9-9.3-6

EE24BTECH11049
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QUESTION

Using integration, find the area of the region enclosed by the curve $y = x^2$, the $x - axis$ and the ordinates $x = -2$ and $x = 1$.

SOLUTION:

FUNCTION	FORMULA
$g(x)$	$\mathbf{x^T V x + 2u^T x} + f = 0$
The points of intersection of the line L with the conic section as above are given by $\mathbf{x}_i = \mathbf{h} + \kappa_i \mathbf{m}$	$L : \mathbf{x} = \mathbf{h} + \kappa \mathbf{m}, \kappa \in \mathbb{R}$ $\kappa_i = \frac{1}{\mathbf{m^T V m}} \left(-\mathbf{m^T} \left(\mathbf{Vh + u} \right) \pm \sqrt{\left[\mathbf{m^T} \left(\mathbf{Vh + u} \right)^2 \right] - g(\mathbf{h}) \left(\mathbf{m^T V m} \right)} \right)$

DESCRIPTION
$\mathbf{V} = \|n\|^2 \mathbf{I} - e^2 \mathbf{n n^T}$
$\mathbf{u} = ce^2 \mathbf{n} - \|n\|^2 \mathbf{F}$
$f = \|n\|^2 \|F\|^2 - c^2 e^2$

TABLE 0: Variables Used

Substituting the given values, we have

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{0.1}$$

$$\mathbf{u} = \begin{pmatrix} \frac{-1}{2} \\ 0 \end{pmatrix} \tag{0.2}$$

$$f = 0 \tag{0.3}$$

Substituting the values, we get the point of intersection as

$$\kappa_i = -\begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}\frac{-1}{2} & 0\end{pmatrix} \pm \sqrt{\left[\begin{pmatrix}0 & 1\end{pmatrix}\begin{pmatrix}\frac{-1}{2}\\0\end{pmatrix}\right]^2 + 1\,(1)} \tag{0.4}$$

$$\kappa_i = 1 \tag{0.5}$$

Hence, the point of intersection is $\begin{pmatrix}1\\1\end{pmatrix}$

Similarly, the other point is given by $\begin{pmatrix}-2\\4\end{pmatrix}$.

The area bounded by the curve and the line is

$$\int_{-2}^{1}\left(x^2\right)dx = \frac{1}{3}\left(1-(-8)\right) \tag{0.6}$$

$$= 3 \tag{0.7}$$

Hence the required area is 3 .

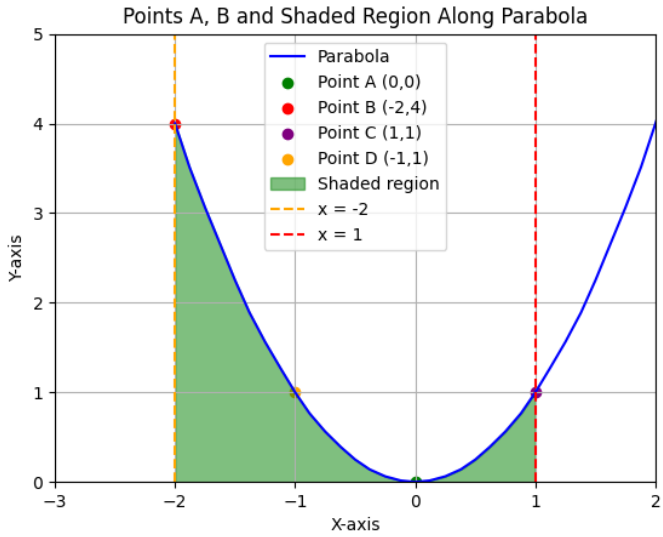

Fig. 0.1: A plot of the given question.