Multi-Period Truck Assignment and Load Balancing Model

Sets and Indices:

$i \in \mathcal{D}$	set of demands (orders)
$k \in \mathcal{K}$	set of trucks
$t \in \mathcal{T} = \{1, 2, \ldots, H\}$	planning horizon days
$\text{dest}(i)$	destination of demand i

Parameters:

w_i	weight of demand i
s_i	size (area) of demand i
travel_days_i	number of days demand i occupies a truck
$\text{feasible}(i) \subseteq \mathcal{T}$	feasible start days for demand i
Q_k	capacity of truck k
S_k	size (area) limit of truck k
M_{stops}	maximum number of stops per truck per day
H	number of planning days
$\text{AvgLoad} = \dfrac{\sum_{i \in \mathcal{D}} w_i}{H}$	average daily total weight
$w_{\text{balance}},\ w_{\text{slack}}$	weights for balance and slack in objective

Decision Variables:

$$x_{i,t,k} = \begin{cases} 1, & \text{if demand } i \text{ assigned to truck } k \text{ starting on day } t, \\ 0, & \text{otherwise;} \end{cases}$$

$$u_{d,t,k} = \begin{cases} 1, & \text{if truck } k \text{ visits destination } d \text{ on day } t, \\ 0, & \text{otherwise;} \end{cases}$$

$$y_{t,k} = \begin{cases} 1, & \text{if truck } k \text{ is active on day } t, \\ 0, & \text{otherwise;} \end{cases}$$

$$\begin{aligned} \text{Load}_t \geq 0 &\ : \text{ total transported weight on day } t, \\ z_t \geq 0 &\ : \text{ deviation from average daily load on day } t, \\ \text{slack}_t \geq 0 &\ : \text{ slack variable for day } t. \end{aligned}$$

Objective Function:

$$\min\ Z = w_{\text{balance}} \sum_{t \in \mathcal{T}} z_t + w_{\text{slack}} \sum_{t \in \mathcal{T}} \text{slack}_t \tag{1}$$

Subject to:

1. Each demand assigned exactly once:

$$\sum_{t \in \text{feasible}(i)} \sum_{k \in \mathcal{K}} x_{i,t,k} = 1, \quad \forall i \in \mathcal{D} \tag{2}$$

2. Truck capacity and size limits (per day):

$$\sum_{i \in \mathcal{D}} \sum_{\substack{s \in \text{feasible}(i) \\ s \leq t < s + \text{travel\_days}_i}} w_i \, x_{i,s,k} \leq Q_k, \qquad \forall k \in \mathcal{K},\ t \in \mathcal{T} \tag{3}$$

$$\sum_{i \in \mathcal{D}} \sum_{\substack{s \in \text{feasible}(i) \\ s \leq t < s + \text{travel\_days}_i}} s_i \, x_{i,s,k} \leq S_k, \qquad \forall k \in \mathcal{K},\ t \in \mathcal{T} \tag{4}$$

3. Daily load definition:

$$\text{Load}_t = \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{D}} \sum_{\substack{s \in \text{feasible}(i) \\ s \leq t < s + \text{travel\_days}_i}} w_i \, x_{i,s,k}, \quad \forall t \in \mathcal{T} \tag{5}$$

4. Visit coupling:

$$u_{\text{dest}(i),t,k} \geq x_{i,t,k}, \quad \forall i \in \mathcal{D},\ t \in \text{feasible}(i),\ k \in \mathcal{K} \tag{6}$$

5. Maximum stops per truck per day:

$$\sum_{d} u_{d,t,k} \leq M_{\text{stops}}, \quad \forall t \in \mathcal{T},\ k \in \mathcal{K} \tag{7}$$

6. Truck availability:

$$y_{t,k} \geq x_{i,s,k}, \quad \forall i \in \mathcal{D},\ k \in \mathcal{K},\ t \in [s, s + \text{travel\_days}_i) \cap \mathcal{T} \tag{8}$$

$$y_{t,k} \leq 1, \quad \forall t \in \mathcal{T},\ k \in \mathcal{K} \tag{9}$$

7. Daily load balance deviation:

$$\text{Load}_t - \text{AvgLoad} \leq z_t, \quad \forall t \in \mathcal{T} \tag{10}$$

$$\text{AvgLoad} - \text{Load}_t \leq z_t, \quad \forall t \in \mathcal{T} \tag{11}$$

