Stream Ciphers

Outline

- One-Time Pad
- Perfect Secrecy
- Pseudorandom Generators and Stream Ciphers
- Attacks
- Security of Pseudorandom Generators
- Semantic Security

Symmetric Ciphers

Definition.

A (symmetric) **cipher** defined over (K, M, C)

is a pair of "efficient" algorithms **(E,D)** where

- **E:** $K \times M \rightarrow C$
- **D:** $K \times C \rightarrow M$

such that $\forall m \in M, \forall k \in K$: $\mathbf{D(k, E(k,m)) = m}$

- E is often **randomized**.
- D is **always deterministic**.

Boolean operation: XOR

XOR of two strings in $\{0,1\}^n$ is their bit-wise addition modulo 2

X	Y	$X \oplus Y$
0	0	0
0	1	1
1	0	1
1	1	0

$$\begin{array}{r} 0\;1\;1\;0\;1\;1\;1 \\ \oplus\quad 1\;0\;1\;1\;0\;1\;0 \\ \hline 1\;1\;0\;1\;1\;0\;1 \end{array}$$

The One-Time Pad (Vernam 1917)

First example of a “secure” cipher

- $K = M = C = \{0,1\}^n$
- $E(k, m) = k \oplus m$
- $D(k, c) = k \oplus c$
- k used **<u>only once</u>**
- k is a **random** key (i.e., **<u>uniform</u>** distribution over K)

$$\begin{array}{l ccccccc l} m: & 0 & 1 & 1 & 0 & 1 & 1 & 1 & \\ k: & 1 & 0 & 1 & 1 & 0 & 1 & 0 & \oplus \\ \hline c: & 1 & 1 & 0 & 1 & 1 & 0 & 1 & \end{array}$$

The One-Time Pad (Vernam 1917)

The one-time pad is a **cipher**:

- $D(k, E(k,m)) =$
- $D(k, k \oplus m) =$
- $k \oplus (k \oplus m) =$
- $(k \oplus k) \oplus m =$
- $0 \oplus m =$
- m

One-time pad definition:

- $E(k, m) = k \oplus m$
- $D(k, c) = k \oplus c$

The One-Time Pad (Vernam 1917)

- **Pro:**
  - Very **fast** encryption and decryption
- **Con:**
  - **Long keys** (as long as the plaintext),
    If Alice wants to send a message to Bob,
    she first has to transmit a key of the same length to Bob **in a secure way**.
    If Alice has a secure mechanism to transmit the key, she might use that same mechanism to transmit the message itself!

Is the OTP secure? **What is a secure cipher?**

What is a secure cipher?

Attacker's abilities: **CipherText (CT) only attack** (for now)

Possible security requirements:

attempt #1: **attacker cannot recover secret key**

$E(k, m) = m$ would be secure

attempt #2: **attacker cannot recover all of plaintext** (partial information)

$E(k, m_0 \| m_1) = m_0 \| k \oplus m_1$ would be secure

Shannon's idea:

CT should reveal no "info" about PT

Information Theoretic Security (Shannon 1949)

Definition.

A cipher (E, D) over (K, M, C) has **perfect secrecy** if

$\forall \mathbf{m_0}, \mathbf{m_1} \in M$ with $\mathbf{len(m_0) = len(m_1)}$ and $\forall \mathbf{c} \in C$

$$\Pr[E(k, m_0)=c] \;=\; \Pr[E(k, m_1)=c]$$

where **k is uniform in K** $(k \longleftarrow K)$

NOTE: there are no computational assumptions about the attacker, this is why this is also called unconditional security or perfect security

Information Theoretic Security

- Given CT, can’t tell if PT is m_0 or m_1 (for all m_0, m_1)
- Most powerful adversary learns nothing about PT from CT
- No CT only attack! (but other attacks are possible…)

Is OTP ''secure''?

OTP has perfect secrecy.

Proof:

$$\forall m, c \qquad \Pr_k[E(k,m) = c] \;=\; \frac{\#keys\ k \in K\ \ s.t.\ E(k,m) = c}{|K|}$$

$$\text{So if } \forall m, c \;\; \#\{k \in K : E(k,m) = c\} = const.$$

$$\Rightarrow \text{Cipher has perfect secrecy}$$

Let **m** ∈ M and **c** ∈ C.

How many OTP keys map **m** to **c** ?

- None
- 1
- 2
- It depends on **m**

m:	0	1	1	0	1	1	1	$\oplus$
k :	?	?	?	?	?	?	?	
c :	1	1	0	1	1	0	1	

Is OTP ‘’secure’’?

OTP has perfect secrecy.

Proof:

$$\forall m, c \quad \Pr_k[E(k,m) = c] = \frac{1}{|K|}$$

$$\text{So if } \forall m, c \;\; \#\{k \in K : E(k,m) = c\} = const.$$

$$\boxed{\Rightarrow \text{Cipher has perfect secrecy}}$$

The bad news …

- OTP drawback: **key-length=msg-length**

- Are there ciphers with perfect secrecy that use shorter keys?

  Theorem: perfect secrecy $\Rightarrow |K| \geq |M|$

  i.e. perfect secrecy $\Rightarrow$ key-length $\geq$ msg-length

- Hard to use in practice!!!!

Pseudorandom Generators and Stream Ciphers

Review

Cipher over (K,M,C): a pair of "efficient" algorithms (E, D) s.t.
$\forall\, m \in M,\ \forall\, k \in K:\quad D(k, E(k, m)) = m$

Weak ciphers: substitution cipher, Vigenère, …

A good cipher: **OTP** $\quad M = C = K = \{0,1\}^n$

$$\mathbf{E(k, m) = k \oplus m \quad , \quad D(k, c) = k \oplus c}$$

OTP has perfect secrecy (i.e., no CT only attacks)

Bad news: perfect-secrecy ⇒ key-len ≥ msg-len

Stream Ciphers: making OTP practical

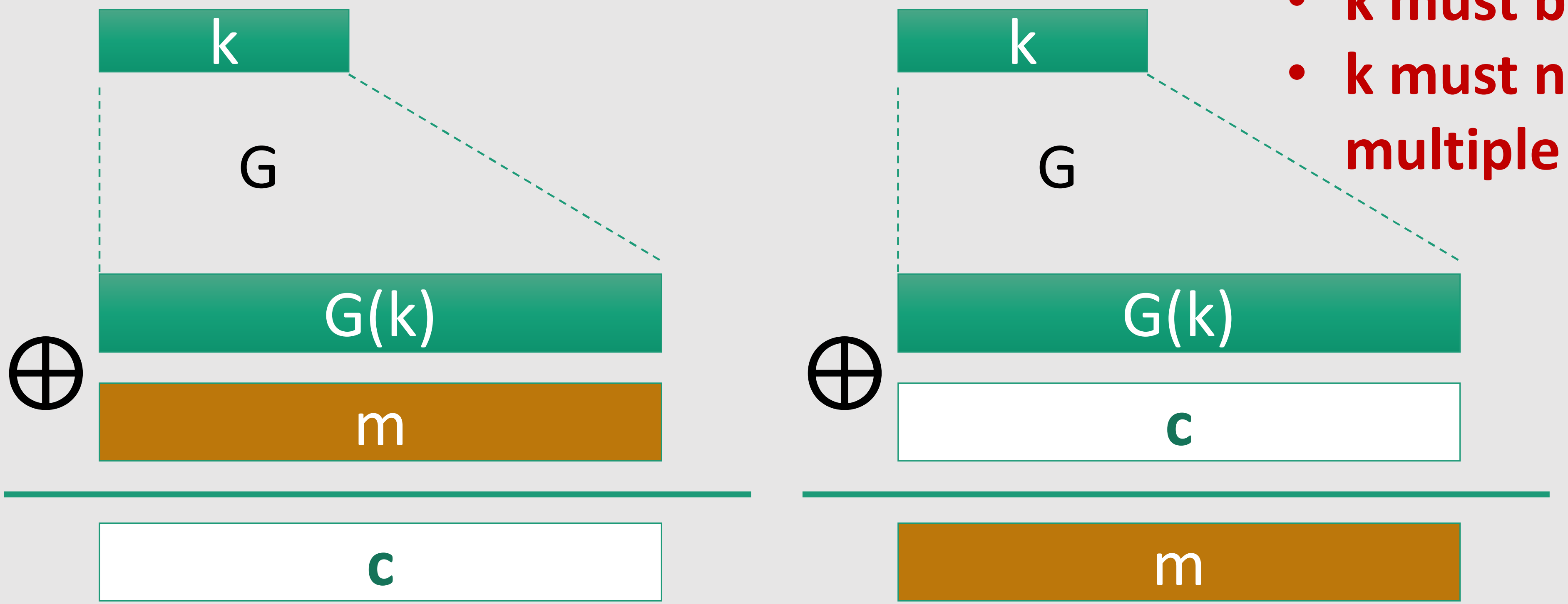
Idea: replace "**random**" key by "**pseudorandom**" key

Pseudorandom Generator (PRG):

PRG is a function $\mathbf{G}: \underbrace{\{0,1\}^s}_{\textbf{seed space}} \to \{0,1\}^n \qquad n \gg s$

(efficiently computable by a <u>deterministic</u> algorithm)

Stream Ciphers: making OTP practical



- **k must be random**
- **k must not be used multiple times**

$E(k, m) = G(k) \oplus m$ $\qquad$ $D(k, c) = G(k) \oplus c$

Can a **stream cipher** have perfect secrecy?

