Assignment-7

EE24BTECH11049

16) Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$ be an ellipse, whose eccentricity is $\frac{1}{\sqrt{2}}$ and the length of latus rectum is $\sqrt{14}$. Then the square of the eccentricity of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is : [Feb-2024]
    a) 3
    b) $\frac{7}{2}$
    c) $\frac{3}{2}$
    d) $\frac{5}{2}$
17) Let $3, a, b, c$ be in $A.P.$ and $3, a-1, b+1, c+9$ be in $G.P.$ Then, the arithmetic mean of a, b and c is : [Feb-2024]
    a) -4
    b) -1
    c) 13
    d) 11
18) Let $C: x^2+y^2=4$ and $C\prime: x^2+y^2-4\lambda x+9=0$ be two circles.If the set of all values of λ so that the circles C and $C\prime$ intersect at two distinct points, is $R-[a,b]$, then the point $(8a+12, 16b-20)$ lies on the curve : [Feb-2024]
    a) $x^2+2y^2-5x+6y=3$
    b) $5x^2-y=-11$
    c) $x^2-4y^2=7$
    d) $6x^2+y^2=42$
19) Let $5f(x)+4f\left(\frac{1}{x}\right)=x^2-2, \forall x \neq 0$ and $y=9x^2 f(x)$, then y is strictly increasing in : [Feb-2024]
    a) $\left(0, \frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$
    b) $\left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$
    c) $\left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$
    d) $\left(-\infty, -\frac{1}{\sqrt{5}}\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$
20) If the shortest distance between the lines $\frac{x-\lambda}{-2}=\frac{y-2}{1}=\frac{z-1}{1}$ and $\frac{x-\sqrt{3}}{1}=\frac{y-1}{-2}=\frac{z-2}{1}$ is 1, then the sum of all possible values of λ is : [Feb-2024]
    a) 0
    b) $2\sqrt{3}$
    c) $3\sqrt{3}$
    d) $-2\sqrt{3}$
21) If $x=x(t)$ is the solution of the differential eqution
    $(t+1)\,dx=\left(2x+(t+1)^4\right)dt, x(0)=2$, then, $x(1)$ equals … [Feb-2024]

22) The number of elements in the set
$S = \{(x, y, z) : x, y, z \in Z, x + 2y + 3z = 42, x, y, z \geq 0\}$ equals ... [Feb-2024]

23) If the coefficient of x^{30} in the expansion of $\left(1 + \frac{1}{x}\right)^6 \left(1 + x^2\right)^7 \left(1 - x^3\right)^8 ; x \neq 0$ is α then $|\alpha|$ equals ... [Feb-2024]

24) Let $3, 11, 15, \ldots, 403$ and $2, 5, 8, 11, \ldots, 404$ be two arithmetic progressions. Then the sum, of the common terms in them,is equal to ... [Feb-2024]

25) Let $\{x\}$ denote the fractional part of x and $f(x) = \frac{\cos^{-1}\left(1-\{x\}^2\right)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}, x \neq 0$. If L and R respectively denote the left and right hand limit of $f(x)$ at $x = 0$, then $\frac{32}{\pi^2}\left(L^2 + R^2\right)$ is equal to ... [Feb-2024]

26) Let the line L: $\sqrt{2}x + y = \alpha$ passes through the point of the intersection $P\,(in\,the\,first\,quadrant)$ of the circle $x^2 + y^2 = 3$ and the parabola $x^2 = 2y$. Let the line L touch two circles C_1 and C_2 of equal radius $2\sqrt{3}$. If the centres Q_1 and Q_2 of the circles C_1 and C_2 lie on the $y-axis$, then the square of the area of triangle PQ_1Q_2 is equal to ... [Feb-2024]

27) Let $P = \{z \in C\colon\ |z + 2 - 3i| \leq 1\}$ and $Q = \{z \in C\colon\ |z(1 + i) + \bar{z}(1 - i)| \leq -8\}$. Let in $P \cap Q, |z - 3 + 2i|$ be maximum and minimum at z_1 and z_2 respectively. If $|z_1|^2 + 2|z|^2 = \alpha + \beta\sqrt{2}$, where α, β are integers, then $\alpha + \beta$ equals ... [Feb-2024]

28) If $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x}{\left(1+e^{\sin x}\right)\left(1+\sin^4 x\right)}\,dx = \alpha\pi + \beta\log_e\left(3 + 2\sqrt{2}\right)$, where α, β are integers, then $\alpha^2 + \beta^2$ equals ... [Feb-2024]

29) Let the line of shortest distance between the lines
$L_1 : \overrightarrow{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$ and
$L_2 : \overrightarrow{r} = \left(4\hat{i} + 5\hat{j} + 6\hat{k}\right) + \mu\left(\hat{i} + \hat{j} - \hat{k}\right)$
intersect L_1 and L_2 at P and Q respectively. If (α, β, γ) is the midpoint of the line segment PQ, then $2(\alpha + \beta + \gamma)$ is equal to ... [Feb-2024]

30) Let $A = \{1, 2, 3, \ldots, 20\}$. Let R_1 and R_2 be two relation on A such that
$R_1 = \{(a, b) : b\,is\,divisible\,by\,a\}$
$R_2 = \{(a, b) : a\,is\,an\,integral\,multiple\,of\,b\}$.
Then, the number of elements in $R_1 - R_2$ is equal to ... [Feb-2024]