Variable Domains:

$$x_{i,t,k},\ u_{d,t,k},\ y_{t,k} \in \{0,1\},$$
$$\text{Load}_t,\ z_t,\ \text{slack}_t \geq 0.$$

Capacitated Vehicle Routing Problem (CVRP) Model

Sets and Indices:

$$
\begin{aligned}
&i, j \in N = \{0, 1, 2, \ldots, n\} && \text{(set of nodes; 0 is the depot)} \\
&k \in K = \{1, 2, \ldots, m\} && \text{(set of vehicles)}
\end{aligned}
$$

Parameters:

C_{ij}	:distance (or travel cost) between node i and j,
q_i	:demand at node i, $q_0 = 0$,
Q_k	:capacity of vehicle k,
c_k	:cost coefficient of vehicle k,
S	:service cost per stop,
$\text{ready}_i, \text{due}_i$	:time window for customer i,
SERVICE_TIME_PER_STOP	:service time at each stop.

Decision Variables:

$$
x_{ijk} = \begin{cases} 1, & \text{if vehicle } k \text{ travels from } i \text{ to } j, \\ 0, & \text{otherwise;} \end{cases}
$$

$$
f_{ijk} \geq 0 \quad : \text{load flow of vehicle } k \text{ on arc } (i, j),
$$

$$
\text{visit}_{ik} = \begin{cases} 1, & \text{if vehicle } k \text{ visits customer } i, \\ 0, & \text{otherwise.} \end{cases}
$$

Objective Function:

$$
\begin{aligned}
\min \quad Z = &\sum_{k \in K} \sum_{i \in N} \sum_{j \in N,\, i \neq j} c_k \cdot \frac{C_{ij}}{\max C}\, x_{ijk} + S \sum_{k \in K} \sum_{j \in N \setminus \{0\}} \text{visit}_{jk} \\
&+ 0.01 \sum_{k \in K} \sum_{i \in N} \sum_{j \in N,\, i \neq j} f_{ijk}
\end{aligned} \tag{12}
$$

Subject to:

1. Departures from depot = number of vehicles:

$$
\sum_{k \in K} \sum_{j \in N \setminus \{0\}} x_{0jk} = |K| \tag{13}
$$

2. Each customer is visited exactly once:

$$
\sum_{k \in K} \text{visit}_{ik} = 1, \quad \forall i \in N \setminus \{0\} \tag{14}
$$

3. Flow conservation for each vehicle and node:

$$\sum_{i \in N,\, i \neq h} x_{ihk} = \sum_{j \in N,\, j \neq h} x_{hjk}, \quad \forall h \in N,\, k \in K \tag{15}$$

4. Linking travel and visit variables:

$$\sum_{j \in N,\, j \neq i} x_{ijk} = \mathrm{visit}_{ik}, \quad \forall i \in N,\, k \in K \tag{16}$$

5. Maximum number of stops per vehicle:

$$\sum_{j \in N \setminus \{0\}} \mathrm{visit}_{jk} \leq \mathrm{MAX\_STOPS}, \quad \forall k \in K \tag{17}$$

6. Load conservation:

$$\sum_{j \in N,\, j \neq i} f_{jik} - \sum_{j \in N,\, j \neq i} f_{ijk} = q_i \cdot \mathrm{visit}_{ik}, \quad \forall i \in N \setminus \{0\},\, k \in K \tag{18}$$

7. Lower and upper bounds on vehicle load:

$$f_{ijk} \geq q_i\, x_{ijk}, \qquad \forall i, j \in N,\, i \neq j,\, k \in K \tag{19}$$

$$f_{ijk} \leq (Q_k - q_i)\, x_{ijk}, \qquad \forall i, j \in N,\, i \neq j,\, k \in K \tag{20}$$

8. Depot load constraint:

$$f_{0jk} \leq Q_k\, x_{0jk}, \quad \forall j \in N \setminus \{0\},\, k \in K \tag{21}$$

Variable domains:

$$\begin{aligned} x_{ijk} &\in \{0, 1\}, \\ \mathrm{visit}_{ik} &\in \{0, 1\}, \\ f_{ijk} &\geq 0. \end{aligned}$$