{W}_c^{\frac{1}{2}}\left(\boldsymbol{x} - \boldsymbol{h}\boldsymbol{p}^l - \boldsymbol{x}_t + \boldsymbol{\lambda}_c^l\right)\right|^2$$

$$\Downarrow$$

Complementary Problem

$$\begin{cases} \boldsymbol{u} = \boldsymbol{Q}\boldsymbol{\gamma}^l + \frac{1}{h}\boldsymbol{J}\left(\boldsymbol{x}^{l+1} - \boldsymbol{x}_t + \boldsymbol{\lambda}_c^l\right) \\ (\boldsymbol{u}, \boldsymbol{\gamma}) \in K_\mu \end{cases}$$

Relative velocity   Contact force

Delassus Operator

$$\boldsymbol{Q} = \boldsymbol{J}\boldsymbol{W}_c^{-1}\boldsymbol{J}^T$$

sparse

$\boldsymbol{W}_c^{-1}$ is a diagonal matrix

Solve the Complementary Problem

$$\begin{cases} \boldsymbol{u} = \boldsymbol{Q}\boldsymbol{\gamma}^l + \frac{1}{h}\boldsymbol{J}(\boldsymbol{x}^{l+1} - \boldsymbol{x}_t + \boldsymbol{\lambda}_c^l) \\ (\boldsymbol{u}, \boldsymbol{\gamma}) \in K_\mu \end{cases}$$

- Projected Gauss-Seidel
  - **Enumerate Method**: for Contact projection [Daviet, 2020]
    - Coulomb friction law
    - Signorini's condition
  - **Kaczmarz method**: avoid explicitly calculate Delassus operator $\boldsymbol{Q}$
    - track $\boldsymbol{p} = \mathbf{J}^{\mathbf{T}}\mathbf{u}$ in time
  - **Contact stabilization**
    - Constraint Force Mixing (CFM)

PGS initialization

$$\mathbf{u}_g^{k+1} = \begin{cases} q_{g,d}\boldsymbol{\gamma}_g^k + \Sigma_o q_{g,o}\boldsymbol{\gamma}_o^l + \mathbf{b}^{l+1} & \text{if } k > 0, \\ \mathbf{J}\mathbf{p} & \text{if } k = 0. \end{cases}$$

$$\mathbf{u}_g^* = \mathbf{u}_g^{k+1} - q_{g,d}\boldsymbol{\gamma}_g^k = \mathbf{J}\mathbf{p}^k - q_{g,d}\boldsymbol{\gamma}_g^k$$

Classify the contact case (Enumerate Method)

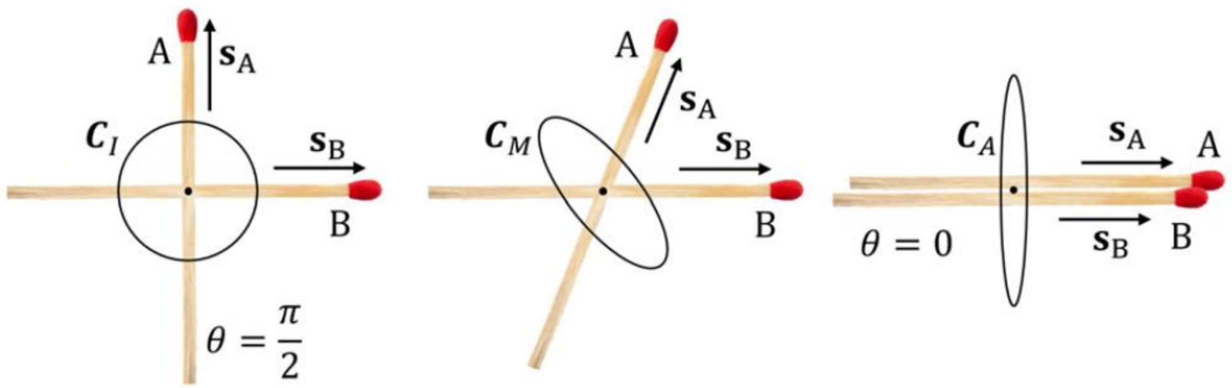
1. Separate: $\boldsymbol{u}_\mathrm{g}^* \cdot \boldsymbol{n} > \mathbf{0} \rightarrow \boldsymbol{u}_g^{k+1} = \boldsymbol{u}_g^*$
2. Slide: $\boldsymbol{u}_\mathrm{g}^* \in \boldsymbol{K}_\mu \rightarrow \boldsymbol{u}_g^{k+1} = \mathbf{0}$
3. Stick: not 1, 2 then $\boldsymbol{u}_\mathrm{g}^{\mathrm{k}+1} = u_{g,T}^* + \frac{\mu |u_{g,N}^*|\, u_{g,T}^*}{|u_{g,T}^*|}$

