Assignment 1
Chapter-11:
Limits, Continuity and
Differentiability

EE24BTECH11049

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D: MCQs with One or More than One correct

1) Let $g(x) = xf(x)$, where $f(x) = \begin{cases} x\sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. At $x = 0$

(1995)

   a) g is differentiable but g' is not continuous
   b) g is differentiable while f is not
   c) both f and g are differentiable
   d) g is differentiable and g' is continuous

2) The function $f(x) = \max\{(1-x),(1+x),2\}$, $x \in (-\infty, \infty)$

(1995)

   a) continuous at all points
   b) differentiable at all points
   c) differentiable at all points except at $x = l$ and $x = -1$
   d) continuous at all points except at $x = l$ and $x = -1$ where it is discontinuous

3) Let $h(x) = \min\{x, x^2\}$

(1998-2 marks)

   a) h is continuous for all x
   b) h is differentiable for all x
   c) $h'(t) = 1$, for all $x > 1$
   d) h is not differentiable at two values of x

4) $\lim_{x \to 1} \frac{\sqrt{1-\cos 2(x-1)}}{x-1}$

(1998-2 marks)

   a) exists and it equals $\sqrt{2}$
   b) exists and it equals $-\sqrt{2}$
   c) does not exist because $x - 1 \to 0$
   d) does not exist because the left-hand limit is not equal to the right-hand limit

5) If $f(x) = \min\{1, x^2, x^3\}$

(2006, 5M, -1)

a) $f(x)$ is continuous $\forall x \in \mathbb{R}$

b) $f(x)$ is continuous and differentiable everywhere

c) $f(x)$ is not diferentiable at two points

d) $f(x)$ is not differentiable at one point

6) Let $L = \lim_{x \to 0} \frac{a - \sqrt{a^2 - x^2 - \frac{x^2}{4}}}{x^4}, a > 0$. If L is finite, then

(2009)

    a) $a = 2$ b) $a = 1$ c) $L = \frac{1}{64}$ d) $L = \frac{1}{32}$

7) Let $f : \mathbb{R} \mapsto \mathbb{R}$ be a function such that $f(x + y) = f(x) + f(y)$, $\forall x, y \in \mathbb{R}$. If $f(x)$ is differentiable at $x = 0$, then

(2011)

    a) $f(x)$ is differentiable only in a finite interval containing zero

    b) $f(x)$ is continuous $\forall x \in \mathbb{R}$

    c) $f\prime(x)$ is constant $\forall x \in \mathbb{R}$

    d) $f(x)$ is differentiable except at finitely many points

8) If $f(x) = \begin{cases} -x - \frac{\pi}{2}, & x \le \frac{\pi}{2} \\ -\cos x, & \frac{\pi}{2} < x \le 0 \\ x - 1, & 0 < x \le 1 \\ \ln x, & x > 1 \end{cases}$

(2011)

    a) $f(x)$ is continuous at $x = \frac{\pi}{2}$

    b) $f(x)$ is not differentiable at $x = 0$

    c) $f\prime(x)$ is differentiable at $x = 1$

    d) $f(x)$ is differentiable at $x = \frac{3}{2}$

9) For every integer n, let a_n and b_n, be real numbers. Let function $f(x) : \mathbb{R} \mapsto \mathbb{R}$ be given by $f(x) = \begin{cases} a_n + \sin \pi x, & for x \in [2n, 2n + 1] \\ b_n + \cos \pi x, & for x \in (2n - 1, 2n) \end{cases}$ for all integers n. If f is continuous, then which of the following hold(s) for all n

(2012)

    a) $a_{n-1} - b_{n-1} = 0$

    b) $a_n - b_n = 1$

    c) $a_n - b_{n+1} = 1$

    d) $a_{n-1} - b_n = -1$

10) For $a \in \mathbb{R}$ (the set of all real numbers),

$$a \ne -1, \lim_{n \to \infty} \frac{(1^a + 2^a + \cdots + n^a)}{(n + 1)^a \, [(na + 1) + (na + 2) + \cdots + (na + n)]} = \frac{1}{60} \text{ Then } a =$$

