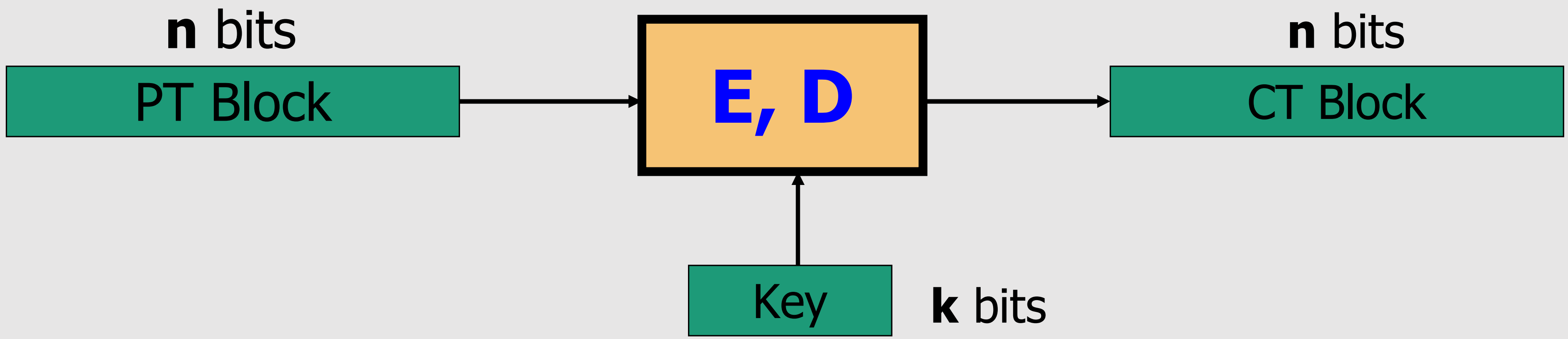
Block Ciphers

Outline

- Block Ciphers
- Pseudo Random Functions (PRFs)
- Pseudo Random Permutations (PRPs)
- DES – Data Encryption Standard
- AES – Advanced Encryption Standard

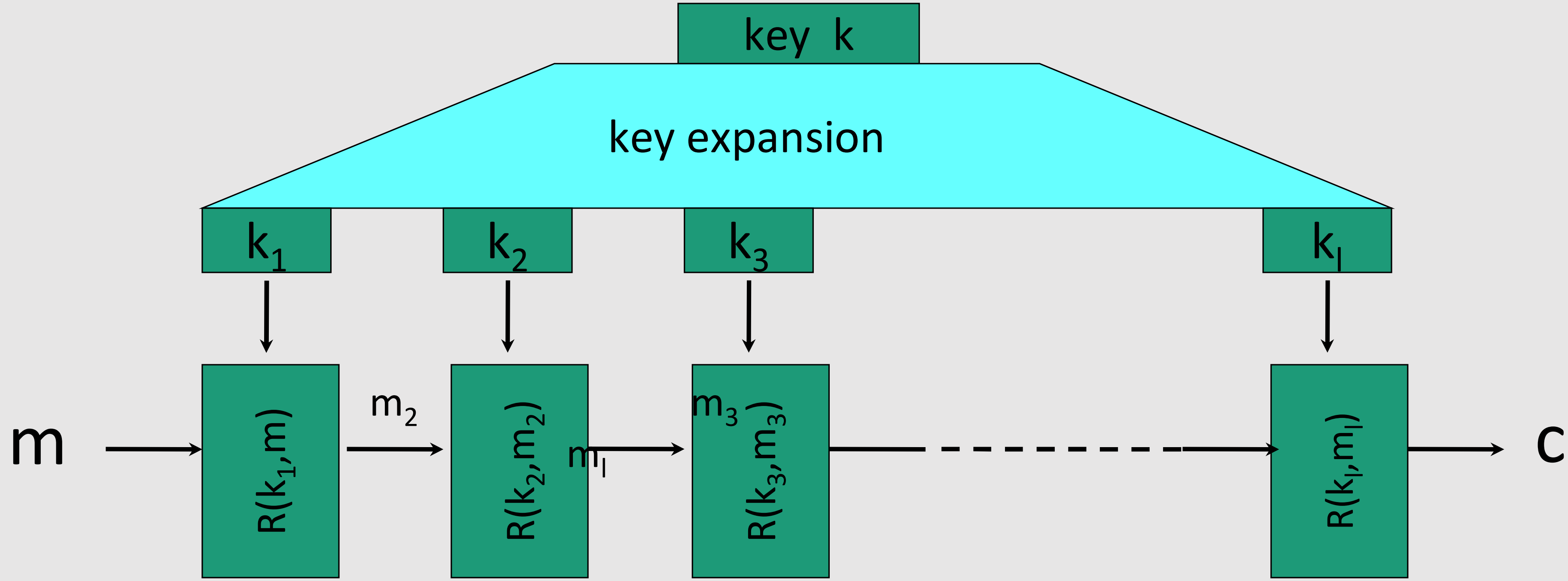
Block Ciphers: crypto work horse



Canonical examples:

- **DES**: $n = 64$ bits, $k = 56$ bits
- **3DES**: $n = 64$ bits, $k = 168$ bits
- **AES**: $n = 128$ bits, $k = 128, 192, 256$ bits

Block Ciphers Built by Iteration



R(k,m) is called a **round function**

for 3DES (l=48), for AES-128 (l=10)

Performance: Crypto++ 5.6.0 [Wei Dai]

AMD Opteron, 2.2 GHz (Linux)

	Cipher	Block/key size	Speed (MB/sec)
stream	RC4		126
	Salsa20/12		643
	Sosemanuk		727
block	3DES	64/168	13
	AES-128	128/128	109

Abstractly: PRPs and PRFs

- **Pseudo Random Function (PRF)** defined over (K,X,Y):

$$F: K \times X \rightarrow Y$$

such that there exists **"efficient" algorithm** to evaluate F(k,x)

- **Pseudo Random Permutation (PRP)** defined over (K,X):

$$E: K \times X \rightarrow X$$

such that:

1. There exists **"efficient"** **deterministic** algorithm to evaluate E(k,x)
2. The function $E(k, \cdot)$ is **one-to-one** (for every k)
3. There exists "efficient" **inversion algorithm** D(k,y)

Running example

- Example PRPs: 3DES, AES, …

  AES: $K \times X \rightarrow X$ where $K = X = \{0,1\}^{128}$

  3DES: $K \times X \rightarrow X$ where $X = \{0,1\}^{64}$, $K = \{0,1\}^{168}$

- Functionally, any **PRP is also a PRF.**
  - A PRP is a PRF where X=Y and is efficiently invertible.