Update p

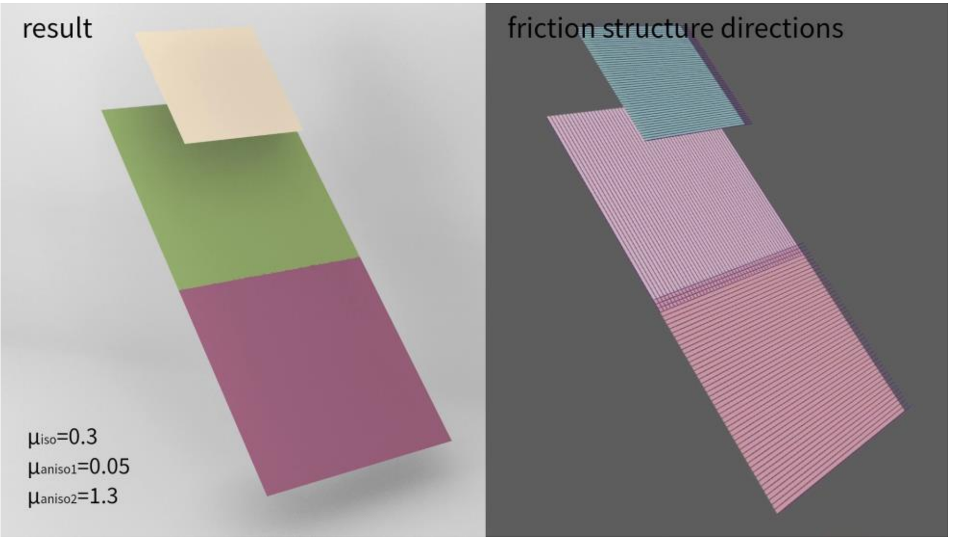
$$\mathbf{p}_i^{k+1} = \mathbf{p}_i + \frac{J_{g,i}}{w_i}(\boldsymbol{\gamma}_g^{k+1} - \boldsymbol{\gamma}_g).$$

