

9-9.3-6

EE24BTECH11049
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QUESTION

Using integration, find the area of the region enclosed by the curve $y = x^2$, the x -axis and the ordinates $x = -2$ and $x = 1$.

SOLUTION:

Given	formula
$y = x^2$	$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0$
$x = -2$	$\begin{pmatrix} -2 \\ 4 \end{pmatrix}$
$x = 1$	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Substituting the given values, we have

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad (0.1)$$

$$\mathbf{u} = \begin{pmatrix} -\frac{1}{2} \\ 0 \end{pmatrix} \quad (0.2)$$

$$f = 0 \quad (0.3)$$

we get the equation of curve as

$$\mathbf{y} = \mathbf{x}^T \mathbf{V} \mathbf{x} \quad (0.4)$$

Line equation of form $\mathbf{x} = \mathbf{h} + k\mathbf{m}$

If a line intersects the conic, k value of intersecting point is given by,

$$k_i = \frac{-\mathbf{m}^T (\mathbf{V} \mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^T (\mathbf{V} \mathbf{h} + \mathbf{u})]^2 - g(h) (\mathbf{m}^T \mathbf{V} \mathbf{m})}}{\mathbf{m}^T \mathbf{V} \mathbf{m}} \quad (0.5)$$

Substituting the values, we get the point of intersection as

$$\kappa_i = -\begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} -\frac{1}{2} & 0 \end{pmatrix} \pm \sqrt{\left[\begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} -\frac{1}{2} \\ 0 \end{pmatrix} \right]^2 + 1} (1) \quad (0.6)$$

$$\kappa_i = 1 \quad (0.7)$$

Hence, the point of intersection is $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Similarly, the other point is given by $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$.

The area bounded by the curve and the line is

$$\int_{-2}^1 (x^2) dx = \frac{1}{3} (1 - (-8)) \quad (0.8)$$

$$= 3 \quad (0.9)$$

Hence the required area is 3 .

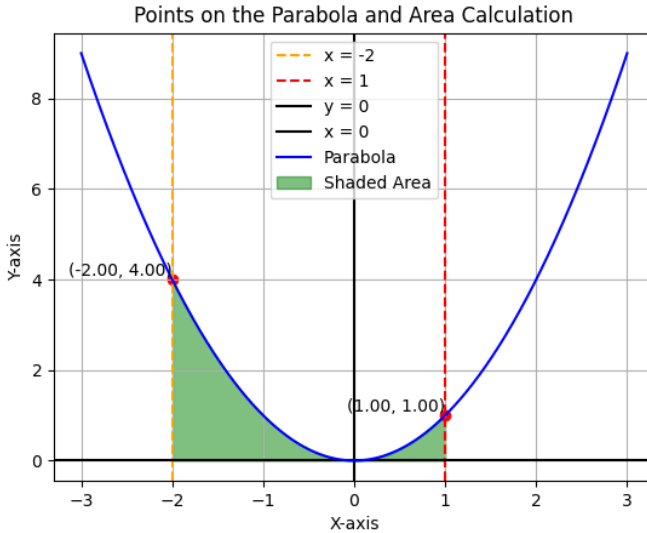


Fig. 0.1: A plot of the given question.