Quadratic Equations

A quadratic eqution is of the form $ax^2 + bx + c = 0$, where x is an unknown variable, a,b,c are constants and a ≠ 0

$ax^2 + bx + c = 0$ is the standard form of quadratic equation.

Root of Quadratic Equation

It is the value of the unknown variable for which the quadratic equation holds true. A quadratic equation has two roots.

Nature of Roots

For equation $ax^2 + bx + c = 0$, expression $b^2 - 4ac$ is called Discriminant and denoted by D.

Value of Discriminant	Nature of roots
D < 0	Unequal and imaginary
D = 0	Real and equal
D > 0 and is a perfect square	Real, unequal and rational
D > 0 and not a perfect square	Real, unequal and irrational

Solution of Quadratic Equation

$$x^2 - 6x + 5 = 0$$

Factorization

- Not conveniently applicable for all quadratic equations
- Simplest method

Example:

$x^2 - 6x + 5 = 0$

$x^2 - 5x - x + 5 = 0$

$x(x - 5) - 1(x - 5) = 0$

$(x - 5)(x - 1) = 0$

x = 5   or   x = 1

Completing square

Example:

$x^2 - 6x + 5 = 0$; Standard form

$x^2 - 6x + 5 + 4 - 4 = 0$

; Add and subtract square of half coefficient of x

$x^2 - 6x + 9 = 4$; Rearrange terms

$(x - 3)^2 = (2)^2$; Making whole squares as applicable

$(x - 3) = \pm 2$; Taking square root

x = 3 + 2 or   x = 3 - 2 ; Simplifying

x = 5   or   x = 1

Quadratic formula

- Formula used to calculate,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Applicable for all equations

Example:

$x^2 - 6x + 5 = 0$

Compare with

$ax^2 + bx + c = 0$

To get a = 1, b = -6, c = 5

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{6 \pm \sqrt{36 - 20}}{2}$$

$$x = \frac{6 \pm \sqrt{16}}{2} = \frac{6 \pm 4}{2}$$

$x = 3 \pm 2$

x = 5   or   x = 1