

Assignment 1

2021-March

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EE24BTECH11049
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1) Consider three observations a , b and c such that $b = a + c$. If the standard deviation of $a + 2$, $b + 2$, $c + 2$ is d , then which of the following is true?

a) $b^2 = a^2 + c^2 + 3d^2$

c) $b^2 = 3(a^2 + c^2) + 9d^2$

b) $b^2 = 3(a^2 + c^2) - 9d^2$

d) $b^2 = 3(a^2 + c^2 + d^2)$

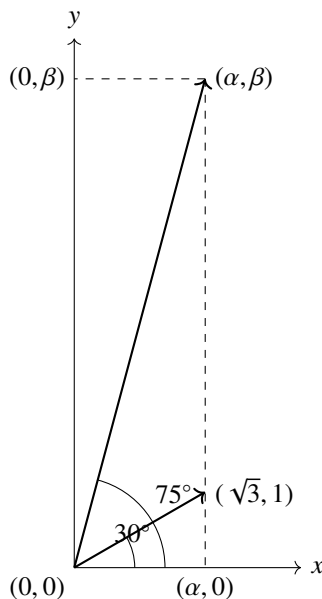
2) Let a vector $\alpha\hat{i} + \beta\hat{j}$ be obtained by rotating the vector $\sqrt{3}\hat{i} + \hat{j}$ by an angle 45° about the origin in counterclockwise direction in the first quadrant. Then the area of triangle having vertices (α, β) , $(0, \beta)$ and $(0, 0)$ is equal to:

a) 1

b) $\frac{1}{2}$

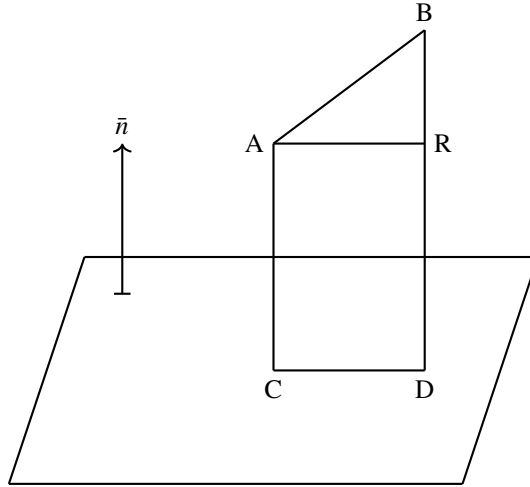
c) $\frac{1}{\sqrt{2}}$

d) $2\sqrt{2}$



- 3) If for $a > 0$, the feet of perpendiculars from the points $A(a, -2a, 3)$ and $B(0, 4, 5)$ on the plane $lx + my + nz = 0$ are points $C(0, -a, -1)$ and D respectively, then the length of line segment CD is equal to:

- a) $\sqrt{41}$ b) $\sqrt{55}$ c) $\sqrt{31}$ d) $\sqrt{66}$



- 4) The range of $a \in \mathbf{R}$ for which the function

$$f(x) = (4a - 3)(x + \log_e 5) + (a - 7) \cot\left(\frac{x}{2}\right) \sin^2\left(\frac{x}{2}\right),$$

$x \neq 2n\pi, n \in \mathbf{N}$ has critical points, is

- a) $\left[-\frac{4}{3}, 2\right]$ b) $[1, \infty)$ c) $(-\infty, -1]$ d) $(-3, 1)$

- 5) Let the functions $f : \mathbf{R} \mapsto \mathbf{R}$ and $g : \mathbf{R} \mapsto \mathbf{R}$ be defined as: $f(x) = \begin{cases} x + 2, & x \leq 0 \\ x^2, & x \geq 0 \end{cases}$ and $g(x) = \begin{cases} x^3, & x < 1 \\ 3x - 2, & x \geq 1 \end{cases}$. Then, the number of points in \mathbf{R} where $(f \circ g)(x)$ is NOT differentiable is equal to:

- a) 1 b) 2 c) 3 d) 0

- 6) Let a complex number $z, |z| \neq 1$, satisfy $\log_{\frac{1}{\sqrt{2}}} \left[\frac{(|z|+11)}{(|z|-1)^2} \right] \leq 2$. Then, the largest value of $|z|$ is equal to

