The Effects of Targeted Incentives for Charter Schools to Expand Capacity: a Dynamic Analysis

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Abstract

Policymakers in the U.S. have introduced initiatives to incentivize charter schools with high academic performance levels and authorize them with eligibility to expand enrollment capacity. I leverage one such state-wide policy reform to evaluate its influence on education access and quality via decisions of charter schools and traditional public schools. I develop and estimate a dynamic model that highlights both the adjustments of the charter school capacity, the performance of both types of schools, and their dynamic responses to competitive pressure. The estimates reveal that the existing incentive scheme reduces adjustment costs when charter schools expand capacity. I use the model to investigate two counterfactual incentive schemes: giving additional expansion eligibility to high value-added charter schools and unconditional deregulation of all charter schools.

Keywords: School Choice, Charter School Capacity, Dynamic Structural Model

JEL Classification: H75, I28, L51

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1 Introduction

The ultimate goal of school choice policies is to improve education quality in the aggregate. Proponents of school choice programs mainly highlight two central mechanisms that support this objective. First, school choice programs may increase quality, variety, and access to the alternatives to students' assigned options.[1] Second, such programs might have competitive spillovers on traditional public schools ("TPS" henceforth) and influence their productivity.[2] In the U.S., charter schools are publicly funded (and tuition-free) but are privately run, often by for-profit enterprises.They serve as a primary instrument for providing school choice. Consequently, proper regulations could help *incentivize* charter schools to raise quality and accessibility, and can trigger competitive spillovers across the entire education sector, creating a "tide that lifts all boats" (Hoxby 2002).

In this paper, I use detailed administrative data to analyze a large-scale policy that incentivizes certain charter schools to increase their performance. The policy does so by conditioning expansion eligibility on past performance. The primary goals of the paper are to assess the policy effects on students' academic performance and access to high-quality education and explore alternative incentive schemes that do better. The policy I focus on is the introduction of the Florida High-performing Charter School Statute in 2012. This statute gives "high-performing" ("HP" henceforth) designation to charter schools with three consecutive years of exemplary performance. Such HP charter schools are authorized to expand enrollment capacity without obtaining approval from local districts. I show that HP charter schools increase the number of classrooms for instruction upon being designated. More importantly, using a difference-in-difference analysis, I find that: following the introduction of the policy, student test scores in TPSs subject to more competitive pressure from neighboring HP charter schools increase more.

Two underlying mechanisms are potentially critical to explain these patterns. First, the policy could, by eliminating the adjustment costs imposed by the regulatory constraint, motivate HP charter schools to expand capacity. Second, the potential competitive pressure of future expansion of HP charter schools may push TPSs to improve their performance. Both mechanisms dynamically influence the charter sector's capacity and the overall quality provision of all schools. Although these patterns are suggestive in terms of the underlying mechanisms at work, they are of limited use in understanding the aggregate effect on all schools and disentangling the importance of each mechanism quantitatively without further structure being imposed. A better understanding of the ex-

[1] Some studies have found that highly effective charter programs lead to improvements in students' test scores and future life choices (Abdulkadiroglu et al. 2011; Booker et al. 2011; Angrist et al. 2016; Dobbie and Fryer 2020; Cohodes et al. 2021; Cohodes and Feigenbaum 2021).

[2] Some studies have found that TPSs increase performance when facing competitive pressure from the choice programs in various contexts (Figlio and Hart 2014; Mehta 2017; Gilraine et al. 2021; Gilarine et al. 2023).

tent to which these mechanisms are essential is helpful for the primary goal of this paper, namely, improving policy. Therefore, to achieve this goal, I develop and estimate a dynamic model of schools' decision-making. I explicitly model the dependence of schools' decisions to expand and exert effort (in improving performance) on the adjustment costs and competitive pressures they face.

To estimate the model, I assemble and examine a rich dataset for Florida that tracks the annual operation of 630 regular charter schools and 2411 TPSs serving K-8 grades from 2006-7 to the 2018-19 school year. The dataset provides a comprehensive history of each school's number of classrooms for instruction, performance level, educational effort measured by schools' value-added, operating cost, HP designation status, local demographics, and competitive pressure. These supply-side dynamics can be further linked to student enrollment changes within schools.

The dynamic model I develop maps schools' two key decisions, capacity expansion and educational effort (or inputs), to the distribution of schools' performance and capacity. Every period, charter schools choose educational effort, which determines students' performance, as well as their capacity to expand. TPSs only choose educational efforts. Both decisions are subject to adjustment costs. Furthermore, the incentive scheme induced by the HP statute is modeled as follows: Charter schools can earn HP designation by performing well, and this designation reduces their future cost of adjusting capacity. Schools' decisions on HP status thus affect their future capacity, performance levels, designation status, and, importantly, the competitive pressure in the market where they compete. All of these influence their future enrollment, one of the primal components of their objective functions, via the demand side.

The simulation of the model present several empirical challenges. First, modeling schools' strategic interaction in a dynamic game using MPNE, or Markov Perfect Nash Equilibrium (Ericson and Pakes 1995), is computationally prohibitive. In typical urban school districts, such as the Miami-Dade School District, the average number of neighboring schools within 3 miles of a school is more than 20. Further, by allowing for rich school heterogeneity, the dynamic game framework generates a particularly high-dimensional state space of the school.[3] To alleviate the computational burden while allowing the model to be rich enough for reasonable counterfactual analysis, I make the

[3]Aguirregabiria et al. (2021) use a numerical example of Pakes and McGuire's (1994) model to illustrate the large state space problem. The model, with ten firms choosing only 20 different quality levels in a dynamic game, has over 10 trillion states. In the context of this paper, a model allowing a school to have four performance levels (e.g., A, B, C, D grades), three capacity levels (e.g., less than 10, between 10 and 20, and more than 20 classrooms) and ten competitors has more than 60 billion states. Additionally, it is possible to find populous communities of relatively small area size with many schools, as in the example of Miami-Dade School District. Schools in these populous regions typically have overlapping sets of neighboring schools. This fact implies that a local school's demand can be influenced by schools far away, which further increases the number of potential competitors for the local school, escalating the computation burden.

following assumptions about schools' beliefs and responses to their enrollment. First, I assume that each school only uses its own states and a uni-dimensional state about the market to calculate their current and future enrollment. This assumption is analogous to what is done in a static monopolistic competition model. Second, each school forms beliefs on the transition of this uni-dimensional state that are consistent with how the market evolves given these beliefs. The first assumption assures tractability by reducing the dimensionality of the state space required by MPNE. The second assumption endogenizes competitive pressure characterized by the uni-dimensional state, allowing schools to alter their beliefs on future competitive pressures as policies change, especially when they dramatically influence the regulatory environment.

Given the primary goals of the paper, I first use the model to evaluate the existing policy. I compare the existing policy with the "no-HP" designation scheme to quantify the policy effects. To do so, I set the "no-HP" scheme to eliminate the benefit of expanding the capacity of the HP charter schools. I focus on the change in the distribution of schools' performance under the counterfactual policy and decompose the differences in outcomes in terms of the contributions of each economic force. In particular, I focus on the relative importance of the designation system's incentive channel (reduction in adjustment cost for charter schools if high-performing) and its competition channel (pressure from expanding neighbors) on the distribution of schools' performance.

Furthermore, I use the model to explore alternative policies that target high value-added charter schools and grant them more opportunities to earn expansion eligibility than the existing scheme. To do so, I modify the designation criteria by allowing charter schools having value-added higher than a cutoff to be also eligible to expand easily, as the current HP charter schools do. This counterfactual policy is motivated by the concern that the existing scheme may exacerbate inequality in access to high value-added charter schools across different SES groups. Many charter schools not designated as HP indeed exert high value-added. These schools typically serve lower SES households and do not achieve the required performance levels for HP designation. Therefore, in this counterfactual, I specifically inspect the inequality of allocating the expansion eligibility rooted in the existing scheme and how equalizing the eligibility can influence access to charter schools and the quality of education in low SES regions.

The design and implementation of school choice programs are at the forefront of education research. For example, using vouchers to increase choice has been extensively researched and evidence shows that they are effective in countries where private education accounts for a large market share (see Hsieh and Urquiola, 2006; Neilson 2013; Arcidiacono et al. 2021). The underlying idea of vouchers is to increase students' alternatives to expensive private schools. Therefore, relaxing capacity constraints for HP charter schools is like increasing the number of vouchers for such schools. To some extent, capacity regulation of this kind is a more "controlled" way to direct students toward *targeted* schools aligned with the policymakers' goal. Therefore, comparing the capacity regulation of charter schools with extensively researched voucher systems can improve the understanding of both policy tools for scholars. Nevertheless, perhaps more importantly, the market environment of voucher systems contrasts dramatically with the public education market, where the government fully funds tuition. The "non-price" nature of public education markets prohibits policy tools like vouchers to incentivize charter and TPSs. Therefore, taking advantage of the incentive for capacity expansion is more practically relevant and can enrich the toolbox for policymakers in the public education market. Leveraging the policy reform, this paper is the first attempt to investigate such policies using an empirical strategy to both establish novel facts and develop a dynamic quantitative model to evaluate policy effects as well as explore alternative schemes that might do better. Finally, if proven beneficial, such policy reforms may be easier to do than the extensively researched public education policies that increase spending in public schools as they do not involve increasing expenditure.[4] It is an incentive scheme that imposes rules in influencing schools' decisions and does not explicitly require increasing or redistributing money across schools. [5]

Contribution and Related Literature. This paper contributes to an extensive literature on school choice in the public-private education systems. Milton Friedman argued that the market-based school choice through vouchers for private school attendance would facilitate Tiebout-style competition without necessitating community relocation. It would extend educational choices to previously underserved families and theoretically enhance the overall quality of education (Friedman 1955; Hoxby 2003). School choice programs typically take the form of charter schools in the U.S. education market. Many studies, especially those using "lottery estimates" (Hoxby and Murarka 2009, Abdulkadiroglu al. 2011, Angrist et al. 2016), have shown that the impact of charter schools on student achievement can be both significantly positive and sizable, especially for the "No-excuses" charter school model (Cohodes and Parhams, 2021). However, these charter programs and many charter schools are under capacity constraints.[6] Therefore, it is natu-

[4]These policies focused on increasing spending are widely discussed by Ellini et al. (2010), Martorell et al. (2015), Jackson et al. (2016), Dinerstein and Smith (2021), and Asker et al. (2022), among others.

[5]This view of the policy potentially ignores the fiscal externality imposed on the TPSs and school districts. Previous research has found that charter expansion could reduce the district funding to TPSs or alter TPSs' spending structure (Ridley and Terrier 2018; Mumma 2020; Singleton and Ladd 2020). Fully internalizing these costs in designing school choice policies is not a focus of this paper and can be a potential direction for future economic research in broader settings. Similar policies have been seen in Florida, Massachusetts (Ridley and Terrier 2018), Missouri, and Louisiana. For a report on the implementation in these states, see the National Association of Charter School Authorizers' 2019 report on "Expanding Access to High-Performing Charter Schools 2019".

[6]In 2012, 61% of Florida charter schools were oversubscribed. Among these, 40% received applications 1.5 times the year's enrolling target, and over half were rated as "A," marking their top-tier academic per-

ral for scholars and policymakers to consider policies that alleviate the capacity constraint for charter schools. By analyzing this novel policy, my work casts light on the trade-off involved in different ways of charter expansion. Specifically, I address the importance of capacity deregulation in designing large-scale school choice policies and provide a quantitative framework enabling comparison of schools' performance across policy schemes.

Within the landscape of school choice literature, this paper relates to the strand that analyzes the policy effects of charter school expansion. Existing research on charter school expansion has focused almost solely on the impact of *entry* of charter schools, in particular, on the competitive pressure it places on TPSs (Imberman 2011; Figlio and Hart 2014; Mehta 2017; Gilraine et al. 2021).[7] However, the form of charter expansion in this paper is novel: it leverages the eligibility to expand *capacity* for certain charter schools. Therefore, by thoroughly analyzing the novel policy, I address the importance of charter capacity previously ignored in the charter expansion literature. Relatedly, under the theme of expanding and replicating charters, another niche literature looks at the specific practices for replicating effective charter programs (Zimmer and Buddin 2007; Angrist et al. 2013; Fryer 2014; Cohodes et al. 2021). I differ from this literature by using evidence from a large-scale state policy that could generate a spillover effect across sectors, particularly on TPSs.

The causal inference strategy applied in this paper builds on the literature that identifies the competitive spillovers of charter schools. This literature has typically found contextual and sometimes conflicting results on competitive spillovers by charter expansion across studies, as Figlio et al. (2021), a closely related paper, points out.[8] Moreover, because of data limitations and lack of policy variation, causal studies in this literature are scarce.[9] In this paper, I tackle these empirical challenges by gathering grade-subject level test scores and taking advantage of the natural experiment created by the policy change in the charter sector. I develop a difference-in-difference specification and utilize a unique feature of the test score data to identify the competitive responses and attribute them to schools' input change. I provide the first estimates of the competitive spillovers on TPSs' test scores of the new policy scheme focusing on charter capacity regulation.

formance. Additionally, among the oversubscribed schools, 46% are located in lower-than-median income regions.

[7]Other papers following this strand also discuss charter expansion in its consequences for inequality in charter access (Singleton 2019), its effect on racial segregation (Monarrez et al. 2022), and its influence on district budgets for TPSs (Baker et al. 2015; Epple et al. 2016; Buerger and Bifulco 2019; Mumma 2020; Singleton and Ladd 2020).

[8]For example, Hoxby 2003; Sass 2006; Zimmer and Buddin 2009; Bettinger 2005; Imberman 2011; Winters 2012; Cordes 2018; Ridley and Terrier 2018; Mann and Bruno 2020; Gilraine et al. 2021.

[9]Figlio et al. (2021) have briefly surveyed the current state of the literature. They claim that several of the studies have been limited to single districts or a small set of districts (e.g., Zimmer and Buddin 2009; Winters 2012; Cordes 2018), while studies that have used statewide data generally look at the very early years of charter policies and over short periods (e.g., Bettinger 2005; Bifluco and Ladd 2006; Sass 2006). Other studies that take a national perspective are limited to district-level data (Han and Keefe 2020).

The results suggest a new source of competitive pressure imposed on TPSs: neighboring charter schools' eligibility to expand capacity. Furthermore, the competitive responses are larger than those obtained in similar contexts (Figlio and Hart 2014; Figlio et al. 2021).

The structural modeling approach followed puts this paper in the growing literature that focuses on analyzing the industrial organization of the supply of education. Papers in this literature are typically model-driven and explicitly quantify students' choice of schools and education providers' responses, such as increasing quality, entry, and exit. These papers further link the supply and demand in an equilibrium model to generate policy-relevant outcomes.[10] However, my work is the first to develop a quantitative dynamic model incorporating decisions on capacity and performance in the K-12 setting. Moreover, the model is designed to be computationally tractable and address schools' strategic considerations in a dynamic setting. My work hence follows recent attempts to apply quantitative dynamic models to study education markets. [11]

The remainder of the paper proceeds as follows. Section 2 provides industry background. Section 3 introduces data sources and the sample under inspection. Section 4 shows descriptive patterns in the Florida education market and evidence of the policy effects. Section 5 introduces the current version of the quantitative model. Section 6 introduces empirical strategy in estimating the model. Section 7 shows estimates of the model. Section 8 displays simulations based on counterfactual policies. Section 9 concludes and discusses the direction of the next version of the paper.

