Assignment 3
2024-April
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EE24BTECH11049
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1 MCQ

1) The integral

$$\int_0^{\frac{\pi}{4}} \frac{136\sin x}{3\sin x + 5\cos x}\,dx$$

is equal to:

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a) $3\pi - 10\log_e\left(2\sqrt{2}\right) + 10\log_e 5$
b) $3\pi - 50\log_e 2 + 20\log_e 5$
c) $3\pi - 30\log_e 2 + 20\log_e 5$
d) $3\pi - 25\log_e 2 + 10\log_e 5$

2) If $y = y(x)$ is the solution of the differential equation

$$\frac{dy}{dx} + 2y = \sin(2x),$$

$y(0) = \frac{3}{4}$, then $y\left(\frac{\pi}{8}\right)$ is equal to:

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a) $e^{\frac{\pi}{8}}$
b) $e^{-\frac{\pi}{8}}$
c) $e^{\frac{\pi}{4}}$
d) $e^{-\frac{\pi}{4}}$

3) Let two straight line drawn from the origin **O** intersect the line $3x + 4y = 12$ at the points **P** and **Q** such that ΔOPQ is an isosceles triangle and $\angle POQ = 90^\circ$. If $l = \mathbf{OP}^2 + \mathbf{PQ}^2 + \mathbf{QO}^2$, then the greatest integer less than or equal to l is:

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a) 44
b) 48
c) 42
d) 46

4) If the function

$$f(x) = \frac{\sin 3x + \alpha\sin x - \beta\cos 3x}{x^3},\ x \in \mathbf{R}$$

is continuous at $x = 0$, then $f(0)$ is equal to:

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a) -4 b) 4 c) 2 d) -2

5) consider the following statements:

Statement I :For any two complex numbers z_1, z_2,

$$(|z_1| + |z_2|)\left|\frac{z_1}{|z_1|} + \frac{z_2}{|z_2|}\right| \leq 2(|z_1| + |z_2|), \text{ and}$$

Statement II :If x, y, z and three distinct complex numbers and a, b, c are three positive real numbers such that

$$\frac{a}{|y-z|} = \frac{b}{|z-x|} = \frac{c}{|x-y|}, \text{ then}$$

$$\frac{a^2}{y-z} + \frac{b^2}{z-x} + \frac{c^2}{x-y} = 1.$$

Between the above two statements:

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a) statement I is correct but statement II is incorrect.
b) both statement I and statement II are correct.
c) statement I is incorrect but statement II is correct.
d) both statement I and statement II are incorrect.

2 INTEGER

1) Let $a_1, a_2, a_3, \ldots$ be in arithmetic progression progression of positive terms. Let

$$A_k = a_1^2 - a_2^2 + a_3^2 - a_4^2 + \cdots + a_{2k-1}^2 - a_{2k}^2.$$

If $A_3 = -153$, $A_5 = -435$ and $a_1^2 + a_2^2 + a_3^2 = 66$, then $a_{17} - A_7$ is equal to _____

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2) Suppose **AB** is focal chord of the parabola $y^2 = 12x$ of length l and slope $m < \sqrt{3}$, If the distance of the chord **AB** from the origin is d, then ld^2 is equal to _____

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3) The number real roots of the equation $|x|\,|x+2| - 5\,|x+1| - 1 = 0$ is _____

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4) If

$$S = \{a \in \mathbf{R} : |2a-1| = 3\,[a] + 2\,\{a\}\}, A = 72\sum_{a \in S} a,$$

where $[t]$ denotes the greatest integer less than or equal to t and $\{t\}$ represents the fractional part of t, then A is equal to _____

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5) Let $\bar{\mathbf{a}} = \hat{i} - 3\hat{j} + 7\hat{k}$, $\bar{\mathbf{b}} = 2\hat{i} - \hat{j} + \hat{k}$ and $\bar{\mathbf{c}}$ be a vector such that $\left(\bar{\mathbf{a}} + 2\bar{\mathbf{b}}\right) \times \bar{\mathbf{c}} = 3(\bar{\mathbf{c}} \times \bar{\mathbf{a}})$. If $\bar{\mathbf{a}}.\bar{\mathbf{c}} = 130$, then $\bar{\mathbf{b}}.\bar{\mathbf{c}}$ is equal to _____

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6) The area of the region enclosed by the parabolas $y = x^2 - 5x$ and $y = 7x - x^2$ is _____

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7) let f be a differentiable function in the interval $(0, \infty)$ such that $f(1) = 1$ and

$$\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1, \forall x > 0.$$

Then $2f(2) + 3f(3)$ is equal to _____

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8) If the constant term in the expansion of

$$\left(1 + 2x - 3x^3\right)\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9 \text{ is } p,$$

then $108p$ is equal to _____

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9) From a lot of 10 items, which include 3 defective items, a sample of 5 items is drawn at random. Let the random variable X denote the numbers of defective items in the sample. If the variance of X is σ, then $96\sigma^2$ is equal to _____

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10) The number of ways of getting sum 16 on throwing a dice four times is _____

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