- a) 5 b) 8 c) 6 d) 7

7) A pack of cards has one card missing. Two cards are drawn randomly and are found to be spades. The probability that the missing card is not a spade is:

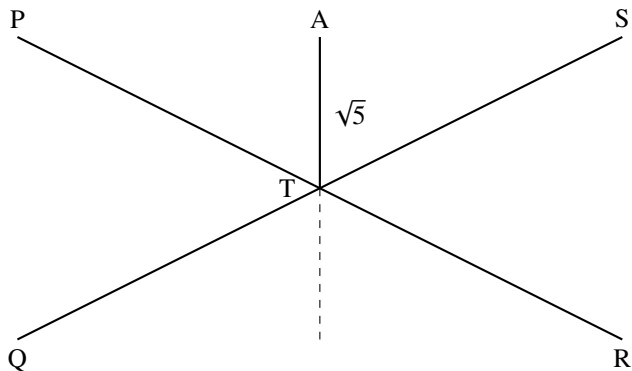
- a) $\frac{3}{4}$ b) $\frac{52}{867}$ c) $\frac{39}{50}$ d) $\frac{22}{425}$

8) If n is the number of irrational terms in the expansion of $\left[3^{\frac{1}{4}} + 5^{\frac{1}{8}}\right]^{60}$, then $(n-1)$ is divisible by

- a) 8 b) 26 c) 7 d) 30

9) Let the position vectors of two points P and Q be $3\hat{i} - \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} - 4\hat{k}$ respectively. Let R and S be two points such that the direction ratios of lines \mathbf{PR} and \mathbf{QS} are $(4, -1, 2)$ and $(-2, 1, -2)$ respectively. Let lines \mathbf{PR} and \mathbf{QS} intersect at T . If the vector \mathbf{TA} is perpendicular to both \mathbf{PR} and \mathbf{QS} and the length of vector \mathbf{TA} is $\sqrt{5}$ units, then the modulus of a position vector of A is:

- a) $\sqrt{5}$ b) $\sqrt{171}$ c) $\sqrt{227}$ d) $\sqrt{482}$



10) If the three normals drawn to the parabola, $y^2 = 2x$ pass through the point $(a, 0)$ $a \neq 0$, then ' a ' must be greater than

- a) 1 b) $\frac{1}{2}$ c) $-\frac{1}{2}$ d) -1

11) let

$$S_K = \sum_{r=1}^K \tan^{-1} \left[\frac{(6^r)}{(2^{r+1} + 3^{2r+1})} \right]. \text{ Then } \lim_{k \rightarrow \infty} S_k =$$

- a) $\tan^{-1}\left(\frac{3}{2}\right)$ b) $\cot^{-1}\left(\frac{3}{2}\right)$ c) $\frac{\pi}{2}$ d) $\tan^{-1}(3)$

12) The number of roots of the equation, $(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$ in the interval $[0, \pi]$ is equal to :

- a) 3 b) 2 c) 4 d) 8

13) If $y = y(x)$ is the solution of the differential equation,

$$\frac{dy}{dx} + 2y \tan x = \sin x, y\left(\frac{\pi}{3}\right) = 0$$

, then the maximum value of the function $y(x)$ over \mathbf{R} is equal to :

- a) 8 b) $\frac{1}{2}$ c) $-\frac{15}{4}$ d) $\frac{1}{8}$

14) Which of the following Boolean expression is a tautology?

- a) $(p \wedge q) \wedge (p \rightarrow q)$ c) $(p \wedge q) \vee (p \rightarrow q)$
 b) $(p \wedge q) \vee (p \vee q)$ d) $(p \wedge q) \rightarrow (p \rightarrow q)$

15) let $A = \begin{pmatrix} \iota & -\iota \\ -\iota & \iota \end{pmatrix}$. Then, the system of linear equations $A^8 \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 64 \end{pmatrix}$ has

- a) No solution c) A unique solution
 b) Exactly two solutions d) Infinitely many solutions