- Yes, if the PRG is really “secure”
- No, there are no ciphers with perfect secrecy
- Yes, every cipher has perfect secrecy
- No, since the key is shorter than the message

Can a stream cipher have perfect secrecy?

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Stream Ciphers: making OTP practical

Stream ciphers cannot have perfect secrecy !!

- Need a different definition of security
- Security will **depend on specific PRG**

Weak PRGs (do not use for crypto)

Linear congruential generator with parameters a, b, p:
(a, b are integers, p is a prime)

r[0] := seed
r[i] ← a r[i-1] + b mod p
output few bits of r[i]
i++

has some good statistical properties
But it's easy to predict

glibc random():

$r[i] \leftarrow (r[i-3] + r[i-31]) \ \% \ 2^{32}$

output r[i] >> 1

Do not use random() for crypto
(e.g., Kerberos v4)

Attacks on OTP and Stream Ciphers

Review

- **One-time pad**:
  - $E(k,m) = \mathbf{k} \oplus m$
  - $D(k,c) = \mathbf{k} \oplus c$

- **k** is random (**uniform**)
- **k** used only once

- **Stream ciphers**
  making OTP practical using a **PRG** $G: K \longrightarrow \{0,1\}^n$
  - $E(k,m) = \mathbf{G(k)} \oplus m$
  - $D(k,c) = \mathbf{G(k)} \oplus c$

Attack 1: **two time** pad is insecure !!

Never use stream cipher **key more than once** !!

$$c_1 \leftarrow m_1 \oplus PRG(k)$$
$$c_2 \leftarrow m_2 \oplus PRG(k)$$

Eavesdropper does:

$$c_1 \oplus c_2 \rightarrow$$

Enough redundancy in English and ASCII encoding that:

$$m_1 \oplus m_2 \rightarrow m_1 , m_2$$

Venona project (1941 – 1980)

American National Security Agency decrypted Soviet messages that were transmitted in the 1940s.

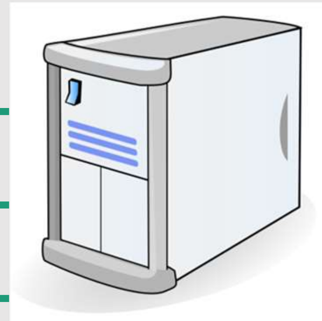
That was possible because the Soviets ***reused*** the keys in the one-time pad scheme.

Real-world examples

MS-PPTP (windows NT):

$k = (k_{C\longrightarrow S},\ k_{S\longrightarrow C})$

k  m_1 m_2 m_3

s_1 s_2 s_3  **k**

$[\ m_1 \,||\, m_2 \,||\, m_3\ ] \oplus \boxed{\text{PRG}(k)}$

$[\ s_1 \,||\, s_2 \,||\, s_3\ ] \oplus \boxed{\text{PRG}(k)}$

Need different keys for C⟶S and S⟶C

Real-world examples

k: LONG-TERM KEY

802.11b WEP:

m | CRC(m)

$\oplus$

PRG(IV ll k)

k 

Client

IV | ciphertext

 k

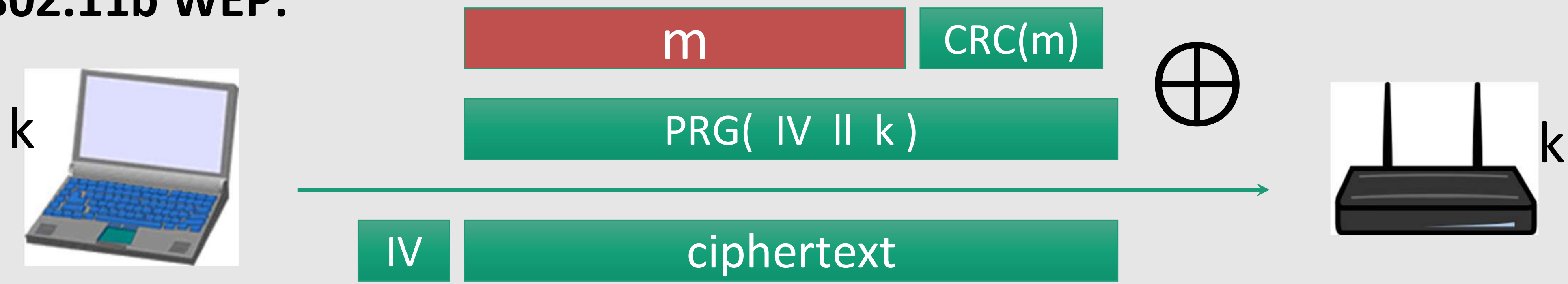
Access Point

Length of IV: 24 bits

- Repeated IV after $2^{24} \approx 16$M frames
- On some 802.11 cards: IV resets to 0 after power cycle

Avoid related keys

802.11b WEP:



24 bits   104 bits

key for frame #1:   (1 ll k)

key for frame #2:   (2 ll k)

⋮

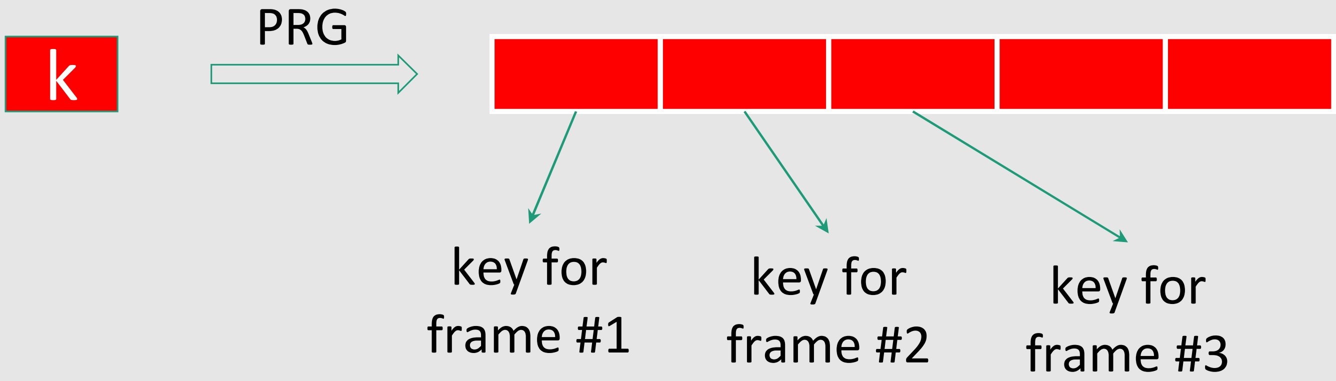
Very related keys!!
Not random keys!

The PRG used in WEP (called RC4) is not secure for such related keys

- Attack that can recover k after 10^6 frames (FMS 2001)
- Recent attack => 40.000 frames

A better construction

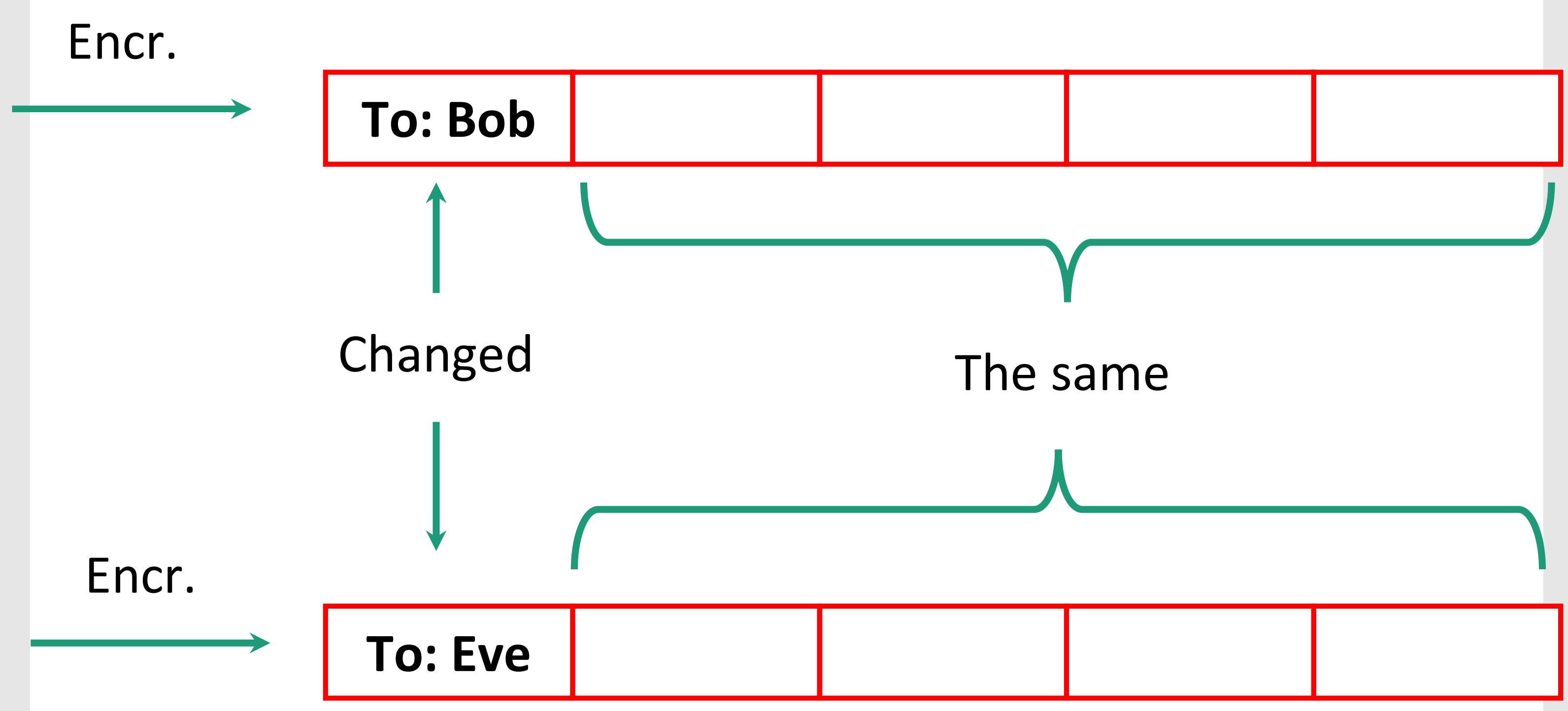
k



⇒ now each frame has a pseudorandom key

better solution: use stronger encryption method (as in WPA2)

