Targeted Incentives for Charter Schools to Expand Capacity: a Dynamic Analysis

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Abstract

The central question of the policy debate on public education is how to get more for less. Charter schools have been a tool in this arena to pressure traditional public schools to improve or lose students to them. Moreover, charter schools designated as "High-performing" have recently been allowed to expand capacity at will in Florida, while the remainder need to request for such permission. I leverage this policy reform and evaluate its influence on education access and quality. I develop and estimate a tractable dynamic model that highlights both the (costly) adjustment of schools' capacity and their "effort" to improve quality, as well as their dynamic response to competitive pressure. I find evidence that obtaining "High-performing" designation reduces adjustment costs of capacity, which is valuable to charter schools. More importantly, such charter schools exert pressure on traditional public schools nearby. Through simulation exercises, I show that targeting value-added, not just performance level, would improve the mean performance of the entire education sector and enhance equity of access.

Keywords: School Choice, Charter School Capacity, Dynamic Structural Model

JEL Classification: H75, I28, L51

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1 Introduction

The ultimate goal of school choice policies is to improve education quality in the aggregate. Proponents of school choice programs mainly highlight two central mechanisms that support this objective. First, school choice programs may increase quality, variety, and access to the alternatives to students' assigned options.[1] Second, such programs might have competitive spillovers on traditional public schools ("TPS" henceforth) and influence their productivity.[2] In the U.S., charter schools are publicly funded (and tuition-free) but are privately run, often by for-profit enterprises.They serve as a primary instrument for providing school choice. Consequently, proper regulations could help *incentivize* charter schools to raise quality and accessibility, and can trigger competitive spillovers across the entire education sector, creating a "tide that lifts all boats" (Hoxby 2003).

In this paper, I use detailed administrative data to analyze a large-scale policy that incentivizes certain charter schools to increase their performance. The policy does so by conditioning expansion eligibility on past performance. The primary goals of the paper are to assess the policy effects on students' academic performance and access to high-quality education and explore alternative incentive schemes that do better. The policy I focus on is the introduction of the Florida High-performing Charter School Statute in 2012. This statute gives "High-performing" ("HP" henceforth) designation to charter schools with three consecutive years of exemplary performance. Such HP charter schools are authorized to expand enrollment capacity without obtaining approval from local districts. I show that HP charter schools increase the number of classrooms for instruction upon being designated. More importantly, using a difference-in-difference analysis, I find that following the introduction of the policy, student test scores increase more in traditional public schools that are subject to more competitive pressure from neighboring High-performing charter schools.

Two underlying mechanisms are potentially critical to explain these patterns. First, the policy could, by eliminating the adjustment costs imposed by the regulatory constraint, motivate HP charter schools to expand capacity. Second, the potential competitive pressure of future expansion of HP charter schools may push TPSs to improve their performance. Both mechanisms dynamically influence the charter sector's capacity and the overall quality provision of all schools. Although these patterns are suggestive in terms of the underlying mechanisms at work, they are of limited use in understanding the aggregate effect on all schools and disentangling the importance of each mechanism

[1] Some studies have found that highly effective charter programs lead to improvements in students' test scores and future life choices (Abdulkadiroğlu et al. 2011; Booker et al. 2011; Angrist et al. 2016; Dobbie and Fryer 2020; Cohodes et al. 2021; Cohodes and Feigenbaum 2021).

[2] Some studies have found that TPSs increase performance when facing competitive pressure from the choice programs in various contexts (Figlio and Hart 2014; Mehta 2017; Gilraine et al. 2021; Gilraine et al. 2023).

quantitatively without further structure being imposed. A better understanding of the relative importance of these mechanisms is helpful for the primary goal of this paper, namely, improving policy. To achieve this goal, I develop and estimate a dynamic model of schools' decision-making. I explicitly model the dependence of schools' decisions to expand and exert effort (in improving performance) on the adjustment costs and competitive pressures they face.

To estimate the model, I assemble and examine a rich dataset for Florida that tracks the annual operation of 630 regular charter schools and 2411 TPSs serving K-8 grades from 2006-7 to the 2018-19 school year. The dataset provides a comprehensive history of each school's number of classrooms for instruction, performance level, educational effort measured by schools' value-added, operating cost, HP designation status, local demographics, and competitive pressure. These supply-side dynamics can be further linked to student enrollment changes within schools.

The dynamic model I develop maps schools' two key decisions, capacity expansion and educational effort (or inputs), to the distribution of schools' performance and capacity. Each period, charter schools choose educational effort, which determines students' performance, as well as their capacity to expand. TPSs only choose educational effort. Both decisions are subject to adjustment costs. Furthermore, the incentive scheme induced by the HP statute is modeled as follows: Charter schools can earn HP designation by performing well, and this designation reduces their future cost of adjusting capacity. Finally, schools' decisions thus affect their future capacity, performance levels, designation status, and, importantly, the competitive pressure in the market where they compete. All of these factors influence their future enrollment, one of the primal components of their objective functions, via the demand side.

The simulation of the model presents several empirical challenges. First, modeling schools' strategic interaction in a dynamic game using MPNE, or Markov Perfect Nash Equilibrium (Maskin and Tirole 1988a; Maskin and Tirole 1988b), is computationally prohibitive. In typical urban school districts, such as the Miami-Dade School District, the average number of neighboring schools within 3 miles of a school is more than 20. Further, by allowing for rich school heterogeneity, the dynamic game framework generates a particularly high-dimensional state space of the school.[3] To alleviate the computational bur-

[3]Aguirregabiria et al. (2021) use a numerical example of Pakes and McGuire (1994) model to illustrate the large state space problem. The model, with ten firms choosing only 20 different quality levels in a dynamic game, has over 10 trillion states. In the context of this paper, a model allowing a school to have four performance levels (e.g., A, B, C, D grades), three capacity levels (e.g., less than 10, between 10 and 20, and more than 20 classrooms) and ten competitors has more than 60 billion states. Additionally, it is possible to find populous communities of relatively small area size with many schools, as in the example of Miami-Dade School District. Schools in these populous regions typically have overlapping sets of neighboring schools. This fact implies that a local school's demand can be influenced by schools far away, which further increases the number of potential competitors for the local school, escalating the computation burden.

den while allowing the model to be rich enough for reasonable counterfactual analysis, I make the following assumptions about schools' beliefs and responses to their enrollment. First, I assume that each school only uses its own state and a uni-dimensional summary statistics about the market conditions it faces to calculate their current and future enrollment. This assumption is analogous to what is done in a static monopolistic competition model. Second, each school forms beliefs on the transition of this uni-dimensional state that are roughly consistent with how the market evolves. The first assumption assures tractability by reducing the dimensionality of the state space required by MPNE. The second assumption endogenizes competitive pressure characterized by the uni-dimensional state, allowing schools to alter their beliefs on future competitive pressures as policies change, especially when they dramatically influence the regulatory environment.

Using the estimated models, I conduct two comparisons of incentive schemes. I first compare the existing HP scheme with the "no-HP" scheme to explore the policy effects. The no-HP scheme eliminates the existing designation system. Additionally, I compare an alternative scheme, named "Target Value-Added", that targets high value-added charter schools and grants them more opportunities to earn expansion eligibility. This alternative scheme is motivated by the concern that the existing scheme may exacerbate inequality in access to high value-added charter schools across different SES groups. Under the existing scheme, many charter schools not designated as HP do have high value-added. These schools typically serve lower SES households and do not achieve the required performance levels for HP designation. For this reason, I focus on the inequality associated with the existing scheme versus the counterfactuals under inspection.

I forward-simulate the evolution of the largest Florida school district starting in 2012. The mean performance of the entire education sector increases the most under the Target Value-Added scheme, and the existing HP also outperforms the no-HP scheme in the same way. The increase of the charter sector accessibility (in terms of capacity) is also the highest under the Target Value-Added scheme. In explaining these differences in performance and accessibility across schemes, I find that the incentive channel accounts relatively more for the accessibility difference. In contrast, the competition channel explains the performance difference. Digging into the variance of performance and accessibility at the end of the inspection window, I find that the Target Value-Added scheme reduces the variance of performance across schools and improves equity of access by allowing for more expansion of high value-added charter schools in the lower SES regions, compared to the existing HP scheme.

The design and implementation of school choice programs are at the forefront of education research. For example, using vouchers to increase choice has been extensively researched and evidence shows that they are effective in countries where private education accounts for a large market share (see Hsieh and Urquiola (2006), Neilson (2021), and Arcidiacono et al. (2021). The underlying idea of vouchers is to increase students' alternatives to expensive private schools. Therefore, relaxing capacity constraints for HP charter schools is like increasing the number of vouchers for such schools. To some extent, capacity regulation of this kind is a more "controlled" way to direct students toward *targeted* schools aligned with the policymakers' goal. Therefore, comparing the capacity regulation of charter schools with extensively researched voucher systems can improve the understanding of both policy tools for scholars.

However, the market environment of voucher systems contrasts dramatically with the public education market, where the government fully funds tuition. The "non-price" nature of public education markets prohibits policy tools like vouchers to incentivize charter and TPSs. Therefore, taking advantage of the incentive for capacity expansion is more practically relevant and can enrich the toolbox for policymakers in public education markets. In this regard, this paper is the first attempt to investigate such policies using an empirical strategy to both establish novel facts and develop a dynamic quantitative model to explore alternative schemes that do better. Furthermore, if proven beneficial, such policy reforms may be easier to implement than the extensively researched public education policies that increase spending in public schools as they do not involve increasing expenditure.[4] It is an incentive scheme that imposes rules in influencing schools' decisions and does not explicitly require increasing or redistributing money across schools. [5]

Contribution and Related Literature. This paper contributes to an extensive literature on school choice in the public-private education systems. Milton Friedman argued that the market-based school choice through vouchers for private school attendance would facilitate Tiebout-style competition without necessitating community relocation. It would extend educational choices to previously underserved families and theoretically enhance the overall quality of education (Friedman 1955; Hoxby 2003). School choice programs typically take the form of charter schools in the U.S. education market. Many studies, especially those using "lottery estimates" (Hoxby and Murarka 2009; Abdulkadiroğlu et al. 2011; Angrist et al. 2016), have shown that the impact of charter schools on student achievement can be both significantly positive and sizable, especially for the "No-excuses" charter school model (Cohodes and Parham 2021). However, these charter pro-

[4]These policies focused on increasing spending are widely discussed by Cellini et al. (2010), Martorell et al. (2015), Jackson et al. (2016), Dinerstein and Smith (2021), and Asker et al. (2022), among others.

[5]This view of the policy potentially ignores the fiscal externality imposed on the TPSs and school districts. Previous research has found that charter expansion could reduce the district funding to TPSs or alter TPSs' spending structure (Ridley and Terrier 2018; Slungaard Mumma 2022; Ladd and Singleton 2020). Fully internalizing these costs in designing school choice policies is not a focus of this paper and can be a potential direction for future economic research in broader settings. Similar policies have been seen in Florida, Massachusetts (Ridley and Terrier 2018), Missouri, and Louisiana. For a report on the implementation in these states, see the National Association of Charter School Authorizers' 2019 report on "Expanding Access to High-Performing Charter Schools 2019."

grams and many charter schools are under capacity constraints.[6] Therefore, it is natural for scholars and policymakers to consider policies that alleviate the capacity constraint for charter schools. By analyzing this novel policy, my work casts light on the trade-off involved in different ways of charter expansion. Specifically, I address the importance of capacity deregulation in designing large-scale school choice policies and provide a quantitative framework enabling comparison of schools' performance across policy schemes.

Within the school choice literature, this paper relates to the strand that analyzes the policy effects of charter school expansion. Existing research on charter school expansion has focused almost solely on the impact of *entry* of charter schools, in particular, on the competitive pressure it places on TPSs (Imberman 2011; Figlio and Hart 2014; Mehta 2017; Gilraine et al. 2021).[7] However, the form of charter expansion in this paper is novel: it leverages the eligibility to expand *capacity* for certain charter schools. Therefore, by thoroughly analyzing the novel policy, I address the importance of charter capacity previously ignored in the charter expansion literature. Relatedly, under the theme of expanding and replicating charters, another niche literature looks at the specific practices for replicating effective charter programs (Zimmer and Buddin 2007; Angrist et al. 2013; Fryer 2014; Cohodes et al. 2021). I differ from this literature by using evidence from a large-scale state policy that could generate a spillover effect across sectors, particularly on TPSs.

The causal inference strategy applied in this paper builds on the literature that identifies the competitive spillovers of charter schools. This literature has typically found contextual and sometimes conflicting results on competitive spillovers by charter expansion across studies, as Figlio et al. (2021), a closely related paper, point out.[8] Moreover, because of data limitations and lack of policy variation, causal studies in this literature are scarce.[9] In this paper, I tackle these empirical challenges by gathering grade-subject level test scores and taking advantage of the natural experiment created by the policy change in the charter sector. I develop a difference-in-difference specification and utilize a unique feature of the test score data to identify the competitive responses and attribute

[6]In 2012, 61% of Florida charter schools were oversubscribed. Among these, 40% received applications 1.5 times the year's enrolling target, and over half were rated as "A," marking their top-tier academic performance. Additionally, among the oversubscribed schools, 46% are located in lower-than-median income regions.

[7]Other papers following this strand also discuss charter expansion in its consequences for inequality in charter access (Singleton 2019), its effect on racial segregation (Monarrez et al. 2022), and its influence on district budgets for TPSs (Baker et al. 2015; Epple et al. 2015; Buerger and Bifulco 2019; Slungaard Mumma 2022; Ladd and Singleton 2020).

[8]For examples, see Hoxby and Murarka (2009), Sass (2006), Zimmer and Buddin (2007), Bettinger (2005), Imberman (2011), Winters (2012), Cordes (2018), Ridley and Terrier (2018), and Gilraine et al. (2021).

[9]Figlio et al. (2021) have briefly surveyed the current state of the literature. They claim that several of the studies have been limited to single districts or a small set of districts (e.g., Zimmer and Buddin (2007), Winters (2012), and Cordes (2018)), while studies that have used statewide data generally look at the very early years of charter policies and over short periods (e.g., Bettinger (2005), Bifulco and Ladd (2006), and Sass (2006)). Other studies that take a national perspective are limited to district-level data (Han and Keefe 2020).

them to schools' input change. I provide the first estimates of the competitive spillovers on TPSs' test scores of the new policy scheme focusing on charter capacity regulation. The results suggest a new source of competitive pressure imposed on TPSs: neighboring charter schools' eligibility to expand capacity. Furthermore, the competitive responses are larger than those obtained in similar contexts (Figlio and Hart 2014; Figlio et al. 2021).

The structural modeling approach puts this paper in the growing literature that focuses on the industrial organization of the education supply. Papers in this literature are typically model-driven and explicitly quantify students' choice of schools and education providers' responses, such as increasing quality, entry, and exit. These papers further link the supply and demand in an equilibrium model to generate policy-relevant outcomes.[10] However, my work is the first to develop a quantitative dynamic model incorporating decisions on capacity and performance in the K-12 setting. Moreover, the model is designed to be computationally tractable and address schools' strategic considerations in a dynamic setting. My work hence follows recent attempts to apply quantitative dynamic models to study education markets.[11]

The remainder of the paper proceeds as follows. Section 2 provides industry background. Section 3 introduces data sources and the sample under inspection. Section 4 shows descriptive patterns in the Florida education market and evidence of the policy effects. Section 5 introduces the current version of the quantitative model. Section 6 introduces empirical strategy in estimating the model. Section 7 shows estimates of the model. Section 8 displays simulations based on counterfactual policies. Section 9 concludes and discusses the direction of the next version of the paper.

2 Industry Background

In this section, I introduce the industry background of the Florida public education market and the relevant institutional background related to the policy I focus on.

2.1 The Florida Public Education Market

Florida has one of the largest public school enrollments in both the traditional and charter sectors across all states. It also has sound charter laws and relatively lenient entry screen-

[10]These outcomes include students' welfare, test scores, access to schools, and segregation (Hastings et al. 2009; Neilson 2021; Ferreyra and Kosenok 2018; Mehta 2017; Singleton 2019; Allende 2019; Dinerstein and Smith 2021; Arcidiacono et al. 2021; Bau 2022; Dinerstein et al. 2022).

[11]For example, Larroucau and Rios (2022) investigate the effects of centralized assignment mechanisms in influencing outcomes and choices after their initial assignment to college. Hahm and Park (2022) explore how preference for high school characteristics influences students' choices in middle school. Bodere (2022) looks at the effects of government subsidies in childcare on the entry, exit, and quality investment of private pre-schools.

ing (Singleton 2019), making it a state with one of the highest numbers of charter schools and charter enrollment shares in the United States. Additionally, Floridian students can choose any public school or charter school if they are not capacity-constrained through a process known as "controlled open enrollment."[12] These unique features of the Florida public education market amplify the potential impact of policies targeting the charter sector on the overall landscape of public education. Therefore, this makes Florida an ideal state for evaluating the effects of charter school policies.

Regarding accountability, Florida has implemented a system that assesses and gives performance scores to nearly all charter and TPSs annually. This system assigns accountability scores or letter grades to schools, ranging from A (highest) to F (lowest), based on the same criteria applied to both charter and TPS. Notably, while the rating system aims to consider students' achievements and learning gains relative to their previous scores, it still places more emphasis on absolute achievements. This emphasis is evident in the criteria used to assess schools' learning gains, where a school can receive a high score if its students maintain their test scores at a sufficiently high level, regardless of their individual progress. Among all schools in my sample in the 2018-2019 school year, the letter grade distribution is approximately 34% A, 26% B, 32% C, and the remaining 8% are D, F, or missing.

2.2 The New Statute and Charter Expansion Management

In July 2011, Florida enacted the High-Performing Charter School Statute, which remains in effect today. The statute defines HP charter schools as those with three consecutive years of exemplary performance,[13] two As and no grades below B ("2A1B" rule henceforth),[14] marking satisfactory student achievement and progress in standardized tests. An HP charter school can keep its HP designation until receiving two C grades or worse. In such cases, its HP designation can be revoked. However, such cases were rare in the sample.[15] Among all charter schools in the sample, approximately 20% held HP designation in 2012, and this percentage increased to 40% by 2019.

