

Singular Value Decomposition

An application to Big Data

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Outline

1 Singular Value Decomposition

- What is it?
- How can singular values be computed?

Definition of SVD

Theorem

Given a matrix $A \in \mathbb{R}^{m \times n}$, it can always be found a decomposition such that

$$A = U\Sigma V^T$$

where $U \in \mathbb{R}^{m \times m}$, $V \in \mathbb{R}^{n \times n}$ and $\Sigma \in \mathbb{R}^{m \times n}$.

U and V are two orthogonal matrices and Σ is a diagonal matrix, namely:

$$(\Sigma)_{ij} = \begin{cases} 0, & i \neq j \\ \sigma_i, & i = j \end{cases}$$

where $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_p \geq 0$, $p = \min\{m, n\}$.

Definition of SVD

The non-zero entries of Σ , denoted by σ_i , are called *singular values*. They are arranged in a nonincreasing order by convention. The column vectors \mathbf{u}_i of U are called *left singular vectors* and those \mathbf{v}_i of V are called *right singular vectors*.

Since in general $m \neq n$, we have:

$$A = \sum_{i=1}^p \mathbf{u}_i \sigma_i \mathbf{v}_i^T$$




Definition of SVD

Singular values computation

To compute the singular values, consider the transpose of A given its decomposition:

$$A^T = (U\Sigma V^T)^T = V\Sigma^T U^T$$

References

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