PHY985 Nuclear Dynamics - Homework 9

(Fanurs) Chi-En Teh (A56423387)

Due on Dec 10, 2019 (Tuesday)

1. A particle with mass m is moving in mean field potential of the form $U(n,p^2)$, where I have chosen the notation where $n(\vec{r}) = \rho(\vec{r})/\rho_0$ is the ratio of the nucleon density ρ divided by the saturation density ρ_0. I have chosen this notation in order to minimize confusion between the density and the momentum. In relativistic mean field theory, this form might be

$$U(n(\vec{r}),p^2) = U_v n(\vec{r}) + \frac{U_s n(\vec{r})}{\sqrt{1+\frac{p^2}{m^2c^2}}} \ ,$$

where $U_v = 300$ MeV and $U_s = -420$ MeV.

a) *Expand the mean field potential to order p^2 around $p=0$. Your result should be of the form $U(n,p^2) = U_0(n) + U'(n)\cdot p^2$. Assume the kinetic energy to be of the form $T = p^2/2m$. Obtain the Hamiltonian for this single particle.*

b) *Using Hamilton's equations, obtain the time derivatives of the position r and the momentum p. During this step, also identify the force accelerating this particle. Use the equation for the time derivative of the position to identify the effective mass.*

c) *Combine these two equations to find the acceleration in terms of the force. Put the equation in the form:*

$$m_{\text{eff}}\ddot{\vec{r}} + \dot{m}_{\text{eff}}\dot{\vec{r}} = \vec{F} \ .$$

a) Expansion around $p=0$ gives

$$\begin{aligned} U(n(\vec{r}),p^2) &= U_v n(\vec{r}) + \frac{U_s n(\vec{r})}{\sqrt{1+\frac{p^2}{m^2c^2}}} \\ &\approx U_v n(\vec{r}) + U_s n(\vec{r})\left(1 - \frac{p^2}{2m^2c^2}\right) \\ &= [U_v n(\vec{r}) + U_s n(\vec{r})] - U_s n(\vec{r})\frac{p^2}{2m^2c^2} \ . \end{aligned}$$

Hence, the single-particle Hamiltonian is

$$H = \frac{p^2}{2m} + [U_v n(\vec{r}) + U_s n(\vec{r})] - U_s n(\vec{r})\frac{p^2}{2m^2c^2} \ .$$

b) Recall Hamilton's equations, $\nabla_{\vec{r}} H = -\dot{\vec{p}}$ and $\nabla_{\vec{p}} H = \dot{\vec{r}}$. Hence, we have

$$-\dot{\vec{p}} = (U_v + U_s)\nabla_{\vec{r}} n(\vec{r}) - U_s \nabla_{\vec{r}} n(\vec{r}) \cdot \frac{p^2}{2m^2c^2}$$

and

$$\dot{\vec{r}} = \frac{\vec{p}}{m} - U_s n(\vec{r})\frac{\vec{p}}{m^2c^2} \ .$$

In the expression of $\dot{\vec{r}}$, if we require it to take the form of $\dot{\vec{r}} \equiv \vec{p}/m_{\text{eff}}$, then we may identify the effective mass as

$$m_{\text{eff}} \equiv \left[\frac{1}{m} - U_s n(\vec{r})\frac{1}{m^2c^2}\right]^{-1} \ .$$

c) The L.H.S. is just

$$\frac{d}{dt}(m_{\text{eff}}\dot{\vec{r}}) = \frac{d}{dt}\vec{p}\ .$$

This verifies that $m_{\text{eff}}\ddot{\vec{r}} + \dot{m}_{\text{eff}}\dot{\vec{r}} = \vec{F}$. To find the acceleration $\ddot{\vec{r}}$ in terms of the force $\vec{F} \equiv \dot{\vec{p}}$, we take the time derivative of $\dot{\vec{r}}$,

$$\begin{aligned}
\ddot{\vec{r}}(\vec{r},\vec{p}) &= \frac{\dot{\vec{p}}}{m} - U_s n(\vec{r})\frac{\dot{\vec{p}}}{m^2c^2} - U_s\left[\nabla_{\vec{r}} n(\vec{r})\cdot\dot{\vec{r}}\right]\frac{1}{m^2c^2} \\
&= \frac{\dot{\vec{p}}}{m_{\text{eff}}} - \frac{U_s}{m^2c^2}\left[\nabla_{\vec{r}} n(\vec{r})\cdot\frac{\vec{p}}{m_{\text{eff}}}\right] \\
&= \frac{1}{m_{\text{eff}}}\left[\vec{F} - \frac{U_s}{m^2c^2}\nabla_{\vec{r}} n(\vec{r})\cdot\vec{p}\right]\ .
\end{aligned}$$

2. It was first pointed out by Bethe and Butler [45] that one can verify and determine the shell-model single-particle level spectrum through the study of stripping reactions. Using the appropriate selection rules, illustrate this in the case of $^{10}B(d,p)^{11}B$ and compare with the actual experimental data [46].

[45] Bethe H A and Butler S T 1952 *Phys. Rev.* **85** 1045

[46] Ajzenberg-Selove F 1960 *Nuclear Spectroscopy* part B (New York: Academic)

According to the shell model, the required ℓ for n captured by ^{11}B should be at the $1p_{3/2}$ orbital, i.e. $\ell_n = 1$. Also, $J^\pi = \frac{3}{2}^-$ is predicted for ^{11}B.

Now, let us look at the experimental data. According to the triangle rule, the allowed ℓ_n values of n are governed by

$$\left| |J_i - \ell_n| - \frac{1}{2} \right| \leq J_f \leq J_i + \ell_n + \frac{1}{2} \,.$$

NNDC reports $J_i^\pi = 3^+$ and $J_f^\pi = \frac{3}{2}^-$ (matches prediction). Hence, the possible values include $\ell_n = 1, 2, 3, 4, 5$. To check if shell model's prediction is correct, we look up to http://www.tunl.duke.edu/nucldata/HTML/A=11/11B_2012.shtml, where in the 27th item, it states, "The lowest five levels are formed by $\ell_n = 1$ except for ^{11}B*(2.12) which appears to involve a spin-flip process."

As expected, shell model works when we have a nucleon/hole in a closed sub-shell.