Assignment 1

Chapter-11: Limits, Continuity and Differentiability

EE24BTECH11049
Patnam Shariq Faraz Muhammed

D (11 – 25): MCQs with One or More than One correct

1) Let $g(x) = x.f(x)$, where
$$f(x) = \begin{cases} x.\sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}. \text{ At } x = 0$$
(1995)
   (a) g is differentiable but g' is not continuous
   (b) g is differentiable while f is not
   (c) both f and g are differentiable
   (d) g is differentiable and g' is continuous
2) The function $f(x) = \max\{(1-x), (1+x), 2\}$, $x \in (-\infty, \infty)$
(1995)
   (a) continuous at all points
   (b) differentiable at all points
   (c) differentiable at all points except at $x = l$ and $x = -1$
   (d) continuous at all points except at $x = l$ and $x = -1$ where it is discontinuous
3) Let $h(x) = \min\{x, x^2\}$
$(1998 - 2marks)$
   (a) h is continuous for all x
   (b) h is differentiable for all x
   (c) $h'(t) = 1$, for all $x > 1$
   (d) h is not differentiable at two values of x
4) $\lim_{x\to 1} \frac{\sqrt{1-\cos 2(x-1)}}{x-1}$
$(1998 - 2marks)$
   (a) exists and it equals $\sqrt{2}$
   (b) exists and it equals $-\sqrt{2}$
   (c) does not exist because $x - 1 \mapsto 0$
   (d) does not exist because the left-hand limit is not equal to the right-hand limit
5) If $f(x) = \min\{1, x^2, x^3\}$
$(2006, 5M, -1)$
   (a) $f(x)$ is continuous $\forall x \in R$
   (b) $f(x)$ is continuous and differentiable everywhere
   (c) $f(x)$ is not diferentiable at two points
   (d) $f(x)$ is not differentiable at one point
6) Let $L = \lim_{x\to 0} \frac{a-\sqrt{a^2-x^2-\frac{x^2}{4}}}{x^4}, a > 0$.
If L is finite, then
(2009)
   (a) $a = 2$
   (b) $a = 1$
   (c) $L = \frac{1}{64}$
   (d) $L = \frac{1}{32}$
7) Let $f : R \to R$ be a function such that $f(x+y) = f(x) + f(y)$, $\forall x, y \in R$. If $f(x)$ is differentiable at $x = 0$, then
(2011)
   (a) $f(x)$ is differentiable only in a finite interval containing zero
   (b) $f(x)$ is continuous $\forall x \in R$
   (c) $f'(x)$ is constant $\forall x \in R$
   (d) $f(x)$ is differentiable except at finitely many points
8) If $f(x)= \begin{cases} -x - \frac{\pi}{2}, & x \leq \frac{\pi}{2} \\ -\cos x, & \frac{\pi}{2} < x \leq 0 \\ x - 1, & 0 < x \leq 1 \\ \ln x, & x > 1 \end{cases}$
(2011)

(a) $f(x)$ is continuous at $x = \frac{\pi}{2}$
(b) $f(x)$ is not differentiable at $x = 0$
(c) $f'(x)$ is differentiable at $x = 1$
(d) $f(x)$ is differentiable at $x = \frac{3}{2}$

9) For every integer n, let a_n and b_n, be real numbers. Let function $f(x) : IR \mapsto IR$ be given by $f(x)= \begin{cases} a_n + \sin \pi x, & for x \in [2n, 2n+1] \\ b_n + \cos \pi x, & for x \in (2n-1, 2n) \end{cases}$ for all integers n. If f is continuous,then which of the following hold(s) for all n

(2012)

(a) $a_{n-1} - b_{n-1} = 0$
(b) $a_n - b_n = 1$
(c) $a_n - b_{n+1} = 1$
(d) $a_{n-1} - b_n = -1$