The most significant benefit granted by the statute was the authorization for HP charter schools to expand their enrollment capacities without the approval of local school

[12]The capacity constraint does not seem to apply to many TPSs. As the Annual Five Year Plan indicated, it is frequent to have TPSs enrolling more students than their enrollment capacity.

[13]The statute also requires healthy financial conditions. However, this is much easier to be satisfied and almost never binds in giving designation compared to the performance requirement. For all charter schools meeting the performance criteria, there are few cases in which schools fail to satisfy the financial requirement or an incumbent HP school has been deprived of the designation for financial reasons.

[14]The criterion allows charter schools having two years of A level to be designated after 2017.

[15]In my sample, seven charter schools were de-designated from 2012 to 2019, and 179 charter schools were designated and never de-designated. Since the de-designated charter schools account for less than 4% of the designated charter schools, I code them as never designated throughout the paper.

districts. They can increase enrollment capacity once per school year, expand grade span not already served within the range of K-12, or replicate their educational program in any district in Florida.[16] The statute legally prevents local school districts from rejecting these expansion requests made by HP charter schools. On the other hand, districts had the discretion to reject any expansion before the policy's implementation, or after the policy if the non-HP charter schools propose such requests. Hence, the policy essentially introduced a new incentive scheme that links the past performance of charter schools to automatic eligibility for expansion.

I do not directly observe the enrollment capacity measured in student count as written in charter contracts. Thus, I make the critical measurement assumption that the number of classrooms for instruction in a charter school serves as a sufficient statistic for enrollment capacity.[17] Leasing is also notable as the primary ownership type of charter school contract. Leasing is the primary form of ownership for charter schools, and the cost of expanding capacity, i.e., adding classrooms for instruction, is typically associated with leasing more space, renting relocatable classrooms, or renovating existing leased facilities that are not currently utilized. Consequently, modifying capacity in this context can be achieved quickly relative to constructing entirely new facilities.

Throughout this study, I refer to this event as "the policy" or "the statute." Moreover, I refer to the years before 2012 as the "pre-policy" period and the year 2012 and onward as the "post-policy" period.

3 Data and Sample

To conduct this research, I combined digitized government documents, publicly available datasets, and those with limited public access that require requests for disclosure of information. I collected enrollment in each grade and race, location, and activity status for all public schools in Florida from the National Center of Education Statistics' ELSi dataset, which was merged into the Florida School Master File to obtain additional school characteristics. The locations of schools were mapped to census tracts whose geocodes were merged with the U.S. Census Bureau's American Community Survey to acquire granular local demographic information for all schools. The school's location is also valuable for providing the distance students need to travel from each census tract to a particular

[16] Additional benefits for individual HP charter schools include reduced frequency of financial statement reporting to the sponsor, usually the local school district. They also have the opportunity to modify their charter to extend its duration and enjoy a slight reduction in administrative fees.

[17] In this context, enrollment capacity refers to the maximum number of students a charter school can enroll. It should not be confused with facility capacity, which represents the maximum number of students the school's physical facilities can accommodate safely. Naturally, enrollment capacity cannot exceed facility capacity, although the two quantities are correlated due to the costs associated with leasing or owning additional facilities that remain unused.

school and identifying which schools are closely competing with it. I collected schools' performance information, the letter grades, detailed component scores used to produce the letter grades, and standardized test scores from Florida School Grades Archives and the Department of Education's Bureau of K-12 Assessment.

To tailor the analysis to the policy context, I obtained characteristics such as capacity (number of classrooms and buildings), leases, mission statement, education model, management company, staff details, and annual waitlist status of charter schools from Florida charter schools' annual Accountability Reports from the Florida Office of Independent Education and Parental Choice. From the same source, I obtained the annual HP designation status (designated, de-designated). My variables include charter schools' capacity, performance, designation, local demographics, and neighboring schools. These can be mapped to their enrollment volume and composition. Additionally, I obtained annual teacher-subject level value-added estimates from a regression-based statistical model run by the Florida Department of Education, Bureau of Accountability Reporting. I averaged teacher-level value-added scores to the school level according to the teacher-school linkage provided by the same dataset to measure the educational effort in improving a school's performance level, one of the crucial investment decisions in the model. Lastly, I extended Singleton (2019)'s digitized independent audit data to include more years and the coverage of charter schools than the original paper. The audit, filed by charter schools annually to the Florida Auditor General, reports charter schools' revenue, itemized expenses, and assets. The instructional expenditure is employed in estimating the operating cost function in my quantitative model.

This paper focuses on regular charter and TPSs that serve elementary (K-5) and middle grades (6-8) in Florida from 2007 to 2019.[18] These schools encompass the majority of K-8 public schools and their enrollment in Florida. Schools operating grades from kindergarten to 8^{th} yet running concurrently high school grades (the 9^{th} to 12^{th} grade) during the sample period are excluded. This exclusion was necessary due to the distinct accountability requirements for high schools, which differ from those of elementary and middle schools. By excluding these schools, the statistical analysis becomes easier, and the interpretation of the schools' performance scores is less convoluted. Thus, around 7% of the total K-8 students are not considered during the sample period.

The ultimate sample under examination has 2,411 TPS and 630 charter schools, whose observation counts are 29,333 and 4,483, respectively, at the school-year level. Comparing the sample length (13 years), the median panel length of TPS and charter observations is 12.2 and 7.28 years, respectively.

[18]Regular schools in my selection are all public schools excluding those that are laboratory, municipal, virtual, providing special education, and those charter schools conversed from a TPS.

4 Preliminary Evidence

In this section, I start by introducing TPS and charter schools in Florida, addressing the heterogeneity between non-HP and HP charter schools. Further, I highlight two key findings critical in analyzing the policy effects and the underlying mechanisms at work. First, I provide suggestive evidence that the charter sector responds to the policy by expansion and that students are reallocated across schools and sectors. Second, I identify the competitive responses of TPSs using a difference-in-difference design enabled by the policy shock and matched-cohort test score data. Finally, I motivate an alternative policy by pointing out that the existing policy could advantage the charter schools already serving the high SES regions. For ease of exposition, when describing a school year, I use "2019" to represent the "2018-19 school year."

4.1 Overview of Florida Traditional and Charter Sector

In the sample, charter enrollment accounts for an increasingly larger share of the public K-8 enrollment over time: 3.3% in 2007, 6.5% in 2011, and 11.4% or around 210,000 students in 2019. The number of charter schools in my sample increases as well, from 216 in 2007, 290 in 2011, 376 in 2015, to 436 in 2019. After 2012, charter school exit rates in my sample remained stable at around 3% to 5%, while the entry rates started to drop from around 18% in 2011 to 5% in 2019.[19] Typically, there are more charter schools in districts with highly urbanized regions, and charter schools in these regions tend to be densely distributed. In these large school districts, charter schools account for a higher share of public enrollment (around 20%) and tend to be closer to other charter and TPSs than elsewhere.

There exists considerable heterogeneity between the HP and non-HP charter schools. In Table (1), I compare the mean and standard deviations (in parenthesis) of the non-HP and the HP charter schools in 2015, 4 years after the enactment of the policy. In 2015, among 376 charter schools in my sample, 31.6% were HP: 69 were designated in 2012 and 50 between 2013 and 2015. On average, compared to the non-HP ones, HP charter schools have higher performance scores, capacity, and enrollment. They operate in locations with higher population density, income, students' test scores, and a more white or Hispanic population. Consistent with the demographics of their locations, they serve more white and Hispanic students on average while systemically fewer disadvantaged student populations, including black students and those eligible for free or reduced-price lunch. The type of population served by the HP and non-HP charter schools is reflected in

[19] Exit rate in year t is defined as the ratio between total exits in t and count of charter schools in t. The entry rate is the ratio between the total entries in t and the count in $t-1$. An exit is labeled as in year t if I do not observe enrollment records since $t+1$. Moreover, an entry is labeled as in year t if I start to observe a charter school's enrollment record since t but do not observe the enrollment record before t.

their instructional cost. HP charter schools, on average, have less instructional expenditure per enrollment than the non-HP. This gap may reflect that HP charter schools tend to have greater efficiency in spending and that their students are less expensive to education (Singleton 2019).

Table 1. Summary Statistics of 2015 Charter Schools by HP Status

	non-HP	HP		non-HP	HP
I. School Characteristics			**III. Location (Census Tract) Characteristics**		
Total Performance Score (%)	0.50	0.72	Population Density (1000/square mile)	1.29	1.53
	(0.16)	(0.12)		(0.88)	(1.00)
Enrollment	357.25	560.24	Household Income	62755.03	68443.73
	(330.20)	(349.40)		13625.40	19158.80
Number of Classroom	21.88	33.04	Mean School Reading Score within 5 Miles	-0.23	-0.04
	(16.90)	(19.41)		(0.51)	(0.53)
II. Student Composition			Mean School Math Score within 5 Miles	-0.19	0.01
% of Free/Reduced Price Lunch	0.52	0.40		(0.49)	(0.53)
	(0.30)	(0.27)			
			Number of Traditional Public Schools	24.40	24.60
% of Hispanic	0.32	0.43		(15.39)	(15.44)
	(0.28)	(0.32)			
			IV. Instructional Costs		
% of Black	0.31	0.13	Annual per-enrollment Instructional Cost	4110.00	3838.00
	(0.31)	(0.19)		(2373.00)	(978.00)
% of Whilte	0.31	0.37			
	(0.28)	(0.30)	Number of Observations	257	119

4.2 Charter Expansion, Student Reallocation, and Competition

In this subsection, I first analyze the direct effect of designation on charter schools' capacity and enrollment. I continue the analysis by showing suggestive evidence of the reallocation of students associated with the appearance of HP charter schools. I then show something more causal: after the policy, TPSs with more HP charter school neighbors raise test scores.

HP Charter Expansion How does the charter sector react to the policy in terms of capacity? And to what extent does the reaction influence neighboring schools? To answer these questions, I first investigate the relationship between designation timing and measures of school size and enrollment. In particular, I examine the relationship between the within-school variation in the number of classrooms for instruction, total enrollment (in logarithms), and the number of grades with the time-varying designation status. To do this, I run a two-way fixed effect model as shown in equation (1). The regressor HP_{jt} is the HP-designation status of a charter school j in year t. It gives a value of 1 if a charter school gets or has the designation status maintained in that year and 0 otherwise. Since the policy started in 2012, charter schools are not designated before 2012, i.e., $HP_{jt} = 0, \forall j$ if $t < 2012$. The year fixed effect controls for factors common to all charter schools, such as macroeconomic shocks. The main coefficient of interest is β. It captures the difference in the within-school variation in capacity or enrollment between the pre- and post-designation observations.

$$Y_{jt} = \beta HP_{jt} + FE_j + FE_t + \epsilon_{jt}. \tag{1}$$

Table 2. Correlation of School Size and Designation

	(1) Classrooms	(2) Logenr	(3) GradeSpan	(4) NeighborTPS	(5) Age5-14Stu.	(6) Income
HP	1.84119***	0.10230***	0.01067	-0.09635	1.46084**	2.43518***
	(0.51439)	(0.02053)	(0.05561)	(0.83629)	(0.61285)	(0.72124)
HP X var				0.18860***	0.87852	-0.88358
				(0.06422)	(0.76958)	(0.75209)
Constant	18.51560***	5.12653***	5.25999***	18.58774***	18.52268***	18.53242***
	(0.62777)	(0.02432)	(0.06588)	(0.63082)	(0.62777)	(0.62789)
Observations	4,080	4,483	4,483	4,054	4,080	4,080
R-squared	0.84287	0.90076	0.87469	0.84286	0.84293	0.84294
School FE	Y	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y	Y

Standard errors in parentheses
*** p¡0.01, ** p¡0.05, * p¡0.1

I show results in Table (2). After controlling for two-way fixed effects, the designation status positively correlates with the total number of classrooms and enrollment while not significantly so for the grade span. Notably, charter schools are relatively smaller than TPSs: the average number of classrooms used for instruction in the charter sector steadily increased from 16 in 2007 to 25 in 2015 and 29 in 2019. The estimate on the designation status, roughly 1.8 classroom difference, suggests a sizable within-school expansion between the average capacity between the pre- and post-designation observations.[20] In Appendix A.2, I use an alternative specification to inspect the timing of the expansion after designation. I replace the regressor HP_{jt} with a list of year-to-designation indicators in an event study regression model regarding HP designation as the focal event for a charter school. The result shows that, on average, more classrooms and enrollment are

[20] It is worth pointing out that none of these patterns causally support that the designation induces charter schools to expand. The empirical difficulty is that the designation is an endogenous characteristic of charter schools, and the designation rule applies equally to all charter schools. Therefore, one potential future research could be comparing charter schools in Florida to states where the HP designation system did not exist after 2012. This design would create variation in the policy exposure across different charter schools. However, different states might have distinct education systems, so it is unclear which states are more comparable to Florida before the HP policy. Therefore, to collect evidence that designation induces expansion, I interviewed a few charter school principals and former Florida Department of Education officials. They confirmed that in their cases and most cases they encountered in negotiating expansion, charter schools leveraged the designation to avoid expansion restrictions after the policy's establishment.

added after the first few years of designation, which suggests that the expansion motives might be important behind the designation. [21]

I explore the heterogeneity of the above relationship regarding classroom count by interacting the HP designation with charter schools' local schooling market demographics. As the remaining columns in Table (2) show, the within-school addition of classrooms between the pre- and post-designation is seen significantly more if a charter school is surrounded with more TPSs within 3 miles, as shown in column 4. This suggests that expansion decisions might be based on the local competitive environment. However, this relationship has no significant differences among charter schools with varying local demographic environments from the results in columns 5 and 6. In column 5, I interact HP designation with the local demographic information of charter schools. I interact HP designation with the dummy of whether the mean household income of a 3-mile-neighborhood of a charter school is higher than the median (across all charter school-year observations). In column 6, I apply the above procedure similarly using the age 5-14 population proportion. None of the interaction effects in these tests are significant.

These results suggest that HP designation might reduce the adjustment cost for HP charter schools in expansion. Additionally, there exists heterogeneity in this relationship among charter schools facing different degrees of local market conditions.

Student Reallocation To understand how charter schools' reactions influence neighboring schools, I investigate the source of the increased enrollment in the HP charter schools. I inspect how much student reallocation occurs across sectors as neighboring charter schools are designated as HP. To do so, I run regressions with specification (2). They exploit the cross-sectional and intertemporal variation of the exposure to HP charter schools faced by TPSs, and I correlate them with TPSs' enrollment. The cross-sectional variation of HP exposure comes from the spatial variation in the existence of HP charter schools. In contrast, the intertemporal variation comes from the policy implementation and the increase of HP charter schools as time passes. Particularly, I regress the logarithm of enrollment of TPSs on their exposure to HP charter schools within 3 miles($HPexpoband1_{it}$) and from 3 to 5 miles ($HPexpoband2_{it}$), controlling for school fixed effect and local demographics (D_{it}) faced by the TPS from 2007 to 2019.

$$log(enrollment_{it}) = \beta_1 HPexpoband1_{it} + \beta_2 HPexpoband2_{it} + \gamma D_{it} + FE_i + \epsilon_{it} \tag{2}$$

[21] Note that the designation is endogenous to charter schools' decisions. Therefore, the event study outcome must be interpreted as capturing the variation in classroom counts influenced by the timing of the designation status. Imposing strong assumptions to claim causality would detract from the research focus, and using estimates of event study coefficients to represent dynamic treatment effect is proved to be misleading even with strong assumptions, as Sun and Abraham (2021) pointed out.

In Table (3), I show the results in columns 1 and 2 using the count of HP charter schools as the exposure variable, while in columns 3 and 4 using the total capacity of neighboring charter schools, considering potential expansion behaviors post-designation. In columns 2 and 4, I control for local demographics. Across columns, TPSs' enrollment is negatively and significantly correlated with the higher number or total capacity of neighboring HP charter schools. Conditional on all other covariates, one more HP charter school within 5 miles is associated with 2.5% less TPS enrollment.

Table 3. Effects on Log Enrollment of Exposure to HP Charter Schools

	(1) Count	(2) Count	(3) Capacity	(4) Capacity
HP Charter 0-3 Miles	-0.011***	-0.012***	-0.00034***	-0.00030***
	(0.001)	(0.002)	(0.00010)	(0.00010)
HP Charter 3-5 Miles	-0.011***	-0.013***	-0.00015**	0.00001
	(0.001)	(0.001)	(0.00007)	(0.00007)
Constant	6.486***	6.485***	6.48588***	6.47099***
	(0.003)	(0.027)	(0.00258)	(0.02719)
Observations	29,037	29,037	29,037	29,037
R-squared	0.940	0.940	0.93898	0.93927
School FE	Y	Y	Y	Y
Control	N	Y	N	Y

Robust standard errors in parentheses
*** $p<0.01$, ** $p<0.05$, * $p<0.1$

In Table (4), I show the results of the tests on the correlation between the student composition of these TPSs and the existence of neighboring HP charters.

I run the same specification as equation (2), fixing the measurement of HP exposure to be the count of HP charter schools while testing various outcomes related to the demographic composition of students in a TPS. Each of the columns in Table (4) reports the regression result using the log enrollment or the proportion of a specific type of students as the outcome. As columns 1 and 2 show, TPSs have fewer black and lower-income students (as measured by those who need free and reduced-price lunch) if HP charter schools appear more in their neighborhood. This is consistent with the total enrollment loss in Table (3). However, as shown in column 4, the ratio of lower-income students is higher in these TPSs as HP charter schools appear more in the neighborhood.

