
ONLAY: ONLINE LAYERING FOR SCALABLE ASYNCHRONOUS BFT SYSTEM

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ABSTRACT

This paper presents a new framework, namely ONLAY, for scalable asynchronous distributed systems. In this framework, we propose a consensus protocol L_ϕ , which is based on the Lachesis protocol [1].

At the core of L_ϕ protocol, it introduces to use layering algorithm to achieve practical Byzantine fault tolerance (pBFT) in leaderless asynchronous Directed Acyclic Graph (DAG). Further, we present new online layering algorithms for the evolutionary DAGs across the nodes. Our new protocol achieves deterministic scalable consensus in asynchronous pBFT by using assigned layers and asynchronous partially ordered sets with logical time ordering instead of blockchains. The partial ordering produced by L_ϕ is flexible but consistent across the distributed system of nodes.

We then present the formal model of our layering-based consensus. The model is generalized that can be applied to abstract asynchronous DAG-based distributed systems.

Keywords Consensus algorithm · Byzantine fault tolerance · Online Layering · Layer assignment · Partially ordered sets · Lachesis protocol · OPERA chain · Lamport timestamp · Main chain · Root · Clotho · Atropos · Distributed Ledger · Blockchain

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1 Introduction

Following the widespread success over cryptocurrencies, recent advances in blockchain and its successors have offered secure decentralized consistent transaction ledgers in numerous domains including financial, logistics as well as health care sectors. There have been extensive work [2, 3, 4, 5] addressing some known limitations, such as long consensus confirmation time and high power consumption, of blockchain-powered distributed ledgers.

In distributed database systems, *Byzantine* fault tolerance (BFT) [6] addresses the reliability of the system when up to a certain number (e.g., one-third) of the participant nodes may be compromised. Consensus algorithms [7] ensures the integrity of transactions between participants over a distributed network [6] and is equivalent to the proof of BFT in distributed database systems [8, 9].

For deterministic, completely asynchronous system, Byzantine consensus is not guaranteed with unbounded delays [10]. Though achieving consensus is completely feasible for nondeterministic system. In practical Byzantine fault tolerance (pBFT), all nodes can successfully reach a consensus for a block in the presence of a Byzantine node [11]. Consensus in pBFT is reached once a created block is shared with other participants and the share information is further shared with others [12, 13].

There have been extensive research in consensus algorithms. Proof of Work (PoW) [14], used in the original Nakamoto consensus protocol in Bitcoin, requires exhaustive computational work from participants for block generation. Proof Of Stake (PoS) [15, 16] uses participants' stakes for generating blocks. With recent technological advances and innovations, consensus algorithms [2, 3, 4, 5] have addressed to improve the consensus confirmation time and power consumption over blockchain-powered distributed ledgers. These approaches utilize directed acyclic graphs (DAG) [17, 4, 5, 18, 19] to facilitate consensus. Examples of DAG-based consensus algorithms include Tangle [20], Byteball [21], and Hashgraph [22]. Lachesis protocol [1] presents a general model of DAG-based consensus protocols.

1.1 Motivation

Lachesis protocols, as introduced in our previous paper [1], is a set of consensus protocols that create a directed acyclic graph for distributed systems. We introduced a Lachesis consensus protocol, called L_0 , which is a DAG-based asynchronous non-deterministic protocol that guarantees pBFT. L_0 generates each block asynchronously and uses the OPERA chain (DAG) for faster consensus by confirming how many nodes share the blocks.

In BFT systems, an synchronous approach is to use a broadcast voting approach and ask each node to vote on the validity of each block. Instead, we aim for an asynchronous system where we leverage the concepts of distributed common knowledge and network broadcast to achieve a local view with high probability of being a consistent global view. Each node receives transactions from clients and then batches them into an event block. The new event block is then communicated with other nodes through asynchronous event transmission. During communication, nodes share their own blocks as well as the ones they received from other nodes. Consequently, this spreads all information through the network. The process is asynchronous and thus it can increase throughput near linearly as nodes enter the network.

Lachesis protocols L_0 [1] and L_1 [23] proposed new approaches which are better than the previous DAG-based approaches. However, both protocols L_0 and L_1 have posed some limitations in their algorithms, which can be made simpler and more reliable.

In this paper we are interested in a new consensus protocol that is more reliable to address pBFT in asynchronous scalable DAG. Specifically, we are investigating to use the notion of *graph layering* and *hierarchical graph* in graph theory to develop an intuitive model of consensus.

Let $G=(V,E)$ be a directed acyclic graph. In graph theory, a layering of G is a topological numbering ϕ of G that maps each vertex $v \in V$ of G to an integer $\phi(v)$ such that $\phi(v) \geq \phi(u) + 1$ for every directed edge $(u,v) \in E$. That is, a vertex v is laid at layer j , if $\phi(v)=j$, and the j -th layer of G is $V_j = \phi^{-1}(j)$. In other words, a layering ϕ of G partitions the set of vertices V into a finite number l of non-empty disjoint subsets (called layers) V_1, V_2, \dots, V_l , such that $V = \cup_{i=1}^l V_i$, and for every edge $(u,v) \in E$, $u \in V_i$, $v \in V_j$, $1 \leq i < j \leq l$. For a layering ϕ , $H=(V,E,\phi)$ is called a hierarchical (layered or levelled) graph. An l -layer hierarchical graph can be represented as $H=(V_1, V_2, \dots, V_l; E)$.

Figure 1(a) shows an example of block DAG, which is a local snapshot of a node in a three-node network. Figure 1(b) depicts the result of layering applied on the DAG. In the figure, there are 14 layers. Vertices of the same layer are horizontally aligned. Every edge points from a vertex of a higher layer to a vertex of a lower layer. The hierarchical graph depicts a clear structure of the DAG, in which the dependency between blocks are shown uniformly from top to bottom.

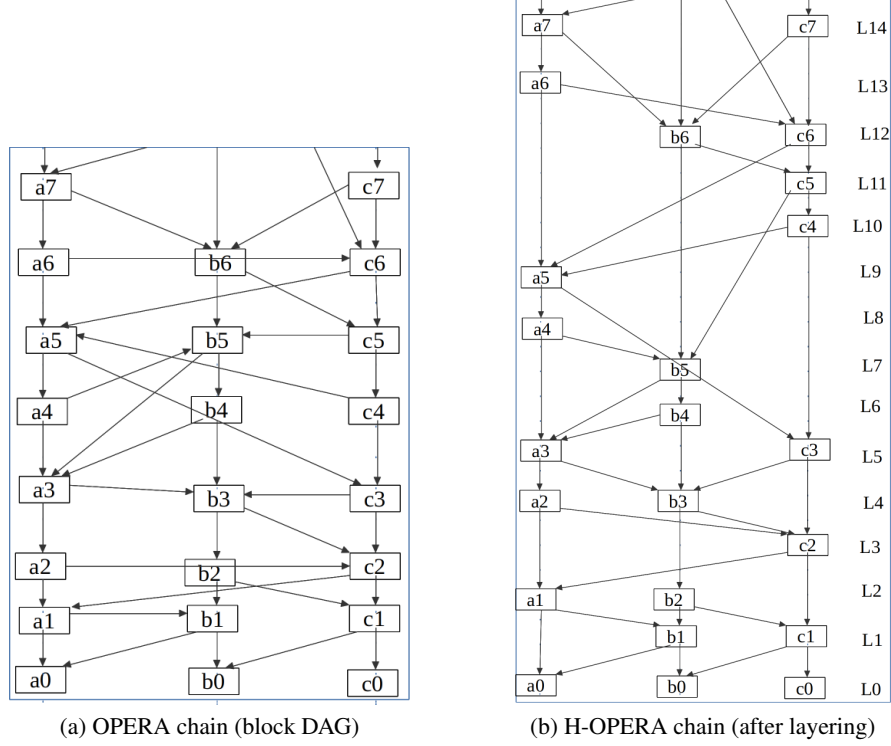


Figure 1: An example of layered OPERA chain

The layering gives better structure of DAG. Thus, we aim to address the following questions: 1) Can we leverage the concepts of layering and hierarchical graph on the block DAGs? 2) Can we extend the layering algorithms to handle block DAGs that are evolving over time, i.e., on block creation and receiving? 3) Can a model use layering to quickly compute and validate the global information and help quickly search for Byzantine nodes within the block DAG? 4) Is it possible to use hierarchical graph to reach consensus of the partial ordering of blocks at finality across the nodes?

1.2 ONLAY framework

In this paper, we present ONLAY, a new framework for asynchronous distributed systems. ONLAY leverages asynchronous event transmission for practical Byzantine fault tolerance (pBFT), similar to our previous Lachesis protocol [1]. The core idea of ONLAY is to create a leaderless, scalable, asynchronous DAG. By computing asynchronous partially ordered sets with logical time ordering instead of blockchains, ONLAY offers a new practical alternative framework for distributed ledgers.

Specifically, we propose a consensus protocol L_ϕ , which is based on Lachesis protocol [1]. In particular, L_ϕ protocol introduces an online layering algorithm to achieve practical Byzantine fault tolerance (pBFT) in leaderless DAG. The protocol achieves deterministic scalable consensus in asynchronous pBFT by using assigned layers and asynchronous partially ordered sets with logical time ordering instead of blockchains. The partial ordering produced by L_ϕ is flexible but consistent across the distributed system of nodes.

We then present a formal model for the L_ϕ protocol that can be applied to abstract asynchronous DAG-based distributed system. The formal model is built upon the model of current common knowledge (CCK) [24].

Figure 2 shows the overview of our ONLAY framework. In the framework, each node stores and maintain its own DAG. There are multiple steps, including layering, selecting roots, assigning frames, selecting Clothos and Atropos, and topologically ordering event blocks to determine consensus. The major step of the framework compared to previous approaches is the layering step.

The main concepts of ONLAY are given as follows:

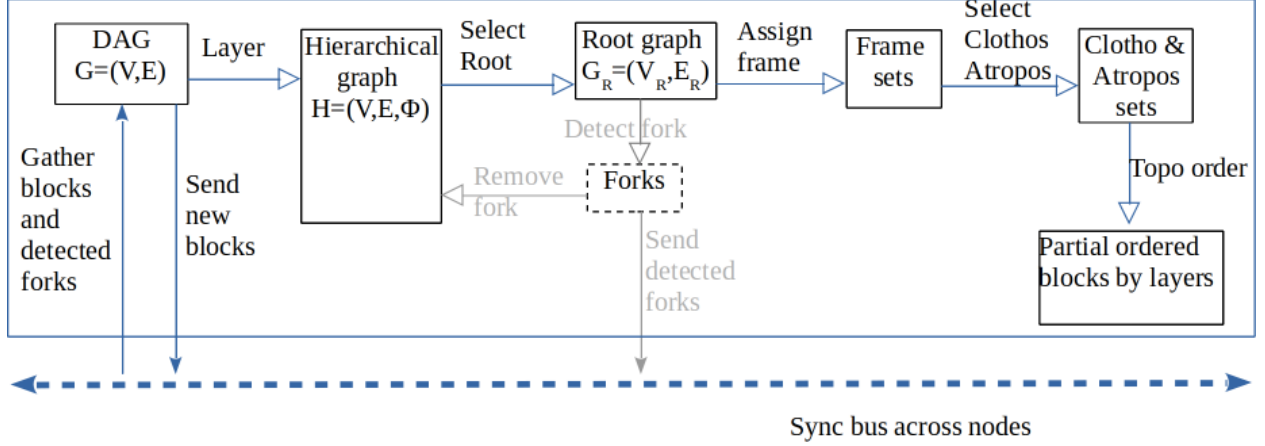


Figure 2: An overview of ONLAY framework

- **Event block** An event block is a holder of a set of transactions created by a node and is then transported to other nodes. Event block includes signature, timestamp, transaction records and referencing hashes to previous (parent) blocks.
- **L_ϕ protocol** sets the rules for event creation, communication and reaching consensus in ONLAY.
- **OPERA chain** is the local view of the DAG held by each node. This local view is used for determine consensus.
- **Layering** Layering assigns every block in OPERA chain a number so that every edge only points from high to low layers.
- **L-OPERA chain** The L-OPERA chain is a hierarchical graph obtained from layering the DAG held by each node.
- **Lamport timestamp** An event block x is said *Happened-before* y , if y was created after x was created. *Lamport timestamp* is a timestamp assigned to each event block by a logical clock of a node. It is used to determine a partial order amongst the blocks.
- **Root** An event block is called a *root* if either (1) it is the first generated event block of a node, or (2) it can reach more than two-thirds of other roots. A *root set* R_s contains all the roots of a frame. A *frame*, denoted by f , is a natural number that is assigned to Root sets (and dependent event blocks).
- **Root graph** Root graph contains roots as vertices and reachability between roots as edges.
- **Clotho** A Clotho is a root satisfying that it is known by more than $2n/3$ nodes and more than $2n/3$ nodes know that information.
- **Atropos** An Atropos is a Clotho that is assigned with a consensus time.
- **ONLAY chain** ONLAY chain is the main subset of the L-OPERA chain. It contains Atropos blocks and the sub-graphs reachable from those Atropos blocks.

1.3 Contributions

In summary, this paper makes the following contributions:

- We propose a new scalable framework, so-called ONLAY, aiming for practical DAG-based distributed ledgers.
- We introduce a novel consensus protocol L_ϕ , which uses layer algorithm and root graphs, for faster root selection. L_ϕ protocol uses layer assignment on the DAG to achieve deterministic and thus more reliable consensus.
- We define a formal model using continuous consistent cuts of a local view to achieve consensus via layer assignment.
- We formalize our proofs that can be applied to any generic asynchronous DAG solution.

1.4 Paper structure

The rest of this paper is organised as follows. Section 2 presents some preliminaries and background for our ONLAY framework. Section 3 introduces our L_ϕ consensus protocol. The section describes our consensus algorithm that uses layering algorithm to achieve a more reliable and scalable solution to the consensus problem in BFT systems. Section 4 presents important details of our ONLAY framework. Section 5 concludes. Proof of Byzantine fault tolerance is described in Section 6.

