TUGAS 3

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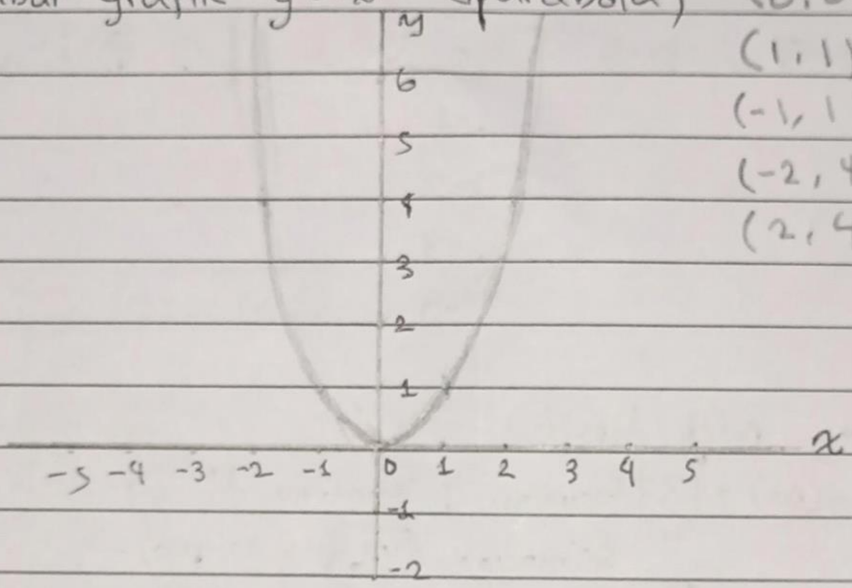
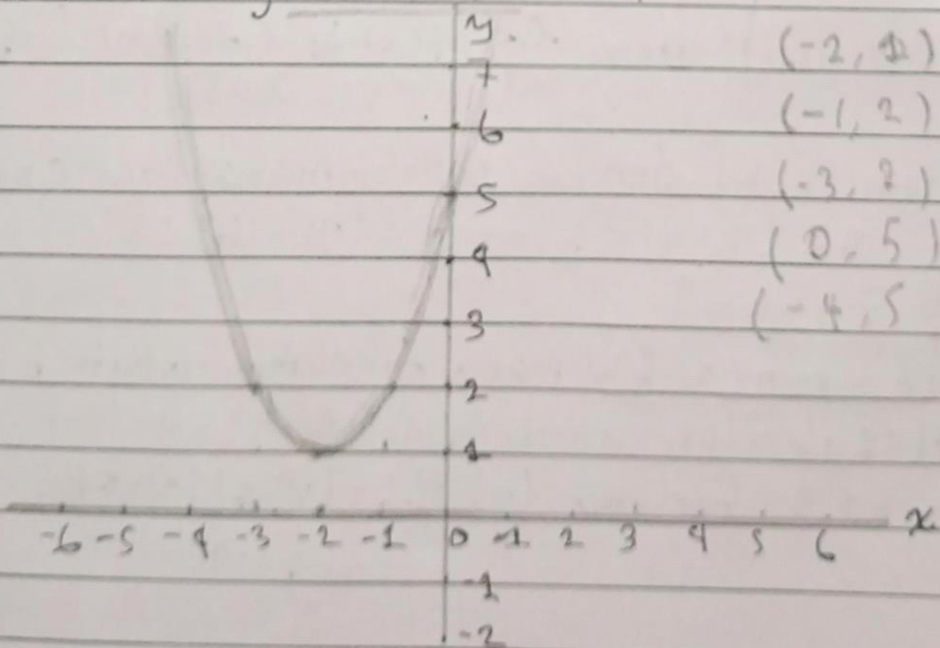
1. Gambar grafik $y = x^2$ (parabola)

$(0,0)$
$(1,1)$
$(-1,1)$
$(-2,4)$
$(2,4)$



Gambar grafik $y = x^2 + 4x + 5 = (x+2)^2 + 1$
yang berarti bentuknya $\approx x^2$, tetapi geser x ke kiri 2 satuan dan y ke atas 1 satuan

