Problem 1

$$s_n = \sum_{i=2}^{n} \frac{1}{i^2 - 1} = \frac{(n-1)(3n+2)}{4n(n+1)}$$

Proof by Induction

We will prove the statement

$$s_n = \sum_{i=2}^{n} \frac{1}{i^2 - 1} = \frac{(n-1)(3n+2)}{4n(n+1)}$$

using mathematical induction.

Base Case

For $n = 2$:

$$s_2 = \sum_{i=2}^{2} \frac{1}{i^2 - 1} = \frac{1}{3}$$

$$\frac{(2-1)(3 \cdot 2 + 2)}{4 \cdot 2 \cdot (2+1)} = \frac{1 \cdot 8}{4 \cdot 2 \cdot 3} = \frac{8}{24} = \frac{1}{3}$$

So, the base case holds true.

Inductive Step

Assume that the statement is true for some $i = 2, 3, 4, ..., n$

$$s_i = \frac{(i-1)(3i+2)}{4i(i+1)}$$

Show that

$$s_{n+1} = \frac{n(3n+5)}{4(n+1)(n+2)}$$

Using the inductive hypothesis:

$$s_{n+1} = \sum_{i=2}^{n+1} \frac{1}{i^2 - 1} = s_n + \frac{1}{(n+1)^2 - 1}$$

$$s_{n+1} = \frac{(n-1)(3n+2)}{4n(n+1)} + \frac{1}{(n+1-1)(n+1+1)} = \frac{(n-1)(3n+2)}{4n(n+1)} + \frac{1}{n(n+2)}$$

To add these fractions, find a common denominator:

$$s_{n+1} = \frac{(n-1)(3n+2)(n+2) + 4(n+1)}{4n(n+1)(n+2)}$$

Simplifying the numerator:

$$s_{n+1} = \frac{(3n^2 - n - 2)(n+2) + 4(n+1)}{4n(n+1)(n+2)}$$

$$s_{n+1} = \frac{(3n^3 + 6n^2 - n^2 - 2n - 2n - 4) + 4n + 4}{4n(n+1)(n+2)}$$

$$s_{n+1} = \frac{(3n^3 + 5n^2 - 4n - 4) + 4n + 4}{4n(n+1)(n+2)}$$

$$s_{n+1} = \frac{3n^3 + 5n^2}{4n(n+1)(n+2)}$$

$$s_{n+1} = \frac{n^2(3n+5)}{4n(n+1)(n+2)}$$

$$s_{n+1} = \frac{n(3n+5)}{4(n+1)(n+2)}$$

This matches the form of $\frac{n(3n+5)}{4(n+1)(n+2)}$, confirming the statement holds for $n+1$.

By the principle of mathematical induction, the statement is true for all $n \geq 2$.

Problem 2

Given sets A and B:

$$A = \{2, 3, 5, 7\}$$

$$B = \{2, 4, 5, 8, 9\}$$

The universal set U is:

$$U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

First, we find the complement of B, denoted as $\overline{B}$:

$$\overline{B} = U - B = \{0, 1, 3, 6, 7, 10\}$$

Next, we find the union of A and $\overline{B}$:

$$A \cup \overline{B} = \{2, 3, 5, 7\} \cup \{0, 1, 3, 6, 7, 10\} = \{0, 1, 2, 3, 5, 6, 7, 10\}$$

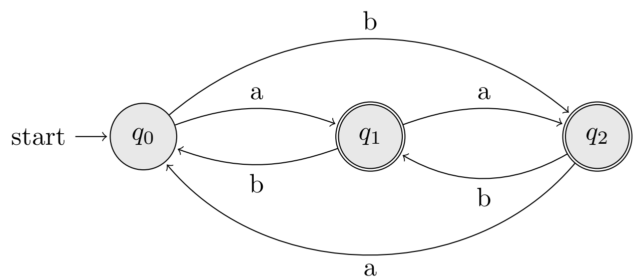
Then, we find the complement of $A \cup \overline{B}$, denoted as $\overline{A \cup \overline{B}}$:

$$\overline{A \cup \overline{B}} = U - (A \cup \overline{B}) = \{4, 8, 9\}$$

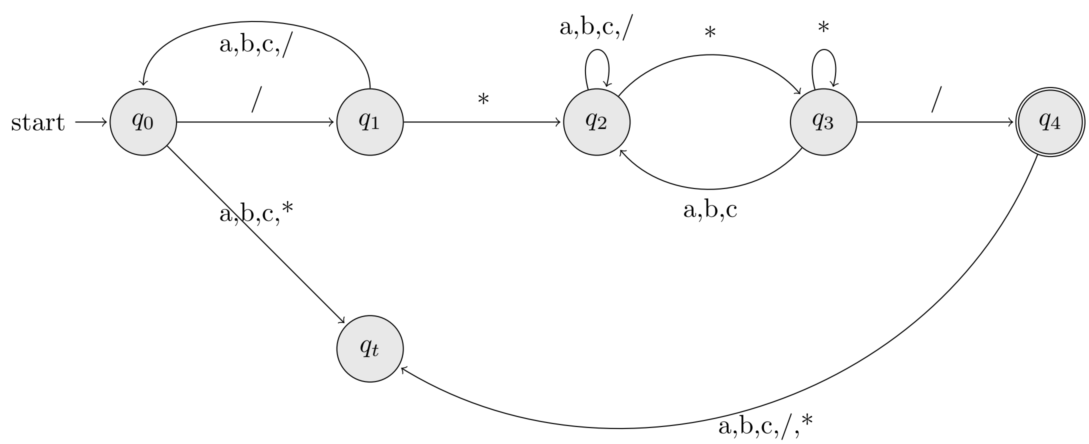
Thus, the set S is:

$$S = \{4, 8, 9\}$$

Problem 3



Problem 4



Problem 5

$$\begin{array}{lcl} S & \rightarrow & 0A \mid 1B \mid \lambda \\ A & \rightarrow & 1A \mid 0B \mid \lambda \\ B & \rightarrow & 0B \mid 1A \end{array}$$

Problem 6

The language L generated by the grammar G is:

$$L = \{w \colon |w| \mod 4 \neq 0\}$$

Short Description of the Conclusion

The outputs of the grammar can be a sequence of 'a' or 'a's, but it follows one of these three rules: the length of 'a's is $4q+1$, $4q+2$, or $4q+3$. This means the length of 'a's is not divisible by 4, and the remainder of this division could be 1, 2, or 3, but not 0. Therefore, the length of the 'a's mod 4 is not equal to zero.

Examples of Derivations

1. Derivations for $S \to A$:

$S \to A \to a$

$S \to aaaaA \to aaaaa$

$S \to aaaaA \to aaaaaaaaA \to aaaaaaaaa$

$S \to aaaaA \to aaaaaaaaA \to aaaaaaaaaaaaA \to aaaaaaaaaaaaa$

and so on.

2. Derivations for $S \to B$:

$S \to B \to aa$

$S \to aaaaB \to aaaaaa$

$S \to aaaaB \to aaaaaaaaB \to aaaaaaaaaa$

$S \to aaaaB \to aaaaaaaaB \to aaaaaaaaaaaaB \to aaaaaaaaaaaaaa$

and so on.

3. Derivations for $S \to C$:

$S \to C \to aaa$

$S \to aaaaC \to aaaaaaa$

$S \to aaaaC \to aaaaaaaaC \to aaaaaaaaaaa$

$S \to aaaaC \to aaaaaaaaC \to aaaaaaaaaaaaC \to aaaaaaaaaaaaaaa$

and so on.