2 Preliminaries

This section presents some background information as well as related terminologies used in ONLAY framework.

2.1 Basic Definitions

The L_ϕ protocol sets rules for all nodes representing client machines that forms a network. In L_ϕ , a (participant) node is a server (machine) of the distributed system. Each node can create messages, send messages to, and receive messages from, other nodes. The communication between nodes is asynchronous.

[Node] Each machine that participating in the protocol is called a *node*. Let n_i denote the node with the identifier of i . Let n denote the total number of nodes.

[Process] A process p_i represents a machine or a *node*. The process identifier of p_i is i . A set $P = \{1, \dots, n\}$ denotes the set of process identifiers.

[Channel] A process i can send messages to process j if there is a channel (i, j) . Let $C \subseteq \{(i, j) \text{ s.t. } i, j \in P\}$ denote the set of channels.

The basic units of the protocol are called event blocks - a data structure created by a single node as a container to wrap and transport transaction records across the network. Each event block references previous event blocks that are known to the node. This makes a stream of DAG event blocks to form a sequence of history.

The history stored on each node can be represented by a directed acyclic graph $G=(V, E)$, where V is a set of vertices and E is a set of edges. Each vertex in a row (node) represents an event. Time flows bottom-to-top (or left-to-right) of the graph, so bottom (left) vertices represent earlier events in history. For a graph G , a path p in G is a sequence of vertices (v_1, v_2, \dots, v_k) by following the edges in E . Let v_c be a vertex in G . A vertex v_p is the *parent* of v_c if there is an edge from v_p to v_c . A vertex v_a is an *ancestor* of v_c if there is a path from v_a to v_c .

[Event block] An event block is a holder of a set of transactions. The structure of an event block includes the signature, generation time, transaction history, and references to previous event blocks. The first event block of each node is called a *leaf event*.

Each node can create event blocks, send (receive) messages to (from) other nodes. Suppose a node n_i creates an event v_c after an event v_s in n_i . Each event block has exactly k references. One of the references is self-reference, and the other $k-1$ references point to the top events of n_i 's $k-1$ peer nodes.

[Top event] An event v is a top event of a node n_i if there is no other event in n_i referencing v .

[Ref] An event v_r is called “ref” of event v_c if the reference hash of v_c points to the event v_r . Denoted by $v_c \hookrightarrow^r v_r$. For simplicity, we can use \hookrightarrow to denote a reference relationship (either \hookrightarrow^r or \hookrightarrow^s).

[Self-ref] An event v_s is called “self-ref” of event v_c , if the self-ref hash of v_c points to the event v_s . Denoted by $v_c \hookrightarrow^s v_s$.

[Self-ancestor] An event block v_a is self-ancestor of an event block v_c if there is a sequence of events such that $v_c \hookrightarrow^s v_1 \hookrightarrow^s \dots \hookrightarrow^s v_m \hookrightarrow^s v_a$. Denoted by $v_c \hookrightarrow^{sa} v_a$.

[Ancestor] An event block v_a is an ancestor of an event block v_c if there is a sequence of events such that $v_c \hookrightarrow v_1 \hookrightarrow \dots \hookrightarrow v_m \hookrightarrow v_a$. Denoted by $v_c \hookrightarrow^a v_a$.

For simplicity, we simply use $v_c \hookrightarrow^a v_s$ to refer both ancestor and self-ancestor relationship, unless we need to distinguish the two cases.

2.2 OPERA chain

L_ϕ protocol uses the DAG-based structure, called the OPERA chain, which was introduced in Lachesis protocols [1]. For consensus, the algorithm examines whether an event block is reached by more than $2n/3$ nodes, where n is the total number of participant nodes.

Let $G=(V,E)$ be a DAG. We extend with $G=(V,E,\top,\perp)$, where \top is a pseudo vertex, called *top*, which is the parent of all top event blocks, and \perp is a pseudo vertex, called *bottom*, which is the child of all leaf event blocks. With the pseudo vertices, we have \perp happened-before all event blocks. Also all event blocks happened-before \top . That is, for all event v_i , $\perp \rightarrow v_i$ and $v_i \rightarrow \top$.

[OPERA chain] The OPERA chain is a graph structure stored on each node. The OPERA chain consists of event blocks and references between them as edges.

The OPERA chain (DAG) is the local view of the DAG held by each node. OPERA chain is a DAG graph $G=(V,E)$ consisting of V vertices and E edges. Each vertex $v_i \in V$ is an event block. An edge $(v_i, v_j) \in E$ refers to a hashing reference from v_i to v_j ; that is, $v_i \hookrightarrow v_j$. This local view is used to identify topological ordering, Root, Clotho and Atropos vertices.

[Leaf] The first created event block of a node is called a leaf event block.

[Root] An event block v is a root if either (1) it is the leaf event block of a node, or (2) v can reach more than $2n/3$ of the roots.

[Root set] The set of all first event blocks (leaf events) of all nodes form the first root set R_1 ($|R_1| = n$). The root set R_k consists of all roots r_i such that $r_i \notin R_i, \forall i = 1..(k-1)$ and r_i can reach more than $2n/3$ other roots in the current frame, $i = 1..(k-1)$.

[Frame] Frame f_i is a natural number that separates Root sets. The root set at frame f_i is denoted by R_i .

[Creator] If a node n_i creates an event block v , then the creator of v , denoted by $cr(v)$, is n_i .

[Clotho] A root r_k in the frame f_{a+3} can nominate a root r_a as Clotho if more than $2n/3$ roots in the frame f_{a+1} dominate r_a and r_k dominates the roots in the frame f_{a+1} .

[Atropos] An Atropos is a Clotho that is decided as final.

Event blocks in the subgraph rooted at the Atropos are also final events. Atropos blocks form a Main-chain, which allows time consensus ordering and responses to attacks.

2.3 Layering Definitions

For a directed acyclic graph $G=(V,E)$, a layering is to assign a layer number to each vertex in G .

[Layering] A layering (or levelling) of G is a topological numbering ϕ of G , $\phi : V \rightarrow \mathbb{Z}$, mapping the set of vertices V of G to integers such that $\phi(v) \geq \phi(u) + 1$ for every directed edge $(u, v) \in E$. If $\phi(v)=j$, then v is a layer- j vertex and $V_j = \phi^{-1}(j)$ is the j th layer of G .

A layering ϕ of G partitions the set of vertices V into a finite number l of *non-empty* disjoint subsets (called layers) V_1, V_2, \dots, V_l , such that $V = \cup_{i=1}^l V_i$. Each vertex is assigned to a layer V_j , where $1 \leq j \leq l$, such that every edge $(u, v) \in E$, $u \in V_i, v \in V_j, 1 \leq i < j \leq l$.

[Hierarchical graph] For a layering ϕ , the produced graph $H=(V,E,\phi)$ is a *hierarchical graph*, which is also called an l -layered directed graph and could be represented as $H=(V_1, V_2, \dots, V_l; E)$.

The spans of an edge $e=(u, v)$ with $u \in V_i$ and $v \in V_j$ is $j-i$ or $\phi(v)-\phi(u)$. If no edge in the hierarchical has a span greater than one then the hierarchical graph is *proper*.

Let HG denote hierarchical graph. Let $V^+(v)$ denote the outgoing neighbours of v ; $V^+(v) = \{u \in V | (v, u) \in E\}$. Let $V^-(v)$ denote the incoming neighbours of v ; $V^-(v) = \{u \in V | (u, v) \in E\}$.

[Height of HG] The height of a hierarchical graph is the number of layers l .

[Width of HG] The width of a hierarchical graph is the number of vertices in the longest layer. that is, $\max_{1 \leq i \leq l} |V_i|$

There are three common criteria for a layering. First, the hierarchical graph should be *compact*. Compactness aims for minimizing the width and the height of the graph. Finding a layering by minimizing the height with respect to a given width is NP-hard. Second, the hierarchical graph should be *proper*, e.g., by introducing dummy vertices into the layering for every long edge (u, v) with $u \in V_i$ and $v \in V_j$ where $i < j-1$. Each long edge (u, v) , $u \in V_i, v \in V_j$ is

replaced by a path $(u, v_1, v_2, \dots, v_l, v)$ where a dummy vertex v_p , $i+1 \leq p \leq j-1$, is inserted in each intermediate layer V_{i+p} . Third, the number of dummy vertices should be kept minimum so as to reduce the running time.

There are several approaches to DAG layering.

2.3.1 Minimizing The Height

[Longest Path layering] The longest path method layers vertex to layer i where i is the length of the longest path from a source.

Longest path layering is a list scheduling algorithm produces hierarchical graph with the smallest possible height [25, 26]. The main idea is that given an acyclic graph we place the vertices on the i -th layer where i is the length of the longest path to the vertex from a source vertex.

Algorithm 1 shows the longest path layering algorithm. The algorithm picks vertices whose incoming edges only come from past layers (Z) and assigns them to the current layer U . When no more vertex satisfying that condition, the layer is incremented. The process starts over again until all vertices have been assigned a layer.

Algorithm 1 Longest Path Layering

```

1: Require: A DAG  $G=(V,E)$ 
2: function LONGESTPATHLAYERING
3:    $U \leftarrow \emptyset; Z \leftarrow \emptyset; l \leftarrow 1$ 
4:   while  $U \neq V$  do
5:     Select vertex  $v \in V \setminus U$  with  $V^+(v) \subseteq Z$ 
6:     if  $v$  has been selected then
7:        $\phi(v) \leftarrow l$ 
8:        $U \leftarrow U \cup \{v\}$ 
9:     if no vertex has been selected then
10:       $l \leftarrow l + 1$ 
11:       $Z \leftarrow Z \cup U$ 

```

The following algorithm similar to the above computes layerings of minimum height. Firstly, all source vertices are placed in the first layer V_1 . Then, the layer $\phi(v)$ for every remaining vertex v is recursively defined by $\phi(v) = \max\{\phi(u) | (u, v) \in E\} + 1$. This algorithm produces a layering where many vertices will stay close to the bottom, and hence the number of layers k is kept minimized. The main drawback of this algorithm is that it may produce layers that are too wide. By using a topological ordering of the vertices [27], the algorithm can be implemented in linear time $O(|V|+|E|)$.

2.3.2 Fixed Width Layering

The longest path layering algorithm minimizes the height, while compactness of HG depends on both the width and the height. The problem of finding a layering with minimum height of general graphs is NP-complete if a fixed width is no less than three [28]. Minimizing the number of dummy vertices guarantees minimum height.

Now we present *Coffman-Graham* (CG) algorithm, which considers a layering with a maximum width [29],

[Coffman-Graham algorithm] Coffman-Graham is a layering algorithm in which a maximum width is given. Vertices are ordered by their distance from the source vertices of the graph, and are then assigned to the layers as close to the bottom as possible.

Coffman-Graham layering algorithm was originated to solve multiprocessor scheduling to minimize the width (regardless of dummy vertices) as well as the height. The Coffman-Graham algorithm is currently the most commonly used layering method.

Algorithm 2 gives the Coffman-Graham algorithm. The algorithm takes as input a reduced graph, i.e., no transitive edges are included in the graph, and a given width w . An edge (u, v) is called transitive if a path $(u=v_1, v_2, \dots, v_k=v)$ exists in the graph. The Coffman-Graham algorithm works in two phases. First, it orders the vertices by their distance from the source vertices of the graph. In OPERA chain, the source vertices are the leaf event blocks. In the second phase, vertices are assigned to the layers as close to the bottom as possible.

Algorithm 2 Coffman-Graham Layering

```

1: Require: A reduced DAG  $G = (V, E)$ 
2: function COFFMANGRAHAM( $W$ )
3:   for all  $v \in V$  do  $\lambda(v) \leftarrow \infty$ 
4:   for  $i \leftarrow 1$  to  $|V|$  do
5:     Choose  $v \in V$  with  $\lambda(v) = \infty$  s.t.  $|V^-(v)|$  is minimised
6:      $\lambda(v) \leftarrow 1$ 
7:    $l \leftarrow 1; L_1 \leftarrow \emptyset; U \leftarrow \emptyset$ 
8:   while  $U \neq V$  do
9:     Choose  $v \in V \setminus U$  such that  $V^+(v) \subseteq U$  and  $\lambda(v)$  is maximised
10:    if  $|L_l| \leq W$  and  $V^+(v) \subseteq L_1 \cup L_2 \cup \dots \cup L_{l-1}$  then
11:       $L_l \leftarrow L_l \cup \{v\}$ 
12:    else
13:       $l \leftarrow l + 1$ 
14:       $L_l \leftarrow \{v\}$ 
15:     $U \leftarrow U \cup \{v\}$ 

```

Lam and Sethi [30] showed that the number of layers l of the computed layering with width w is bounded by $l \leq (2 - 2/w) \cdot l_{opt}$, where l_{opt} is the minimum height of all layerings with width w . So, the Coffman-Graham algorithm is an exact algorithm for $w \leq 2$. In certain applications, the notion of width does not consider dummy vertices.

2.3.3 Minimizing the Total Edge Span

The objective to minimize the total edge span (or edge length) is equivalent to minimizing the number of dummy vertices. It is a reasonable objective in certain applications such as hierarchical graph drawing. It can be shown that minimizing the number of dummy vertices guarantees minimum height. The problem of layering to minimize the number of dummy vertices can be modelled as an ILP, which has been proved to be very efficient experimentally, even though it does not guarantee a polynomial running time.

2.4 Lamport timestamps

Our Lachesis protocol L_ϕ relies on Lamport timestamps to define a topological ordering of event blocks in OPERA chain. By using Lamport timestamps, we do not rely on physical clocks to determine a partial ordering of events.