Anisotropic Friction

- Match Stick Model
  - Friction Cone： Conic K_{μ} → Elliptic cone K_{μ_t,μ_b}

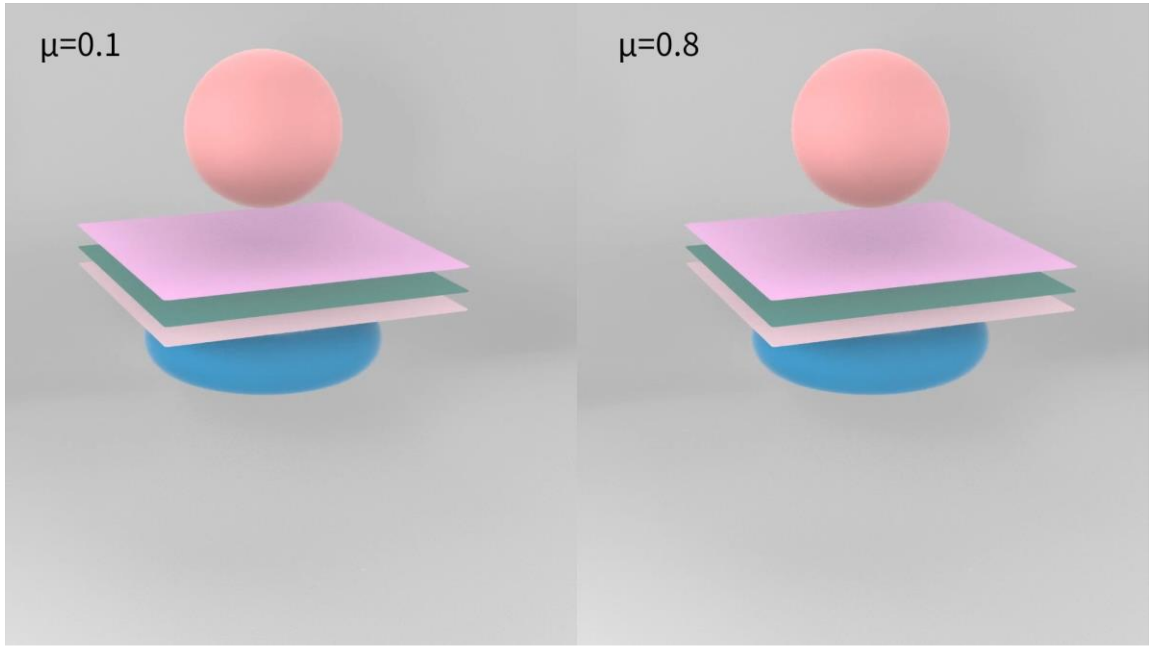


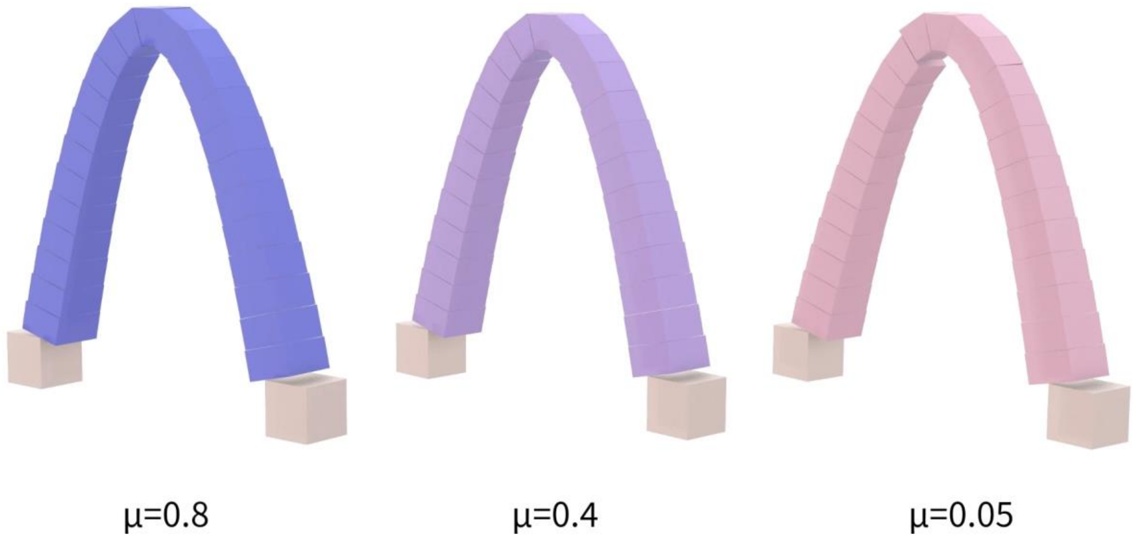
[Erleben et al. 19]



