$x_1 = x_0 f(b) - b f(x_0)$ $= 1.4967...$

Ques1) Find a real root of the equation $x^2 - 5x + 2 = 0$ by Newton Rapson correct to three decimal places by Newton Rapson method.

Soln:

$f(x) = x^2 - 5x + 2$

$f(0) = 2 = +ve$

$f(1) = -2 = -ve$

So, the solution will lie in $[0,1]$.

$f'(x) = 2x - 5$

So, take initial appraximation

$x_0 = 0.5$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= x_0 - \frac{x_0^2 - 5x_0 + 2}{2x_0 - 5}$$

$$= 0.5 - \frac{(0.5)^2 - 5(0.5) + 2}{2(0.5) - 5} = 0.5 - \frac{(-0.25)}{-4}$$

$$= 0.4375$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = x_1 - \frac{x_1^2 - 5x_1 + 2}{2x_1 - 5}$$

$$= 0.4375 - \frac{(0.4375)^2 - 5(0.4375) + 2}{2(0.4375) - 5}$$

$$= 0.4384$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= x_2 - \frac{x_2^2 - 5x_2 + 2}{2x_2 - 5}$$

$$= 0.4384 - \frac{(0.4384)^2 - 5(0.4384) + 2}{2(0.4384) - 5}$$

$$= 0.4384$$

The root in two consecutive solutions is correct upto 3 decimal places. Hence, the solution is correct upto 3 decimal places.

Scanned by CamScanner

$$x_6 = \frac{x_5 f(b) - b f(x_5)}{f(b) - f(x_5)} = \frac{1.6952 \times 2 - 2 \times (-0.0022)}{2 - (-0.0022)}$$

Ques 2) Using Regular-Falsi method, find a real root of the equation $x^3 - x^2 - 2 = 0$ correct upto 3 decimal places.

Soln: $x^3 - x^2 - 2 = 0$ is continuous.

$f(x) = x^3 - x^2 - 2$

$f(0) = -2 = -ve$

$f(1) = -2 = -ve$

$f(2) = 2 = +ve$

So, solution will lie in $[1, 2]$.

Here, $a = 1$, $f(a) = -2$, $b = 2$, $f(b) = 2$

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)} = \frac{1 \times 2 - 2 \times (-2)}{2 - (-2)} = \frac{2+4}{4}$$

$= 1.5$

$f(x_1) = (1.5)^3 - (1.5)^2 - 2 = -0.875 = -ve$

Now, solution will lie in $[\cancel{0.875}\ 1.5, 2]$

$$x_2 = \frac{x_1 f(b) - b f(x_1)}{f(b) - f(x_1)} = \frac{1.5 \times 2 - 2 \times (-0.875)}{2 - (-0.875)}$$

$= 1.6521$

$f(x_2) = (1.6521)^3 - (1.6521)^2 - 2 = -0.2201 = -ve$

Now, solution will lie in $[1.6521, 2]$

$$x_3 = \frac{x_2 f(b) - b f(x_2)}{f(b) - f(x_2)} = \frac{1.6521 \times 2 - 2 \times (-0.2201)}{2 - (-0.2201)}$$

$= 1.6865$

$f(x_3) = (1.6865)^3 - (1.6865)^2 - 2 = -0.0474 = -ve$

Now, solution will lie in $[1.6865, 2]$.

$$x_4 = \frac{x_3 f(b) - b f(x_3)}{f(b) - f(x_3)} = \frac{1.6865 \times 2 - 2 \times (-0.0474)}{2 - (-0.0474)}$$

$= 1.6937$

$f(x_4) = (1.6937)^3 - (1.6937)^2 - 2 = -0.01003 = -ve$

Now, solution will lie in ~~[0.01003~~ $[1.6937, 2]$

$$x_5 = \frac{x_4 f(b) - b f(x_4)}{f(b) - f(x_4)} = \frac{1.6937 \times 2 - 2 \times (-0.01003)}{2 - (-0.01003)}$$

$= 1.6952$

$f(x_5) = (1.6952)^3 - (1.6952)^2 - 2 = -0.0022 = -ve$

Now, solution will lie in $[1.6952, 2]$.

Scanned by CamScanner

$x_3 = \phi(x_2)$

$$x_6 = \frac{x_5 f(b) - b f(x_5)}{f(b) - f(x_5)} = \frac{1.6952 \times 2 - 2 \times (-0.0022)}{2 - (-0.0022)}$$

$x_6 = 1.6955$

$f(x_6) = (1.6955)^3 - (1.6955)^2 - 2 = -0.00063 = -ve$

The root in two consequtive solutions is correct upto 3 decimal places. Hence, the solution is correct upto 3 decimal places.