$(-2,1)$
$(-1,2)$
$(-3,2)$
$(0,5)$
$(-4,5)$



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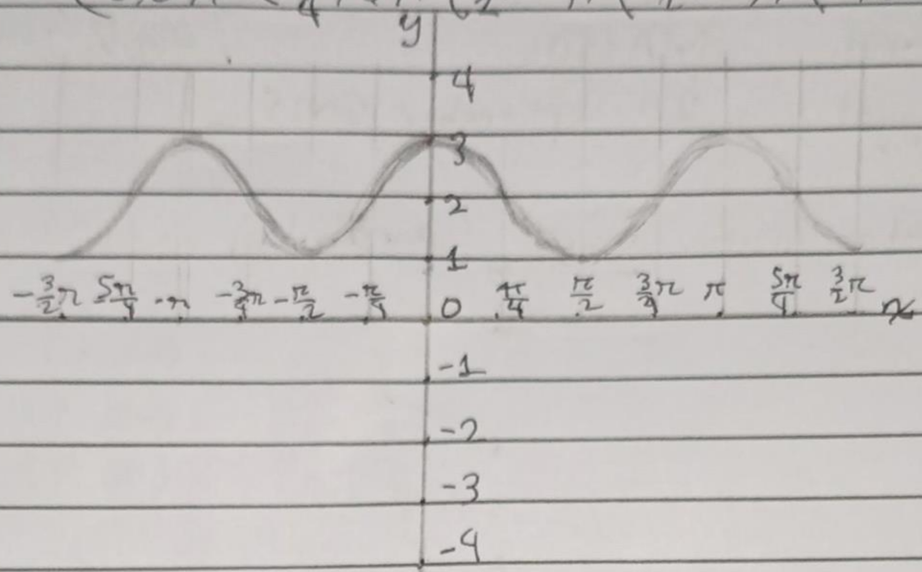
2 Grafik $y = \cos(2x) + 2$, maka geser y ke atas dengan 2 satuan dan periode cos dari $\frac{2\pi}{2} \to \pi$ sehingga ambil titik $(0, 3)$, $(\frac{\pi}{4}, 2)$, $(\frac{\pi}{2}, 1)$, $(\frac{3\pi}{4}, 2)$, $(\pi, 3)$



3 Menghitung nilai limit

(a) $\lim_{x \to 0} f(x) = \boxed{3}$ karena f kontinu di $[0-\varepsilon, 0+\varepsilon]$ sehingga $\lim_{x \to 0} f(x) = f(0) = 3$

(b) $\lim_{x \to 3^+} f(x) = \boxed{4}$ karena dari kanan mendekati 4

(c) $\lim_{x \to 3^-} f(x) = \boxed{1}$ karena dari kiri mendekati 1

(d) $\boxed{\text{Tidak ada}}$ karena $\lim_{x \to 3^+} f(x) \neq \lim_{x \to 3^-} f(x)$ $[4 \neq 1]$

4. Substitusi $f(x)$ dengan $x^2 - 5$, maka soal ekuivalen dengan $0 < |x-4| < \delta \Rightarrow 0 < |x^2 - 16| < \varepsilon$

$\forall \varepsilon > 0, \exists \delta > 0 \ni$

Ambil $\boxed{\delta = \min(1, \frac{\varepsilon}{9})}$, maka diketahui untuk $\delta \leq 1$

$0 < |x-4| < 1 \to$ berlaku untuk $\forall x$, $|x+4| < 9$

Untuk $\delta \leq \frac{\varepsilon}{9}$, berlaku
$$|x^2 - 16| = |x-4||x+4| < \left(\frac{\varepsilon}{9}\right) \cdot 9 = \varepsilon$$

Maka, $\forall \varepsilon > 0$. $\exists \delta > 0 \ni 0 < |x-4| < \delta \Rightarrow 0 < |x^2-16| < \varepsilon$.

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5. $\lim\limits_{x\to 1} f(x) = 2$ dan $\lim\limits_{x\to 1} g(x) = -1$

(a) $\lim\limits_{x\to 1} \dfrac{(x+2)f(x)+2g(x)}{2x+1-g(x)}$

$= \dfrac{\lim\limits_{x\to 1} (x+2)f(x)+2g(x)}{\lim\limits_{x\to 1} 2x+1-g(x)}$

$= \dfrac{(\lim\limits_{x\to 1} x+2)(\lim\limits_{x\to 1} f(x)) + 2(\lim\limits_{x\to 1} g(x))}{\lim\limits_{x\to 1}(2x+1) - \lim\limits_{x\to 1} g(x)}$

$= \dfrac{3(2)+2(-1)}{3-(-1)} = \dfrac{4}{4} = \dfrac{\boxed{1}}{2}$

(b) $\lim\limits_{x\to 1} \sqrt{7f(x)+2(g(x))^2}$

$= \sqrt{\lim\limits_{x\to 1} 7f(x)+2(g(x))^2}$

$= \sqrt{(\lim\limits_{x\to 1} 7f(x)) + (\lim\limits_{x\to 1} 2(g(x))^2)}$

$= \sqrt{7(\lim\limits_{x\to 1} f(x)) + 2((\lim\limits_{x\to 1} g(x))^2)}$

$= \sqrt{7(2) + 2((\lim\limits_{x\to 1} g(x))^2)}$

$= \sqrt{14+2(1)}$

$= \sqrt{16} = \dfrac{\boxed{4}}{2}$

6. Langsung menghitung nilai limit

(a) $\lim\limits_{t\to -3} \dfrac{t^2-9}{2t^3+7t+3}$

$= \lim\limits_{t\to -3} \dfrac{(t-3)(t+3)}{(2t+1)(t+3)}$

$= \lim\limits_{t\to -3} \dfrac{t-3}{2t+1}$

$= \dfrac{(-3)-3}{2(-3)+1} = \dfrac{-6}{-5} = \boxed{\dfrac{6}{5}}$

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(b) $\lim_{u \to 16} \frac{4-\sqrt{u}}{16u-u^2}$