These patterns in charter schools' capacity and student reallocation suggest that the charter sector could respond to the policy by expansion. This suggests that the policy can be thought of as reducing the adjustment cost of expansion for HP charter schools. Moreover, the HP charter schools will likely impose an externality on the nearby TPSs via reallocation of enrollment. Therefore, competitive spillovers might be a crucial mechanism in evaluating the existing or other similar policies. Particularly, to what extent the competitive spillover can push neighboring schools to improve test scores is a policy-relevant question. Additionally, as the above patterns suggest, this policy is associated with student composition change in schools, which can result in test scores even if schools do not change educational inputs. This imposes an empirical challenge in identifying the competitive spillover on test scores. In what follows, I address this empirical challenge using a difference-in-difference design facilitated by the policy and control for the student composition change.

Table 4. Effects on Composition of Students of Exposure to HP Charter Schools

	(1) LogenrBlack	(2) LogenrFRL	(3) RatioBlack	(4) RatioFRL
HP Charter 0-1 Miles	-0.021***	-0.001	0.00054	0.00360***
	(0.006)	(0.004)	(0.00068)	(0.00123)
HP Charter 1-3 Miles	-0.023***	-0.012***	0.00025	0.00218***
	(0.003)	(0.002)	(0.00034)	(0.00055)
HP Charter 3-5 Miles	-0.023***	-0.008***	-0.00055***	0.00364***
	(0.003)	(0.001)	(0.00016)	(0.00035)
Constant	4.583***	5.944***	0.25988***	0.63447***
	(0.005)	(0.003)	(0.00063)	(0.00105)
Observations	28,927	28,967	29,037	29,037
R-squared	0.959	0.910	0.98470	0.92243
School FE	Y	Y	Y	Y

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Competitive Pressure Following the enactment of the High-performing Charter School Statute, a school could face more competitive pressure as more of its neighboring charter schools can expand with less regulatory constraint. Because the future expansion of neighbors might cause fleeing of students from the school. Facing the pressure, schools might increase their input into educating students, as reflected by test scores. Therefore, to explore the potential competitive responses of schools influenced by the policy, I exploit the establishment of the policy as a natural experiment. I focus on the TPS sector for this test: TPSs differing in competitive pressure from local charter schools receive different intensities of pressure after the policy. Those TPSs with more neighboring HP charter schools in 2012 right after the policy faced higher competitive pressure than other TPSs with either no or fewer neighboring HP charter schools in 2012. However, when investigating responses via test scores, they are likely influenced by the change in student composition. Especially, policies like the High-performing Charter School Statute are likely to be associated with student reallocation, as seen in North Carolina (Gilraine et al. 2021; Slungaard Mumma 2022). Therefore, econometricians might pick up the effect of reallocation instead of the increase of inputs of schools if naively regressing test scores on the treatment. Guided by the above design, I develop a specification to explore the causal effects of competitive pressure on TPSs' increases of inputs.

Before introducing the specification, I formalize the notation and measure of the treatment, outcomes, and other critical controls. I define $Treat_i$ as the number of charter schools within 5 miles of a TPS i such that these charter schools would become HP in 2012. I further define $Post_t$ as an indicator of whether an observation from year t is later than 2011. Therefore, the treatment variable $Post_t \times Treat_i$ switches to positive after the policy and is larger if school i faces more HP charter schools. The outcomes under inspection, A_{igkt}, are the normalized average score of subject k of the student cohort in school i in year t of grade g.[22] Therefore, a triple (i, t, g) uniquely identifies a cohort of students. Motivated by the goal of isolating the effects of student reallocation from schools' increase of inputs, I use the matched cohort test score, $A_{igkt}^{LastYear}$, of the previous year's average score of the students that construct the (i, t, g) cohort, to control for the student composition changes. Note that the averaging in both years is over the same students, although students in the cohort may not study in school i in the previous year. [23] As I show the results, I introduce the rest of the covariates, subsumed in $\mathbf{Z}_{igkt}$.

With these notations and measurements in hand, I estimate the following difference-in-difference regression (3) to reveal the causal effect on schools' change of inputs when facing more potentially expanding neighboring charter schools, i.e., the HP charter schools. I restrict my primary analysis to TPSs with a charter school within five miles in 2011, which shrinks the full sample of TPSs by one-third. I implement the tests on the full sample as a robustness check. [24]

$$\underbrace{A_{igkt}}_{\substack{\text{Cohort}(i,g,t) \\ \text{test score}}} = \beta Post_t \times Treat_i + \rho \underbrace{A_{igkt}^{LastYear}}_{\substack{\text{Same}(i,g,t) \\ \text{Last year test score}}} + \alpha Post_t + \eta Treat_i + \gamma \mathbf{Z}_{igkt} + \epsilon_{igkt} \tag{3}$$

In this specification, β is the parameter of interest. It captures the change in the difference between the average test scores of the TPSs facing more pressure from potentially-

[22]The raw data contain the average test score of the cohort and the enrollment size of the cohort. The normalization is across all schools within the grade-subject-year level, with the enrollment size being the weight of each observation in the calculation.

[23]The current scores and their previous year are normalized separately across schools in their corresponding years.

[24]Unfortunately, the matched cohort test scores are no longer publicly available after 2014. Therefore, the analysis of longer-term dynamic effects is not available under the current empirical strategy.

HP charter schools and that of the TPSs facing less such pressure after the policy change (conditional on other controls). Under the assumption that trends in unobservable characteristics that affect test scores are the same across TPSs with varying degrees of such pressure, the estimates of β recover the causal effect of the pressure brought by charter schools' potential expansion.

The results of the tests are shown in Table (5). In the first column, the estimate of β suggests that adding one nearby HP charter school within 5 miles increases test scores significantly by 1.48% standard deviation ("σ" henceforth in this section). The causal effect is not only significant but also larger than the existing findings in the studies on TPSs' competitive responses to choice programs.[25] Although both the samples used to identify the competitive response vary across the study, and the identification strategy is different, I speculate that there may be several critical reasons why my estimate is higher than the ones in the existing studies. First of all, in my context, the competitive pressure is generated by HP charter schools with satisfactory performance records. They are probably more attractive than normal choice programs discussed in the existing studies. Secondly, the expansion eligibility associated with the HP designation strengthens the potential ability of the HP charter schools to attract students from the neighboring TPSs. More importantly, the expansion eligibility signifies a potential threat in the future. As a neighboring TPS, it might feel the pressure of continuing to lose future students. This finding potentially sheds light on the considerable potential of using expansion eligibility to incentivize charter schools because it might also incentivize the neighboring TPSs to increase effort.

Furthermore, the main treatment effect reduces to 0.82%σ but is significant and comparable to existing findings as I control for more covariates such as fixed effects, match rate of the cohort,[26] school student compositions, the count of charter schools within 5 miles, and pupil-teacher ratio. Additionally, I separately run the tests on math and reading scores with the choice of covariates mimicking column (3) of Table (5). The results are shown in the first two columns in Table (B4). The effects on both subjects are positive and significant, while the effect on reading is higher.

[25] For example, Figlio and Hart (2014) find adding one nearby private school increases test scores by only 0.21%σ in using Florida data ranging from 1998 to 2002, also using a difference-in-difference strategy. Figlio et al. (2021), using Florida student-level data from the early 2000 to the late 2010s, show that increasing one charter school within 5 miles increase reading scores by 0.36%σ to 0.98%σ depending on the instruments they use.

[26] The match rate of the cohort measures the proportion of the students in cohort (i, t, g) that also exist in the cohort $(i, t-1, g)$. Following the logic of the analysis, if this number is higher across schools, it means that student reallocation is less intensive across schools and that the students contributing to the average test score of cohort (i, t, g) are more alike with the students contributing to the average score of the cohort $(i, t-1, g)$. This also means the observations with a high match rate support the legitimacy of attributing the causal effect on the test score increase to schools' input increase. I formally test this idea in the robustness check following the main specification.

To test whether there were significant pre-policy differences across TPSs with varying HP charter exposure, I run an event-study specification, i.e., replacing the post-policy indicators $Post_t$ with the list of l-year-to-2011 indicators. I include the most covariates as in column (3). Figure (A1) reports the event-study coefficient plot regarding 2011 as the baseline year. I confirm there is no significant pre-policy differential trend of average test score difference across treatment groups as defined. The results also show that the post-policy dynamic effects built up and then alleviated from 2013 to 2014.

Table 5. TPSs' Responses in Test Score to HP Threat

Outcome: Average Test Score	(1)	(2)	(3)
$Post_t \times Treat_i$	0.0148***	0.0082***	0.0083***
	(0.0017)	(0.0023)	(0.0023)
Constant	0.0098***	37.6187***	32.1766**
	(0.0015)	(13.1661)	(13.1787)
Observations	55,310	55,304	55,304
R-squared	0.8933	0.8972	0.8973
FEs (gt, ig, gk, gt)	Y	Y	Y
Charter Entry + School Demo	N	Y	Y
PT Ratio	N	N	Y

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

I test for the robustness of the findings and show the results in Table (B4). I first change the measurement of $Treat_i$ and re-run the pre-post specification with the most covariates. In the main specification aforementioned, I use the number of charter schools within 5 miles of a TPS i such that they will become HP in 2012. I construct alternative measures by slightly modifying the original one: 3 miles instead of 5 miles, using indicators instead of count, and using the number of A charter schools in 2012 instead of the to-be-HP charter schools.[27] The estimates of β across different measurement choices are almost all positive and significant. Additionally, I test whether results vary if I change samples. Firstly, I run the tests on the full TPS sample. This essentially increases the number of observations in the control group because a TPS having no charter school implies that it has no HP charter schools (within five miles in 2011). I design this test to check whether including TPSs with little charter exposure in the control group could alter the qualitative results. Because these TPSs might not be as comparable to those treated with high HP charter presence as the ones with some charter schools existing in 2011. However,

[27]Potentially, these A schools are candidates for HP charter schools in 2012, and some did become HP in 2012 or later.

the qualitative results do not change. Secondly, I exclude the observations of a cohort if it has a lower than 80% or 90% match rate with its previous year's scores. This means the Department of Education can not track 20% of 10% of the cohort's previous year's test scores. These tests examine whether using the data of cohorts with less attribution due to reallocation across schools will change the results. The estimates from these tests can be more credibly attributed to the change of inputs instead of the reallocation of students. The results show that, although truncating observations at a 90% match rate of the cohort causes considerable data loss, all the qualitative results remain the same. A similar result is found when truncating using 80% as the cutoff. It should be noted that this way of controlling students' reallocation is not perfect due to data limitations. Ideally, if student-level test score is available, one can largely eliminate the reallocation channel by controlling an individual's test score in the previous year. With all these robustness checks, I conclude that the competitive pressure of HP charter school neighbors imposed on the TPSs increases TPSs' inputs into education, which raises their test scores.

4.3 Target Whom? Designation Advantages High SES Charter School

As shown in Table (1), HP charter schools appeared more in higher SES regions. This raises a question: Would charter schools that serve low SES regions get designation by exerting higher value-added, reducing the systematic performance difference observed in Table (1) across charter schools?

Figure (1) answers this question by showing that such differences might be systematically rooted in the designation criteria. It shows the density of specific indicators of student compositions within a charter school among all the charter schools with value-added that is higher than the median in 2015. This figure, therefore, illustrates the distribution of student composition across the non-HP and HP charter schools among the charter schools that exert relatively high effort in educating students. The two indicators of student composition of a charter school are the percentage of students with free or reduced-price lunches (left) and the percentage of black students (right), the two relatively disadvantaged student groups. From Figure (1), among the higher-than-median value-added charter schools, the non-HP tend to serve poor or black students, as the non-HP density curve of these percentages of disadvantaged students is on the right of the HP's curve. The reason could be that the designation criterion, namely "2A1B," relies heavily on the *level* of academic performance of charter schools, less on the *value-added*. This favors charter schools in high SES regions where their students come from more educated families.

This raises a concern about whether the policy could lead to unequal allocation of expansion eligibility, which might result in unequal access to high-quality charter school seats across regions with different SES. Giving charter schools serving the low SES regions with high value-added the opportunity to expand may help reduce the inequality of high-quality charter programs across regions.

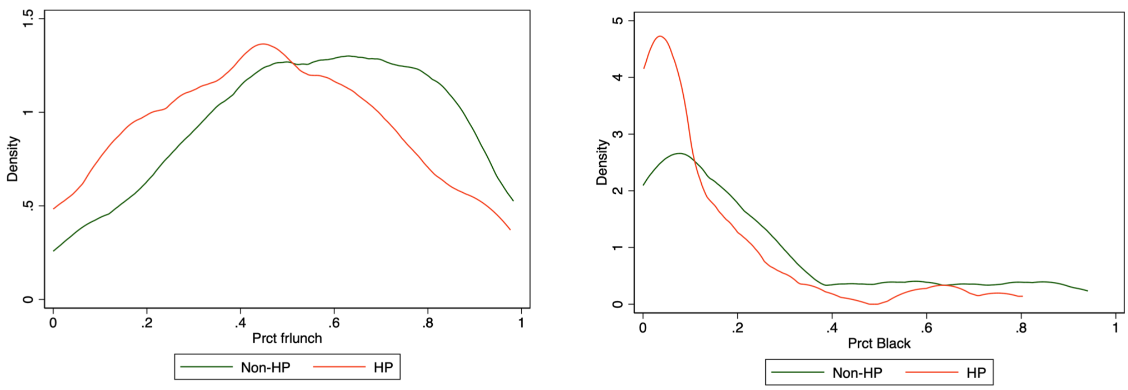


Figure 1. Density of Student Composition Across Higher-than-median Value-added Charter Schools in 2015

In the following sections, I formally investigate this alternative scheme targeting value-added by building a quantitative model to simulate its effects. The model characterizes the key mechanisms informed by the data patterns. One mechanism is the adjustments of capacity and performance. I model them as the two critical decisions made by schools and allow charter schools' capacity adjustment to be influenced by their HP designation status. The other is that competition across schools influences schools' adjustment in performance. I incorporate this mechanism in the model by explicitly modeling the pressure a school faces in competing for students. Both mechanisms are crucial in influencing the distribution of accessibility (i.e., charter and TPSs' capacity) and school performance.

5 Quantitative Model

In this section, I develop an empirical model that characterizes the dynamics of charter and TPSs' performance and capacity. I build the model based on the dynamic oligopoly model developed by Ericson and Pakes (1995) and adapt it to capture the education market and policy context of Florida.

In each period, schools endogenously expand capacity or improve their performance (or both) to maximize their long-term objectives. They make decisions according to their own capacity, performance level, and other time-varying characteristics of themselves and the schooling market where they belong. Then, in the schooling market competition stage, students choose schools based on the schools' characteristics. Because adjustments of capacity and performance are costly, schools consider a trade-off between the ongoing benefits of having higher performance and larger capacity (so as to enroll more) and the one-time adjustment costs involved in both decisions. In addition, charter schools can earn HP designation by accumulating good performance, and the designation can reduce the cost of adjusting capacity. Under this setting, the model links the time-varying operating environment with schools' two key choices and links them with the policy (via the modeling of HP designation) and the competitive environment schools face. It allows schools' endogenous reaction to the change of adjustment cost and competitive environment brought by the HP policy, as informed by the preliminary data patterns.

5.1 Environment

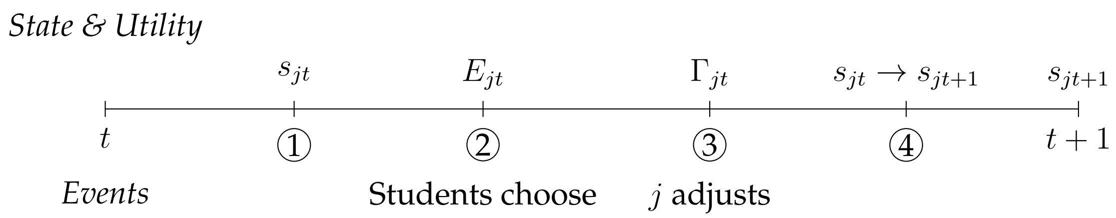
The model describes a regional schooling market. Time is discrete, unbounded, and measured in school years, denoted by $t \in \{1, 2, 3, ...\}$. A school is denoted by j. The number of the operating schools, J, is assumed to be constant over time and schools in the market do not expect entry, exit, or change in ownership. Hence, I also use J to denote the set of schools. Schools are heterogeneous with respect to their own state x_{jt} (to be discussed). The market situation state that j faces, n_{jt}, is a function of all of the schools' own state, i.e., $(x_{jt})_{j=1\sim J}$. It summarizes how school j's utility in period t is influenced by other potentially competing schools' and its own state. The information set of school j at period t is denoted by s_{jt}:

$$s_{jt} = (x_{jt}, n_{jt}).$$

I introduce the functional form used to construct n_{jt} in greater details in the demand subsection. Thus, the market is fully characterized by all schools' information sets: $s_t = (x_{jt}, n_{jt})_{j=1\sim J}$.

The sequence of events within a period is shown in Figure (2). Firstly, school j learns about s_{jt} at the beginning of a period. Secondly, students choose school j according to s_{jt}, resulting in enrollment E_{jt}. Thirdly, school j adjusts its own states by expanding or exerting effort (or both) that incurs adjustment costs Γ_{jt}. Fourthly, the state s_{jt} evolves to its new level s_{jt+1}. Particularly, x_{jt} evolves to x_{jt+1} according to j's adjustment decisions and exogenous state transition rules, and the market situation state n_{jt} evolves to n_{jt+1} according to all j's decisions.

Figure 2. Timing of the Events in the Model



5.2 Demand and the Form of Market Situation State n_{jt}

In this subsection, I introduce the demand model and the construction of the market situation state n_{jt}.

Allowing students to differ according to their residential location is crucial in characterizing school choice (Neilson 2021; Agarwal and Somaini 2018; Allende 2019; Dinerstein et al. 2022; Gilraine et al. 2023). Given this, I build my demand model based on the spatial demand literature (Holmes 2011; Zheng 2016; Ellickson et al. 2020).

The market is endowed with a set of locations l. Let L denote the set of all locations. To be consistent with the empirical implementation, I call a location a census tract. I assume the existence of a representative student in each tract l. Therefore, I index students by their location l. The student population size of tract l in period t is denoted as m_{lt}.