The “happened before” relation, denoted by \rightarrow , gives a partial ordering of events from a distributed system of nodes. For a pair of event blocks v_i and v_j , the relation “ \rightarrow ” satisfies: (1) If v_i and v_j are events of the same node p_i , and v_i comes before v_j , then $v_i \rightarrow v_j$. (2) If v_i is the send(m) by one process and v_j is the receive(m) by another process, then $v_i \rightarrow v_j$. (3) If $v_i \rightarrow v_j$ and $v_j \rightarrow v_k$ then $v_i \rightarrow v_k$.

[Happened-Immediate-Before] An event block v_x is said Happened-Immediate-Before an event block v_y if v_x is a (self-) ref of v_y . Denoted by $v_x \mapsto v_y$.

[Happened-before] An event block v_x is said Happened-Before an event block v_y if v_x is a (self-) ancestor of v_y . Denoted by $v_x \rightarrow v_y$.

Happened-before relation is the transitive closure of happens-immediately-before. “ v_x Happened-before v_y ” means that the node creating v_y knows event block v_x . An event v_x happened before an event v_y if one of the followings happens: (a) $v_y \hookrightarrow^s v_x$, (b) $v_y \hookrightarrow^r v_x$, or (c) $v_y \hookrightarrow^a v_x$. The happened-before relation of events form an acyclic directed graph $G' = (V, E')$ such that an edge $(v_i, v_j) \in E'$ corresponding to an edge $(v_j, v_i) \in E$.

[Concurrent] Two event blocks v_x and v_y are said concurrent if neither of them happened before the other. Denoted by $v_x \parallel v_y$.

Given two vertices v_x and v_y both contained in two OPERA chains (DAGs) G_1 and G_2 on two nodes. We have the following: (1) $v_x \rightarrow v_y$ in G_1 if $v_x \rightarrow v_y$ in G_2 ; (2) $v_x \parallel v_y$ in G_1 if $v_x \parallel v_y$ in G_2 .

[Total ordering] Let \prec denote an arbitrary total ordering of the nodes (processes) p_i and p_j . *Total ordering* is a relation \Rightarrow satisfying the following: for any event v_i in p_i and any event v_j in p_j , $v_i \Rightarrow v_j$ if and only if either (i) $C_i(v_i) < C_j(v_j)$ or (ii) $C_i(v_i) = C_j(v_j)$ and $p_i \prec p_j$.

This defines a total ordering relation. The Clock Condition implies that if $v_i \rightarrow v_j$ then $v_i \Rightarrow v_j$.

2.5 State Definitions

Each node has a local state, a collection of histories, messages, event blocks, and peer information, we describe the components of each.

[State] A (local) state of a process i is denoted by s_j^i consisting of a sequence of event blocks $s_j^i = v_0^i, v_1^i, \dots, v_j^i$.

In a DAG-based protocol, each event block v_j^i is *valid* only if the reference blocks exist before it. From a local state s_j^i , one can reconstruct a unique DAG. That is, the mapping from a local state s_j^i into a DAG is *injective* or one-to-one. Thus, for ONLAY, we can simply denote the j -th local state of a process i by the DAG g_j^i (often we simply use G_i to denote the current local state of a process i).

[Action] An action is a function from one local state to another local state.

Generally speaking, an action can be one of: a $send(m)$ action where m is a message, a $receive(m)$ action, and an internal action. A message m is a triple $\langle i, j, B \rangle$ where $i \in P$ is the sender of the message, $j \in P$ is the message recipient, and B is the body of the message. In ONLAY, B consists of the content of an event block v . Let M denote the set of messages.

Semantics-wise, there are two actions that can change a process's local state: creating a new event and receiving an event from another process.

[Event] An event is a tuple $\langle s, \alpha, s' \rangle$ consisting of a state, an action, and a state. Sometimes, the event can be represented by the end state s' .

The j -th event in history h_i of process i is $\langle s_{j-1}^i, \alpha, s_j^i \rangle$, denoted by v_j^i .

[Local history] A local history h_i of process i is a (possibly infinite) sequence of alternating local states — beginning with a distinguished initial state. A set H_i of possible local histories for each process i in P .

The state of a process can be obtained from its initial state and the sequence of actions or events that have occurred up to the current state. In the Lachesis protocol, we use append-only semantics. The local history may be equivalently described as either of the following: (1) $h_i = s_0^i, \alpha_1^i, \alpha_2^i, \alpha_3^i, \dots$, (2) $h_i = s_0^i, v_1^i, v_2^i, v_3^i, \dots$, (3) $h_i = s_0^i, s_1^i, s_2^i, s_3^i, \dots$.

In Lachesis, a local history is equivalently expressed as: $h_i = g_0^i, g_1^i, g_2^i, g_3^i, \dots$ where g_j^i is the j -th local DAG (local state) of the process i .

[Run] Each asynchronous run is a vector of local histories. Denoted by $\sigma = \langle h_1, h_2, h_3, \dots, h_N \rangle$.

Let Σ denote the set of asynchronous runs. We can now use Lamport's theory to talk about global states of an asynchronous system. A global state of run σ is an n -vector of prefixes of local histories of σ , one prefix per process. The happens-before relation can be used to define a consistent global state, often termed a consistent cut, as follows.

2.6 Consistent Cut

An asynchronous system consists of the following sets: a set P of process identifiers, a set C of channels, a set H_i of possible local histories for each process i , a set A of asynchronous runs, a set M of all messages. Consistent cuts represent the concept of scalar time in distributed computation, it is possible to distinguish between a “before” and an “after”, see CCK paper [24].

[Consistent cut] A consistent cut of a run σ is any global state such that if $v_x^i \rightarrow v_y^j$ and v_y^j is in the global state, then v_x^i is also in the global state. Denoted by $c(\sigma)$.

The concept of consistent cut formalizes such a global state of a run. A consistent cut consists of all consistent DAG chains. A received event block exists in the global state implies the existence of the original event block. Note that a consistent cut is simply a vector of local states; we will use the notation $c(\sigma)[i]$ to indicate the local state of i in cut c of run σ .

The formal semantics of an asynchronous system is given via the satisfaction relation \vdash . Intuitively $c(\sigma) \vdash \phi$, “ $c(\sigma)$ satisfies ϕ ,” if fact ϕ is true in cut c of run σ .

We assume that we are given a function π that assigns a truth value to each primitive proposition p . The truth of a primitive proposition p in $c(\sigma)$ is determined by π and c . This defines $c(\sigma) \vdash p$.

[Equivalent cuts] Two cuts $\mathbf{c}(\sigma)$ and $\mathbf{c}'(\sigma')$ are equivalent with respect to i if:

$$\mathbf{c}(\sigma) \sim_i \mathbf{c}'(\sigma') \Leftrightarrow \mathbf{c}(\sigma)[i] = \mathbf{c}'(\sigma')[i]$$

[i knows ϕ] $K_i(\phi)$ represents the statement “ ϕ is true in all possible consistent global states that include i ’s local state”.

$$\mathbf{c}(\sigma) \vdash K_i(\phi) \Leftrightarrow \forall \mathbf{c}'(\sigma') (\mathbf{c}'(\sigma') \sim_i \mathbf{c}(\sigma) \Rightarrow \mathbf{c}'(\sigma') \vdash \phi)$$

[i partially knows ϕ] $P_i(\phi)$ represents the statement “there is some consistent global state in this run that includes i ’s local state, in which ϕ is true.”

$$\mathbf{c}(\sigma) \vdash P_i(\phi) \Leftrightarrow \exists \mathbf{c}'(\sigma) (\mathbf{c}'(\sigma) \sim_i \mathbf{c}(\sigma) \wedge \mathbf{c}'(\sigma) \vdash \phi)$$

[Majority concurrently knows] The next modal operator is written M^C and stands for “majority concurrently knows.” The definition of $M^C(\phi)$ is as follows.

$$M^C(\phi) =_{def} \bigwedge_{i \in S} K_i P_i(\phi),$$

where $S \subseteq P$ and $|S| > 2n/3$.

This is adapted from the “everyone concurrently knows” in CCK paper [24]. In the presence of one-third of faulty nodes, the original operator “everyone concurrently knows” is sometimes not feasible. Our modal operator $M^C(\phi)$ fits precisely the semantics for BFT systems, in which unreliable processes may exist.

[Concurrent common knowledge] The last modal operator is concurrent common knowledge (CCK), denoted by C^C . $C^C(\phi)$ is defined as a fixed point of $M^C(\phi \wedge X)$.

CCK defines a state of process knowledge that implies that all processes are in that same state of knowledge, with respect to ϕ , along some cut of the run. In other words, we want a state of knowledge X satisfying: $X = M^C(\phi \wedge X)$. C^C will be defined semantically as the weakest such fixed point, namely as the greatest fixed-point of $M^C(\phi \wedge X)$. It therefore satisfies:

$$C^C(\phi) \Leftrightarrow M^C(\phi \wedge C^C(\phi))$$

Thus, $P_i(\phi)$ states that there is some cut in the same asynchronous run σ including i ’s local state, such that ϕ is true in that cut.

Note that ϕ implies $P_i(\phi)$. But it is not the case, in general, that $P_i(\phi)$ implies ϕ or even that $M^C(\phi)$ implies ϕ . The truth of $M^C(\phi)$ is determined with respect to some cut $\mathbf{c}(\sigma)$. A process cannot distinguish which cut, of the perhaps many cuts that are in the run and consistent with its local state, satisfies ϕ ; it can only know the existence of such a cut.

[Global fact] Fact ϕ is valid in system Σ , denoted by $\Sigma \vdash \phi$, if ϕ is true in all cuts of all runs of Σ .

$$\Sigma \vdash \phi \Leftrightarrow (\forall \sigma \in \Sigma) (\forall \mathbf{c}) (\mathbf{c}(a) \vdash \phi)$$

Fact ϕ is valid, denoted $\vdash \phi$, if ϕ is valid in all systems, i.e. $(\forall \Sigma) (\Sigma \vdash \phi)$.

[Local fact] A fact ϕ is local to process i in system Σ if $\Sigma \vdash (\phi \Rightarrow K_i \phi)$.

3 L_ϕ : Layering-based Consensus Protocol

This section will then present the main concepts and algorithms for the L_ϕ of our ONLAY framework. The key idea of our ONLAY framework is to use layering algorithms on the OPERA chain. The assigned layers are then used to determine consensus of the event blocks.

For a DAG $G=(V,E)$, a layering ϕ of G is a mapping $\phi : V \rightarrow \mathbb{Z}$, such that $\phi(v) \geq \phi(u) + 1$ for every directed edge $(u,v) \in E$. If $\phi(v)=j$, then v is a layer- j vertex and $V_j=\phi^{-1}(j)$ is the j -th layer of G . In terms of happened before relation, $v_i \rightarrow v_j$ iff $\phi(v_i) < \phi(v_j)$. Layering ϕ partitions the set of vertices V into a finite number l of *non-empty* disjoint subsets (called layers) V_1, V_2, \dots, V_l , such that $V = \cup_{i=1}^l V_i$. Each vertex is assigned to a layer V_j , where $1 \leq j \leq l$, such that every edge $(u,v) \in E$, $u \in V_i$, $v \in V_j$, $1 \leq i < j \leq l$. Section 2.3 gives more details about layering.

3.1 H-OPERA chain

Recall that OPERA chain, which is a DAG $G=(V,E)$ stored in each node. We introduce a new notion of H-OPERA chain, which is built on top of the OPERA chain. By applying a layering ϕ on the OPERA chain, one can obtain the hierarchical graph of G , which is called H-OPERA chain.

Definition 3.1 (H-OPERA chain). *An H-OPERA chain is the result hierarchical graph $H = (V, E, \phi)$.*

Thus, H-OPERA chain can also be represented by $H=(V_1, V_2, \dots, V_l; E)$. In H-OPERA chain, vertices are assigned with layers such that each edge $(u,v) \in E$ flows from a higher layer to a lower layer i.e., $\phi(u) > \phi(v)$.

Note that, the hierarchical graph H-OPERA chain produced by ONLAY is not proper. This is because there always exist a long edge $(u,v) \in E$ that spans multiple layers i.e. $\phi(u) - \phi(v) > 1$. Generally speaking, a layering can produce a proper hierarchical graph by introducing dummy vertices along every long edge. However, for consensus protocol, such dummy vertices in the resulting proper hierarchical graph are also not ideal as they can increase the computational cost of the HG itself and of the following steps to determine consensus. Thus, in ONLAY we consider layering algorithms that do not introduce dummy vertices.

A simple approach to compute H-OPERA chain is to consider OPERA chain as a static graph G and then apply a layering algorithm such as either LongestPathLayer (LPL) algorithm or Coffman-Graham (CG) algorithm on the graph G . LPL algorithm can achieve HG in linear time $O(|V|+|E|)$ with the minimum height. CG algorithm has a time complexity of $O(|V|^2)$, for a given W . The LPL and CG algorithms are given in Section 2.3.

The above simple approach that takes a static G works well when G is small, but then not applicable when G becomes larger and larger. As time goes, the OPERA chain becomes larger and larger, it takes more and more time to process.

3.2 Online Layer Assignment

To overcome the limitation of the above simple approach, we introduce online layering algorithms to apply on the evolutionary OPERA chain to reduce the computational cost.

In ONLAY, the OPERA chain in each node keeps evolving rather than being static. A node can introduce a new vertex and few edges when a new event block was created. The OPERA chain can also be updated with new vertices and edges that a node receives from the other nodes in the network.

Definition 3.2 (Dynamic OPERA chain). *OPERA chain is a dynamic DAG which is evolving as new event block is created and received.*

We consider a simple more of dynamic OPERA chain. Let DAG $G=(V,E)$ be the current OPERA chain of a node. Let $G'=(V',E')$ denote the *diff graph*, which consists of the changes to G at a time, either at block creation or block arrival. We can assume that the vertex sets V and V' are disjoint, similar to the edges E and E' . That is, $V \cap V' = \emptyset$ and $E \cap E' = \emptyset$. At each graph update, the updated OPERA chain becomes $G_{new}=(V \cup V', E \cup E')$.

