Personal Thesis

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I believe that **algebraic formulas** are notional concepts that describe the **properties of the concept of number**.
The concept of number is, to me, a **measurable and conserved concept**, which can be judged through discrete logical statements.
An algebraic formula contains **logical flows**, and the following is a list of formulas derived from those logical flows.

Representing Logical Operations via Arithmetic Operations

First, define some functions to be used:

- $\mathrm{Solve}(F)\{\bar{x} \mid F(\bar{x}) = 0\}$
- $\mathrm{boolf}(S)(x)(x \in S)$
- $\mathrm{bool}(x)(x \neq 0)$

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1. 3-variable: $x, y \to z$

Logical value assignments:

$$
\begin{aligned}
0000 &: \quad z = 0 \\
0001 &: \quad z = 1 + xy - (x + y) \\
0010 &: \quad z = y - xy \\
0011 &: \quad z = 1 - x \\
0100 &: \quad z = x - xy \\
0101 &: \quad z = 1 - y \\
0110 &: \quad z = x + y - 2xy \\
0111 &: \quad z = 1 - xy \\
1000 &: \quad z = xy \\
1001 &: \quad z = 1 + 2xy - (x + y) \\
1010 &: \quad z = y \\
1011 &: \quad z = 1 + xy - x \\
1100 &: \quad z = x \\
1101 &: \quad z = 1 + xy - y \\
1110 &: \quad z = x + y - xy \\
1111 &: \quad z = 1
\end{aligned}
$$

2. 2-variable: $x \to y$

Logical value assignments:

$$\begin{aligned} 00: &\quad y = 0 \\ 01: &\quad y = 1 - x \\ 10: &\quad y = x \\ 11: &\quad y = 1 \end{aligned}$$

3. 0-variable Linear Mapping xe

- Logical assignment 0 maps to 0
- Logical assignment 1 maps to 1

Model Theory and Equations

To judge **equality of values**, define:

$$\delta_i(x) \lim_{n \to (x-i)^+} 0^n$$

By interpreted proposition $P(x) = 0$ into the logical truth value which is $\{0, 1\}$, then the composed function $\delta_0 \circ P$ computes the truth of the proposition $P(x) = 0$.

Key Insight: Logical Meaning of an Equation: The "Existential ($\exists$) Condition"

About function P, "the equation which is introduced by function P" is a solution (root) to the equation $P(x) = 0$.

Thus, "the equation which is introduced by function P" is logically equivalent to the predicate logic statement:

$$(\exists x)(P(x) = 0)$$

Which can also be written using composed functions:

$$(\exists x)((\mathrm{bool} \circ \delta_0 \circ P)(x))$$

Background: Logical Consequence

If an assignment $\bar{x}$ satisfies a proposition p, we write $\bar{x} \vDash p$, and call this a **satisfying relationship**.

- For a proposition p, define its **model set**:

$$\mathrm{Mod}(p)\{\bar{x} \mid \bar{x} \vDash p\}$$

- For a set of propositions $\Phi = \{p_1, p_2, \ldots, p_n\}$:

$$\mathrm{Mod}(\Phi) \bigcap \mathrm{Mod}(p)$$

- The **logical consequence** relation $A \vDash B$ means:

$$\mathrm{Mod}(A) \subseteq \mathrm{Mod}(B)$$

Key Insight: Algebraic Expression of Logical Consequence

The logical consequence relation $A \vDash B$ can be expressed arithmetically using the divisibility operator:

$$\mathrm{bool}^{-1}(A) \mid \mathrm{bool}^{-1}(B)$$

Note: Since $x \equiv y \pmod{m}$ means $m \mid (x - y)$,
we can interpret $f \mid g$ as $g \equiv 0 \pmod{f}$.

Thus, if $g \bmod f = 0$, then g is a logical consequence of f, and this itself ($g \bmod f = 0$) is an equation.

Key Insight: Logical Meaning of an Identity ($\forall$ Condition)

If $f(x) = 0$ is an identity (always true), then $f(x) \neq 0$ is **inconsistent**.

Thus, $(\exists x)(\mathrm{bool}((1 - \delta_0 \circ f)(x)))$ is always false, and
$\neg(\exists x)(\mathrm{bool}((1 - \delta_0 \circ f)(x)))$ is always true.

Hence, the identity f can be expressed as:

$$\neg(\exists x)(\mathrm{bool}((1 - \delta_0 \circ f)(x)))$$

Applying "Logical Flows in Algebraic Formulas" to Arithmetize Logic

Covering:

- Equations (including inconsistent, identity, and standard types)
- Functions (predicates, operators)
- Propositional logical connectives
- Logical consequence in model theory
- Equations as existential statements
- Identity equations as universal statements

Examples:

$$\begin{aligned} \text{Solve}(\lambda x.1) &= \mathbb{R} \quad \text{(the universal set, all real numbers)} \\ \text{Solve}(\lambda x.0) &= \varnothing \quad \text{(empty set)} \\ \text{Solve}(\lambda x.ax^2+bx+c) &= \left\{ \frac{-b \pm \sqrt{b^2-4ac}}{2} \right\} \end{aligned}$$

Also:

$$(\text{bool}^{-1} \circ \text{boolf})(S) = 1_S$$

$$(\text{bool}^{-1} \circ \text{boolf} \circ \text{Solve})(x) = 1_{\text{solve}(x)}$$

Therefore, by using the function composition $\text{bool}^{-1} \circ \text{boolf} \circ \text{Solve}$, one can perform **predicate logic using arithmetic**, without having to define natural-language predicates. . . . (1)

Further, for any natural-language predicate P, one can **port** it into arithmetic using $\text{bool}^{-1} \circ P$. . . . (2)

Hence, by conclusions (1) and (2), **logical operations can be expressed via algebraic operations**.

Note on Conclusion (1)

This works because for any algebraic operation f, the function $\text{Solve}(f)$ already serves the role of $\text{boolf}^{-1} \circ \text{bool}^{-1}$.

Therefore, unless external set theory is used, Solve-defined sets are **set-theoretically implicit**, not external.

Bonus 1

This logical framework can be smoothly extended to:

- **Fuzzy set logic** (membership degree, like probability)
- **Multiset logic** (multiplicity degree)
- **Fuzzy multiset logic** (membership + multiplicity)

Bonus 2

By using **adjacency matrices** from linear algebra and assigning objects to node sequences, this framework can incorporate **graphs**.

This allows the use of:

- Category theory
- Fuzzy/multiset/general sets (via subset notation)
- Graph representations dependent on natural-language predicates and node structures

Caveat: When extended to graphs, the underlying structure becomes **linear algebra**, thus using **tensors as algebraic entities**, which **goes beyond high school mathematics**.

That would deviate from the **secondary goal**:

> *"A complete mathematical logic system expressible via pure high school algebra."*

This is why I'm trying to resolve things using only **Euclidean space**.

Final Thought

I see:

- Numbers as **measurable conserved concepts**
- Algebraic formulas as **statements about numbers**
- Logic embedded in those formulas as the **bridge between mathematical logic and symbolic logic**
- Mathematical logic as the **descriptive system of the logical context implied by numbers**