Yet another example: disk encryption



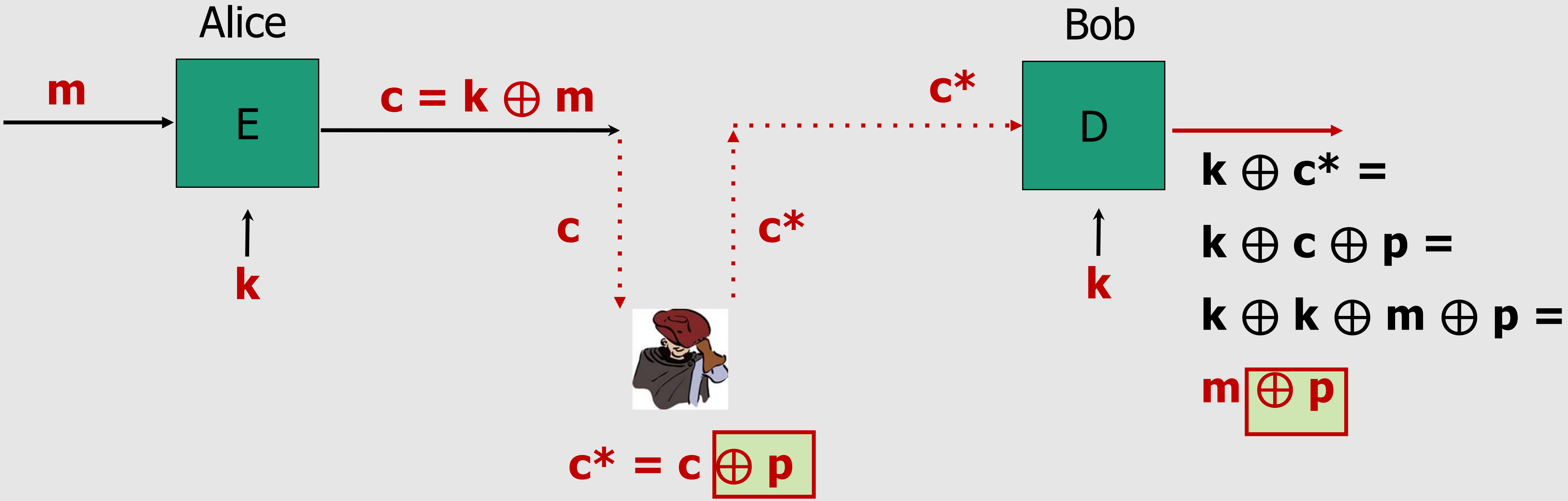
Two time pad: summary

Never use stream cipher key **more than once** !!

- Network traffic: negotiate a new key for every session (e.g. TLS)
  - One key (or ‘’sub-key’’) for traffic **from Client to Server**
  - One key (or ‘’sub-key’’) for traffic **from Server to Client**

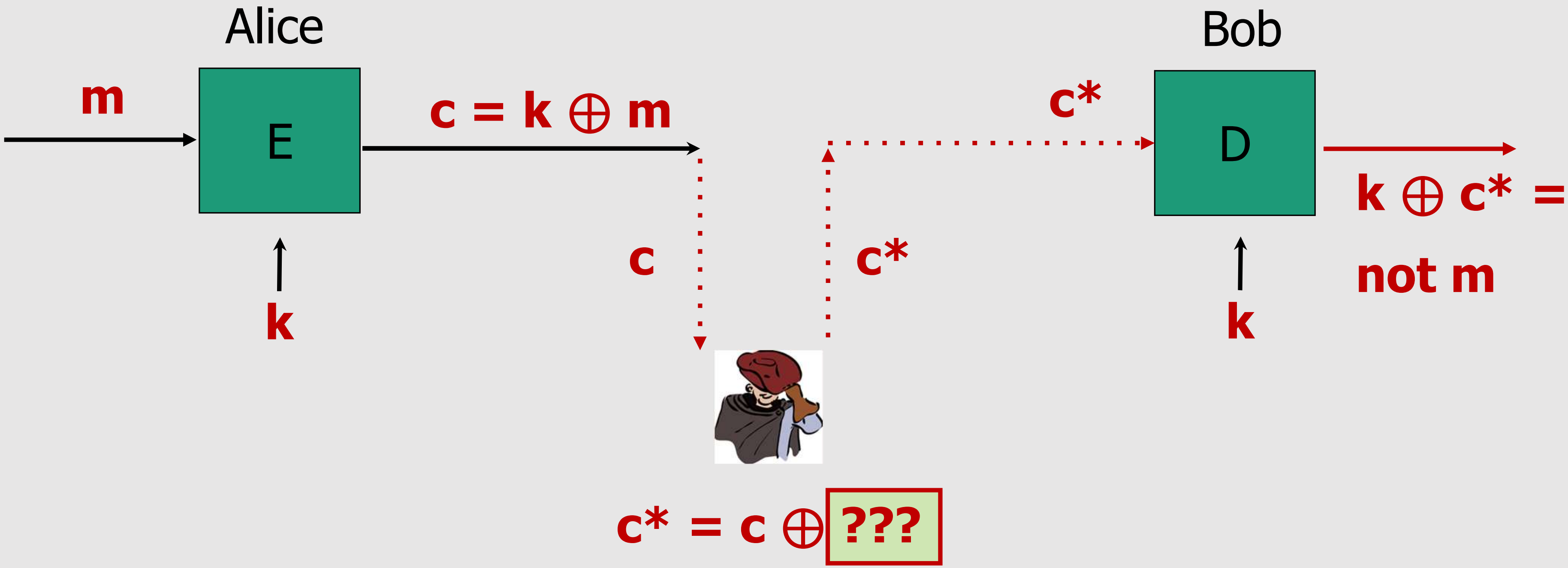
- Disk encryption: typically do not use a stream cipher

Attack 2: no integrity (OTP is **malleable**)



Modifications to ciphertext are **undetected** and have **predictable** impact on plaintext

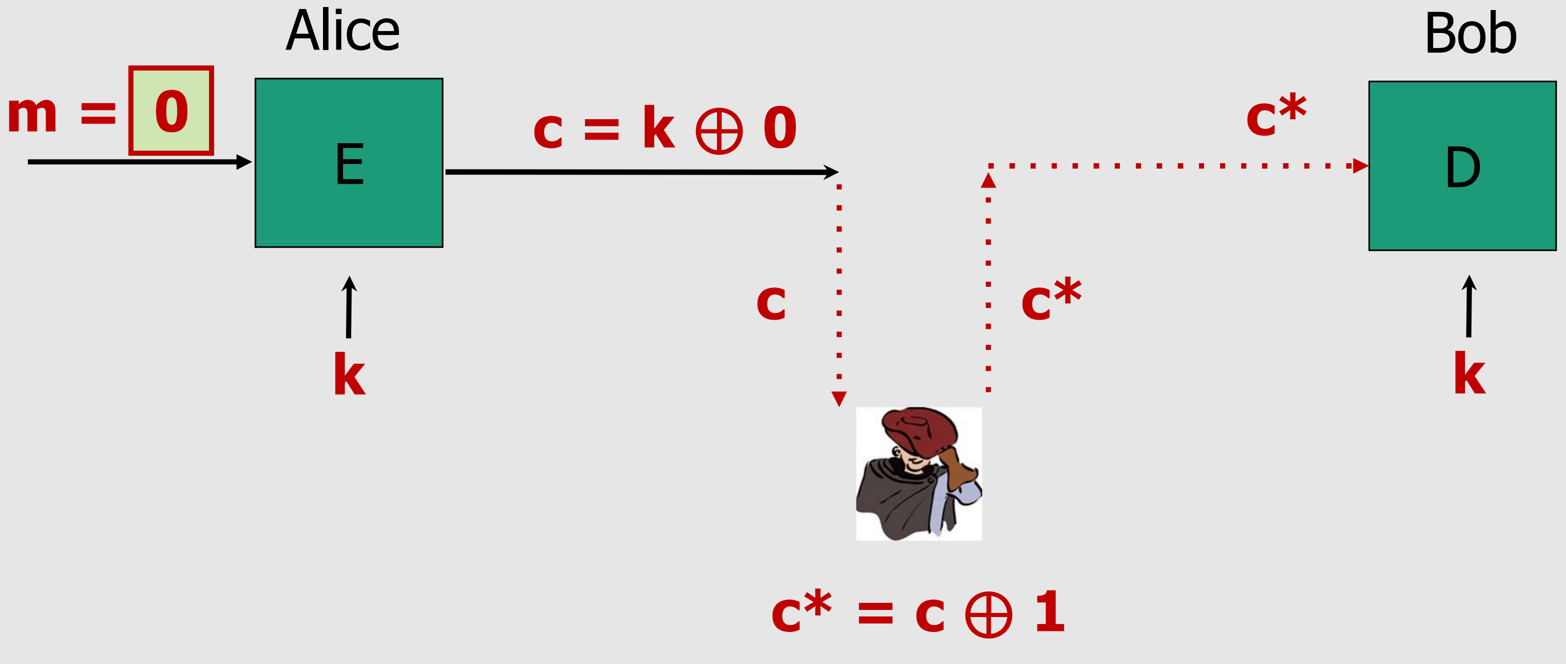
Attack 2: no integrity (OTP is **malleable**)