The online versions of layering take as input a new change to OPERA change and the current layering information and then efficiently compute the layers of new vertices.

Definition 3.3 (Online Layering). *Online layering is a modified version to compute layering of a dynamic DAG G efficiently as G evolves.*

Specifically, we propose two online layering algorithms. *Online Longest Path Layering* (O-LPL) is an improved version of the LPL algorithm. *Online Coffman-Graham* (O-CG) algorithm is a modified version of the original CG

algorithm. The online layering algorithms assign layers to the vertices in diff graph $G'=(V',E')$ consisting of new self events and received unknown events.

3.2.1 Online Longest Path Layering

Definition 3.4 (Online LPL). *Online Longest Path Layering (O-LPL) is a modified version of LPL to compute layering of a dynamic DAG G efficiently.*

Algorithm 3 gives our new algorithm to compute LPL layering for a dynamic OPERA chain. The O-LPL algorithm takes as input the following information: U is the set of processed vertices, Z is the set of already layered vertices, l is the current height (maximum layer number), V' is the set of new vertices, and E' is the set of new edges.

Algorithm 3 Online Longest Path Layering

```

1: Require: A DAG  $G=(V,E)$  and  $G'=(V',E')$ 
2: function ONLINELONGESTPATHLAYERING( $U, Z, l, V', E'$ )
3:    $V_{new} \leftarrow V \cup V', E_{new} \leftarrow E \cup E'$ 
4:   while  $U \neq V_{new}$  do
5:     Select vertex  $v \in V_{new} \setminus U$  with  $V_{new}^+(v) \subseteq Z$ 
6:     if  $v$  has been selected then
7:        $l' \leftarrow \max\{\phi(u) | u \in V_{new}^+(v)\} + 1$ 
8:        $\phi(v) \leftarrow l'$ 
9:        $U \leftarrow U \cup \{v\}$ 
10:    if no vertex has been selected then
11:       $l \leftarrow l + 1$ 
12:       $Z \leftarrow Z \cup U$ 

```

The algorithm assumes that the given value of width W is sufficiently large. Section 3.3 gives some discussions about choosing a value of W . The algorithm has a worst case time complexity of $O(|V'| + |E'|)$.

3.2.2 Online Coffman-Graham Algorithm

Definition 3.5 (Online CG). *Online Coffman-Graham (O-CG) Algorithm is a modified version of the CG algorithm to compute layering of a dynamic DAG G efficiently.*

Algorithm 4 gives a new layering algorithm that is based on the original Coffman-Graham algorithm. The algorithm takes as input the following: W is the fixed maximum width, U is the unprocessed vertices, l is the current height (maximum layer), ϕ is the layering assignment, V' is the set of new vertices, E' is the set of new edges. The algorithm has a worst case time complexity of $O(|V'|^2)$.

Algorithm 4 Online Coffman-Graham Layering

```

1: Require: A reduced DAG  $G = (V, E)$  and  $G' = (V', E')$ 
2: function ONLINECOFFMANGRAHAM( $W, U, l, \phi, V', E'$ )
3:   for all  $v \in V'$  do  $\lambda(v) \leftarrow \infty$ 
4:   for  $i \leftarrow 1$  to  $|V'|$  do
5:     Choose  $v \in V'$  with  $\lambda(v) = \infty$  s.t.  $|V^-(v)|$  is minimised
6:      $\lambda(v) \leftarrow 1$ 
7:    $V_{new} \leftarrow V \cup V'; E_{new} \leftarrow E \cup E'$ 
8:   while  $U \neq V_{new}$  do
9:     Choose  $v \in V_{new} \setminus U$  such that  $V_{new}^+(v) \subseteq U$  and  $\lambda(v)$  is maximised
10:     $l' \leftarrow \max\{\phi(u) | u \in V_{new}^+(v)\} + 1$ 
11:    if  $|\phi_{l'}| \leq W$  and  $V_{new}^+(v) \subseteq \phi_1 \cup \phi_2 \cup \dots \cup \phi_{l'-1}$  then
12:       $\phi_{l'} \leftarrow \phi_{l'} \cup \{v\}$ 
13:    else
14:       $l \leftarrow l + 1$ 
15:       $\phi_l \leftarrow \{v\}$ 
16:       $U \leftarrow U \cup \{v\}$ 

```

3.3 Layer Width in BFT

This section presents our formal analysis of the layering with respect to the Byzantine fault tolerance.

In distributed database systems, *Byzantine* fault tolerance addresses the functioning stability of the network in the presence of at most $n/3$ of Byzantine nodes, where n is the number of nodes. A Byzantine node is a node that is mal-functioned, dies or adversarially targeted. Such nodes can also be called dishonest or faulty.

To address BFT of L_ϕ protocol, we present some formal analysis of the layering results of OPERA chain. For simplicity, let ϕ_{LP} denote the layering of G obtained from LongestPathLayering algorithm. Let $\phi_{CG(W)}$ denote the layering of G obtained from the Coffman-Graham algorithm with some fixed width W .

3.3.1 Byzantine free network

We consider the case in which the network is free from any faulty nodes. In this setting, all nodes are honest and thus zero forks exist.

Theorem 3.1. *In the absense of dishonest nodes, the width of each layer L_i is at most n .*

Since there are n nodes, there are n leaf vertices. We can prove the theorem by induction. The width of the first layer L_1 is n at maximum, otherwise there exists a fork, which is not possible. We assume that the theorem holds for every layer from 1 to i . That is, the L_i at each layer has the width of at most n . We will prove it holds for layer $i + 1$ as well. Since each honest node can create at most one event block on each layer, the width at each layer is at most n . We can prove by contradiction. Suppose there exists a layer $|\phi_{i+1}| > n$. Then there must exist at least two events v_p, v_q on layer ϕ_{i+1} such that v_p and v_q have the same creator, say node n_i . That means v_p and v_q are the forks of node n_i . It is a contradiction with the assumption that there is no fork. Thus, the width of each layer $|\phi_{i+1}| \leq n$. The theorem is proved.

Theorem 3.2. *LongestPathLayering(G) and CoffmanGraham(G, n) computes the same layer assignment. That is, ϕ_{LP} and $\phi_{CG(n)}$ are the same.*

Proposition 3.3. *For each vertex $v \in G$, $\phi_{LP}(v) = \phi_{CG(n)}(v)$.*

3.3.2 1/3-BFT

Now, let us consider the layering result for the network in which has at most $n/3$ dishonest nodes.

Without loss of generality, let w_p be the probability that a node can create fork. Let w_c be the maximum number of forks a faulty node can create at a time. The number of forked events a node can create at a time is $w_p w_c$. The number of fork events at each layer is given by

$$W_{fork} = \frac{1}{3} N w_p w_c$$

Thus, the maximum width of each layer is given by:

$$W_{max} = N + W_{fork} = N + \frac{1}{3} N w_p w_c$$

Thus, if we set the maximum width high enough to tolerate the potential existence of forks, we can achieve BFT of the layering result. The following theorem states the BFT of LPL and CG algorithms.

Theorem 3.4. *LongestPathLayering(G) and CoffmanGraham(G, W_{max}) computes the same layer assignment. That is, ϕ_{LP} and $\phi_{CG(W_{max})}$ are identical.*

Proposition 3.5. *For each vertex $v \in G$, $\phi_{LP}(v) = \phi_{CG(W_{max})}(v)$.*

3.4 Root Graph

We propose to use a new data structure, called *root graph*, which is used to determine frames.

Definition 3.6 (Root graph). *A root graph $G_R = (V_R, E_R)$ is a directed graph consisting of vertices as roots and edges represent their reachability.*

In the root graph G_R , the set of roots V_R is a subset of V . The set of edges E_R is the reduced edges from E , in that $(u, v) \in E_R$ only if u and v are roots and u can reach v following edges in E i.e., $v \rightarrow u$.

A root graph contains the n genesis vertices i.e., the n leaf event blocks. A vertex v that can reach at least $2/3n + 1$ of the current set of all roots V_R is itself a root. For each root r_i that new root r reaches, we include a new edge (r, r_i)

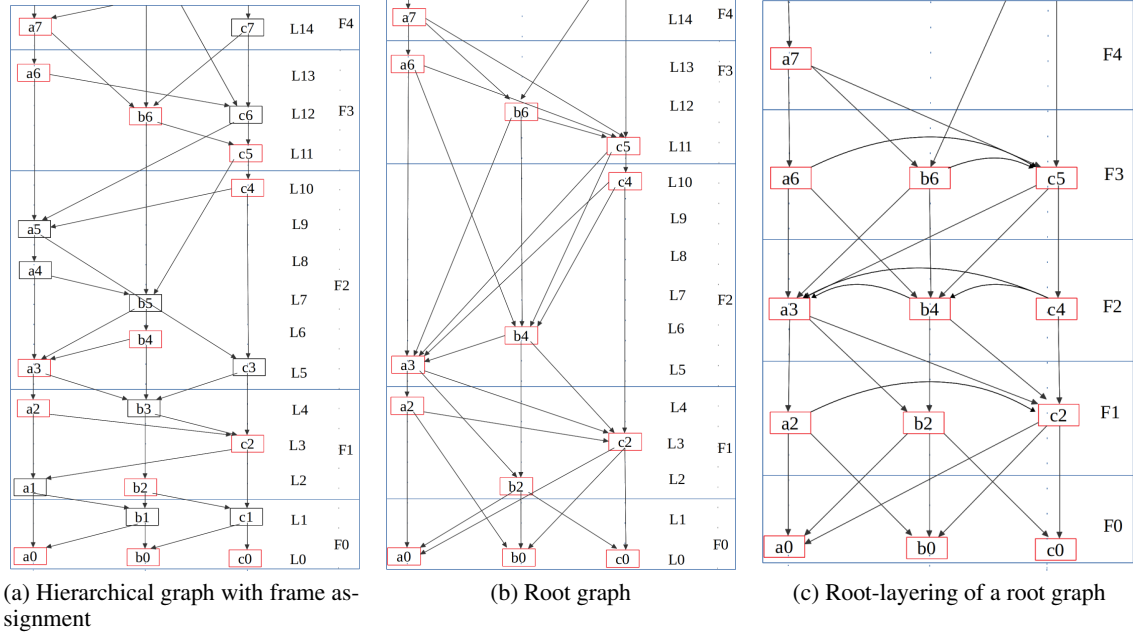


Figure 3: An example of hierarchical graph and root graph computed from the OPERA chain in Figure 1

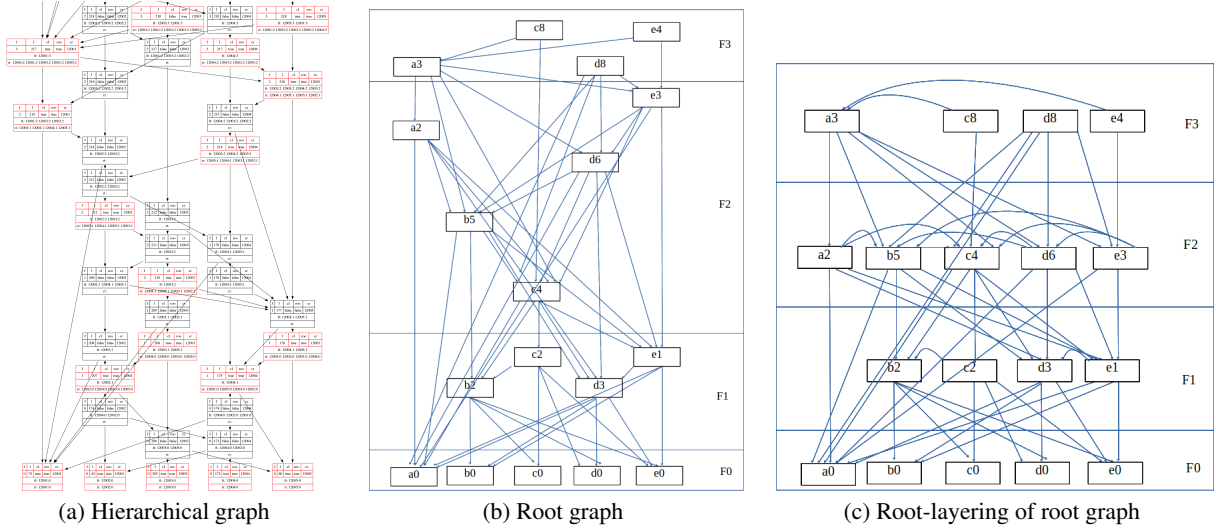
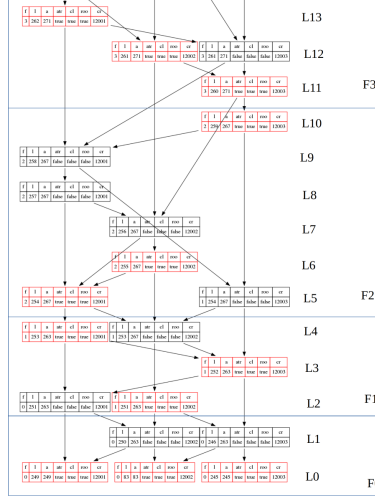


Figure 4: An example of OPERA chain on a node in a 5-node network. (a) hierarchical graph, (b) root graph, (c) layering root frame for frame assignment.

into the set of root edges E_R . Note that, if a root r reaches two other roots r_1 and r_2 of the same node, then we retain only one edge (r, r_1) or (r, r_2) if $\phi(r_1) > \phi(r_2)$. This requirement makes sure each root of a node can have at most one edge to any other node.

