Exploratory Analysis

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Introduction

Our dataset comes from a longitudinal study conducted in the United States. Researchers observed the time until divorce of 3371 couples, and tracked three covariates listed below. The data's time variable is measured in years with up to three decimals of precision. The event indicator is labeled 0 for censorship, and 1 for divorce.

- The husband's education level, coded as...
  - 0 -> less than 12 years (only high school)
  - 1 -> 12 to 15 years (only bachelors or equivalent)
  - 2 -> 16 or more years (some form of graduate studies)
- The husband's race, coded as...
  - 1 if the husband is black
  - 0 otherwise
- Whether or not both partners are black, coded as...
  - 1 if both partners are **not** black
  - 0 if one parter is not black, and the other is

We also encoded two new variables using the above data, *wblack* and *couple*.

- The wife's race, coded as...
  - 1 if the wife is black
  - 0 otherwise
- The couple's racial makeup, encoded as...
  - BB if both are black
  - BO if the husband is black, and the wife is not
  - OB if the wife is black, and the husband is not
  - OO if both are not black

Research question

Our team was primarily interested in how the husband's highest education level affects time until divorce, using the various racial covariates as controlling variables. Our secondary goal was to investigate the influence of racial makeups as a stratified variable.

Methodology

After an exploratory analysis, our team began modeling the data with a base Cox-PH model including only the husband's education level. We then constructed various control models using the racial covariates. After establishing that the base model was still significant under the controls, we turned our interest to the possible influence of these racial predictors as stratified variables. Finally, our team investigated the necessity of a time-transform to compensate for the near non-proportionality between two levels of the education predictor. Sadly, the time-transformation offered no additional insight.

Exploratory Analysis of Covariates

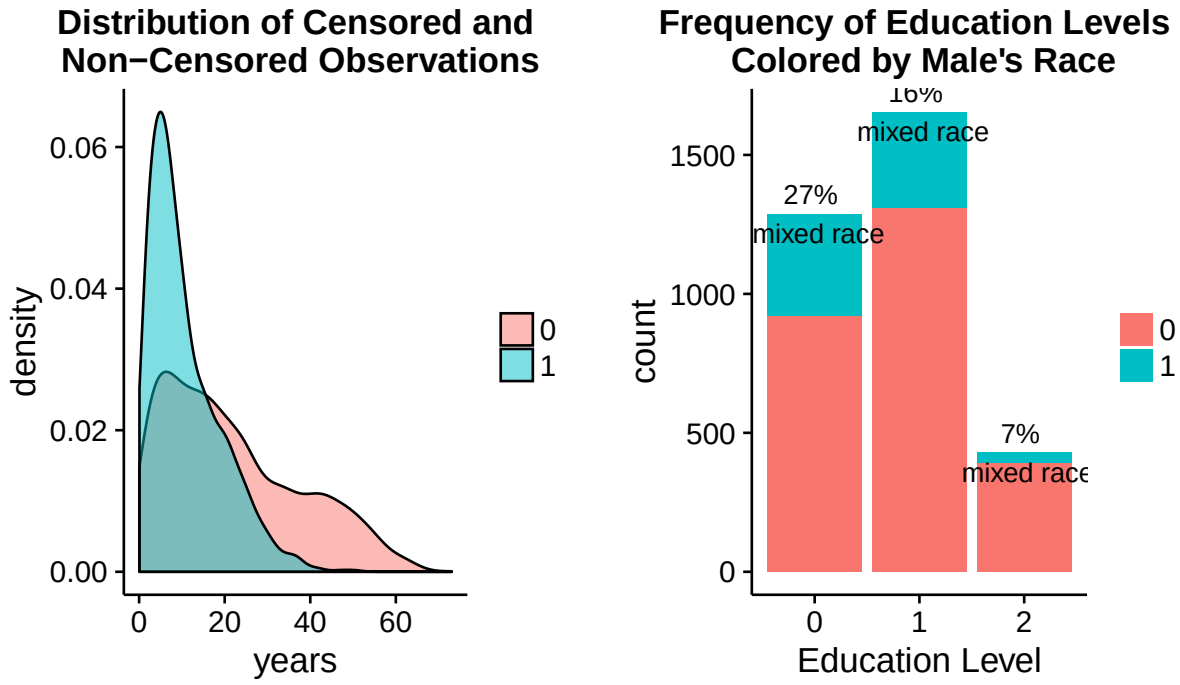
The following plots explore the distributions of our covariates.

```
# Loading necessary libraries
library(ggplot2)
library(survival)
library(cowplot)
library(devtools)
library(ggkm)

# ggkm is not on CRAN, using devtools we can install it from GitHub
# devtools::install_github("sachsmc/ggkm")
# https://github.com/sachsmc/ggkm
```

```
# Load data
colNames <- c("id", "edu", "hblack", "mixed", "years", "div")
divorce <- read.table(file = "divorce.txt", header = F, col.names = colNames)

# Coerce numeric variables
divorce$edu <- as.factor(divorce$edu)
divorce$hblack <- as.factor(divorce$hblack)
divorce$mixed <- as.factor(divorce$mixed)

# Encode race variables
divorce <- femalecol(divorce)
divorce <- couple_column(divorce)

head(divorce)
```

```
##   id edu hblack mixed  years div wblack couple
## 1  9   1      0     0 10.546   0      0     00
## 2 11   0      0     0 34.943   0      0     00
## 3 13   0      0     0  2.834   1      0     00
## 4 15   0      0     0 17.532   1      0     00
## 5 33   1      0     0  1.418   0      0     00
## 6 36   0      0     0 48.033   0      0     00
```