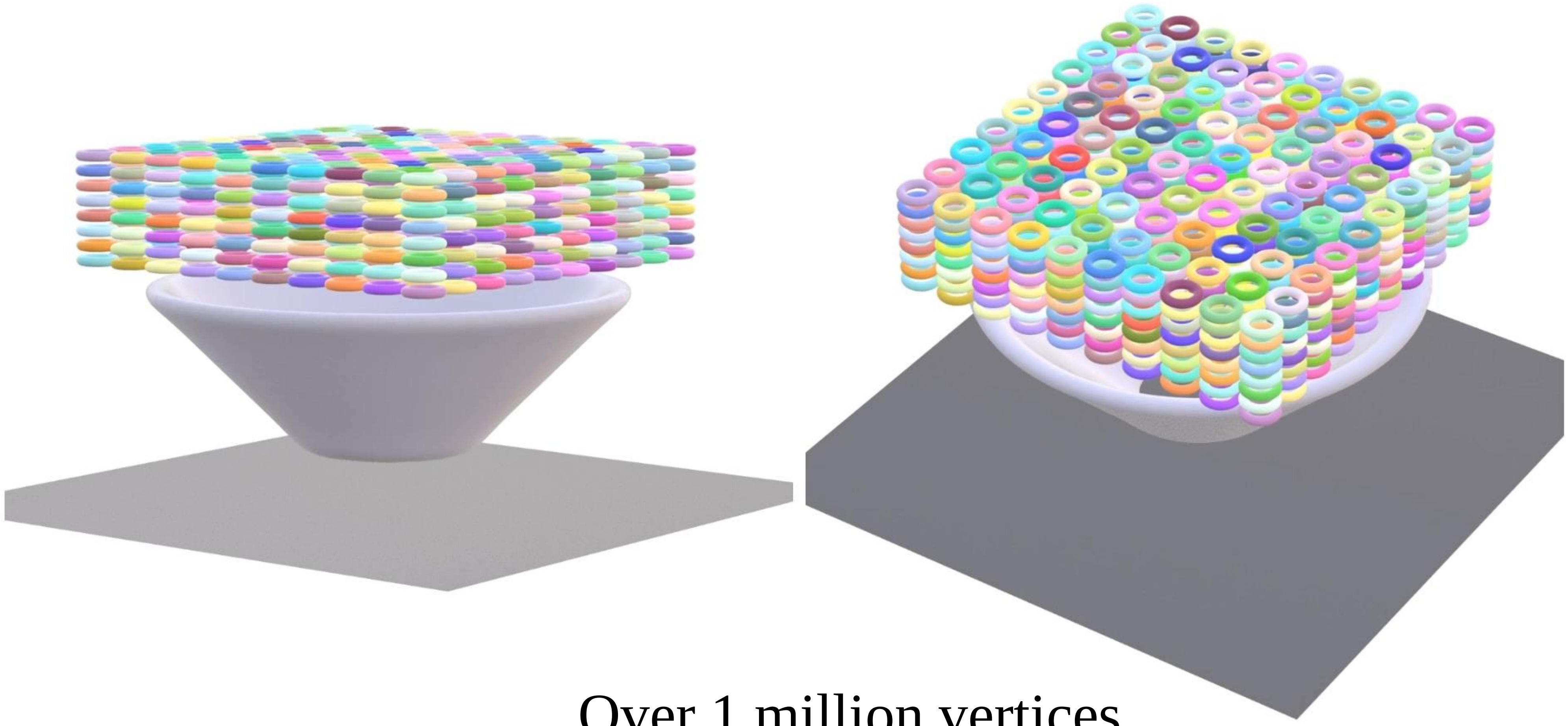
Our method with Anisotropic Friction

Result *different friction coefficients*





Result *large scale*



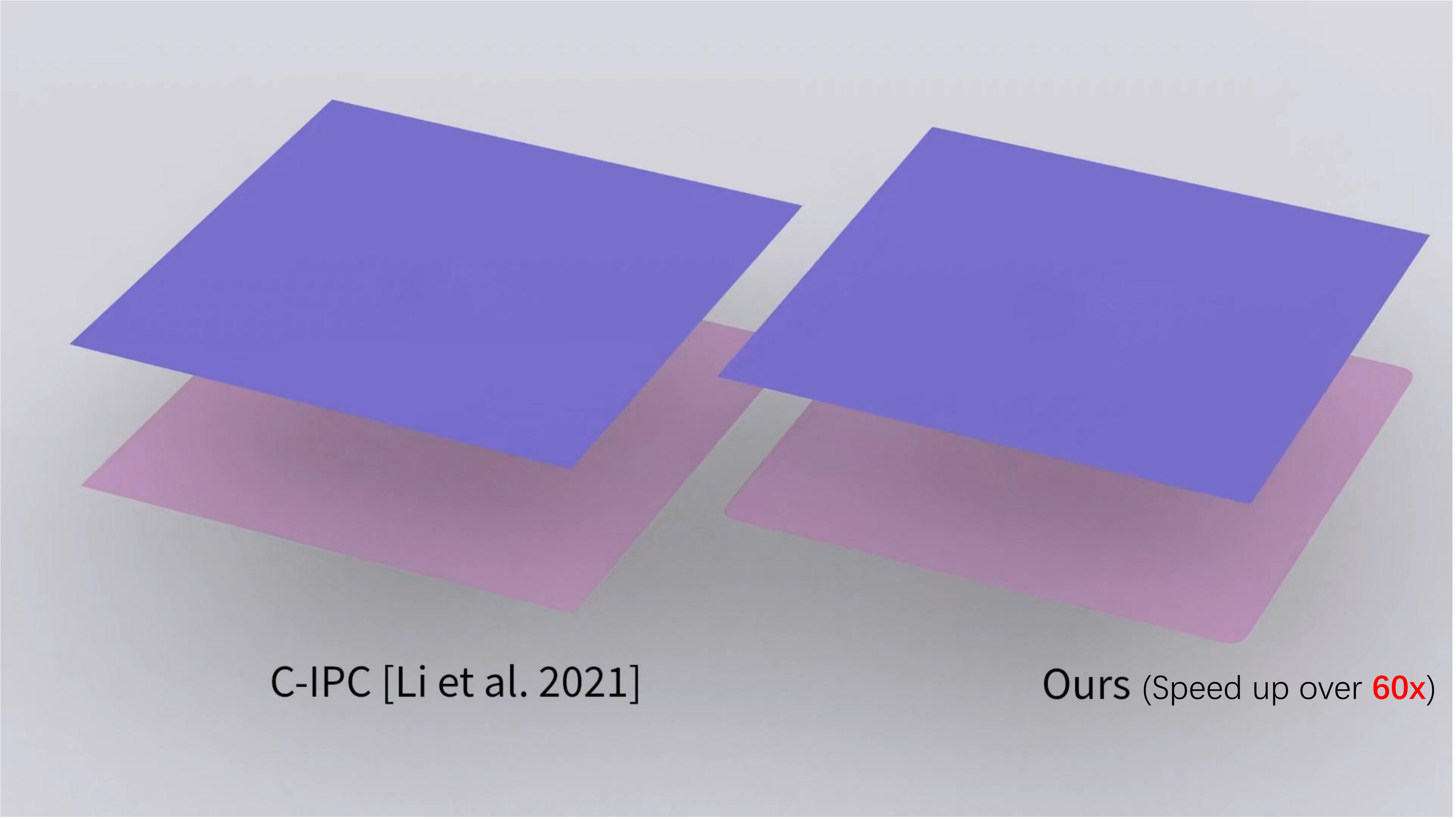
Over 1 million vertices

Result *compare with IPC*

IPC[Li et al. 2020]

Ours (Speed up **10x**)

