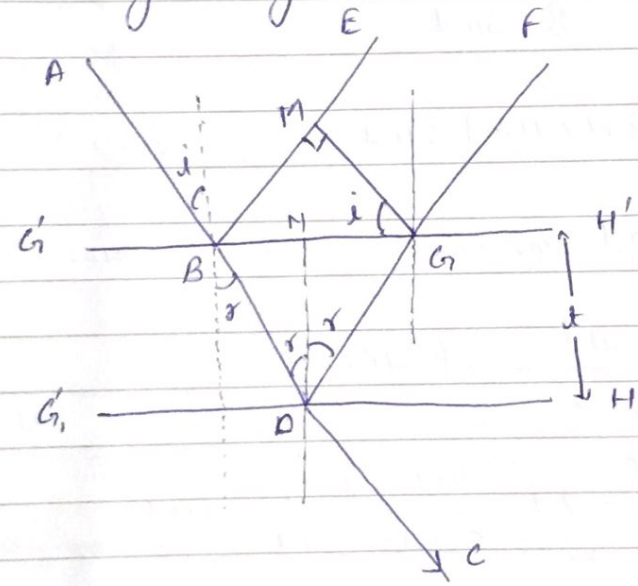
Interference in thin films: Interference can take place due to the light reflected or transmitted through a thin film.

1) Thin film of Uniform thickness:-

(i) Due to reflected light:- Consider a transparent film of uniform thickness t, and refractive index μ. A light beam is incident at an angle i on the upper surface of the film is partly reflected along BE and partly refracted along BD at any angle r.



At point D, the ray BD is again reflected from the second surface along DG and partly refracted along DC and so on. In this condition the interference takes place between BE and GF

The path difference between the reflected rays

$$\Delta = \mu(BD + DG) - BM \quad \text{—— (1)}$$

In ΔBDN

$$\cos r = \frac{ND}{BD} = \frac{\text{Base}}{\text{hypo.}}$$

$$BD = \frac{t}{\cos r} \quad \text{also} \quad BD = DG$$

$$\therefore \quad \Delta = \mu\left(\frac{2t}{\cos r}\right) - BM \qquad \text{— (2)}$$

$$\text{and} \quad \sin i = \frac{\text{Perpendicular}}{\text{hypo.}} = \frac{BM}{BG}$$

$$BM = BG \sin i$$

$$BM = (BN + NG)\sin i \qquad \tan r = \frac{\text{Par}}{\text{base}}$$

$$\therefore \quad BM = 2t \tan r \sin i \qquad \tan r = \frac{BN}{t}$$

$$\therefore \quad \Delta = \frac{2\mu t}{\cos r} - 2t \tan r \sin i \qquad \mu = \frac{\sin i}{\sin r}$$

$$\Delta = \frac{2\mu t}{\cos r} - 2t \frac{\sin r}{\cos r} \cdot \frac{\sin i}{\sin r} \cdot \sin r$$

$$\Delta = \frac{2\mu t}{\cos r} - 2t\mu \frac{\sin^2 r}{\cos r}$$

$$\Delta = \frac{2\mu t}{\cos r}\left[1 - \sin^2 r\right]$$

$$\boxed{\Delta = 2\mu t \cos r} \qquad \text{— (3)}$$

According to stoke's law, the reflected rays suffers a phase change of π or $\frac{\lambda}{2}$ (path difference).

Therefore, the effective path difference is

$$\boxed{\Delta = 2\mu t \cos r + \frac{\lambda}{2}}$$

Condition for Maxima;

$$\Delta = n\lambda$$

$$n\lambda = 2\mu t \cos r + \frac{\lambda}{2}$$

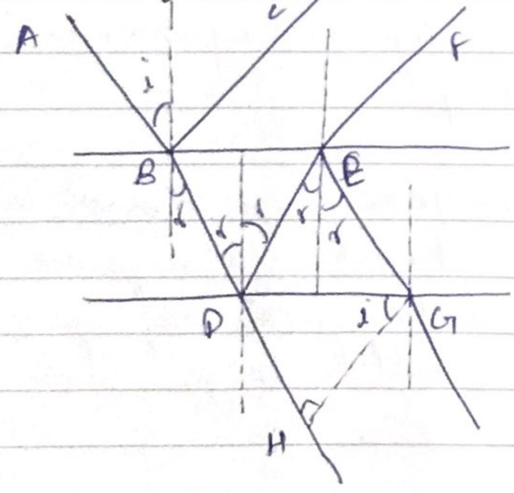
$$\boxed{(2n-1)\frac{\lambda}{2} = 2\mu t \cos r} \qquad n = 0, 1, 2, \ldots$$

Condition for minima

$$\Delta = (2n+1)\frac{\lambda}{2} = 2\mu t \cos r + \frac{\lambda}{2}$$

$$\boxed{n\lambda = 2\mu t \cos r} \qquad n = 0, 1, 2, \ldots$$

(ii) Interference due to transmitted light:-



path difference

$$\Delta = \mu(DE + EG) - DH$$

$$= 2\mu DE - DH$$

$$= \frac{2\mu t}{\cos r} - \cancel{DH}\ DG \sin i$$

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$2\mu t \cos r - 2t \tan r \sin i$

$$\Delta = 2\mu t \cos r$$

For maxima $\boxed{n\lambda = 2\mu t \cos r}$

for minima = $\boxed{(2n+1)\frac{\lambda}{2} = 2\mu t \cos r}$