1-1.7-9

EE24BTECH11049

Patnam Shariq Faraz Muhammed

QUESTION

If the points $A = (k + 1, 2k)$, $B = (3k, 2k + 3)$, and $C = (5k - 1, 5k)$ are collinear, then find the value of k.

SOLUTION:

Variables	Description	Formula
A	A point in the 2-D plane whose coordinates are $(k + 1, 2k)$	$\begin{pmatrix} k + 1 \\ 2k \end{pmatrix}$
B	A point in the 2-D plane whose coordinates are $(3k, 2k + 3)$	$\begin{pmatrix} 3k \\ 2k + 3 \end{pmatrix}$
C	A point in the 2-D plane whose coordinates are $(5k - 1, 5k)$	$\begin{pmatrix} 5k - 1 \\ 5k \end{pmatrix}$

TABLE 0: Variables Used

Points A, B, C are defined to be collinear if

$$Rank \begin{pmatrix} B - A & C - A \end{pmatrix}^T$$

is equal to 1.

$$\begin{pmatrix} B - A & C - A \end{pmatrix}^T = \begin{pmatrix} 2k - 1 & 3 \\ 4k - 2 & 3k \end{pmatrix} \tag{0.1}$$

$$\xrightarrow{R_2 = R_2 - 2R_1} \begin{pmatrix} k - 1 & 3 \\ 0 & 3k - 6 \end{pmatrix} \tag{0.2}$$

For the rank of the matrix to be 1 it should've one non-zero row.

make the elements of the 2nd row zero.
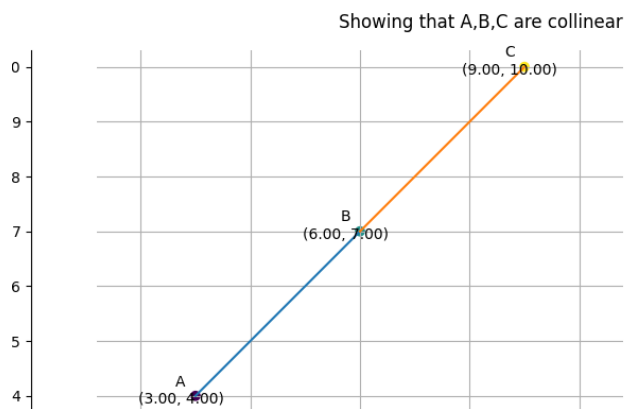
$$\implies 3k - 6 = 0$$
$$\implies k = 2$$

Fig. 0.1