2 Industry Background

In this section, I introduce the industry background of the Florida public education market and the relevant institutional background of the policy of focus.

2.1 The Florida Public Education Market

Florida has one of the largest public school enrollments in both the traditional and charter sectors across all states. It also has sound charter laws and relatively lenient entry screening (Singleton 2019), making it a state with one of the highest numbers of charter

[10]The inspected outcomes include students' welfare, test scores, access to schools, and segregation (Hasting et al., 2009; Neilson 2013; Ferreyra and Kosenok, 2018; Mehta 2017; Singleton 2019; Allende 2019; Dinerstein and Smith 2021; Arcidiacono et al. 2021; Bau 2022; Dinerstein et al. 2022).

[11]For example, Larroucau and Rios (2021) investigate the effects of centralized assignment mechanisms in influencing outcomes and choices after their initial assignment to college. Hahm and Park (2022) explore how preference for high school characteristics influences students' choices in middle school. Bodéré (2022) looks at the effects of government subsidies in childcare on the entry, exit, and quality investment of private pre-schools.

schools and charter enrollment shares in the United States. Additionally, Floridian students can choose any public school or charter school if they are not capacity-constrained through a process known as "controlled open enrollment".[12] These unique features of the Florida public education market amplify the potential impact of policies targeting the charter sector on the overall landscape of public education. Therefore, Florida becomes an ideal state for evaluating the effects of charter school policies.

Regarding accountability, Florida has implemented a system that assesses and gives performance scores to nearly all charter and TPSs annually. This system assigns accountability scores or letter grades to schools, ranging from A (highest) to F (lowest), based on the same criteria applied to both charter and TPS. Notably, while the rating system aims to consider students' achievements and learning gains relative to their previous scores, it still places more emphasis on absolute achievements. This emphasis is evident in the criteria used to assess schools' learning gains, where a school can receive a high score if its students maintain their test scores at a sufficient level, regardless of their individual growth. Among all schools in my sample in the 2018-2019 school year, the letter grade distribution is approximately 34% A, 26% B, 32% C, and the rest 8% are D, F, or missing.

2.2 The New Statute and Charter Expansion Management

In July 2011, Florida enacted the High-Performing Charter School Statute, which remains in effect to today. The statute defines HP charter schools as those with three consecutive years of exemplary performance,[13] two As and no grades below B ("2A1B" rule henceforth),[14] marking satisfactory achievement and progress of performance in standardized tests of the students in the school. An HP charter school can keep the designation until receiving two C grades or worse. In such cases, its HP designation can be revoked. However, such cases were rare in the sample.[15] Among all charter schools in the sample, approximately 20% held HP designation in 2012, and this percentage increased to 40% by 2019.

The most significant benefit granted by the statute was the authorization for HP charter schools to expand their enrollment capacities without the approval of local school districts. They can increase enrollment capacity once per school year, expand grade span

[12] The capacity constraint does not seem to apply to many TPSs. As Annual Five Year Plan indicated, it is frequent to have TPSs enrolling more students than their enrollment capacity.

[13] The statute also requires healthy financial conditions. However, this is much easier to be satisfied and almost never binds in giving designation as compared to the performance requirement. For all charter schools meeting the performance criteria, there are few cases in which schools fail to satisfy the financial requirement or an incumbent HP school has been deprived of the designation for financial reasons.

[14] The criterion allows charter schools having two years of A level to be designated after 2017.

[15] In my sample, seven charter schools were de-designated from 2012 to 2019, and 179 charter schools that were designated and never de-designated. Since the de-designated charter schools account for less than 4% of the designated charter schools, I code them as never designated throughout the paper.

not already served within the range of K-12, or replicate its educational program in any district in Florida.[16] The statute legally prevents local school districts from rejecting these expansion requests made by HP charter schools. On the other hand, districts had the discretion to reject any expansion before the policy's implementation, or after the policy if the non-HP charter schools propose such requests. Hence the policy essentially introduced a new incentive system that links the past performance of charter schools to the automatic eligibility for expansion.

I do not directly observe the enrollment capacity measured in student count as written in charter contracts. Thus, I make a critical measurement assumption that the number of classrooms for instruction in a charter school serves as a sufficient statistic for enrollment capacity.[17] Leasing is also notable as the primary ownership type of charter schools contract. Leasing is the primary form of ownership for charter schools, and the cost of expanding capacity, i.e., adding classrooms for instruction, is typically associated with leasing more spaces, renting relocatable classroom, or renovating existing leased facilities that are not yet utilized. Consequently, modifying capacity in this context can be achieved relatively quickly compared to constructing entirely new facilities.

Throughout this study, I interchangeably use the term "the policy" or "the statute" to refer to this event. Moreover, I refer to the years before 2012, the "pre-policy" period, and the year 2012 and onward, the "post-policy" period.

3 Data and Sample

I introduce the data and the sample of targeted schools and years in this section. To conduct this research, I combined digitized government documents, publicly available datasets, and those with limited public access that require requests for disclosure of information. I collected enrollment of each grade and race, location, and activity status for all public schools in Florida from the National Center of Education Statistics' ELSi dataset, which was merged into the Florida School Master File to obtain additional school characteristics. The locations of schools were mapped to census tracts whose geocodes were merged with the U.S. Census Bureau's American Community Survey to acquire granular local demographics for all schools. The school's location is also valuable for providing the distance students need to travel from each census tract to a particular school and iden-

[16] Additional benefits for individual HP charter schools include reduced frequency of financial statement reporting to the sponsor, usually the local school district. They also have the opportunity to modify their charter to extend its duration and enjoy a slight reduction in administrative fees.

[17] In this context, enrollment capacity refers to the maximum number of students a charter school can enroll. It should not be confused with facility capacity, which represents the maximum number of students the school's physical facilities can accommodate safely. Naturally, enrollment capacity cannot exceed facility capacity, although the two quantities are correlated due to the costs associated with leasing or owning additional facilities that remain unused.

tifying which schools are closely competing with it. I collected schools' performance information, the letter grades, detailed component scores used to produce the letter grades, and standardized test scores from Florida School Grades Archives and the Department of Education's Bureau of K-12 Assessment.

To tailor the analysis to the policy context, I obtained characteristics such as capacity (number of classrooms and buildings), lease, mission statement, education model, management company, staff details, and annual waitlist status of charter schools from Florida charter schools' annual Accountability Reports from the Florida Office of Independent Education and Parental Choice. From the same source, I obtained the annual HP designation status (designated, de-designated). With all these variables, I can characterize a complete history of the supply side by each charter school's capacity, performance, designation, local demographics, and neighboring schools that can be mapped to its enrollment volume and composition. Additionally, I obtained annual teacher-subject level value-added estimates from a regression-based statistical model run by the Florida Department of Education, Bureau of Accountability Reporting. I averaged teacher-level value-added scores to the school level according to the teacher-school linkage provided by the same dataset to measure the educational effort in improving a school's performance level, one of the crucial investment decisions in the model. Lastly, I extended Singleton's (2019) digitized independent audit data to include more years and the coverage of charter schools than the original paper. The audit, filed by charter schools annually to the Florida Auditor General, report charter schools' revenue, itemized expenses, and assets. The instructional expenditure can be conveniently employed in estimating the operating cost function in my quantitative model.

This paper focuses on regular charter and TPSs that serve elementary (K-5) and middle grades (6-8) in Florida from 2007 to 2019.[18] These selected schools encompass the majority of K-8 public schools and their enrollment in Florida. Schools operating grades from kindergarten to 8^{th} yet running concurrently high school grades (the 9^{th} to 12^{th} grade) during the sample period are excluded. This exclusion was necessary due to the distinct accountability requirements for high schools, which differ from those of elementary and middle schools. By excluding these schools, the statistical analysis becomes more convenient, and the interpretation of the schools' performance score is less convoluted. Thus around 7% of the total K-8 students are not considered during the sample period.

The ultimate sample under examination has 2,411 TPS and 630 charter schools, whose observation counts are 29,333 and 4,483, respectively, at the school-year level. Comparing the sample length (13 years), the median panel length of TPS and charter observations is 12.2 and 7.28 years, respectively.

[18]Regular schools in my selection are all public schools excluding those that are laboratory, municipal, virtual, providing special education and those charter schools conversed from a TPS.

4 Preliminary Evidence

In this section, I start by documenting an overview of the TPS and charter sector in Florida, highlighting the heterogeneity between non-HP and HP charter schools. More importantly, I highlight two key mechanisms critical in analyzing the policy effects: charter's adjustment cost in expansion and competition across schools. I first provide suggestive evidence that the charter sector responds to the policy by expansion and that there exist reallocation of students across sectors. Then, I identify the competitive responses of TPSs using a difference-in-difference design enabled by the policy shock and a special test score data. Finally, I motivate an alternative policy by pointing out that the existing policy could advantage the charter schools already serving the high SES regions. For ease of exposition, when describing a school year, I use "2019" to represent the "2018-19 school year."

4.1 Overview of Florida Traditional and Charter Sector

During the sample, charter enrollment accounts for an increasingly larger share of the public K-8 enrollment over time: 3.3% in 2007, 6.5% in 2011, and 11.4% or around 210,000 students in 2019. The number of charter schools in my sample is increasing, too: 216 in 2007, 290 in 2011, 376 in 2015, and 436 in 2019. After 2012, the charter sector's exit rate in my sample remained stable at around 3% to 5%, while the entry rate started to drop from around 18% in 2011 to 5% in 2019.[19] Typically, they are more in the count and more densely distributed in school districts with higher population and population density, usually highly urbanized regions. In these large school districts, charter schools account for a higher share of public enrollment (around 20%) and tend to be proximate closer to other charter and TPSs than other districts.

There exists considerable heterogeneity between the HP and non-HP charter schools. In Table (1), I compare the mean and standard deviations (in parenthesis) of the non-HP and the HP charter schools in 2015, 4 years after the enactment of the policy. In 2015, among 376 charter schools in my sample, 31.6% were HP: 69 were designated in 2012 and 50 in 2013-15. On average, compared to the non-HP ones, HP charter schools have higher performance scores, capacity, and enrollment. They operate in locations with higher population density, income, students' test scores, and a more white or Hispanic population. Consistent with the demographics of their locations, they serve more white and Hispanic students on average while systemically fewer disadvantaged groups, including black students and those eligible for free or reduced-price lunch. The distinction of the type of

[19] Exit rate in year t is defined as the ratio between total exits in t and count of charter schools in t. The entry rate is the ratio between the total entries in t and the count in $t-1$. An exit is labeled as in year t if I do not observe enrollment records since $t+1$. Moreover, an entry is labeled as in year t if I start to observe a charter school's enrollment record since t but do not observe the enrollment record before t.

population served between the HP and non-HP charter schools can also be reflected in their instructional cost. HP charter schools, on average, spend less per enrollment on students than non-HP charter schools. This phenomenon may jointly reflect the efficiency of spending and the cost differentials across student populations (Singleton, 2019).

Table 1. Sum. Stat. of 2015 Charter Schools by HP status

	non-HP	HP		non-HP	HP
I. School Characteristics			**III. Location (Census Tract) Characteristics**		
Total Performance Score (%)	0.50	0.72	Population Density (1000/square mile)	1.29	1.53
	(0.16)	(0.12)		(0.88)	(1.00)
Enrollment	357.25	560.24	Household Income	62755.03	68443.73
	(330.20)	(349.40)		13625.40	19158.80
Number of Classroom	21.88	33.04	Mean School Reading Score within 5 Miles	-0.23	-0.04
	(16.90)	(19.41)		(0.51)	(0.53)
II. Student Composition			Mean School Math Score within 5 Miles	-0.19	0.01
% of Free/Reduced Price Lunch	0.52	0.40		(0.49)	(0.53)
	(0.30)	(0.27)			
			Number of Traditional Public Schools	24.40	24.60
% of Hispanic	0.32	0.43		(15.39)	(15.44)
	(0.28)	(0.32)			
			IV. Instructional Costs		
% of Black	0.31	0.13	Annual per-enrollment Instructional Cost	4110.00	3838.00
	(0.31)	(0.19)		(2373.00)	(978.00)
% of Whilte	0.31	0.37			
	(0.28)	(0.30)	Number of Observations	257	119

4.2 Charter Expansion, Student Reallocation, and Competition

In this subsection, I first analyze the direct effect of designation on charter schools' capacity and enrollment. I continue the analysis by raising suggestive evidence of the associated reallocation of students associated with the appearance of HP charter schools. Then, I provide a causal inference on the competitive spillover reflected in TPSs' test scores.

HP Charter Expansion How does the charter sector react to the policy in terms of capacity, and to what extent does the reaction influence neighboring schools? To answer these questions, I first analyze the relationship between designation timing and measures of school size and enrollment. I correlate the within-school variation in the number of classrooms for instruction, total enrollment (in logarithm), and the number of grades with the time-varying designation status. To do this, I run a two-way fixed effect model as shown in equation (1). The regressor HP_{jt} is the HP-designation status of a charter school j in year t. It gives value to one if a charter school gets or has the designation status maintained in that year. All charter schools before 2012 are set to not have the designation, i.e., $HP_{jt} = 0, \forall j$ if $t < 2012$.The year fixed effect controls for factors that are common to all charter schools, such as macro economy shock. The main coefficient of interest is β.

$$Y_{jt} = \beta HP_{jt} + FE_j + FE_t + \epsilon_{jt}. \tag{1}$$

Table 2. Correlation of School Size and Designation

	(1) Classrooms	(2) Logenr	(3) GradeSpan	(4) NeighborTPS	(5) Age5-14Stu.	(6) Income
HP	1.84119***	0.10230***	0.01067	-0.09635	1.46084**	2.43518***
	(0.51439)	(0.02053)	(0.05561)	(0.83629)	(0.61285)	(0.72124)
HP X var				0.18860***	0.87852	-0.88358
				(0.06422)	(0.76958)	(0.75209)
Constant	18.51560***	5.12653***	5.25999***	18.58774***	18.52268***	18.53242***
	(0.62777)	(0.02432)	(0.06588)	(0.63082)	(0.62777)	(0.62789)
Observations	4,080	4,483	4,483	4,054	4,080	4,080
R-squared	0.84287	0.90076	0.87469	0.84286	0.84293	0.84294
School FE	Y	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y	Y

Standard errors in parentheses
*** p¡0.01, ** p¡0.05, * p¡0.1

I show results in Table (2). After controlling for two-way fixed effects, the designation status positively correlates with the total number of classrooms and enrollment while not significantly so for the grade span. Note that charter schools are relatively smaller than TPSs: the average number of classrooms used for instruction in the charter sector steadily increased from 16 in 2007 to 25 in 2015 and 29 in 2019. The estimate on the designation status, roughly 1.8 classroom difference, suggests a sizable within-school expansion between the average capacity between the pre- and post-designation observations.[20]In Appendix B.1, I use an alternative specification to inspect the timing of the expansion after designation. I replace the regressor HP_{jt} with a list of year-to-designation indicators in an event study regression model regarding HP designation as the focal event for a charter school. The result shows that, on average, more classrooms and enrollment are added after the first few years of designation, which suggests that the expansion motives might be important behind the designation. [21]

[20]It is worth pointing out that none of these patterns causally support designation, which induces charter schools to expand. The difficulty of designing a test to justify is because the designation is an endogenous characteristic of charter schools, and the designation rule applies equally to all charter schools. Therefore, one potential future research could be to compare charter schools in Florida to states in which the HP designation system did not exist after 2012. This at least creates variation in the policy exposure across different charter schools. Given the empirical difficulty, I interviewed a few charter school principals and former charter school officials of the Florida Department of Education. They confirmed that in their cases and for most of the cases they encountered in negotiating expansion, charter schools leveraged the designation to avoid expansion restrictions after the policy's establishment.

