9-9.3-6

EE24BTECH11049
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QUESTION
Using integration, find the area of the region enclosed by the curve $y = x^2$, the $x - axis$ and the ordinates $x = -2$ and $x = 1$.
SOLUTION:

Given	formula
$y = x^2$	$\mathbf{x^TVx} + \mathbf{2u^Tx} + f = 0$
$x = -2$	$\begin{pmatrix} -2 \\ 4 \end{pmatrix}$
$x = 1$	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Substituting the given values, we have

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{0.1}$$

$$\mathbf{u} = \begin{pmatrix} \frac{-1}{2} \\ 0 \end{pmatrix} \tag{0.2}$$

$$f = 0 \tag{0.3}$$

we get the equation of curve as

$$\mathbf{y} = \mathbf{x^TVx} \tag{0.4}$$

Line equation of form $\mathbf{x} = \mathbf{h} + k\mathbf{m}$
If a line intersects the conic, k value of intersecting point is given by,

$$k_i = \frac{-\mathbf{m}^\top (\mathbf{Vh} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^\top (\mathbf{Vh} + \mathbf{u})]^2 - g(h)(\mathbf{m}^\top \mathbf{Vm})}}{\mathbf{m}^\top \mathbf{Vm}} \tag{0.5}$$

Substituting the values, we get the point of intersection as

$$\kappa_i = -\begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}\frac{-1}{2} & 0\end{pmatrix} \pm \sqrt{\left[\begin{pmatrix}0 & 1\end{pmatrix}\begin{pmatrix}\frac{-1}{2}\\0\end{pmatrix}\right]^2 + 1\,(1)} \tag{0.6}$$

$$\kappa_i = 1 \tag{0.7}$$

Hence, the point of intersection is $\begin{pmatrix}1\\1\end{pmatrix}$

Similarly, the other point is given by $\begin{pmatrix}-2\\4\end{pmatrix}$.

The area bounded by the curve and the line is

$$\int_{-2}^{1}\left(x^2\right)dx = \frac{1}{3}\left(1-(-8)\right) \tag{0.8}$$

$$= 3 \tag{0.9}$$

Hence the required area is 3 .

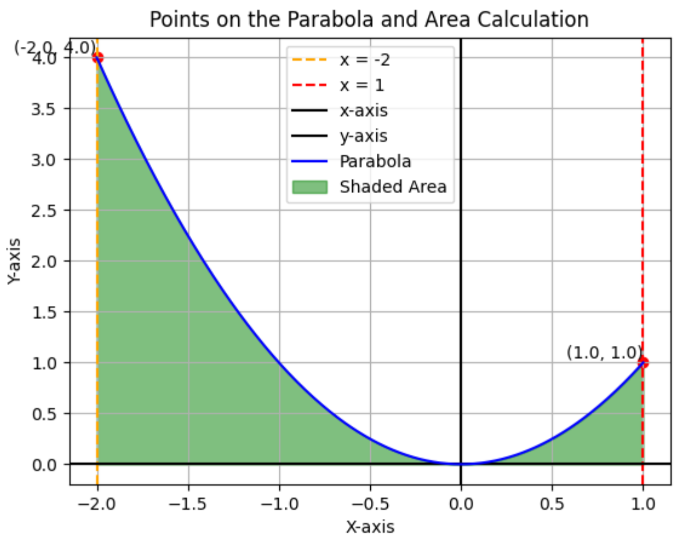

Fig. 0.1: A plot of the given question.