Ques 3) Find the root of the equation $\log x - \cos x = 0$ by using bisection method.

Solu^n:

$f(x) = \log x - \cos x$

$f(1) = -0.5403 = -ve$

$f(2) = 0.7171 = +ve$

So, the solution will lie in $[1, 2]$

Let initial approximation be x_0

$$x_0 = \frac{1+2}{2} = 1.5$$

$f(1.5) = \log(1.5) - \cos(1.5)$

$= 0.1053$ solution will lie in $[1, 1.5]$

So, next approximation is x_1.

$$x_1 = \frac{1 + 1.5}{2}$$

$= 1.25$

$f(1.25) = \log(1.25) - \cos(1.25) = -0.2184 = -ve$

solution will lie in $[1.25, 1.5]$

So, next approximation is x_2.

$$x_2 = \frac{1.25 + 1.5}{2} = 1.375$$

$f(1.375) = \log(1.375) - \cos(1.375) = -0.0562 = -ve$

Solution will lie in $[1.375, 1.5]$.

So, next approximation is x_3.

$$x_3 = \frac{1.375 + 1.5}{2} = 1.4375$$

$f(1.4375) = \log(1.4375) - \cos(1.4375) = 0.0247 = +ve$

Solution will lie in $[1.375, 1.4375]$

So, next approximation is x_4.

$$x_4 = \frac{1.375 + 1.4375}{2} = 1.4062$$

$f(1.4062) = \log(1.4062) - \cos(1.4062) = -0.0158 = -ve$

Solution will lie in $[1.4062, 1.4375]$

So, next approximation is x_5.

Scanned by CamScanner

$x_5 = \frac{1.4062 + 1.4375}{2}$

$= 1.4218$

$f(1.4218) = \log(1.4218) - \cos(1.4218) = 0.0043 = +ve$

solution will lie in $[1.4062, 1.4218]$.

x_6 is the next approximation.

$x_6 = \frac{1.4062 + 1.4218}{2} = 1.414$

$f(1.414) = \log(1.414) - \cos(1.414) = -ve$

solution will lie in $[1.414, 1.4218]$

So, the next approximation is x_7.

$x_7 = \frac{1.414 + 1.4218}{2} = 1.4179$

The root in two consecutive solutions is correct upto 2 decimal places. Hence, the solution is correct upto 2 decimal places.

Ques 4) Find the cube root of 48 correct upto 4 decimal places by using iteration method.

Soln:

$f(x) = x^3 - 48$

$f(0) = -48$; $f(1) = -47$, $f(2) = -40$, $f(3) = -21$,

$f(4) = 16$

So, solution will lie between $[3, 4]$

Now, for $x = \phi(x)$ we have

$x^3 - 48 = 0 \Rightarrow x^2 \cdot x = 48$

$x^2 = \frac{48}{x} \Rightarrow x = \sqrt{\frac{48}{x}}$

$\phi(x) = \frac{4\sqrt{3}}{\sqrt{x}}$

$\phi'(x) = -\frac{\cancel{4}^{2}\sqrt{3}}{\cancel{2}x^{3/2}} \Rightarrow \phi'(x) = -\frac{2\sqrt{3}}{x^{3/2}}$

Now, for a and b, that is, 3 and 4 is

$|\phi'(3)| = \left|\frac{-3.464}{5.196}\right| = |-0.66| = 0.66 < 1$

$|\phi'(4)| = \left|\frac{-3.464}{8}\right| = |-0.433| = 0.433 < 1$

Now, we can apply iterative method is

$x_0 = 3.5$

$x_1 = \phi(x_0)$

$= \frac{4\sqrt{3}}{\sqrt{3.5}} = 3.70328$

$x_2 = \phi(x_1) = 3.60020544$

Scanned by CamScanner

Ques6) Find the value of tan 0.13 by using the table:

$x_3 = \phi(x_2)$
$= 3.6513$

$x_4 = \phi(x_3)$
$= 3.6257$

$x_5 = \phi(x_4)$
$= 3.6385$

$x_6 = \phi(x_5)$
$= 3.6321$

$x_7 = \phi(x_6)$
$= 3.6353$

$x_8 = \phi(x_7)$
$= 3.6337$

$x_9 = \phi(x_8)$
$= 3.6345$

$x_{10} = \phi(x_9)$
$= 3.6341$

The root in two consecutive solutions is correct upto 3 decimal places. Hence, the solution is correct upto 3 decimal places.