10) For $a \in R$ $(the set of all real numbers)$, $a \neq -1$ $\lim_{n\to\infty} \frac{(1^a+2^a+\cdots+n^a)}{(n+1)^a.[(na+1)+(na+2)+\cdots+(na+n)]} = \frac{1}{60}$ Then $a =$

(*JEE Adv.*2013)

(a) 5
(b) 7
(c) $\frac{-15}{2}$
(d) $\frac{-17}{2}$

11) Let $f : [a, b] \mapsto [1, \infty)$ be a continuous function and let $g : R \mapsto R$ be defined as $f(x)= \begin{cases} 0, & if x < a, \\ \int_a^x f(t)\, dt, & if a \leq x \leq b \\ \int_a^b f(t)\, dt, & if x > b \end{cases}$; then

(*JEE Adv.*2013)

(a) $g(x)$ is continuous but not differentiable at a
(b) $g(x)$ is differentiable on R
(c) $g(x)$ is continuous but not differentiable at b
(d) $g(x)$ is continuous and differentiable at either (a) or (b) but not both

12) For every pair of continuous functions $f, g : [0, 1] \mapsto R$ such that max $\{f(x) : x \in [0, 1]\}=$ max $\{g(x) : x \in [0, 1]\}$, the correct statement(s) is(are):

(*JEE Adv.*2014)

(a) $(f(c))^2 + 3.f(c) = (g(c))^2 + 3.g(c)$ for some $c \in [0, 1]$
(b) $(f(c))^2 + f(c) = (g(c))^2 + 3.g(c)$ for some $c \in [0, 1]$
(c) $(f(c))^2 + 3.f(c) = (g(c))^2 + g(c)$ for some $c \in [0, 1]$
(d) $(f(c))^2 = (g(c))^2$ for some $c \in [0, 1]$

13) Let $g : R \mapsto R$ be a differentiable function with $g(0) = 0$, $g'(0) = 0$ and $g'(1) \neq 0$. Let $f(x) = \begin{cases} \frac{x}{|x|}.g(x), & x \neq 0 \\ 0, & x = 0 \end{cases}$ and $h(x) = e^{|x|}$ for all $x \in R$. Let $(f \circ h)(x)$ denote $f(h(x)))$ and $(h \circ f)(x)$ denote $h(f(x))$. Then which of the following is(are) true?

(*JEE Adv.*2015)

(a) f is differentiable at $x = 0$
(b) h is differentiable at $x = 0$
(c) $f \circ h$ is differentiable at $x = 0$
(d) $h \circ f$ is differentiable at $x = 0$

14) Let $a, b \in R$ and $f : R \mapsto R$ be defined by $f(x) = a.\cos\left(|x^3 - x|\right) + b.|x|.\sin\left(|x^3 + x|\right)$. Then f is

(*JEE Adv.*2016)

(a) differentiable at $x = 0$ if $a = 0$ and $b = 1$
(b) differentiable at $x = 1$ if $a = 1$ and $b = 0$
(c) NOT differentiable at $x = 0$ if $a = 1$ and $b = 0$
(d) NOT differentiable at $x = 1$ if $a = 0$ and $b = 1$

15) Let $f : \left[-\frac{1}{2}, 2\right] \mapsto R$ and $g : \left[-\frac{1}{2}, 2\right] \mapsto R$ be functions defined by $f(x) = \left[x^2 - 3\right]$ and $g(x) = |x|.f(x) + |4x - 7|.f(x)$, where $[y]$ denotes the greatest integer less than or equal to y for $y \in R$. Then

(*JEE Adv.*2016)

(a) f is discontinuous exactly at three points in $\left[-\frac{1}{2}, 2\right]$
(b) f is discontinuous exactly at four points in $\left[-\frac{1}{2}, 2\right]$
(c) g is NOT differentiable exactly at four points in $\left[-\frac{1}{2}, 2\right]$
(d) g is NOT differentiable exactly at five points in $\left[-\frac{1}{2}, 2\right]$