# Assignment 1

# Chapter-11: Limits, Continuity and Differentiability

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## D (11 – 25): MCQs with One or More than One correct

1) Let $g(x) = x.f(x)$, where
$f(x) = \begin{cases} x.\sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. At $x = 0$

(1995)

   (a) $g$ is differentiable but $g'$ is not continuous
   (b) $g$ is differentiable while $f$ is not
   (c) both $f$ and $g$ are differentiable
   (d) $g$ is differentiable and $g'$ is continuous

2) The function $f(x) = \max\{(1-x), (1+x), 2\}$, $x \in (-\infty, \infty)$

(1995)

   (a) continuous at all points
   (b) differentiable at all points
   (c) differentiable at all points except at $x = l$ and $x = -1$
   (d) continuous at all points except at $x = l$ and $x = -1$ where it is discontinuous

3) Let $h(x) = \min\left\{x, x^2\right\}$

$(1998 - 2marks)$

   (a) $h$ is continuous for all $x$
   (b) $h$ is differentiable for all $x$
   (c) $h'(t) = 1$, for all $x > 1$
   (d) $h$ is not differentiable at two values of $x$

4) $\lim_{x\to 1} \frac{\sqrt{1-\cos 2(x-1)}}{x-1}$

$(1998 - 2marks)$

   (a) exists and it equals $\sqrt{2}$
   (b) exists and it equals $-\sqrt{2}$
   (c) does not exist because $x - 1 \mapsto 0$
   (d) does not exist because the left-hand limit is not equal to the right-hand limit

5) If $f(x) = \min\left\{1, x^2, x^3\right\}$

$(2006, 5M, -1)$

   (a) $f(x)$ is continuous $\forall x \in R$
   (b) $f(x)$ is continuous and differentiable everywhere
   (c) $f(x)$ is not diferentiable at two points
   (d) $f(x)$ is not differentiable at one point

6) Let $L = \lim_{x\to 0} \frac{a-\sqrt{a^2-x^2-\frac{x^2}{4}}}{x^4}, a > 0.$
If L is finite, then

(2009)

   (a) $a = 2$
   (b) $a = 1$
   (c) $L = \frac{1}{64}$
   (d) $L = \frac{1}{32}$

7) Let $f : R \to R$ be a function such that $f(x+y) = f(x) + f(y)$, $\forall x, y \in R$. If $f(x)$ is differentiable at $x = 0$, then

(2011)

   (a) $f(x)$ is differentiable only in a finite interval containing zero
   (b) $f(x)$ is continuous $\forall x \in R$
   (c) $f'(x)$ is constant $\forall x \in R$
   (d) $f(x)$ is differentiable except at finitely many points

8) If $f(x) = \begin{cases} -x - \frac{\pi}{2}, & x \leq \frac{\pi}{2} \\ -\cos x, & \frac{\pi}{2} < x \leq 0 \\ x - 1, & 0 < x \leq 1 \\ \ln x, & x > 1 \end{cases}$

(2011)

(a) $f(x)$ is continuous at $x=\frac{\pi}{2}$
(b) $f(x)$ is not differentiable at $x=0$
(c) $f'(x)$ is differentiable at $x=1$
(d) $f(x)$ is differentiable at $x=\frac{3}{2}$

9) For every integer $n$, let $a_n$ and $b_n$, be real numbers. Let function $f(x): IR \mapsto IR$ be given by $f(x)= \begin{cases} a_n+\sin \pi x, & for x \in [2n, 2n+1] \\ b_n+\cos \pi x, & for x \in (2n-1, 2n) \end{cases}$ for all integers $n$. If $f$ is continuous,then which of the following hold($s$) for all $n$

(2012)

(a) $a_{n-1}-b_{n-1}=0$
(b) $a_n-b_n=1$
(c) $a_n-b_{n+1}=1$
(d) $a_{n-1}-b_n=-1$

10) For $a \in R$ $(the set of all real numbers)$, $a \neq -1$ $\lim_{n\to\infty} \frac{(1^a+2^a+\cdots+n^a)}{(n+1)^a.[(na+1)+(na+2)+\cdots+(na+n)]}=\frac{1}{60}$ Then $a=$