[21]Note that the designation is endogenous to charter schools' decisions. Therefore, the event study out-

I continue to inspect the heterogeneity of the above relationship regarding classroom count by interacting the HP designation with local schooling market demographics of charter schools. As the rest of the columns in Table (1) show, the within-school addition of classrooms between the pre- and post-designation is seen significantly more if a charter school is surrounded with more TPSs within 3 miles, as shown in column 4. This suggests that expansion decisions might be based on the local competitive environment. However, there are no significant differences in this relationship among charter schools with varying local demographic environments. In column 5, I interact HP designation with the dummy of whether the mean household income of a 3-mile-neighborhood of a charter school is higher than the median (across all charter school-year observations). In column 6, I apply the above procedure similarly using the proportion of the age 5-14 population. None of the interaction effects in these tests are significant.

These results suggest that HP designation might reduce the adjustment cost for HP charter schools in expansion. Additionally, there exists heterogeneity in this relationship among charter schools facing different degrees of local market conditions.

Student Reallocation I continue to investigate the source of the increased enrollment in the HP charter schools. Mainly, I inspect how much reallocation of students across sectors happens as neighboring charter schools are designated as HP. To do so, I run regressions with specification (2). It exploits the cross-sectional and intertemporal variation of the exposure to HP charter schools faced by TPSs and correlates them with TPSs' enrollment. The cross-sectional variation of HP exposure comes from the spatial variation in the existence of HP charter schools. In contrast, the intertemporal variation comes from the policy implementation and the increase of HP charter schools as time passes. Particularly, I regress the logarithm of enrollment of TPSs on the exposure of HP charter schools within ($HPexpoband1_{it}$) and that in 3 to 5 miles ($HPexpoband2_{it}$), controlling for school fixed effect and local demographics (D_{it}) faced by the TPS from 2007 to 2019.

$$Logenrollment_{it} = \beta_1 HPexpoband1_{it} + \beta_2 HPexpoband2_{it} + \gamma D_{it} + FE_i + \epsilon_{it} \qquad (2)$$

In Table (3), I show results in columns 1 and 2 using the count of HP charter schools as the exposure variable, while in columns 3 and 4 using the total capacity of neighboring charter schools, considering potential expansion behaviors post-designation. In columns 2 and 4, I control for local demographics. Across columns, TPSs' enrollment is negatively

come must be interpreted as capturing the variation in classroom counts influenced by the timing of the designation status. Imposing strong assumptions to claim causality would detract from the research focus, and using estimates of event study coefficients to represent dynamic treatment effect is proved to be misleading even with strong assumptions, as Abraham and Sun (2021) pointed out.

and significantly correlated with more existence or higher capacity of neighboring HP charter schools. Conditional on all other covariates, one more HP charter school within 5 miles is associated with 2.5% less TPS enrollment.

Table 3. Effects on Log Enrollment of Exposure to HP Charter Schools

	(1) Count	(2) Count	(3) Capacity	(4) Capacity
HP Charter 0-3 Miles	-0.011***	-0.012***	-0.00034***	-0.00030***
	(0.001)	(0.002)	(0.00010)	(0.00010)
HP Charter 3-5 Miles	-0.011***	-0.013***	-0.00015**	0.00001
	(0.001)	(0.001)	(0.00007)	(0.00007)
Constant	6.486***	6.485***	6.48588***	6.47099***
	(0.003)	(0.027)	(0.00258)	(0.02719)
Observations	29,037	29,037	29,037	29,037
R-squared	0.940	0.940	0.93898	0.93927
School FE	Y	Y	Y	Y
Control	N	Y	N	Y

Robust standard errors in parentheses
*** $p<0.01$, ** $p<0.05$, * $p<0.1$

In Table (4), I continue to show the results of the tests on the correlation between the student composition of these TPSs and the existence of neighboring HP charters.

I run the same specification, fixing the measurement of HP exposure to be the count of HP charter schools while testing various outcomes related to the demographic composition of students in a TPS. For all the columns in Table (4), each of them reports the regression result using the log enrollment or the proportion of a specific type of students as the outcome. As columns 1 and 2 show, TPSs have less enrollment of black and lower-income students (as measured by those who need free and reduced-price lunch) if HP charter schools appear more, as consistent total enrollment loss in Table (3). However, as shown in column 4, the ratio of lower-income students is higher in these TPSs as HP charter schools appear more in the neighborhood.

These patterns on charter schools' capacity and student reallocation suggest that the charter sector could respond to the policy by expansion, which essentially reduces the adjustment cost of expansion for HP charter schools. Concomitantly, the HP charter schools are likely to impose an externality on the nearby TPSs via reallocation of enrollment. Therefore, competitive spillovers might be a crucial mechanism in evaluating the existing or other similar policies. Particularly, to what extent the competitive spillover can push neighboring schools to improve test scores is a policy-relevant question. Additionally, as the above patterns suggest, this policy is associated with student composition change in schools, which can result in changes in test scores even if schools do not make any changes in educational inputs. This imposes an empirical challenge in identifying the competitive spillover on test scores. In what follows, I address this empirical challenge using a difference-in-difference design facilitated by the policy with proper data.

Table 4. Effects on Composition of Students of Exposure to HP Charter Schools

	(1) LogenrBlack	(2) LogenrFRL	(3) RatioBlack	(4) RatioFRL
HP Charter 0-1 Miles	-0.021***	-0.001	0.00054	0.00360***
	(0.006)	(0.004)	(0.00068)	(0.00123)
HP Charter 1-3 Miles	-0.023***	-0.012***	0.00025	0.00218***
	(0.003)	(0.002)	(0.00034)	(0.00055)
HP Charter 3-5 Miles	-0.023***	-0.008***	-0.00055***	0.00364***
	(0.003)	(0.001)	(0.00016)	(0.00035)
Constant	4.583***	5.944***	0.25988***	0.63447***
	(0.005)	(0.003)	(0.00063)	(0.00105)
Observations	28,927	28,967	29,037	29,037
R-squared	0.959	0.910	0.98470	0.92243
School FE	Y	Y	Y	Y

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Competitive Pressure Following the enactment of the High-performing Charter School Statute, a school could face more competitive pressure as more of its neighboring charter schools can expand with less regulatory constraint. Because the future expansion of neighbors might cause fleeing of students from the school. Facing the pressure, schools might increase their input into educating students, as reflected by test scores. Therefore, to explore the potential competitive responses of schools influenced by the policy, I exploit the establishment of the policy as a natural experiment. I focus on the TPS sector for this test: TPSs differing in competitive pressure from local charter schools receive different intensities of pressure after the policy. Those TPSs with more neighboring HP charter schools in 2012 right after the policy faced higher competitive pressure than other TPSs with either no or fewer neighboring HP charter schools in 2012. However, when investigating responses via test scores, they are likely influenced by the change in student composition. Especially, policies like the Statue that essentially aims at expanding the charter sector are likely to be associated with student reallocation, as seen in North Carolina (Gilraine et al., 2010; Mumma, 2022). Therefore, econometricians might pick up the effect of reallocation instead of the increase of inputs of schools if naively regressing test scores on the treatment. Guided by the above design, I develop a specification to explore the causal effects of competitive pressure on TPSs' increases of inputs.

Before introducing the specification, I formalize the notation and measure of the treatment, outcomes, and other critical controls. I define $Treat_i$ as the number of charter schools within 5 miles of a TPS i such that these charter schools would become HP in 2012. I further define $Post_t$ as an indicator of whether an observation from year t is later than 2011. Therefore, the treatment variable $Post_t \times Treat_i$ switches to positive after the policy and is larger if school i faces more HP charter schools. The outcomes under inspection, A_{igkt}, are the normalized average score of subject k of the student cohort in school i in year t of grade g.[22] Therefore, a triple (i, t, g) uniquely identifies a cohort of students. As motivated by the goal of isolating the effects of student reallocation from schools' increase of inputs, I use the matched cohort test score, $A_{igkt}^{LastYear}$, the last year, average score of the students that construct the (i, t, g) cohort, to control for the student composition changes. Note that the averaging in both years is over the same students, although students in the cohort may not study in school i in the last year. The current and past scores are normalized separately. As I show the results, I introduce the rest of the covariates, subsumed in $\mathbf{Z}_{igkt}$.

With these notations and measurements in hand, I estimate the following difference-in-difference regression (3) to reveal the causal effect on schools' change of inputs when facing more potentially expanding neighboring charter schools, i.e., the HP charter schools. I restrict my primary analysis to TPSs with a charter school within five miles in 2011, which shrinks the full sample of TPSs by one-third. I implement the tests on the full sample as a robustness check. Unfortunately, the matched cohort test scores are no longer publicly available after 2014. Therefore, the analysis of longer-term dynamic effects is prohibitive under the current empirical strategy.

$$\underbrace{A_{igkt}}_{\substack{\text{Cohort}(i,g,t)\\ \text{test score}}} = \beta Post_t \times Treat_i + \rho \underbrace{A_{igkt}^{LastYear}}_{\substack{\text{Same}(i,g,t)\\ \text{Last year test score}}} + \alpha Post_t + \eta Treat_i + \gamma \mathbf{Z}_{igkt} + \epsilon_{igkt} \tag{3}$$

In this specification, β is the parameter of interest. It captures the change in the difference between the average test scores of the TPSs facing more pressure from potentially-HP charter schools and that of the TPSs facing less such pressure after the policy change (conditional on other controls). Under the assumption that trends in unobservable characteristics that affect test scores are the same across TPSs with varying degrees of such pressure, the estimates of β recover the causal effect of the pressure brought by charter schools' potential expansion.

[22]The raw data contain the average test score of the cohort and the enrollment size of the cohort. The normalization is across all schools within the grade-subject-year level, with the enrollment size being the weight of each observation in the calculation.

The results of the tests are shown in Table (5). In the first column, the estimate of β suggests that adding one nearby HP charter school within 5 miles increases test scores significantly by 1.48% standard deviation ("σ" henceforth in this section). The causal effect is not only significant but also considerable compared to existing findings in the studies on TPSs' competitive responses to choice programs.[23] Although both the samples used to identify the competitive response vary across the study, and the identification strategy is different, I postulate that there are several critical reasons why my estimate is higher than the ones in the existing studies. First of all, in my context, the competitive pressure is generated by HP charter schools with satisfactory performance records. They are probably more attractive than normal choice programs discussed in the existing studies. Secondly, the expansion eligibility associated with the HP designation strengthens the potential ability of the HP charter schools to attract students from the neighboring TPSs. More importantly, the expansion eligibility signifies a potential threat in the future. As a neighboring TPS, it might feel the pressure of continuing to lose future students. This finding potentially sheds light on the considerable potential of using expansion eligibility to incentivize charter schools because it might also incentivize the neighboring TPSs to increase effort.

Furthermore, the main treatment effect, although alleviated to 0.82%σ, is still significant and large as I control for more covariates such as fixed effects, match rate of the cohort,[24] school student compositions, the count of charter schools within 5 miles, and pupil-teacher ratio. Additionally, I separately run the tests on math and reading scores with the choice of covariates mimicking column (3) of Table (5). The results are shown in the first two columns in Table (B3). The effects on both subjects are positive and significant, while the effect on reading is higher.

To test whether there were significant pre-policy differences across TPSs with varying HP charter exposure, I run an event-study specification, i.e., replacing the post-policy indicators $Post_t$ with the list of l-year-to-2011 indicators. I include the most covariates as in column (3). Figure (A1) reports the event-study coefficient plot regarding 2011 as the baseline year, I confirm there is no significant pre-policy differential trend of average

[23]For example, Figlio and Hart (2014) find adding one nearby private school increases test scores by only 0.21%σ in using Florida data ranging from 1998 to 2002, also using a difference-in-difference strategy. Figlio et al. (2021), using Florida student-level data from the early 2000 to the late 2010s, show that increasing one charter school within 5 miles increase reading scores by 0.36%σ to 0.98%σ depending on the instruments they use.

[24]The match rate of the cohort measures the proportion of the students in cohort (i, t, g) that also exist in the cohort $(i, t-1, g)$. Following the logic of the analysis, if this number is higher across schools, it means that student reallocation is less intensive across schools and that the students contributing to the average test score of cohort (i, t, g) are more alike with the students contributing to the average score of the cohort $(i, t-1, g)$. This also means the observations with a high match rate support the legitimacy of attributing the causal effect on the test score increase to schools' input increase. I formally test this idea in the robustness check following the main specification.

test score difference across treatment groups as defined. The results also show that the post-policy dynamic effects built up and then alleviated from 2013 to 2014.

Table 5. TPSs' Responses in Test Score to HP Threat

Outcome: Test Score	(1)	(2)	(3)
$Post_t \times Treat_i$	0.0148***	0.0082***	0.0083***
	(0.0017)	(0.0023)	(0.0023)
Constant	0.0098***	37.6187***	32.1766**
	(0.0015)	(13.1661)	(13.1787)
Observations	55,310	55,304	55,304
R-squared	0.8933	0.8972	0.8973
FEs (gt, ig, gk, gt)	Y	Y	Y
Charter Entry + School Demo	N	Y	Y
PT Ratio	N	N	Y

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

I test for the robustness of the findings and conclude results in Table (B3). I first change the measurement of $Treat_i$ and re-run the pre-post specification with the most covariates. In the main specification aforementioned, I use the number of charter schools within 5 miles of a TPS i such that they will become HP in 2012. I construct alternative measures by slightly modifying the original one: 3 miles instead of 5 miles, using indicators instead of count, using the number of A charter schools in 2012 instead of the to-be-HP charter schools.[25] Although the estimates of β vary across measurement choices, they are almost all positive and significant. Additionally, I test whether results vary if I change samples. Firstly, I run the tests on the full TPS sample. This essentially increases the number of observations in the control group because a TPS having no charter school implies that it has no HP charter schools (within five miles in 2011). I design this test to check whether including TPSs in charter "desert" could alter the qualitative results. Because these TPSs might not be as comparable to those treated with high HP charter presence as the ones with some charter schools existing in 2011. However, the qualitative results do not change. Secondly, I exclude the score of a cohort (i, t, g) if it has a lower than 80% or 90% match rate of the cohort. This means the Department of Education can not track 20% of 10% of the cohort's past test scores. These tests examine whether using the data of cohort with less attribution due to reallocation across schools will change the results. The estimates from these tests can be more credibly attributed to the change of inputs in-

[25]Potentially these A schools are candidates for HP charter schools in 2012 and some of them did become HP in 2012 or later years.

