

# Assignment 1

## Chapter-11:

## Limits, Continuity and Differentiability

EE24BTECH11049  
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D: MCQs WITH ONE OR MORE THAN ONE CORRECT

1) Let  $g(x) = xf(x)$ , where

$$f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases} \text{ . At } x = 0$$

(1995)

- a)  $g$  is differentiable but  $g'$  is not continuous
- b)  $g$  is differentiable while  $f$  is not
- c) both  $f$  and  $g$  are differentiable
- d)  $g$  is differentiable and  $g'$  is continuous

2) The function  $f(x) = \max\{(1-x), (1+x), 2\}$ ,  $x \in (-\infty, \infty)$

(1995)

- a) continuous at all points
- b) differentiable at all points
- c) differentiable at all points except at  $x = 1$  and  $x = -1$
- d) continuous at all points except at  $x = 1$  and  $x = -1$  where it is discontinuous

3) Let  $h(x) = \min\{x, x^2\}$

(1998 – 2marks)

- a)  $h$  is continuous for all  $x$
- b)  $h$  is differentiable for all  $x$
- c)  $h'(t) = 1$ , for all  $x > 1$
- d)  $h$  is not differentiable at two values of  $x$

4)  $\lim_{x \rightarrow 1} \frac{\sqrt{1 - \cos 2(x-1)}}{x-1}$

(1998 – 2marks)

- a) exists and it equals  $\sqrt{2}$
- b) exists and it equals  $-\sqrt{2}$
- c) does not exist because  $x - 1 \mapsto 0$
- d) does not exist because the left-hand limit is not equal to the right-hand limit

5) If  $f(x) = \min\{1, x^2, x^3\}$

(2006, 5M, -1)

- a)  $f(x)$  is continuous  $\forall x \in R$   
 b)  $f(x)$  is continuous and differentiable everywhere  
 c)  $f(x)$  is not differentiable at two points  
 d)  $f(x)$  is not differentiable at one point

6) Let  $L = \lim_{x \rightarrow 0} \frac{a - \sqrt{a^2 - x^2 - \frac{x^2}{4}}}{x^4}$ ,  $a > 0$ .

If  $L$  is finite, then

(2009)

- a)  $a = 2$                       b)  $a = 1$                       c)  $L = \frac{1}{64}$                       d)  $L = \frac{1}{32}$

- 7) Let  $f : R \mapsto R$  be a function such that  $f(x + y) = f(x) + f(y)$ ,  $\forall x, y \in R$ . If  $f(x)$  is differentiable at  $x = 0$ , then

(2011)

- a)  $f(x)$  is differentiable only in a finite interval containing zero  
 b)  $f(x)$  is continuous  $\forall x \in R$   
 c)  $f'(x)$  is constant  $\forall x \in R$   
 d)  $f(x)$  is differentiable except at finitely many points

8) If  $f(x) = \begin{cases} -x - \frac{\pi}{2}, & x \leq \frac{\pi}{2} \\ -\cos x, & \frac{\pi}{2} < x \leq 0 \\ x - 1, & 0 < x \leq 1 \\ \ln x, & x > 1 \end{cases}$

(2011)

- a)  $f(x)$  is continuous at  $x = \frac{\pi}{2}$   
 b)  $f(x)$  is not differentiable at  $x = 0$   
 c)  $f'(x)$  is differentiable at  $x = 1$   
 d)  $f(x)$  is differentiable at  $x = \frac{3}{2}$

- 9) For every integer  $n$ , let  $a_n$  and  $b_n$ , be real numbers. Let function  $f(x) : IR \mapsto IR$  be given by  $f(x) = \begin{cases} a_n + \sin \pi x, & \text{for } x \in [2n, 2n + 1] \\ b_n + \cos \pi x, & \text{for } x \in (2n - 1, 2n) \end{cases}$  for all integers  $n$ . If  $f$  is continuous, then which of the following hold(s) for all  $n$

(2012)

- a)  $a_{n-1} - b_{n-1} = 0$   
 b)  $a_n - b_n = 1$   
 c)  $a_n - b_{n+1} = 1$   
 d)  $a_{n-1} - b_n = -1$