The student i who lives in l can choose schools $j = 0, 1, 2, 3..., J$, where $j = 0$ indicates the option of homeschooling or attending private schools. According to Florida's open enrollment policy, a student can enroll in any charter or TPS in Florida. Therefore, I model students' choice set to be J. The student needs to travel tract-school specific distance, $dist_{jl}$, to a school j. The geography of the market is assumed to be fixed over time, and hence, all distances are time-invariant.

The utility for the student i residing in l is:

$$w_{ijlt} = \delta(x_{jt}; \alpha) + \lambda dist_{jl} + \zeta_{ijlt}.$$

The first term, $\delta(x_{jt}; \alpha)$, is the mean utility a student gets from enrolling in school j in period t, parametrized by α. As the name suggests, the mean utility is common to all students. The second term, $\lambda dist_{jl}$, captures students' disutility from traveling. The third term, ζ_{jti}, is an idiosyncratic taste shock. I assume the ζ_{jti} is distributed as i.i.d. Type-I Extreme Value.[28] The outside option is assumed to have zero mean utility: $\delta(x_{0t}) = 0$. Notably, I allow capacity, a critical component in x_{jt}, to influence j's enrollment. I explain the functional form of the mean utility $\delta(.)$ and the contents in x_{jt} relevant for characterizing schooling demand in the estimation section.

Given the assumption imposed on ζ_{jti}, the choice probability of students living in l choosing j is:

$$\frac{\exp(\delta(x_{jt}; \alpha) + \lambda dist_{jl})}{1 + \sum_{j' \in J} \exp(\delta(x_{j't}; \alpha) + \lambda dist_{j'l})}.$$

Therefore, the enrollment of school j in period t, E_{jt}, is obtained by adding all the students

[28]The i.i.d. assumption reflects the random variations in the student's schooling experience that do not persist across time because of schools' culture, security concern, atmosphere, and so forth. Additionally, I assume there are a large number of students in each tract, so schools perfectly predict their market share based on their states. Therefore, ζ is not assumed to be a state variable for schools in this model.

that j enrolls across all tracts:

$$E_{jt} = \sum_{l \in L} m_{lt} \cdot \frac{\exp(\delta(x_{jt}; \alpha) + \lambda dist_{jl})}{1 + \sum_{j' \in J} \exp(\delta(x_{j't}; \alpha) + \lambda dist_{j'l})}. \tag{4}$$

One can alternatively write equation (4) as:

$$E_{jt} = \exp(\delta(x_{jt}; \alpha)) \cdot \sum_{l \in L} m_{lt} \cdot \frac{\exp(\lambda dist_{jl})}{1 + \sum_{j' \in J} \exp(\delta(x_{j't}; \alpha) + \lambda dist_{j'l})}.$$

I define the market situation state variable n_{jt} as:

$$n_{jt} = \sum_{l \in L} m_{lt} \cdot \frac{\exp(\lambda dist_{jl})}{1 + \sum_{j' \in J} \exp(\delta(x_{j't}; \alpha) + \lambda dist_{j'l})}.$$

Under this definition, enrollment is

$$E_{jt} = \exp(\delta(x_{jt}; \alpha)) \cdot n_{jt}. \tag{5}$$

In my empirical context, there are potentially many schools that are heterogeneous in their x_{jt}, such as performance and capacity. Without further assumptions, the current setting implies the state space for school j is the Cartesian product of the state space of all schools' own state x. Therefore, the state space for each school expands rapidly in J. This "curse of dimensionality" imposes a challenge in computing the MPNE, i.e., Markov Perfect Nash Equilibrium, in this model. Therefore, I make a simplifying assumption on j's state space and define an alternative equilibrium concept (to be discussed) to facilitate the computation of the model. This assumption is stated as follows.

Assumption "Inclusiveness". *Each school's belief in its demand is represented by equation (5) and is summarized by states characterizing schools' own state x and a uni-dimensional state n characterizing the market situation faced by each school.*

This assumption reduces the dimensionality of the state space for a school. It also implies that schools have limited cognitive ability to track all their competitors' states over time to predict their future enrollment. If J is large, this is a reasonable assumption. This setting still preserves schools' competitive responses by allowing their decisions to depend on their beliefs over the time-varying market situation via the summary statistics, n, that summarizes the relative "attractiveness" of other competing schools.[29]

[29]This modeling device and the formula generated from a demand model are shared by other industrial organization research using a dynamic model (Hendel and Nevo 2006; Gowrisankaran and Rysman 2012) and static models used in the economics of education setting (Sanchez 2018; Dinerstein et al. 2022)

5.3 Schools' Dynamic Programming Problems

State Space Schools are heterogeneous in many dimensions. In each period t, their own states consist of the following:

$$x_{jt} = (o_j, q_{jt}, k_{jt}, hp_{jt}, d_{jt}, \xi_{jt}, \epsilon_{jt}).$$

Except for the ϵ_{jt}, all state variables in x_{jt} are observable to the econometrician. The ϵ_{jt} are distributed i.i.d., across schools and periods. They capture the unobserved heterogeneity of schools' adjustment costs and hence allow for gaps between the model-predicted and observed decisions of schools. I discuss the economic interpretation of ϵ_{jt} in greater detail, along with introducing schools' adjustment process.

The time-invariant state o denotes the school type, either charter or TPS. Since the government regulates TPSs and charter schools differently, their decision-makers are distinct in objectives and the tools that they can use to influence schools' development. Therefore, by breaking into two types of schools, the model allows school types to govern different constraints on schools' state space, action space, and objectives. Accordingly, all the parameters in the following are allowed to be different by type and, hence, are estimated separately for each type.

The state variables q, performance, and k, capacity, influence the school's enrollment, a component of both types of schools' objectives. The state variable hp, HP designation status, influences the adjustment cost of charter schools' capacity. The states q, k, and hp are the core endogenous states directly influenced by a school's decisions.

The state variable d characterizes the local operating environment schools face, such as racial composition and household income level. This state variable allows charter schools' operating costs to vary across demographics, as in Singleton (2019). It therefore allows, for example, for the return of adding a classroom or exerting effort to depend on demographics. Since I do not model schools' entry and exit decisions, d is assumed to be exogenous and independent of schools' decisions.

Finally, the one-dimensional state variable ξ represents all other aspects of school quality that can shift students' demands. It will be recovered from demand estimates and hence is assumed to be observable to the econometrician.

School's Flow Utility and Adjustment Decisions Schools make two adjustment decisions in each period to maximize expected utility over time. The two decisions are educational effort, v, and capacity expansion e. Particularly, the variable v represents schools' decisions on value-added. It is a scalar summarizing all the schools' inputs that are invested in improving students' test scores. It can include spending on the professional development of teachers, teacher coaches, better leadership, and administrative support.

The decision e_t represents the school's extra capacity to expand (or shrink) in period t.

Charter schools are allowed to make both decisions, while TPSs in this model are assumed to have a fixed capacity, i.e., $e_{jt} = 0, \forall t$, and can only decide on value-added. I make this assumption because TPSs do not change the enrollment capacity frequently or by a large proportion over time in my data.[30] Decisions of adjustment are defined as the mappings from states to actions:

$$v : (s_{jt}) \to v_{jt}$$
$$e : (s_{jt}) \to e_{jt}.$$

These adjustments are costly and jointly influence all the endogenous variables.

I assume charter schools operate as for-profit organizations.[31] Their flow utility u_{jt} has the following form:

$$u_{jt} = rE(s_{jt}) - \Psi(E_{jt}, s_{jt}) - \Gamma(v_{jt}, e_{jt}, hp_{jt}, \epsilon_{jt}).$$

Enrollment $E(s_{jt})$ is a function of the state variables. It summarizes the demand side of the schooling market. $rE(s_{jt})$ represents the total revenue charter schools get from enrolling $E(s_{jt})$ students. In practice, charter schools get revenue from the government

[30] I do not have complete and high-quality capacity data for TPS. However, I manage to get long panels of TPSs' capacity in Lee and Palm Beach counties, measured by student station. I find that most of the change in capacity is either zero or not empirically relevant in magnitude. Take TPSs from the Palm Beach County as an example. From 2011 to 2020, over the 2659 observations of annual change of capacity compared to the previous year, 85% show zero change, and 5% shows less than 1% change (compared to the previous year's capacity). A similar qualitative conclusion can be found in inspecting Lee County's panels. Potentially, one can digitize the Annual Five Year Plan document published by the local school districts to obtain all TPSs' capacity. However, the document does not provide the unique school ID number. Moreover, it does not use the same name as the school that appeared in NCES or Florida Master File data, making the exact merge across datasets almost impossible. Based on the data I can digitize and merge, I conclude that the facility in TPSs does not frequently change over time. In the empirical implementation, I impute their capacity using their in-sample largest enrollment divided by a constant to measure their capacity.

[31] Although all charter schools in Florida operate as non-profit organizations, around 40% to 50% of charter enrollment is in charter schools that sign contracts with private management companies to operate the daily business in my sample from 2012 to 2019. The pressure of making a profit may come from payments to these private companies. I label these charter schools as for-profit, which is aligned with the definitions used by Singleton (2017) and Singleton (2019). Singleton (2019) also defines two types of charter schools: the "no-excuses" and the "other" charter schools. The "no-excuses" charter schools follow an educational philosophy emphasizing high expectations, comportment, and traditional math and reading skills. The rest are in the "other" category. Using his definition of the labels, I discover that in my sample, the "no-excuses" and the "other" charter schools account for around 15% and 35% charter enrollment in recent years. According to Singleton (2019), no-excuses charter schools are considerably less sensitive to variable costs and large enrollment than the other two types of charter schools. Although the no-excuses charter schools have different objectives, given their relatively lower market share, I focus on the other two types of charter schools that operate in Florida. Furthermore, I do not distinguish the "other" type of charter schools from the for-profit ones in their objectives, because the heterogeneity of charter schools is not the paper's focus. In future versions of the paper, I can allow these two types of charter schools to have distinct model primitives.

according to a per-enrollment reimbursement rate r,[32] which is known to the econometrician. The function $\Psi(.)$ captures the variable cost of maintaining daily operation and instruction, e.g., teachers' salary, rent, staff compensation, and maintenance. The functional forms of $E(.)$ and $\Psi(.)$ will be described in estimation. The function $\Gamma(.)$ represents the adjustment costs charter schools pay to change future capacity and performance.

As for TPSs, I assume they operate as non-profit organizations. Their flow utility is a weighted sum of enrollment, performance, and the adjustment cost of improving performance:

$$u_{jt} = r^E E(s_{jt}) + r^q q_{jt} - \Gamma(v_{jt}, \epsilon_{jt}).$$

In this specification, r^E and r^q indicate the relative weight of enrollment $E(s_{jt})$ and performance q_{jt}. They are enumerated in terms of the TPSs' valuation of adjustment cost. This reflects the principal's objective in maintaining enrollment and performance: if the school constantly performs badly or not enough students attend the school, its principal can be fired. I assume the econometrician knows r^E and r^q because these two parameters can not be separately identified with the adjustment cost using the TPSs' value-added decisions.[33] I calibrate both parameters according to Mehta's (2017) structural estimates with slight modifications.

The most critical component in both types of schools' flow utility is their adjustment costs. For charter schools, I model their adjustment cost function as:

$$\Gamma(v_{jt}, e_{jt}, hp_{jt}, \epsilon_{jt}) = \gamma_v v_{jt} + 1_{\{e_{jt} \geq 0\}} \left(\overbrace{\gamma_1(\epsilon_{jt}) + \underset{\substack{\uparrow \\ \text{HP effect}}}{\gamma_2} \cdot \; hp_{jt}}^{\text{Fixed Costs}} \; + \; \overbrace{\gamma_3(\epsilon_{jt}) \cdot e_{jt} + \underset{\substack{\uparrow \\ \text{HP effect}}}{\gamma_4} \cdot \; e_{jt} \cdot hp_{jt}}^{\text{Variable Costs}} \right)$$
$$+ 1_{\{e_{jt} < 0\}} \gamma_5 \cdot e_{jt}. \tag{6}$$

The γ_v captures the per-unit cost of value-added from all inputs a school spends to improve students' scores. The per-unit cost of capacity change is captured by γ_3. It includes spending on purchasing furniture, hiring designers, and building extra classrooms. I also consider fixed costs of increasing capacity as indicated by γ_1. Introducing the fixed costs rationalizes the lumpiness in adjustment in capacity, as observed in the data. Furthermore, γ_1 also captures the reality that capacity adjustment is associated with hiring lawyers to negotiate and re-contract with the government, regardless of the size of the expansion. Furthermore, the HP designation is modeled as influencing both fixed and variable expansion costs for designated charter schools via γ_2 and γ_4. Finally, I allow the unobserved heterogeneity ϵ_{jt} to influence charter schools' adjustment cost of expansion.

[32] Although the per-enrollment reimbursement rate evenly changes across years, I do not model it as so for convenience.

[33] For example, low adjustment cost of exerting value-added or prioritizing in getting high performance can both generate high value-added decisions.

This heterogeneity exists because charter schools execute different modes of expanding capacity, which can involve different costs. For example, charter schools could renovate five classrooms within the existing facilities or add a floor to their existing building with five classrooms. The former is usually less costly. In the data, I do not observe the mode of expansion. Therefore, I model the expansion decisions to depend on the unobserved heterogeneity to rationalize the discrepancy between the policy function estimated and charter schools' expansion data. Therefore, for both γ_1 and γ_3, I assume they are drawn each period from normal distributions common across all charter schools in all years.

Since I do not allow TPSs to alter their capacity in the model, the adjustment cost functions for TPSs are simply:

$$\Gamma(v_{jt}) = \gamma_v v_{jt} \tag{7}$$

State Transitions of Individual States Capacity evolves in a deterministic way. Future capacity is a sum of current capacity and expansion

$$k_{jt+1} = k_{jt} + e_{jt}.$$

Performance evolves according to the current performance and the value-added into the next period performance, captured by the function $\tau(.)$:

$$q_{jt+1} = \tau(v_{jt}, q_{jt}).$$

In my application, this corresponds to the following production process of academic performance: Students attend school and perform in standardized tests, earning the school a rating of q_{jt} in period t. The school decides to put in v_{jt} amount of value-added to promote students' academic performance in $t+1$, resulting in schools earning q_{jt+1}. This transition rule applies to both charter and TPSs.

Designation of charter schools evolves as a function of period t's performance level and the HP status, namely:

$$hp_{jt+1} = \eta(q_{jt}, hp_{jt}).$$

I regard hp_{jt+1} as a passively evolving endogenous variable unaffected by decisions *directly*. This assumption reflects the nature of the statute that designation is not dependent on value-added directly. I also abstract away from the actual policy, which requires three years of satisfactory performance, by assuming that the determination of future designation depends on current performance to avoid unnecessary complications. In practice, predicting future designation with only current performance and designation has acceptable accuracy. Additionally, as shown by the data, de-designation happens extremely infrequently. Therefore, I set $hp_{jt+1} = 1$ if $hp_{jt} = 1$.

For the rest of the components in a school's own states, namely d_{jt} and ξ_{jt}, I assume they all independently follow AR(1) processes.

State Transitions of Market States Given the "Inclusiveness" assumption, I constrain how schools form beliefs about the n_{jt}'s evolution, denoted as $\nu(.)$, in the following assumption.

Assumption "Consistent Belief". *Each school forms a rational expectation that $\nu(.)$ is an autoregressive process with one lag, i.e., AR(1), and its belief is consistent with how the market would evolve when the school itself and its competitors make optimal dynamic decisions given their beliefs $\nu(.)$.*

This assumption requires that schools have no strategic consideration about n_{jt} (i.e., they believe their own decisions do not directly change n_{jt}) and that their beliefs on n_{jt} have to be consistent with how the market evolves. This assumption is established to allow schools' beliefs about the competitive environment to change under the alternative supply-side policy. Think, for example, a simulation exercise in which the econometrician expects to test the value-added response by TPSs under a counterfactual policy. The policy does not allow the HP designation system to exist and imposes more constraints on the extent to which charter schools can expand. Even though the traditional sector is not directly targeted, they should predict a less "aggressive" expansion of neighboring charter schools under this counterfactual environment. The "Consistent Belief" assumption allows schools to alter beliefs in a way consistent with how the market evolves. This assumption requires jointly considering schools' optimal decisions according to the dynamic programming problems and their beliefs about the evolution of the market environment. This implies an iterative algorithm to find a fixed point of $\nu(.)$ that satisfies the "Consistent Belief" assumption. More computation details are explained in section 8.

Based on the assumptions "Inclusiveness" and "Consistent Belief", I introduce the dynamic programming problems faced by both types of schools and the equilibrium concept.

Schools' Dynamic Programming Problem and Equilibrium With all model components specified, the maximization problem faced by a charter school is summarized by (8). I denote β as the discount factor. I omit subscript j.

$$\begin{aligned} V(s_t) = & \max_{v_t, e_t} rE_t(s_t) - \Psi(E_t, s_t) - \Gamma(v_t, e_t, s_t) + \beta \mathbb{E} V(s_{t+1}|s_t) \\ s.t. \quad & q_{t+1} = \tau(v_t, q_t),\ k_{t+1} = k_t + e_t,\ hp_{t+1} = \eta(q_t, hp_t), \\ & d_t, n_t, \xi_t \sim AR(1), n_t \text{ transition satisfies Consistent Belief} \\ & \epsilon_t \sim i.i.d. \end{aligned} \tag{8}$$

The maximization problem faced by a TPS is summarized by (9).

$$\begin{aligned}
V(s_t) = \max_{v_t} \; & r^E E(s_t) + r^q q_t - \Gamma(v_t, s_t) + \beta \mathbb{E} V(s_{t+1}|s_t) \\
s.t. \quad & q_{t+1} = \tau(v_t, q_t), k_{t+1} = \bar{k}, hp_{t+1} = 0, \\
& d_t, n_t, \xi_t \sim AR(1), n_t \text{ transition satisfies Consistent Belief} \\
& \epsilon_t \sim i.i.d.
\end{aligned} \tag{9}$$

I define the equilibrium below to close the model. To facilitate exposition, first, denote z as a school's strategy, i.e., $z = (v(.), e(.)) \in Z$ and define the expected value function implied by each school's own ($\tilde{z}$) and other schools' strategy (z) as

$$\bar{V}_{\tilde{z},z}(s) = \mathbb{E}_\epsilon V_{\tilde{z},z}(s) = \mathbb{E}_\epsilon \left[\max_{\tilde{z}(s)} \pi(s) - \Gamma(s, \tilde{z}(s)) + \beta \mathbb{E}_{\tilde{z},z} V(s'|s) \right].$$

Definition. *An equilibrium of a market is characterized by a strategy z such that:*

1. *(Optimality) z satisfies the optimality condition. That is, for every state $s \in \mathcal{S}$, for every school,*
$$\sup_{\tilde{z} \in Z} \bar{V}_{\tilde{z},z}(s) = \bar{V}_{z,z}(s).$$
2. *(Consistent Belief) Each school forms rational expectation on the perceived transition, $\nu(.)$, of competitive pressure n, s.t. $\nu(.)$ is consistent with how the market evolves based on this belief. That is,*
$$\tilde{\nu}^z(.) = \nu(.),$$
where $\tilde{\nu}^z(.)$ is the transition of n when all schools play strategy z.