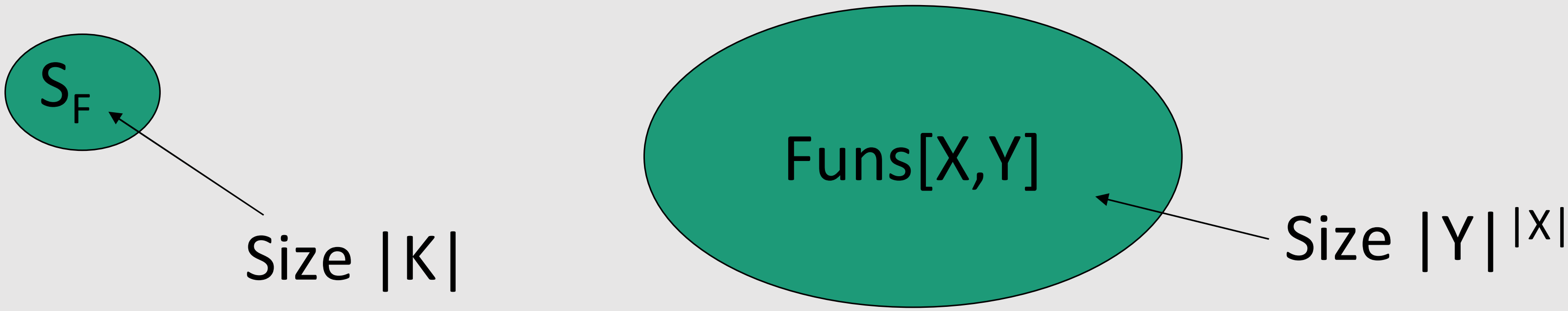
Secure PRFs

- Let $F: K \times X \rightarrow Y$ be a PRF. Set some notation:

  Funs[X,Y]: the set of **all** functions from X to Y

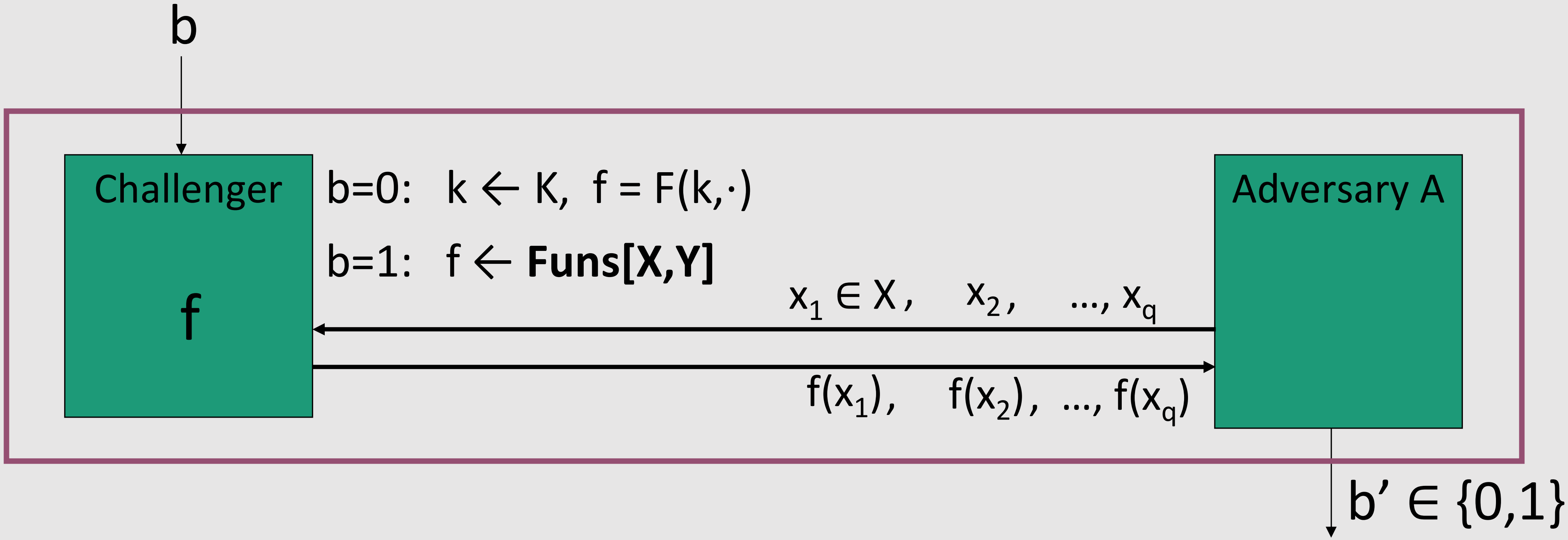
  $S_F = \{ F(k,\cdot) \text{ s.t. } k \in K \} \subseteq$ Funs[X,Y]

- **Intuition:** a PRF is **secure** if a random function in Funs[X,Y] is "indistinguishable" from a random function in S_F



Secure PRF: definition

- Consider a PRF **F : K × X → Y.** For b=0,1 define experiment EXP(b) as:



Definition: F is a **secure PRF** if for all "efficient" adversary A:

$$\mathrm{Adv}_{\mathrm{PRF}}[A,F] := \Big|\Pr[\mathrm{EXP}(0)=1] - \Pr[\mathrm{EXP}(1)=1]\Big| \text{ is "negligible".}$$

Secure PRPs (secure block cipher)

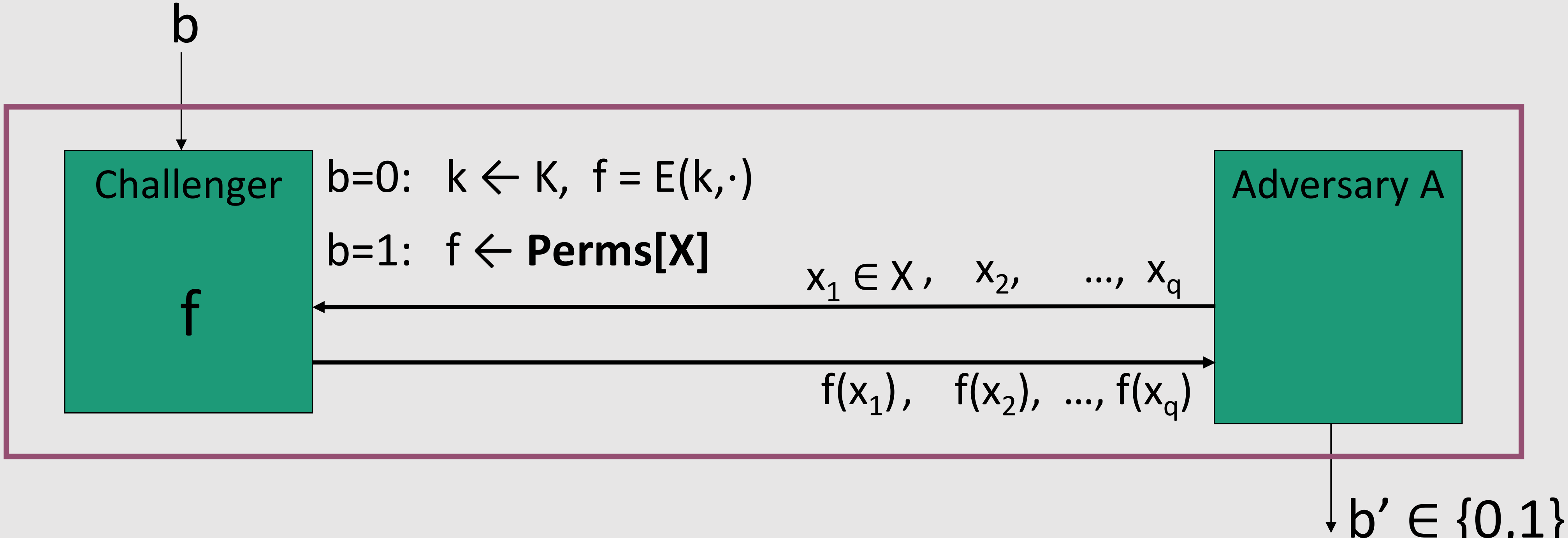
- Let $E: K \times X \rightarrow X$ be a PRP