Result *compare with C-IPC*



Result *compare with XPBD*

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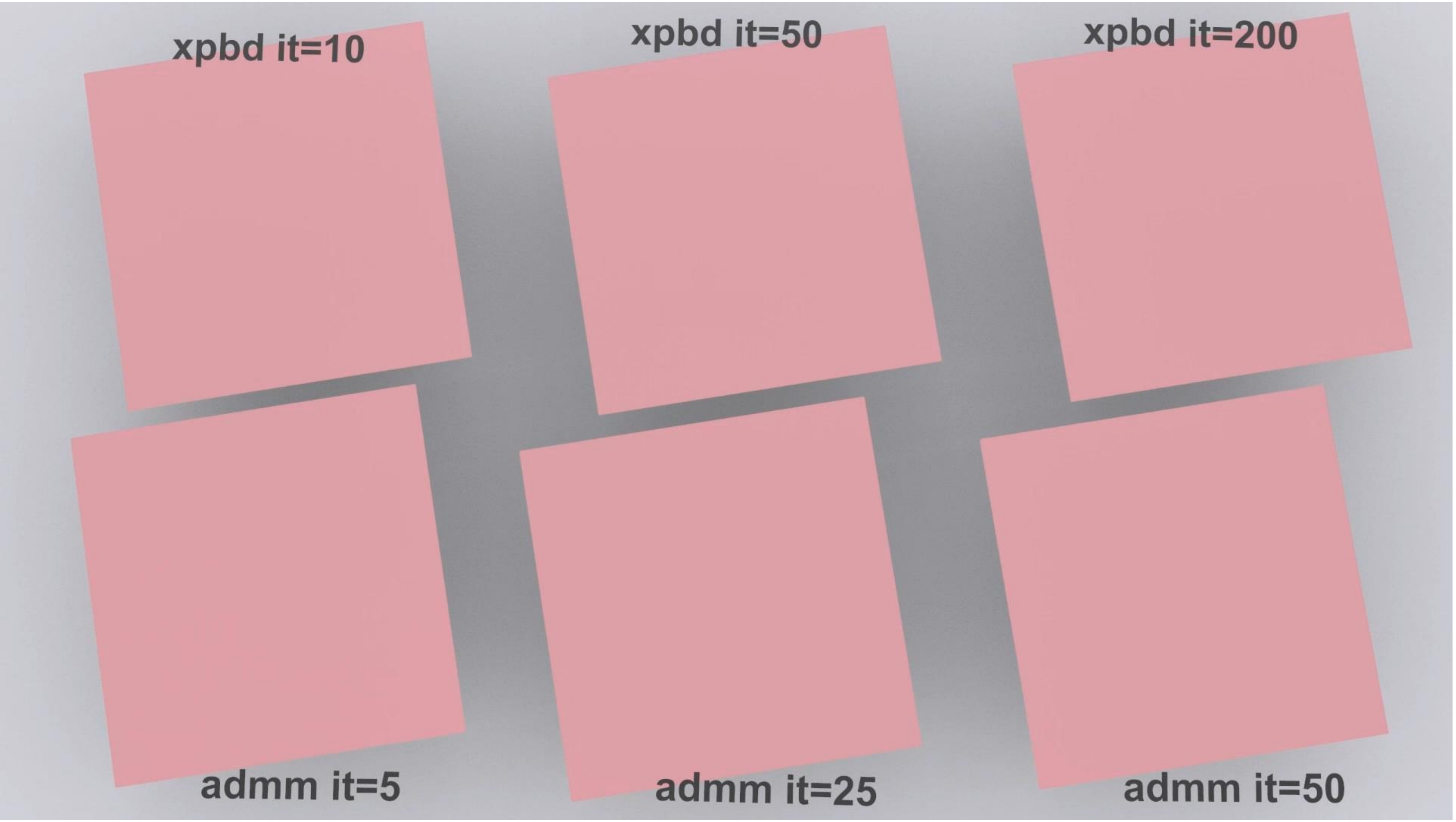