This equilibrium concept and the implied iterative algorithm used to solve the model are similar to the Moment-based Markov Equilibrium (Ifrach and Weintraub 2017) in which agents' strategies are assumed to depend on summary statistics of the distribution of other agents' states. The Moment-based Markov Equilibrium, along with other equilibrium concepts following the work by Weintraub et al. (2008), are attempts to address the computation burden created by using MPNE as the solution concept of a dynamic game.

5.4 Analysis of Mechanisms

The model captures two key mechanisms that govern schools' decisions: incentives in adjustment and competition.

Firstly, I explicitly model the adjustment costs to influence schools' intertemporal decisions. Adjustments are costly at the moment but can benefit the school by increasing future enrollment. Furthermore, the model introduces the HP designation hp in the adjustment cost function of charter schools. This enables the evaluation of the direct policy effect. In one of the counterfactual simulations, I compare the observed outcomes of interest to the predicted ones when the designation-related benefits and transitions are eliminated from the model. Conceivably, under the existing policy, non-HP charter schools can accumulate high performance to change their HP status in future periods, thereby reducing the adjustment cost for expansion. The existing HP scheme naturally interacts with the two decisions of charter schools. Since investing in the performance may reduce future costs for expansion, the effort choices of charter schools can be influenced accordingly by the existing HP scheme.

Secondly, because competitive pressure n enters the demand function, schools' decisions can respond to local competitive pressure from neighboring schools. These responses can be further influenced in the future according to what schools believe about the evolution of the market situation. More importantly, incorporating competitive responses is crucial in quantifying the effects of large-scale counterfactual policies, such as deregulating all charter schools. Such policies will likely change schools' beliefs about the evolution of the competitive pressure they face. To properly characterize how schools change a belief about the evolution of their competitive pressure, the "Consistent Belief" assumption is critical.

Finally, the model allows for decisions of both charter and TPSs to be responsive to demographic heterogeneity d. Particularly, $\Psi(.)$ can depend on local demographics. This heterogeneity is important for counterfactual policy evaluation. As is also shown in the data, educating students with low SES can involve higher instructional expenditure per enrollment. Modeling the dependence on local conditions can help evaluate the heterogeneous responses of different schools that operate in various demographic conditions across different regions. Specifically, to evaluate whether an alternative policy that gives more expansion eligibility to charter schools in low SES regions requires scrutiny of the estimates of the operating cost function. Such a policy may not trigger charter schools to expand capacity as expected if the charter schools in these regions may not have the incentive to expand due to high operation costs.

6 Empirical Strategy

This section first introduces the two-step estimation strategy. Then, it presents measurements, estimation samples, and empirical specifications, with a particular focus on the demand. And then, it continues to introduce the identification of the adjustment cost function.

6.1 Overview of Estimation Strategy

I calibrate the reimbursement rate r and the utility weights for TPSs (r^E, r^q) directly from Florida laws and Mehta (2017), respectively. For charter schools, the per-enrollment reimbursement rate r is set to be \$8000 a year.[34] For TPSs, I calibrate the utility weights according to Mehta (2017)'s structural estimates. In the paper, enrollment is set to be the numéraire, and his estimates show that TPSs put weight 19.634 on their average test scores. Therefore, I set $r^q = 20 * r^E$, approximating Mehta (2017)'s results. I further set $r^E = r$. This is an innocuous assumption as long as the ratio between r^q and r^E is appropriate. Setting $r^E = r$ not only reflects that charter and TPSs are reimbursed under the same formula,[35] but it also makes the estimates in the adjustment costs for value-added between charter and TPSs comparable. The discount rate β is set to be 0.9.

I use the simulation-based algorithm developed by Bajari et al. (2007) henceforth referred to as BBL, to estimate the structural parameters. These include the enrollment function $E(.)$, operating cost function $\Psi(.)$, adjustment cost function $\Gamma(.)$, and all the transition functions. BBL propose a two-step procedure that avoids directly solving the policy functions of the agents in conducting estimation.

In the first step, I use appropriate functional forms to estimate the demand, operating cost, policy functions, and transition functions. In this step, I characterize the agents' decisions and flow utility as functions of the state variables. In the second step, I use the estimated policy functions in the first stage, denoted as $\hat{v}(.)$ and $\hat{e}(.)$, and their perturbed versions $\tilde{v}(.)$ and $\tilde{e}(.)$ to compute the expected discounted sum of the flow utility for large enough periods T. The estimator will search for the parameter $\hat{\Gamma}$ of the adjustment cost function $\Gamma(.)$ that minimizes the profitable deviations with perturbed policy functions $(\tilde{v}_j(.), \tilde{e}_j(.))$ from the optimal policies estimated in the first stage:

$$\hat{\Gamma} = \arg\min \sum_j \sum_i \min\{0, \bar{V}(s_{i0}; \hat{v}(.), \hat{e}(.); \hat{\Gamma}) - \bar{V}(s_{i0}; \tilde{v}_j(.), \tilde{e}_j(.); \hat{\Gamma})\}^2, \tag{10}$$

where

$$\bar{V}(s_{i0}; v(.), e(.), \hat{\Gamma}) = \frac{1}{NS} \sum_{ns} \sum_{t=0}^{T} \beta^t u(s_{it}; \hat{\Gamma}) \text{ s.t. } v(.) \text{ and } e(.) \text{ governs the evolution of } s_{it}.$$

[34] I choose this per-enrollment reimbursement rate to approximate \$8143, a number provided by the latest state budget release (for a source, see Florida Charter School Alliance's report). Note that the per-enrollment reimbursement rate tends to increase evenly every year. Therefore, the actual rates during my sample period might be below this number.

[35] According to Florida law, charter schools are funded through the Florida Education Finance Program in the same way as all other public schools in the school district. The charter school receives operating funds from the Florida Education Finance Program (FEFP) based on the number of full-time (FTE) students enrolled. Notably, a recent report "Charter School Funding: Inequity Surges in the Cities" finds charter schools receive less reimbursement compared to TPSs in states that apply this equal-reimbursement law. Therefore, accounting for this might raise the estimate for TPSs' adjustment cost of value added.

Here, i denotes a specific initial state randomly picked, and j indexes a perturbed policy function that slightly and randomly changes the actions predicted by $\hat{v}(.)$ and $\hat{e}(.)$. Note that an ns indexes a simulation and signifies that the goal is to get the *expected* discounted sum. I estimate charter and TPSs separately, following the same procedure.

6.2 Measurement

In Table (B5), I integrate the measurement of relevant variables in the model and their coverage of years and schools. Unless specified otherwise, all measures are available throughout the sample period. In the model, a period corresponds to a school year, where the label for the year follows a format where the 2013-2014 school year is labeled as $t = 2014$. Each school in the dataset is identified by a unique school ID. I highlight several measurement assumptions below. I calculate the average teacher value-added score within a school to measure educational effort. I consider the accountability score in the previous year of t as the performance state variable in t. This choice is motivated by the fact that schools and students are unaware of the schools' accountability scores for the upcoming school year during the recruitment season of the previous year. Hence, the accountability score in the previous year is a more suitable measure variable for the contemporaneous performance state.[36] As for the capacity measure of TPSs, although I do not have the number of classrooms directly, I impute a TPS's capacity using the largest enrollment observed in a school divided by 22. Because TPSs are not often capacity-constrained and are subject to a regulated middle school class size of 22 students per class. For all information from the American Community Survey, I particularly use its 5-year Data Profile, where the middle year of the 5-year data serves as the year label for a certain variable. For the measurement of all variables related to the demand estimation, I leave them as I introduce the demand estimation.

6.3 Estimation Sample

The sample used for structural estimation consists of a selected set of charter and TPSs. First, for both types of schools, I exclude those that only run grades from K-2 for most of the sample period, those with a short sample length, or schools with a small average enrollment per grade. These exclusions are necessary because the excluded schools may have objectives that differ significantly from the rest. Moreover, they are systematically more likely to have missing variables. For example, schools that constantly run K-2 do not

[36]Here is an example: The enrollment of $t = 2012$, i.e., the school year 2011-2012, is determined in the recruitment season of 2011, in spring. At that time, students did not know the schools' accountability scores for the upcoming 2011-2012 school year starting 2011 in the summer. Therefore, a more appropriate measure for the state variable of performance level is the accountability score 2011, which has been made public to schools and students since the start of the 2010-2011 school year.

participate in standardized tests and hence do not have a reliable source of performance evaluation.

Particularly for charter schools, I exclude observations from specific charter schools. When estimating the policy functions of charter schools, I only include observations from charter schools that have been operational for more than three years. This selection criterion aligns with the model's focus on characterizing the relatively mature operation of charter schools after their entry. Additionally, the expansion in a charter school's early life cycle is predetermined and negotiated prior to entry, independent of post-entry factors such as designation and performance level. Therefore, including observations from this period would not be appropriate.

When it comes to TPSs, I select the set of schools used in showing the main results of the difference-in-difference analysis. That is, all the TPSs that had no charter schools within 5 miles in 2011 are excluded from the structural estimation. Since the model allows both types of schools to respond to competitive pressure endogenously affected by the policy change, for TPSs with no charter competitors in a reasonably large neighborhood, it is less suitable to characterize their behaviors in such a competitive environment in the model.

Finally, I choose post-policy observations to estimate the structural model.[37] As the model requires, all schools are assumed to know the existence of the HP designation system, and their belief about its existence remains unchanged. Therefore, the post-policy period is more suitable for estimating the model, particularly because the operation of the designation system is commonly known during this period and undergoes minimal changes.

I compare the charter and TPS samples in conducting preliminary data analysis and estimation in Table (B1) in the appendix. In the end, around ten thousand charter and TPS observations exist in the structural estimation from 28 districts.

6.4 Empirical Specification

In this subsection, I introduce the definition of a market and the empirical specifications used in the estimation of the offline functions. These include the demand, operating cost, transition, and policy functions.

Market Definition and Fundamentals. I regard a school district as a market in the model. Florida has 67 school districts, whose sizes are similar to U.S. counties. I as-

[37] There are exceptions in which I also include pre-policy data in estimation to get more statistical power in implementation. For example, I estimate the operating cost of charter schools using all data without conditioning on the HP status. I essentially assume the operating cost does not depend on the belief about the designation system.

sume students do not travel across districts to choose schools. Because the demand is set to be static in the model, I regard the total number of public and private enrollment as a district-year's market size and define schools' share accordingly.[38] I regard each geography unit, i.e., l, in the model as a census tract. The crow-fly distance between a census tract centroid and a school hence measures travel distance to school. [39] Accordingly, m_l, the student population size of a census tract l is then measured by the total number of K-8 students. Since the district-year market size and the census tract demand size come from different data sources, I apply Ferreyra and Kosenok (2018)'s method to moderate the tract demand size data.[40] Essentially, I impose that the sum of the tract demand size of all tracts in a district of a year is equal to the market size constructed by adding up all the charter, traditional, and private school enrollment of the district in that year. I show the total number of schools by type, population density, and the average household income for each district in 2015 in Table (B2) in the appendix.

Ideally, one would solve out for the equilibrium of every district. This implies that each district should have their district-specific offline functions. The data limit such an approach. Particularly in districts with not a lot of charter schools e.g., less than 10, it is impractical to estimate their policy functions due to lack of statistical power. Therefore, in what follows, except for the transition rule $\nu(.)$ of the market situation state n, I estimate all offline functions, including the demand, by pooling observations from all districts of all years. The reason of estimating the district-specific AR(1) process for each district is explained below in the estimation result section.

Demand Function $E(.)$ and Demand-based Measures ξ and n. The main empirical challenge I need to tackle in the demand estimation is the existence of capacity-constrained charter schools in the market. One might underestimate students' preference over schools' performance if the capacity-constrained ones tend to be preferred. Notably, the existing literature on the industrial organization of the U.S. education market (Hastine et al. 2009; Ferreyra and Kosenok 2018; Singleton 2019; Dinerstein and Smith 2021) has not treated such capacity constraints explicitly in their demand models. To account for capacity con-

[38]Due to the sample selection for the empirical implementation, a district's "inside" option, i.e., the charter and TPS enrollment, is from the selected set of schools inside of the districts. When calculating a district's "outside" option, i.e., private enrollment, I therefore also constrained to the district's private schools that only appear in the neighborhood of these selected charter and TPSs. Additionally, other major forms of schooling, such as home-schooling, are missing in measuring the total demand size. Evidence suggests that they accounted for less than 3% of the total Florida public enrollment in 2013: https://www.fldoe.org/core/fileparse.php/5606/urlt/Home-Ed-Annual-Report-2022-23.pdf.

[39]In the U.S., census tracts are "designed to be relatively homogeneous units with respect to population characteristics, economic status, and living conditions" and "average about 4,000 inhabitants." Florida have 4245 census tracts in the 2010 Census. Therefore, it is relatively accurate to measure travel distance as the distance between a tract centroid and a school.

[40]In their application, they also need to impute demand size for charter and TPSs from each census tract of Washington, D.C., using only enrollment and tract demographics.

straint, I model students' preferences for schools depending on class size. This treatment helps explain the low enrollment in constrained schools as students dislike larger class sizes, thereby better approximating students' choice under capacity constraint.[41] Another way to properly accounting for capacity constraint in influencing rationing of students is to impose structure on students' feasible choice set, i.e., what charter schools they applied have given them offers, as in Walters (2018). This treatment requires granular data on students applications and acceptance, which I do not have for Florida students.

Following the notation in the model, I use the specification in (11) to represent the utility of a representative student i living in census tract l in enrolling in school j in year t, i.e.,w_{ijlt} :

$$\begin{aligned} w_{ijlt} &= \delta(s_{jt};\alpha) + \lambda dist_{jl} + \zeta_{ijlt} \\ &= \alpha_1 ClassSize_{jt} + \alpha_2 \left(ClassSize_{jt} \cdot o_j\right) + \alpha_3 q_{jt} + \alpha_4 o_j + \xi_{jt} + \lambda dist_{jl} + \zeta_{ijlt}, \end{aligned} \tag{11}$$

where $ClassSize_{jt}$ is defined by the enrollment per classroom, i.e., $\frac{E_{jt}}{k_{jt}}$.

And therefore, given the distributional assumption on ζ_{ijlt}, the enrollment of each school-year is:

$$E_{jt} = \sum_{l \in L} m_{lt} \cdot \frac{\exp(\alpha x_{jt}^{\text{demand}} + \lambda dist_{jl})}{1 + \left(\sum_{j' \in J} \exp(\alpha x_{j't}^{\text{demand}} + \lambda dist_{j'l})\right)}, \tag{12}$$

where J denotes all the schools in the district, and L denotes all the relevant census tracts of a district.[42] And x^{demand}, the individual state variable used in demand estimation, includes therefore $(o_j, k_{jt}, q_{jt}, \xi_{jt})$.

However, incorporating class size into students' preferences introduces correlations between class size and hidden school quality ξ. To address this issue, I use a specific instrument for class size, following the empirical strategy by Bayer and Timmins (2007).[43] To adopt this instrument and the estimation procedure proposed by the authors, I use a two-step approach. In the first step, I run Non-linear Least Square (NLS) on a demand model that is identical to (11) except that the class size terms and ξ are excluded from the specification.[44] Then, the implied estimates are used to form a predictor for class size from

[41] The consideration of class size in students' preferences is inspired by Urquiola and Verhoogen (2009), who developed a model to study the sorting of Chilean schools under class-size caps.

[42] After I select all the schools, for the relevant census tracts of a district, I include all census tracts whose 5-mile radius neighborhood has at least one school in the district.

[43] Richards-Shubik et al. (2021) estimate a discrete choice model in which patients select specialists. In the model, a similar "congestion effect" is added to patients' preferences to characterize patients' unwillingness to wait in long queues. They use the instrument proposed by Bayer and Timmins (2007) to deal with the endogeneity problem similar to my context.

[44] Since there is no ξ in such a model, one does not need to apply the inversion technique (Berry 1994), and NLS is the appropriate method.

the model just estimated. In the second step, this predictor, along with other instruments I pick, is used to form the moment conditions used in estimating a Generalized Method of Moment (GMM) objective function. It aims to find the optimal $\hat{\alpha}$ and $\hat{\lambda}$ that minimizes the correlation between the instruments and the ξ_{jt}. In this step, I use the nested fixed point algorithm, as in Berry et al. (1995), to conduct the GMM. I explain details of the Nested Fixed Point algorithm used in the second step, the moment conditions used to construct the GMM objective and the testing of whether instruments are weak in Appendix C.1. Note that once the $\hat{\alpha}$ and $\hat{\lambda}$ are found, one can back out ξ_{jt} by standard inversion technique introduced in Berry (1994). Finally, given $\hat{\alpha}$ and $\hat{\lambda}$, I can then use the following formula,

$$n_{jt} = \sum_{l \in L} m_{lt} \cdot \frac{\exp(\hat{\lambda} dist_{jl})}{1 + \sum_{j' \in J} \exp(\hat{\alpha} x^{\text{demand}}_{j't} + \hat{\lambda} dist_{j'l})},$$

to construct the market situation variable, n_{jt}, faced by each school j at year t. As the "Consistent Belief" requires, the market situation variable constructed can be used to calculate schools' beliefs about the evolution rule. Since this estimated rule is assumed to be the belief schools hold to make decisions, it can further be used to back out the adjustment cost functions with which schools make decisions.