Figure 3a shows an example of the H-OPERA chain, which is the result hierarchical graph from a laying algorithm. Vertices of the same layer are placed on a horizontal line. There are 14 layers which are labelled as L0, L1, ..., to L14. Roots are shown in red colors. Figure 3b gives an example of the root graph constructed from the H-OPERA chain.

Figure 4a shows a H-OPERA chain (hierarchical graph resulted from layering an OPERA chain) on a single node in a 5-node network. Figure 4b depicts the root graph from the H-OPERA chain. There are 14 layers in the example.

Figure 5: An example of H-OPERA with frame assignment in L_ϕ protocol.

3.5 Frame Assignment

We then present a deterministic approach to frame assignment, which assigns each event block a unique frame number. First, we show how to assign frames to root vertices via the so-called *root-layering*. Second, we then assign non-root vertices to the determined frame numbers with respect to the topological ordering of the vertices.

Definition 3.7 (Root-layering). *A root layering assigns each root with a unique layer (frame) number.*

Here, we model root layering problem as a modified version of the original layering problem. For a root graph $G_R = (V_R, E_R)$, root-layering ϕ_R is a layering that assigns each root vertex v_r of V_R with an integer $\phi_R(v_r)$ such that:

- $\phi_R(u) \geq \phi_R(v) + 1$, for each edge $e = (u, v) \in E_R$.
- if u reaches at least $2/3n + 1$ of the roots of frame i , then $\phi_R(v) = i + 1$.

If $\phi_R(v) = j$, then v is a frame- j vertex and $V_j = \phi_R^{-1}(j)$ is the j -th frame of G . In terms of happened before relation, $v_i \rightarrow v_j$ iff $\phi_R(v_i) < \phi_R(v_j)$. Similarly to the original layering problem, root layering ϕ_R partitions the set of vertices V_R into a finite number l of *non-empty* disjoint subsets (called frames) V_1, V_2, \dots, V_l , such that $V_R = \cup_{i=1}^l V_i$. Each vertex is assigned to a root-layer V_j , where $1 \leq j \leq l$, such that every edge $(u, v) \in E$, $u \in V_i, v \in V_j, 1 \leq i < j \leq l$.

Figure 3c depicts an example of root layering for the root graph in Figure 3b. There are five frames assigned to the root graph. The frames are labelled as F0, F1, F2, F3 and F4. Figure 4c depicts an example of root layering for the root graph in Figure 4b. There are four frames whose labels are F0, F1, F2 and F3.

We now present our new approach that uses root layering information to assign frames to non-root vertices.

Definition 3.8 (Frame assignment). *A frame assignment assigns every vertex v in H-OPERA chain H a frame number $\phi_F(v)$ such that*

- $\phi_F(u) \geq \phi_F(v)$ for every directed edge (u, v) in H ;
- for each root r , $\phi_F(r) = \phi_R(r)$;
- for every pair of u and v : if $\phi(u) \geq \phi(v)$ then $\phi_F(u) \geq \phi_F(v)$; if $\phi(u) = \phi(v)$ then $\phi_F(u) = \phi_F(v)$.

Figure 5 shows an example of the H-OPERA chain with frame assignment of the OPERA chain in Figure 3a. Each vertex shows the assigned frame number, Lamport timestamp, consensus timestamp, flags to show whether they are atropos, root or clotho, and its creator's identifier.

3.6 Consensus

The protocol L_ϕ uses several novel concepts such as OPERA chain, H-OPERA chain, Root graph, Frame assignment to define deterministic solution for the consensus problem. We now present our formal model for the consistency of

knowledge across the distributed network of nodes. We use the consistent cut model, described in Section 2.6. For more details, see the CCK paper [24].

For an OPERA chain G , let $G[v]$ denote the subgraph of G that contains nodes and edges reachable from v .

Definition 3.9 (Consistent chains). *For two chains G_1 and G_2 , they are consistent if for any event v contained in both chains, $G_1[v] = G_2[v]$. Denoted by $G_1 \sim G_2$.*

For any two nodes, if their OPERA chains contain the same event v , then they have the same k hashes contained within v . A node must already have the k references of v in order to accept v . Thus, both OPERA chains must contain k references of v . Presumably, the cryptographic hashes are always secure and thus references must be the same between nodes. By induction, all ancestors of v must be the same. When two consistent chains contain the same event v , both chains contain the same set of ancestors for v , with the same reference and self-ref edges between those ancestors. Consequently, the two OPERA chains are consistent.

Definition 3.10 (Global OPERA chain). *A global consistent chain G^C is a chain such that $G^C \sim G_i$ for all G_i .*

Let $G \sqsubseteq G'$ denote that G is a subgraph of G' . Some properties of G^C are given as follows:

- (1) $\forall G_i (G^C \sqsubseteq G_i)$.
- (2) $\forall v \in G^C \forall G_i (G^C[v] = G_i[v])$.
- (3) $(\forall v_c \in G^C) (\forall v_p \in G_i) ((v_p \rightarrow v_c) \Rightarrow v_p \in G^C)$.

The layering of consistent OPERA chains is consistent itself.

Definition 3.11 (Consistent layering). *For any two consistent OPERA chains G_1 and G_2 , layering results ϕ^{G_1} and ϕ^{G_2} are consistent if $\phi^{G_1}(v) = \phi^{G_2}(v)$, for any vertex v common to both chains. Denoted by $\phi^{G_1} \sim \phi^{G_2}$.*

Theorem 3.6. *For two consistent OPERA chains G_1 and G_2 , the resulting H-OPERA chains using layering ϕ_{LPL} are consistent.*

The theorem states that for any event v contained in both OPERA chains, $\phi_{LPL}^{G_1}(v) = \phi_{LPL}^{G_2}(v)$. Since $G_1 \sim G_2$, we have $G_1[v] = G_2[v]$. Thus, the height of v is the same in both G_1 and G_2 . Thus, the assigned layer using ϕ_{LPL} is the same for v in both chains.

Proposition 3.7 (Consistent root graphs). *Two root graphs G_R and G'_R from two consistent H-OPERA chains are consistent.*

Definition 3.12 (Consistent root). *Two chains G_1 and G_2 are root consistent, if for every v contained in both chains, v is a root of j -th frame in G_1 , then v is a root of j -th frame in G_2 .*

Proposition 3.8. *For any two consistent OPERA chains G_1 and G_2 , they are root consistent.*

By consistent chains, if $G_1 \sim G_2$ and v belongs to both chains, then $G_1[v] = G_2[v]$. We can prove the proposition by induction. For $j = 0$, the first root set is the same in both G_1 and G_2 . Hence, it holds for $j = 0$. Suppose that the proposition holds for every j from 0 to k . We prove that it also holds for $j = k + 1$. Suppose that v is a root of frame f_{k+1} in G_1 . Then there exists a set S reaching 2/3 of members in G_1 of frame f_k such that $\forall u \in S (u \rightarrow v)$. As $G_1 \sim G_2$, and v in G_2 , then $\forall u \in S (u \in G_2)$. Since the proposition holds for $j=k$, As u is a root of frame f_k in G_1 , u is a root of frame f_k in G_2 . Hence, the set S of 2/3 members u happens before v in G_2 . So v belongs to f_{k+1} in G_2 .

Thus, all nodes have the same consistent root sets, which are the root sets in G^C . Frame numbers are consistent for all nodes.

Proposition 3.9 (Consistent Clotho). *A root r_k in the frame f_{a+3} can nominate a root r_a as Clotho if more than $2n/3$ roots in the frame f_{a+1} dominate r_a and r_k dominates the roots in the frame f_{a+1} .*

Proposition 3.10 (Consistent Atropos). *An Atropos is a Clotho that is decided as final. Event blocks in the subgraph rooted at the Atropos are also final events. Atropos blocks form a Main-chain, which allows time consensus ordering and responses to attacks.*

Proposition 3.11 (Consistent Main-chain). *Main-chain is a special sub-graph of the OPERA chain that stores Atropos vertices.*

An event block is called a *root* if the event block is linked to more than two-thirds of previous roots. A leaf vertex is also a root itself. With root event blocks, we can keep track of “vital” blocks that $2n/3$ of the network agree on. Each participant node computes to the same Main chain, which is the consensus of all nodes, from its own event blocks.

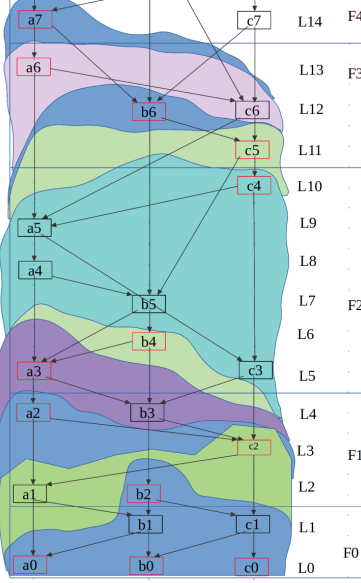


Figure 6: Peeling the subgraphs in topological sorting

3.7 Topological sort

In this section, we present an approach to ordering event blocks that reach finality. The new approach is deterministic subject to any deterministic layering algorithm.

After layers are assigned to achieve H-OPERA chain, root graphs are constructed. Then frames are determined for every event block. Once certain roots become Clothos, which are known by $2/3n + 1$ of the roots that are in turn known by $2/3n + 1$ of the nodes. The final step is to compute the ordering of event blocks that are final.

Figure 6 shows H-OPERA chain of event blocks. The Clothos/Atropos vertices are shown in red. For each Atropos vertex, we compute the subgraph under it. These subgraphs are shown in different colors. We process Atropos from lower layer to high layer.

Our algorithm for topological ordering the consensed event blocks is given in Algorithm 5. The algorithm requires that H-OPERA chain and layering assignment from ϕ are precomputed. The algorithm takes as input the set of Atropos vertices, denoted by A . We first order the atropos vertices using *SortVertexByLayer* function, which sorts vertices based on their layer, then lamport timestamp and then hash information of the event blocks. Second, we process every Atropos in the sorted order, and then compute the subgraph $G[a] = (V_a, E_a)$ under each Atropos. The set of vertices V_u contains vertices from V_a that is not yet process in U . We then apply *SortVertexByLayer* to order vertices in V_u and then append the ordered result into the final ordered list S .

Algorithm 5 TopoSort

- 1: Require: H-OPERA chain H , ϕ , ϕ_F .
 - 2: **function** TOPOSORT(A)
 - 3: $S \leftarrow$ empty list
 - 4: $U \leftarrow \emptyset$
 - 5: $Q \leftarrow \text{SortVertexByLayer}(A)$
 - 6: **for** Atropos $a \in Q$ **do**
 - 7: Compute the graph $G[a] = (V_a, E_a)$
 - 8: $V_u \leftarrow V_a \setminus U$
 - 9: $T \leftarrow \text{SortVertexByLayer}(V_u)$
 - 10: Append T into the end of the ordered list S .
 - 11: **function** SORTVERTEXBYLAYER(V)
 - 12: Sort the vertices in V_u by layer, lamport timestamp and hash in that order.
-

4 ONLAY Framework

We now present our ONLAY framework, which is a practical DAG-based solution to distributed ledgers.

Algorithm 6 shows the main function, which is the entry point to launch ONLAY framework. There are two main loops in the main function and they run asynchronously in parallel. The first loop attempts to create new event blocks and then communicate with other nodes about the new blocks. The second loop will accept any incoming sync request from other nodes. The node will retrieve the incoming event blocks and will send responses that consist of its known events.

Algorithm 6 ONLAY Main Function

```

1: function MAIN FUNCTION
2: loop:
3:    $\{n_i\} \leftarrow k\text{-PeerSelectionAlgo}()$ 
4:   Request sync to each node in  $\{n_i\}$ 
5:   (SyncPeer) all known events to each node in  $\{n_i\}$ 
6:   Create new event block:  $\text{newBlock}(n, n_i)$ 
7:   (SyncOther) Broadcast out the message
8:   Apply layering to obtain H-OPERA chain
9:   Root selection
10:  Compute root graph
11:  Compute global state
12:  Clotho selection
13:  Atropos selection
14:  Order vertices (and assign consensus time)
15:  Compute Main chain
16: loop:
17:   Accepts sync request from a node
18:   Sync all known events by Lachesis protocol

```

Specifically, in the first loop, a node makes synchronization requests to k other nodes and it then creates event blocks. Once created, the blocks are broadcasted to all other nodes. In line 3, a node runs the Node Selection Algorithm, to select k other nodes that it will need to communicate with. In line 4 and 5, the node sends synchronization requests to get the latest OPERA chain from the other nodes. Once receiving the latest event blocks from the responses, the node creates new event blocks (line 6). The node then broadcasts the created event block to all other nodes (line 7). After new event block is created, the node updates its OPERA chain, then will then apply layering (line 8). After layering is performed, H-OPERA chain is obtained. It then checks whether the block is a root (line 9) and then computes the root graph (line 10). In line 11, it computes global state from its H-OPERA chain. Then the node decides which roots are Clothos (line 12) and which then becomes Atropos (line 13). Once Atropos vertices are confirmed, the algorithm runs a topological sorting algorithm to get the final ordering for vertices at consensus (line 14). The main chain is constructed from the Atropos vertices that are found (line 15).

4.1 Peer selection algorithm

In order to create a new event block, a node needs to synchronize with k other nodes to get their latest top event blocks. The peer selection algorithm computes such set of k nodes that a node will need to make a synchronization request.

There are multiple ways to select k nodes from the set of n nodes. An simple approach can use a random selection from the pool of n nodes. A more complex approach is to define a cost model for the node selection. In general, L_ϕ protocol does not depend on how peer nodes are selected.

In ONLAY, we have tried a few peer selection algorithms, which are described as follows: (1) Random: randomly select a peer from n peers; (2) Least Used: select the least use peer(s). (3) Most Used (MFU): select the most use peer(s). (4) Fair: select a peer that aims for a balanced distribution. (5) Smart: select a peer based on some other criteria, such as successful throughput rates, number of own events.