  $\left\{ \begin{array}{l} \text{Perms[X]: the set of } \textbf{all one-to-one} \text{ functions from X to X} \\ \qquad\qquad\qquad \text{(i.e., } \textbf{permutations}\text{)} \\ \\ S_E = \{ E(k,\cdot) \text{ s.t. } k \in K \} \subseteq \text{Perms[X]} \end{array} \right.$

- **Intuition:** a PRP is **secure** if a random function in Perms[X] is "indistinguishable" from a random function in S_E

Secure PRP (secure block cipher)

- Consider a PRP **E : K × X → X.** For b=0,1 define experiment EXP(b) as:



Definition. E is a **secure PRP** if for all "efficient" adversary A:

$\mathbf{Adv_{PRP}[A,E]} = \left| \mathbf{Pr[EXP(0)=1] - Pr[EXP(1)=1]} \right|$ is "negligible".

Data Encryption Standard (DES)

The Data Encryption Standard (DES)

- Early 1970s: **Horst Feistel** designs Lucifer at IBM

  key-length = 128 bits ; block-length = 128 bits

- 1973: NBS (nowadays called NIST) asks for block cipher proposals.

  IBM submits variant of Lucifer.

- 1976: NBS adopts DES as a federal standard

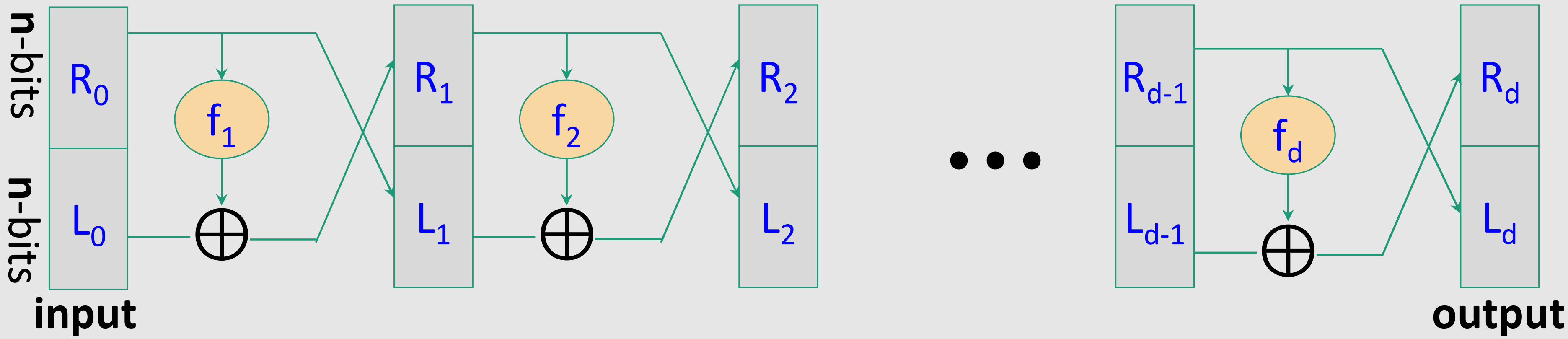
  key-length = 56 bits ; block-length = 64 bits

- 1997: DES broken by exhaustive search (***brute-force attack***)
  - The problem of short keys
- 2000: NIST adopts Rijndael as AES to replace DES

DES: core idea – Feistel Network

Given functions $f_1, \ldots, f_d$: $\{0,1\}^n \longrightarrow \{0,1\}^n$ (not necessarily invertible)

Goal: build an **invertible** function **F**: $\{0,1\}^{2n} \longrightarrow \{0,1\}^{2n}$



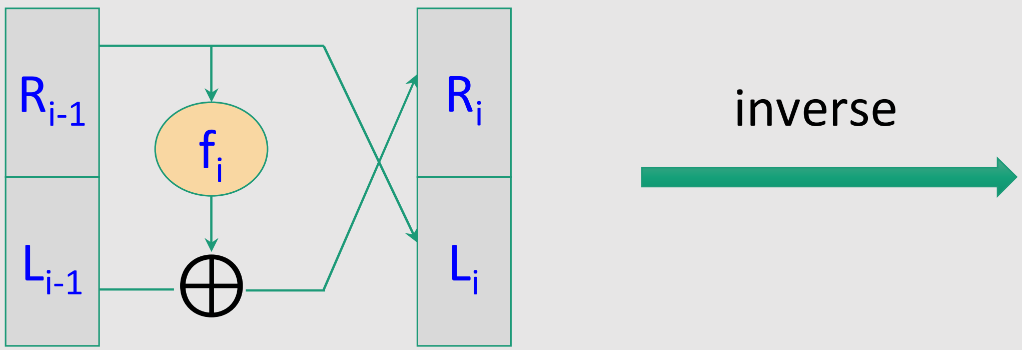
In symbols:
$$R_i = f_i(R_{i-1}) \oplus L_{i-1}$$
$$L_i = R_{i-1}$$

Feistel network is invertible

Claim: for all (**arbitrary**) $f_1, \ldots, f_d$: $\{0,1\}^n \longrightarrow \{0,1\}^n$

Feistel network **F**: $\{0,1\}^{2n} \longrightarrow \{0,1\}^{2n}$ is **invertible**

Proof: construct inverse



$R_{i-1} = L_i$

$L_{i-1} =$

Feistel network is invertible

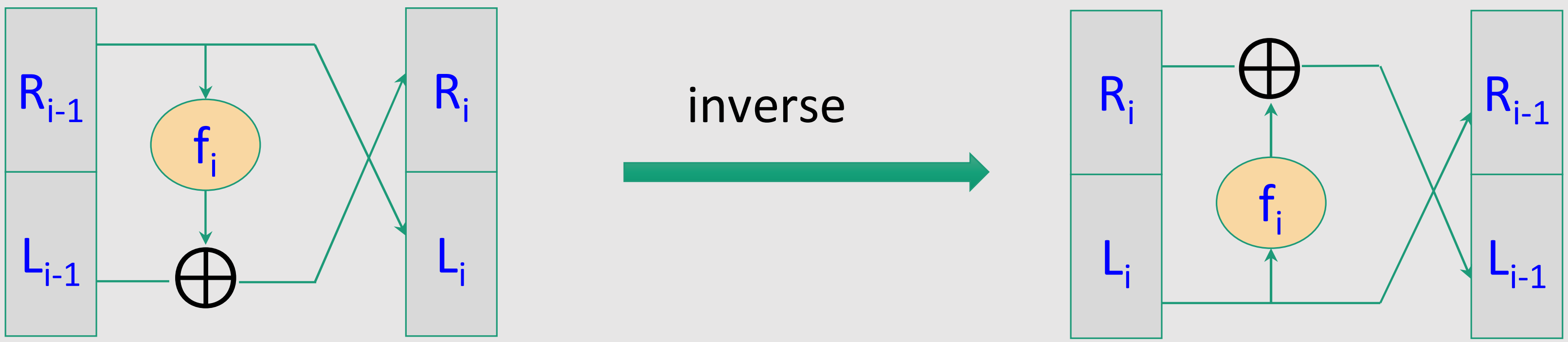
Claim: for all (**arbitrary**) $f_1, \ldots, f_d$: $\{0,1\}^n \longrightarrow \{0,1\}^n$