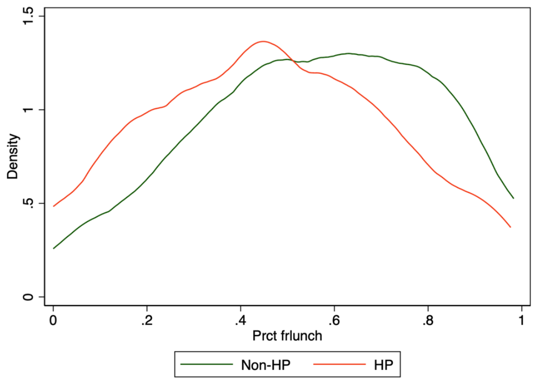
stead of reallocation of students. The results show that, although truncating observations at a 90% match rate of the cohort causes considerable data loss, all the qualitative results maintain. Similar result is found when truncating using 80% as the cutoff. Notably, this way of controlling for students' reallocation is not perfect due to data limitation. Ideally, if student level test score is available, one can largely eliminate the reallocation channel by controlling individual's test score in the last year. With all these robustness checks, I conclude that the competitive pressure of HP charter school neighbors imposed on the TPSs increases TPSs' inputs into education, which raises their test scores.

4.3 Target Whom: Designation Advantages High SES Charter School

As shown in Table (1), HP charter schools appeared more in higher SES regions. This raises a question: Would charter schools that serve low SES regions get designation by exerting higher value-added, reducing the systematic performance difference observed in Table (1) across charter schools? The following figures show that such differences might be systematically rooted in the designation criteria.

Figure (1) shows the density of specific indicators of student compositions within a charter school among all charter schools with higher-than-median value-added in 2015. This figure, therefore, illustrates the distribution of student composition across the non-HP and HP charter schools among charter schools that pay relatively high effort. The two indicators of student composition of a charter school are the percentage of students with free or reduced-price lunches (left) and the percentage of black students (right), the two relatively disadvantaged student groups. From Figure (1), among the higher-than-median value-added charter schools, the non-HP tend to serve poor or black students, as the non-HP density curve of these percentages of disadvantaged students is on the right of the HP's curve. The reason could be that the designation criterion, namely "2A1B", relies heavily on the *level* of academic performance of charter schools, less on the *value-added*. This favors charter schools in high SES regions where their students come from more educated families.

This raises a concern about whether the policy could lead to unequal allocation of expansion eligibility, which might result in unequal access to high-quality charter school seats across regions with different SES. Giving charter schools serving the low SES regions with high value-added the opportunity to expand might help reduce the inequality of high-quality charter programs across regions.



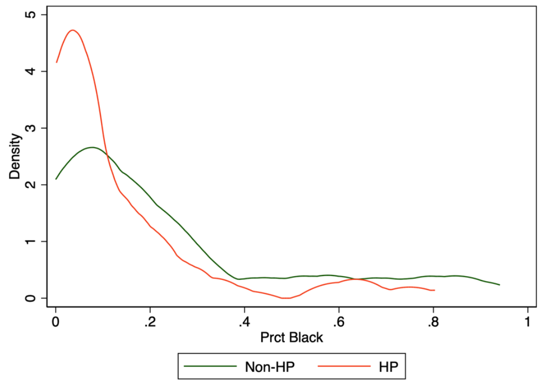


Figure 1. Density of Student Composition Across Higher-than-median Value-added Charter Schools in 2015

5 Quantitative Model

In this section, I develop an empirical model that characterizes charter and TPSs' supply of performance and capacity in a dynamic framework. I built the model based on the dynamic oligopoly model developed by Ericson and Pakes (1995) and adapted it to feature the education market and policy context of Florida.

In each period, schools endogenously expand capacity or improve their performance (or both) to maximize their long-term objectives. They make decisions according to their own capacity, performance level, and other time-varying characteristics of themselves and the schooling market. In the schooling market competition stage, students choose schools according to these schools' characteristics. Because schools' adjustments of capacity and performance are costly, they consider a trade-off between the lasting benefits of having higher performance and larger capacity (so as to enroll more) and the adjustment costs involved in both decisions. In addition, charter schools can earn the HP designation by accumulating good performance and can reduce the cost of adjusting capacity. Therefore, the model links the time-variant operating environment with schools' two key choices and links them with the policy (via the modeling of HP designation) and competitive environment schools face. The model, therefore, can predict schools' endogenous reaction to the change of adjustment cost and competitive environment brought by the HP policy, as informed by the preliminary data patterns.

In what follows, I first describe environment in 5.1. I then describe a simplified demand model in 5.2 to streamline the subsequent introduction of schools' state space, dynamic programming problems and equilibrium in 5.3. I analyze the core mechanisms of the model in 5.4 and conclude by introducing the more realistic demand model used in the estimation and simulation in 5.5.

5.1 Environment

The building block of this model is a regional schooling market. Time is discrete and unbounded, denoted as $t \in \{1, 2, 3, ...\}$. Each decision period of a school is thought of as a school year. A school is denoted as j. The number of the operating schools J is assumed to be constant overtime and schools in the market do not expect entry, exit, or change in ownership. Hence, I also use J to denote the set of schools. Schools are heterogeneous with respect to their school-specific state x_{jt}, which I will discuss its contents throughout this section. Then, the market that school j faces can be characterized by:

$$s_t = ((x_{jt})_{j=1\sim J}, m_t, n_t).$$

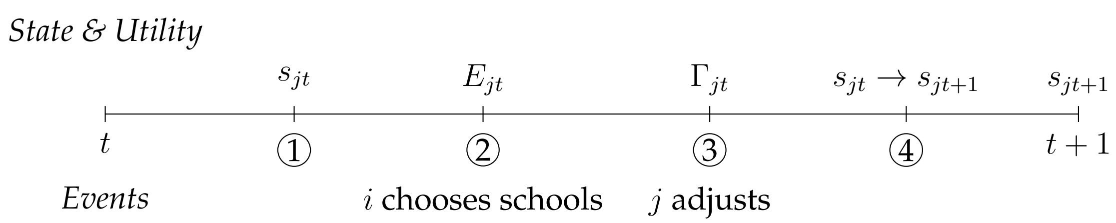
The market size m_t characterizes the total number of students in the market. The market situation state, n_t, is a function of all schools' own state, i.e., $(x_{jt})_{j=1\sim J}$, that summarizes how school j's utility in period t is influenced by other competing schools' and its own states. Therefore, I denote s_{jt} as:

$$s_{jt} = (x_{jt}, m_t, n_t),$$

to represent all the information j needs to keep track of overtime to make decision. I introduce the functional form used to construct n_t in greater details in the demand subsection.

In each period, the sequence of the events happen as follows, as shown in Figure (2).

Figure 2. Timing of the Events in the Model



Firstly, s_{jt}, is known to school j. Secondly, students choose school j according to s_{jt}, resulting in enrollment E_{jt}, which, together with s_{jt}, brings parts of the utility j gets in t. Thirdly, school j adjusts its own states by expanding or exerting effort (or both) that incur adjustment costs Γ_{jt}, which determine the rest part of the flow utility of j. Fourthly, the state s_{jt} evolves to its new level s_{jt+1}. Particularly, x_{jt} evolves to x_{jt+1} according to j's adjustment decisions and exogenous state transition rules, and the market state n_t evolves to n_{t+1} according all j's decisions.

5.2 Demand and Competitive Pressure

In this subsection, I illustrate students' school choice using a Logit model for the ease of exposition. This implies that students in the market have no difference in taste in school characteristics. I explain the deviation to a more realistic demand in 5.5.

In the market, a representative student i can choose among schools $j = 0, 1, 2, 3..., J$, where $j = 0$ indicates the option of homeschooling or attending private schools. I specify the student's utility of enrolling in a school j in time t by the following indirect utility w_{jti}:

$$w_{jti} = \delta(x_{jt}; \alpha) + \zeta_{jti}.$$

The above specification characterizes students' utility by the sum of school j's mean utility $\delta(x_{jt}; \alpha)$, which is a function of x_{jt}, and students' taste shock ζ_{jti}. I assume ζ_{jti} to have i.i.d. Type-I Extreme Value distribution.[26] The taste parameters in the mean utility $\delta(x_{jt}; \alpha)$ are summarized by α. The outside option is assumed to have zero mean utility: $\delta(x_{0t}) = 0$. Note that I allow for capacity, a critical component in x_{jt}, to influence j's enrollment. I explain the functional form of the mean utility $\delta(.)$ and the contents in x_{jt} relevant for characterizing schooling demand in the estimation section.

The distribution of ζ_{jti} implies that, given that the market has m_t students, the enrollment of school j in t, denoted by E_{jt}, is

$$E_{jt} = E(x_{jt}, n_t; \alpha) = m_t \cdot \frac{\exp(\delta(x_{jt}; \alpha))}{1 + \exp(n_t)}, \tag{4}$$

where

$$n_t = \log \left(\sum_{j' \in J} \exp(\delta(x_{j't}; \alpha)) \right). \tag{5}$$

I refer to n_t henceforth as the "competitive pressure" faced by school j from its neighbors in time t. As the form shows, if J is relatively large, it captures j's belief about the inclusive value of the mean utility of its competitors. As J or some competitor's mean utility becomes larger, school j believes its market share shrinks more.

5.3 Schools' Dynamic Programming Problems

State Space Schools are heterogeneous in rich dimensions. In each period t, the states that j needs to keep track of, namely s_{jt}, consists of the following:

[26] The i.i.d. assumption reflects the random variations in the students' schooling experience that do not persist across time because of schools' culture, security concern, atmosphere, and so forth. Additionally, I assume there are a large number of students. Hence, schools predict perfectly about their market share based on their states. Therefore, ζ is not assumed to be a state variable for schools.

$$s_{jt} = (x_{jt}, m_t, n_t) = (o_j, q_{jt}, k_{jt}, hp_{jt}, d_{jt}, \xi_{jt}, \epsilon_{jt}, m_t, n_t).$$

I introduce below each component of the state x_{jt}. Except for the ϵ_{jt}, the unobserved heterogeneity, all other state variables are observable to the econometrician. The ϵ_{jt} are distributed i.i.d (across schools and periods). It captures the unobserved heterogeneity of schools' adjustment costs and hence allows for gaps between the model-predicted and observed decisions of schools. I introduce the economics of ϵ_{jt} with greater details, along with introducing schools' adjustment process.

The time-invariant state o denotes the school type, either charter or traditional. Since the government regulates TPSs and charter schools differently, their decision-makers are distinct in objectives and the tools that they can use to influence schools' development. Therefore, by breaking into two types of schools, the model allows school types to govern different constraints on schools' state space, action space, and objectives. Accordingly, all the parameters in the following are allowed to be different by type and, hence, are estimated separately for each type.

The state variables q, performance, and k, capacity, influence the school's enrollment, a component of both types of schools' objectives. The state variable hp, HP designation status, influences the adjustment cost of charter schools' capacity. The states q, k, and hp are the core endogenous states directly influenced by a school's decisions in the school's own state x.

The state variable d characterizes the demographic heterogeneity the school faces. They represent schools' local demographics, such as income. Mainly, this allows charter schools' operating costs to vary across demographics, as previous literature found (Singleton 2019). Since I do not model schools' entry and exit decisions, d is assumed to be exogenous, independent of schools' decisions.

The state variable ξ, the underlying quality of schools, characterizes a one-dimensional level of time-varying quality shock a school gets in each period, assumed to be independent of observable schools' states. It is assumed to be observable to students and econometricians, and in empirical implementation, I recover it from demand estimates.

School's Flow Utility and Adjustment Decisions Schools make two adjustment decisions to maximize expected utility over time. The two decisions are educational effort, v, and capacity expansion e. Particularly, the decision v_{jt} represents schools' decisions on value-added. It is a uni-dimensional variable summarizing all the schools' inputs that are particularly invested in improving students' test scores. It can include spending on the professional development of teachers, teacher coaches, better leadership, and administrative support. The decision e_t represents the school's extra capacity to expand (or shrink) in period t.

Charter schools are allowed to make both decisions, while TPSs in this model are assumed to have a fixed capacity, i.e., $e_{jt} = 0, \forall t$, and can only decide on value-added. I make this assumption because I do not get complete and high-quality capacity data of TPS. With my limited data, TPSs do not change the enrollment capacity frequently or by a large proportion over time. In the empirical implementation, I impute their capacity using their in-sample largest enrollment divided by a constant to measure their capacity. [27] Decisions of adjustment are defined as the mappings from states to actions:

$$v : (s_{jt}) \to v_{jt}$$
$$e : (s_{jt}) \to e_{jt}.$$

These adjustments are costly and jointly influence all the endogenous variables, hence the flow utility.

I assume charter schools operate as for-profit organizations.[28] Their flow utility u_{jt} has the following form:

$$u_{jt} = rE(s_{jt}) - \Psi(E_{jt}, s_{jt}) - \Gamma(v_{jt}, e_{jt}, hp_{jt}, \epsilon_{jt}).$$

Enrollment $E(s_{jt})$ is a function of the state variable. It summarizes the demand side of the schooling market. Hence, $rE(s_t)$ represents the total revenue charter schools get from en-

[27] Although I do not get the universal capacity data for TPSs in my sample, I manage to get long panels of TPSs' capacity in Lee and Palm Beach counties, measured by student station. I find that most of the change in capacity is either zero or not empirically relevant in magnitude. Take TPSs from the Palm Beach County as an example. From 2011 to 2020, over the 2659 observations of annual change of capacity compared to the previous year, 85% show zero change, and 5% shows less than 1% change (compared to the previous year's capacity). A similar qualitative conclusion can be found in inspecting Lee County's panels. Potentially, one can digitize the Annual Five Year Plan document published by the local school districts to obtain all TPSs' capacity. However, the document does not provide the unique school ID number. Moreover, it does not use the same name as the school that appeared in NCES or Florida Master File data, making the exact merge across datasets almost impossible. Based on the data I can digitize and merge, I conclude that the facility in TPSs does not frequently change over time.

[28] Although all charter schools in Florida operate as non-profit organizations, around 40% to 50% of charter enrollment is in charter schools that sign contracts with private management companies to operate the daily business in my sample from 2012 to 2019. The pressure of making a profit may come from payments to these private companies. I label these charter schools as for-profit, which is aligned with Singleton's (2017; 2019) definition. In Singleton (2019), the paper also defines two types of charter schools: the "no-excuses" and the "other" charter schools. The "no-excuses" charter schools follow an educational philosophy emphasizing high expectations, comportment, and traditional math and reading skills. The rest are in the "other" category. Using his definition of the labels, I discover that in my sample, the "no-excuses" and the "other" charter schools account for around 15% and 35% charter enrollment in recent years. According to Singleton (2019), no-excuses charter schools are considerably less sensitive to variable costs and large enrollment than the other two types of charter schools. Although the no-excuses charter schools have different objectives, given their relatively lower market share, I focus on the other two types of charter schools that operate in Florida. Furthermore, I do not distinguish the "other" type of charter schools from the for-profit ones in their objectives, mainly because the heterogeneity of charter schools is not the paper's focus. In future versions of the paper, I can allow these two types of charter schools to have distinct model primitives.

rolling $E(s_{jt})$ students. In practice, charter schools get the revenue from the government according to a per-enrollment reimbursement rate r,[29] which is known to the econometricians. The function $\Psi(.)$ captures the variable cost of maintaining daily operation and instruction, e.g., teachers' salary, rent, staff compensation, and maintenance. The functional forms of $E(.)$ and $\Psi(.)$ will be exploited during estimation. The $\Gamma(.)$ represents the adjustment costs charter schools pay to change future capacity and performance.