Ques5) Solve the system of equations by method of factorization:

$$x + 2y + 3z = 14$$
$$2x + 3y + 4z = 20$$
$$3x + 4y + z = 14$$

Soln: From the equation we conclude that

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 1 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 14 \\ 20 \\ 14 \end{bmatrix}$$

Let

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \quad U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$LU = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$LU = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix}$$

On comparing with A,

$$A = LU$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 1 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix}$$

$\boxed{u_{11} = 1}$, $\boxed{u_{12} = 2}$, $\boxed{u_{13} = 3}$

$l_{21}u_{11} = 2$, $l_{21}u_{12} + u_{22} = 3$, $l_{21}u_{13} + u_{23} = 4$

$\boxed{l_{21} = 2}$ $\boxed{u_{22} = -1}$ $\boxed{u_{23} = -2}$

$l_{31}u_{11} = 3$, $l_{31}u_{12} + l_{32}u_{22} = 4$, $l_{31}u_{13} + l_{32}u_{23} + u_{33} = 1$

$\boxed{l_{31} = 3}$ $\boxed{l_{32} = 2}$ $\boxed{u_{33} = -4}$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & -4 \end{bmatrix}$$

Now, we have $AX = B$

$LUX = B$

$LY = B$ where $UX = Y$ where $Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

For, $LY = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 14 \\ 20 \\ 14 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ 2y_1 + y_2 \\ 3y_1 + 2y_2 + y_3 \end{bmatrix} = \begin{bmatrix} 14 \\ 20 \\ 14 \end{bmatrix}$$

$\boxed{y_1 = 14}$, $2y_1 + y_2 = 20$, $3y_1 + 2y_2 + y_3 = 14$

$\boxed{y_2 = -8}$ $\boxed{y_3 = -12}$

For, $UX = Y$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ -8 \\ -12 \end{bmatrix}$$

$$\begin{bmatrix} x + 2y + 3z \\ -y - 2z \\ -4z \end{bmatrix} = \begin{bmatrix} 14 \\ -8 \\ -12 \end{bmatrix}$$

$-4z = -12$, $-y - 2z = -8$, $x + 2y + 3z = 14$

$\boxed{z = 3}$ $\boxed{y = 2}$ $\boxed{x = 1}$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$ Ans.

Scanned by CamScanner

Ques 16) Find the value of tan 0.13 by using the table :

x	.10	.15	.20	.25	.30
$y = \tan x$	.1003	.1511	.2027	.2553	.3093

Solu^n: $x = 0.13$, $x_0 = 0.10$, $h = 0.05$

$$u = \frac{x - x_0}{h} = \frac{0.13 - 0.10}{0.05} = 0.6$$

Difference table is :

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0.10	0.1003	0.0508	0.0008	0.0002	0.0002
0.15	0.1511	0.01516	0.001	0.0004	
0.20	0.2027	0.0526	0.0014		
0.25	0.2553	0.054			
0.30	0.3093				

By Newton Forward difference formula is :

$$f(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{\lfloor 2}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{\lfloor 3}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{\lfloor 4}\Delta^4 y_0$$

$$= 0.1003 + 0.6 \times 0.0508 + \frac{0.6(0.6-1)}{2} \times 0.0008 + \frac{0.6(0.6-1)(0.6-2)}{6} \times 0.0002 + \frac{0.6(0.6-1)(0.6-2)(0.6-3)}{24} \times 0.0002$$

$$= 0.1003 + 0.03048 - 0.00096 + 0.0000112 - 0.00000672$$

$$= 0.13068 \quad \text{Ans.}$$

Ques 17) From the following table find the pressure at temperature 175.

Temp. (in C)	140	150	160	170	180
Pressure (in $\frac{kg}{cm^2}$)	3.685	4.854	6.302	8.076	10.225

Solu^n: Since, we have to find the temperature at 175. 175 is close to last value, so we will apply Newton Backward.

$x = 175$, $x_n = 180$, $h = 10$

Scanned by CamScanner

$$u = \frac{x - x_n}{h} = \frac{175-180}{10} = -0.5$$

Difference table is:

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
140	3.685				
150	4.854	1.169			
160	6.302	1.448	0.279		
170	8.076	1.774	0.326	0.047	
180	10.225	2.149	0.375	0.049	0.002

By Newton Backward formula,

$$y(x) = y_n + u\nabla y_n + \frac{u(u+1)}{\lfloor 2}\nabla^2 y_n + \frac{u(u+1)(u+2)}{\lfloor 3}\nabla^3 y_n + \frac{u(u+1)(u+2)(u+3)}{\lfloor 4}\nabla^4 y_n$$

$$= 10.225 + (-.5)\times 2.149 + \frac{(-.5)\times .5}{2}\times 0.375 + \frac{(-.5)(.5)(1.5)}{6}\times 0.049 + \frac{(-.5)(.5)(1.5)(2.5)}{24}\times 0.002$$

$$= 10.225 - 1.0745 - 0.0468 - 0.00306 - 0.00007$$

$$= 10.225 - 1.12443$$

$$= 9.1005 \text{ Ans.}$$

Ques 8) Given $\sqrt{100} = 10$, $\sqrt{110} = 10.489$, $\sqrt{120} = 10.954$, $\sqrt{130} = 11.402$, and $\sqrt{140} = 11.832$, find $\sqrt{122}$.