- Alice has to answer yes (**1**) or no (**0**) to Bob's invitation. She'll encrypt the answer with OTP.
- The attacker cannot recover Alice's answer from CT.
- **Still, can the attacker "flip" Alice's answer?**

  Yes !! Apply ⊕ 1 to the intercepted CT

Attack 2: no integrity (OTP is **malleable**)



$k \oplus c^* =$

$k \oplus c \oplus 1 =$

$k \oplus k \oplus 0 \oplus 1 =$

$0 \oplus 0 \oplus 1 =$

$\boxed{1}$

Attack 2: no integrity (OTP is **malleable**)

Alice

$m = \boxed{1}$ → E ← k

$c = k \oplus 1$

c

$c^* = c \oplus 1$

c^*

Bob

c^* → D ← k

$k \oplus c^* =$

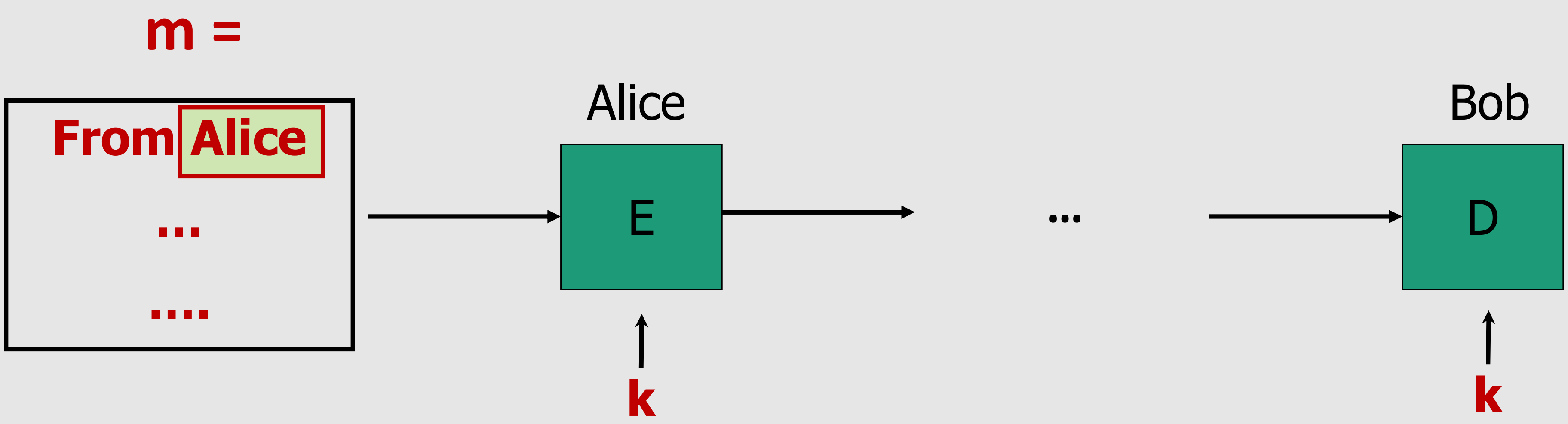
$k \oplus c \oplus 1 =$

$k \oplus k \oplus 1 \oplus 1 =$

$0 \oplus 1 \oplus 1 =$

$\boxed{0}$

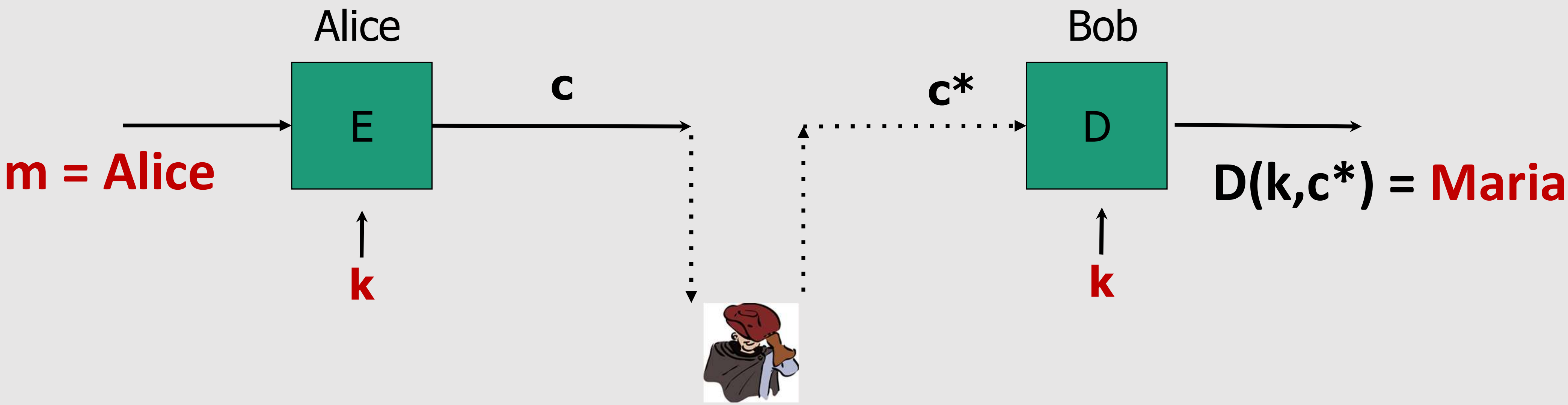
Attack 2: no integrity   (OTP is **malleable**)



Attacker wants to change **Alice** into **Maria**.

Can he do that?

Attack 2: no integrity (OTP is **malleable**)



$c^* = c \oplus \boxed{???}$

Attacker wants to change **Alice** into **Maria**.

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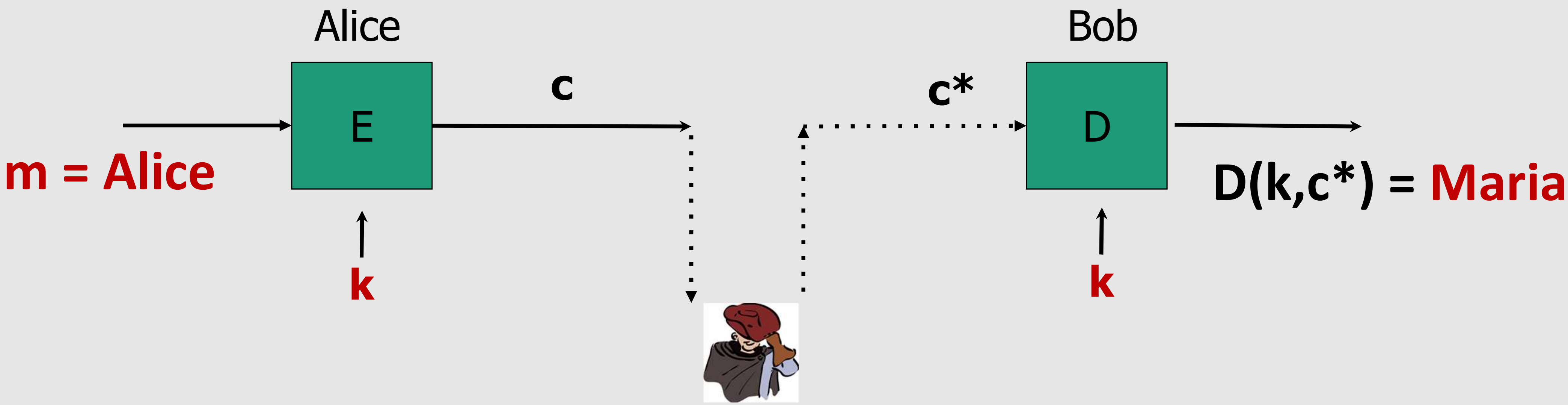

$c^* = c \oplus \boxed{\text{Alice} \oplus \text{Maria}}$

Attacker wants to change **Alice** into **Maria**.

Can he do that?