The functions, *femalecol( )* and *couple_column( )* encode the wife's race and the couple's racial makeup respectively. Their levels are explained in the Introduction above.

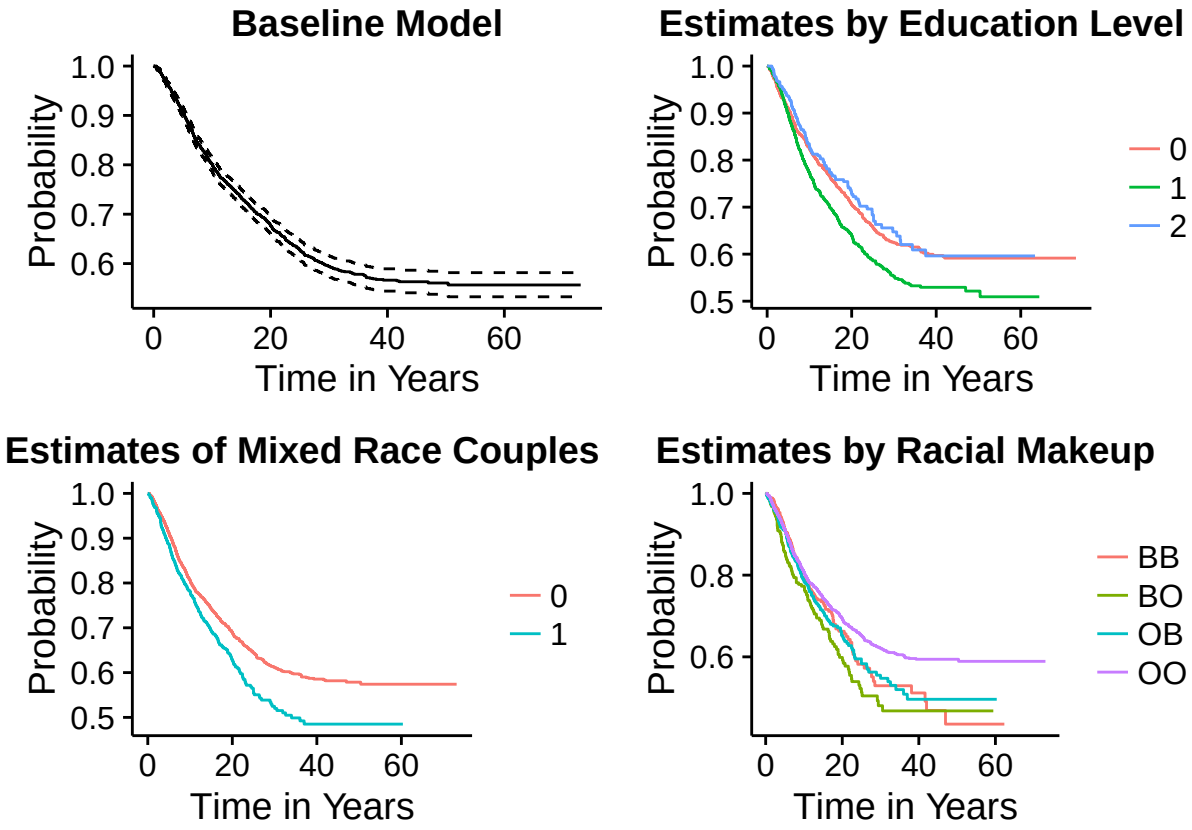


From left to right

Fig. 1 We can see that the bulk of our observed divorces, colored in blue, occur around the ten year mark. Our censored observations, colored in orange, drop steeply at 30 years into the study.

