Reference Manual for the Matlab program

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Introduction

This is a reference manual for the Matlab program "Evaluation of state estimators," which includes all experiments implemented in the manuscript "Theoretical Relationship of Set-Based Robust State Estimators: An Optimality Perspective" submitted to IEEE Transactions on Automatic Control. Please test programs to complement the manual.

Installation

The employment of this package needs two Matlab toolboxes:

- MPT3: https://www.mpt3.org/Main/Installation
- CVX: http://cvxr.com/cvx/download/

Additional care should be taken to the MPT3 toolbox. We detected a bug in the file "mtimes.m", which is in the installation subdirectory "\tbxmanager\toolboxes\mpt\3.2.1\all\mpt3-3_2_1\mpt\modules\geometry\sets\@Polyhedron\mtimes.m" of the MPT3 toolbox. This function deals with the operation that a scalar α times a polyhedron $P = \{x|Ax \leq b\}$. The 125th line makes $\alpha P = \{x|Ax \leq \alpha b\}$, which is only valid when $\alpha \geq 0$. The correct form is $\alpha P = \{x|\frac{1}{\alpha}Ax \leq b\}$, $\forall \alpha \neq 0$.

Program structure and function reference

This program has one main directory with eight subdirectories, whose roles are listed as follows.

- Main directory: The main files that conduct large-scale numerical tests on the systems.
  - Twotank_intense_test.m: Run large-scale tests on the two-tank system.
  - Fourtank_intense_test.m: Run large-scale tests on the four-tank system.
  - LTV_test.m: Run one test on the linear-time varying (LTV) system. Please see reply **R3-2** in the Answer sheet for more details.

  - Check.m: Run one sampling case to debug or visualize the result.
  - Verification.m: Compute the approximation accuracy of the proposed algorithms by optimization programs. Please see reply **R1-5** and Sec. V in the revised version for more details.

- Data: The stored numerical results of the experiments described in the manuscript and Answer sheet.
  - intense_twotank.mat: Results of the large-scale test on the two-tank system.
  - intense_fourtank.mat: Results of the large-scale test on the four-tank system.
  - data_load.m: Plot the average computational time $\bar{\mathbf{t}}_{com}$, the average error $\bar{\mathbf{e}}_r$, and the number of facets and vertices of the non-conservative state estimation sets generated in the tests.
  - Two_tank.mat: The experimental result of one test on the two-tank system, whose visualization by the function "Plot_sets.m" has been shown as Fig. 5 in the main manuscript's body.
  - LTV.mat: One sampling case of the LTV system. The reviewer can call the function "Plot_LTV_system.m" to visualize the result, which is shown as Fig. 2-4 in the Answer sheet.
  - Random_4dim_k 10.mat, Random_5dim_k 1.mat, Random_ k_2 failure.mat: The experimental results of the random system. Please see reply **R1-26** for detailed descriptions.

- Error_data_fourtank, Error_data_fourtank: The intermediate results of some cases in which the MPT3 reports errors. These data are reserved for debugging and illustrating the limits of the MPT3. The causes have been analyzed in reply **R1-26** of the Answer sheet. We have not uploaded all the error cases due to the file size limit.

- Warning_data_fourtank, Warning_data_twotank: The intermediate results of cases in which the numerical errors of the proposed algorithms are relatively large. The leading cause is the numerical inaccuracy of the MPT3 toolbox. Please see reply **R1-26** for details. We have not uploaded all the warning cases due to the file size limit.

- Fig: The Matlab figures illustrated in the main manuscript's body and Answer sheet.

- Systems: The parameters of the numerical systems, including the two-tank, four-tank, LTV system, and random system.
  - RandOrthMat.m: Generates a random $n \times n$ orthogonal real matrix as the eigenbasis of the random system matrix A. See reply **R1-26**.
  - Random_system_generating.m: Generates a random system matrix A.
  - The other system types can be easily inferred from their file names.

- Toolbox: All functions utilized in the program.
  - Box: Generates the minimal interval that encloses a given polytope P, i.e., $\square(P)$.
  - Compute_SESs: Computes the state estimation sets (SES) X_{k+1}^P and X_{k+1}^I given the observer bundles $\mathbb{L}_k$ and $\mathbb{LS}_k$.

- IO_LTV_opt.m: Solve $\mathbb{LS}_k$ of the LTV system.
- IO_opt.m: Solve $\mathbb{LS}_k$ of the linear-time invariant (LTI) system.
- L_opt.m: Solve $\mathcal{L}(v; X_k)$ given the direction v and the SES X_k. See Theorem 4.1 for details.
- mRep.m: Solve the mH-Rep and mV-rep of a given polytope. This procedure is intended to reduce the representational complexity of each intermediate polytope in the SME process.
- Normalize: Normalize the outward normals of the polytope ($\|h_i\| = 1$) to make sure the numerical errors of all facets are of the same scale. See Sec. V in the manuscript.
- Plot_LTV_system.m: Plot the resulting SESs X_{k+1}^M, X_{k+1}^P, and X_{k+1}^I of the LTV system.
- Plot_sets.m: Plot the resulting SESs X_{k+1}^M, X_{k+1}^P, and X_{k+1}^I of the LTI system.
- Select_vertices.m: Solve the vertice $P(v; X_k)$ given the direction v and SES X_k. See Lemma 2.4 in the new version.
- Smatrix.m: Solve the coordinate transformation matrix $T_k = \mathcal{T}(v, L_k)$ given the direction v and observer gain L_k. See Lemma 4.7 in the main content.
- Support_func.m: Compute the support function $S(v; X)$.
- SVO_LTV_opt.m: Solve $\mathbb{L}_k$ of the LTV system.
- SVO_opt.m: Solve $\mathbb{L}_k$ of the LTI system.
- zonotope.m: Generates a zonotope with G-rep $Z(c, G) = \{c + G\xi | \|\xi\|_\infty \leq 1\}$. This function can be utilized to generate symmetric and complicated uncertainty sets.

Quick start

The reviewers are suggested to run "Check.m" to see the experimental result of one sampling case on the two-tank system. The numerical accuracy of the proposed algorithms is reported, and the resulting SESs are plotted. The reviewers can also load the intense_two_tank.mat or intense_four_tank.mat in the subdirectory "Data" and run "data_load.m" to visualize the outcomes of the large-scale numerical tests.

Some remarks

- Some notations in the Matlab program follow the old manuscript. Specifically, the coordinate transformations are S_k, and the measurement noises are d_k, which have been replaced by T_k and η_k in the revised version. Although the effectiveness of the program is unaffected, we will ensure the consistency of the denotations in the following updates of the code.
- The authors will work on finding better numerical techniques to stabilize the MPT3 toolbox so that the applicability of the proposed algorithms can be enhanced.