- 10) For  $a \in R$  (the set of all real numbers),  $a \neq -1$

$$\lim_{n \rightarrow \infty} \frac{(1^a + 2^a + \dots + n^a)}{(n+1)^a [(na+1) + (na+2) + \dots + (na+n)]} \cdot \frac{1}{60} \text{ Then } a =$$

(JEE Adv. 2013)

a) 5

b) 7

c)  $\frac{-15}{2}$

d)  $\frac{-17}{2}$

11) Let  $f : [a, b] \mapsto [1, \infty)$  be a continuous function and let  $g : R \mapsto R$  be defined as

$$f(x) = \begin{cases} 0, & \text{if } x < a, \\ \int_a^x f(t) dt, & \text{if } a \leq x \leq b; \\ \int_a^b f(t) dt, & \text{if } x > b \end{cases}$$

(JEEAdv.2013)

a)  $g(x)$  is continuous but not differentiable at  $a$

b)  $g(x)$  is differentiable on  $R$

c)  $g(x)$  is continuous but not differentiable at  $b$

d)  $g(x)$  is continuous and differentiable at either  $(a)$  or  $(b)$  but not both

12) For every pair of continuous functions  $f, g : [0, 1] \mapsto R$  such that  $\max\{f(x) : x \in [0, 1]\} = \max\{g(x) : x \in [0, 1]\}$ , the correct statement(s) is(are):

(JEEAdv.2014)

a)  $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$  for some  $c \in [0, 1]$

b)  $(f(c))^2 + f(c) = (g(c))^2 + 3g(c)$  for some  $c \in [0, 1]$

c)  $(f(c))^2 + 3f(c) = (g(c))^2 + g(c)$  for some  $c \in [0, 1]$

d)  $(f(c))^2 = (g(c))^2$  for some  $c \in [0, 1]$

13) Let  $g : R \mapsto R$  be a differentiable function with  $g(0) = 0$ ,  $g'(0) = 0$  and  $g'(1) \neq 0$ .

Let  $f(x) = \begin{cases} \frac{x}{|x|} g(x), & x \neq 0 \\ 0, & x = 0 \end{cases}$  and  $h(x) = e^{|x|}$  for all  $x \in R$ . Let  $(f \circ h)(x)$  denote  $f(h(x))$  and  $(h \circ f)(x)$  denote  $h(f(x))$ . Then which of the following is(are) true?

(JEEAdv.2015)

a)  $f$  is differentiable at  $x = 0$

b)  $h$  is differentiable at  $x = 0$

c)  $f \circ h$  is differentiable at  $x = 0$

d)  $h \circ f$  is differentiable at  $x = 0$

14) Let  $a, b \in R$  and  $f : R \mapsto R$  be defined by  $f(x) = a \cos(|x^3 - x|) + b|x| \sin(|x^3 + x|)$ . Then  $f$  is

(JEEAdv.2016)

a) differentiable at  $x = 0$  if  $a = 0$  and  $b = 1$

b) differentiable at  $x = 1$  if  $a = 1$  and  $b = 0$

c) NOT differentiable at  $x = 0$  if  $a = 1$  and  $b = 0$

d) NOT differentiable at  $x = 1$  if  $a = 0$  and  $b = 1$

15) Let  $f : [-\frac{1}{2}, 2] \mapsto R$  and  $g : [-\frac{1}{2}, 2] \mapsto R$  be functions defined by  $f(x) = [x^2 - 3]$  and  $g(x) = |x|f(x) + |4x - 7|f(x)$ , where  $[y]$  denotes the greatest integer less than or equal to  $y$  for  $y \in R$ . Then

(JEEAdv.2016)

a)  $f$  is discontinuous exactly at three points in  $[-\frac{1}{2}, 2]$

b)  $f$  is discontinuous exactly at four points in  $[-\frac{1}{2}, 2]$

c)  $g$  is NOT differentiable exactly at four points in  $[-\frac{1}{2}, 2]$

d)  $g$  is NOT differentiable exactly at five points in  $[-\frac{1}{2}, 2]$