Executive Summary: Analog Bistable Systems for Combinatorial Optimization

Project Overview

The Ising model is fundamentally important in both condensed matter physics and computational science. In physics background, it is a mathematical representation of ferromagnetism in statistical mechanics. It is widely used to study quantum phase transitions, spin glasses and other phenomena in condensed matter physics [1, 2]. In quantum computing, similar models are used to understand quantum bits (qubits) and can exhibit complex interactions.

Besides, in the field of combinatorial optimisation, efficiently solving the Ising model is one of the major challenges [3, 4, 5, 6]. By finding optimal or near-optimal solutions from a vast set of possibilities, combinatorial optimization helps areas such as supply chain management, telecommunications, and artificial intelligence. Although impressive progress has been made in solving these problems (including traditional, classical and quantum methods), there is still room for innovation [7, 8, 9].

The problem of finding the ground state of the Ising model is classified as NP-hard, which means there is no known algorithm that can efficiently solve all instances of this problem in polynomial time. One of the approaches is to use analogue systems, such as optical parametric oscillator (OPO) networks. This is because they are particularly promising due to their parallel processing capabilities and potential for efficient hardware implementation [10, 11, 12].

Therefore, this project explores the possibility of using analogue bistable systems (in particular OPO networks) to search for Ising Hamiltonian low-energy states. The Coherent Ising Machine (CIM) is a simulated spin system that efficiently simulates and searches for low-energy states of the Ising Hamiltonian using OPO networks.

Based on the idea of utilizing CIM, Leleu et al. proposed a novel method to improve the performance of simulated spin systems for solving the Ising problem by addressing the amplitude inhomogeneity [13].

The main aim of this project is to replicate the main findings and methods outlined in Leleu et al. using classical computational simulations. This includes the implementation of the Chaotic Amplitude Control (CAC) mechanism in a simulated network of OPOs (which is the CIM). Then the performance of the results found by using CIM with CAC is compared with state-of-the-art heuristics. In this way, this project provides insights to use an unconventional classical method to solve for the Ising problem, even more, it can be a novel strategy in the wider picture of combinatorial optimization.

Methodologies

The project replicated and extended innovative methods – the Chaotic Amplitude Control (CAC) mechanism – to improve the performance of simulated spin systems of the Ising model. Various state-of-the-art heuristics such as Breakout Local Search (BLS) [14], [14], GRASP-Tabu Search Algorithm (GRASP-TS) [15], Scattered Spot Search (SS) [16], Rank-2 Relaxation Heuristic (CirCut) [17] were benchmarked against the CIM's performance. This study is written is Python. The specific implementation of CAC includes three main parts. Firstly, we need to initialize the Analog Bistable Units x_i, which represent the spins in the Ising model. Then the System Dynamics describes the time evolution of the spins, which is a differential equation. At last but not the least, the time-dependent error signal e_i is introduced for correction of amplitude heterogeneity to prevent the system from stabilizing in local minima, which is also the innovation point of the CAC method. The usage of the algorithm is:

```
results = CIM_AHC_GPU(T_time=20,
                  J=J,
                  batch_size=1,
                  time_step=0.01,
```

```
                        beta = 0.05,
                        mu = 0.5,
                        noise = 0,
                        custom_fb_schedule=func_of_eps,
                        custom_pump_schedule=func_of_r)
spin_config, x_trajectory, t, energy_plot_data, error_var_data, divg, kappa = results
```

To make sure the algorithm finds the best ground state for a given Ising model, it is important to tune these five hyperparameters. There are two appraches for finding the best combination of hyperparameters (β, μ, $noise$, ϵ_0 and r_0) for each given instance. Firstly, M-LOOP package (a machine learning optimization tool) is used to scan thoroughly through the ranges of all the parameters, and find the best values for a combination of them. Then, for simplicity, the combination of ϵ_0 and r_0 are scanned again for fine tuning.

Key Findings

All the MAX-CUT instances and smaller Gsets (800 vertices) found the best known ground states by using the algorithm within 40 seconds. This is impressive because for other more sophisticated algorithms like GRASP-TS (100 seconds), SS (139 seconds), CirCut (352 seconds), much more time were taken to reach the ground state for the same instance (G1 set). This efficiency is not only for G1 set, but also for all G sets tested with 800 vertices.

Table 1: Comparison of Max-Cut Energy Obtained by Coherent Ising Machine and Breakout Local Search, where $|V|$ is the number of vertices in the problem, f_{avg} and σ are the average max-cut energy and corresponding standard deviation found for the given instance, f_{best} is the best max-cut energy found by CIM, and f_{bls} is the best max-cut energy found by BLS. $t(s)$ is the time taken for CIM to converge to the best values.

Name	$\|V\|$	f_{avg}	σ	f_{best}	f_{bls}	$t(s)$
G1	800	11567	40	11624	11624	33
G2	800	11576	31	11620	11620	39
G3	800	11578	35	11622	11622	20
G23	2000	13262	44	13307	13344	37
G36	2000	7596	17	7620	7678	37
G56	5000	3830	19	3845	4012	37

However, as the size of the graph increases, BLS tends to outperform CIM in both finding a lower ground state and faster convergence. This means the CIM did not use the best sets of hyperparameters before running the simulation, or the algorithm has scalability issues for larger instances. The results demonstrated that CIM, enhanced with the CAC method, effectively destabilizes local minima for most of the Ising problems and common Gsets (N=800), improving the probability of reaching the global minimum. The algorithm is comparable with other state-of-art algorithms for smaller instances, and it even outperforms some of them.

Conclusion

This research has undertaken a comprehensive evaluation of the Coherent Ising Machine (CIM) for solving the Max-Cut problem, contrasting its efficacy with established heuristics. The findings reveal that while the CIM performs on par with BLS for smaller graph instances, its effectiveness diminishes with increasing graph complexity.

The consistent execution times across different graph sizes underline the CIM's computational robustness, indicating that its core architecture is well-suited for parallel computation tasks. However, the increased ρ values in more complex scenarios suggest that current algorithms within CIM might not scale as effectively in handling intricate computations or large datasets.

Furthermore, the study exposes critical areas for improvement, particularly the need for algorithmic enhancements to boost scalability and efficiency. The research indicates that integrating adaptive strategies, such as dynamic parameter tuning and enhanced error correction mechanisms, could significantly mitigate the observed performance drop-offs in larger graphs.

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