Feistel network $F: \{0,1\}^{2n} \longrightarrow \{0,1\}^{2n}$ is **invertible**

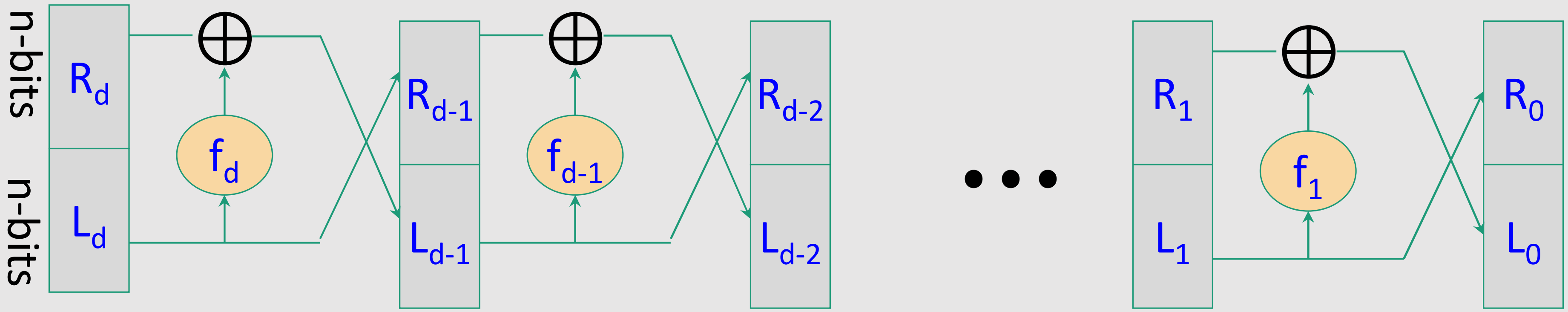
Proof: construct inverse

$R_{i-1} = L_i$

$L_{i-1} = f_i(L_i) \oplus R_i$



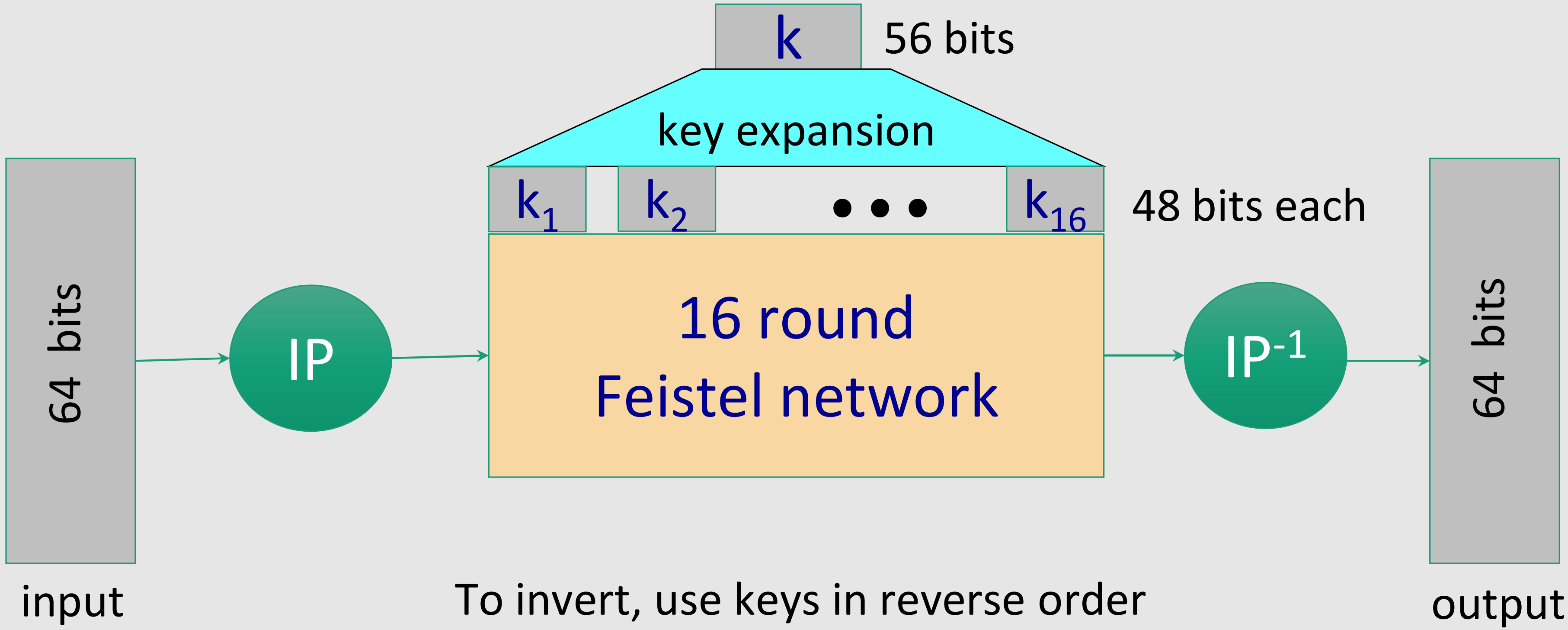
Decryption circuit



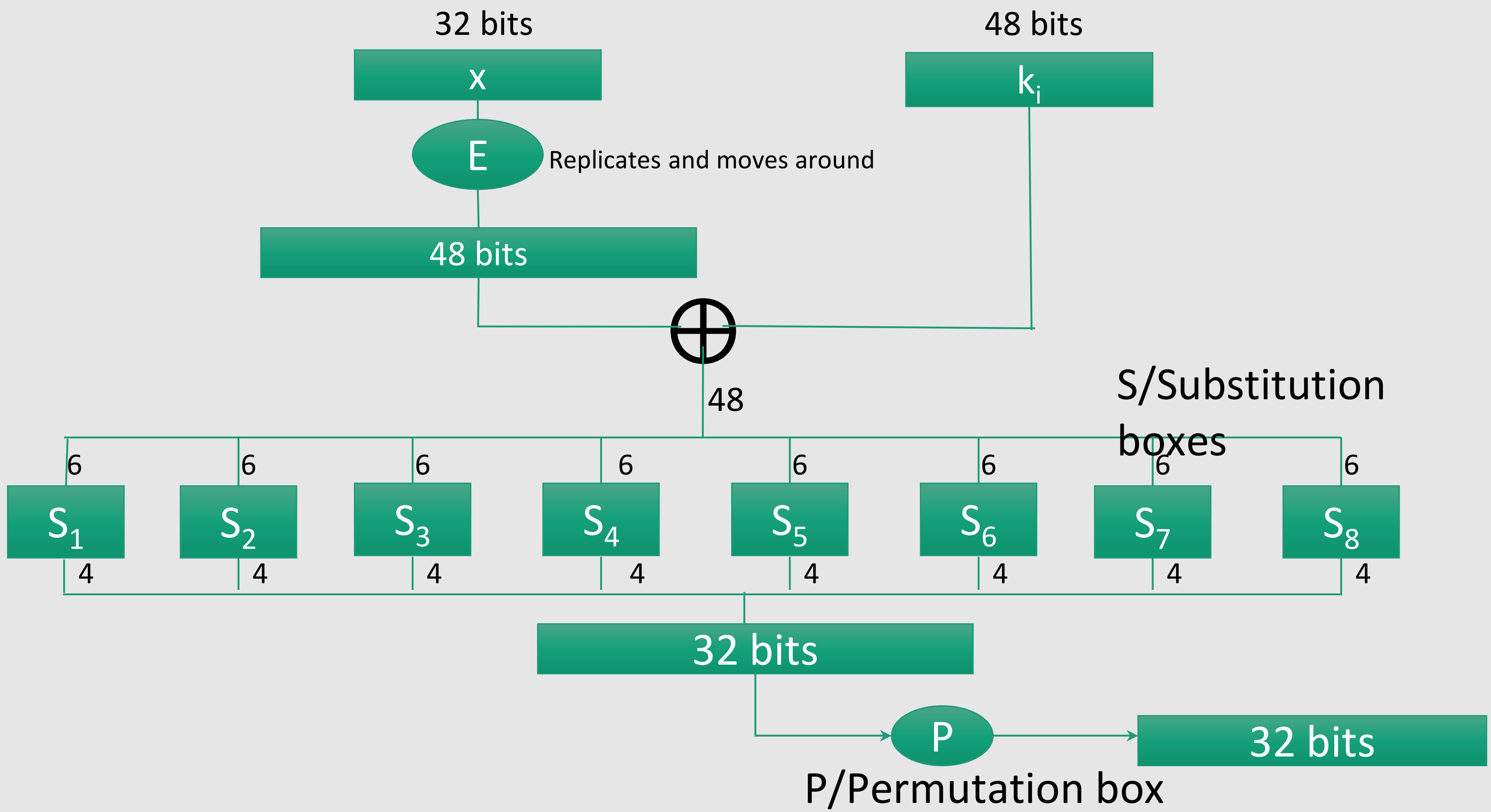
- Inversion is basically the same circuit, with $f_1, \ldots, f_d$ applied in reverse order
- General method for building invertible functions (block ciphers) from arbitrary functions.
- Used in many block ciphers … *but not AES*

DES: 16 round Feistel network

$f_1, \ldots, f_{16}: \ \{0,1\}^{32} \longrightarrow \{0,1\}^{32} \quad , \quad f_i(x) = F(k_i, x)$



The function $f_i(x) = F(k_i, x)$



S-box: function $\{0,1\}^6 \longrightarrow \{0,1\}^4$, implemented as look-up table.

The S-boxes (substitution boxes)

$$S_i: \{0,1\}^6 \longrightarrow \{0,1\}^4$$

S_5		Middle 4 bits of input															
		0000	**0001**	**0010**	**0011**	**0100**	**0101**	**0110**	**0111**	**1000**	**1001**	**1010**	**1011**	**1100**	**1101**	**1110**	**1111**
Outer bits	**00**	0010	1100	0100	0001	0111	1010	1011	0110	1000	0101	0011	1111	1101	0000	1110	1001
	01	1110	1011	0010	1100	0100	0111	1101	0001	0101	0000	1111	1010	0011	1001	1000	0110
	10	0100	0010	0001	1011	1010	1101	0111	1000	1111	1001	1100	0101	0110	0011	0000	1110
	11	1011	1000	1100	0111	0001	1110	0010	1101	0110	1111	0000	1001	1010	0100	0101	0011

$$S_5(0\overbrace{1101}^{\text{the column}}1) \longrightarrow 1001$$

the row

Choosing the S-boxes and P-box

- Choosing the S-boxes and P-box at random would result in an insecure block cipher (*key recovery after ≈2^{24} outputs*)
- Several rules are used in the choice of S and P boxes:
  - No output bit should be close to a linear func. of the input bits