As for TPSs, I assume they operate as non-profit organizations. Their flow utility is a weighted sum of enrollment, performance, and the adjustment cost of improving performance, different from charter schools:

$$u_{jt} = r^E E(s_{jt}) + r^q q_{jt} - \Gamma(v_{jt}, \epsilon_{jt}).$$

In this specification, r^E and r^q indicate the relative weight of enrollment $E(s_t)$ and performance q_t. They are enumerated in terms of the TPSs' valuation of adjustment cost. This reflects the principal's objective in maintaining the position: if the school constantly performs badly or not enough students attend the school, its principal can be fired. I assume econometricians know r^E and r^q because these two parameters can not be separately identified with the adjustment cost using the TPSs' value-added decisions.[30] I calibrate both parameters according to Mehta's (2017) structural estimates with slight modifications.

The most critical component in both types of schools' flow utility is their adjustment costs. For charter schools, I model their adjustment cost function as follows in (6):

$$\begin{aligned}\Gamma(v_{jt}, e_{jt}, hp_{jt}) = \gamma_v v_{jt} + 1_{\{e_{jt}\geq 0\}} \Bigg(& \overbrace{\gamma_1 + \underset{\substack{\uparrow \\ \text{HP effect}}}{\gamma_2} \cdot \; hp_{jt}}^{\text{Fixed Costs}} + \overbrace{\gamma_3 e_{jt} + \underset{\substack{\uparrow \\ \text{HP effect}}}{\gamma_4} \cdot \; e_{jt} \cdot hp_{jt}}^{\text{Variable Costs}} \Bigg) \\ & + 1_{\{e_{jt}<0\}} \gamma_5 e_{jt}. \end{aligned} \tag{6}$$

The γ_v captures the per-unit cost of value-added that comes from all inputs a school spends in improving students' scores. The per-unit cost of capacity change is captured by γ_3. It includes spending on purchasing furniture, hiring designers, and building extra classrooms. I also consider fixed costs of increasing capacity as indicated by γ_1. Introducing the fixed costs rationalizes the lumpiness in adjustment in capacity, as observed in the data. Furthermore, γ_1 also captures the reality that capacity adjustment is associated with hiring lawyers to negotiate and re-contract with the government, regardless of the amount of capacity expected to change. Furthermore, the HP policy is modeled

[29] Although the per-enrollment reimbursement rate evenly changes across years, I do not model it as so for convenience.

[30] For example, low adjustment cost of exerting value-added or putting priority in getting high performance can both generate high value-added decisions.

as potentially influencing both fixed and variable expansion costs for designated charter schools via γ_2 and γ_4. Finally, I allow the unobserved heterogeneity ϵ_{jt} to influence charter schools' adjustment cost of expansion. This heterogeneity exists because charter schools execute different modes of expanding capacity, which can involve different costs. For example, charter schools could renovate five classrooms with new facilities or add a floor to their existing building with five classrooms. The former is usually less costly. In the data, I do not observe the mode of expansion. Therefore, I model the expansion decisions to depend on the unobserved heterogeneity to rationalize the discrepancy between the policy function estimated and charter schools' expansion data. Therefore, for both γ_1 and γ_3 , I assume they are drawn each period from normal distributions that are common across all charter schools in all years.

Since I do not allow TPSs to alter their capacity in the model, the adjustment cost functions for TPSs are simply:

$$\Gamma(v_{jt}) = \gamma_v v_{jt} \tag{7}$$

State Transitions of Individual States Capacity evolves in a deterministic way. Future capacity is a sum of current capacity and expansion

$$k_{jt+1} = k_{jt} + e_{jt}.$$

Performance evolves according to the current performance and the value-added into the next period performance, captured by the function: $\tau(.)$:

$$q_{jt+1} = \tau(v_{jt}, q_{jt}).$$

In the application, this corresponds to the following production process of academic performance: Students come and perform in standardized tests, earning the school a rating of q_{jt} in period t. Moreover, the school decides to put in v_{jt} amount of value-added to promote students' academic performance in $t + 1$, resulting in schools earning q_{jt+1}. This transition rule applies to both charter and TPSs.

Designation of charter schools evolves as a function of period t's performance level and the HP status, namely:

$$hp_{jt+1} = \eta(q_{jt}, hp_{jt}).$$

I regard hp_{jt+1} as a passively evolving endogenous variable that is unaffected by decisions *directly*. This assumption reflects the nature of the statute that designation is not dependent on value-added directly. I also abstract away from the actual policy, which requires three years of satisfactory performance, by assuming that the determination of future designation depends on current performance to avoid unnecessary complications.

In practice, predicting future designation with only current performance and designation has acceptable accuracy. Additionally, as shown by the data, de-designation happens extremely infrequently. Therefore, I set $hp_{jt+1} = 1$ if $hp_{jt} = 1$.

For the rest of the components in a school's individual states, namely d_{jt} and ξ_{jt}, I assume they all independently follow AR(1) processes.

State Transitions of Market States As is introduced, n_t summarizes the competitive pressure j faces, and by allowing schools' decisions to depend on n_t, the model, therefore, allows competition as a core mechanism that influences the dynamics of schools. However, without adding further assumptions, the state n_t is potentially high-dimensional. Because there are potentially many schools competing in a market, and they are heterogeneous in rich dimensions. This "curse of dimensionality" imposes a challenge in computing the MPNE, i.e., Markov Perfect Nash Equilibrium (Ericson and Pakes 1995), in this model. I, therefore, make a simplifying assumption on the competitive pressure n_t to facilitate computation.

The assumption, "Inclusiveness", is made to make schools' decisions dependent on other schools' states only via a uni-dimensional state variable n.

Assumption "Inclusiveness". *Each school's belief in its demand is represented by equation (4) and is summarized by states characterizing schools' own operational status x and a uni-dimensional state n characterizing the competitive pressure imposed by their neighbors in the market.*

This assumption dramatically reduces the dimensionality of the state space for a school. It also implies that schools have limited cognitive ability to track all their competitors' states over time to predict their future enrollment. This setting still preserves schools' competitive responses by allowing their decisions to depend on their beliefs on the time-varying competitive pressure via summary statistics of the "attractiveness" of other competing schools. This modeling device and the formula generated from a demand model are shared by other industrial organization research using a dynamic model (Hendel and Nevo 2006; Gowrisankaran and Rysman 2012) and static models used in the economics of education setting (Sanchez 2018; Dinerstein et al. 2022).

Following the "Inclusiveness" assumption, I continue to make another critical assumption on how schools form beliefs about the n_t's evolution, denoted as $\nu(.)$.

Assumption "Consistent Belief". *Each school forms a rational expectation that $\nu(.)$ is an autoregressive process with one lag, i.e., AR(1), and its belief is consistent with what it believes about how the market would evolve when the school itself and its competitors make optimal dynamic decisions given their beliefs $\nu(.)$.*

This assumption requires that schools have no strategic consideration about n_t (i.e., they believe their own decisions do not directly change n_t) and that their beliefs on n have to be consistent with how the market evolves. This assumption is established to allow schools' beliefs about the competitive environment to change under the alternative supply-side policy. Think of a simulation exercise in which the econometrician expects to test the value-added response by TPSs under a counterfactual policy. The policy does not allow the HP designation system to exist and imposes more constraints on the extent to which charter schools can expand. Even though the traditional sector is not directly targeted, they should predict a less "aggressive" expansion of neighboring charter schools under this counterfactual environment. Then, the "Consistent Belief" assumption hence allows schools to alter belief in a way that is consistent with their perceived change of how the market evolves. Under this assumption, the model can, therefore, make a more realistic prediction. This assumption, although it adds empirical relevance to the model, requires jointly considering schools' optimal decisions according to the dynamic programming problems but also how their beliefs about the market environment change. This implies an iterative algorithm to find a fixed point of $\nu(.)$ that satisfies the above assumptions. More details of the computation algorithm are explained in section 8.

Based on these assumptions, I introduce the dynamic programming problems faced by both types of schools and the equilibrium concept.

Schools' Dynamic Programming Problem and Equilibrium With all model components specified, the maximization problem faced by a charter school is summarized by (8). I denote β as the discount factor. I omit subscript j.

$$\begin{aligned} V(s_t) &= \max_{v_t, e_t} \; rE_t(s_t) - \Psi(E_t, s_t) - \Gamma(v_t, e_t, s_t) + \beta \mathbb{E} V(s_{t+1}|s_t) \\ s.t. \quad & q_{t+1} = \tau(v_t, q_t), \; k_{t+1} = k_t + e_t, \; hp_{t+1} = \eta(q_t, hp_t), \\ & d_t, n_t, \xi_t \sim AR(1), n_t \text{ transition satisfies Consistent Belief} \\ & \epsilon_t \sim i.i.d. \end{aligned} \tag{8}$$

The maximization problem faced by a TPS is summarized by (9).

$$\begin{aligned} V(s_t) &= \max_{v_t} \; r^E E(s_t) + r^q q_t - \Gamma(v_t, s_t) + \beta \mathbb{E} V(s_{t+1}|s_t) \\ s.t. \quad & q_{t+1} = \tau(v_t, q_t), k_{t+1} = \bar{k}, hp_{t+1} = 0, \\ & d_t, n_t, \xi_t \sim AR(1), n_t \text{ transition satisfies Consistent Belief} \\ & \epsilon_t \sim i.i.d. \end{aligned} \tag{9}$$

I define the equilibrium below to close the model. To facilitate exposition, first, denote

z as a school's strategy, i.e., $z = (v(.), e(.)) \in Z$. And define the expected value function implied by each school's own ($\tilde{z}$) and other schools' strategy (z) as

$$\bar{V}_{\tilde{z},z}(s) = \mathbb{E}_{\epsilon} V_{\tilde{z},z}(s) = \mathbb{E}_{\epsilon} \left[\max_{\tilde{z}(s)} \pi(s) - \Gamma(s, \tilde{z}(s)) + \beta \mathbb{E}_{\tilde{z},z} V(s'|s) \right]$$

Definition. *An equilibrium of a market is characterized by a strategy z such that:*

1. *(Optimality) z satisfies the optimality condition. That is, for every state $s \in \mathcal{S}$, for every school,*

$$\sup_{\tilde{z} \in Z} \bar{V}_{\tilde{z},z}(s) = \bar{V}_{z,z}(s).$$

2. *(Consistent Belief) Each school forms rational expectation on the perceived transition, $\nu(.)$, of competitive pressure n, s.t.: $\nu(.)$ is consistent with how the market evolves based on this belief. That is,*

$$\tilde{\nu}^z(.) = \nu(.),$$

where $\tilde{\nu}^z(.)$ is the transition of n when all schools play strategy z.

This equilibrium concept and the implied iterative algorithm used in this model are similar to the Moment-based Markov Equilibrium (Ifrach and Weintraub 2017) in which agents' strategy is assumed to depend on summary statistics of the distribution of other agents' states. The Moment-based Markov Equilibrium, along with other equilibrium concepts following the seminal work by Weintraub et al. (2008), are the attempts to address the computation burden created by using MPNE as the solution concept of a dynamic game.

5.4 Analysis of Mechanisms

The model captures two key mechanisms that govern schools' decisions: incentives in adjustment and competition.

Firstly, I explicitly model the adjustment costs to influence schools' intertemporal decisions. Adjustments are costly at the moment yet beneficial to future revenues. Further, the model introduces the HP designation hp in the adjustment cost function of charter schools. This design enables the evaluation of the direct policy effect. In one of the counterfactual simulations, I compare the observed outcomes of interest to the predicted ones when the designation-related benefits and transitions are eliminated from the model. Conceivably, under the existing policy, non-HP charter schools can accumulate high performance to change their HP status in future periods, thereby reducing the adjustment cost for expansion. The existing HP scheme naturally interacts with the two decisions of charter schools. Furthermore, since investing in the effort has lower future costs for expansion, the effort choices of charter schools can be influenced accordingly.

Secondly, because competitive pressure n enters the demand function, schools' decisions can respond to local competitive pressure from neighboring schools. These responses can be further influenced in the future according to how schools believe about the evolution of their competitive pressure. More importantly, incorporating competitive responses is crucial in quantifying the effects of large-scale counterfactual policies, such as deregulating all charter schools. Such policies will likely change schools' beliefs about the evolution of the competitive pressure they face. To properly characterize how schools change a belief about the evolution of their competitive pressure, the "Consistent Belief" assumption is critical.

Finally, the model allows for decisions of both charter and TPSs to be responsive to rich demographic heterogeneity d. Particularly, $\Psi(.)$ can depend on local demographics. I address that this heterogeneity is also important in policy evaluation. As is also shown in the data, educating students with low SES can involve higher operational expenditure per enrollment. Modeling the dependence on local conditions can help evaluate the heterogeneous responses of different schools that operate in various demographic conditions across different regions. Specifically, to evaluate whether an alternative policy that gives more expansion eligibility to charter schools in low SES regions needs scrutiny of the estimates of the operating cost function. Such policy may not trigger charter expanding capacity as expected if charter schools operating in these regions do not want to expand due to the high operation cost.

5.5 Extension to Heterogeneous Agent Demand

According to Florida's open enrollment policy, a student can enroll in any charter or TPS in Florida. Therefore, allowing for spatial heterogeneity, especially the distance to school, in the indirect utility of schooling is crucial in characterizing school choices (Neilson, 2013; Agarwal and Somaini, 2018; Allende, 2019; Dinerstein et al. 2022; Gilraine et al. 2022). Following this idea, the spatial demand model with no market boundary and consumers having their own choice set (Holmes, 2011; Zheng 2016; Ellickson et al. 2020) serves as a more proper benchmark than the Logit model introduced in 5.2. In the subsequent empirical implementation, I, therefore, use a spatial demand model to characterize students' choices.

All the set-up in the Logit demand about the regional schooling market maintains in this setting. The core difference is that students are heterogeneous in their residential locations. For illustration, assume the market is constructed by two census tracts: $L = l_1, l_2$, which has respectively $m_{m_l 1}$ and m_{l_2} demand size, and distance $dist_{jl_1}$, $dist_{jl_2}$ to a school j. Assume these spatial characteristics are time-invariant for simplicity. Then, similar to before, the utility for a student i located in l receive $w_{ijlt} = \alpha x_{jt} + \lambda dist_{jl} + \zeta_{ijlt}$

utility. Imposing i.i.d. Type I Extreme Value distribution on ζ_{ijlt}, one can derive

$$E_{jt} = \sum_{l=l_1,l_2} m_l \cdot \frac{\exp(\alpha x_{jt} + \lambda dist_{jl})}{1 + \left(\sum_{j' \in J} \exp(\alpha x_{j't} + \lambda dist_{j'l})\right)}. \tag{10}$$

Note that using a similar inclusive value formula to define competitive pressure, namely $\log\left(\sum_{j' \in J} \exp(\alpha x_{j't} + \lambda dist_{j'l})\right)$, still introduces high computational costs. Because a school needs to keep track of the l-specific competitive pressure and other information (e.g., market size and distance) for each location. As the number of such relevant locations increases, the number of states in schools' state space grows accordingly. Therefore, I use an alternative formula for characterizing the market situation state. Note that,

$$\begin{aligned} E_{jt} &= \sum_{l \in L} m_l \cdot \frac{\exp(\alpha x_{jt} + \lambda dist_{jl})}{1 + \sum_{j' \in J} \exp(\alpha x_{j't} + \lambda dist_{j'l})} \\ &= \exp(\alpha x_{jt}) \cdot \sum_{l \in L} m_l \cdot \frac{\exp(\lambda dist_{jl})}{1 + \sum_{j' \in J} \exp(\alpha x_{j't} + \lambda dist_{j'l})}, \end{aligned}$$

therefore, $E_{jt} = \exp(\alpha x_{jt}) \cdot \bar{n}_{jt}$ where,

$$\bar{n}_{jt} = \sum_{l \in L} m_l \cdot \frac{\exp(\lambda dist_{jl})}{1 + \sum_{j' \in J} \exp(\alpha x_{j't} + \lambda dist_{j'l})}.$$

By imposing the same set of assumptions on $\bar{n}_{jt}$, the model is still computationally tractable. In this case, the formula conveys the idea that, for the market situation state that j needs to keep track of overtime, j will put the weight on each location according to its characteristics. For example, if a location is far away, it is less important to influence j's enrollment if λ is negative. This inclusive value, therefore, internalizes richer information and still possesses the functionality the previous "competitive pressure" has. Therefore, in empirical implementation, I use this inclusive value to estimate the model and conduct a counterfactual simulation. I introduce more implementation details as I advance to the estimation and counterfactual simulation parts of the paper.

