

# Assignment-6

EE24BTECH11049

- 1) A plane P is parallel to two lines whose direction ratios are  $(-2, 1, 3)$  and  $(-1, 2, -2)$  and it contains the point  $(2, 2, -2)$ . Let P intersect the co-ordinate axes at the points A, B, C making the intercepts  $\alpha, \beta, \gamma$ . If V is the volume of the tetrahedron OABC, where O is the origin and  $p = \alpha + \beta + \gamma$ , then the ordered pair  $(V, p)$  is equal to : [July 2022]
- a)  $(48, -13)$       b)  $(24, -13)$       c)  $(48, 11)$       d)  $(24, -5)$
- 2) Let S be the set of all  $a \in \mathbf{R}$  for which the angle between the vectors  $\mathbf{u} = a(\log_e b)\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}$  and  $\mathbf{v} = \log_e b\mathbf{i} + 2\mathbf{j} + 2a\log_e b\mathbf{k}$ , ( $b > 1$ ) is acute. Then S is equal to [July 2022]
- a)  $(-\infty, -\frac{4}{3})$       b)  $\phi$       c)  $(-\frac{4}{3}, 0)$       d)  $(\frac{12}{7}, \infty)$
- 3) A horizontal park is in the shape of a triangle OAB with  $AB = 16$ . A vertical lamp post OP is erected at the point O such that  $\angle PAO = \angle PBO = 15^\circ$  and  $\angle PCO = 45^\circ$ , where C is the mid-point of AB. Then  $(OP)^2$  is equal to [July 2022]
- a)  $\frac{32}{\sqrt{3}}(\sqrt{3} - 1)$       b)  $\frac{32}{\sqrt{3}}(2 - \sqrt{3})$       c)  $\frac{16}{\sqrt{3}}(\sqrt{3} - 1)$       d)  $\frac{16}{\sqrt{3}}(2 - \sqrt{3})$
- 4) Let A and B be two events such that  $P(B|A) = \frac{2}{3}$ ,  $P(A|B) = \frac{1}{7}$  and  $P(A \cap B) = \frac{1}{9}$ . Consider  
**S1** :  $P(A' \cup B) = \frac{5}{6}$   
**S2** :  $P(A' \cap B') = \frac{1}{18}$  [July 2022]
- a) Both S1 and S2 are true .  
 b) Both S1 and S2 are false.  
 c) S1 is true, but S2 is false.  
 d) S1 is false, but S1 is true.
- 5) Let  
**p** : Ramesh listens to music.  
**q** : Ramesh is out of his village.  
**r** : It is Sunday.  
**s** : It is Saturday.  
 Then the statement "Ramesh listens to music only if he is in his village and it is Sunday or Saturday" can be expressed as [July 2022]
- a)  $((\neg q) \wedge (r \vee s)) \implies p$       c)  $p \implies (q \wedge (r \vee s))$   
 b)  $(q \wedge (r \vee s)) \implies p$       d)  $p \implies ((\neg q) \wedge (r \vee s))$

## I. INTEGER-TYPE QUESTIONS

- 1) Let the coefficients of the middle terms in the expansion of  $(\frac{1}{\sqrt{6}} + \beta x)^4$ ,  $(1 - 3\beta x)^2$  and  $(1 - \frac{\beta}{2}x)^6$ , ( $\beta \geq 0$ ), respectively form the first three terms of an A.P. If d is the common difference of this A.P., then the value of  $50 - \frac{2d}{\beta^2}$  is equal to : [July 2022]
- 2) A class contains b boys and g girls. If the number of ways of selecting 3 boys and 2 girls from the class is 168, then  $b + 3g$  is equal to : [July 2022]
- 3) Let the tangents at the points P and Q on the ellipse  $\frac{x^2}{2} + \frac{y^2}{4} = 1$  meet at the point  $\mathbf{R}(\sqrt{2}, 2\sqrt{2} - 2)$ . If S is the focus of the ellipse on its negative major axis, then  $(SP)^2 + (SQ)^2$  is equal to : [July 2022]

- 4) If  $1 + (2 + {}^{49}C_1 + {}^{49}C_2 + \cdots + {}^{49}C_{49})({}^{50}C_2 + {}^{50}C_4 + \cdots + {}^{50}C_{50})$  is equal to  $2^n m$ , where  $m$  is odd, then  $n + m$  is equal to : [July 2022]
- 5) Two tangent lines  $l_1$  and  $l_2$  are drawn from the point  $(2, 0)$  to the parabola  $2y^2 = x$ . If the lines  $l_1$  and  $l_2$  are also tangent to the circle  $(x - 5)^2 + y^2 = r$ , then  $17r$  is equal to : [July 2022]
- 6) If  $\frac{6}{3^{12}} + \frac{10}{3^{11}} + \frac{20}{3^{10}} + \frac{40}{3^9} + \cdots + \frac{10240}{3} = 2^n m$ , where  $m$  is odd, then  $m \cdot n$  is equal to : [July 2022]
- 7) Let  $S = \left[-\pi, \frac{\pi}{2}\right) - \left\{\frac{-\pi}{2}, \frac{-\pi}{4}, \frac{-3\pi}{4}, \frac{\pi}{4}\right\}$ . Then the number of elements in the set

$$A = \{\theta \in S : \tan \theta (1 + \sqrt{5} \tan 2\theta) = \sqrt{5} - \tan 2\theta\}$$

is :

[July 2022]

- 8) Let  $z = a + ib, b \neq 0$  be complex numbers satisfying  $z^2 = \bar{z}2^{1-|z|}$ . Then the least value of  $n \in \mathbf{N}$  such that  $z^n = (z + 1)^n$  is equal to : [July 2022]
- 9) A bag contains white and 6 black balls. Three balls are drawn at random from the bag. Let  $X$  be the number of white balls, among the drawn balls. If  $\sigma^2$  is the variance of  $X$ , then  $100 \sigma^2$  is equal to [July 2022]
- 10) The value of the integral  $\int_0^{\frac{\pi}{2}} 60 \frac{\sin 6x}{\sin x} dx$  is equal to : [July 2022]