Result *compare with XPBD*

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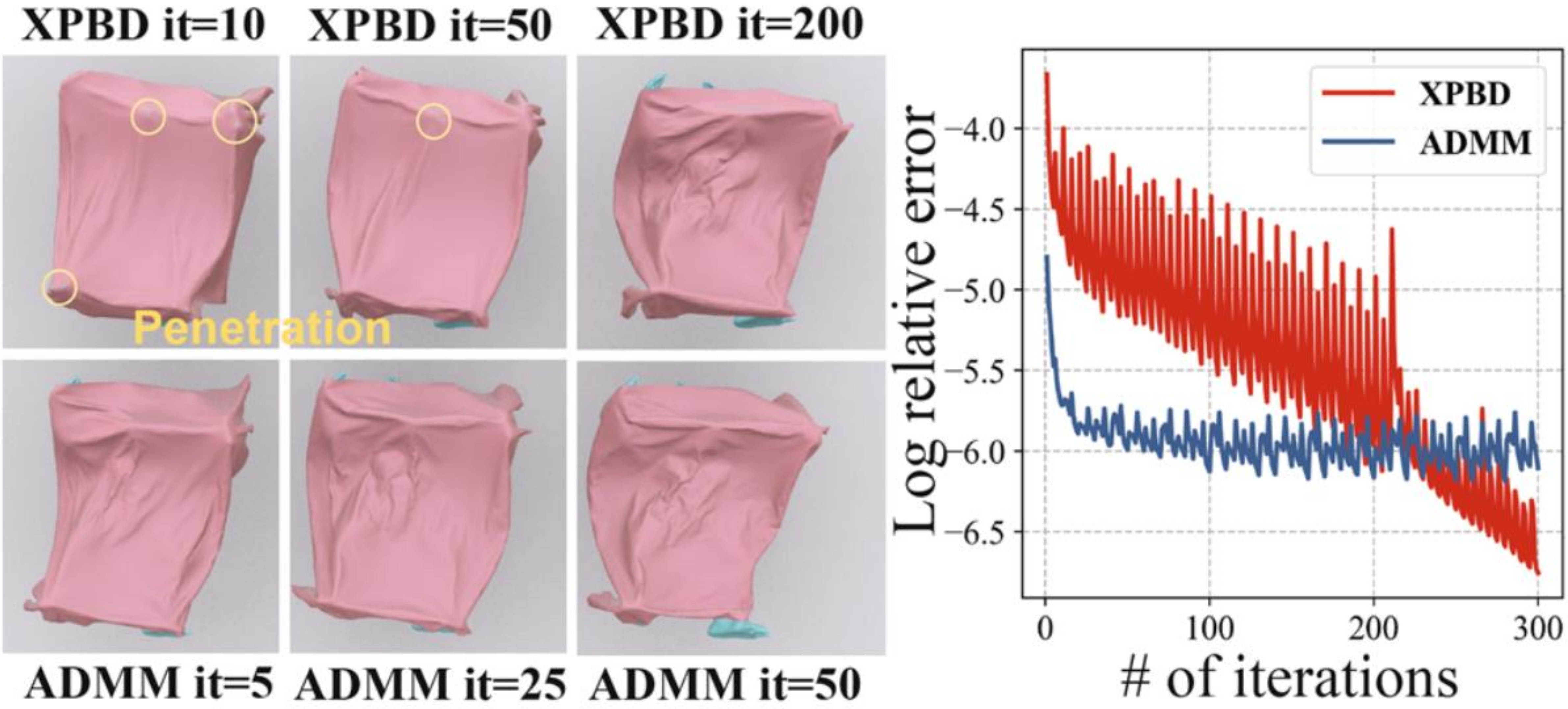
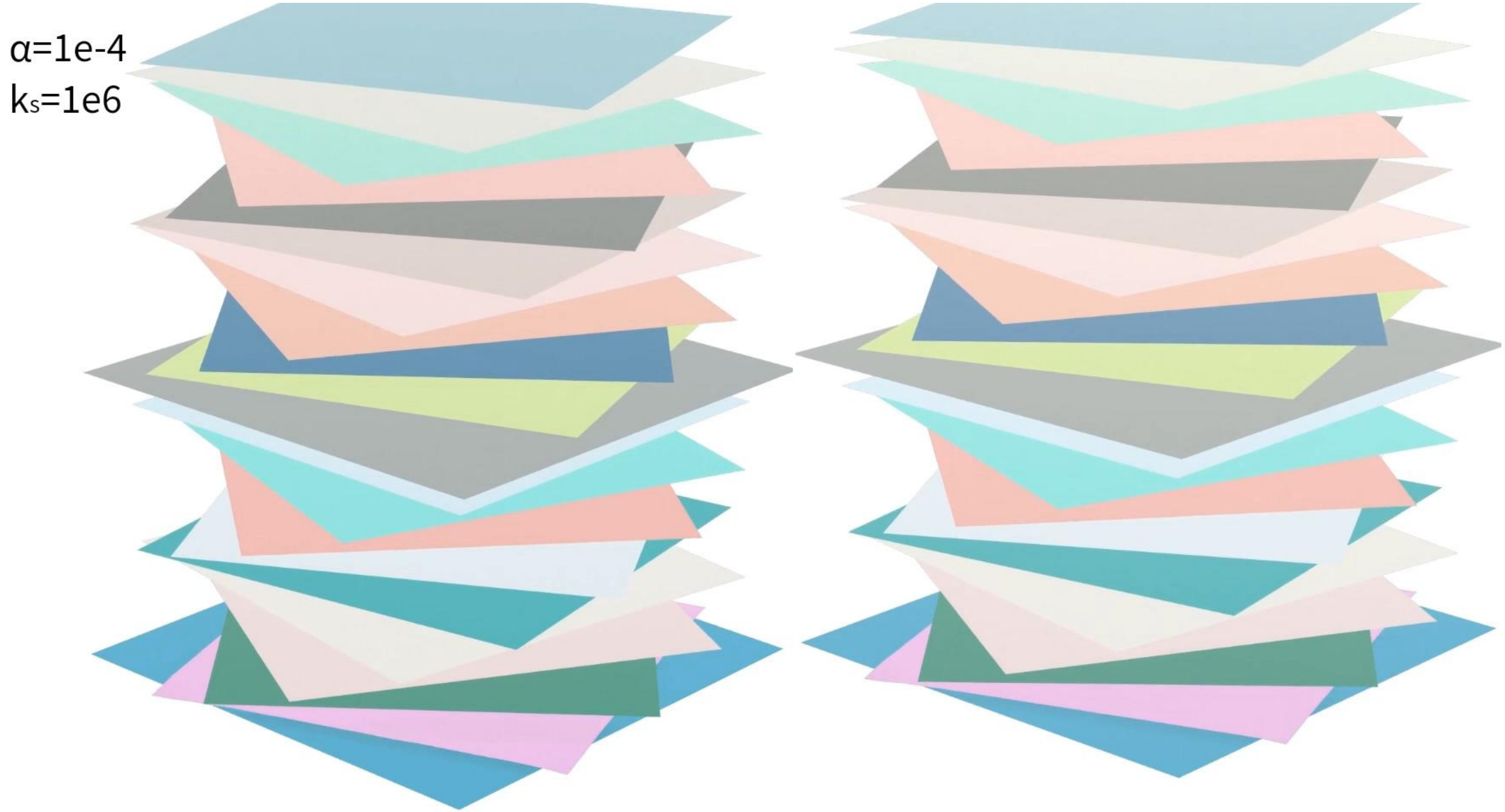