Operating Cost $\Psi(.)$ and Transitions. To estimate the operating cost of charter schools, I regress the logarithm of instructional cost from charter audit reports on the relevant state variables (and polynomials of these variables) and the logarithm of enrollment. Particularly, I include the local demographics of schools to reflect the cost differentials in operating charter schools across different regions, as Singleton (2019) points out. To estimate the transition function of school performance q, I regress a school's performance score on its lag performance score, value added, and their interaction. The interaction term captures the differentials across different performance levels in the degree of value-added inputs needed to boost the same amount of performance score. To estimate the transition of the designation status hp, I exploit the empirical transitions to the contemporaneous designation status across charter schools conditional *only* on their past performance and designation status. This simplifies the modeling of the "2A1B" rule, which, if modeled precisely as the existing scheme, requires the contemporaneous designation status to depend on three years of past performances. This increases the dimensionality of the state space dramatically. I also assume that a charter school does not lose the designation as long as it is designated.[45] The rest of the transition functions, i.e., the transition of n, ξ,

[45] As explained in the industry background, de-designation is rare in the sample. Furthermore, I rarely observe that eligible (i.e., those that pass the "2A1B" requirement) charter schools are not designated. These observations might reflect that they do not apply for the designation. Since they are rare, I exclude them from the estimation sample.

and d, are all estimated as AR(1) processes, respectively.

Policy Functions When it comes to the estimation of the expansion policy function of charter schools, note that fixed costs are involved with increasing the number of classrooms for instruction, supported by the process of drafting new contracts and obtaining approval from local school districts. This can also be shown in the lumpiness in the adjustment of charter school classroom count. In the structural estimation sample, approximately 83% of charter school observations indicate no adjustment (i.e., an increase or decrease in the classroom count) throughout the selected sample period. Thus, to characterize such a feature of adjusting capacity, I adopt the (S, s) rule following Attanasio (2000) and Ryan (2012). Ryan (2012) utilizes this decision scheme to estimate cement manufacturers' capacity adjustment policy function for his dynamic game model. In my context, the (S, s) rule states that each charter school j sets a target k^*_{jt}, a lower band $\underline{k}_{jt}$, and an upper band $\bar{k}_{jt}$, in year t based on a statistical rule whose parameters are to be estimated. According to the rule, a charter school increases classrooms to reach its target only when it falls below the lower band: $e_{jt} = \underline{k}_{jt} - k^*_{jt}$ if $k^*_{jt} < \underline{k}_{jt}$. It decreases classrooms to reach its target only when it exceeds the upper band: $e_{jt} = \bar{k}_{jt} - k^*_{jt}$ if $k^*_{jt} > \bar{k}$. Therefore, when the target stays within the bands, charter school j in that year t remains inactive. Therefore, this decision rule can characterize the lumpiness in the expansion adjustment data. Following their specification, I use a flexible functional form of the state variables (the $h(.)$ functions below) to estimate both the target and bands, as shown in (13). [46]

$$\begin{aligned}
k^*_{jt} &= h_1(s_{jt}) + u^*_{jt} \\
\underline{k}_{jt} &= k^*_{jt} - \exp\left(h_2(s_{jt}) + \underline{u}^b_{jt}\right) \\
\bar{k}_{jt} &= k^*_{jt} + \exp\left(h_2(s_{jt}) + \bar{u}^b_{jt}\right)
\end{aligned} \tag{13}$$

Same as Attanasio (2000) and Ryan (2012), I use only the observations that involve non-zero adjustments of capacity to estimate (13). In particular, in estimating the target equation, I regress the $t+1$ number of classrooms on state variables of t, and in estimating the band equations, I regress the difference between $t+1$ and t in the number of classrooms on current state variables, both using flexible functional forms.[47] I also consider the residuals $u^*_{jt}, \bar{u}^b_{jt}$, and $\underline{u}^b_{jt}$ as structural errors, as to capture the discrepancy between the estimated policy functions in adjusting capacity and the model-predicted adjustment processes. As emphasized in the adjustment costs of charter schools in (6), this discrepancy

[46] The exponential functional form guarantees that the target is always between the lower and upper bands.

[47] Ideally, upper and lower bands should be estimated by the shrinkage and the expansion data separately. However, because shrinkage, i.e., the decrease in classrooms, is much less common than expansion, I assume that the shrinkage decision shares the same statistical relationship with the expansion decision and estimate the two bands by pooling all observations of expansion and shrinkage.

may exist due to the unobserved mode of capacity adjustment. I assume the different structural errors all follow an i.i.d. zero-mean normal distribution with variance (same across schools) to be estimated, independent of each other.

6.5 Identification of the Adjustment Cost Function

The identification of the key structural parameters in equation (6) for both types of schools relies on the policy shock and the functional form assumptions imposed on $\Gamma(.)$.

For charter schools, the cost of exerting v amount of value-added, namely γ_v, and the HP-related cost effects, namely γ_2 and γ_4, jointly govern the value-added decisions. These parameters can be separately identified by exploiting the policy shock. The early designated charter schools, e.g., those designated in 2012, do not need to adjust their value-added to secure future designation since they can never be de-designated, as the model imposes. Hence, the difference in value-added choices between these and later-designated schools helps separate the HP-related cost effects and γ_v. The separable form of the adjustment costs separately identifies γ_v, γ_2, and γ_4. Specifically, γ_v is separately identified from γ_2 and γ_4 by the variation in a school's performance in the following school year when its capacity remains unchanged. This is because γ_2 and γ_4 only affect adjustment costs when charter schools expand. The identification of γ_v for TPSs follows a similar logic.

To separately identify the fixed and variable costs of expansion, note that conditional on expansion, the fixed cost, γ_1, does not influence the expansion volume. Therefore, γ_3 can be separately identified by the variation in the magnitudes of expansions across or within schools. Subsequently, γ_1, the fixed cost of expansion, is identified by the frequency of charter schools initiating an expansion. As is set up in the model, γ_1 and γ_3 are assumed to follow normal distributions with mean zero and to-be-estimated variance. These variance coefficients are identified by the variances in magnitude and frequency of expansion conditional on the state variables as the (S, s) policy functions specify.

Finally, γ_1 and γ_3 can be separately identified from the γ_2 and γ_4, the HP-related effects. This is so by comparing the difference in expansion choices across charter schools or within those that experience a change in their HP status in the sample. Identifying the remaining parameters follows standard practices in the literature.

7 Structural Estimation Results

In this section, I provide the results of the structural estimation. I first show the results of the offline functions estimated in the BBL's first stage, then I proceed to show the results of the adjustment cost functions estimated in the BBL's second stage.

7.1 First Stage: Demand Function and the Implied Transition of n and ξ

I show the results of the demand estimation in Table (6). Panel A shows the estimates under various specifications of the demand model to facilitate comparison. Panel B shows the statistics school individual state x_{jt} across schools and years in the estimation sample, including the implied market situation n and the hidden quality ξ. Panel C shows the transitions of ξ and n of some districts.

In terms of the demand estimates, I show in the column (1) of panel A the result of estimating the spatial demand model using the proposed instruments in the empirical strategy with 9,921 school-year enrollment. In total, the coefficients illustrate that households prefer schools of smaller class sizes, higher performance scores, traditional types, and less distant schools. I compare these results with another model in which I do not assume the ξ is correlated with class size. The estimates of this model are shown in column (2). Compared to such a model, I get a larger estimate of students' taste in performance and a smaller one of their taste in distance. The difference in these estimates addresses the role of capacity constraint in estimating the demand for schooling. Schools with higher performance scores and closer schools are favored more; hence, it is easier to hit the enrollment capacity limit. Not accounting for these facts result in underestimating the taste for performance score and distaste for distance. In what follows, I regard the results in column (1) as the structural estimates of demand for the second step of BBL.[48]

Importantly, the adjustment cost estimates hinge on the implied enrollment elasticity regarding capacity and performance score. Because they convey the messages of the perceived marginal return of expansion and effort of value-added, if the demand elasticity concerning performance score is underestimated, potentially because high-scored charter schools are capacity-constrained, one might underestimate the adjustment cost of exerting value-added. In this regard, I estimate using the same charter school data a log-linear demand model, as in Singleton (2019), with a flexible functional form. Then, I calculate the elasticity of interest implied by the adopted model and this log-linear model at certain values of capacity and performance score. I find that, for a medium-sized charter school having 20 classrooms and 400 students, with a performance score of 0.6 (B grade), the adopted model predicts demand elasticities amount to 0.58 and 1.20 for classrooms and performance scores, respectively.[49] While the log-linear demand model implies a

[48] One might question the existence of equilibrium of this demand model and how to deal with it in forward-simulating schools' enrollment both in estimating the model and in simulation. Bayer and Timmins (2005) have provided the condition of the existence and uniqueness of equilibrium. In the case of this paper, given the inclusiveness assumption imposed on the demand, one can easily prove that as long as the the taste parameter on class size has negative coefficient, the equilibrium exists and it is unique.

[49] This implies that, for example, increasing 10% classrooms (in this case, $2 = 20 \times 10\%$ classrooms), increase students by $23.2 = 400 \times 10\% \times 0.58$. While the log-linear demand predicts 48 more students enrolled. Given that 48 students can be put into two classrooms, the latter model predicts that charter schools can increase capacity and automatically enroll more students. It should be emphasized that, although the com-

Table 6. Demand Estimates and the Implied Transitions of ξ and n

Panel A			
	Demand with ξ	Demand with no ξ	
Variable	(1)	(2)	
Class Size	-0.073	0.0076	
	(0.0160)	(0.0034)	
Performance Score	4.891	0.938	
	(0.470)	(0.398)	
Charter	-0.43	-0.814	
	(0.0863)	(0.0407)	
Distance	-0.538	-0.0005	
	(0.0538)	(0.0003)	
Panel B			
Variables	Mean	Variance	Median
Class Size	18.56	11.20	18.39
Performance Score	0.60	0.12	0.61
Charter	0.15	0.36	0
ξ	0.07	2.63	-0.52
n	421.56	448.47	260.79
Panel C			
Functions	Slope	Intercept	Observations
Transition of ξ	0.980	0.167	8036
	(0.0026)	(0.0059)	
Transition of n			
Miami-Dade	1.002	-0.529	1525
	(0.007)	(7.905)	
Pinellas	0.689	134	461
	(0.028)	(13.608)	
Polk	0.888	0.634	256
	(0.154)	(0.238)	

larger capacity elasticity, at 1.26, and smaller performance score elasticity, at 0.51. This illustrates that, if regarding the adopted model as a benchmark, using a "less-structured" demand model, such as the log-linear demand, will underestimate the demand elasticity of performance score while overestimating that of capacity.

I show in Panel B that the averages of n and ξ across all school-year are 421.56 and 0.07, respectively. Particularly, there exists a large variance in terms of n. This can also be seen in the implied transition function across different districts. This is especially so in the district-specific intercept estimates of the AR(1) process, as shown in Panel C. This difference emphasizes the necessity of estimating a district-specific evolution rule of schools' belief on n.[50]

7.2 First Stage: Other Offline Functions

I display all the estimates of the offline functions and the related statistics in the tables in Appendix C.2.

Operating Cost Functions The estimated operating cost function is shown in Table (C1). It shows close constant returns to scale because the coefficient on the logarithm of enrollment is close to one. Additionally, all else equal, a higher performance score is associated with less instructional expenditure on average, although the negative relationship is less in magnitude as the performance score gets higher. For a charter school with an average performance score of 0.62 (full score is 1), its instructional cost goes down by 0.097 percent as its performance score goes up by 0.1. For capacity, a marginal increase in classroom holding does not significantly influence the instructional cost of an average-sized charter school, all else equal. Notably, the estimate also shows that operations under different local demographic situations, as measured by local income level (measured in logarithm), involve differential cost, a result similar to Singleton (2019). In particular, a 10 percent increase in mean household income within a 3-mile radius of a charter school is associated with roughly 0.4 percent less total instructional expenditure, holding other regressors constant. This cost differential may explain part of the variation expansion patterns across different demographic environments.

parison is useful, it is not necessary that the model I adopt outperformed the log-linear model in improving the estimates of the adjustment cost. There is a lack of empirical work that provides benchmark elasticities that I can compare my estimates with.

[50] I regress the implied n to schools' local environment and, not surprisingly, find that the market situation measures school j faces in year t is positively correlated with its local population density, household income, and educational background while negatively correlated with the number of schools in its neighborhood.

Policy Functions Table (C2) and Table (C3) show the estimates of all policy functions, including both types of schools' value-added policy functions and charter schools' (S, s) components (target and band). Particularly, I use second-order polynomials in capacity and performance with rich interaction of the designation status (of charter schools only). Across the board, all results show the highly non-linear relationship between schools' decisions on state variables.

In Table (C2), the value-added policy functions of charter and TPSs show the distinctive relationship between the value-added and state variables across the two sectors. As shown in column (2), a TPS's value-added is negatively associated with the classroom, although less so in magnitude if performance is higher, all else equal. This pattern may reflect the underlying teachers' production function of test scores. Teachers' effort at traditional schools might be less if they need to teach many classes, and this burden might be alleviated if schools' management is more efficient, as reflected in high performance levels. Notably, for TPSs, value-added is significantly associated with higher market situation n, all else equal. This is similar to the relationship between local household income and TPSs' value-added. When it comes to charter schools, the estimated value-added policy function shows, in general, less dependence on the selected state variables. As shown in column (1), the value-added decisions of charter schools are positively associated with the interaction of HP status and market situation and the squares of performance level, all else equal. For the HP status, performance, and capacity, their respective marginal changes do not appreciably change the value-added of charter schools. This might primarily be due to the specification choice. A standard F-test rejects the null hypothesis that the coefficients on all these state variables and their interactions are jointly zero (p-value$<$0.001). Additionally, this might reflect that the pattern of value-added decisions is less systematic across various charter schools in terms of how their value-added decisions are dependent on the selected state variables.

In Table (C3), I show the estimates of the expansion policy function of charter schools using the (S, s) rule. For the specifications of the target and the band equations, I apply second-order polynomials and rich interaction of the state variables. Notably, the results manifest differences in patterns of expansion across non-HP and HP charter schools in both the target equation and the band equation, as is shown by the significant coefficients on the HP dummy (1 for HP, 0 for non-HP) and its correlation with other state variables. The results suggest that HP charter schools tend to set larger targets and, once initiating an expansion, expand more than the non-HP charter schools, all else equal. Such effects vary across HP charter schools in regions with different income and market situations. The estimates of the target equation also suggest that larger charter schools tend to have larger capacity targets. [51]

[51] It should be emphasized that some of the coefficients in the expansion rules are not precisely estimated.

Transition Functions I show all the rest of the transition functions in Table (C.4). Notably, the performance transition function suggests that schools' performance score positively correlates with the past score and the value-added. All else equal, for a school performed at the average (0.62), the marginal increase of the average teacher value-added by 1 unit, and the performance score increased by 0.152. Higher past performance is associated with a higher contribution of value-added in future performance scores, which might suggest that schools' effort in maintaining effective teachers and the students' past performances are complementary in producing test scores. As for the HP designation, the estimates reflect the empirical transition of charter schools into the designation: among all the non-HP charter schools, "A" charter schools get designated in the next period with a probability of 0.345, B with 0.037, and C or below with zero probability. Finally, the estimates of the household income transition show that it is relatively persistent over time.

7.3 Second Stage: Adjustment Cost Function

With the offline functions estimated, I show in Table (7) the estimates of the adjustment cost function using the BBL estimator. The computation details in this BBL second stage, such as the selected initial states, implementation of the perturbation on the policy functions, and the simulation parameters, are in Appendix C.3.

I run the structural estimation separately for charter and TPSs using their observations separately. Table (7) concludes the structural estimates for the adjustment cost function $\Gamma(.)$. All estimates are in terms of cost, and hence, a negative number means a reduction of cost. The effect of HP designation on the variable cost of expansion, γ_4, is positive and precisely estimated, indicating that the HP designation decreases the fixed cost of an expansion. Combined, these results align with the policy contents: The policy facilitates expansion ($\gamma_4 < 0$) for the HP charter schools and does so as charter schools expand more. Charter schools have reduced variable costs of expanding capacity by 4.7%. With a back-of-envelope calculation, holding fixed the expansion choices HP charter schools actually make, removing the HP designation saves their total expansion cost by 4.4%. The estimates of γ_v show that exerting value-added is costly for both charter schools and TPS. However, charter schools have lower costs. This might imply charter schools' higher efficiency in managing teachers in directing teaching goals to test scores. As expected, the fixed cost of expansion, γ_1, and the variable cost of expansion in increasing one classroom,

This is due to data limitations. Ideally, one could get a more precisely estimated expansion rule if adjustments of classrooms are observed more or, in this context, the capacity is measured in the student count instead of classrooms for instruction. To ensure the (S, s) rule is an acceptable approximation of how charter schools make expansion decisions, I test the in-sample fitness of the (S, s) rule. From the sample I use to estimate the (S, s) rule, I pick all charter school observations that have next-period data available, plug their states into the estimated (S, s) rule to predict their expansion behaviors, and compare the predicted behaviors with the actual expansion behavior in the next period. The estimated (S, s) rule approximates the mean expansion well in extensive and intensive margins.

γ_3, are smaller than the cost of value-added per unit of change.