($JEE$ $Adv.$2013)

(a) 5
(b) 7
(c) $\frac{-15}{2}$
(d) $\frac{-17}{2}$

11) Let $f : [a,b] \mapsto [1\,,\infty)$ be a continuous function and let $g: R \mapsto R$ be defined as $f(x)= \begin{cases} 0, & if x < a, \\ \int_a^x f(t)\,dt, & if a \le x \le b \\ \int_a^b f(t)\,dt, & if x > b \end{cases}$; then

($JEE$ $Adv.$2013)

(a) $g(x)$ is continuous but not differentiable at $a$
(b) $g(x)$ is differentiable on $R$
(c) $g(x)$ is continuous but not differentiable at $b$
(d) $g(x)$ is continuous and differentiable at either ($a$) or ($b$) but not both

12) For every pair of continuous functions $f, g : [0,1] \mapsto R$ such that max $\{f(x) : x \in [0,1]\}=$ max $\{g(x) : x \in [0,1]\}$, the correct statement($s$) is($are$):

($JEE$ $Adv.$2014)

(a) $(f(c))^2+3.f(c)=(g(c))^2+3.g(c)$ for some $c \in [0,1]$
(b) $(f(c))^2+f(c)=(g(c))^2+3.g(c)$ for some $c \in [0,1]$
(c) $(f(c))^2+3.f(c)=(g(c))^2+g(c)$ for some $c \in [0,1]$
(d) $(f(c))^2=(g(c))^2$ for some $c \in [0,1]$

13) Let $g : R \mapsto R$ be a differentiable function with $g(0)=0$, $g'(0)=0$ and $g'(1)\neq 0$. Let $f(x)= \begin{cases} \frac{x}{|x|}.g(x), & x \neq 0 \\ 0, & x=0 \end{cases}$ and $h(x)=e^{|x|}$ for all $x \in R$. Let $(f \circ h)(x)$ denote $f(h(x))$ and $(h \circ f)(x)$ denote $h(f(x))$. Then which of the following is($are$) true?

($JEE$ $Adv.$2015)

(a) $f$ is differentiable at $x=0$
(b) $h$ is differentiable at $x=0$
(c) $f \circ h$ is differentiable at $x=0$
(d) $h \circ f$ is differentiable at $x=0$

14) Let $a, b \in R$ and $f : R \mapsto R$ be defined by $f(x)=a.\cos\left(|x^3-x|\right)+b.|x|.\sin\left(|x^3+x|\right)$. Then $f$ is

($JEE$ $Adv.$2016)

(a) differentiable at $x=0$ if $a=0$ and $b=1$
(b) differentiable at $x=1$ if $a=1$ and $b=0$
(c) NOT differentiable at $x=0$ if $a=1$ and $b=0$
(d) NOT differentiable at $x=1$ if $a=0$ and $b=1$

15) Let $f : \left[-\frac{1}{2}, 2\right] \mapsto R$ and $g : \left[-\frac{1}{2}, 2\right] \mapsto R$ be functions defined by $f(x)=\left[x^2-3\right]$ and $g(x)=|x|.f(x)+|4x-7|.f(x)$, where $[y]$ denotes the greatest integer less than or equal to $y$ for $y \in R$. Then

($JEE$ $Adv.$2016)

(a) $f$ is discontinuous exactly at three points in $\left[-\frac{1}{2}, 2\right]$
(b) $f$ is discontinuous exactly at four points in $\left[-\frac{1}{2}, 2\right]$
(c) $g$ is NOT differentiable exactly at four points in $\left[-\frac{1}{2}, 2\right]$
(d) $g$ is NOT differentiable exactly at five points in $\left[-\frac{1}{2}, 2\right]$