Imp

* Heisenberg Uncertainity Principle:

De-Broglie proposed that the motion of a particle with a velocity v is controlled by wave packet with a group velocity v_g.

$$v_g = \frac{d\omega}{dk}$$

He further suggested that there is a limit beyond which we can not determine simultaneously both the momentum and the position of the particle. This is known as uncertainty principle.

The uncertainty in momentum Δp_x and uncertainty in position Δx are related as

$$\Delta x \, \Delta p_x \geq \hbar \qquad \left(\hbar = \frac{h}{2\pi}\right)$$

$$\Delta x \, \Delta p_x \geq \frac{h}{2\pi}$$

Similarly $\boxed{\Delta J \, \Delta \phi \geq \hbar}$ $\Delta J \rightarrow$ uncertainity in angular momentum

$\Delta \phi \rightarrow$ uncertainity in angular position

& $\boxed{\Delta E \, \Delta t \geq \hbar}$

$\Delta E \rightarrow$ uncertainty in energy

$\Delta t \rightarrow$ uncertainty in time.

* Application of Uncertainty principle:

(i) Non-existance of free electrons in the Nucleus:-

1→ The maximum kinetic energy of an electron emitted by radio-active nuclei is 4 MeV.

2- Rest mass of e^- is $m_0 = 9.11 \times 10^{-31}$ kg

3- ~~Dia~~ Diameter of nucleus $= 2 \times 10^{-14}$ m.

If the e^- exist in the nucleus, it can be anywhere within the diameter of the nucleus. Therefore. Therefore maximum uncertainty Δx in position of the e^- is same as the diameter of the nucleus.

$$\Delta x = 2 \times 10^{-14} \text{ m.}$$

A/c to Heisenberg Uncertainty principle

$$\Delta x \cdot \Delta p_x \geq \hbar \cancel{/2}$$

$$\Delta p_x \geq \frac{\hbar}{\cancel{2}\Delta x}$$

The minimum uncertainty in momentum is given by

$$\Delta p_x = \frac{\hbar}{\cancel{2}\Delta x} \Rightarrow \Delta p_x = \frac{\hbar}{\Delta x}$$

$$\Delta p_x = \frac{1.0536 \times 10^{-34}}{2 \times 10^{-14}} \qquad \hbar = 1.056 \times 10^{-34}$$

$$\Delta p_x = 5.28 \times 10^{-21} \ kg \cdot m/sec.$$

The kinetic energy of the mass

$$K.E. = \frac{p^2}{2m} = \frac{(\Delta p_x)^2}{2 m_0}$$

$$\boxed{K.E. \cong 97 MeV}$$

This mean that if the e^- exist inside the nucleus, their kinetic energy must be of the order of 97 MeV. But no e^- posses the energy greater than 4 MeV. Thus e^- does not reside in the nucleus.

Radius of Bhor's First Orbit: If Δq and Δp are uncertainties in determening the position and momentum of e^- in first orbit, then

$$\Delta x \Delta p \cong \hbar$$

$$\Delta p = \frac{\hbar}{\Delta x}$$

The uncertainty in K.E. may be calculated as follows

$$T = K.E. = p^2/2m$$

uncertainty in K.E. of e^-

$$\Delta T = \frac{(\Delta p)^2}{2m} = \frac{1}{2m}\left(\frac{\hbar}{\Delta x}\right)^2$$

The potential energy of e^- is

$$V = \frac{1}{4\pi\epsilon_0}\frac{(Ze)(-e)}{(\Delta x)} = -\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{\Delta x}$$

Uncertainty in total energy;

$$\Delta E = \Delta T + \Delta V$$

$$\Delta E = \frac{\hbar^2}{2m(\Delta x)^2} - \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{\Delta x}$$

for minimum or maximum value of E, $\frac{dE}{dx} = 0$

$$\frac{d(\Delta E)}{d(\Delta x)} = \frac{-\hbar^2 2}{2m(\Delta x)^3} + \frac{1}{4\pi\epsilon_0}\left(\frac{Ze^2}{(\Delta x)^2}\right) = 0$$

$$\Delta x = 4\pi\epsilon_0\frac{\hbar^2}{mZe^2}$$

Radius of Bohr's first orbit

$$r = \Delta x = \frac{4\pi\epsilon_0\hbar^2}{mZe^2} \qquad \hbar = \frac{h}{2\pi}$$

$$\boxed{r = \frac{\epsilon_0 h^2}{\pi m Z e^2}}$$