$= \lim_{u \to 16} \frac{4-\sqrt{u}}{u(16-u)} = \lim_{u \to 16} \frac{4-\sqrt{u}}{u(4+\sqrt{u})(4-\sqrt{u})}$

$= \lim_{u \to 16} \frac{1}{u(4+\sqrt{u})}$

$= \frac{1}{16\cdot(4+\sqrt{16})} = \frac{1}{16(8)} = \boxed{\frac{1}{128}}$

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(c) $\lim_{t \to 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right)$

$= \lim_{t \to 0} \frac{\sqrt{1+t}-1}{-t\sqrt{1+t}} = \lim_{t \to 0} \frac{1-\sqrt{1+t}}{t\sqrt{1+t}}$

$= \lim_{t \to 0} \frac{(1-\sqrt{1+t})(1+\sqrt{1+t})}{t\sqrt{1+t}\,(1+\sqrt{1+t})}$

$= \lim_{t \to 0} \frac{1-(1+t)}{t\sqrt{1+t}\,(1+\sqrt{1+t})} = \lim_{t \to 0} \frac{-t}{t\sqrt{1+t}\,(1+\sqrt{1+t})}$

$= \lim_{t \to 0} \frac{-1}{\sqrt{1+t}\,(1+\sqrt{1+t})}$

$= \frac{-1}{\sqrt{1+0}\,(1+\sqrt{1+0})} = \boxed{-\frac{1}{2}}$

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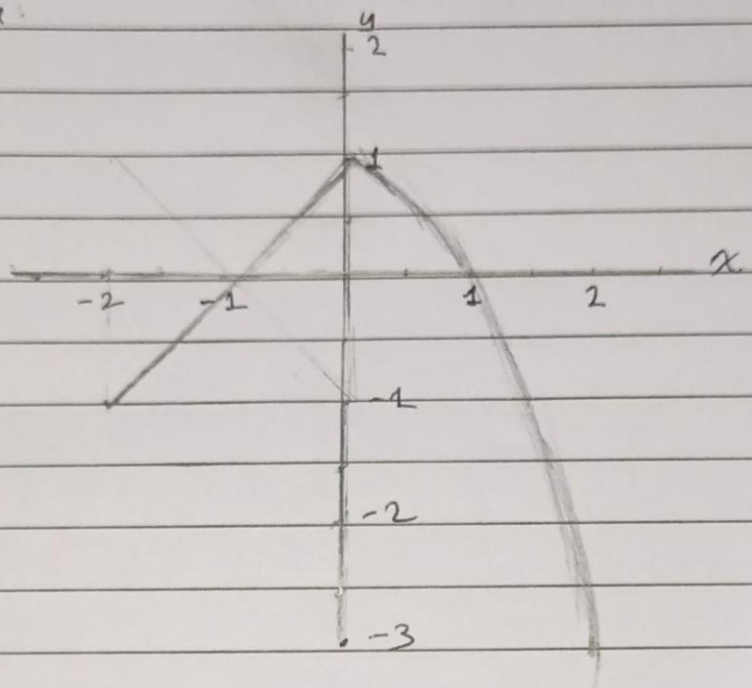
7. $f(x) = \begin{cases} x+1, & -2 \le x < 0 \\ 1-x^2 & 0 \le x \le 2 \end{cases}$ $\begin{matrix} \to (-2,-1), (-1,0), (0,1) \\ \to (0,1), (1,0), (2,-3) \end{matrix}$

Sketsa:



Nilai limit dari $\boxed{\lim_{x \to 0} f(x) = 1}$ karena limit kanan sama dengan limit kiri

8. $\lim_{x \to 1} \frac{\sqrt{2x}\,|x-1|}{x-1}$ itu $\boxed{\text{tidak ada}}$

Akan dibuktikan dengan melihat tanda dari

$\lim_{x \to 1^+} \frac{\sqrt{2x}\,|x-1|}{x-1}$ dan $\lim_{x \to 1^-} \frac{\sqrt{2x}\,|x-1|}{x-1}$

Perhatikan bahwa $\sqrt{2x}\,|x-1| > 0$, tetapi $x-1 < 0 \iff x < 1$

$x-1 > 0 \iff x > 1$

Dari kiri $(x<1)$, nilai limitnya negatif, tetapi dari kanan $(x>1)$, nilai limitnya positif. Agar ada, kanan = kiri $\Rightarrow$ positif = negatif. kontradiksi sehingga nilai limitnya $\boxed{\text{tidak ada}}$

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