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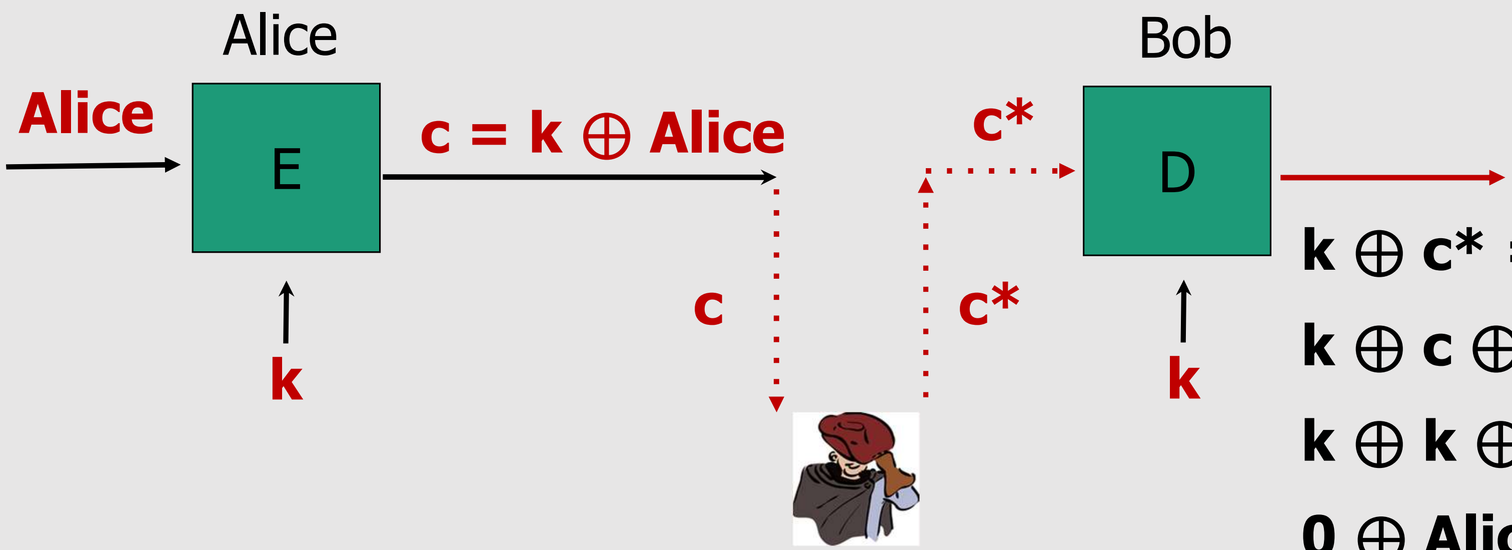


$c^* = c \oplus \text{Alice} \oplus \text{Maria}$

$k \oplus c^* =$

$k \oplus c \oplus \text{Alice} \oplus \text{Maria} =$

$k \oplus k \oplus \text{Alice} \oplus \text{Alice} \oplus \text{Maria} =$

$0 \oplus \text{Alice} \oplus \text{Alice} \oplus \text{Maria} =$

$0 \oplus 0 \oplus \text{Maria} =$

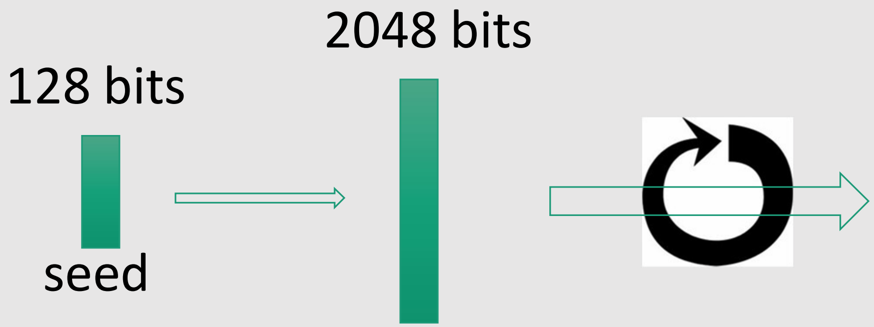
Maria

Consider the bank account number in a wire transfer…

Real-world Stream Ciphers

Old example (software): RC4 (1987)

Variable size seed (e.g., 128 bits)



1 byte per round

- RC4: Rivest Cipher 4, and it was designed by Ron Rivest of RSA Security in 1987.
- Used in
  - HTTPS
  - WEP (Wired Equivalent Privacy) to secure Wireless/Wi-Fi communications

RC4 PRG

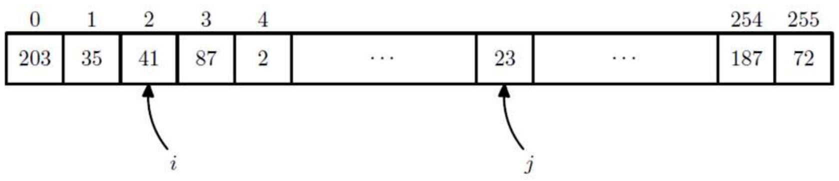


Figure 3.12: An example RC4 internal state

- The RC4 stream cipher key ***s*** is a seed for the PRG and is used to initialize the array **S** to a pseudo-random permutation of the numbers 0 : : : 255.
- Initialization is performed using the following **setup algorithm**:

input: string of bytes s

for $i \leftarrow 0$ to 255 do: $\quad S[i] \leftarrow i$

$j \leftarrow 0$
for $i \leftarrow 0$ to 255 do
  $k \leftarrow s\big[i \bmod |s|\big]$ // *extract one byte from seed*
  $j \leftarrow \big(\, j + S[i] + k \,\big) \bmod 256$
  swap$(S[i], S[j])$

- During the loop the index ***i*** runs linearly through the array while the index ***j*** jumps around.
- At each iteration the entry at index ***i*** is swapped with the entry at index ***j***.

RC4 PRG

- Once the array S is initialized, the PRG generates pseudo-random output one byte (*one byte of the key*) at a time using the following **stream generator**:

$$
\begin{array}{l}
i \leftarrow 0, \quad j \leftarrow 0 \\
\text{repeat} \\
\qquad i \leftarrow (i+1) \bmod 256 \\
\qquad j \leftarrow (j + S[i]) \bmod 256 \\
\qquad \text{swap}(S[i], S[j]) \\
\qquad \text{output} \quad S\big[\ (S[i] + S[j]) \bmod 256\ \big] \\
\text{forever}
\end{array}
$$

- The procedure runs for as long as necessary. Again, the index ***i*** runs linearly through the array while the index ***j*** jumps around.
- Swapping S[i] and S[j] continuously shuffles the array S.

Security of RC4

Weaknesses:

1. Bias in initial output: let us assume that the RC4 **setup algorithm is perfect** and generates a uniform permutation from the set of all 256! permutations.

Mantin and Shamir showed that, even assuming perfect initialization, the output of RC4 is biased: Pr[2^{nd} byte = 0] = 2/256 => RC4-drop[n]

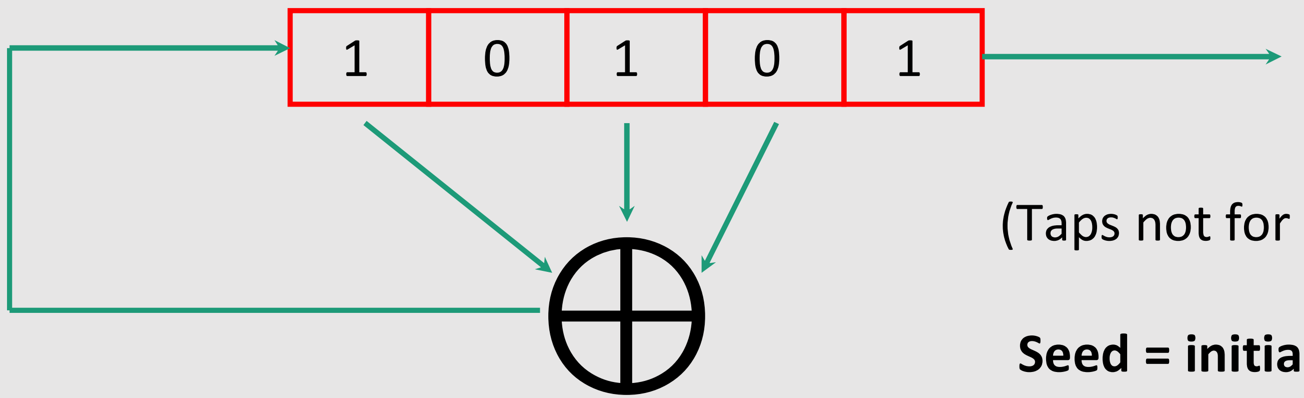
2. Fluhrer and McGrew: Prob. of (0,0) is $1/256^2 + 1/256^3$

3. Related key attacks: attack on WEP

Old example (hardware): CSS (badly broken)

Content Scrambling System

Linear feedback shift register (LFSR):



(Taps not for all cells)