6 Empirical Strategy

This section first introduces the two-step estimation strategy. Then, it presents measurements, estimation samples, and empirical specifications, with a particular focus on the demand. And then, it continues to introduce the identification of the adjustment cost function.

6.1 Overview of Estimation Strategy

I calibrate the reimbursement rate r and the utility weights for TPSs (r^E, r^q) directly from Florida laws and Mehta (2017), respectively. For charter schools, the per-enrollment reimbursement rate r is set to be \$8000 a year.[31] For TPSs, I calibrate the utility weights according to Mehta's (2017) structural estimates. In the paper, enrollment is set to be the numéraire, and his estimates show that TPSs put weight 19.634 on their average test scores. Therefore, I set $r^q = 20 * r^E$, approximating Mehta's (2017) results. I further set $r^E = r$. This is an innocuous assumption as long as the ratio between r^q and r^E is appropriate. Setting $r^E = r$ not only reflects that charter and TPSs are reimbursed under the same formula,[32] but it also makes the estimates in the adjustment costs for value-added between charter and TPSs comparable. The discount rate β is set to be 0.9.

I use the simulation-based algorithm developed by Bajari et al. (2007), henceforth referred to as BBL, to estimate the structural parameters. These include the enrollment function $E(.)$, operating cost function $\Psi(.)$, adjustment cost function $\Gamma(.)$, and all the transition functions. BBL propose a two-step procedure that avoids directly solving the policy functions of the agents in conducting estimation.

In the first step, I use appropriate functional forms to estimate the demand, operating cost, policy functions, and transition functions. In this step, I characterize the agents' decisions and flow utility as functions of the state variables. In the second step, I use the estimated policy functions in the first stage, denoted as $\hat{v}(.)$ and $\hat{e}(.)$, and their perturbed versions $\tilde{v}(.)$ and $\tilde{e}(.)$ to compute the expected discounted sum of the flow utility for large enough periods T. The estimator will search for the parameter $\hat{\Gamma}$ of the adjustment cost function $\Gamma(.)$ that minimizes the profitable deviations with perturbed policy functions $(\tilde{v}_j(.), \tilde{e}_j(.))$ from the optimal policies estimated in the first stage:

$$\hat{\Gamma} = \arg\min \sum_j \sum_i \min\{0, \bar{V}(s_{i0}; \hat{v}(.), \hat{e}(.); \hat{\Gamma}) - \bar{V}(s_{i0}; \tilde{v}_j(.), \tilde{e}_j(.); \hat{\Gamma})\}^2, \tag{11}$$

where

$$\bar{V}(s_{i0}; v(.), e(.), \hat{\Gamma}) = \frac{1}{NS} \sum_{ns} \sum_{t=0}^{T} \beta^t u(s_{it}; \hat{\Gamma}) \text{ s.t. } v(.) \text{ and } e(.) \text{ governs the evolution of } s_{it}.$$

[31] I choose this per-enrollment reimbursement rate to approximate \$8143, a number provided by the latest state budget release (for a source, see Florida Charter School Alliance's report). Note that the per-enrollment reimbursement rate tends to increase evenly every year. Therefore, the actual rates during my sample period might be below this number.

[32] According to Florida law, charter schools are funded through the Florida Education Finance Program in the same way as all other public schools in the school district. The charter school receives operating funds from the Florida Education Finance Program (FEFP) based on the number of full-time (FTE) students enrolled. Notably, a recent study (DeAngelis et al. 2020) finds charter schools receive less reimbursement compared to TPSs in states that apply this equal-reimbursement law. Therefore, accounting for this might raise the estimate for TPSs' adjustment cost of value added.

Here, i denotes a specific initial state randomly picked, and j indexes a perturbed policy function that slightly and randomly changes the actions predicted by $\hat{v}(.)$ and $\hat{e}(.)$. Note that an ns indexes a simulation and signifies that the goal is to get the *expected* discounted sum. I estimate charter and TPSs separately, following the same procedure.

6.2 Measurement

In Table (B4), I integrate the measurement of relevant variables in the model and their coverage of years and schools. Unless specified otherwise, all measures are available throughout the sample period. In the model, a period corresponds to a school year, where the label for the year follows a format where the 2013-2014 school year is labeled as $t = 2014$. Each school in the dataset is identified by a unique school ID. I highlight several measurement assumptions below. I calculate the average teacher value-added score within a school to measure educational effort. I consider the accountability score in the last year of t as the performance state variable in t. This choice is motivated by the fact that schools and students are unaware of the schools' accountability scores for the upcoming school year during the recruitment season of the previous year. Hence, the accountability score in the previous year is a more suitable measure variable for the contemporaneous performance state.[33] As for the capacity measure of TPSs, although I do not have the number of classrooms directly, I impute a TPS's capacity using the largest enrollment observed in a school divided by 22. Because TPSs are not often capacity-constrained and are subject to a regulated middle school class size of 22 students per class. For all information from the American Community Survey, I particularly use its 5-year Data Profile, where the middle year of the 5-year data serves as the year label for a certain variable. For the measurement of all variables related to the demand estimation, I leave them as I introduce the demand estimation.

6.3 Estimation Sample

The sample used for structural estimation consists of a selected set of charter and TPSs. First, for both types of schools, I exclude those that only run grades from K-2 for most of the sample period, those with a short sample length, or schools with a small average enrollment per grade. These exclusions are necessary because the excluded schools may have objectives that differ significantly from the rest. Moreover, they are systematically more likely to have missing variables. For example, schools that constantly run K-2 do not

[33] Here is an example: The enrollment of $t = 2012$, i.e., the school year 2011-2012, is determined in the recruitment season of 2011, in spring. At that time, students did not know the schools' accountability scores for the upcoming 2011-2012 school year starting 2011 in the summer. Therefore, a more appropriate measure for the state variable of performance level is the accountability score 2011, which has been made public to schools and students since the start of the 2010-2011 school year.

participate in standardized tests and hence do not have a reliable source of performance evaluation.

Particularly for charter schools, I exclude observations from specific charter schools. When estimating the policy functions of charter schools, I only include observations from charter schools that have been operational for more than three years. This selection criterion aligns with the model's focus on characterizing the relatively mature operation of charter schools after their entry. Additionally, the expansion in a charter school's early life cycle is predetermined and negotiated prior to entry, independent of post-entry factors such as designation and performance level. Therefore, including observations from this period would not be appropriate.

When it comes to TPSs, I select the set of schools used in showing the main results of the difference-in-difference analysis. That is, all the TPSs that had no charter schools within 5 miles in 2011 are excluded from the structural estimation. Since the model allows both types of schools to respond to competitive pressure endogenously affected by the policy change, for TPSs with no charter competitors in a reasonably large neighborhood, it is less suitable to characterize their behaviors in such a competitive environment in the model.

Finally, I choose post-policy observations to estimate the structural model. [34] As the model requires, all schools are assumed to know the existence of the HP designation system, and their belief about its existence remains unchanged. Therefore, the post-policy period is more suitable for estimating the model, particularly because the operation of the designation system is commonly known during this period and undergoes minimal changes.

I compare the charter and TPS samples in conducting preliminary data analysis and estimation in Table (B1) in the appendix. In the end, around ten thousand charter and TPS observations exist in the structural estimation from 28 districts.

6.4 Empirical Specification

In this subsection, I introduce the empirical specifications used in the estimation of the offline functions and the policy functions.

Demand Function $E(.)$ and Demand-based Measures ξ and n. Florida has 67 school districts, whose sizes are similar to counties. I assume students do not travel across districts to choose schools. Hence, I define a market by a school district-year pair and mea-

[34] There are exceptions in which I also include pre-policy data in estimation to get more statistical power in implementation. For example, I estimate the operating cost of charter schools using all data without conditioning on the HP status. I essentially assume the operating cost does not depend on the belief about the designation system.

sure market size by the total number of public and private enrollment in a district of a year.[35] I regard each location, i.e., l, in the model as characterized by a census tract. The crow-fly distance between a census tract centroid and a school hence measures travel distance to school. Accordingly, m_l, the demand size of a census tract l is then measured by the total number of K-8 students. Since the district market size and the census tract demand size come from different data sources, I apply Ferreyra and Kosenok's (2018) method to moderate the tract demand size data.[36] Essentially, I impose that the sum of the tract demand size of all tracts in a market is equal to the market size constructed by adding up all the charter, traditional, and private school enrollment of the market. I show the total number of schools by type, population density, and the average household income for each district in 2015 in Table (B2) in the appendix.

Following the notation in the model, I use the specification in (12) to represent w_{ijlt} :

$$\begin{aligned} w_{ijlt} &= \delta(s_{jt};\alpha) + \lambda dist_{jl} + \zeta_{ijlt} \\ &= \alpha_1 ClassSize_{jt} + \alpha_2 \left(ClassSize_{jt} \cdot o_j\right) + \alpha_3 q_{jt} + \alpha_4 o_j + \xi_{jt} + \lambda dist_{jl} + \zeta_{ijlt}, \end{aligned} \tag{12}$$

where $ClassSize_{jt}$ is defined by the enrollment per classroom, i.e., $\frac{E_{jt}}{k_{jt}}$.

And therefore, given the distributional assumption on ζ_{ijlt}, the enrollment of each school-year is:

$$E_{jt} = \sum_{l \in L} m_l \cdot \frac{\exp(\alpha x^{\text{demand}}_{jt} + \lambda dist_{jl})}{1 + \left(\sum_{j' \in J} \exp(\alpha x^{\text{demand}}_{j't} + \lambda dist_{j'l})\right)}, \tag{13}$$

where J denotes all the schools in the district, and L denotes all the relevant census tracts of this market.[37] And x^{demand}, the individual state variable used in demand estimation, includes therefore $(o_j, k_{jt}, q_{jt}, \xi_{jt})$.

To account for capacity constraints, especially in charter schools, I model students' preferences for schools depending on class size. This treatment helps explain the low enrollment in constrained schools as students dislike larger class sizes, thereby correcting biased estimates of other school characteristics. The consideration of class size in students' preferences is inspired by Urquiola and Verhoogen (2008), who developed a

[35] Due to the sample selection for the empirical implementation, a district's "inside" option, i.e., the charter and TPS enrollment, is from the selected set of schools inside of the districts. When calculating a district's "outside" option, i.e., private enrollment, I therefore also constrained to the district's private schools that only appear in the neighborhood of these selected charter and TPSs. Additionally, other major forms of schooling, such as home-schooling, are missing in measuring the market size. Evidence suggests that they accounted for less than 3% of the total Florida public enrollment in 2013: https://www.fldoe.org/core/fileparse.php/5606/urlt/Home-Ed-Annual-Report-2022-23.pdf.

[36] In their application, they also need to impute demand size for charter and TPSs from each census tract of Washington, D.C., using only enrollment and tract demographics.

[37] After I select all the schools, for the relevant census tracts of a district, I include all census tracts whose 5-mile radius neighborhood has at least one school in the district.

model to study the sorting of Chilean schools under class-size caps. However, incorporating class size into students' preferences introduces correlations between class size and hidden school quality ξ. To address this issue, I use a specific instrument for class size, following the empirical strategy by Bayer and Timmins (2007). [38]

To adopt this instrument and the estimation procedure proposed by Bayer and Timmins (2007), I use a two-step approach. In the first step, I run Non-linear Least Square (NLS) on a demand model that is identical to (12) except that the class size terms and ξ are excluded from the specification. Since there is no ξ in such a model, one does not need to apply the inversion technique (Berry 1994), and NLS is the appropriate method. Then, the implied estimates are used to form a predictor for class size from the model just estimated. In the second step, this predictor, along with other instruments I pick, is used to form the moment conditions used in estimating a Generalized Method of Moment (GMM) objective function. It aims to find the optimal $\hat{\alpha}$ and $\hat{\lambda}$ that minimizes the correlation between the instruments and the ξ_{jt}. In this step, I use the nested fixed point algorithm, as in Berry et al. (1995), to conduct the GMM. I explain all the details in Appendix C.1, including the moment conditions, testing whether instruments are weak, and the step-by-step estimation procedures. Note that once the $\hat{\alpha}$ and $\hat{\lambda}$ are found, one can back out ξ_{jt} by standard inversion procedure, as in Berry (1994). Finally, given $\hat{\alpha}$ and $\hat{\lambda}$, I can then use the following formula,

$$\bar{n}_{jt} = \sum_{l \in L} m_l \cdot \frac{\exp(\hat{\lambda} dist_{jl})}{1 + \sum_{j' \in J} \exp(\hat{\alpha} x_{j't}^{\text{demand}} + \hat{\lambda} dist_{j'l})},$$

to construct the market situation variable, $\bar{n}_{jt}$ faced by each school j at year t. As the "Consistent Belief" requires, the market situation variable constructed can be used to calculate schools' beliefs about the evolution rule. Since this estimated rule is assumed to be the belief schools hold to make decisions, it can further be used to back out the adjustment cost functions with which schools make decisions.

Operating Cost $\Psi(.)$ and Transitions. To estimate the operating cost of charter schools, I regress the logarithm of instructional cost from charter audit reports on the relevant state variables (and polynomials of these variables) and the logarithm of enrollment. Particularly, I include the local demographics of schools to reflect the cost differentials in operating charter schools across different regions, as Singleton (2019) points out. To estimate the transition function of school performance q, I regress a school's performance score on

[38] Richards-Shubik et al. (2021) estimate a discrete choice model in which patients select specialists. In the model, a similar "congestion effect" is added to patients' preferences to characterize patients' unwillingness to wait in long queues. They use the instrument proposed by Bayer and Timmins (2007) to deal with the endogeneity problem similar to my context.

its lag performance score, value added, and their interaction. The interaction term captures the differentials across different performance levels in the degree of value-added inputs needed to boost the same amount of performance score. To estimate the transition of the designation status hp, I exploit the empirical transitions to the contemporaneous designation status across charter schools conditional *only* on their past performance and designation status. This simplifies the modeling of the "2A1B" rule, which, if modeled precisely as the existing scheme, requires three years of past performances. I also assume that a charter school does not lose the designation as long as it is designated.[39] The rest of the transition functions, i.e., the transition of $\bar{n}$, ξ, and d, are all estimated as AR(1) processes.