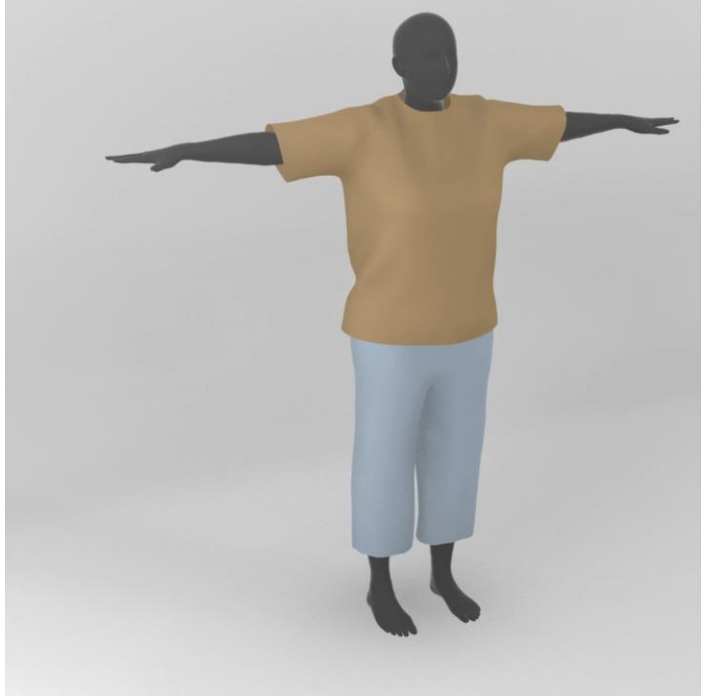
Result *Ablation: Contact Stabilization*