Fig. 2 This plot shows the frequency of different education levels in our data. The bars are colored by their proportion of black husbands within the education level. So we can see that our data is largely composed of couples with husbands that are not African American. Lastly, the percentages over each bar report the percent of mixed race coupled within their respective education bracket. For example, 27% of couples in education bracket **0** are mixed.

Exploratory Survival Curves



Plot analysis clockwise from top left.

Fig. 3 Thanks to the large number of observations, our KM estimate is nearly a smooth line.

Fig. 4 This plot is especially interesting, as one would expect there to be a linear relationship between the education levels and the respective hazard rates... for example, $S_1(X) < S_2(X) < S_3(X)$ (where S_i denotes the survival function of the ith education level). Interestingly, we do not see this trend. The survival rates of couples with a college-educated husband are visibly lower than that of couples with either a high-school educated or graduate-educated husband.

Fig. 5 The *couple* predictor gives us a greater insight than *mixed*. Interestingly, we can see that couples with neither partner being black have a higher survival function.

Fig. 6 Non-mixed couples are likely to survive longer, shown by their higher survival curve and that their marriages carried on longer into the study.

Modeling

```
# Base model using only education
edu_coxph <- coxph(Surv(years, div) ~ edu, data = divorce)
edu_coxph
```

```
## Call:
## coxph(formula = Surv(years, div) ~ edu, data = divorce)
```

```
##
##         coef exp(coef) se(coef)     z       p
## edu1  0.2388    1.2697   0.0669  3.57 0.00036
## edu2 -0.0778    0.9251   0.1080 -0.72 0.47106
##
## Likelihood ratio test=17.4  on 2 df, p=0.00017
## n= 3371, number of events= 1032
```

Although the P-value for the ratio between education levels 0 and 2 (highschool and graduate) is not significant, the model as a whole attains a significant P-value of 0.017%. The high P-value for education levels 0 and 2 is not surprising, however, given how close the two curves are in **Figure 4**.

Control for Racial Makeup

Here we add the racial covariates to previous. Our goal is to ensure that *education* is still a significant predictor in all models.

```
eh_coxph <- coxph(Surv(years,div) ~ edu + hblack, data=divorce)
em_coxph <- coxph(Surv(years,div) ~ edu + mixed, data=divorce)
emb_coxph <- coxph(Surv(years,div) ~ edu + mixed + hblack , data=divorce)
embw_coxph <- coxph(Surv(years,div) ~ edu + mixed + hblack + wblack, data=divorce)
```

```
## Call:
## coxph(formula = Surv(years, div) ~ edu + hblack, data = divorce)
##
##            coef exp(coef) se(coef)     z       p
## edu1     0.2683    1.3077   0.0677  3.97 7.3e-05
## edu2    -0.0210    0.9792   0.1097 -0.19  0.8481
## hblack1  0.2460    1.2789   0.0765  3.22  0.0013
##
## Likelihood ratio test=27.3  on 3 df, p=5.17e-06
## n= 3371, number of events= 1032
```

```
## Call:
## coxph(formula = Surv(years, div) ~ edu + mixed, data = divorce)
##
##            coef exp(coef) se(coef)     z       p
## edu1    0.27822   1.32077  0.06789  4.10 4.2e-05
## edu2   -0.00771   0.99232  0.10993 -0.07 0.94405
## mixed1  0.28333   1.32754  0.07603  3.73 0.00019
##
## Likelihood ratio test=30.6  on 3 df, p=1.03e-06
## n= 3371, number of events= 1032
```

```
## Call:
## coxph(formula = Surv(years, div) ~ edu + mixed + hblack, data = divorce)
##
##           coef exp(coef) se(coef)    z       p
## edu1    0.2928    1.3401   0.0682 4.29 1.8e-05
## edu2    0.0217    1.0220   0.1107 0.20  0.8444
## mixed1  0.2342    1.2640   0.0791 2.96  0.0031
## hblack1 0.1829    1.2008   0.0796 2.30  0.0216
```

```
##
## Likelihood ratio test=35.7  on 4 df, p=3.28e-07
## n= 3371, number of events= 1032
```

```
## Call:
## coxph(formula = Surv(years, div) ~ edu + mixed + hblack + wblack,
##     data = divorce)
##
##           coef exp(coef) se(coef)    z       p
## edu1    0.2936    1.3412   0.0683 4.30 1.7e-05
## edu2    0.0236    1.0239   0.1112 0.21  0.8318
## mixed1  0.2298    1.2583   0.0825 2.78  0.0054
## hblack1 0.1789    1.1959   0.0824 2.17  0.0300
## wblack1 0.0154    1.0156   0.0822 0.19  0.8511
##
## Likelihood ratio test=35.8  on 5 df, p=1.05e-06
## n= 3371, number of events= 1032
```