Solu^n: $x = 122$, $x_0 = 120$, $h = 10$

$$u = \frac{x - x_0}{h} = \frac{122-120}{10} = 0.2$$

Since, $u < 1$ so, we will apply gauss forward formula. Difference table is

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_{-2}	100	10	0.489	-0.024	0.007	-0.008
x_{-1}	110	10.489	0.465	-0.017	0.001	
x_0	120	10.954	0.448	-0.018		
x_1	130	11.402	0.430			
x_2	140	11.832				

Scanned by CamScanner

By Gauss forward interpolation method,

$$y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{\lfloor 2}\Delta^2 y_{-1} + \frac{(u+1)u(u-1)}{\lfloor 3}\Delta^3 y_{-1} +$$

$$\frac{(u+1)u(u-1)(u-2)}{\lfloor 4}\Delta^4 y_{-2}$$

$$y(x) = 10.954 + 0.2 \times 0.448 + \frac{0.2(0.2-1) \times (-0.17)}{2} +$$

$$\frac{(0.2+1)(0.2)(0.2-1)}{6} \times (-0.001) + \frac{(0.2+1)0.2(0.2-1)(0.2-2)}{24} \times (-0.008)$$

$$= 10.954 + 0.0896 + 0.00136 + 0.000032 - 0.0001152$$

$$= 11.0448 \text{ Ans.}$$

Ques 9) Find the cubic polynomial form the following data by Lagrange's formula:

x	0	1	2	5
y	2	3	12	47

Solu^n: By Lagrange Interpolation formula,

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} f(x_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$$

$$f(x_1) + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} f(x_2) +$$

$$\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} f(x_3)$$

$$= \frac{(x-1)(x-2)(x-5)}{(0-1)(0-2)(0-5)} \times 2 + \frac{(x-0)(x-2)(x-5)}{1 \times -1 \times -4} \times 3 +$$

$$\frac{(x-0)(x-1)(x-5)}{2 \times 1 \times -3} \times 12 + \frac{(x-0)(x-1)(x-2)}{5 \times 4 \times 3} \times 47$$

$$= \frac{x^3 - 8x^2 - 17x - 10}{-5} + 3\frac{x^3 - 21x^2 - 30x}{4} -$$

$$2x^3 + 12x^2 - 10x + \frac{47x^3 - 141x^2 + 94x}{60}$$

$$= \frac{1}{60}\left[-12x^3 + 96x^2 - 204x + 120 + 45x^3 - 315x^2 + 450x - 120x^3 + 720x^2 - 600x + 47x^3 - 141x^2 + 94x\right]$$

$$= \frac{-40x^3 + 360x^2 - 260x + 120}{60} \text{ Ans.}$$

SEMES
4nth
NO:

Ques 10) From the following table, find $f(x)$ and hence find $f(6)$ by using Newton's Interpolation formula:

x	1	2	7	8
y	1	5	5	4

Soln: Divided difference table is:

x	$f(x)$	$\Delta f(x_0)$	$\Delta^2 f(x_0)$	$\Delta^3 f(x_0)$
1	1	4	$-2/3$	$1/14$
2	5	0	$-1/6$	
7	5	-1		
8	4			

By Newton divided difference formula,

$$f(x) = f(x_0) + (x-x_0)\Delta f(x_0) + (x-x_0)(x-x_1)\Delta^2 f(x_0) + (x-x_0)(x-x_1)(x-x_2)\Delta^3 f(x_0)$$

$$= 1 + (x-1)4 + (x-1)(x-2)(-2/3) + (x-1)(x-2)(x-7)\tfrac{1}{14}$$

$$= 1 + 4x - 4 + (x^2-3x+2)\left(\frac{-2}{3}\right) + \frac{x^3}{14} - \frac{10x^2}{14} + \frac{24x}{14} - \frac{21}{14}$$

$$= 1 + 4x - 4 - \frac{2}{3}x^2 + 2x - \frac{4}{3} + \frac{x^3}{14} - \frac{10}{14}x^2 + \frac{23}{14}x - 1$$

$$= \frac{x^3}{14} - \frac{29}{21}x^2 + \frac{107}{14}x - \frac{16}{3}$$

$$\therefore f(6) = \frac{(6)^3}{14} - \frac{29}{21}(6)^2 + \frac{107}{14}\times 6 - \frac{16}{3}$$

$$= 6.2380 \text{ Ans.}$$

Scanned by CamScanner