Seed = initial state of the LFSR

DVD encryption (CSS): 2 LFSRs

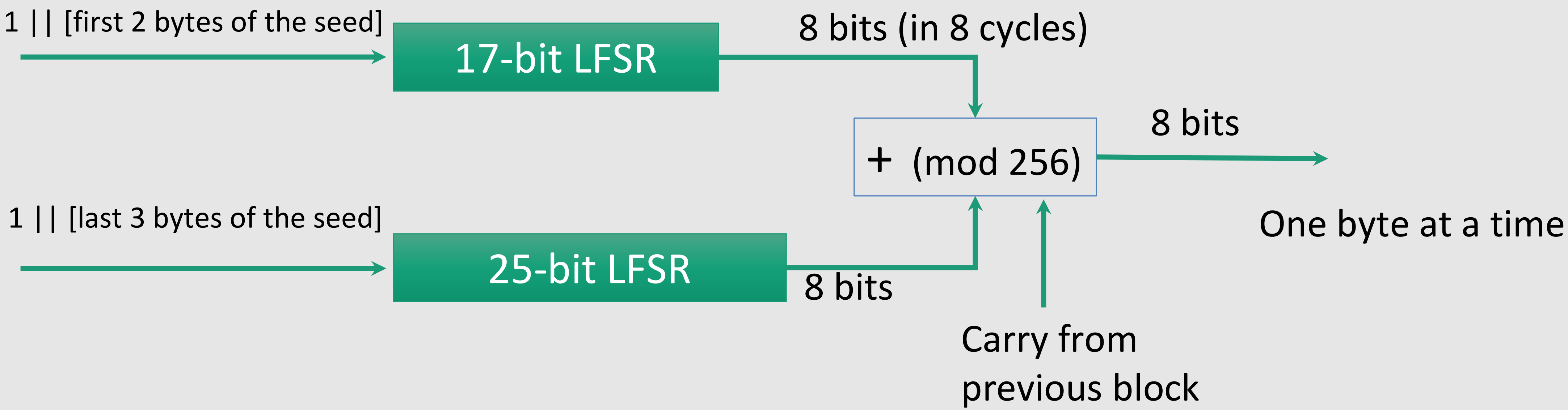
GSM encryption (A5/1,2): 3 LFSRs

Bluetooth (E0): 4 LFSRs

all broken

Old example (hardware): CSS (badly broken)

CSS: seed = 5 bytes = 40 bits



Easy to break in time $\approx 2^{17}$

Modern stream ciphers: eStream

PRG: $\{0,1\}^s \times R \longrightarrow \{0,1\}^n$ $\quad n>>s$

Seed ***Nonce***

Nonce: a non-repeating value for a given key, that is **a pair (k,r) which is never used more than once**

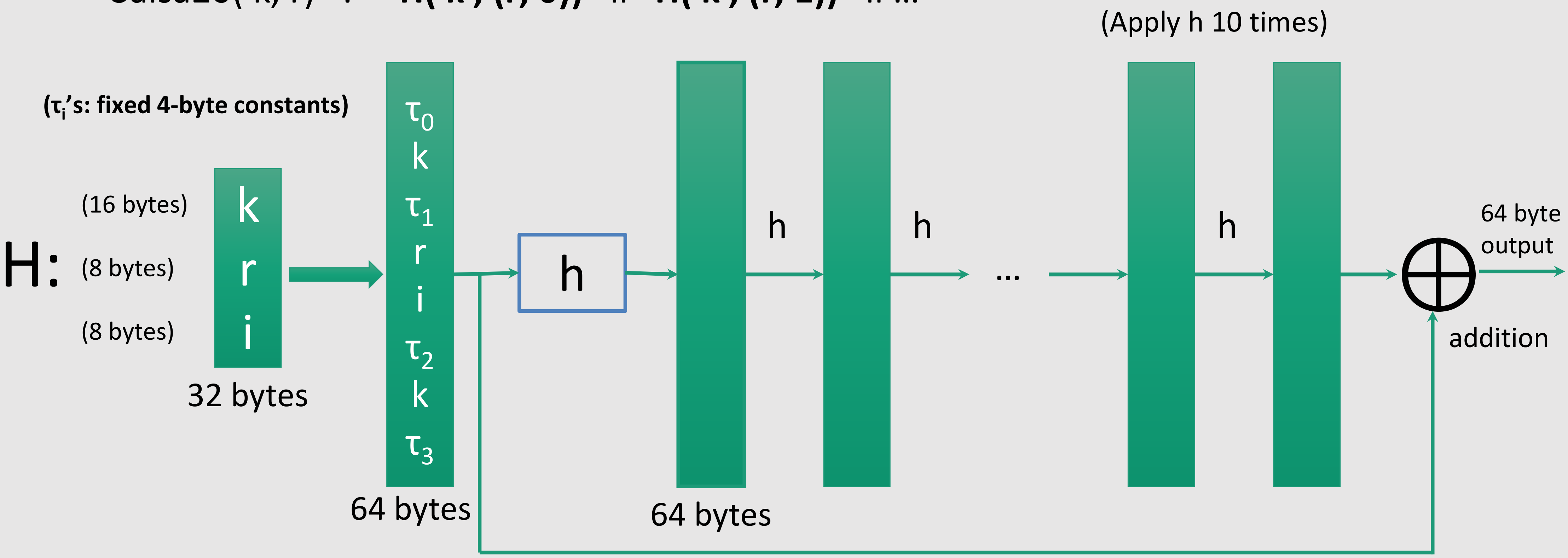
=> can re-use the key as long as the nonce changes

$E(k, m, r) = m \oplus PRG(k, r)$

eStream: Salsa 20 (SW+HW)

Salsa20: $\{0,1\}^{128 \text{ or } 256} \times \{0,1\}^{64} \longrightarrow \{0,1\}^{n}$ (max n = 2^{73} bits)

Salsa20(k, r) := **H(k , (r, 0)) ‖ H(k , (r, 1)) ‖ …**



h: invertible function. designed to be fast on x86 (SSE2)

Performance: Crypto++ 5.6.0 [Wei Dai]

AMD Opteron, 2.2 GHz (Linux)

	PRG	**Speed (MB/sec)**
	RC4	126
eStream	Salsa20/12	643
eStream	Sosemanuk	727

When is a PRG ‘‘secure’’?

When is a PRG ‘‘secure’’?

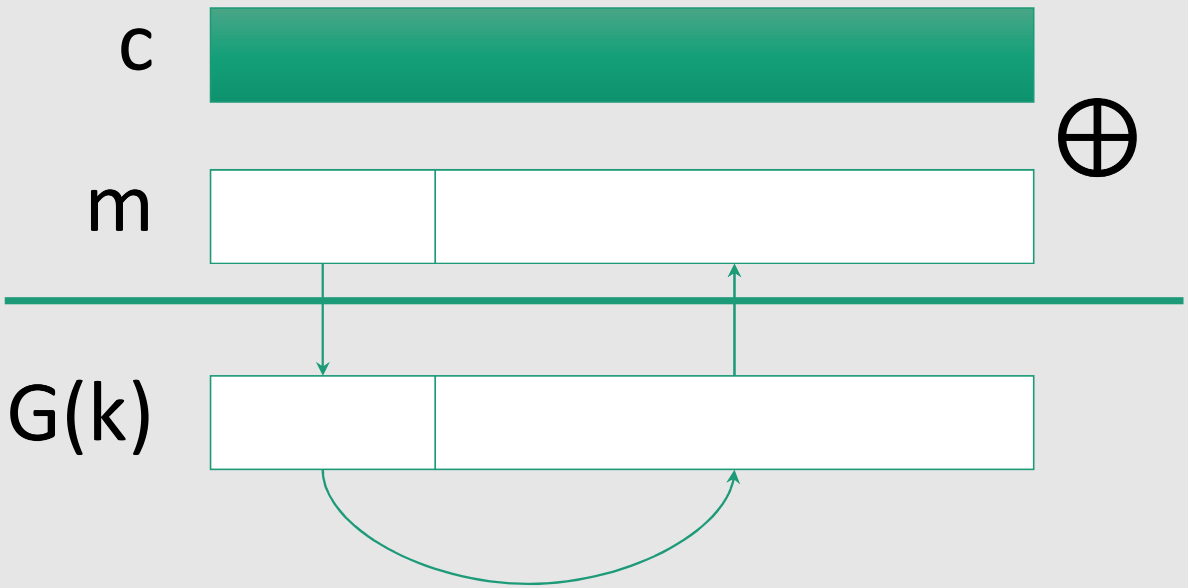
1. **Unpredictable** PRG
2. **Secure** PRG

We’ll see that they are **equivalent** notions

PRG must be unpredictable

Suppose PRG is **predictable**:

$$\exists i : \quad G(k)\big|_{1,\ldots,i} \xrightarrow{Alg} G(k)\big|_{i+1,\ldots,n}$$



Even

$$G(k)\big|_{1,\ldots,i} \xrightarrow{Alg} G(k)\big|_{i+1}$$

is a problem

PRG must be unpredictable

We say that G: K ⟶ $\{0,1\}^n$ is **predictable** if:

$\exists$ "efficient" algorithm A and $\exists 1 \leq i \leq n-1$ s.t.

$$\Pr_{k \leftarrow K}\left[A(G(k)|_{1,\ldots,i}) = G(k)|_{i+1}\right] > \frac{1}{2} + \epsilon$$

for non-negligible ϵ (e.g., $\epsilon = \frac{1}{2^{30}}$)

PRG is **unpredictable** if it **is not predictable**

⇒ ∀i: no "efficient" adversary can predict bit (i+1) for "non-neg" ε

- Suppose $G:K \longrightarrow \{0,1\}^n$ is such that for all **k**: **XOR(G(k)) = 1**
- Is G predictable ??

