# 9-9.3-6

EE24BTECH11049
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**QUESTION**

Using integration, find the area of the region enclosed by the curve $y = x^2$ , the $x - axis$ and the ordinates $x = -2$ and $x = 1$.

**SOLUTION:**

| FUNCTION | FORMULA |
|---|---|
| $g(x)$ | $\mathbf{x^T V x} + \mathbf{2u^T x} + f = 0$ |
| The points of intersection of the line L with the conic section as above are given by $\mathbf{x}_i = \mathbf{h} + \kappa_i \mathbf{m}$ | $L : \mathbf{x} = \mathbf{h} + \kappa \mathbf{m}, \kappa \in \mathbb{R}$ <br> $\kappa_i = \frac{1}{\mathbf{m^T V m}} \left( -\mathbf{m^T} \left( \mathbf{Vh} + \mathbf{u} \right) \pm \sqrt{\left[ \mathbf{m^T} \left( \mathbf{Vh} + \mathbf{u} \right)^2 \right] - g(\mathbf{h}) \left( \mathbf{m^T V m} \right)} \right)$ |

TABLE 0: Variables Used

| DESCRIPTION |
|---|
| $\mathbf{V} = \|\|n\|\|^2 \mathbf{I} - e^2 \mathbf{nn^T}$ |
| $\mathbf{u} = ce^2 \mathbf{n} - \|\|n\|\|^2 \mathbf{F}$ |
| $f = \|\|n\|\|^2 \|\|F\|\|^2 - c^2 e^2$ |

TABLE 0: Variables Used

Substituting the given values, we have

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{0.1}$$

$$\mathbf{u} = \begin{pmatrix} \frac{-1}{2} \\ 0 \end{pmatrix} \tag{0.2}$$

$$f = 0 \tag{0.3}$$

$$\tag{0.4}$$

Substituting the values, we get the point of intersection as

$$\kappa_i = -\begin{pmatrix} 0 \\ 1 \end{pmatrix}\begin{pmatrix} \frac{-1}{2} & 0 \end{pmatrix} \pm \sqrt{\left[\begin{pmatrix} 0 & 1 \end{pmatrix}\begin{pmatrix} \frac{-1}{2} \\ 0 \end{pmatrix}\right]^2 + 1\,(1)} \tag{0.5}$$

$$\kappa_i = 1 \tag{0.6}$$

$$\tag{0.7}$$

Hence, the point of intersection is $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Similarly, the other point is given by $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$ .

The area bounded by the curve and the line is

$$\int_{-2}^{1} \left(x^2\right) dx = \frac{1}{3}\left(1 - (-8)\right) \tag{0.8}$$

$$= 3 \tag{0.9}$$

$$\tag{0.10}$$

Hence the required area is 3 .

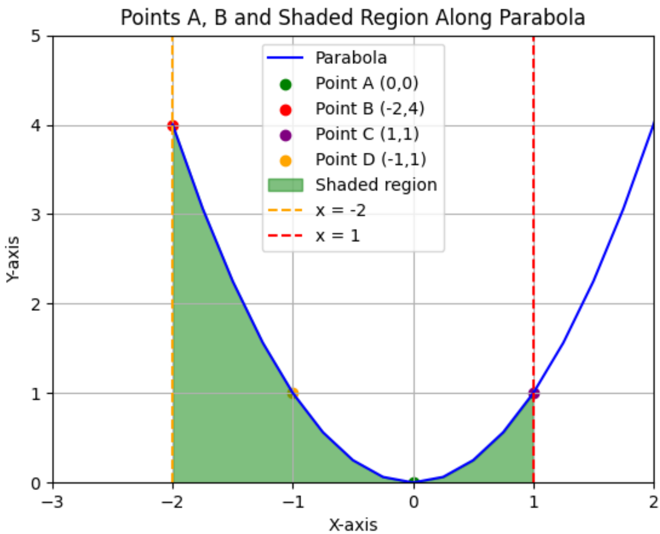


Fig. 0.1: A plot of the given question.