The R output above confirms that *education* is a significant predictor in all control situations. Now that we have established that *education* alone is a stable covariate, our team decided to investigate the above models further. Our motivation is that one (or more) of the models may in fact be a better choice than *education* alone.

Effect of Racial Makeup

```
edu_BIC <- BIC(edu_coxph)
eh_BIC <- BIC(eh_coxph)
em_BIC <- BIC(em_coxph)
emb_BIC <- BIC(emb_coxph)
embw_BIC <- BIC(embw_coxph)

# A new model using the new "couple"" covariate
edu_couple_coxph <- coxph(Surv(years, div) ~ edu + couple, data = divorce)
edu_couple_BIC <- BIC(edu_couple_coxph)
```

```
## Base model = 15685.14
## Edu + Husband = 15682.16
## Edu + Mixed = 15678.84
## Edu + Mixed + Husband = 15680.64
## Edu + Mixed + Husband + Wife = 15687.54
## Edu + Couple = 15687.54
```

Although the BIC on the model using *education* and *mixed* was the lowest, we decided to continue our analysis using a new variable, *couple* which encodes all the racial makeups into one predictor. Our reasoning is that *couple* encodes far more information than *mixed* and may provide a more insightful model.

To justify our decision, we used a likelihood ratio test to ensure there is no significant difference between the model using the newly encoded predictor, and the model which attained the lowest BIC (em_coxph).

```r
chi_test <- 2*(edu_couple_coxph$loglik[[2]] - em_coxph$loglik[[2]])

# 5 parameters in the bigger model (lots of factor levels)
# 3 parameters in the restricted model
1 - pchisq(chi_test, df = 5-3)
```

```
## [1] 0.07526716
```

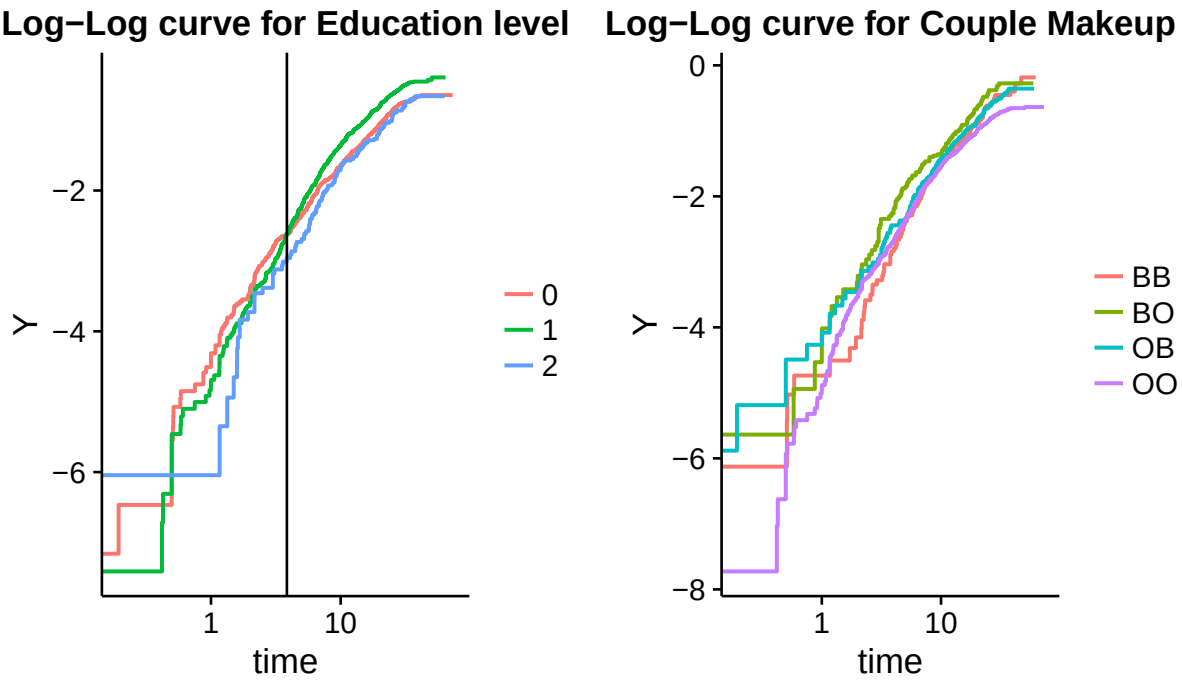
The P-value is 0.075, which is above any common significance level. Therefore, there is no significant difference between the two models using...