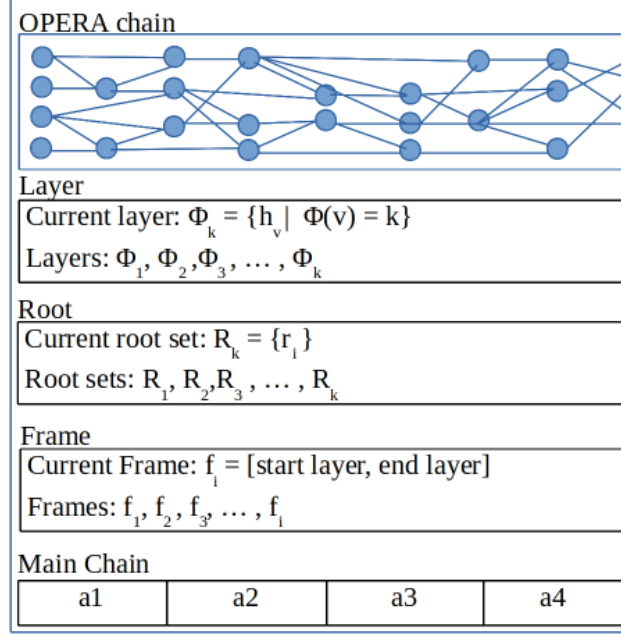


Figure 7: ONLAY node structure

4.2 Peer synchronization

Now we describe the steps to synchronize events between the nodes, as presented in Algorithm 7. Each node n_1 selects a random peer n_2 and sends a sync request indicating the local known events of n_1 to n_2 . Upon receiving the sync request from n_1 , n_2 will perform an event diff with its own known events and will then return the unknown events to n_1 .

Algorithm 7 EventSync

- 1: **function** SYNC-EVENTS()
 - 2: n_1 selects a random peer n_2 to synchronize with
 - 3: n_1 gets local known events (map[int]int)
 - 4: n_1 sends a RPC Sync request to peer
 - 5: n_2 receives a RPC Sync request
 - 6: n_2 does an EventDiff check on the known map (map[int]int)
 - 7: n_2 returns the unknown events to n_1
-

The algorithm assumes that a node always needs the events in topological ordering (specifically in reference to the lamport timestamps). Alternatively, one can simply use a fixed incrementing index to keep track of the top event for each node.

4.3 Node Structure

This section gives an overview of the node structure in ONLAY. A node in ONLAY system is a machine that participates with other nodes in the network.

Figure 7 shows the structure of a node. Each node consists of an OPERA chain of event blocks, which is the local view of the DAG. In each node, it stores the layering information (H-OPERA chain), roots, root graph, frames, Clotho set and Atropos set. The top event block indicates the most recently created event block by this node. Each frames contains the root set of that frame. A root can become a Clotho if it is known by $2/3n + 1$ of the roots, which are then known by another root. When a root becomes Clotho, its Clotho flag is set on. A Clotho is then promoted to an Atropos, once it is assigned with a final ordering number. The Main-chain is a data structure storing hash values of the Atropos blocks.

4.4 Event block creation

Like previous Lachesis protocol, L_ϕ protocol allows every node to create an event block. A new event block refers to the top event blocks of k other nodes by using their hash values. One of the references is a self-ref that references to an event block of the same node. The other $k-1$ references refer to the top event nodes of other $k-1$ nodes. An event block can only be created if the referencing event blocks exist.

4.5 Root selection

There is a Root selection algorithm given in Fantom paper [23]. In ONLAY, we propose a new approach that uses root graph and frame assignment (see Section 3.4 and Section 3.5).

The root graph $G_R = (V_R, E_R)$ contains vertices as roots $V_R \subseteq V$, and the set of edges E_R the reduced edges from E , such that $(u, v) \in E_R$ only if u and v are roots and there is a path from u to v following edges in E .

A root graph contains the n genesis vertices i.e., the n leaf event blocks. A vertex v that can reach at least $2/3n + 1$ of the current set of all roots V_R is itself a root. For each root r_i that new root r reaches, we include a new edge (r, r_i) into the set of root edges E_R . Note that, if a root r reaches two other roots r_1 and r_2 of the same node, and $\phi(r_1) > \phi(r_2)$ then we include only one edge (r, r_1) in E_R . This constraint ensures that each root of a node can have at most one edge to any other node.

The steps to build the root graph out of an H-OPERA chain is given in Algorithm 8. The algorithm processes from layer 0 to maximum layer l . Line 3, R is the set of active roots, we will update the set after finding new roots at each layer. Line 4 and 5 set the initial value of the vertex set and edge set of the root graph. For each layer i , it runs the steps from line 8 to 18. Z is the set of new roots at layer i , in which Z is initially set to empty (Line 8). We then process each vertex v in the layer ϕ_i (line 9). We compute the set S of all active roots in R that v can reach. If more than $2/3n$ of R member is reached, we can promote v to be a new root (line 11). For every root v_i in S that is reachable from v , we add an add (v, v_i) in the E_R . In line 14-15, when v becomes a new root, we add v into V_R of the root graph, and also into Z which keeps track of the new roots found at layer i . The second inner loop will update the current active set R with the new roots found at layer i (line 16-18).

Algorithm 8 Root graph algorithm

```

1: Require: H-OPERA chain
2: Output: root graph  $G_R = (V_R, E_R)$ 
3:  $R \leftarrow$  set of leaf events
4:  $V_R \leftarrow R$ 
5:  $E_R \leftarrow \emptyset$ 
6: function BUILDROOTGRAPH( $\phi, l$ )
7:   for each layer  $i=1..l$  do
8:      $Z \leftarrow \emptyset$ 
9:     for each vertex  $v$  in layer  $\phi_i$  do
10:       $S \leftarrow$  the set of vertices in  $R$  that  $v$  reaches
11:      if  $|S| > 2/3n$  then
12:        for each vertex  $v_i$  in  $S$  do
13:           $E_R \leftarrow E_R \cup \{(v, v_i)\}$ 
14:           $V_R \leftarrow V_R \cup \{v\}$ 
15:           $Z \leftarrow Z \cup \{v\}$ 
16:      for each vertex  $v_j$  in  $Z$  do
17:        Let  $v_{old}$  be a root in  $R$  such that  $cr(v_j) = cr(v_{old})$ 
18:         $R \leftarrow R \setminus \{v_{old}\} \cup \{v_j\}$ 

```

The algorithm always updates the active set R of roots at each layer. The number of active roots in R is always n , though the active roots can be from different layers.

After the root graph is constructed, we can assign frame numbers to the roots as specified in Section 3.5). Frames are assigned to the roots using root-layering algorithm. When a new root reaches more than $2/3n$ of the roots of a frame i , the root is assigned to the next frame.

4.6 Fork detection and removal

When a node n receives Sync request from another node, n will check if the received events would cause a fork. In a 1/3-BFT, we proved in our previous paper [23] that there exists an honest node that sees the fork. It will remove one of the forked events from its OPERA chain, and then notifies the fork to all other nodes.

When a root is selected, it is certain that if there exists a fork in the subgraph under the root event block, the fork should be detected and removed.

Proposition 4.1 (Fork-free). *For any Clotho v_c , the subgraph $G[v_c]$ is fork-free.*

When a root r becomes a Clotho, the subgraph $G[r]$ has already detected forks, if any. The fork was already removed and the node already synchronized with other nodes about the fork. By definition of Clotho, the r is known by more than $2/3n$ events, which are in turn known by another root event. Thus, more than $2/3n$ knew about and removed the detected forks that were synchronized.

Proposition 4.2 (Fork-free Global chain). *The global consistent OPERA chain G^C is fork-free.*

Recall that the global consistent chain G^C consists of the finalised events i.e., the Clothos and its subgraphs that get final ordering at consensus. Those finally ordered Clothos are Atropos vertices. From the above proposition, for every Clotho v_c , there is no fork in its subgraph $G[v_c]$. Thus, we can prove by induction. We

4.7 Compute global state

From the local view H-OPERA chain of its own, a node can estimate the subgraph of H that other nodes know at the time. The intuition is that a node n_i , from its history H , conservatively knows that the other node n_j may just know the event blocks under the top event of n_j . Let t_i denote the top event of a node n_i in the local chain H . From the local state H , a node can determine all top events of all nodes in H .

Algorithm 9 shows an algorithm that a node can estimate a global state from a local state of a node. The steps show how to compute the global state H^C from the local state H . The set of processed nodes S is initially empty (line 2). Let L be the stack of all the layers so far. The processed set S has less than n nodes, we process the layers one at a time in the while loop. We pop the stack L to get the highest layer L_i (line 4). For each block v in L_i , v is removed from L_i if the the creator of v is already processed (line 5-7). After that, the set L_i contains only the event blocks whose creators do not belong to S . Line 8 randomly picks a block v from the remainder in L_i . Then the processed set S is appended with the creator of v . Once all nodes are processed, the global state of the node is the induced subgraph from the remainder nodes in L , which excludes the nodes removed by the algorithm.

Algorithm 9 Estimate a Global State from Local State

```

1: procedure GLOBALSTATE( $G, L$ )
2:    $S \leftarrow \emptyset$ 
3:   while  $|S| < n$  do
4:      $L_i \leftarrow \text{pop}(L)$ 
5:     for  $v \in L_i$  do
6:       if  $cr(v) \in S$  then
7:         Remove  $v$  from  $L_i$ 
8:     Remove  $v$  (randomly) from  $L_i$ 
9:      $S \leftarrow S \cup cr(v)$ 
10:   $G^C \leftarrow$  induced graph from remainder nodes in  $L$ 

```

When a node n_i has an event block v , it knows that node all other nodes n_j may not concurrently know the event block v . The above algorithm determines the highest event block v that n_i know for sure all the other nodes, from n_i 's view, already knows it. One can repeat the algorithm several times, to determine n_i knows that n_j knows that n_k knows a certain event v .

4.8 Clotho selection

For practical BFT, consensus is reached once an event block is known by more than $2/3n$ of the nodes and that information is further known by $2/3n$ of the nodes. Similarly, we define the consensus of a root, at which condition the root becomes a Clotho, if it can be reached by more than $2/3n$ of the roots and the information is known by another $2/3n$ of the roots.

For a root r at frame f_i , if there exists a root r' at a frame f_j such that $j \geq i+2$, then the root r reaches pBFT consensus. The root r' is called the *nominator*, which nominates r to become a Clotho.

Proposition 4.3 (Global Clotho). *For any two honest nodes n_i and n_j , if c is a Clotho in H_i of n_i , then c is also a Clotho in H_j of n_j .*

4.9 Atropos Selection

After a new Clotho is found, we will assign the final *consensus time* for it. When a Clotho is assigned with a consensus time, it becomes an Atropos.

In order to assign consensus time for an Clotho, Atropos selection process first sorts the Clothos based on topological ordering. Section 3.7 gives more details on the topological ordering and consensus time assignment.

Proposition 4.4 (Global Atropos). *For any two honest nodes n_i and n_j , if a is an Atropos in H_i of n_i , then a is also an Atropos in H_j of n_j .*

After Atropos consensus time is computed, the Clotho is nominated to Atropos and each node stores the hash value of Atropos and Atropos consensus time in Main-Chain (blockchain). The Main-chain is used for time order between event blocks. The proof of Atropos consensus time selection is shown in the section 6.2.

4.10 Transaction confirmations

Here are some steps for a transaction to reach finality in our system. First, when a user submits a transaction into a node, a successful submission receipt will be issued to the client as a confirmation of the submitted transaction. Second, the node will batch the user submitted transaction(s) into a new event block amended into a DAG structure of event blocks and then will broadcast the event block to all other nodes of the system. The node will send a receipt confirming that the containing event block identifier is being processed. Third, when the event block is known by majority of the nodes (e.g., it becomes a Root event block), or being known by such a Root block, the node will send a confirmation that the event block is acknowledged by a majority of the nodes. Fourth, our system will determine the condition at which a Root event block becomes a Clotho for being further acknowledged by a majority of the nodes. A confirmation is then sent to the client to indicate that the event block has come to the semi-final stage as a Clotho or being confirmed by a Clotho. Fifth, after the Clotho stage, we will determine the consensus timestamp for the Clotho and its dependent event blocks. Once an event block gets the final consensus timestamp, it is finalized and a final confirmation will be issued to the client that the transaction has been successfully finalized. Thus, there will be a total of five confirmations to be sent and the fifth receipt is the final confirmation of a successful transaction.

There are some cases that a submitted transaction can fail to reach finality. Examples include a transaction does not pass the validation, e.g., insufficient account balance, or violation of account rules. The other kind of failure is when the integrity of DAG structure and event blocks is not complied due to the existence of compromised or faulty nodes. In such unsuccessful cases, the event blocks are marked for removal and detected issues are notified to all nodes. Receipts of the failure will be sent to the client.

5 Conclusion

In this paper, we present a new framework, namely ONLAY, for a scalable asynchronous distributed system with practical BFT. We propose a new consensus protocol, called L_ϕ , to address a more reliable consensus compared to precedent DAG-based approaches. The new consensus protocol uses the well-known concept of layering of DAG to achieve a deterministic consensus on the OPERA chain.

We further propose new approaches to optimizing the OPERA chain such as using H-OPERA, root graph, root layering, improved frame assignment, improved Clotho and Atropos selection, and Main-chain. These new models, data structures and algorithm deliver a more intuitive model and a faster more reliable consensus in a distributed system.

We have presented a formal definitions and semantics for L_ϕ protocol in Section 2. Our formal proof of pBFT for our Lachesis protocol is given in Section 6. Our work extends the formal foundation established in Fantom paper [23], which is the first that studies concurrent common knowledge semantics [24] in DAG-based protocols.

6 Appendix

6.1 Basic Definitions

Each node stores a local view of the history built from the L_ϕ protocol. We cover some basic definitions used in our previous Lachesis papers [1, 23]. We also include definitions of layering, which our ONLAY framework and L_ϕ protocol is based on.