  - S-boxes are 4-to-1 maps (4 pre-images for each output)
  - …

Exhaustive Search for block cipher key

Goal: given a few input output pairs $\big(m_i,\ c_i = E(k, m_i)\big)$ i=1,..,3
find key k.

Exhaustive Search for block cipher key

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find key k.

Lemma: Suppose DES is an ***ideal cipher***
(2^{56} random invertible functions $\Pi_1, \ldots, \Pi_{2\text{^}56} : \{0,1\}^{64} \to \{0,1\}^{64}$)

Then $\forall$ m, c there is at most **<u>one</u>** key k s.t. c = DES(k, m)

with prob. $\geq 1 - 1/256 \approx 99.5\%$

Proof:

$\Pr[\ \exists\ k' \neq k\!: c=DES(k,m)=DES(k',m)] \ \leq\ \sum_{k' \in \{0,1\}^{56}} \Pr[DES(k,m) = DES(k',m)] \ \leq$
$2^{56} \times 1/(2^{64}) = 1/(2^8) = 1/256$

Exhaustive Search for block cipher key

For two DES pairs $\big(m_1, c_1{=}DES(k, m_1)\big), \big(m_2, c_2{=}DES(k, m_2)\big)$

unicity prob. $\approx 1 - 1/2^{71}$

For AES-128: given two input/output pairs, unicity prob. $\approx 1 - 1/2^{128}$

$\Rightarrow$ two input/output pairs are enough for exhaustive key search.

Exhaustive Search Attacks

DES challenge

8 byte blocks, encrypted with the same key

m_1 m_2 m_3

msg = "`The unknown messages are: XXXX` ... "

CT = c_1 c_2 c_3 c_4

Goal: find $k \in \{0,1\}^{56}$ s.t. $DES(k, m_i) = c_i$ for $i=1,2,3$ and decrypt $c_4, c_5 \ldots$

1997: Internet search -- **3 months**

1998: EFF machine (deep crack) -- **3 days** (250K $)

1999: combined search -- **22 hours**

2006: COPACOBANA (120 FPGAs) **-- 7 days** (10K $)

$\Rightarrow$ 56-bit ciphers should not be used !!

Strengthening DES against exhaustive search

- Method 1: **Triple-DES**
- Method 2: **DESX**
- General construction that can be applied to other block ciphers as well.