Table 7. Estimates of $\Gamma(.)$ and Standard Errors

	Adjustment Cost $\Gamma(.)$	
	TPS	Charter
Value-added Cost (γ_v)	23.08	19.458
	(2.21)	(4.37)
Mean of Fixed Cost (γ_1^μ)		1.185
		(1.603)
Std. Deviation of Fixed Cost (γ_1^σ)		0.363
		(0.10)
Mean of Variable Cost (γ_3^μ)		6.612
		(1.42)
Std. Deviation of Variable Cost (γ_3^σ)		0.0648
		(0.032)
HP's Effect in Variable Cost (γ_4)		-0.3113
		(0.134)
Variable Cost of Shrinkage (γ_5)		7.04
		(2.20)

Note: Standard errors (in parenthesis) are obtained by bootstrap. I re-sample half of the initial states randomly 100 times with the same set of perturbed policy functions. All parameters are estimated assuming discount factor $\beta = 0.9$, per-enrollment reimbursement $r = r^E = 0.08$ representing eight thousand per student, and utility weight on school performance score $r^q = 1.6$. All parameters can be regarded as measured in hundreds of thousands of dollars.

All the estimates can be expressed in dollar terms since the revenue and cost of charter schools (and the imputation scale applied to the TPSs accordingly) are all measured in dollars. The result suggests that, on average, for the non-HP charter schools, adding one classroom costs around \$661200, equivalent to \$734.6 per square foot for a 900-square-foot classroom size. This cost number is, therefore, slightly higher than the average construction cost of education facilities found in some major U.S. cities.[52] This is reasonable because the estimated costs also include all the associated adjustments equipping the capacity expansion. Compared to the variable cost of adjustment, the fixed cost of increasing the classroom and the value-added costs have fewer accounting estimates to compare with. The fixed cost of increasing capacity is \$118500. Increasing 1 unit of value-added, i.e., mean teacher value-added score, costs about \$1.95 million for charter and \$2.30 million for TPS. This may seem relatively high. However, according to the estimated performance transition function, this can move a school from grade C to almost grade A (by purely increasing the effectiveness of teachers).[53]

[52] See for example:https://www.statista.com/statistics/830447/construction-costs-of-educational-buildings-in-us-cities/.

[53] It should be emphasized that these estimates should be more properly thought of as the difference

Finally, I compare my cost estimates with Singleton (2019) 's estimate of entry cost. His estimate suggests around $10 million entry cost for Florida charter schools having 250 students. In Florida context, this roughly implies $858000 costs per classroom (of around 20 students), 1.3 times larger than my cost estimate of expanding capacity. This might suggest adding the same capacity at the intensive margin (by adding classrooms) is less costly than extensive margin expansion (by entry).[54] Therefore, capacity deregulation policy, especially the focused incentive scheme in this paper, might be a more effective way to increase charter sector provision of access.

8 Policy Counterfactuals

The primary goals of this paper are to evaluate the policy effects of the existing HP scheme and explore alternative schemes that aim at the supply of quality education at the aggregate. In the following counterfactual policy experiments, I anchor the idea of incentivizing by authorizing expansion eligibility of charter schools,[55] and hence focus on deviating the existing scheme by targeting differently on "who should expand more easily" while holding fixed the other model primitives. Particularly, I propose two schemes: the no-HP scheme and the scheme that gives additional expansion eligibility to high value-added charter schools. I compare them, respectively, with the existing HP scheme. The former comparison aims to decompose the effect of introducing the policy in students' access and education quality and provide an aggregate analysis on the entire education sector. The latter aims to explore whether targeting value-added increases specifically the quality of education and accessibility for disadvantaged households. In future versions of the paper, I plan to consider more counterfactual experiments, for example, deregulate all charter schools in expansion eligibility or limit the expansion eligibility to charter schools locating in areas of low-performing TPSs.

8.1 Analyzing Framework

There are three components of the models for charter schools and TPSs change across schemes: the adjustment cost of expansion, the transition of the HP designation, and the

between schools' certain decisions and doing nothing, i.e., creating zero value-added and not adjusting capacity. Therefore, I implicitly assume that exerting zero value-added and adjusting no capacity have zero adjustment costs. However, it is difficult to justify whether this is appropriate because, particularly, there exists limited research on how costly it is to increase mean teacher value-added within a school, the measure adopted in this paper.

[54] It should be emphasized that the adjustment cost might not be linear in the additional classroom, as the functional form adopted in this paper. In future versions, I will experiment with more, especially nonlinear, forms of adjustment cost function.

[55] Although I focus on the charter sector policy change in this paper, this model can also be used to analyze the supply side changes on the traditional sector or on both sectors.

belief about the competitive pressure. as shown in Table (8). The no-HP scheme eliminates the possibility of the expansion benefit for the eligible charter schools as well as the designation system ($hp_t = 0, \forall t$). For the target-va scheme, it remains the designation system while changing the targeted schools from only the High-performing charter schools to, additionally, high value-added charter schools. Specifically, if a charter school's value-added is higher than the 50% percentile, denoted as $\tilde{v}$, of the average value-added of the entire charter sector (in all the observed years), the charter school gets the designation and enjoys the same cost reduction as the HP charter schools in expansion benefit. In all schemes, the equilibrium belief on the market state needs to be recalculated to satisfy the "Consistent Belief" assumption. In what follows, I denote such beliefs as respective, ν_{HP}, $\nu_{\text{no-H}}$, and ν_{TVA} for the existing HP scheme, no-HP scheme, and the scheme that target high value-added.

Table 8. Changes of Primitives of Policy Counterfactuals

	Existing HP Scheme	"No-HP"	"Target-va"
Γ^{charter}	$\gamma_2 = \hat{\gamma}_2, \gamma_4 = \hat{\gamma}_4$	$\gamma_2 = 0, \gamma_4 = 0$	
η	$hp_{t+1} = \hat{\eta}(hp_t, q_t)$	$hp_t = 0, \forall t$	$hp_{t+1} = 1$ if $v_t \geq \tilde{v}$
ν	ν_{HP}	$\nu_{\text{no-HP}}$	ν_{TVA}
	$\nu_{\text{HP}} =: \hat{\nu}(n_{jt})$	Change according to the "Consistent Belief" Assumption.	

When comparing schemes, I focus on two channels. One channel is the change of the target of the charter designation system and the adjustment costs of expansion. The other channel is the associated change on the competition environment, as characterized by the change of belief on the market situation state n. I call the former "incentive channel" and the "latter competition channel." The incentive channel describes the effect particularly on charter schools by changing their incentive to exert effort. The competition channel characterizes how both types of schools, in equilibrium, change their effort given the change in the charter sector.

Particularly, I use the formula in (14) to decompose channels. Take the comparison between the no-HP scheme with the existing HP scheme as an example. Denote Y^{noHP} and Y^{HP} as an outcome benchmark, Y, of the market of interest under the no-HP scheme and the existing HP scheme at the equilibrium. Then the total effect of the existing HP scheme transforming from the no-HP scheme is $Y^{HP} - Y^{noHP}$. Denote $Y^{HP}_{\nu_{\text{no-HP}}}$ as the outcome under the existing HP scheme while the belief $\nu(.)$ maintaining at the no-HP scheme. In other words, schools do not believe that the policy change will alter the evolution of the market state. I refer $Y^{HP} - Y^{HP}_{\nu_{\text{no-HP}}}$ to be the competition effect, as the difference"controls" for the scheme change, while $Y^{HP}_{\nu_{\text{no-HP}}} - Y^{no-HP}$ as the incentive effect, as the difference holds constant the equilibrium belief. From the equation, if the outcome benchmark is constrained to the traditional sector, the incentive effect is zero.[56]

$$Y^{HP} - Y^{noHP} = \underbrace{Y^{HP} - Y^{HP}_{\nu_{\text{no-HP}}}}_{\text{Competition Effect}} + \underbrace{Y^{HP}_{\nu_{\text{no-HP}}} - Y^{noHP}}_{\text{Incentive Effect}} \tag{14}$$

8.2 Implementation Methodology

The equilibrium concept adopted in the model requires an iterative process of getting the consistent belief that is consistent with the belief on which all schools in the market make decisions based over time. Therefore, the computation procedure adopted in this paper is built on previous work on using simplified state space to calculate dynamic equilibrium (Krusell and Smith 1998; Ifrach and Weintraub 2017). However, the procedure I adopt differs in an important way. Particularly, I pick specific districts and set the initial states to be the districts' 2012 states. Additionally when schools update belief, instead of using a stream of steady states to update the belief on the market state, as previous work did, I use a stream of states 20-year forward of the market. I choose this deviation because the primary goal of the paper is on accessing relatively short term (e.g., less than a decade) transitional dynamics of these policy changes instead of their longer term implications on the steady state of schools' performance.[57] I explain more details of the computation procedure in Appendix D. In this version of the paper, I pick the largest school district of Florida, the Miami-Dade district, to conduct my counterfactual simulation. It not only accounts for almost 20% of enrollment in my sample but it has relatively higher charter market share. Accordingly, I adopt the estimated transition rule of the market situation n of Miami-Dade. I show the summary statistics of Miami-Dade schools in 2012 in Table (B3). All the rest of the computation details are in Appendix D.1.

8.3 Results

In this subsection, I show results of two comparisons of the focused charter incentive schemes. The outcome benchmarks are the evolution of the distribution of decisions (value-added and expansion), and the implied performance and capacity transition of each sector of the education market.

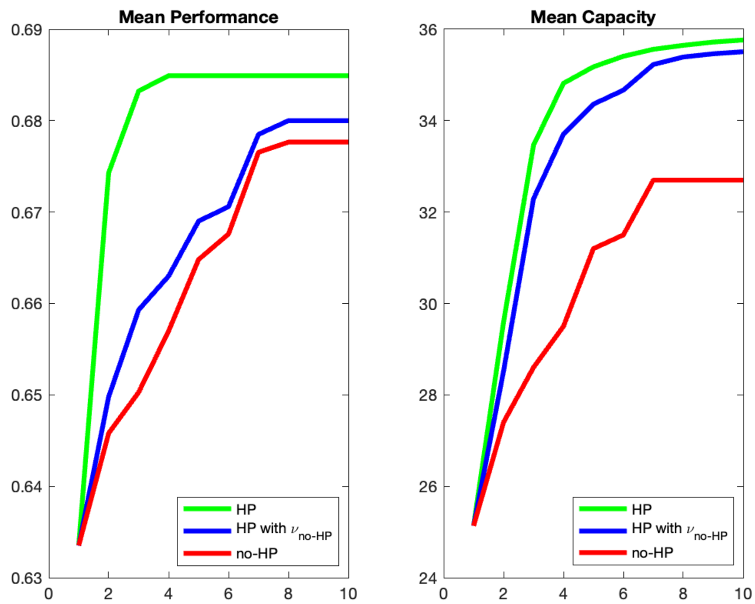
[56] It should be emphasized that there exist other important ways in the real world in which charter policy affects the traditional sector. For example, **SingletonLadd2020** find that the removal of statewide cap on charter school entry in 2011 imposes large and negative fiscal impact in excess of $500 per traditional public school pupil.

[57] This research goal is therefore distinct from the previous work using the similar algorithm that focus more on the steady state analysis of an industry.

8.3.1 No-HP v.s. HP

I show in Figure (3) the result of comparing the no-HP scheme with the HP scheme in the mean performance of the charter sector and its mean access provision (i.e., capacity). The green lines indicate the trend under the existing HP scheme, while the red line represents the situation of the no-HP scheme. As is implied by equation (14), to decompose channels, I draw a blue line to represent $Y^{HP}_{\nu_{\text{no-HP}}}$, namely a certain outcome under the HP scheme with the belief not yet updated from the non-HP scheme. Therefore, the difference between the green and blue lines represents the competition effect, and the difference between the blue and red lines represents the incentive effect. I use ten years to be the inspection window.

Figure 3. No-HP v.s. HP: Mean Performance and Capacity Transition Over 10 Years



From the figure, under the HP scheme, the performance of the charter sector increases over time, more so compared to the non-HP scheme. The competition accounts for a larger increase in influencing the mean performance. At the end of the inspection window, the mean performance score of the charter sector under the HP scheme is higher than that of the non-HP scheme by 0.073, and competition accounts for 67.8% of such difference. When it comes to the access provision, the charter sector also has a higher speed of expansion under the HP scheme at the beginning of the inspection window. The increase of the mean capacity in 10 years is 11 and 8, respectively, in classrooms under the HP and no-HP schemes. The incentive effect accounts for most of the expansion growth, contributing to 91.6% of the mean capacity difference between the HP and no-HP at the end of the inspection window. Combining the results, the HP scheme increases both the provision of seats and the mean performance of the charter sector. The increase of charter capacity brought more "directly" by the policy influences the competition environment of the entire education sector and "ripples" back to influence the charter sector itself in its performance.

Regarding the traditional sector, the mean performance trends under both schemes show similar patterns as that of the charter sector. However, the difference between the mean performance of the two schemes increases to 0.056 in 10 years, slightly lower than the difference seen in the charter sector. I show more details in Figure (**??**). As is explained, the competition channel accounts for all the effects. Given that the traditional sector shows similar performance trends, it serves as a "magnifier" of the charter sector policy change.

I show in Figure (**??**) the mean trends of value-added and expansion under both schemes. Consistent with the patterns in Figure (3), it generally holds that relatively high-volume changes (of both performance and capacity) happen to the charter sector at the early inspection window. Following that, accumulated capacity and performance drive down the mean intensity of the expansion and value-added in the sector, slowing down the changes.

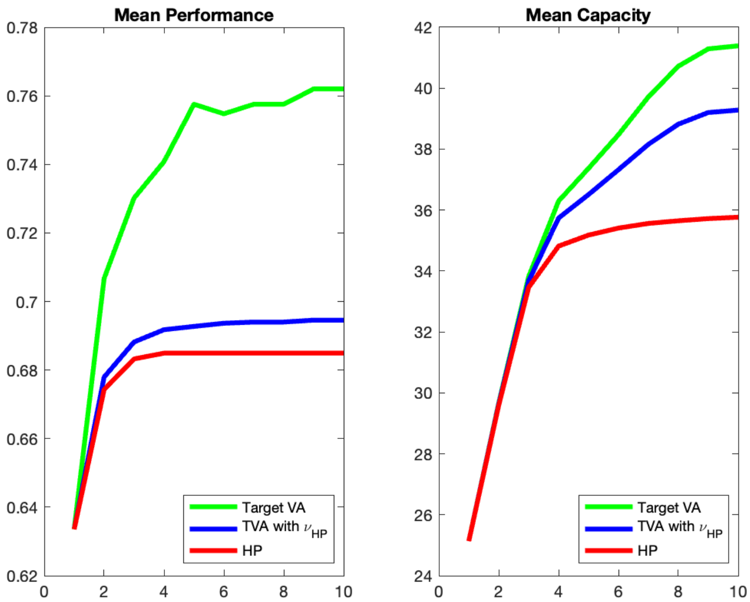
8.3.2 HP v.s. Target Value-Added

I simulate a counterfactual policy that gives additional expansion eligibility to charter schools once observed to exert higher-than-median value-added observed in the data. An immediate effect of this new designation system is the increased proportion of charter schools with the HP designation in a short period compared to the existing scheme. By inspecting the additional charter schools designated over the years, compared to the existing scheme, they are roughly 30% more likely to have medium or low neighborhood income. Therefore, the Target Value-Added scheme essentially gives more designation to low-performing and high value-added schools.

It is an empirical question whether these designated charter schools, especially those not designed (or not designated earlier during the inspection window) under the existing HP scheme, increase their enrollment capacity after their designation under the new scheme. As shown in the offline function estimate in Table (C1), operating cost is higher in lower-income regions. Therefore, given this demographic heterogeneity, it is ambiguous whether giving these charter schools designation does increase the provision of seats.

Repeating the same dimensions of comparison, I show in Figure (4) that the new policy does increase the sector's capacity. Compared with the existing HP scheme (red), the Target Value-Added scheme (green) results in 5.62 more classrooms in the mean capacity at the end of the inspection window. Moreover, the contribution of such additional difference can be shown to come additionally from the charter schools serving the lower income households. Similarly to the patterns observed, the mean capacity difference across the Target Value-Added scheme and the existing HP scheme comes more from the incentive effect.

Figure 4. HP v.s. Target Value-Added: Mean Perf. and Capacity Transition Over 10 Years



The mean performance under the Target Value-Added scheme is also higher than the existing HP scheme over time, amounting to 0.077 in the final year. Still, the decomposition shows competition accounts for a larger effect in influencing value-added. Similar results can also be found in the traditional sector, given the updated belief in the target value-added scheme. I show in **??** that traditional schools, on average, increase performance scores by 0.04 compared to the existing HP scheme, which again supports that the traditional sector is a magnifier of the competitive spillover across schools.

8.3.3 Compare of Variance in Performance and Equity of Charter Access

It is an empirical question whether, under the Target Value-Added scheme, the discrepancy of school performance across high- and low-income neighborhoods increases or decreases, as compared to the existing scheme. On the one hand, under the Target Value-Added scheme, getting a designation is easier for the charter schools serving low-income regions; they might exert little effort after the designation. On the other hand, the potential increase of the charter capacity in the low-income region might trigger more competition given the belief about the newly designated charter schools' future expansion.

This might push charter and traditional public schools in those regions to increase performance. Which direction in the simulation hinges on the perceived returns and cost of exerting value-added (of both types of schools) and adjusting capacity (of charter schools). This justifies using a structural model to quantify these "deep" parameters.

Given the estimates, the simulation results support that the variance of performance across schools under the Target Value-Added Scheme is lower. The main driving force underlying this result is that: such a scheme incentivizes more expansion of high-value-added charter schools in the lower-income regions. Under the existing scheme, these charter schools do not get expansion eligibility. They are not "High-performing" not because they do not have high value-added on students' test scores but because the local students enrolled are more disadvantaged, pushing these schools' performance down. By giving more eligibility instead to the high value-added charter schools, charter schools in lower-income regions have more incentive to increase performance, reducing the variance of performance across schools. This also increases the equity of access because the increase of the charter capacity under the Target Value-Added scheme is closer to the level of the higher-income regions under the existing scheme.

