Assignment 1 Chapter-11: Limits, Continuity and Differentiability

EE24BTECH11049
Patnam Shariq Faraz Muhammed

D: MCQs with One or More than One correct

1) Let $g(x) = x.f(x)$, where
$f(x) = \begin{cases} x.\sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. At $x = 0$
(1995)
   a) g is differentiable but g' is not continuous
   b) g is differentiable while f is not
   c) both f and g are differentiable
   d) g is differentiable and g' is continuous
2) The function $f(x) = \max\{(1-x),(1+x),2\}$, $x \in (-\infty, \infty)$
(1995)
   a) continuous at all points
   b) differentiable at all points
   c) differentiable at all points except at $x = l$ and $x = -1$
   d) continuous at all points except at $x = l$ and $x = -1$ where it is discontinuous
3) Let $h(x) = \min\left\{x, x^2\right\}$
$(1998 - 2marks)$
   a) h is continuous for all x
   b) h is differentiable for all x
   c) $h'(t) = 1$, for all $x > 1$
   d) h is not differentiable at two values of x
4) $\lim_{x\to1} \frac{\sqrt{1-\cos 2(x-1)}}{x-1}$
$(1998 - 2marks)$
   a) exists and it equals $\sqrt{2}$
   b) exists and it equals $-\sqrt{2}$
   c) does not exist because $x - 1 \mapsto 0$
   d) does not exist because the left-hand limit is not equal to the right-hand limit
5) If $f(x) = \min\left\{1, x^2, x^3\right\}$
$(2006, 5M, -1)$
   a) $f(x)$ is continuous $\forall x \in R$
   b) $f(x)$ is continuous and differentiable everywhere
   c) $f(x)$ is not diferentiable at two points
   d) $f(x)$ is not differentiable at one point
6) Let $L = \lim_{x\to0} \frac{a-\sqrt{a^2-x^2-\frac{x^2}{4}}}{x^4}, a > 0.$
If L is finite, then
(2009)
   a) $a = 2$
   b) $a = 1$
   c) $L = \frac{1}{64}$
   d) $L = \frac{1}{32}$
7) Let $f : R \mapsto R$ be a function such that $f(x+y) = f(x) + f(y)$, $\forall x, y \in R$. If $f(x)$ is differentiable at $x = 0$, then
(2011)
   a) $f(x)$ is differentiable only in a finite interval containing zero
   b) $f(x)$ is continuous $\forall x \in R$
   c) $f\prime(x)$ is constant $\forall x \in R$
   d) $f(x)$ is differentiable except at finitely many points
8) If $f(x) = \begin{cases} -x - \frac{\pi}{2}, & x \leq \frac{\pi}{2} \\ -\cos x, & \frac{\pi}{2} < x \leq 0 \\ x - 1, & 0 < x \leq 1 \\ \ln x, & x > 1 \end{cases}$
(2011)
   a) $f(x)$ is continuous at $x = \frac{\pi}{2}$
   b) $f(x)$ is not differentiable at $x = 0$
   c) $f\prime(x)$ is differentiable at $x = 1$
   d) $f(x)$ is differentiable at $x = \frac{3}{2}$
9) For every integer n, let a_n and b_n, be real numbers. Let function $f(x) : IR \mapsto IR$ be given by $f(x) = \begin{cases} a_n + \sin \pi x, & for x \in [2n, 2n+1] \\ b_n + \cos \pi x, & for x \in (2n-1, 2n) \end{cases}$
for all integers n. If f is continuous,then which of the following hold(s) for all n

(2012)

a) $a_{n-1} - b_{n-1} = 0$
b) $a_n - b_n = 1$
c) $a_n - b_{n+1} = 1$
d) $a_{n-1} - b_n = -1$

10) For $a \in R$ (the set of all real numbers), $a \neq -1$
$$\lim_{n\to\infty} \frac{(1^a + 2^a + \cdots + n^a)}{(n+1)^a . [(na+1) + (na+2) + \cdots + (na+n)]}$$
$= \frac{1}{60}$ Then $a =$

($JEEAdv$.2013)

a) 5
b) 7
c) $\frac{-15}{2}$
d) $\frac{-17}{2}$

11) Let $f : [a,b] \mapsto [1\ ,\infty)$ be a continuous function and let $g : R \mapsto R$ be defined as $f(x)$=
$$\begin{cases} 0, & if x < a, \\ \int_a^x f(t)\ dt, & if a \leq x \leq b; \text{ then} \\ \int_a^b f(t)\ dt, & if x > b \end{cases}$$

($JEEAdv$.2013)

a) $g(x)$ is continuous but not differentiable at a
b) $g(x)$ is differentiable on R
c) $g(x)$ is continuous but not differentiable at b
d) $g(x)$ is continuous and differentiable at either (a) or (b) but not both

12) For every pair of continuous functions $f, g : [0,1] \mapsto R$ such that max $\{f(x) : x \in [0,1]\}$= max $\{g(x) : x \in [0,1]\}$, the correct statement(s) is(are):

($JEEAdv$.2014)

a) $(f(c))^2 + 3.f(c) = (g(c))^2 + 3.g(c)$ for some $c \in [0,1]$
b) $(f(c))^2 + f(c) = (g(c))^2 + 3.g(c)$ for some $c \in [0,1]$
c) $(f(c))^2 + 3.f(c) = (g(c))^2 + g(c)$ for some $c \in [0,1]$
d) $(f(c))^2 = (g(c))^2$ for some $c \in [0,1]$

13) Let $g : R \mapsto R$ be a differentiable function with $g(0) = 0$, $g'(0) = 0$ and $g'(1) \neq 0$. Let $f(x) = \begin{cases} \frac{x}{|x|}.g(x), & x \neq 0 \\ 0, & x = 0 \end{cases}$ and $h(x) = e^{|x|}$ for all $x \in R$. Let $(f \circ h)(x)$ denote $f(h(x))$ and $(h \circ f)(x)$ denote $h(f(x))$. Then which of the following is(are) true?

($JEEAdv$.2015)

a) f is differentiable at $x = 0$
b) h is differentiable at $x = 0$
c) $f \circ h$ is differentiable at $x = 0$
d) $h \circ f$ is differentiable at $x = 0$

14) Let $a, b \in R$ and $f : R \mapsto R$ be defined by $f(x) = a.\cos\left(\left|x^3 - x\right|\right) + b.|x|.\sin\left(\left|x^3 + x\right|\right)$. Then f is

($JEEAdv$.2016)

a) differentiable at $x = 0$ if $a = 0$ and $b = 1$
b) differentiable at $x = 1$ if $a = 1$ and $b = 0$
c) NOT differentiable at $x = 0$ if $a = 1$ and $b = 0$
d) NOT differentiable at $x = 1$ if $a = 0$ and $b = 1$

15) Let $f : \left[-\frac{1}{2}, 2\right] \mapsto R$ and $g : \left[-\frac{1}{2}, 2\right] \mapsto R$ be functions defined by $f(x) = \left[x^2 - 3\right]$ and $g(x) = |x|.f(x) + |4x - 7|.f(x)$, where $[y]$ denotes the greatest integer less than or equal to y for $y \in R$. Then

($JEEAdv$.2016)

a) f is discontinuous exactly at three points in $\left[-\frac{1}{2}, 2\right]$
b) f is discontinuous exactly at four points in $\left[-\frac{1}{2}, 2\right]$
c) g is NOT differentiable exactly at four points in $\left[-\frac{1}{2}, 2\right]$
d) g is NOT differentiable exactly at five points in $\left[-\frac{1}{2}, 2\right]$