Assignment 1
Chapter-11:
Limits, Continuity and Differentiability

EE24BTECH11049
Patnam Shariq Faraz Muhammed

D: MCQs with One or More than One correct

1) Let $g(x) = xf(x)$, where $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. At $x = 0$

(1995)

   a) g is differentiable but g' is not continuous
   b) g is differentiable while f is not
   c) both f and g are differentiable
   d) g is differentiable and g' is continuous

2) The function $f(x) = max\{(1-x),(1+x),2\}$, $x \in (-\infty, \infty)$

(1995)

   a) continuous at all points
   b) differentiable at all points
   c) differentiable at all points except at $x = l$ and $x = -1$
   d) continuous at all points except at $x = l$ and $x = -1$ where it is discontinuous

3) Let $h(x) = \min\left\{x, x^2\right\}$

$(1998 - 2marks)$

   a) h is continuous for all x
   b) h is differentiable for all x
   c) $h'(t) = 1$, for all $x > 1$
   d) h is not differentiable at two values of x

4) $\lim_{x\to1} \frac{\sqrt{1-\cos 2(x-1)}}{x-1}$

$(1998 - 2marks)$

   a) exists and it equals $\sqrt{2}$
   b) exists and it equals $-\sqrt{2}$
   c) does not exist because $x - 1 \to 0$
   d) does not exist because the left-hand limit is not equal to the right-hand limit

5) If $f(x) = \min\left\{1, x^2, x^3\right\}$

$(2006, 5M, -1)$

a) $f(x)$ is continuous $\forall x \in \mathbb{R}$
b) $f(x)$ is continuous and differentiable everywhere
c) $f(x)$ is not diferentiable at two points
d) $f(x)$ is not differentiable at one point

6) Let $L = \lim_{x\to 0} \frac{a-\sqrt{a^2-x^2-\frac{x^2}{4}}}{x^4}, a > 0$. If L is finite, then

(2009)

a) $a = 2$ b) $a = 1$ c) $L = \frac{1}{64}$ d) $L = \frac{1}{32}$

7) Let $f : \mathbb{R} \mapsto \mathbb{R}$ be a function such that $f(x+y) = f(x) + f(y)$, $\forall x, y \in \mathbb{R}$. If $f(x)$ is differentiable at $x = 0$, then

(2011)

a) $f(x)$ is differentiable only in a finite interval containing zero
b) $f(x)$ is continuous $\forall x \in \mathbb{R}$
c) $f\prime(x)$ is constant $\forall x \in \mathbb{R}$
d) $f(x)$ is differentiable except at finitely many points

8) If $f(x) = \begin{cases} -x - \frac{\pi}{2}, & x \le \frac{\pi}{2} \\ -\cos x, & \frac{\pi}{2} < x \le 0 \\ x - 1, & 0 < x \le 1 \\ \ln x, & x > 1 \end{cases}$

(2011)

a) $f(x)$ is continuous at $x = \frac{\pi}{2}$
b) $f(x)$ is not differentiable at $x = 0$
c) $f\prime(x)$ is differentiable at $x = 1$
d) $f(x)$ is differentiable at $x = \frac{3}{2}$

9) For every integer n, let a_n and b_n, be real numbers. Let function $f(x) : \mathbb{IR} \mapsto \mathbb{IR}$ be given by $f(x)$= $\begin{cases} a_n + \sin \pi x, & forx \in [2n, 2n+1] \\ b_n + \cos \pi x, & forx \in (2n-1, 2n) \end{cases}$ for all integers n. If f is continuous,then which of the following hold(s) for all n

(2012)

a) $a_{n-1} - b_{n-1} = 0$
b) $a_n - b_n = 1$
c) $a_n - b_{n+1} = 1$
d) $a_{n-1} - b_n = -1$