Definition 6.1 (node). *Each machine, which is a participant in the Lachesis protocol, is called a node.*

Let n be the number of nodes in a network running ONLAY.

Definition 6.2 (peer node). *A node n_i has $n-1$ peer nodes.*

The history can be represented by a directed acyclic graph $G = (V, E)$, where V is a set of vertices and E is a set of edges. Each vertex in a row (node) represents an event. Time flows left-to-right of the graph, so left vertices represent earlier events in history. A path p in G is a sequence of vertices (v_1, v_2, \dots, v_k) by following the edges in E . Let v_c be a vertex in G . A vertex v_p is the *parent* of v_c if there is an edge from v_p to v_c . A vertex v_a is an *ancestor* of v_c if there is a path from v_a to v_c .

6.1.1 OPERA chain

Definition 6.3 (event block). *Each node can create event blocks, send (receive) messages to (from) other nodes.*

Suppose a node n_i creates an event v_c after an event v_s in n_i . Each event block has exactly k references. One of the references is self-reference, and the other $k-1$ references point to the top events of n_i 's $k-1$ peer nodes.

Definition 6.4 (OPERA chain). *OPERA chain is a DAG graph $G = (V, E)$ consisting of V vertices and E edges. Each vertex $v_i \in V$ is an event block. An edge $(v_i, v_j) \in E$ refers to a hashing reference from v_i to v_j ; that is, $v_i \hookrightarrow v_j$.*

Definition 6.5 (vertex). *An event block is a vertex of the OPERA chain.*

Definition 6.6 (Leaf). *The first created event block of a node is called a leaf event block.*

Definition 6.7 (top event). *An event v is a top event of a node n_i if there is no other event in n_i referencing v .*

Definition 6.8 (self-ref). *An event v_s is called “self-ref” of event v_c , if the self-ref hash of v_c points to the event v_s . Denoted by $v_c \hookrightarrow^s v_s$.*

Definition 6.9 (ref). *An event v_r is called “ref” of event v_c if the reference hash of v_c points to the event v_r . Denoted by $v_c \hookrightarrow^r v_r$.*

For simplicity, we can use \hookrightarrow to denote a reference relationship (either \hookrightarrow^r or \hookrightarrow^s).

Definition 6.10 (self-ancestor). *An event block v_a is self-ancestor of an event block v_c if there is a sequence of events such that $v_c \hookrightarrow^s v_1 \hookrightarrow^s \dots \hookrightarrow^s v_m \hookrightarrow^s v_a$. Denoted by $v_c \hookrightarrow^{sa} v_a$.*

Definition 6.11 (ancestor). *An event block v_a is an ancestor of an event block v_c if there is a sequence of events such that $v_c \hookrightarrow v_1 \hookrightarrow \dots \hookrightarrow v_m \hookrightarrow v_a$. Denoted by $v_c \hookrightarrow^a v_a$.*

For simplicity, we simply use $v_c \hookrightarrow^a v_s$ to refer both ancestor and self-ancestor relationship, unless we need to distinguish the two cases.

Definition 6.12 (creator). *If a node n_x creates an event block v , then the creator of v , denoted by $cr(v)$, is n_x .*

We introduce pseudo vertices, *top* and *bot*, of the OPERA chain G .

Definition 6.13 (pseudo top). *A pseudo vertex, called top, is the parent of all top event blocks. Denoted by \top .*

Definition 6.14 (pseudo bottom). *A pseudo vertex, called bottom, is the child of all leaf event blocks. Denoted by \perp .*

With the pseudo vertices, we have \perp happened before all event blocks. Also all event blocks happened before \top . That is, for all event v_i , $\perp \rightarrow v_i$ and $v_i \rightarrow \top$.

Here, we introduce a new idea of subgraph rooted at a vertex v .

Definition 6.15 (subgraph). *For a vertex v in a DAG G , let $G[v] = (V_v, E_v)$ denote an induced subgraph of G such that V_v consists of all ancestors of v including v , and E_v is the induced edges of V_v in G .*

6.1.2 Happened-Before relation

Definition 6.16 (Happened-Immediate-Before). *An event block v_x is said Happened-Immediate-Before an event block v_y if v_x is a (self-) ref of v_y . Denoted by $v_x \mapsto v_y$.*

Definition 6.17 (Happened-Before). *An event block v_x is said Happened-Before an event block v_y if v_x is a (self-) ancestor of v_y . Denoted by $v_x \rightarrow v_y$.*

The happens-before relation is the transitive closure of happens-immediately-before. An event v_x happened before an event v_y if one of the followings happens: (a) $v_y \hookrightarrow^s v_x$, (b) $v_y \hookrightarrow^r v_x$, or (c) $v_y \hookrightarrow^a v_x$. We come up with the following proposition:

Proposition 6.1 (Happened-Immediate-Before OPERA). *$v_x \mapsto v_y$ iff $v_y \hookrightarrow v_x$ iff edge $(v_y, v_x) \in E$ of OPERA chain.*

Lemma 6.2 (Happened-Before Lemma). *$v_x \rightarrow v_y$ iff $v_y \hookrightarrow^a v_x$.*

Definition 6.18 (concurrent). *Two event blocks v_x and v_y are said concurrent if neither of them happened before the other. Denoted by $v_x \parallel v_y$.*

Given two vertices v_x and v_y both contained in two OPERA chains G_1 and G_2 on two nodes. We have the following: (1) $v_x \rightarrow v_y$ in G_1 iff $v_x \rightarrow v_y$ in G_2 ; (2) $v_x \parallel v_y$ in G_1 iff $v_x \parallel v_y$ in G_2 .

6.1.3 Layering Definitions

For a directed acyclic graph $G=(V,E)$, a layering is to assign a layer number to each vertex in G .

[Layering] A layering (or levelling) of G is a topological numbering ϕ of G , $\phi : V \rightarrow \mathbb{Z}$, mapping the set of vertices V of G to integers such that $\phi(v) \geq \phi(u) + 1$ for every directed edge $(u, v) \in E$. If $\phi(v)=j$, then v is a layer- j vertex and $V_j = \phi^{-1}(j)$ is the j th layer of G .

A layering ϕ of G partitions the set of vertices V into a finite number l of *non-empty* disjoint subsets (called layers) V_1, V_2, \dots, V_l , such that $V = \cup_{i=1}^l V_i$. Each vertex is assigned to a layer V_j , where $1 \leq j \leq l$, such that every edge $(u, v) \in E$, $u \in V_i$, $v \in V_j$, $1 \leq i < j \leq l$.

Definition 6.19 (Hierarchical graph). *For a layering ϕ , the produced graph $H=(V, E, \phi)$ is a hierarchical graph, which is also called an l -layered directed graph and could be represented as $H=(V_1, V_2, \dots, V_l; E)$.*

6.2 Proof of Byzantine Fault Tolerance for L_ϕ Algorithm

This section presents a proof of our L_ϕ . We aim to show that our consensus is Byzantine fault tolerant when at most one-third of participants are compromised. We first provide some definitions, lemmas and theorems. Then we validate the Byzantine fault tolerance.

We introduce a new notion of H-OPERA chain, which is built on top of the OPERA chain. By applying a layering ϕ on the OPERA chain, one can obtain the hierarchical graph of G , which is called H-OPERA chain.

Definition 6.20 (H-OPERA chain). *An H-OPERA chain is the result hierarchical graph $H = (V, E, \phi)$.*

Definition 6.21 (Root). *The leaf event block of a node is a root. When an event block v can reach more than $2n/3$ of the roots in the previous frames, v becomes a root.*

Definition 6.22 (Root set). *The set of all first event blocks (leaf events) of all nodes form the first root set R_1 ($|R_1| = n$). The root set R_k consists of all roots r_i such that $r_i \notin R_i$, $\forall i = 1..(k-1)$ and r_i can reach more than $2n/3$ other roots in the current frame, $i = 1..(k-1)$.*

Definition 6.23 (Root graph). *A root graph $G_R=(V_R, E_R)$ is a directed graph consisting of vertices as roots and edges represent their reachability.*

In the root graph G_R , the set of roots V_R is a subset of V . The set of edges E_R is the reduced edges from E , in that $(u, v) \in E_R$ only if u and v are roots and u can reach v following edges in E i.e., $v \rightarrow u$.

Definition 6.24 (Frame). *Frame f_i is a natural number that separates Root sets.*

The root set at frame f_i is denoted by R_i .

Definition 6.25 (consistent chains). *OPERA chains G_1 and G_2 are consistent iff for any event v contained in both chains, $G_1[v] = G_2[v]$. Denoted by $G_1 \sim G_2$.*

When two consistent chains contain the same event v , both chains contain the same set of ancestors for v , with the same reference and self-ref edges between those ancestors:

Theorem 6.3. *All nodes have consistent OPERA chains.*

Proof. If two nodes have OPERA chains containing event v , then they have the same k hashes contained within v . A node will not accept an event during a sync unless that node already has k references for that event, so both OPERA chains must contain k references for v . The cryptographic hashes are assumed to be secure, therefore the references must be the same. By induction, all ancestors of v must be the same. Therefore, the two OPERA chains are consistent. \square

Definition 6.26 (fork). *The pair of events (v_x, v_y) is a fork if v_x and v_y have the same creator, but neither is a self-ancestor of the other. Denoted by $v_x \pitchfork v_y$.*

For example, let v_z be an event in node n_1 and two child events v_x and v_y of v_z . if $v_x \hookrightarrow^s v_z, v_y \hookrightarrow^s v_z, v_x \not\hookrightarrow^s v_y, v_y \not\hookrightarrow^s v_x$, then (v_x, v_y) is a fork. The fork relation is symmetric; that is $v_x \pitchfork v_y$ iff $v_y \pitchfork v_x$.

Lemma 6.4. $v_x \pitchfork v_y$ iff $cr(v_x) = cr(v_y)$ and $v_x \parallel v_y$.

Proof. By definition, (v_x, v_y) is a fork if $cr(v_x) = cr(v_y)$, $v_x \not\hookrightarrow^a v_y$ and $v_y \not\hookrightarrow^a v_x$. Using Happened-Before, the second part means $v_x \not\rightarrow v_y$ and $v_y \not\rightarrow v_x$. By definition of concurrent, we get $v_x \parallel v_y$. \square

Lemma 6.5. (fork detection). *If there is a fork $v_x \pitchfork v_y$, then v_x and v_y cannot both be roots on honest nodes.*

Proof. Here, we show a proof by contradiction. Any honest node cannot accept a fork so v_x and v_y cannot be roots on the same honest node. Now we prove a more general case. Suppose that both v_x is a root of n_x and v_y is root of n_y , where n_x and n_y are honest nodes. Since v_x is a root, it reached events created by more than $2/3$ of member nodes. Similarly, v_y is a root, it reached events created by more than $2/3$ of member nodes. Thus, there must be an overlap of more than $n/3$ members of those events in both sets. Since we assume less than $n/3$ members are not honest, so there must be at least one honest member in the overlap set. Let n_m be such an honest member. Because n_m is honest, n_m does not allow the fork. This contradicts the assumption. Thus, the lemma is proved. \square

Each node n_i has an OPERA chain G_i . We define a consistent chain from a sequence of OPERA chain G_i .

Definition 6.27 (consistent chain). *A global consistent chain G^C is a chain if $G^C \sim G_i$ for all G_i .*

We denote $G \sqsubseteq G'$ to stand for G is a subgraph of G' .

Lemma 6.6. $\forall G_i (G^C \sqsubseteq G_i)$.

Lemma 6.7. $\forall v \in G^C \forall G_i (G^C[v] \sqsubseteq G_i[v])$.

Lemma 6.8. $(\forall v_c \in G^C) (\forall v_p \in G_i) ((v_p \rightarrow v_c) \Rightarrow v_p \in G^C)$.

Now we state the following important propositions.

Definition 6.28 (consistent root). *Two chains G_1 and G_2 are root consistent, if for every v contained in both chains, and v is a root of j -th frame in G_1 , then v is a root of j -th frame in G_2 .*

Proposition 6.9. *If $G_1 \sim G_2$, then G_1 and G_2 are root consistent.*

Proof. By consistent chains, if $G_1 \sim G_2$ and v belongs to both chains, then $G_1[v] = G_2[v]$. We can prove the proposition by induction. For $j = 0$, the first root set is the same in both G_1 and G_2 . Hence, it holds for $j = 0$. Suppose that the proposition holds for every j from 0 to k . We prove that it also holds for $j = k + 1$. Suppose that v is a root of frame f_{k+1} in G_1 . Then there exists a set S reaching $2/3$ of members in G_1 of frame f_k such that $\forall u \in S (u \rightarrow v)$. As $G_1 \sim G_2$, and v in G_2 , then $\forall u \in S (u \in G_2)$. Since the proposition holds for $j=k$, As u is a root of frame f_k in G_1 , u is a root of frame f_k in G_2 . Hence, the set S of $2/3$ members u happens before v in G_2 . So v belongs to f_{k+1} in G_2 . The proposition is proved. \square

From the above proposition, one can deduce the following:

Lemma 6.10. G^C is root consistent with G_i for all nodes.

Thus, all nodes have the same consistent root sets, which are the root sets in G^C . Frame numbers are consistent for all nodes.

Definition 6.29 (Clotho). A root r_k in the frame f_{a+3} can nominate a root r_a as Clotho if more than $2n/3$ roots in the frame f_{a+1} Happened-Before r_a and r_k Happened-Before the roots in the frame f_{a+1} .

Lemma 6.11. For any root set R , all nodes nominate same root into Clotho.

Proof. Based on Theorem 6.14, each node nominates a root into Clotho via the flag table. If all nodes have an OPERA chain with same shape, the values in flag table should be equal to each other in OPERA chain. Thus, all nodes nominate the same root into Clotho since the OPERA chain of all nodes has same shape. \square

Lemma 6.12 (Fork Lemma). For any Clotho v_c in an honest node, there does not exist a pair of forked event blocks (v_i, v_j) under $G[v_c]$.