1. Yes, given the first bit I can predict the second
2. No, G is unpredictable
3. Yes, given the first (n-1) bits I can predict the n-th bit
4. It depends

- Suppose $G: K \longrightarrow \{0,1\}^n$ is such that for all **k**: **XOR(G(k)) = 1**
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4. It depends

One more definition of ‘‘secure’’ PRG

Let **G:K** $\longrightarrow$ **$\{0,1\}^n$** be a PRG

Goal:

define what it means that

$$[k \leftarrow K, \;\; \text{output } G(k)]$$

is “indistinguishable” from

$$[r \leftarrow \{0,1\}^n, \;\; \text{ouput } r]$$

G: $\{0,1\}^{10} \longrightarrow \{0,1\}^{1000}$

$[k \longleftarrow \{0,1\}^{10}$, output G(k)]

$[r \longleftarrow \{0,1\}^{1000}$, output r]

Note

A minimum security requirement for a PRG is that the length **s** of the random seed should be **sufficiently large**

so that a search over $\mathbf{2^s}$ elements (the total number of possible seeds) is infeasible for the adversary.

Statistical Tests

Statistical test on $\{0,1\}^n$:

An algorithm A s.t. A(x) outputs "0" or "1",

that is $\mathbf{A : \{0,1\}^n \longrightarrow \{0,1\}}$

Examples:

1. A(x)=1 iff $|\#0(x) - \#1(x)| \leq 10 \sqrt{n}$
2. A(x)=1 iff $|\#00(x) - n/4| \leq 10 \sqrt{n}$
3. A(x)=1 iff max-run-of-0(x) $< 10 \log_2(n)$ ...

Advantage

- Let **G:K ⟶{0,1}^{n}** be a **PRG**
- Let **A: {0,1}^{n} ⟶ {0,1}** be a **statistical test** on {0,1}^{n}

Define: $$Adv_{PRG}[A,G] = \left| \Pr_{k \leftarrow K}[A(G(k))=1] - \Pr_{r \leftarrow \{0,1\}^n}[A(r)=1] \right| \in [0,1]$$

- Adv close to 0 => A cannot distinguish G from random
- Adv non-negligible => A can distinguish G from random
- Adv close to 1 => A can distinguish G from random very well

A silly example: A(x) = 0 ⇒ Adv_{PRG} [A,G] =

Example of Advantage

- Suppose $G: K \longrightarrow \{0,1\}^n$ satisfies **msb(G(k)) = 1** for 2/3 of keys in K
- Define statistical test $A(x)$ as:

if [msb(x)=1] output "1" else output "0"

Then

$$Adv_{PRG}[A,G] = \left| \Pr[A(G(k))=1] - \Pr[A(r)=1] \right| =$$

$$\left| 2/3 - 1/2 \right| = 1/6$$

A breaks G with advantage 1/6 (which is not negligible)
hence **G is not a good PRG**

Secure PRGs: crypto definition

Definition:

We say that **$G : K \longrightarrow \{0,1\}^n$** is a **secure PRG** if

for every "*efficient*" statistical test **A**, **$Adv_{PRG}[A,G]$ is "negligible"**

Are there provably secure PRGs? Unknown (=> P ≠ PN)

A secure PRG is unpredictable

We show: PRG predictable $\Rightarrow$ PRG is insecure

Suppose *A* is an efficient algorithm s.t.

$$\Pr_{k \leftarrow K}\left[A(G(k)|_{1,\ldots,i}) = G(k)|_{i+1}\right] > \frac{1}{2} + \epsilon$$

for non-negligible ε (e.g. ε = 1/1000)

A secure PRG is unpredictable

Define statistical test B as:

$$B(X) = \begin{cases} \text{if } A(X|_{1,\ldots,i}) = X_{i+1} \text{ output } 1 \\ \text{else output } 0 \end{cases}$$

$$k \leftarrow K: \ Pr[B(G(k)) = 1] > \frac{1}{2} + \epsilon$$

$$r \leftarrow \{0,1\}^n: \ Pr[B(r) = 1] = \frac{1}{2}$$

$$\Rightarrow Adv_{PRG}[B,G] = |Pr[B(G(k)) = 1] - Pr[B(r) = 1]| > \epsilon$$

Thm (Yao'82): an unpredictable PRG is secure

Let **G : K ⟶ $\{0,1\}^n$** be **PRG**

"Thm": if **$\forall$ i $\in$ {0, … , n-1}** **G** is **unpredictable** at position **i**

then **G** is a **secure PRG**.

If next-bit predictors cannot distinguish G from random

then no statistical test can !!

More Generally

Let **P_1** and **P_2** be two distributions over $\{0,1\}^n$

We say that **P_1** and **P_2** are **computationally indistinguishable** (denoted $P_1 \approx_p P_2$)

$$\text{if } \forall \text{ "efficient" statistical test } A$$

$$\left| \Pr_{X \leftarrow P_1}[A(X)=1] - \Pr_{X \leftarrow P_2}[A(X)=1] \right| < \text{ negligible}$$

Example: a PRG is secure if **$\{ k \longleftarrow K : G(k) \} \approx_p \text{uniform}(\{0,1\}^n)$**

Semantic Security

What is a secure cipher?

Attacker's abilities: **CT only attack: obtains one ciphertext**

Possible security requirements:

attempt #1: **attacker cannot recover secret key**

$E(k, m) = m$ would be secure

attempt #2: **attacker cannot recover all of plaintext**

$E(k, m_0 \| m_1) = m_0 \| k \oplus m_1$ would be secure

Shannon's idea:

CT should reveal no "info" about PT

Recall Shannon's perfect secrecy

Let (E,D) be a cipher over (K,M,C)

Shannon's perfect secrecy:

(E,D) has perfect secrecy if $\forall\ m_0, m_1 \in M \quad (\ |m_0| = |m_1| \ )$

$$\{ E(k,m_0) \} \quad = \quad \{ E(k,m_1) \} \qquad \text{where} \quad k \longleftarrow K$$

- The two distributions must be **identical**
- Too strong definition
- It requires long keys
- Stream Ciphers can't satisfy it

Weaker Definition:

(E,D) has perfect secrecy if $\forall\ m_0, m_1 \in M \quad (\ |m_0| = |m_1| \ )$

$$\{ E(k,m_0) \} \ \approx_p \ \{ E(k,m_1) \} \qquad \text{where} \quad k \longleftarrow K$$

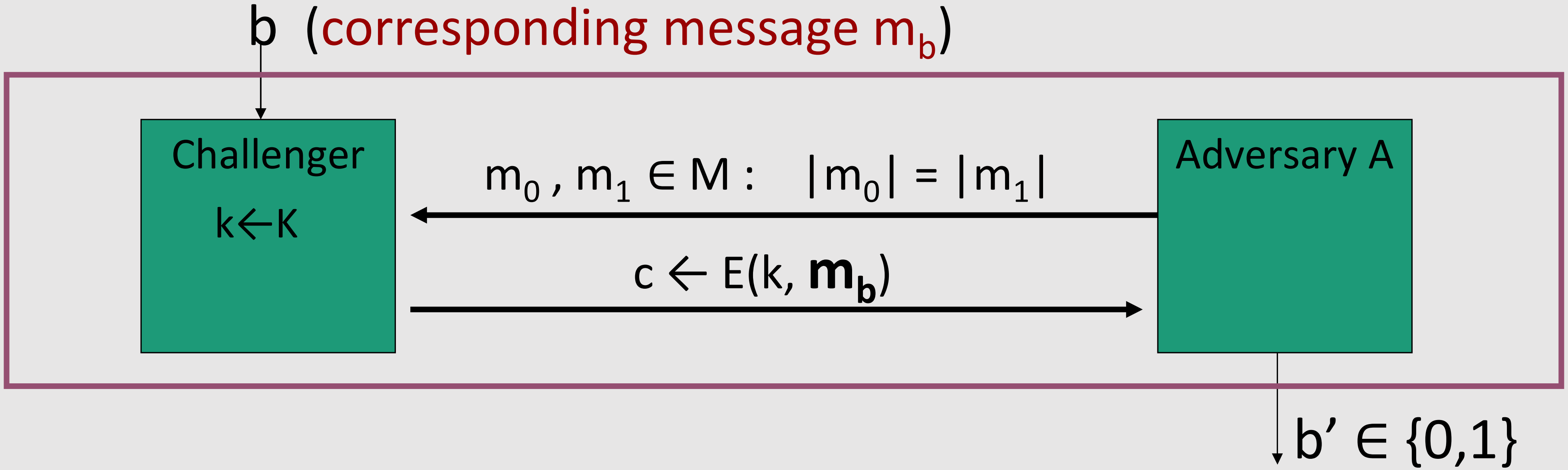
Rather than requiring the two distributions to be identical, we require them to be **COMPUTATIONALLY INDISTINGUISHABLE**

(One more requirement) … but also need adversary to exhibit $m_0, m_1 \in M$ explicitly

Semantic Security (one-time key)

