

Assignment 1

Chapter-11:

Limits, Continuity and Differentiability

EE24BTECH11049
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D: MCQs WITH ONE OR MORE THAN ONE CORRECT

- 1) Let $g(x) = xf(x)$, where $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. At $x = 0$ (1995)
- a) g is differentiable but g' is not continuous
 - b) g is differentiable while f is not
 - c) both f and g are differentiable
 - d) g is differentiable and g' is continuous
- 2) The function $f(x) = \max\{(1-x), (1+x), 2\}$, $x \in (-\infty, \infty)$ (1995)
- a) continuous at all points
 - b) differentiable at all points
 - c) differentiable at all points except at $x = 1$ and $x = -1$
 - d) continuous at all points except at $x = 1$ and $x = -1$ where it is discontinuous
- 3) Let $h(x) = \min\{x, x^2\}$ (1998-2M)
- a) h is continuous for all x
 - b) h is differentiable for all x
 - c) $h'(t) = 1$, for all $x > 1$
 - d) h is not differentiable at two values of x
- 4) $\lim_{x \rightarrow 1} \frac{\sqrt{1 - \cos 2(x-1)}}{x-1}$ (1998-2M)
- a) exists and it equals $\sqrt{2}$
 - b) exists and it equals $-\sqrt{2}$
 - c) does not exist because $x - 1 \rightarrow 0$
 - d) does not exist because the left-hand limit is not equal to the right-hand limit
- 5) If $f(x) = \min\{1, x^2, x^3\}$ (2006, 5M, -1)

- a) $f(x)$ is continuous $\forall x \in \mathbb{R}$
 b) $f(x)$ is continuous and differentiable everywhere
 c) $f(x)$ is not differentiable at two points
 d) $f(x)$ is not differentiable at one point

6) Let $L = \lim_{x \rightarrow 0} \frac{a - \sqrt{a^2 - x^2 - \frac{x^2}{4}}}{x^4}$, $a > 0$. If L is finite, then (2009)

- a) $a = 2$ b) $a = 1$ c) $L = \frac{1}{64}$ d) $L = \frac{1}{32}$

7) Let $f: \mathbb{R} \mapsto \mathbb{R}$ be a function such that $f(x+y) = f(x) + f(y)$, $\forall x, y \in \mathbb{R}$. If $f(x)$ is differentiable at $x = 0$, then (2011)

- a) $f(x)$ is differentiable only in a finite interval containing zero
 b) $f(x)$ is continuous $\forall x \in \mathbb{R}$
 c) $f'(x)$ is constant $\forall x \in \mathbb{R}$
 d) $f(x)$ is differentiable except at finitely many points

8) If $f(x) = \begin{cases} -x - \frac{\pi}{2}, & x \leq \frac{\pi}{2} \\ -\cos x, & \frac{\pi}{2} < x \leq 0 \\ x - 1, & 0 < x \leq 1 \\ \ln x, & x > 1 \end{cases}$ (2011)

- a) $f(x)$ is continuous at $x = \frac{\pi}{2}$
 b) $f(x)$ is not differentiable at $x = 0$
 c) $f'(x)$ is differentiable at $x = 1$
 d) $f(x)$ is differentiable at $x = \frac{3}{2}$

9) For every integer n , let a_n and b_n , be real numbers. Let function $f(x): \mathbb{R} \mapsto \mathbb{R}$ be given by $f(x) = \begin{cases} a_n + \sin \pi x, & \forall x \in [2n, 2n+1] \\ b_n + \cos \pi x, & \forall x \in (2n-1, 2n) \end{cases}$ for all integers n . If f is continuous, then which of the following hold(s) for all n (2012)

- a) $a_{n-1} - b_{n-1} = 0$
 b) $a_n - b_n = 1$
 c) $a_n - b_{n+1} = 1$
 d) $a_{n-1} - b_n = -1$

10) For $a \in \mathbb{R}$ (the set of all real numbers),

$$a \neq -1, \lim_{n \rightarrow \infty} \frac{(1^a + 2^a + \dots + n^a)}{(n+1)^a [(na+1) + (na+2) + \dots + (na+n)]} = \frac{1}{60} \text{ Then } a =$$

(JEE Adv.2013)

a) 5

b) 7

c) $\frac{-15}{2}$

d) $\frac{-17}{2}$

11) Let $f : [a, b] \mapsto [1, \infty)$ be a continuous function and let $g : \mathbb{R} \mapsto \mathbb{R}$ be defined as

$$f(x) = \begin{cases} 0, & \text{if } x < a, \\ \int_a^x f(t) dt, & \text{if } a \leq x \leq b; \\ \int_a^b f(t) dt, & \text{if } x > b \end{cases}$$

(JEE Adv.2013)

a) $g(x)$ is continuous but not differentiable at a

b) $g(x)$ is differentiable on \mathbb{R}

c) $g(x)$ is continuous but not differentiable at b

d) $g(x)$ is continuous and differentiable at either (a) or (b) but not both

12) For every pair of continuous functions $f, g : [0, 1] \mapsto \mathbb{R}$ such that $\max\{f(x) : x \in [0, 1]\} = \max\{g(x) : x \in [0, 1]\}$, the correct statement(s) is(are):

(JEE Adv.2014)

a) $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0, 1]$

b) $(f(c))^2 + f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0, 1]$

c) $(f(c))^2 + 3f(c) = (g(c))^2 + g(c)$ for some $c \in [0, 1]$

d) $(f(c))^2 = (g(c))^2$ for some $c \in [0, 1]$

13) Let $g : \mathbb{R} \mapsto \mathbb{R}$ be a differentiable function with $g(0) = 0$, $g'(0) = 0$ and $g'(1) \neq 0$.

Let $f(x) = \begin{cases} \frac{x}{|x|} g(x), & x \neq 0 \\ 0, & x = 0 \end{cases}$ and $h(x) = e^{|x|}$ for all $x \in \mathbb{R}$. Let $(f \circ h)(x)$ denote $f(h(x))$ and $(h \circ f)(x)$ denote $h(f(x))$. Then which of the following is(are) true?

(JEE Adv.2015)

a) f is differentiable at $x = 0$

b) h is differentiable at $x = 0$

c) $f \circ h$ is differentiable at $x = 0$

d) $h \circ f$ is differentiable at $x = 0$

14) Let $a, b \in \mathbb{R}$ and $f : \mathbb{R} \mapsto \mathbb{R}$ be defined by $f(x) = a \cos(|x^3 - x|) + b|x| \sin(|x^3 + x|)$. Then f is

(JEE Adv.2016)

a) differentiable at $x = 0$ if $a = 0$ and $b = 1$

b) differentiable at $x = 1$ if $a = 1$ and $b = 0$

c) NOT differentiable at $x = 0$ if $a = 1$ and $b = 0$

d) NOT differentiable at $x = 1$ if $a = 0$ and $b = 1$

15) Let $f : [-\frac{1}{2}, 2] \mapsto \mathbb{R}$ and $g : [-\frac{1}{2}, 2] \mapsto \mathbb{R}$ be functions defined by $f(x) = [x^2 - 3]$ and $g(x) = |x|f(x) + [4x - 7]f(x)$, where $[y]$ denotes the greatest integer less than or equal to y for $y \in \mathbb{R}$. Then

(JEE Adv.2016)

a) f is discontinuous exactly at three points in $[-\frac{1}{2}, 2]$

b) f is discontinuous exactly at four points in $[-\frac{1}{2}, 2]$

c) g is NOT differentiable exactly at four points in $[-\frac{1}{2}, 2]$

d) g is NOT differentiable exactly at five points in $[-\frac{1}{2}, 2]$