(JEE Adv.2013)

a) 5   b) 7   c) $\frac{-15}{2}$ d) $\frac{-17}{2}$

11) Let $f : [a, b] \mapsto [1, \infty)$ be a continuous function and let $g : \mathbb{R} \mapsto \mathbb{R}$ be defined as

$$f(x) = \begin{cases} 0, & if\, x < a, \\ \int_a^x f(t)\, dt, & if\, a \le x \le b; \text{ then} \\ \int_a^b f(t)\, dt, & if\, x > b \end{cases}$$

(JEE Adv.2013)

   a) $g(x)$ is continuous but not differentiable at a
   b) $g(x)$ is differentiable on $\mathbb{R}$
   c) $g(x)$ is continuous but not differentiable at b
   d) $g(x)$ is continuous and differentiable at either (a) or (b) but not both

12) For every pair of continuous functions $f, g : [0, 1] \mapsto \mathbb{R}$ such that max $\{f(x) : x \in [0, 1]\} =$ max $\{g(x) : x \in [0, 1]\}$, the correct statement(s) is(are):

(JEE Adv.2014)

   a) $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0, 1]$
   b) $(f(c))^2 + f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0, 1]$
   c) $(f(c))^2 + 3f(c) = (g(c))^2 + g(c)$ for some $c \in [0, 1]$
   d) $(f(c))^2 = (g(c))^2$ for some $c \in [0, 1]$

13) Let $g : \mathbb{R} \mapsto \mathbb{R}$ be a differentiable function with $g(0) = 0$, $g'(0) = 0$ and $g'(1) \ne 0$.

Let $f(x) = \begin{cases} \frac{x}{|x|} g(x), & x \ne 0 \\ 0, & x = 0 \end{cases}$ and $h(x) = e^{|x|}$ for all $x \in \mathbb{R}$. Let $(f \circ h)(x)$ denote $f(h(x))$ and $(h \circ f)(x)$ denote $h(f(x))$. Then which of the following is(are) true?

(JEE Adv.2015)

   a) f is differentiable at $x = 0$
   b) h is differentiable at $x = 0$
   c) $f \circ h$ is differentiable at $x = 0$
   d) $h \circ f$ is differentiable at $x = 0$

14) Let $a, b \in \mathbb{R}$ and $f : \mathbb{R} \mapsto \mathbb{R}$ be defined by $f(x) = a \cos\left(\left|x^3 - x\right|\right) + b|x| \sin\left(\left|x^3 + x\right|\right)$. Then f is

(JEE Adv.2016)

   a) differentiable at $x = 0$ if $a = 0$ and $b = 1$
   b) differentiable at $x = 1$ if $a = 1$ and $b = 0$
   c) NOT differentiable at $x = 0$ if $a = 1$ and $b = 0$
   d) NOT differentiable at $x = 1$ if $a = 0$ and $b = 1$

15) Let $f : \left[-\frac{1}{2}, 2\right] \mapsto \mathbb{R}$ and $g : \left[-\frac{1}{2}, 2\right] \mapsto \mathbb{R}$ be functions defined by $f(x) = \left[x^2 - 3\right]$ and $g(x) = |x| f(x) + |4x - 7| f(x)$, where $[y]$ denotes the greatest integer less than or equal to y for $y \in \mathbb{R}$. Then

(JEE Adv.2016)

   a) f is discontinuous exactly at three points in $\left[-\frac{1}{2}, 2\right]$
   b) f is discontinuous exactly at four points in $\left[-\frac{1}{2}, 2\right]$
   c) g is NOT differentiable exactly at four points in $\left[-\frac{1}{2}, 2\right]$
   d) g is NOT differentiable exactly at five points in $\left[-\frac{1}{2}, 2\right]$