

# 8.ex.8

EE24BTECH11049 - Patnam Shariq Faraz Muhammed

## Question:

Find the area enclosed between the ellipse  $\frac{x^2}{4} + \frac{y^2}{36} = 1$  and the line  $\frac{x}{2} + \frac{y}{6} = 1$ .

## Solution:

Given	Formula
$\frac{x^2}{4} + \frac{y^2}{36} = 1$	$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f$
$\frac{x}{2} + \frac{y}{6} = 1$	$\mathbf{x} = \mathbf{h} + \kappa \mathbf{m}$

TABLE 0: Equations

Substituting the given values of we have

## Conic:

$$\mathbf{V} = \begin{pmatrix} 36 & 0 \\ 0 & 4 \end{pmatrix} \quad (0.1)$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (0.2)$$

$$f = -144 \quad (0.3)$$

## Line:

$$\mathbf{h} = \begin{pmatrix} 0 \\ 6 \end{pmatrix} \quad (0.4)$$

$$\mathbf{m} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} \quad (0.5)$$

If a line intersects a conic, the  $\kappa$  value of the intersection points is given by

$$\kappa_i = \frac{-\mathbf{m}^T (\mathbf{V} \mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^T (\mathbf{V} \mathbf{h} + \mathbf{u})]^2 - g(h) (\mathbf{m}^T \mathbf{V} \mathbf{m})}}{\mathbf{m}^T \mathbf{V} \mathbf{m}} \quad (0.6)$$

Substituting the given values, we get  $\kappa$  of the points of intersections as

$$\kappa_i = 0, 2 \quad (0.7)$$

Hence the points of intersection are  $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$  and  $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

Now  $\frac{x^2}{4} + \frac{y^2}{36} = 1$  gives  $y = \pm 3 \sqrt{4 - x^2}$ . But the common area lies in the first quadrant because the the points of intersection are on positive  $x$  and  $y$  axes.

The area bounded by the curve and the line is

**Numerical solution:**

$$= \int_0^2 3 \sqrt{4 - x^2} - (6 - 3x) dx \quad (0.8)$$

$$= 3 \left[ \frac{x}{2} \sqrt{4 - x^2} + 2 \sin^{-1} \frac{x}{2} \right]_0^2 - \left[ 6x - \frac{3x^2}{2} \right]_0^2 \quad (0.9)$$

$$= 3 \left[ 0 + 2 \sin^{-1} (1) \right] - [12 - 6] \quad (0.10)$$

$$= 3\pi - 6 \approx 3.42 \quad (0.11)$$

**Computational method:**

- Split the interval  $[0, 2]$  into  $N$  parts

$$h = \frac{2 - 0}{N} \quad (0.12)$$

- Consider the points

$$x_0 = 0 \quad (0.13)$$

$$x_N = 2 \quad (0.14)$$

$$x_{i+1} = x_i + h \quad (0.15)$$

- **Trapezoidal rule**

Summing the areas of the trapezoids formed, we approximate the area between the line and curve Let

$$A = \int_0^2 (3 \sqrt{4 - x^2} - (6 - 3x)) dx \quad (0.16)$$

- It can be approximated as

$$f(x) = 3 \sqrt{4 - x^2} - 6 + 3x \quad (0.17)$$

$$A \approx \frac{h}{2} \sum_{i=1}^N (f(x_{i-1}) + f(x_i)) \quad (0.18)$$

$$j_{i+1} = j_i + \frac{h}{2} (f(x_i) + f(x_{i+1})) \quad (0.19)$$

$$j_{i+1} = j_i + \frac{h}{2} \left( 3 \sqrt{4 - x_i^2} - 6 + 3x_i + 3 \sqrt{4 - x_{i+1}^2} - 6 + 3x_{i+1} \right) \quad (0.20)$$

**Result:**

Theoretical Area: 3.4247779607693793

Computed Area: 3.4247767135897336

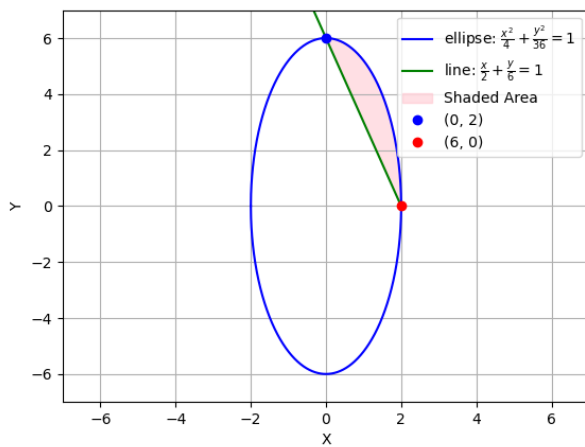


Fig. 0.1: Area between line and ellipse