Triple DES

- Consider a **block cipher**

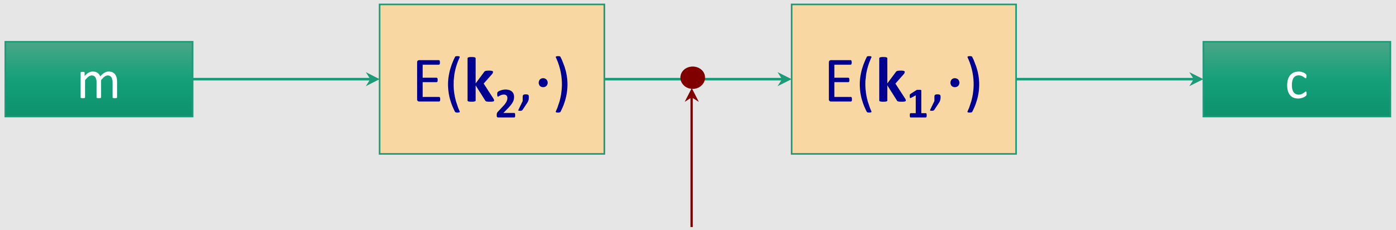
  E : $K \times M \longrightarrow M$

  D : $K \times M \longrightarrow M$
- Define **3E: $K^3 \times M \longrightarrow M$** as

  $$\mathbf{3E\ (k_1,k_2,k_3,\ m) = E(k_1,\ D(k_2,\ E(k_3,m)))}$$
- For **3DES** (or **Triple DES**)
  - **key length** = 3×56 = **168 bits**.
  - **3×slower** than DES.
  - $k_1=k_2=k_3 \Rightarrow$ **single DES**
  - **simple attack in time $\approx 2^{118}$** (more on this later …)

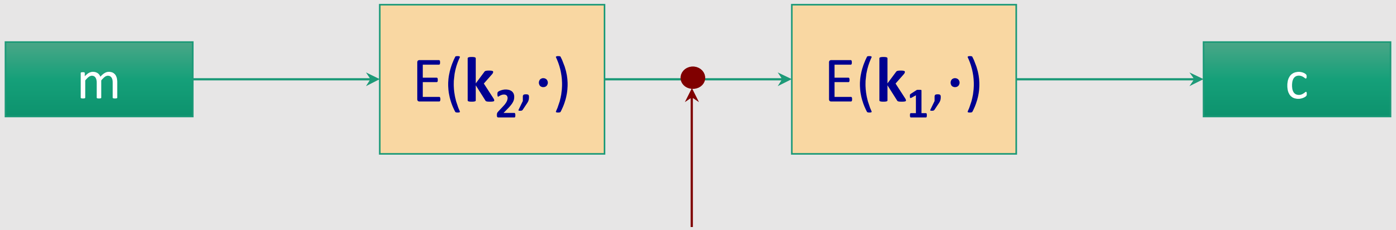
Why not double DES?

- Given a block cipher **E**, define **2E(k_1, k_2, m) = E(k_1 , E(k_2 , m))**
- **Double DES:** **2DES(k_1, k_2, m) = E(k_1 , E(k_2 , m))**
  key-length = 112 bits for 2DES
- **Attack:** Given **m** and **c** the goal is to
  find (k_1,k_2) s.t. **E(k_1, E(k_2,m)) = c** **or equivalently**
  find (k_1,k_2) s.t. **E(k_2,m) = D(k_1,c)**



Meet in the middle (MITM) attack

- **Attack:** Given **m** and **c** the goal is to
  find (k_1,k_2) s.t. $\mathbf{E(k_1, E(k_2,m)) = c}$ **or equivalently**
  find (k_1,k_2) s.t. $\mathbf{E(k_2,m) = D(k_1,c)}$



- **Attack involves TWO STEPS**

Meet in the middle attack

Step 1:

- build table.
- sort on 2nd column

$k^0 = 00\ldots00$	$E(k^0, m)$
$k^1 = 00\ldots01$	$E(k^1, m)$
$k^2 = 00\ldots10$	$E(k^2, m)$
$\vdots$	$\vdots$
$k^N = 11\ldots11$	$E(k^N, m)$

2^{56} entries

Meet in the middle attack

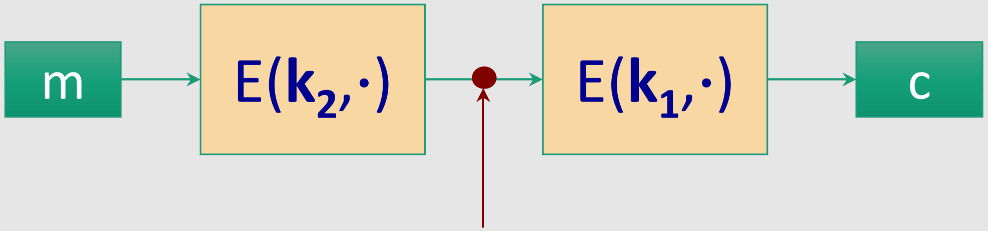
Step 2:

- for each $k \in \{0,1\}^{56}$ do:

  test if $D(k, c)$ is in the 2^{nd} column of the table

  If so, then $\mathbf{E(k^i,m) = D(k,c)} \quad \Rightarrow \quad \mathbf{(k^i,k) = (k_2,k_1)}$

$k^0 = 00\ldots00$	$E(k^0, m)$
$k^1 = 00\ldots01$	$E(k^1, m)$
$\vdots$	$\vdots$
$k^i = 00\ldots\ldots.$	$E(k^i, m)$
$\vdots$	$\vdots$
$k^N = 11\ldots11$	$E(k^N, m)$



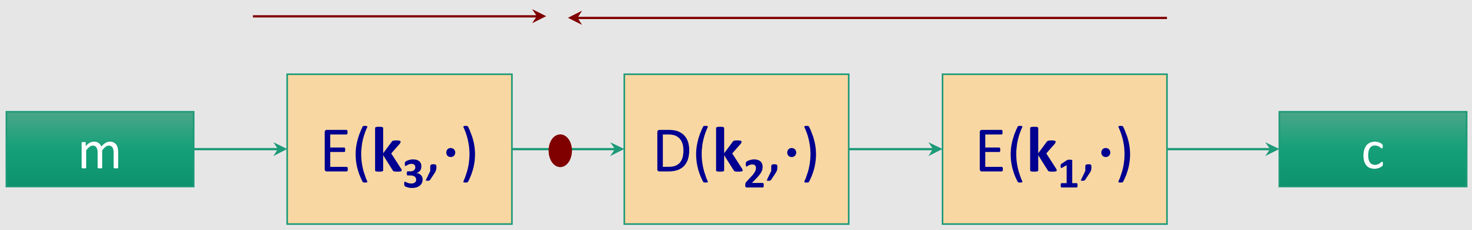
Meet in the middle attack

$$\text{Time} = \underbrace{2^{56}\log(2^{56})}_{\text{build + sort table}} + \underbrace{2^{56}\log(2^{56})}_{\text{search in table}} < 2^{63} \ll 2^{112} \ ,$$

$$\text{Space} \approx 2^{56}$$

Meet in the middle attack

Same attack on 3DES:



Time = 2^{118} , space ≈ 2^{56}

$$\text{Time} = \underbrace{2^{56}\log(2^{56})}_{\text{build + sort table}} + \underbrace{2^{112}\log(2^{56})}_{\text{search in table}} < 2^{118}$$

DESX

- Consider a **block cipher**

  $\mathbf{E} : K \times M \longrightarrow M$

  $\mathbf{D} : K \times M \longrightarrow M$

- Define **EX** as

$$\mathbf{EX}(k_1, k_2, k_3, m) = k_1 \oplus \mathbf{E}(k_2, m \oplus k_3)$$

- For **DESX**
  - key-len = **64**+**56**+**64** = 184 bits $\qquad \mathbf{k_1 \oplus E(k_2, m \oplus k_3)}$
  - … but easy attack in time $2^{64+56} = 2^{120}$

- Note: $k_1 \oplus E(k_2, m)$ and $E(k_2, m \oplus k_1)$ insecure !!

