Quantum Information and Computing (QIaC)

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Theory

The system under investigation is the **transverse-field Ising model** (TFI), which describes a sequence of N spins under the presence of a transverse magnetic field, with Hamiltonian:

$$\hat{H}_N = -J \sum_{i=1}^{N-1} \hat{\sigma}_x^i \hat{\sigma}_x^{i+1} + \lambda \sum_{i=1}^{N} \hat{\sigma}_z^i \tag{1}$$

The **Real Space Renormalization Group** algorithm (RSRA) has been developed in order to efficiently approximate the diagonalization process for Hamiltonians of big dimensions. Starting from a system A (size N), the ground state of the bigger system $A + B$ (size $2N$) is composed of **few low energy eigenstates** of each subsystem of size N. The algorithm works as follows:

1. Double the system size $N \to 2N$ and consider the Hamiltonian of the full system

$$\tilde{H}_{2N} = \hat{H}_N^A \otimes \mathbb{1}_N^B + \mathbb{1}_N^A \otimes \hat{H}_N^B + \mathbb{1}_{N-1}^A \otimes \hat{\sigma}_x^A \otimes \hat{\sigma}_x^B \otimes \mathbb{1}_{N-1}^B \in \mathcal{H}^{2N \times 2N} \tag{2}$$

2. Diagonalize $\tilde{H}$ and build the matrix $\hat{P}$ out of its first N smallest eigenvectors. Project then the full matrix $\tilde{H}_{2N}$, and the two full interaction matrices H_A, H_B into the subspace spanned by its first N eigenvalues:

$$\hat{H}_N^T = \hat{P}^T \tilde{H}_{2N} \hat{P} \quad \hat{H}_N^{A,T} = \hat{P}^T (H_A \otimes \mathbb{1}_N^B) \hat{P} \quad \hat{H}_N^{B,T} = \hat{P}^T (\mathbb{1}_N^A \otimes H_B) \hat{P} \tag{3}$$

We use the last matrices as inputs to create another $\tilde{H}_{2N}$ and iterate the algorithm up until the **variation** of the ground state energy is smaller than $\delta = 10^{-12}$.

