Assignment 4

EE24BTECH11049
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1 MA: Mathematics

1.1 Carry one mark each

1) If $\left(x_1^*, x_2^*\right)$ is an optional solution of the linear programming problem, minimize $x_1 + 2x_2$ subject to

$$\begin{aligned} 4x_1 - x_2 &\geq 8 \\ 2x_1 + x_2 &\geq 10 \\ -x_1 + x_2 &\leq 7 \\ x_1, x_2 &\geq 0 \end{aligned}$$

and $\left(\lambda_1^*, \lambda_2^*, \lambda_3^*\right)$ is an optional solution of its dual problem, then $\sum_{i=1}^{2} x_i^* 0 + \sum_{j=1}^{3} \lambda_j^*$ is equal to __________ (correct upto one decimal place)

2) let $a, b, c \in \mathbf{R}$ be such that the quadrature rule

$$\int_{-1}^{1} f(x)\, dx \approx af(-1) + bf(0) + cf'(1)$$

is exact for all polynomials of degree less than or equal to 2. Then b is equal to __________ (rounded off to two decimal places)

3) Let $f(x) = x^4$ and let $p(x)$ be the interpolating polynomial of f at nodes 1,2 and 3. Then $p(0)$ is equal to __________

4) For $n \geq 2$, define the sequece $\{x_n\}$ by

$$x_n = \frac{1}{2\pi} \int_0^{\frac{\pi}{2}} tan^{\frac{1}{n}} t\, dt.$$

Then the sequence $\{x_n\}$ converges to __________
(correct up to two decimal places)

5)

$$L^2[0, 10] = \left\{ f : [0, 10] \mapsto \mathbf{R} : f \text{ is lebesgue measurable and } \int_0^{10} f^2 dx < \infty \right\}$$

equipped with the norm $\|f\| = \left(\int_0^{10} f^2\, dx\right)^{\frac{1}{2}}$ and let T be the linear functional on $L^2[0, 10]$ given by

$$T(f) = \int_0^2 f(x)\, dx - \int_3^{10} f(x)\, dx.$$

Then $\|T\|$ is equal to __________

6) if $\{x_{13}, x_{22}, x_{23} = 10, x_{31}, x_{32}, x_{34}\}$ is the set of basic variable of balanced transportation problem seeking to minimize cost of transportation from origins to destinations, where the cost matrix is, and $\lambda, \mu \in \mathbf{R}$ is equal to ________
7) Let $\mathbf{Z}_{225}$ be the ring of integers modulo 225. If x is the number of prime ideals and y is the number of non trivial units $\mathbf{Z}_{225}$, then $x + y$ is equal to ________
8) let $u(x,t)$ be the solution of

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0, u(x,0) = f(x), \frac{\partial u}{\partial t}(x,0) = 0, x \in \mathbf{R}, t > 0,$$

Where f is twice continuously differentiable function. If $f(-2) = 4, f(0) = 0$ and $u(2,2) = 8$, then each value of $u(1,3)$ is ________

1.2 Carry two marks each

9) Let $\{e_n\}_{n=1}^{\infty}$ be an orthomormal basis for a separable Hilbert space H with the inner product $\langle \cdot, \cdot \rangle$. Define

$$f_n = e_n - \frac{1}{n+1} e_{n+1} \text{ for } n \in \mathbf{N}$$

Then
a) the closure of the span $\{f_n : n \in \mathbf{N}\}$ equals H
b) $f = 0$ if $\langle f, f_n \rangle = \langle f, e_n \rangle$ for all $n \in \mathbf{N}$
c) $\{f_n\}_{n=1}^{\infty}$ is an orthogonal subset of H
d) there does not exist non zero $f \in H$ such that $\langle f, e_2 \rangle = \langle f, f_2 \rangle$

10) Suppose V is finite dimensional non-zero vector space over $\mathbf{C}$ and $T : V \mapsto V$ is a linear transformation such that Range(T) = Nullspace(T). Then which of following statements is FALSE?
a) The dimensions of V is even
b) 0 is the only eigenvalue of T
c) Both 0 and 1 are the eigen values of T
d) $T^2 = 0$

11) Let $P \in M_{m \times n}(R)$. Consider the following statements: Then

a) both I and II are true
b) I is true but II is false
c) I is false but II is true
d) both I and II are false

12) For $n \in \mathbf{N}$, let $T_n : \left(l^1, \|\cdot\|_1\right) \mapsto (l^\infty, \|\cdot\|_\infty)$ and $T : \left(l^1, \|\cdot\|_1\right) \mapsto (l^\infty, \|\cdot\|_\infty)$ be the bounded linear operators defined by

$$T_n(x_1, x_2, \dots) = (y_1, y_2, \dots), \text{ where } y_j = \begin{cases} x_j, & j \le n \\ x_n, & j > n \end{cases}$$

and

$$T(x_1, x_2, \dots) = (x_1, x_2, \dots)$$

Then

a) $||T_n||$ does not converge to $||T||$ as $n \to \infty$
b) $||T_n - T||$ converges to zero as $n \to \infty$
c) for all $x \in l^1$, $||T_n(x) - T(x)||$ converges to zero as $n \to \infty$
d) for each non-zero $x \in l^1$, there exists a continuous linear functional g on l^∞ such that $g(T_n(x))$ does not converge to $g(T(x))$ as $n \to \infty$

13) Let $P(\mathbf{R})$ denote the power set of $\mathbf{R}$, equipped with the metric

$$d(U, V) = \sup_{x \in \mathbf{R}} |\chi_U(x) - \chi_V(x)|,$$

where χ_U and χ_V denote the characteristic function of subsets U and V, respectively of $\mathbf{R}$. The set $\{\{m\} : m \in \mathbf{Z}\}$ in the metric space $(P(\mathbf{R}), d)$ is

a) bounded but not totally bounded
b) totally bounded but not compact
c) compact
d) not bounded