  (XOR outside) or (XOR inside) ⇒ As weak as E w.r.t. exhaustive search

Few other attacks on block ciphers

Linear attacks on DES

A tiny bit of linearly in S_5 lead to a 2^{43} time attack.

Total attack time $\approx 2^{43}$ ($<< 2^{56}$) with 2^{42} random inp/out pairs

Quantum attacks

Generic search problem:

Let $f: X \longrightarrow \{0,1\}$ be a function.

Goal: find $x^* \in X$ s.t. $f(x^*)=1$.

Classical computer: best generic algorithm **time = O(|X|)**

Quantum computer [Grover '96] : **time = O($|X|^{1/2}$)**

Quantum exhaustive search

Given **m** and **c = E(k,m)** define

$$\text{For } k \in K,\ \ f(k) = \begin{cases} 1 & \text{if } E(k,m) = c \\ 0 & \text{otherwise} \end{cases}$$

Grover $\Rightarrow$ quantum computer can find k in time $O(\ |K|^{1/2}\ )$

DES: time $\approx 2^{28}$, AES-128: time $\approx 2^{64}$

Quantum computer $\Rightarrow$ 256-bits key ciphers (e.g., AES-256)

Advanced Encryption Standard (AES)

The AES process

- 1997: **NIST** (the National Institute of Standards and Technology, https://www.nist.gov/) publishes a request for proposals.
  - **Goal**: replace DES for both government and private-sector encryption.
  - The algorithm must implement symmetric key cryptography as a block cipher and (at a minimum) support block sizes of 128- bits and key sizes of 128-, 192- , and 256-bits.
- 1998: NIST selects 15 AES candidate algorithms.
- 1999: NIST chooses 5 finalists
- 2000: NIST chooses Rijndael as AES (designed in Belgium)
  - Key sizes: 128, 192, 256 bits. Block size: 128 bits

AES

- Is <u>not</u> a Feistel cipher.
  - It works somehow in parallel over the whole input block.
- Designed to be efficient both in hardware and software across a variety of platforms.
- Is a block cipher which works *iteratively*
  - Block size: 128 bits (but also 192 or 256 bits)
  - Key length: 128, 192, or 256 bits
  - Number of rounds: 10, 12 o 14
  - Key scheduling: 44, 52 or 60 subkeys having length = 32 bits
- Each **round** (except the last one) is a uniform and parallel composition of 4 steps
  - **SubBytes** (byte-by-byte substitution using an S-box)
  - **ShiftRows** (a permutation, which cyclically shifts the last three rows in the State)
  - **MixColumns** (substitution that uses Galois Fields, corps de Galois, GF(2^8) arithmetic)
  - **Add Round key** (bit-by-bit XOR with an expanded key)

AES Parameters

- Block size: 128 bits (Nb=4 words)
- 1 word = 32 bit

	Key Length *(Nk words)*	Block Size *(Nb words)*	Number of Rounds *(Nr)*
AES-128	4	4	10
AES-192	6	4	12
AES-256	8	4	14

AES keys

With 128 bits: 2^{128} = $3.4x\ 10^{38}$ possible keys

- A PC that tries 2^{55} keys per second needs 149.000 billion years to break AES