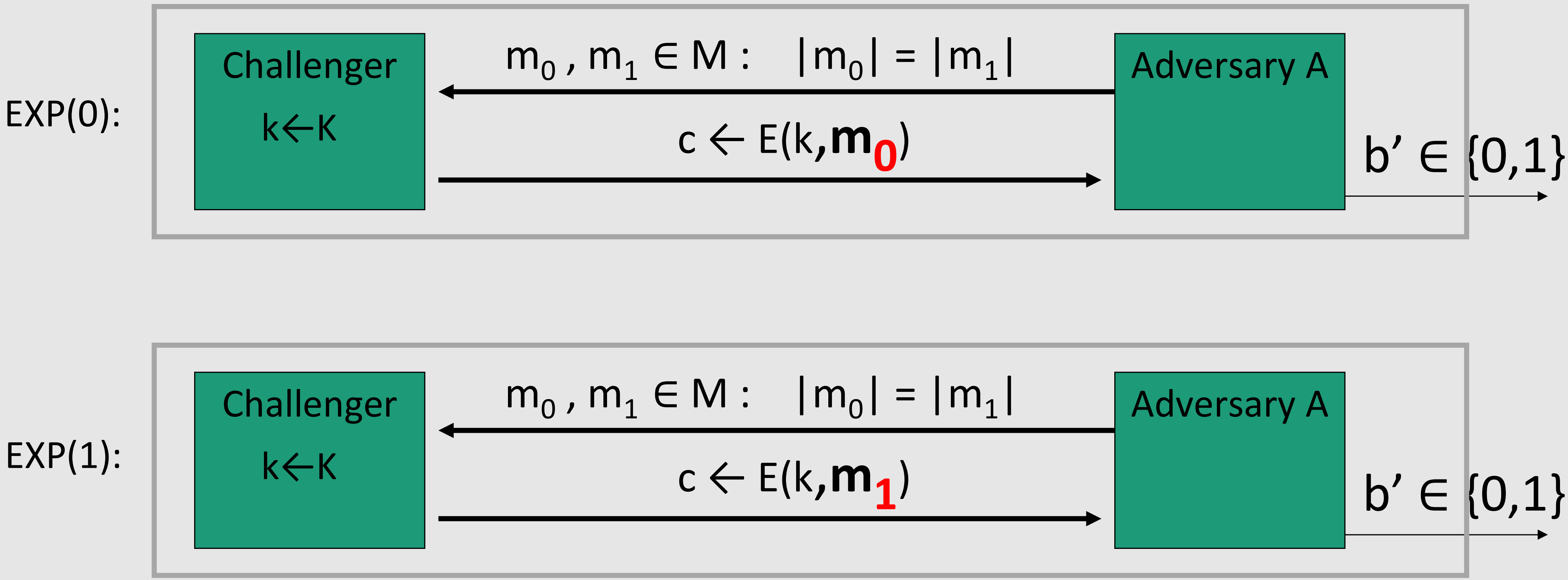
For a cipher **Q = (E,D)** and an adversary **A** define a game as follows.

For b=0,1 define experiments EXP(0) and EXP(1) as:



$$\mathbf{Adv_{SS}[A,Q] := |\ Pr[EXP(0)=1] - Pr[EXP(1)=1]\ |}$$

Semantic Security (one-time key)



$\mathrm{Adv}_{SS}[A,Q] = \left|\, \Pr[\, \mathbf{EXP(0)}=1 \,] - \Pr[\, \mathbf{EXP(1)}=1 \,] \,\right|$ should be "negligible" for all "efficient" A

Semantic Security (one-time key)

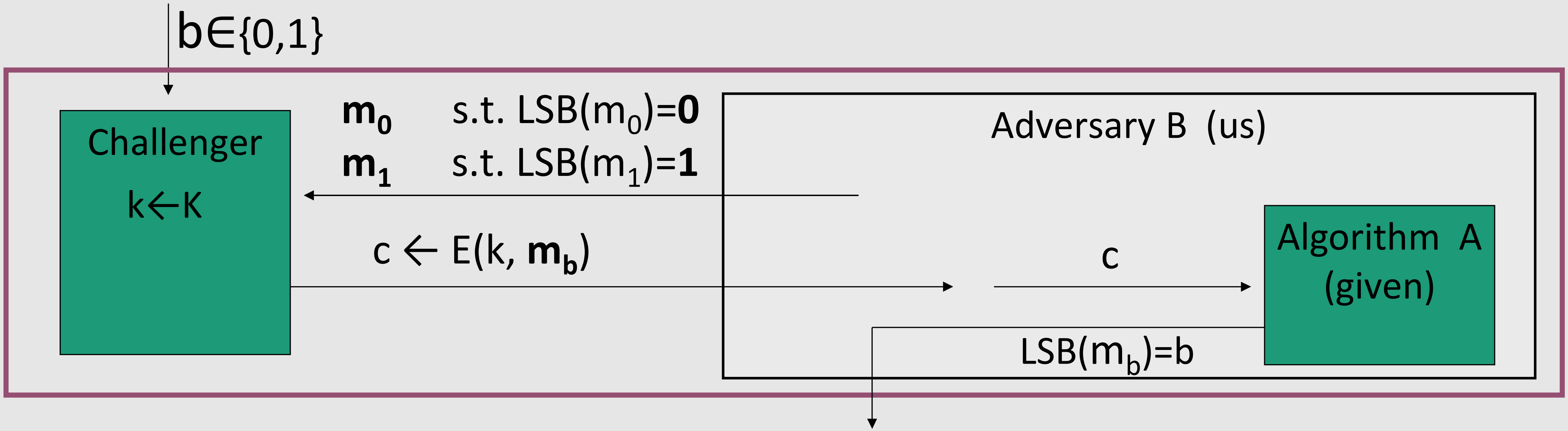
Definition:

Q is **semantically secure** if for all "efficient" **A**,

$$\mathbf{Adv_{SS}[A,Q]\ is\ \text{“negligible”}.}$$

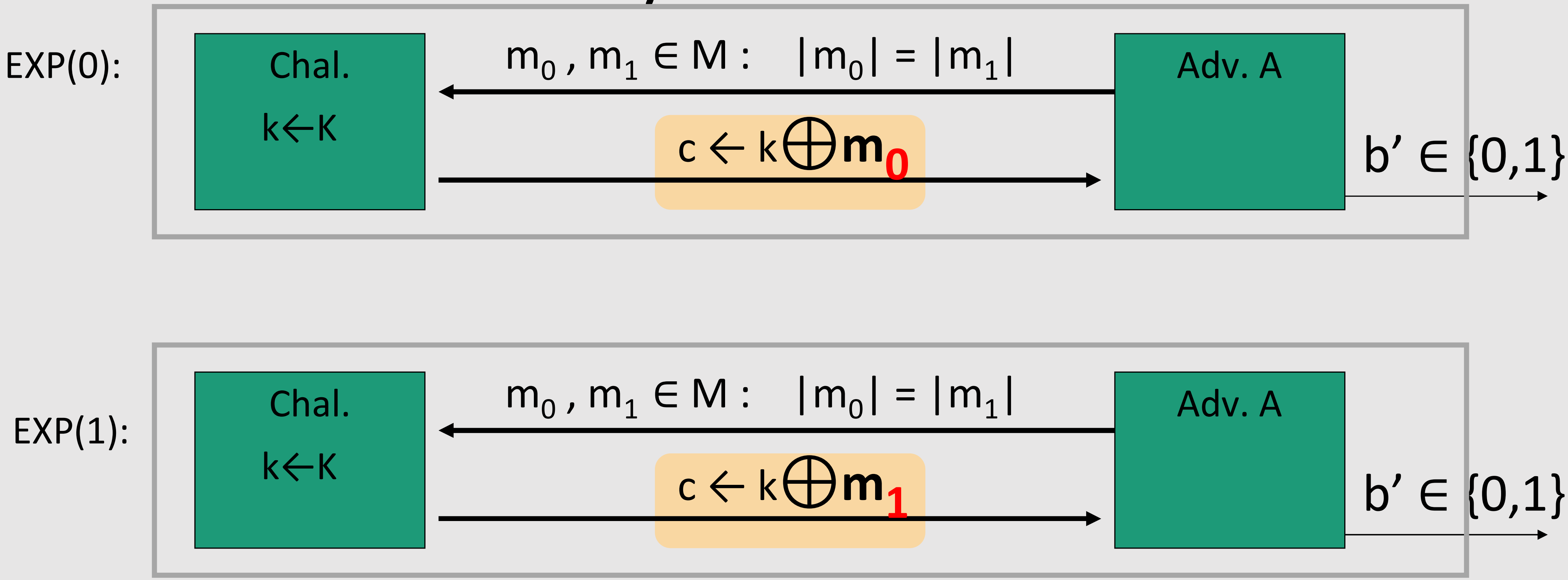
Example

Suppose efficient **A can always deduce LSB of PT from CT**
⇒ **Q** is **not** semantically secure.



Then $\textbf{Adv}_{SS}[B,Q] = \left| \Pr[\,\textbf{EXP(0)}=1\,] - \Pr[\,\textbf{EXP(1)}=1\,] \right| =$

OTP is semantically secure: at home



For **<u>all</u>** A: $\quad Adv_{SS}[A,OTP] = \Big|\, \Pr[\, \mathbf{A(k \oplus m_0)}=1 \,] - \Pr[\, \mathbf{A(k \oplus m_1)}=1 \,] \,\Big| =?$

Stream ciphers are semantically secure

Theorem:

G is a **secure PRG** $\Rightarrow$ stream cipher **Q** derived from G is **semantically secure**

In particular:

$\forall$ semantic security adversary **A**, $\exists$ a PRG adversary **B** (i.e., a statistical test) s.t.

$$\mathbf{Adv_{SS}[A,Q] \le 2 \cdot Adv_{PRG}[B,G]}$$