9-9.3-6

EE24BTECH11049
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QUESTION

Using integration, find the area of the region enclosed by the curve $y = x^2$, the $x - axis$ and the ordinates $x = -2$ and $x = 1$.

SOLUTION:

Given	**formula**
$y = x^2$	$\mathbf{x^T V x + 2u^T x} + f = 0$
$x = -2$	$\begin{pmatrix} -2 \\ 4 \end{pmatrix}$
$x = 1$	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Substituting the given values, we have

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{0.1}$$

$$\mathbf{u} = \begin{pmatrix} \frac{-1}{2} \\ 0 \end{pmatrix} \tag{0.2}$$

$$f = 0 \tag{0.3}$$

we get the equation of curve as

$$\mathbf{y = x^T V x} \tag{0.4}$$

Line equation of form $\mathbf{x} = \mathbf{h} + k\mathbf{m}$

If a line intersects the conic, k value of intersecting point is given by,

$$k_i = \frac{-\mathbf{m}^\top (\mathbf{Vh} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^\top (\mathbf{Vh} + \mathbf{u})]^2 - g(h)(\mathbf{m}^\top \mathbf{Vm})}}{\mathbf{m}^\top \mathbf{Vm}} \tag{0.5}$$

Substituting the values, we get the point of intersection as

$$\kappa_i = -\begin{pmatrix} 0 \\ 1 \end{pmatrix}\begin{pmatrix} \frac{-1}{2} & 0 \end{pmatrix} \pm \sqrt{\left[\begin{pmatrix} 0 & 1 \end{pmatrix}\begin{pmatrix} \frac{-1}{2} \\ 0 \end{pmatrix}\right]^2 + 1\,(1)} \tag{0.6}$$

$$\kappa_i = 1 \tag{0.7}$$

Hence, the point of intersection is $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Similarly, the other point is given by $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$.

The area bounded by the curve and the line is

$$\int_{-2}^{1} \left(x^2\right) dx = \frac{1}{3}\,(1 - (-8)) \tag{0.8}$$

$$= 3 \tag{0.9}$$

Hence the required area is 3 .

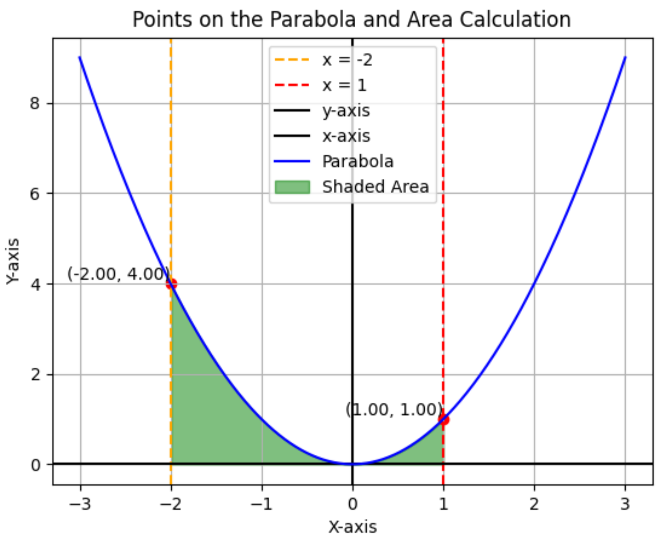

Fig. 0.1: A plot of the given question.