With 192 bits: 2^{192} = $6.2x\ 10^{57}$ possible keys

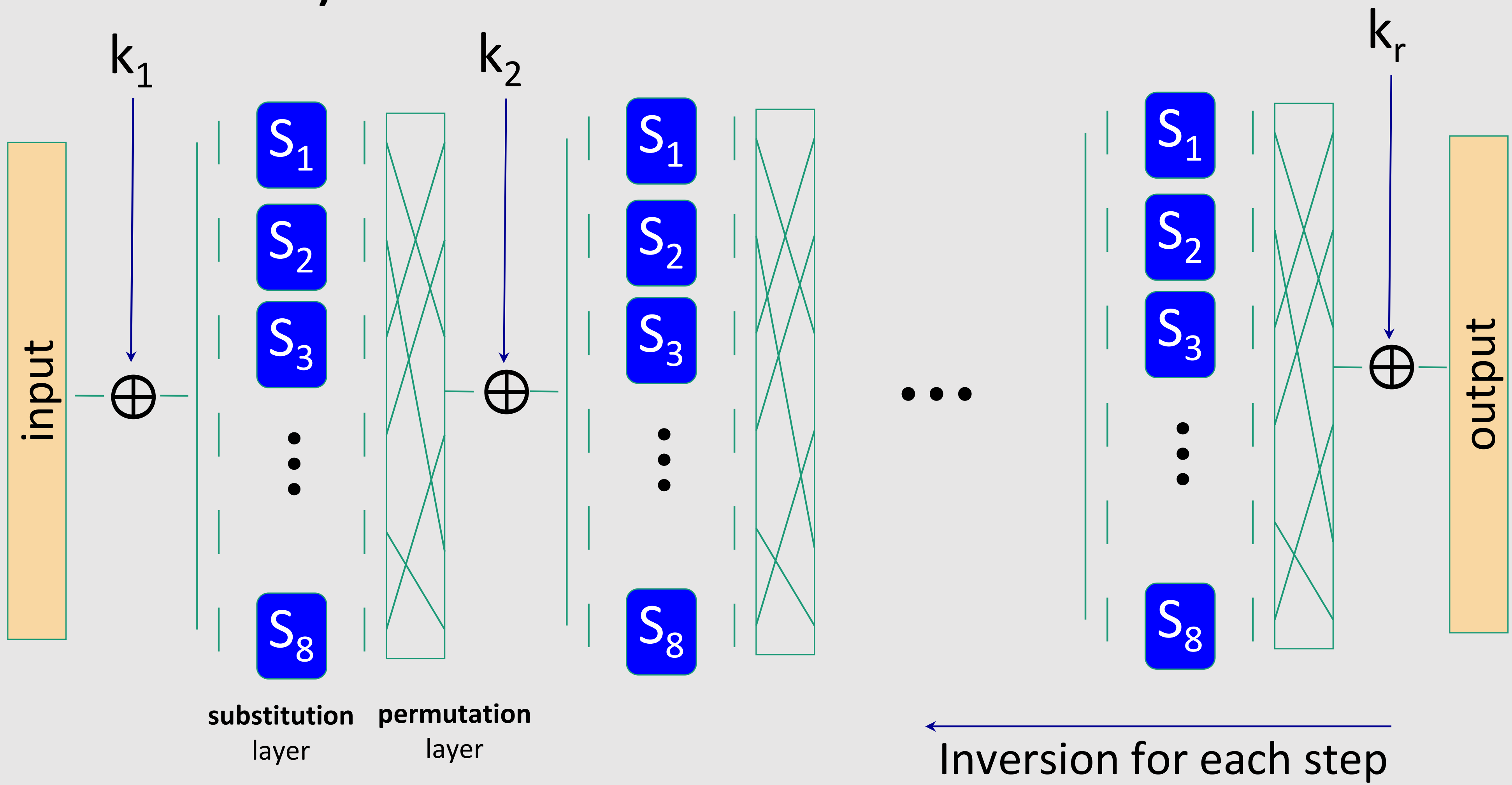
- …

With 256 bits: 2^{256} = $1.1x\ 10^{77}$ possible keys

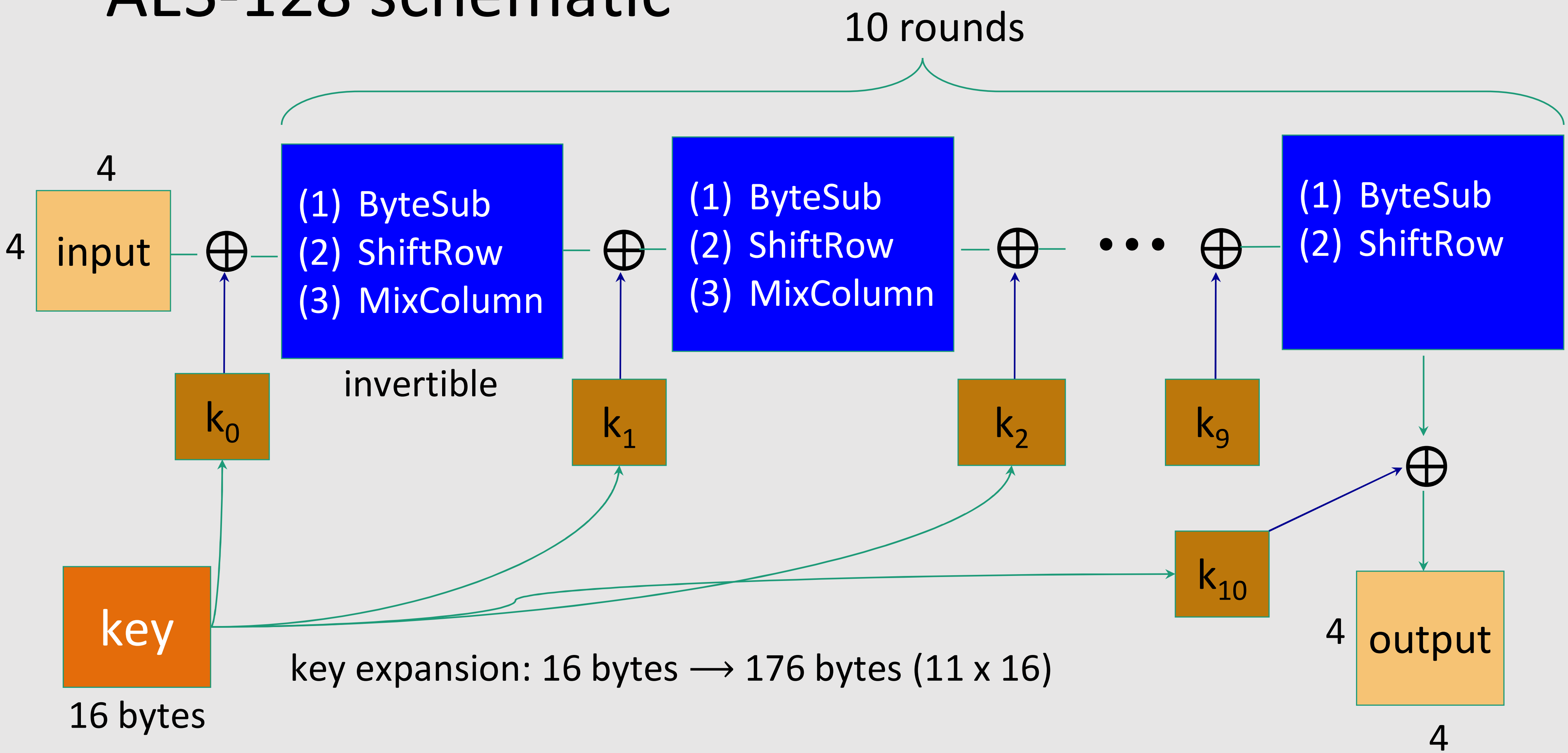
- …

Probably AES will stay secure for at least 20 years

AES is a Substitution–permutation Network (not Feistel)



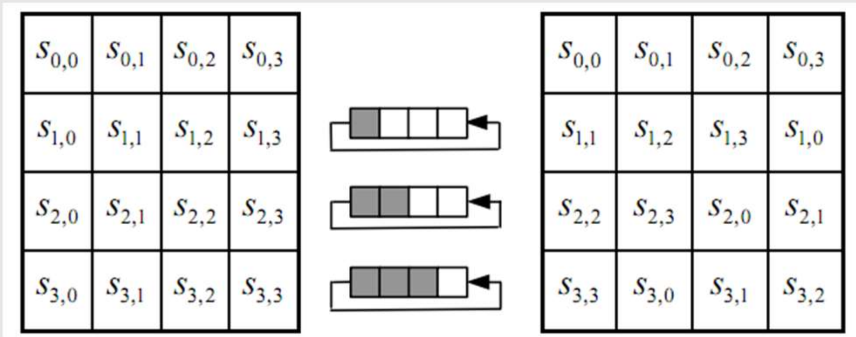
AES-128 schematic



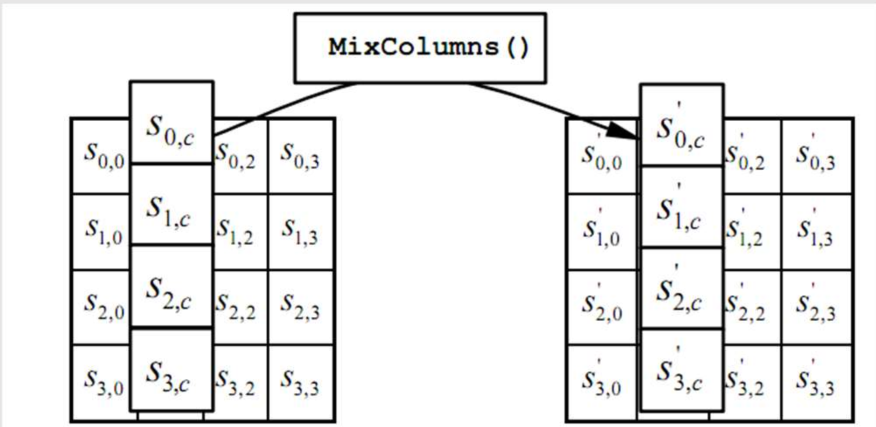
The round function

- **ByteSub**: a 1 byte S-box. 256 byte table (easily computable)
  - Apply S-box to each byte of the 4x4 input A, i.e., A[i,j] = S[A[i,j]], for 1 ≤i,j≤4

- **ShiftRows**:



- **MixColumns**:



Attacks

- Best key recovery attack:

  four times better than ex. search [BKR'11]

- Related key attack on AES-256:   [BK'09]

  Given 2^{99} inp/out pairs from **four related keys** in AES-256

  can recover keys in time $\approx 2^{99}$