Assignment 1

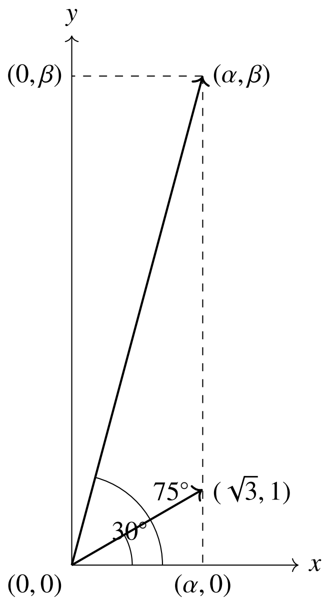
EE24BTECH11049

1) Consider three observations a, b and c such that $b = a + c$. If the standard deviation of $a + 2$, $b + 2$, $c + 2$ is d, then which of the following is true?
(2021-Mar)

   a) $b^2 = a^2 + c^2 + 3d^2$
   b) $b^2 = 3\left(a^2 + c^2\right) - 9d2$
   c) $b^2 = 3\left(a^2 + c^2\right) + 9d^2$
   d) $b^2 = 3\left(a^2 + c^2 + d^2\right)$

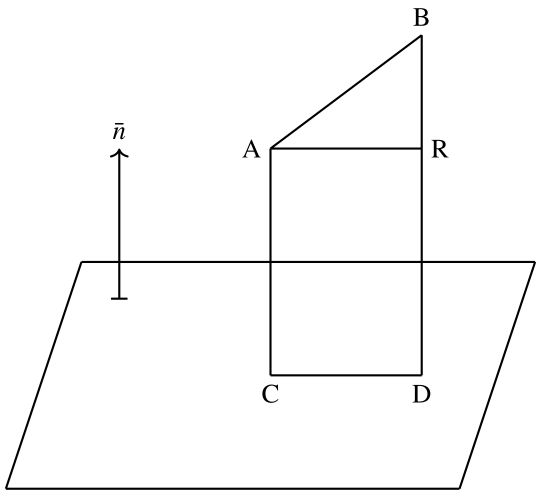
2) Let a vector $\alpha\hat{\mathbf{i}} + \beta\hat{\mathbf{j}}$ be obtained by rotating the vector $\sqrt{3}\hat{\mathbf{i}} + \hat{\mathbf{j}}$ by an angle 45° about the origin in counterclockwise direction in the first quadrant. Then the area of triangle having vertices (α, β), $(0, \beta)$ and $(0, 0)$ is equal to: (2021-Mar)

   a) 1
   b) $\frac{1}{2}$
   c) $\frac{1}{\sqrt{2}}$
   d) $2\sqrt{2}$



3) If for $a > 0$, the feet of perpendiculars from the points $\mathbf{A}(a, -2a, 3)$ and $\mathbf{B}(0, 4, 5)$ on the plane $lx + my + nz = 0$ are points $\mathbf{C}(0, -a, -1)$ and $\mathbf{D}$ respectively, then the length of line segment $\mathbf{CD}$ is equal to:
(2021-Mar)

a) $\sqrt{41}$ b) $\sqrt{55}$ c) $\sqrt{31}$ d) $\sqrt{66}$



4) The range of $a \in R$ for which the function

$$f(x) = (4a-3)(x + \log_e 5) + (a-7)\cot\left(\frac{x}{2}\right)\sin^2\left(\frac{x}{2}\right),$$

$x \neq 2n\pi$, $n \in \mathbf{N}$ has critical points, is

(2021-Mar)

a) $\left[-\frac{4}{3}, 2\right]$ b) $[1\ ,\ \infty)$ c) $(-\infty\ ,\ -1]$ d) $(-3, 1)$

5) Let the functions $f : \mathbf{R} \mapsto \mathbf{R}$ and $g : \mathbf{R} \mapsto \mathbf{R}$ be defined as: $f(x) = \begin{cases} x+2, & x \leq 0 \\ x^2, & x \geq 0 \end{cases}$ and $g(x) = \begin{cases} x^3, & x < 1 \\ 3x-2, & x \geq 1 \end{cases}$ Then, the number of points in R where $(f \circ g)(x)$ is NOT differentiable is equal to:

(2021-Mar)

a) 1 b) 2 c) 3 d) 0

6) Let a complex number z, $|z| \neq 1$, satisfy $\log_{\frac{1}{\sqrt{2}}}\left[\frac{(|z|+11)}{(|z|-1)^2}\right] \leq 2$. Then, the largest value of $|z|$ is equal to

(2021-Mar)

a) 5 b) 8 c) 6 d) 7

7) A pack of cards has one card missing. Two cards are drawn randomly and are found to be spades. The probability that the missing card is not a spade is:

a) $\frac{3}{4}$ b) $\frac{52}{867}$ c) $\frac{39}{50}$ d) $\frac{22}{425}$

8) If n is the number of irrational terms in the expansion of $\left[3^{\frac{1}{4}} + 5^{\frac{1}{8}}\right]^{60}$, then $(n-1)$ is divisible by

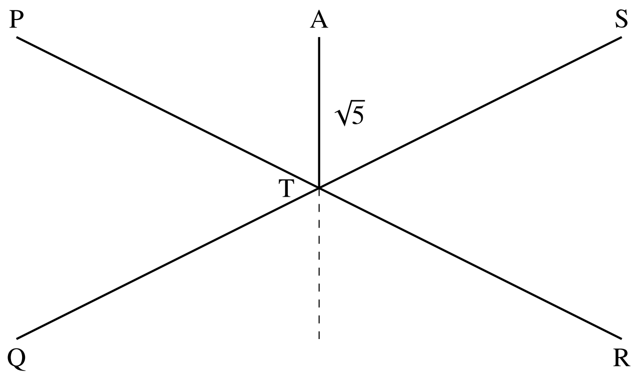
(2021-Mar)

a) 8 b) 26 c) 7 d) 30

9) Let the position vectors of two points **P** and **Q** be $\mathbf{3\hat{i} - \hat{j} + 2\hat{k}}$ and $\mathbf{\hat{i} + 2\hat{j} - 4\hat{k}}$ respectively. Let **R** and **S** be two points such that the direction ratios of lines **PR** and **QS** are $(4, -1, 2)$ and $(-2, 1, -2)$ respectively. Let lines **PR** and **QS** intersect at **T**. If the vector **TA** is perpendicular to both **PR** and **QS** and the length of vector **TA** is $\sqrt{5}$ units, then the modulus of a position vector of **A** is:

(2021-Mar)

a) $\sqrt{5}$ b) $\sqrt{171}$ c) $\sqrt{227}$ d) $\sqrt{482}$



10) If the three normals drawn to the parabola, $y^2 = 2x$ pass through the point $(a, 0)$ $a \neq 0$, then $'a'$ must be greater than

(2021-Mar)

a) 1 b) $\frac{1}{2}$ c) $-\frac{1}{2}$ d) -1

11) let

$$S_K = \sum_{r=1}^{k} \tan^{-1}\left[\frac{(6^r)}{(2^{r+1} + 3^{2r+1})}\right]. \text{ Then } \lim_{k\to\infty} S_k =$$

(2021-Mar)

a) $\tan^{-1}\left(\frac{3}{2}\right)$ b) $\cot^{-1}\left(\frac{3}{2}\right)$ c) $\frac{\pi}{2}$ d) $\tan^{-1}(3)$

12) The number of roots of the equation, $(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$ in the interval $[0, \pi]$ is equal to :

(2021-Mar)

a) 3 b) 2 c) 4 d) 8

13) If $y = y(x)$ is the solution of the differential equation,

$$\frac{dy}{dx} + 2y\tan x = \sin x,$$

$y\left(\frac{\pi}{3}\right) = 0$, then the maximum value of the function $y(x)$ over $\mathbf{R}$ is equal to :

(2021-Mar)

a) 8 b) $\frac{1}{2}$ c) $-\frac{15}{4}$ d) $\frac{1}{8}$

14) Which of the following Boolean expression is a tautology?

(2021-Mar)

a) $(p \wedge q) \wedge (p \rightarrow q)$
b) $(p \wedge q) \vee (p \vee q)$
c) $(p \wedge q) \vee (p \rightarrow q)$
d) $(p \wedge q) \rightarrow (p \rightarrow q)$

15) let $A = \begin{pmatrix} \iota & -\iota \\ -\iota & \iota \end{pmatrix}$. Then, the system of linear equations $A^8 \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 64 \end{pmatrix}$ has

(2021-Mar)

a) No solution
b) Exactly two solutions
c) A unique solution
d) Infinitely many solutions