9 Conclusion

In this paper, I exploit a novel policy incentivizing charter schools with expansion eligibility. I leverage the policy to explore the design of a charter capacity regulation policy that can potentially increase education equality and provide more access to under-served students. I collect administrative data and use them to conduct statistical analysis and run policy simulations with a model. I find suggestive evidence that charter capacity adjustment cost might be substantial and that alleviating such cost creates competitive spillover across sectors. I highlight these two mechanisms in a dynamic model of school decision-making to explore further the aggregate policy effect and explore the implication of targeting value-added to allocate expansion eligibility. I find that the existing scheme and the scheme that targets value-added increase the mean performance and accessibility of the charter sector, as well as the mean performance of the traditional sector. However, the existing scheme can be improved by targeting better, e.g., the value-added. Such scheme improves equity of access to high quality education by increasing the performance and accessibility of the schools serving the lower-income neighborhoods.

The current model restricts the dimension of students' heterogeneity only to their residential location. This does not allow the model to answer questions such as how the student demographic distribution will be changed if the policy were not implemented. As the preliminary evidence shows, TPSs tend to have a higher proportion of students who need free and reduced-price lunch as they are surrounded by a higher number of HP charter schools. This is suggestive evidence that, as charter schools expand, more students might resort from their original TPS to nearby, expanded HP charter schools. The sorting within local neighborhoods can be critical to the distribution of performance across schools. To allow this into the model, I need to allow for students to differ in terms of demographics additional to residential location, such as race, and factor such demographic differences in their taste parameters to schools' characteristics, such as performance. In this way, the demand can capture the differences in the taste between high and low-income families in their taste of school performance scores, which can be influenced by schools. In this regard, such richer model might contribute to the dynamic sorting (Bayer et al. 2016; Hahn and Park 2022) literature by allowing for endogenous school adjustment decisions.

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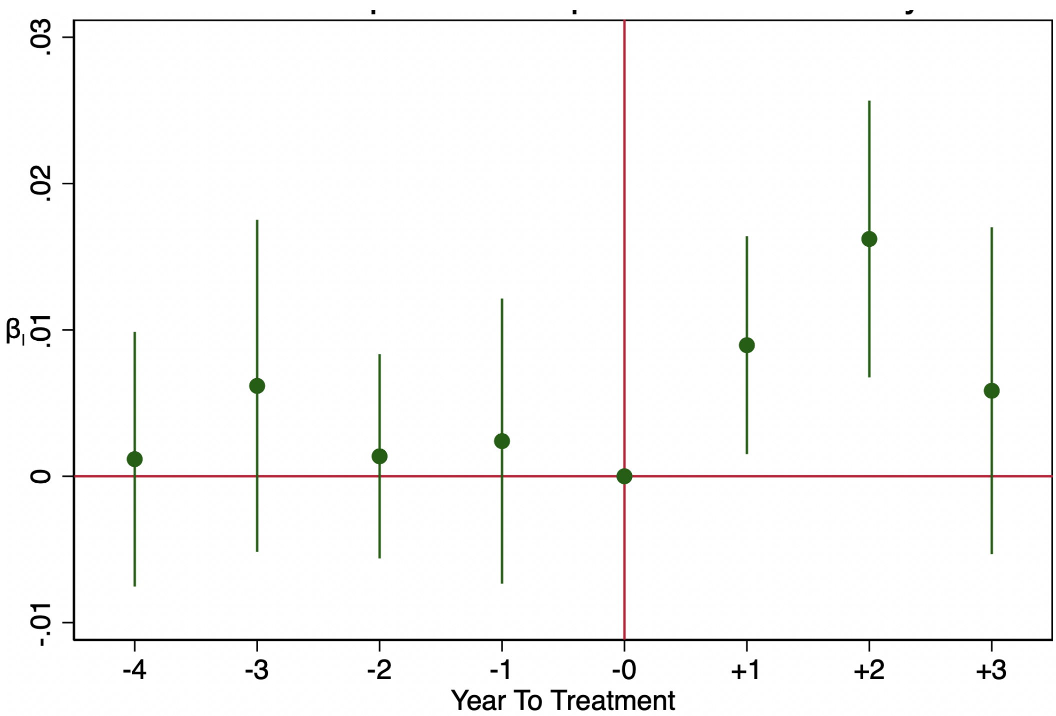
A Figure Appendix

A.1 Event Study of Competitive Spillover on the TPSs

$$A_{igkt} = \sum_{\ell=-4}^{3} \beta_\ell 1_{\ell=t-2011} \times Treat_i + \rho A_{igkt-1} + \sum_{\ell=-4}^{3} \alpha_\ell 1_{\ell=t-2011} +$$
$$\eta Treat_i + \gamma \mathbf{Z}_{igkt} + \epsilon_{igkt}$$

Figure A1. Event Study of TPS Competition Responses



A.2 Event Study of HP Designation on Charter Capacity and Enrollment

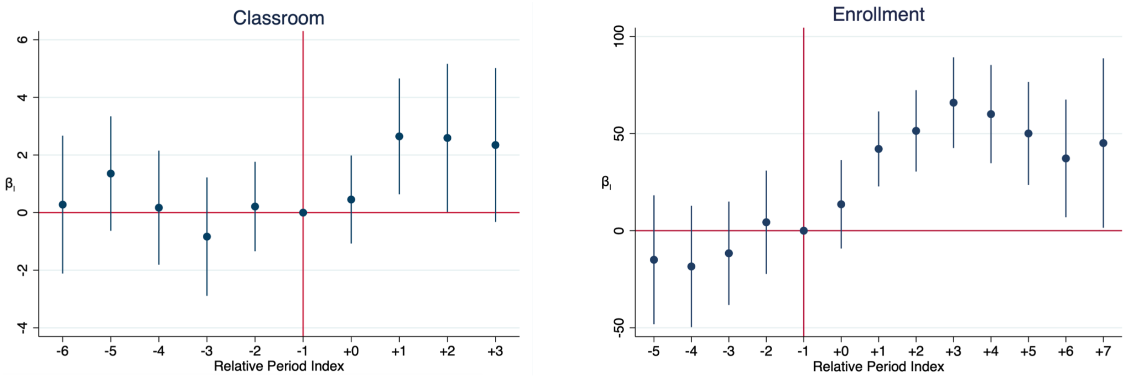


Figure A2. Coefficient Plots of β_ℓs for Classroom Count $2007-15$ and Enrollment, $2007-19$

B Table Appendix

B.1 Summary Statistics

Table B1. Summary Statistics of Different Samples

Table B2. Summary Statistics of the Selected Districts for Structural Estimation in 2015

Table B3. Summary Statistics of the Selected Markets for Simulation in 2012

B.2 Robustness Tests on the Diff-in-Diff Results

Table B4. Other Variants of the TPS Competition Response Tests

Outcome: Test Score	By Subject		Alternative Treatment Measure					Sample Selection		
	Read	Math	#HP in 3	Exist in 3	Exist in 5	#A in 3	#A in 5	>80 Match	>90 Match	Full Sample
$Post_t \times Treat_i$	0.0090***	0.0076**	0.0132***	0.0189***	0.0176***	0.0055**	0.0031	0.0082***	0.0088***	0.0097***
	(0.0024)	(0.0033)	(0.0033)	(0.0066)	(0.0062)	(0.0028)	(0.0023)	(0.0024)	(0.0032)	(0.0023)
Constant	30.5105**	37.2682**	32.0312**	29.9242**	28.8405**	29.5594**	29.3216**	29.9680**	25.6203	25.8730**
	(13.6531)	(18.7661)	(13.1629)	(13.1426)	(13.1283)	(13.1501)	(13.1679)	(13.6499)	(21.1103)	(10.1978)
Observations	27,593	27,593	55,304	55,304	55,304	55,304	55,304	52,286	27,599	83,004
R-squared	0.9504	0.9013	0.8973	0.8973	0.8973	0.8973	0.8972	0.8985	0.9097	0.8976
Fixed Effects	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Charter Entry + School Demo	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
PT Ratio	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y

Standard errors in parentheses
*** p¡0.01, ** p¡0.05, * p¡0.1

B.3 Measurement

Table B5. Full List of Variables with Measurement and Availability

Variable	Meaning	Measurment	Data Availability
Part A. Endogeneous States and Decisions			
k_t	Capacity	For charter schools, this is the number of classrooms in year t. For TPSs, this is the largest enrollment observed during 2007-2019 divided by 22.	
q_t	Performance level	For both types of schools, this is the accountability score in year $t-1$	
hp_t	Designation	For charter schools, this is the designation status in year t. For TPS, this is zero in all situations.	
e_t	Increment in classrooms	For charter schools, this is the first-difference of classrooms in t+1 and t. For TPSs, this is zero in all situations.	
v_t	Effort in performance	For both types of schools, this is the average teacher value-added score within a school in year t	2012-2019
$\bar{n}_{jt}$	Inclusive value about the market situation	Demand model-implied	Estimated
Part B. Other State Variables			
d_{jt}	Local demographics	The mean household income of all census tracts within 3-mile radius of a school.	
ξ_{jt}	Hidden quality	Demand model-implied	Estimated
ϵ_t	Unobserved heterogeneity	Random normal	
Part C. Other Variables in the Model			
m_{lt}	Local market size	ACS tract level school attendance to K-8 grades of tract l in t, tuned according to private school enrollment using Ferreyra and Kosenok (2018) method	
$dist_{jl}$	Travel distance to school	Crowfly distance between school j and tract l	
E_{jt}	Enrollment	For both types of schools, this is the total enrollment of K-8 grades from the NCES and Florida Master Files	
Ψ_{jt}	Operating cost of charter school	For charter schools, this is the total instructional expenditure	2007-2015

C Model and Estimation Appendix

C.1 Spatial Demand Estimation

The Nested fixed point (NFP) algorithm follows Berry et al. (1995). This algorithm finds the optimal $\hat{\lambda}$ that minimizes the correlation between the instrument Z and the derived $\hat{\xi}$ coming from the Berry (1994) inversion. That is, in the inner loop, I match the market share with the derived $\hat{\delta}$ given a guess of $\hat{\delta}$. And get $\hat{\alpha}$ by two stage least square. In the outer loop, the GMM objective is minimized wrt. $\hat{\delta}$:

$$\min_{\hat{\delta}} \hat{\xi}(\hat{\hat{\delta}})' Z W Z' \hat{\xi}(\hat{\hat{\delta}})$$

.

I use four sets of instruments $Z = \{x^{\text{demand}}, Z^{BT}, Z^{BLP}, Z^{demo}\}$. The demand inputs x^{demand} is independent with ξ_{jt} because I assume ξ_{jt} exogenously evolve as an AR(1), as in Sweeting (2013). Given the assumption on x the validity of the Z^{BT} is followed by construction. It is a predicted enrollment $\hat{E}_{jt}$ divided by k_{jt} where the construction of $\hat{E}_{jt}$ follows the following procedure. I run non-linear least square (NLS) estimation on a model that is identical to the original model except that there exists no ξ_{jt} or $ClassSize_{jt}$ in students' indirect utility specification. This $\hat{E}_{jt}$ therefore is independent with ξ_{jt} by construction. The set of instruments Z^{BLP} includes the number of charter and TPSs within 5 miles, and the total capacity of those schools. I call it BLP instrument because it shares the similarity of making using of the information from other firms' characteristics. These characteristics influence j's class size in year t but are assumed to be independent with ξ_{jt}. I also add local demographics Z^{demo} of j in year t as part of the instruments. I regress the class size of j in t on only the Bayer and Timmins (2007) instruments and all the instruments, respectively. I run F tests on both regressions. The results reject (p-value<0.001) on both cases the hypothesis that all coefficients are jointly zeros.

C.2 BBL Estimation Results

Table C1. Estimate of Operation Cost

VARIABLES	(1) logExpenditure
logEnrollment	0.997***
	(0.015)
Performance	-2.057***
	(0.452)
Performance^2	1.748***
	(0.368)
#Classroom	-0.002
	(0.002)
#Classroom^2	0.000
	(0.000)
mean_hh_income_in_3	-0.039**
	(0.016)
Constant	9.229***
	(0.246)
Observations	1,312
R-squared	0.917

Table C2. Estimate of Value-added Functions

VARIABLES	(1) Charter	(2) TPS
HP_status_real = 1	0.108	
	(0.328)	
n_clssrm	0.000	-0.008***
	(0.003)	(0.001)
c.n_clssrm#c.n_clssrm	0.000	0.000
	(0.000)	(0.000)
gscorel1	-0.779	0.185
	(0.630)	(0.143)
c.gscorel1#c.gscorel1	1.100**	-0.121
	(0.515)	(0.117)
1.HP_status_real#c.n_clssrm	0.004	
	(0.005)	
1.HP_status_real#c.gscorel1	-0.011	
	(0.279)	
c.n_clssrm#c.gscorel1	-0.002	0.011***
	(0.004)	(0.001)
1.HP_status_real#c.n_clssrm#c.gscorel1	-0.006	
	(0.007)	
heteroD_n	0.000	0.000***
	(0.000)	(0.000)
mean_hh_income_in_3	0.024	0.025***
	(0.019)	(0.004)
1.HP_status_real#c.heteroD_n	0.000*	
	(0.000)	
1.HP_status_real#c.mean_hh_income_in_3	-0.008	
	(0.026)	
Constant	-0.158	-0.280***
	(0.309)	(0.064)
Observations	1,430	9,555
R-squared	0.126	0.101

Table C3. Estimate of Expansion Functions

VARIABLES	(1) y_target	(2) y_band
HP Status = 1	73.349**	4.935**
	(37.133)	(2.345)
#Classroom	0.890***	0.017
	(0.250)	(0.016)
c.n_clssrm#c.n_clssrm	0.000	-0.000
	(0.003)	(0.000)
Performance	-15.698	1.118
	(41.591)	(2.627)
c.gscorel1#c.gscorel1	15.654	-0.863
	(35.882)	(2.266)
1.HP_status_real#c.n_clssrm	-0.194	-0.046
	(0.702)	(0.044)
1.HP_status_real#c.gscorel1	-8.932	-0.883
	(28.580)	(1.805)
c.n_clssrm#c.gscorel1	-0.145	-0.006
	(0.357)	(0.023)
1.HP_status_real#c.n_clssrm#c.gscorel1	0.017	0.068
	(0.996)	(0.063)
Neighborhood Income	0.001	0.000
	(0.002)	(0.000)
Neighborhood Income	1.591	0.239**
	(1.479)	(0.093)
1.HP_status_real#c.heteroD_n	-0.006*	0.000
	(0.003)	(0.000)
1.HP_status_real#c.mean_hh_income_in_3	-5.083*	-0.370*
	(2.984)	(0.188)
Constant	-6.150	-1.209
	(21.839)	(1.379)
Observations	352	352
R-squared	0.546	0.068

C.3 BBL Estimation Details

I pick the states in 2012 for both types of schools to be the initial states. For every initial state, I forward simulate 100 periods with 100 draws. For charter schools, I generate 500 perturbed policies in which I randomly pick one of the following estimated value-added policy function, band equation, and target equation to perturb. For TPSs, I just perturb their value-added policy function. To construct the perturbed policy, I simply add to the estimated functions a normal error, with variances chosen to be relatively small and close to Ryan (2012)'s corresponding choices. To get the standard errors of the structural estimates, I bootstrap 100 times using half of the initial states with the same set of perturbed policy functions.

D Computation and Simulation Appendix

D.1 Computation Algorithm

I first start with a belief of $\nu(.)$ in an iteration step. Then I solve the dynamic programming problems for schools given the $\nu(.)$. This step gives me the policy functions implied by $\nu(.)$, with which I can forward simulate the stream of the states, including the market state n. With this, I update schools' belief about the n and carry that forward to the next iteration step until the the updated $\nu(.)$ is close enough to the one used to produce it.

Particularly, this implies the following procedure.

1. Start from an initial guess of $\nu^1(n)$. Solve the implied expected value function $\bar{V}^{(\nu^1)}(s)$. Pick a market whose state is

$$s_0 = (o, q_0, k_0, hp_0, d_0, \xi_0, m_0, n_0)$$

2. Simulate one path for horizon T of interest, starting from s_0 for L times under the belief $\nu^i(n)$, the i's iterate of n's transition
   - Regard heterogeneity deterministic at the estimated mean
   - Solve for $z^{(\nu_1)}(s)$ by value function iteration
   - For each school, use $z^{(\nu_1)}(s)$ and get one path of n according to the inclusive value formula:

   $$\{\hat{n}_t : t = 0, \ldots, T\}$$

   - Get $\nu^{i+1}(n)$ by estimating an AR(1) using the this path of $\hat{n}$
3. Repeat until $\nu^{i+1}(n)$ is close enough to $\nu^i(n)$. Denote the converged transition as: $\nu(n)$
4. Use the model under $\nu(n)$ and the initial state s_0 to simulate outcomes of this market. Repeat the above procedure for each picked market.

In solving the dynamic programming problem, I use discretization method and value function iteration. In implementation, due to the long computation time, I have to balance the computation budget and the granularity of the state space. Therefore, when deciding the number of discrete values for each state variable, I intentionally allow for more values on the market situation state n. The running model uses the following specification of the state space for charter schools. For TPSs, their state space is similar but they have only one value of hp state and that their capacity space is allowed to be wider but coarser. Under this specification, solving the value function one time costs 33 minutes under the 1e-4 tolerance level with the absolute norm criterion.

Table D1. Evenly Distanced Grids of Each State of Charter School

Endogenous States				Exogenous States			
State	Min	Max	# Grid	State	Min	Max	# Grid
q	0.4	0.9	6	d	10.97	12.18	4
k	1	61	13	ξ	-2	8	6
hp	0	1	2				
n	300	1300	21				

D.2 Other Simulation Results

[A figure of the x and z trends under all schemes]

Table D2. Variance of Performance under HP and no-HP Scheme

Table D3. Variance of Performance under HP and Target-va Scheme