10) For $a \in \mathbb{R}$ (the set of all real numbers),

$$a \neq -1, \lim_{n\to\infty} \frac{(1^a + 2^a + \cdots + n^a)}{(n+1)^a \ [(na+1) + (na+2) + \cdots + (na+n)]} = \frac{1}{60} \text{ Then } a =$$

($JEEAdv.$2013)

a) 5 b) 7 c) $\frac{-15}{2}$ d) $\frac{-17}{2}$

11) Let $f : [a, b] \mapsto [1\ , \infty)$ be a continuous function and let $g : \mathbb{R} \mapsto \mathbb{R}$ be defined as $f(x) = \begin{cases} 0, & if x < a, \\ \int_a^x f(t)\ dt, & if a \leq x \leq b \\ \int_a^b f(t)\ dt, & if x > b \end{cases}$; then

$(JEEAdv.2013)$

a) $g(x)$ is continuous but not differentiable at a
b) $g(x)$ is differentiable on $\mathbb{R}$
c) $g(x)$ is continuous but not differentiable at b
d) $g(x)$ is continuous and differentiable at either (a) or (b) but not both

12) For every pair of continuous functions $f, g : [0, 1] \mapsto \mathbb{R}$ such that max $\{f(x) : x \in [0, 1]\}$= max $\{g(x) : x \in [0, 1]\}$, the correct statement(s) is(are):

$(JEEAdv.2014)$

a) $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0, 1]$
b) $(f(c))^2 + f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0, 1]$
c) $(f(c))^2 + 3f(c) = (g(c))^2 + g(c)$ for some $c \in [0, 1]$
d) $(f(c))^2 = (g(c))^2$ for some $c \in [0, 1]$

13) Let $g : \mathbb{R} \mapsto \mathbb{R}$ be a differentiable function with $g(0) = 0$, $g'(0) = 0$ and $g'(1) \neq 0$. Let $f(x) = \begin{cases} \frac{x}{|x|} g(x), & x \neq 0 \\ 0, & x = 0 \end{cases}$ and $h(x) = e^{|x|}$ for all $x \in \mathbb{R}$. Let $(f \circ h)(x)$ denote $f(h(x))$ and $(h \circ f)(x)$ denote $h(f(x))$. Then which of the following is(are) true?

$(JEEAdv.2015)$

a) f is differentiable at $x = 0$
b) h is differentiable at $x = 0$
c) $f \circ h$ is differentiable at $x = 0$
d) $h \circ f$ is differentiable at $x = 0$

14) Let $a, b \in \mathbb{R}$ and $f : \mathbb{R} \mapsto \mathbb{R}$ be defined by $f(x) = a\cos\left(\left|x^3 - x\right|\right) + b\,|x|\sin\left(\left|x^3 + x\right|\right)$. Then f is

$(JEEAdv.2016)$

a) differentiable at $x = 0$ if $a = 0$ and $b = 1$
b) differentiable at $x = 1$ if $a = 1$ and $b = 0$
c) NOT differentiable at $x = 0$ if $a = 1$ and $b = 0$
d) NOT differentiable at $x = 1$ if $a = 0$ and $b = 1$

15) Let $f : \left[-\frac{1}{2}, 2\right] \mapsto \mathbb{R}$ and $g : \left[-\frac{1}{2}, 2\right] \mapsto \mathbb{R}$ be functions defined by $f(x) = \left[x^2 - 3\right]$ and $g(x) = |x|\,f(x) + |4x - 7|\,f(x)$, where $[y]$ denotes the greatest integer less than or equal to y for $y \in \mathbb{R}$. Then

$(JEEAdv.2016)$

a) f is discontinuous exactly at three points in $\left[-\frac{1}{2}, 2\right]$
b) f is discontinuous exactly at four points in $\left[-\frac{1}{2}, 2\right]$
c) g is NOT differentiable exactly at four points in $\left[-\frac{1}{2}, 2\right]$
d) g is NOT differentiable exactly at five points in $\left[-\frac{1}{2}, 2\right]$