Proof. Assume there is a Clotho v_c in node n_i such that there exists a pair of forked events (v_i, v_j) under $G[v_c]$.

Recall that from previous Fork detection lemma, if an honest node contains any fork (v_i, v_j) , then they cannot both be roots. One of the forked events was removed and the detected fork was notified across other nodes. Thus, v_i and v_j cannot both be roots. Let r_i denote a root at lowest layer that knows both v_i and v_j . It is obvious that r_i belongs to $G[v_c]$. Since v_c is a Clotho, let assume there is a root r_j that nominates v_c . There are two cases: (1) r_i is different from both v_i and v_j ; (2) r_i is one of v_i and v_j . If r_i is part of the fork, we can detect in r_i and completely remove the fork in r_j . \square

Lemma 6.13. For a fork that happened-before a root v in OPERA chain, any root u must have seen the fork before nominating v to be a Clotho.

Proof. Suppose that a node creates forked event blocks (v_x, v_y) . Suppose root v happens before v_x and v_y . Suppose there is a root u that nominates v as a Clotho. To create two Clothos that can reach both events, the event blocks should reach by more than $2n/3$ nodes. Therefore, the OPERA chain can structurally detect the fork before roots become Clotho. \square

Theorem 6.14 (OPERA consistency). All nodes grows up into same consistent OPERA chain G^C , which is fork-free.

Proof. Suppose that there are two event blocks v_x and v_y contained in both G_1 and G_2 , and their path between v_x and v_y in G_1 is not equal to that in G_2 . We can consider that the path difference between the nodes is a kind of fork attack. Based on Lemma 6.12, if an attacker forks an event block, each chain of G_1 and G_2 can detect and remove the fork before the Clotho is generated. Thus, any two nodes have consistent OPERA chain. \square

Theorem 6.15 (H-OPERA consistency). All nodes grows up into same consistent H-OPERA chain H^C .

Proof. The layering algorithm in L_ϕ when applies to the same graph G , produces the same layering information. Since all nodes builds into the same consistent OPERA chain G^C . The layering on G^C is consistent across the nodes. \square

Proposition 6.16. All nodes have the same Clothos and Atropos.

Theorem 6.17. All nodes have consistent Main chains.

Using the global consistent graph, all nodes grows into the same G^C . Thus, the set of Clothos and Atropos is consistent in all the nodes.

Theorem 6.18 (Finality). All nodes produce the same ordering and consensus time for finalized event blocks.

Proof. Given the set of Clotho is consistent and the layering is consistent across the nodes, the topological sorting of Clotho based on layering is consistent. Thus, the ordering of vertices in each the subgraphs under a Clotho is the same across the nodes. Hence, the consensus time of final event blocks which is assigned based on the topological ordering, is consistent. \square

6.3 Semantics of Lachesis protocol

This section gives the formal semantics of Lachesis consensus protocol. We use CCK model [24] of an asynchronous system as the base of the semantics of our Lachesis protocol. Events are ordered based on Lamport's happens-before relation. In particular, we use Lamport's theory to describe global states of an asynchronous system.

We present notations and concepts, which are important for Lachesis protocol. In several places, we adapt the notations and concepts of CCK paper to suit our Lachesis protocol.

An asynchronous system consists of the following:

Definition 6.30 (process). *A process p_i represents a machine or a node. The process identifier of p_i is i . A set $P = \{1, \dots, n\}$ denotes the set of process identifiers.*

Definition 6.31 (channel). *A process i can send messages to process j if there is a channel (i, j) . Let $C \subseteq \{(i, j) \text{ s.t. } i, j \in P\}$ denote the set of channels.*

Definition 6.32 (state). *A local state of a process i is denoted by s_j^i .*

A local state consists of a sequence of event blocks $s_j^i = v_0^i, v_1^i, \dots, v_j^i$.

In a DAG-based protocol, each v_j^i event block is valid only the reference blocks exist before it. From a local state s_j^i , one can reconstruct a unique DAG. That is, the mapping from a local state s_j^i into a DAG is *injective* or one-to-one. Thus, for Lachesis, we can simply denote the j -th local state of a process i by the OPERA chain g_j^i (often we simply use G_i to denote the current local state of a process i).

Definition 6.33 (action). *An action is a function from one local state to another local state.*

Generally speaking, an action can be either: a $send(m)$ action where m is a message, a $receive(m)$ action, and an internal action. A message m is a triple $\langle i, j, B \rangle$ where $i \in P$ is the sender of the message, $j \in P$ is the message recipient, and B is the body of the message. Let M denote the set of messages. In Lachesis protocol, B consists of the content of an event block v . Semantics-wise, in Lachesis, there are two actions that can change a process's local state: creating a new event and receiving an event from another process.

Definition 6.34 (event). *An event is a tuple $\langle s, \alpha, s' \rangle$ consisting of a state, an action, and a state.*

Sometimes, the event can be represented by the end state s' . The j -th event in history h_i of process i is $\langle s_{j-1}^i, \alpha, s_j^i \rangle$, denoted by v_j^i .

Definition 6.35 (local history). *A local history h_i of process i is a (possibly infinite) sequence of alternating local states — beginning with a distinguished initial state. A set H_i of possible local histories for each process i in P .*

The state of a process can be obtained from its initial state and the sequence of actions or events that have occurred up to the current state. In Lachesis protocol, we use append-only semantics. The local history may be equivalently described as either of the following:

$$h_i = s_0^i, \alpha_1^i, \alpha_2^i, \alpha_3^i \dots$$

$$h_i = s_0^i, v_1^i, v_2^i, v_3^i \dots$$

$$h_i = s_0^i, s_1^i, s_2^i, s_3^i, \dots$$

In Lachesis, a local history is equivalently expressed as:

$$h_i = g_0^i, g_1^i, g_2^i, g_3^i, \dots$$

where g_j^i is the j -th local OPERA chain (local state) of the process i .

Definition 6.36 (run). *Each asynchronous run is a vector of local histories. Denoted by $\sigma = \langle h_1, h_2, h_3, \dots, h_N \rangle$.*

Let Σ denote the set of asynchronous runs.

We can now use Lamport's theory to talk about global states of an asynchronous system. A global state of run σ is an n -vector of prefixes of local histories of σ , one prefix per process. The happens-before relation can be used to define a consistent global state, often termed a consistent cut, as follows.

Definition 6.37 (Consistent cut). *A consistent cut of a run σ is any global state such that if $v_x^i \rightarrow v_y^j$ and v_y^j is in the global state, then v_x^i is also in the global state. Denoted by $c(\sigma)$.*

By Theorem 6.3, all nodes have consistent local OPERA chains. The concept of consistent cut formalizes such a global state of a run. A consistent cut consists of all consistent OPERA chains. A received event block exists in the global state implies the existence of the original event block. Note that a consistent cut is simply a vector of local states; we will use the notation $\mathbf{c}(\sigma)[i]$ to indicate the local state of i in cut \mathbf{c} of run σ .

The formal semantics of an asynchronous system is given via the satisfaction relation \vdash . Intuitively $\mathbf{c}(\sigma) \vdash \phi$, “ $\mathbf{c}(\sigma)$ satisfies ϕ ,” if fact ϕ is true in cut \mathbf{c} of run σ . We assume that we are given a function π that assigns a truth value to each primitive proposition p . The truth of a primitive proposition p in $\mathbf{c}(\sigma)$ is determined by π and \mathbf{c} . This defines $\mathbf{c}(\sigma) \vdash p$.

Definition 6.38 (equivalent cuts). *Two cuts $\mathbf{c}(\sigma)$ and $\mathbf{c}'(\sigma')$ are equivalent with respect to i if:*

$$\mathbf{c}(\sigma) \sim_i \mathbf{c}'(\sigma') \Leftrightarrow \mathbf{c}(\sigma)[i] = \mathbf{c}'(\sigma')[i]$$

We introduce two families of modal operators, denoted by K_i and P_i , respectively. Each family indexed by process identifiers. Given a fact ϕ , the modal operators are defined as follows:

Definition 6.39 (i knows ϕ). $K_i(\phi)$ represents the statement “ ϕ is true in all possible consistent global states that include i ’s local state”.

$$\mathbf{c}(\sigma) \vdash K_i(\phi) \Leftrightarrow \forall \mathbf{c}'(\sigma') (\mathbf{c}'(\sigma') \sim_i \mathbf{c}(\sigma) \Rightarrow \mathbf{c}'(\sigma') \vdash \phi)$$

Definition 6.40 (i partially knows ϕ). $P_i(\phi)$ represents the statement “there is some consistent global state in this run that includes i ’s local state, in which ϕ is true.”

$$\mathbf{c}(\sigma) \vdash P_i(\phi) \Leftrightarrow \exists \mathbf{c}'(\sigma) (\mathbf{c}'(\sigma) \sim_i \mathbf{c}(\sigma) \wedge \mathbf{c}'(\sigma) \vdash \phi)$$

The next modal operator is written M^C and stands for “majority concurrently knows.” This is adapted from the “everyone concurrently knows” in CCK paper [24]. The definition of $M^C(\phi)$ is as follows.

Definition 6.41 (majority concurrently knows).

$$M^C(\phi) =_{def} \bigwedge_{i \in S} K_i P_i(\phi),$$

where $S \subseteq P$ and $|S| > 2n/3$.

In the presence of one-third of faulty nodes, the original operator “everyone concurrently knows” is sometimes not feasible. Our modal operator $M^C(\phi)$ fits precisely the semantics for BFT systems, in which unreliable processes may exist.

The last modal operator is concurrent common knowledge (CCK), denoted by C^C .

Definition 6.42 (concurrent common knowledge). $C^C(\phi)$ is defined as a fixed point of $M^C(\phi \wedge X)$

CCK defines a state of process knowledge that implies that all processes are in that same state of knowledge, with respect to ϕ , along some cut of the run. In other words, we want a state of knowledge X satisfying: $X = M^C(\phi \wedge X)$. C^C will be defined semantically as the weakest such fixed point, namely as the greatest fixed-point of $M^C(\phi \wedge X)$. It therefore satisfies:

$$C^C(\phi) \Leftrightarrow M^C(\phi \wedge C^C(\phi))$$

Thus, $P_i(\phi)$ states that there is some cut in the same asynchronous run σ including i ’s local state, such that ϕ is true in that cut.

Note that ϕ implies $P_i(\phi)$. But it is not the case, in general, that $P_i(\phi)$ implies ϕ or even that $M^C(\phi)$ implies ϕ . The truth of $M^C(\phi)$ is determined with respect to some cut $\mathbf{c}(\sigma)$. A process cannot distinguish which cut, of the perhaps many cuts that are in the run and consistent with its local state, satisfies ϕ ; it can only know the existence of such a cut.

Definition 6.43 (global fact). *Fact ϕ is valid in system Σ , denoted by $\Sigma \vdash \phi$, if ϕ is true in all cuts of all runs of Σ .*

$$\Sigma \vdash \phi \Leftrightarrow (\forall \sigma \in \Sigma) (\forall \mathbf{c}) (\mathbf{c}(\sigma) \vdash \phi)$$

Definition 6.44. *Fact ϕ is valid, denoted $\vdash \phi$, if ϕ is valid in all systems, i.e. $(\forall \Sigma) (\Sigma \vdash \phi)$.*

Definition 6.45 (local fact). *A fact ϕ is local to process i in system Σ if $\Sigma \vdash (\phi \Rightarrow K_i \phi)$*

Theorem 6.19. *If ϕ is local to process i in system Σ , then $\Sigma \vdash (P_i(\phi) \Rightarrow \phi)$.*

Lemma 6.20. *If fact ϕ is local to 2/3 of the processes in a system Σ , then $\Sigma \vdash (M^C(\phi) \Rightarrow \phi)$ and furthermore $\Sigma \vdash (C^C(\phi) \Rightarrow \phi)$.*

Definition 6.46. A fact ϕ is attained in run σ if $\exists \mathbf{c}(\sigma)(\mathbf{c}(\sigma) \vdash \phi)$.

Often, we refer to “knowing” a fact ϕ in a state rather than in a consistent cut, since knowledge is dependent only on the local state of a process. Formally, i knows ϕ in state s is shorthand for

$$\forall \mathbf{c}(\sigma)(\mathbf{c}(\sigma)[i] = s \Rightarrow \mathbf{c}(\sigma) \vdash \phi)$$

For example, if a process in Lachesis protocol knows a fork exists (i.e., ϕ is the existence of fork) in its local state s (i.e., g_j^i), then a consistent cut contains the state s will know the existence of that fork.

Definition 6.47 (i learns ϕ). Process i learns ϕ in state s_j^i of run σ if i knows ϕ in s_j^i and, for all previous states s_k^i in run σ , $k < j$, i does not know ϕ .

The following theorem says that if $C_C(\phi)$ is attained in a run then all processes i learn $P_i C^C(\phi)$ along a single consistent cut.

Theorem 6.21 (attainment). If $C^C(\phi)$ is attained in a run σ , then the set of states in which all processes learn $P_i C^C(\phi)$ forms a consistent cut in σ .

We have presented a formal semantics of Lachesis protocol based on the concepts and notations of concurrent common knowledge [24]. For a proof of the above theorems and lemmas in this Section, we can use similar proofs as described in the original CCK paper.

With the formal semantics of Lachesis, the theorems and lemmas described in Section 6.2 can be expressed in term of CCK. For example, one can study a fact ϕ (or some primitive proposition p) in the following forms: ‘is there any existence of fork?’. One can make Lachesis-specific questions like ‘is event block v a root?’, ‘is v a clothe?’, or ‘is v a atropos?’. This is a remarkable result, since we are the first that define such a formal semantics for DAG-based protocol.

7 Reference

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