Properties of Wave function:

The probability that a particle will be found at a given place in space at a given instant of time is characterized by the function $\psi(x, y, z, t)$. It is called the wavefunction. This function can be real or complex.

The wavefunction ψ itself has no direct physical significance. There is a simple reason why ψ can not be interpreted in terms of experiment. The probability that the particle at a certain place at a time must be lie between 0 and 1.

The square of the absolute value of the wavefunction which is known as probability density. It is a real quantity and given by

$$\rho = \psi\psi^* = |\psi|^2$$

Total probability of finding the particle in a given space is written as

$$\rho = \int_V \psi\psi^* dV = \int_V |\psi|^2 dV$$

Since the particle is found somewhere in space

$$\boxed{\int_V |\psi|^2 dV = 1}$$

A wavefunction satisfies above equation is called a normalized wave function.

If the product of the wavefunction $\psi_1(x)$ and the complex conjugate $\psi_2^*(x)$ of a function $\psi_2(x)$ vanishes when integrated in the interval $[a, b]$ wrt to x, if

$$\int_a^b \psi_2^*(x)\,\psi_1(x)\,dx = 0$$

Then $\psi_1(x)$ and $\psi_2(x)$ are said to be orthogonal in the interval $[a, b]$

Function satisfying both conditions, orthogonality and normality are called orthonormal function and, written as

$$\int \psi_m^*(x)\,\psi_n(x)\,dx = \delta_{mn} \qquad \delta_{mn} \begin{cases} 1 & \text{if } m = n \\ 0 & \text{if } m \neq n \end{cases}$$

If ψ is not normalized wavefunction then it is possible to multiply ψ by a constant known as Normalisation constant.

$$\int \psi^* N^* N \psi\,dx = 1$$

$$N^* N \int \psi^* \psi\,dx = 1$$

$$\boxed{|N|^2 = \frac{1}{\int \psi^* \psi\,dx}}$$