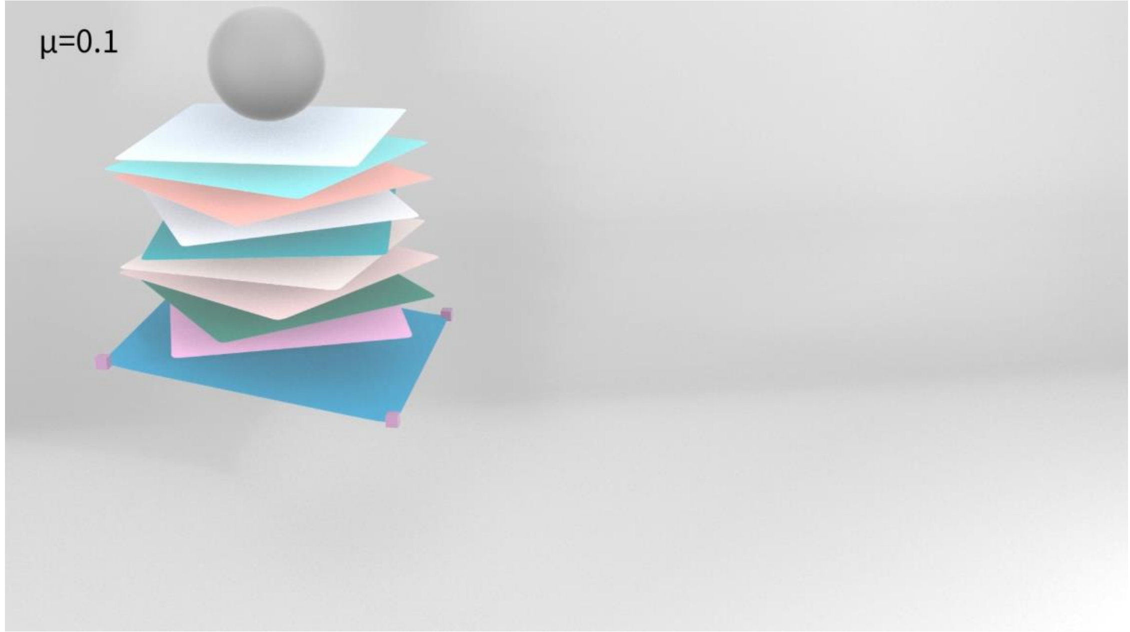


without contact stabilization

with contact stabilization

Result *complex contact*





Conclusion

- Introduce a novel **decoupled scheme** for soft body dynamics with frictional contacts, in the framework of ADMM, combining
  - an **efficient elasticity solver** from PD
  - a **lightweight non-Linear PGS** for contact handling
- Integrate techniques to enhance both reliability and realism
  - Matchstick anisotropic friction
  - Contact stabilization
  - Rayleigh elastic damping (with zero overhead)
- Demonstrating the effectiveness, accuracy, and computational efficiency of our solver

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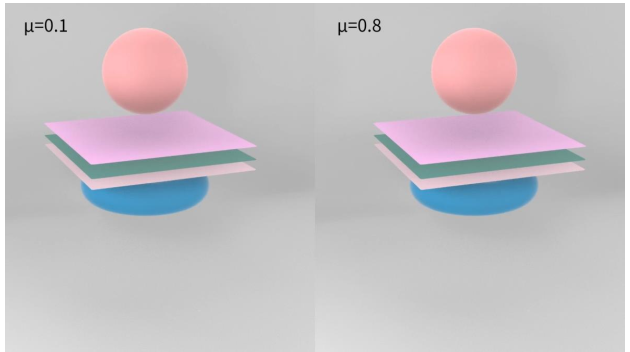

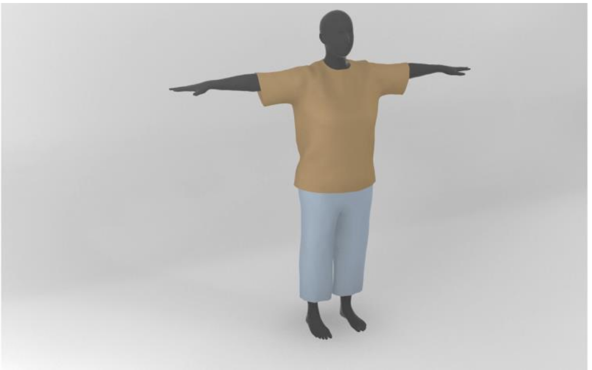

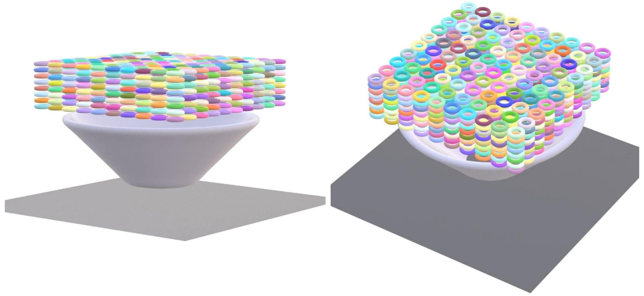

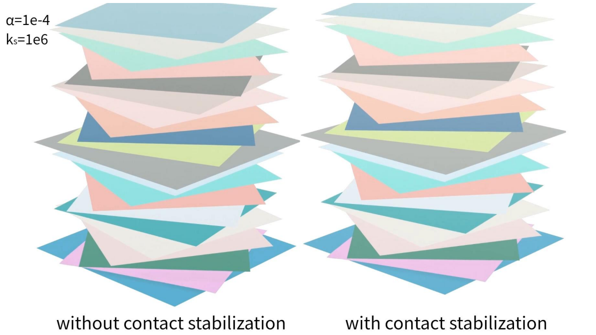

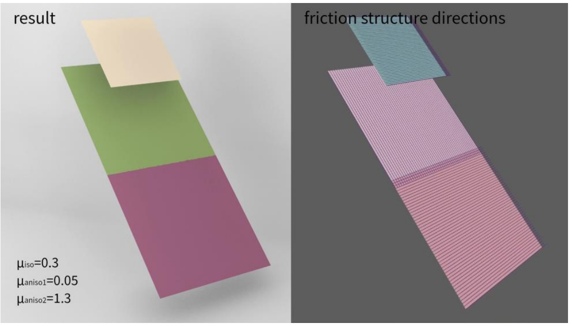

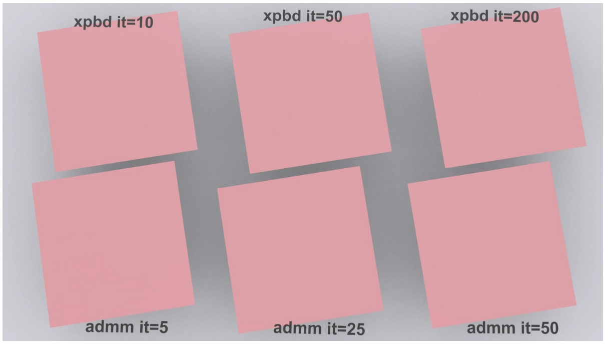

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