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QUESTION

Using integration, find the area of the region enclosed by the curve $y = x^2$, the x - axis and the ordinates x = -2 and x = 1.

SOLUTION:

Given	formula
$y = x^2$	$\mathbf{x}^{\mathbf{T}}\mathbf{V}\mathbf{x} + 2\mathbf{u}^{\mathbf{T}}\mathbf{x} + f = 0$
x = -2	$\begin{pmatrix} -2\\4 \end{pmatrix}$
x = 1	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Substituting the given values, we have

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{0.1}$$

$$\mathbf{u} = \begin{pmatrix} \frac{-1}{2} \\ 0 \end{pmatrix} \tag{0.2}$$

$$f = 0 \tag{0.3}$$

we get the equation of curve as

$$\mathbf{y} = \mathbf{x}^{\mathsf{T}} \mathbf{V} \mathbf{x} \tag{0.4}$$

Line equation of form $\mathbf{x} = \mathbf{h} + k\mathbf{m}$

If a line intersects the conic, k value of intersecting point is given by,

$$k_{i} = \frac{-\mathbf{m}^{\top} \left(\mathbf{V}\mathbf{h} + \mathbf{u}\right) \pm \sqrt{\left[\mathbf{m}^{\top} \left(\mathbf{V}\mathbf{h} + \mathbf{u}\right)\right]^{2} - g\left(h\right)\left(\mathbf{m}^{\top}\mathbf{V}\mathbf{m}\right)}}{\mathbf{m}^{\top}\mathbf{V}\mathbf{m}}$$
(0.5)

Substituting the values, we get the point of intersection as

$$\kappa_i = -\binom{0}{1} \left(\frac{-1}{2} \quad 0\right) \pm \sqrt{\left[\begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{-1}{2} \\ 0 \end{pmatrix}\right]^2 + 1} (1)$$
(0.6)

$$\kappa_i = 1 \tag{0.7}$$

Hence, the point of intersection is $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ Similarly, the other point is given by $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$. The area bounded by the curve and the line is

$$\int_{-2}^{1} (x^2) dx = \frac{1}{3} (1 - (-8))$$

$$= 3$$
(0.8)
(0.9)

Hence the required area is 3.

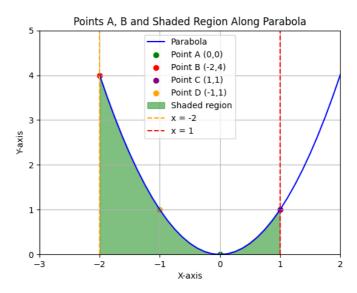


Fig. 0.1: A plot of the given question.