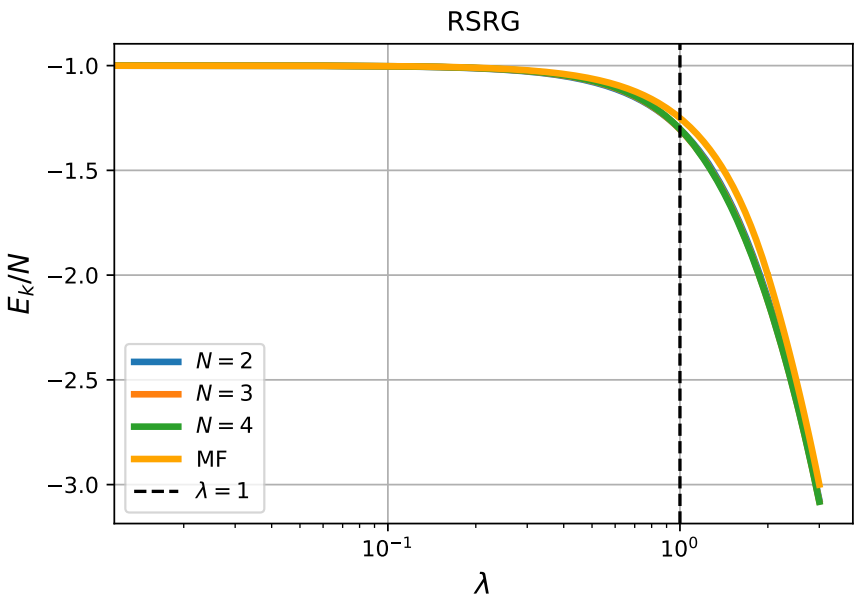
Main code

- We report the code snippet used in order to perform the iterations required by the RSRA algorithm. The idea behind is that we do a RSRA iteration whenever the **difference** between the ground state energy estimated at the previous iterations is smaller than $\delta = 10^{-12}$.

```
H_N_T = TFI_ham_init(N, lambda)                          !TFI hamiltonian for N sites
HabA_T = tp(id_mat_init(N-1), sigma_x())                 !sigma_x on N-th site
HabB_T = tp(sigma_x(), id_mat_init(N-1))                 !sigma_x on (N+1)-th site

DO WHILE(ABS(gs-est_gs)>delta)
    gs = est_gs
    !initialize the extended Hamiltonian, accounting also for the interaction between N and N+1 sites
    H_2N = tp(H_N_T, id_mat_init(N)) + tp(id_mat_init(N), H_N_T) + tp(HabA_T, HabB_T)
    HabA_2N = tp(HabA_T, id_mat_init(N)).                !sigma_x on N-th site for total system
    HabB_2N = tp(id_mat_init(N),HabB_T)                  !sigma_x on (N+1)-th site for total system
    eigvct = H_2N

    CALL ZHEEV('V','U', 2**(2*N), eigvct, 2**(2*N), eigvls, work, lwork, rwork, info)

    sys_size = sys_size * 2                              !the systems size is doubled
    est_gs = eigvls(1)/DBLE(sys_size)
    DO ii=1,SIZE(P,2)
        P(:,ii) = eigvct(:,ii)
    ENDDO
    H_N_T  =  MATMUL(MATMUL(TRANSPOSE(P),H_2N),P)     !projection of H_2N with matrix P
    HabA_T =  MATMUL(MATMUL(TRANSPOSE(P),HabA_2N),P)  !projection of HabA_2N with matrix P
    HabB_T =  MATMUL(MATMUL(TRANSPOSE(P),HabB_2N),P)  !projection of HabB_2N with matrix P
```

Results - Iterations

We used a Python **script** to **automatize** the work and produce the following graphs. Here we show how the ground-state energy (the value when the algorithm converges), normalized to the system size, varies when we change the interaction strength in the range $\lambda \in [0, 3]$.



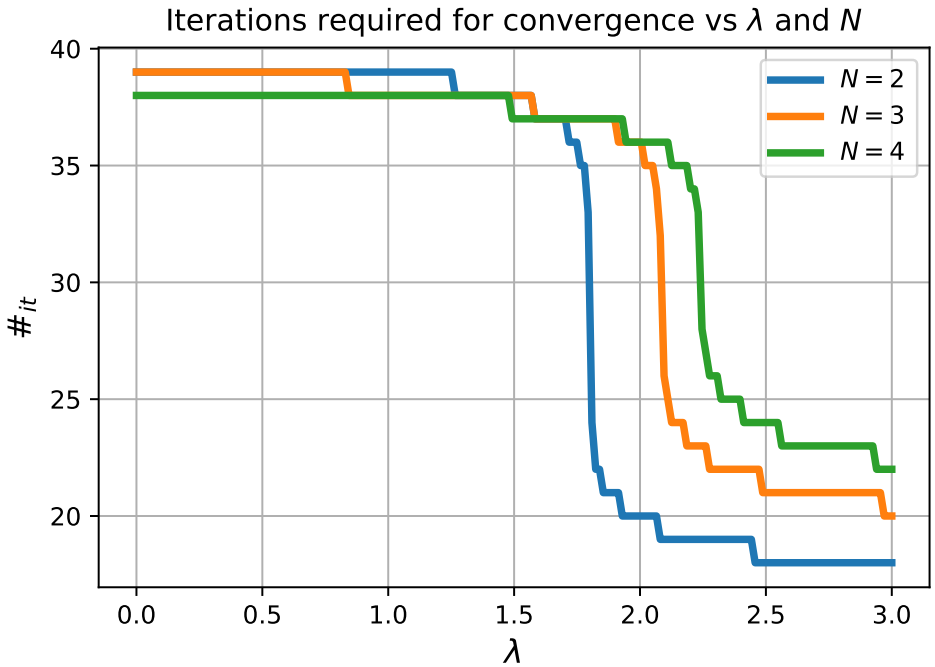
$$E_k/N = \begin{cases} -1 - \frac{\lambda^2}{4}, & \lambda \in [-2, 2] \\ -\lambda, & \lambda \notin [-2, 2] \end{cases} \quad (4)$$

The first thing we notice is that the curves for different N are practically **overlapped**, and this can be explained by the fact that we can diagonalize exactly the Hamiltonian for $2N$ spins. For example, the ground state with $\lambda = 0$ is in modulus equal to the size of the system ($2^{\#_{it}+1}$), and by normalizing it to the size itself we obtain -1.

The results are in perfect accordance with MF for small values of λ, while we can spot a discrepancy as this parameter increases.

Results - Energy

We now want to investigate how the **number of iterations** required by the RSRA for the convergence scales with the interaction strength λ for different N.



A striking feature of this plot is that the number of iterations $\#_{it}$ suddenly drops at a certain point, and that we observe different behaviors for the different sizes. In particular, for $\lambda = 2.5$, we reach the convergence faster with a smaller chain of spins.