Policy Functions When it comes to the estimation of the expansion policy function of charter schools, note that fixed costs are involved with increasing the number of classrooms for instruction, supported by the process of drafting new contracts and obtaining approval from local school districts. This can also be shown in the lumpiness in the adjustment of charter school classroom count. In the structural estimation sample, approximately 83% of charter school observations indicate no adjustment (i.e., an increase or decrease in the classroom count) throughout the selected sample period. Thus, to characterize such a feature of adjusting capacity, I adopt the (S, s) rule following Attanasio (2000) and Ryan (2012). Ryan (2012) utilizes this decision scheme to estimate cement manufacturers' capacity adjustment policy function for his dynamic game model. In my context, the (S, s) rule states that each charter school j sets a target k^*_{jt}, a lower band $\underline{k}_{jt}$, and an upper band $\bar{k}_{jt}$, in year t based on a statistical rule whose parameters are to be estimated. According to the rule, a charter school increases classrooms to reach its target only when it falls below the lower band: $e_{jt} = \underline{k}_{jt} - k^*_{jt}$ if $k^*_{jt} < \underline{k}_{jt}$. It decreases classrooms to reach its target only when it exceeds the upper band: $e_{jt} = \bar{k}_{jt} - k^*_{jt}$ if $k^*_{jt} > \bar{k}$. Therefore, when the target stays within the bands, the charter school j in that year t remains inactive. Therefore, this decision rule matches the lumpiness in the expansion adjustment data. Following their specification, I use a flexible functional form of the state variables (the $h(.)$ functions below) to estimate both the target and bands, as shown in (14). The exponential functional form guarantees that the target is always between the lower and

[39] As explained in the industry background, de-designation is rare in the sample. Furthermore, I rarely observe that eligible (i.e., those that pass the "2A1B" requirement) charter schools are not designated. These observations might reflect that they do not apply for the designation. Since they are rare, I exclude them from the estimation sample.

upper bands.

$$\begin{aligned} k_{jt}^* &= h_1(s_{jt}) + u_{jt}^* \\ \underline{k}_{jt} &= k_{jt}^* - \exp\left(h_2(s_{jt}) + \underline{u}_{jt}^b\right) \\ \bar{k}_{jt} &= k_{jt}^* + \exp\left(h_2(s_{jt}) + \bar{u}_{jt}^b\right) \end{aligned} \tag{14}$$

Same as Attanasio (2000) and Ryan (2012), I use only the observations that involve non-zero adjustments of capacity to estimate (14). In particular, in estimating the target equation, I regress the $t+1$ number of classrooms on state variables of t, and in estimating the band equations, I regress the difference between $t+1$ and t in the number of classrooms on current state variables, both using flexible functional forms.[40] I also consider the residuals $u_{jt}^*, \bar{u}_{jt}^b,$ and $\underline{u}_{jt}^b$ as structural errors, as to capture the discrepancy between the estimated policy functions in adjusting capacity and the model-predicted adjustment processes. As emphasized in the adjustment costs of charter schools in (6), this discrepancy may exist due to the unobserved mode of capacity adjustment. I assume the different structural errors all follow an i.i.d. zero-mean normal distribution with variance (same across schools) to be estimated, independent of each other.

6.5 Identification of the Adjustment Cost Function

The identification of the key structural parameters in equation (6) for both types of schools relies on the policy shock and the functional form assumptions imposed on $\Gamma(.)$.

For charter schools, the cost of exerting v amount of value-added, namely γ_v, and the HP-related cost effects, namely γ_2 and γ_4, jointly govern the value-added decisions. These parameters can be separately identified by exploiting the policy shock. The early designated charter schools, e.g., those designated in 2012, do not need to adjust their value-added to secure future designation since they can never be de-designated, as the model imposes. Hence, the difference in value-added choices between these and later-designated schools helps separate the HP-related cost effects and γ_v. The separable form of the adjustment costs separately identifies γ_v, γ_2, and γ_4. Specifically, γ_v is separately identified from γ_2 and γ_4 by the variation in a school's performance in the following school year when its capacity remains unchanged. This is because γ_2 and γ_4 only affect adjustment costs when charter schools expand. The identification of γ_v for TPSs follows a similar logic.

To separately identify the fixed and variable costs of expansion, note that conditional

[40]Ideally, upper and lower bands should be estimated by the shrinkage and the expansion data separately. However, because shrinkage, i.e., the decrease in classrooms, is much less common than expansion, I assume that the shrinkage decision shares the same statistical relationship with the expansion decision and estimate the two bands by pooling all observations of expansion and shrinkage.

on expansion, the fixed cost, γ_1, does not influence the expansion volume. Therefore, γ_3 can be separately identified by the variation in the magnitudes of expansions across or within schools. Subsequently, γ_1, the fixed cost of expansion, is identified by the frequency of charter schools initiating an expansion. As is set up in the model, γ_1 and γ_3 are assumed to follow normal distributions with mean zero and to-be-estimated variance. These variance coefficients are identified by the variances in magnitude and frequency of expansion conditional on the state variables as the (S, s) policy functions specify.

Finally, γ_1 and γ_3 can be separately identified from the γ_2 and γ_4, the HP-related effects. This is so by comparing the difference in expansion choices across charter schools or within those that experience a change in their HP status in the sample. Identifying the remaining parameters follows standard practices in the literature and is thus omitted here.

7 Structural Estimation Results

(Below are under overhaul)

7.1 First Stage: Demand Function

I display results of the demand estimation in Table (**??**).

Not controlling for the endogeneity caused by the correlation between the underlying quality ξ and class size generates .. ... Use Bayer and Timmins' (2007) instrument ... The result also addresses the role of capacity constraint in the estimation of capacity constrained .... . I pick the results . I also compare the result with the non-linear least square result that does not control for the endogeneity.

Given the above reasons, I use the ... specification to conduct the subsequent analysis.

Relatedly, Ferreyra and Kosenok (2017) and Singleton (2019) might

Implied by the current estimates, the market situation index and hidden quality can be calculated. Table (**??**) shows several summary statistics of these variables. To check the sanity of the constructed n, I regress it to a school's local environment. Not surprisingly, the current market condition index is negatively correlated with denser, richer, more educated neighborhood situation. The mean of n across all school-year observations is .

Notably, the range of n constructed from the demand estimates and the data has important implication in the subsequent simulation. Since I use discretization to solve the value function, from the current formular of enrollment, namely $E_{jt} = exp(\delta_{jt}) \cdot \bar{n}_{jt}$, allowing for wider range of variation in $\bar{n}_{jt}$ is critical in produce variation on E_{jt}. Larger range implied the data requires more grids, hence higher computation "budget" on ... . More discussion of this can be seen in the counterfactual simulation.

7.2 First Stage: Other Offline Functions

Operating Cost Functions The result is shown in table (**??**). It shows almost constant return to scale as the coefficient on the logarithm of enrollment is close to one. Notably, it also shows that operation under different local demographic situation involve differential cost, a result similar to Singleton (2019) in the similar setting. This can be seen from the coefficient on mean household income in 3 miles. Operating in income neighborhood is less costly in terms of instructional expenditure, controlling for enrollment. his cost differential can explain part of the variation in the variation of expansion patterns across different demographic environment. For the relationship between instructional cost and the performance and capacity, within the data range, it is decreasing with..and increasing with capacity. .... (see how John said things about this here: there mus)

Transition Functions I show results for are shown in Table (**??**).

I show the rest of the transition functions in Table (**??**) The implied transition of underlying quality ξ and competitive pressure n shows that . I also show a summary statistics of the estimated ξ and competitive pressure n.

Policy Functions Table (**??**) shows the result of estimating the components of (S, s) rule, the expansion target and the bandwidth of inactivity, using polynomials of the state variables. The results manifest important difference across non-HP and HP charter schools. .... .

Since there is no existing evidence in the literature suggesting that (S, s) rule is an acceptable approximation of how charter schools make expansion decisions, I test the in-sample fitness of the (S, s) rule. From the sample I use to estimate the (S, s) rule, I pick all charter school observations that have next period data available, plug their states into the estimated (S, s) rule to predict their expansion behaviors, and compare the predicted behaviors with the actual expansion behavior in the next period. Table (**??**) in the appendix shows that the estimated (S, s) rule approximates the mean expansion well in both extensive and intensive margin. It also predicts reasonably well the expansion behaviors taken by HP and non-HP charter schools.

Table (**??**) continues to show the estimated policy functions of exerting value-added under different combinations of the state variables. Notably, aligned with the competitive responses by TPS, value-added is positively associated with higher neighboring pressure. (TPS charter comparison)

7.3 Second Stage: Adjustment Cost Function

Table (6) reports the estimates of the adjustment cost function using BBL estimator provided all offline functions estimated in the first stage using the preferred specification. The computation details in this BBL second stage, such as the selected initial states, implementation of the perturbation on the policy functions, and the simulation parameters, are in Appendix C.2.

I run the structural estimation separately for charter and TPSs using charter and TPS observation respectively. Table (6) concludes the structural estimates for adjustment cost function $\Gamma(.)$. From the specification of schools' flow utility, a positive estimate indicates a cost. Notably, γ_2 is positive and precisely estimated, indicating that the HP designation decreases the fixed cost of initiating an expansion. The effect of HP designation on the variable cost of expansion, γ_4, is smaller in magnitude and not estimated with precision, as indicated by the ratio between its estimate and standard errors. Combined, these results align with the policy contents: The policy facilitates expansion ($\gamma_2 < 0$) for the HP charter schools but does not directly support expansion financially as charter schools expand more. The estimates of γ_v show that exerting value-added is costly for both charter schools and TPS. However, charter schools have lower costs. This might imply charter schools' higher efficiency in managing teachers in directing teaching goals to test scores. As expected, the fixed cost of expansion, γ_1, the variable cost of expansion in increasing one unit of a classroom, γ_3, are both larger in magnitude compared to the cost of value-added.

8 Policy Counterfactuals

In this section, I explain the motivation for conducting the counterfactual policy simulations and the implementation details, especially how I update the belief on n_t under the "Consistent Belief" assumption.

My primary interests are to evaluate the policy effects and explore alternative policies that increase the supply of quality education at the aggregate. In the counterfactual policy experiments, I anchor the idea of incentivizing by expansion eligibility and therefore focus on deviating the existing scheme on "who should expand more easily" while holding fixed the other model primitives except for schools' belief on the competitive pressure. Particularly, I test the following alternative schemes, as shown in Table (7). I plan to consider more counterfactual experiments, for example, deregulate all charter schools in expansion eligibility and include it in future versions of the paper.

Compared to the existing policy, three components of the models for charter schools and TPSs change: the adjustment cost of expansion, the transition of the HP designation, and the belief about the competitive pressure. Table (7) lists all the counterfactual policy

Table 6. Estimates of $\Gamma(.)$ and Standard Errors

	Adjustment Cost $\Gamma(.)$	
	TPS	Charter
Value-added Cost (γ_v)	11.08	6.40
	(3.21)	(2.37)
Mean of Fixed Cost (γ_1^μ)		3.88
		(1.60)
Std. Deviation of Fixed Cost (γ_1^σ)		0.42
		(0.20)
HP's Effect in Fixed Cost (γ_2)		5.89
		(2.04)
Mean of Variable Cost (γ_3^μ)		6.53
		(1.42)
Std. Deviation of Variable Cost (γ_3^σ)		0.01
		(0.01)
HP's Effect in Variable Cost (γ_4)		-0.69
		(0.89)
Variable Cost of Shrinkage (γ_5)		4.04
		(2.20)

Note: Standard errors (in parenthesis) are obtained by bootstrap. I re-sample half of the initial states randomly 100 times with the same set of perturbed policy functions. All parameters are estimated assuming discount factor $\beta = 0.9$, per-enrollment reimbursement $r = r^E = 0.08$ representing eight thousand per student, and utility weight on school performance score $r^q = 1.6$. All parameters can be regarded as measured in hundreds of thousands of dollars.

and their change of the primitives, compared to the existing policy.

Table 7. Changes of Primitives of Policy Counterfactuals

	Existing Scheme	"No-HP"	"Target-va"
Γ^{charter}	$\gamma_2 = \hat{\gamma}_2, \gamma_4 = \hat{\gamma}_4$	$\gamma_2 = 0, \gamma_4 = 0$	
η	$hp_{t+1} = \hat{\eta}(hp_t, q_t)$	$hp_t = 0, \forall t$	$hp_{t+1} = 1$ if $v_t \geq \tilde{v}$
ν	$n_{t+1} = \hat{\nu}(n_t)$	Change according to the Consistent Belief Assumption.	

8.1 Implementation Methodology

Under the current framework, I therefore decompose .. . by ... . For example

Finally, I am interested in the shorter run outcomes . The implication on the computation algorithm is reflected on the update rul . Within a market ... . More details about the discretization and computation algorithm are in Appendix D.1 and D.2 respectively.

8.2 Results

In this subsection, I show results of the two counterfactual policy schemes and compare them with the existing scheme.

No-HP In this simulation, the goal is to investigate how the decisions of value-added and expansion will be made across TPS and charter schools if the designation system were not in place. The focused outcomes are the impact on the distribution of schools' performance and provision of access in the charter sector. Intuitively, in the "no-HP" policy, the adjustment costs reduced by the HP status and the designation system are gone, as reflected by essentially making all charter schools non-HP. Therefore, the HP transition rule is modified to make it impossible for schools to be designated, and the parameters γ_2 and γ_4 are set to zero.

To quantify deeper the contributions of each mechanism,

To understand the results,

Firstly, charter schools lose the option value of increasing capacity at lower costs due to the absence of the HP designation system, which reduces their incentive to invest in costly (as γ_v shows) value-added to achieve high performance. Secondly, TPSs may adjust their value-added decisions in response to changes in the performance of surrounding charter schools.

Target Value-Added The simulation of this scheme, "TargetVA" henceforth, explores the inclusion of high value-added as an additional criterion for HP designation and the existing criterion using performance level. Under this alternative policy, if a charter school surpasses a threshold value of value-added (i.e., $\tilde{v}$), it becomes eligible for designation. This essentially gives more designation to low-performing and high value-added schools, presumably located in low-SES regions. It also gives more incentive for charter schools in the high value-added regions to increase performance if they want to expand. This simulation aims to assess how the equality of good education resources across all schools can be improved by granting expansion eligibility to schools with high value-added.

8.3 Analysis

Potential mechanisms

1.First of all, basic heuristics: basic tradeoff, competition return, adjustment cost

2.Further, across all schools, what governs the kala's mechanism, charter incentive effect + TPS magnifying effect

Therefore, the above results show that

Combined all results, I place their central takeaways in the above graphs.

3. Finally, heterogeneity The equality cannot be improved if modifying the designation criteria in such a way merely motivates charter schools serving low SES regions to expand. Therefore, whether this alternative policy improves equality is an empirical question. For example, the operating costs Ψ may vary across different demographic groups, such as the higher costs associated with educating low SES students, which could limit the potential expansion benefits.

This is graph is useful to policymakers.

9 Conclusion

In this paper, I exploit a novel policy that incentivizes charter schools with expansion eligibility. I leverage the policy to explore (research goal) . Motivated by the reduced form evidence that suggests the existence of charter adjustment cost in expansion and competitive spillover across sector, I develop and estimate a tractable dynamic model to explore the policy effects and countererfactual policies that do better.

The main results of this paper show that

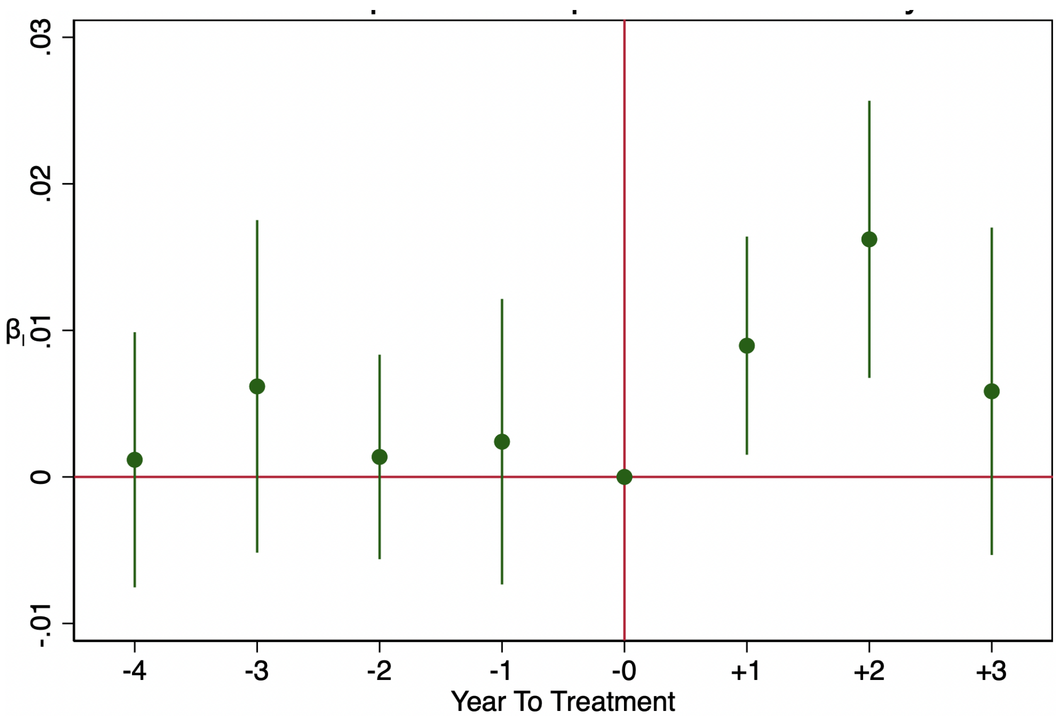
In the current version, heterogeneity in student demographics is limited to their location. This constrains the model to answer questions such as how is the student demographic distribution will be changed if the policy were not implemented. As the preliminary evidence shows, TPSs tend to have higher proportion of students that need free and reduced price lunch as they are surrounded with higher number of charter schools. This is a suggestive evidence that, as charter schools expand, higher students might resort from their original TPS to a nearby, expanded HP charter schools. The sorting within local neighborhood can be important to the distribution of performance across schools. To allow this into the model, I need to allow for students to differ in terms of demographics other than distance, such as income and race and factor such demographic difference in their taste parameters to schools' characteristics, such as performance. Then, the demand can therefore capture difference in the taste between high and low income families in their tastes of school's performance scores. Dynamic sorting (Bayer et al. 2016 ; Hahn and Park 2022)... endogenous to schools' decisions has not been done....

A Figure Appendix

A.1 Event Study of Competitive Spillover on the TPSs

$$A_{igkt} = \sum_{\ell=-4}^{3} \beta_\ell 1_{\ell=t-2011} \times Treat_i + \rho A_{igkt-1} + \sum_{\ell=-4}^{3} \alpha_\ell 1_{\ell=t-2011} +$$
$$\eta Treat_i + \gamma \mathbf{Z}_{igkt} + \epsilon_{igkt}$$

Figure A1. Event Study of TPS Competition Responses



A.2 Event Study of HP Designation on Charter Capacity and Enrollment

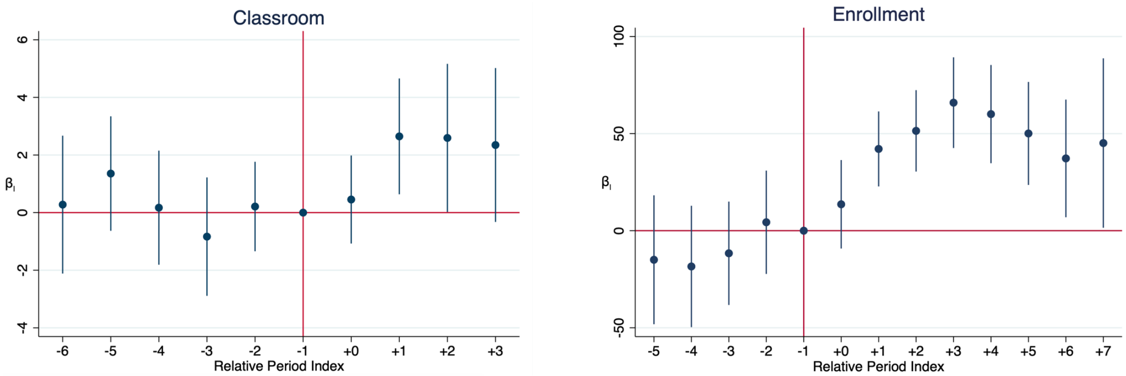


Figure A2. Coefficient Plots of β_ℓs for Classroom Count $2007-15$ and Enrollment, $2007-19$

B Table Appendix

B.1 Summary Statistics

Table B1. Summary Statistics of Different Samples

Table B2. Summary Statistics of the Selected Districts for Structural Estimation in 2015

B.2 Robustness Tests on the Diff-in-Diff Results

Table B3. Other Variants of the TPS Competition Response Tests

Outcome: Test Score	By Subject		Alternative Treatment Measure					Sample Selection		
	Read	Math	#HP in 3	Exist in 3	Exist in 5	#A in 3	#A in 5	>80 Match	>90 Match	Full Sample
$Post_t \times Treat_i$	0.0090***	0.0076**	0.0132***	0.0189***	0.0176***	0.0055**	0.0031	0.0082***	0.0088***	0.0097***
	(0.0024)	(0.0033)	(0.0033)	(0.0066)	(0.0062)	(0.0028)	(0.0023)	(0.0024)	(0.0032)	(0.0023)
Constant	30.5105**	37.2682**	32.0312**	29.9242**	28.8405**	29.5594**	29.3216**	29.9680**	25.6203	25.8730**
	(13.6531)	(18.7661)	(13.1629)	(13.1426)	(13.1283)	(13.1501)	(13.1679)	(13.6499)	(21.1103)	(10.1978)
Observations	27,593	27,593	55,304	55,304	55,304	55,304	55,304	52,286	27,599	83,004
R-squared	0.9504	0.9013	0.8973	0.8973	0.8973	0.8973	0.8972	0.8985	0.9097	0.8976
Fixed Effects	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Charter Entry + School Demo	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
PT Ratio	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y

Standard errors in parentheses
*** p¡0.01, ** p¡0.05, * p¡0.1

B.3 Measurement

Table B4. Full List of Variables with Measurement and Availability

Variable	Meaning	Measurment	Data Availability
Part A. Endogeneous States and Decisions			
k_t	Capacity	For charter schools, this is the number of classrooms in year t. For TPSs, this is the largest enrollment observed during 2007-2019 divided by 22.	
q_t	Performance level	For both types of schools, this is the accountability score in year $t-1$	
hp_t	Designation	For charter schools, this is the designation status in year t. For TPS, this is zero in all situations.	
e_t	Increment in classrooms	For charter schools, this is the first-difference of classrooms in t+1 and t. For TPSs, this is zero in all situations.	
v_t	Effort in performance	For both types of schools, this is the average teacher value-added score within a school in year t	2012-2019
$\bar{n}_{jt}$	Inclusive value about the market situation	Demand model-implied	Estimated
Part B. Other State Variables			
d_{jt}	Local demographics	The mean household income of all census tracts within 3-mile radius of a school.	
ξ_{jt}	Hidden quality	Demand model-implied	Estimated
ϵ_t	Unobserved heterogeneity	Random normal	
Part C. Other Variables in the Model			
m_{lt}	Local market size	ACS tract level school attendance to K-8 grades of tract l in t, tuned according to private school enrollment using Ferreyra and Kosenok (2018) method	
$dist_{jl}$	Travel distance to school	Crowfly distance between school j and tract l	
E_{jt}	Enrollment	For both types of schools, this is the total enrollment of K-8 grades from the NCES and Florida Master Files	
Ψ_{jt}	Operating cost of charter school	For charter schools, this is the total instructional expenditure	2007-2015

C Model and Estimation Appendix

C.1 Spatial Demand Estimation

$\boxed{\text{Estimation and Identification of } E(.)}$

- Nested fixed point (NFP) algorithm, as in Berry et al. (1995)
  - Find the optimal $\hat{\beta^\mu}$ that minimizes the correlation between the Z and the derived $\hat{\xi}$ coming from the Berry inversion
  - Inner loop: match the market share with the derived $\hat{\delta}$ given a guess of $\hat{\beta^\mu}$. Get $\hat{\xi}(\hat{\beta^\mu})$ by 2SLS
  - Outer loop: GMM objective is minimized wrt. $\hat{\beta^\mu}$

$$\min_{\hat{\beta^\mu}} \hat{\xi}(\hat{\beta^\mu})' Z W Z' \hat{\xi}(\hat{\beta^\mu})$$

- Outside the NFP, I run non-linear least square (NLS) on two models that are identical to the original model except
  - no ξ – to get a strawman for the results from NFP
  - no ξ or E/K – to construct the Z^{BT}

$\boxed{\text{Identification Assumptions for Demand}}$

- Instruments $Z = \{Z^{AS}, Z^{BT}, Z^{demo}, X\}$:
  - Z^{BT}: 1 IV from Bayer and Timmins (2007)
  - Z^{AS}: 2 IVs from Agarwal and Somani (2022)
  - Z^{demo}: 7 IVs from Local demographic variables
  - X : the inputs in the δ specification other than the class size and ξ.
- The demand inputs X is independent with ξ_{jt} because I assume ξ_{jt} exogenously evolve as an AR(1), as in Sweeting (2013)
- Z^{BT}: Given the validity of X as IVs, the validity of the Z^{BT} is followed by construction: It is a predicted enrollment E divided by k where the construction of E uses the following formula

$$p_j = \frac{exp(\tilde{X}_j\hat{\beta})}{1 + exp(\tilde{X}_j\hat{\beta})\sum_{j\prime \neq j} exp(\tilde{X}_{j'}\hat{\beta})}, \tilde{X} = (q, k, o, hp)$$

- Z^{AS}: A large previous year's graduating class is not a result of the current period underlying quality ξ_{jt}, yet it decreases the class size of a school j.
- Z^{demo}: this is equivalent to assume no endogenous entry

Lastly, Bayer and Timmins (2006) have provided the condition of the existence and uniqueness of equilibrium. In the case of model of this paper, as long as the the taste parameter on class size has negative coefficient, the equilibrium exists and it is unique.

$\boxed{\text{Elasticity}}$

- Enrollment elasticity w.r.t. capacity (given inclusiveness assumption)

$$\begin{aligned} E_j =& m_j \cdot \frac{\delta_j}{1 + \delta_j + \sum_{j \neq j'} \delta_{j'}} \\ \Rightarrow \frac{\partial E_j}{\partial k_j} =& m_j \cdot \frac{\frac{\partial \delta_j}{\partial k_j}(1 + \delta_j + \sum_{j \neq j'} \delta_{j'}) - \delta_j \frac{\partial \delta_j}{\partial k_j}}{(1 + \delta_j + \sum_{j \neq j'} \delta_{j'})^2}, \end{aligned}$$

where $\frac{\partial \delta_j}{\partial k_j} = \beta \frac{\partial E_j}{\partial k_j} \frac{1}{k_j} - \beta \frac{E_j}{k_j^3}$. Therefore, plug this term in and rearrange, I get:

$$\begin{aligned}
\frac{\partial E_j}{\partial k_j} =& m_j \cdot \frac{(\beta \frac{\partial E_j}{\partial k_j} \frac{1}{k_j} - \beta \frac{E_j}{k_j^3})(1 + \delta_j + \sum_{j \neq j'} \delta_{j'}) - \delta_j (\beta \frac{\partial E_j}{\partial k_j} \frac{1}{k_j} - \beta \frac{E_j}{k_j^3})}{(1 + \delta_j + \sum_{j \neq j'} \delta_{j'})^2} \\
\Rightarrow \frac{\partial E_j}{\partial k_j} =& \frac{\beta E_j m_j (1 + \sum_{j \neq j'} \delta_{j'})}{\beta k_j^2 m_j - k_j^3 (1 + \delta_j + \sum_{j \neq j'} \delta_{j'}) - \beta k_j^2 \delta_j}
\end{aligned}$$

C.2 BBL Estimation Details

D Computation Appendix

D.1 Grids and Computation Time

Grids and Computation Time

- Grids for each state variable for charter schools

Table D1. Evenly Distanced Grids of Each State

Endogenous States				Exogenous States			
State	Min	Max	# Grid	State	Min	Max	# Grid
q	0.4	0.9	11	d			3
k	1	61	21	m	min	max	12k
hp	0	1	2	ξ	min	max	12k
n	1.5	7.5	21				

- TPSs similar. Except for k and hp
- Each value function iteration: 43 m (TolV=1e-4)
- Iterations needed for convergence if L=1: 163 (Tolν=1e-2)
- Simulations are paralleled on 300+ cores

D.2 Computation Algorithm

Computation Framework

- I use a perceived transition of n_t capturing the **short-term transitional dynamics** starting from picked initial states because
  - The goal is to characterize short term ($<$ 20years) effects of a policy change
  - The existing data are perceived as not yet reaching a steady state

- Given this, the belief on the transition of n is made to be aligned with average paths from n_t to n_{t+1} over many short trajectories that start from the picked initial states
  - I modified Ifrach and Weintraub's (2017) algorithm designed for a perceived transition of long-term dynamics
- The initial states are picked to represent typical markets in 2012
  - High income dense urban markets ($J = 50$, $J_{charter} = 20$)
  - Lower income dense urban markets ($J = 40$, $J_{charter} = 10$)
  - Lower income less dense rural markets ($J = 30$, $J_{charter} = 5$)
- Inspection period $T = 20$ years. Simulate $L = 20$ draws.

Simulation Procedures: Simplified Stationary Approach

1. Start from an initial guess of $\nu^1(n)$. Solve the implied expected value function $\bar{V}^{(\nu^1)}(s)$. Pick a market whose state is
$$s_0 = (o, q_0, k_0, hp_0, d_0, \xi_0, m_0, n_0)$$
2. Simulate one path for horizon T of interest, starting from s_0 for L times under the belief $\nu^i(n)$, the i's iterate of n's transition
   - Regard heterogeneity deterministic at the estimated mean
   - Solve for $z^{(\nu_1)}(s)$ by value function iteration
   - For each school, use $z^{(\nu_1)}(s)$ and get one path of n according to the inclusive value formula:
   $$\{\hat{n}_t : t = 0, \ldots, T\}$$
   - Get $\nu^{i+1}(n)$ by estimating an AR(1) using the this path of $\hat{n}$
3. Repeat until $\nu^{i+1}(n)$ is close enough to $\nu^i(n)$. Denote the converged transition as: $\nu(n)$
4. Use the model under $\nu(n)$ and the initial state s_0 to simulate outcomes of this market. Repeat the above procedure for each picked market.