- *base* ~ Education + Couple
- *base* ~ Education + Mixed

This allows us to use a more condensed predictor for later analysis. Lastly, it is interesting to note that the model using Education + Mixed + Hblack + Wblack is in fact the same model as Education + Couple. We expected to see this since they encode the same information, but it is encouraging nonetheless.

Checking Proportional Hazard Assumptions

Graphical Approach



Plot analysis from left to right.

Fig. 7 The intersecting lines are concerning because they indicate a possible violation of the proportionality assumption of the Cox PH model. Our team investigated the necessity of a time-transformation on education to account for the issue.

Fig. 8 Again, the intersecting lines are concerning. In this case, however, we will investigate the necessity of stratifying the racial makeup predictor.

Numerical Approach

```
edu_zph <- cox.zph(edu_coxph)
edu_couple_zph <- cox.zph(edu_couple_coxph)

edu_couple_zph
```

```
##              rho  chisq       p
## edu1      0.0307  0.995 0.31854
## edu2      0.0516  2.818 0.09321
## coupleBO -0.0513  2.720 0.09913
## coupleOB -0.0346  1.239 0.26564
## coupleOO -0.0922  9.031 0.00265
## GLOBAL        NA 11.411 0.04382
```

While our numerical test indicates that the education covariate does **not** violate the proportional hazards assumption, we will still investigate the improvement made by including a time-transformation.

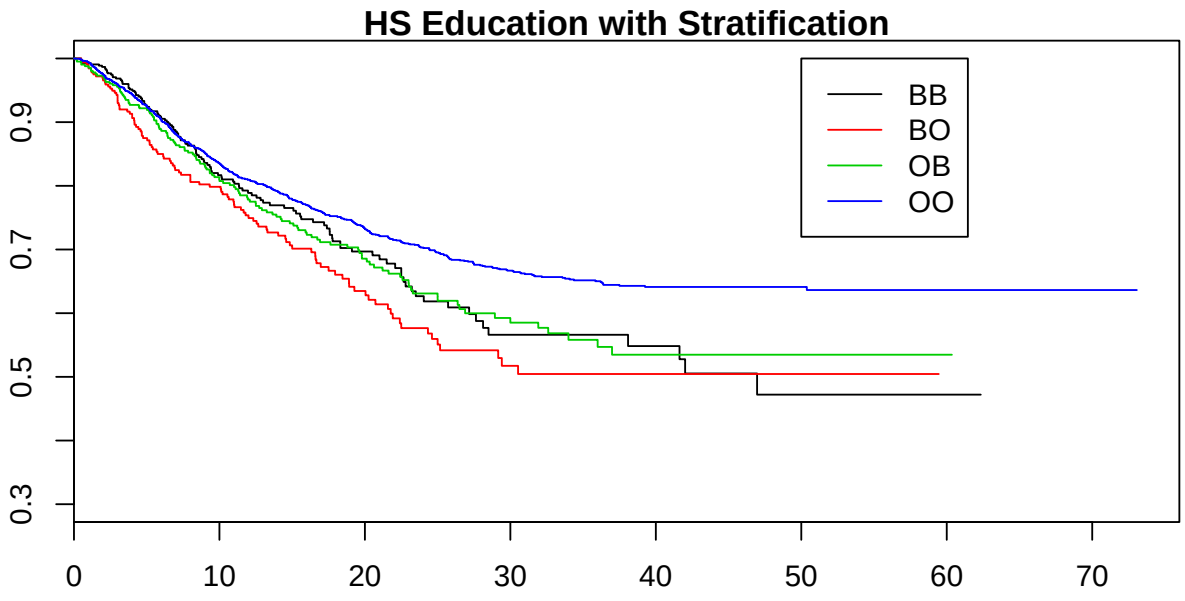
The numerical test for racial makeup of the couple confirms our suspicions from the c-log-log plot. The predictor **does** violate the proportional hazards assumption and we will use it as a stratified variable.

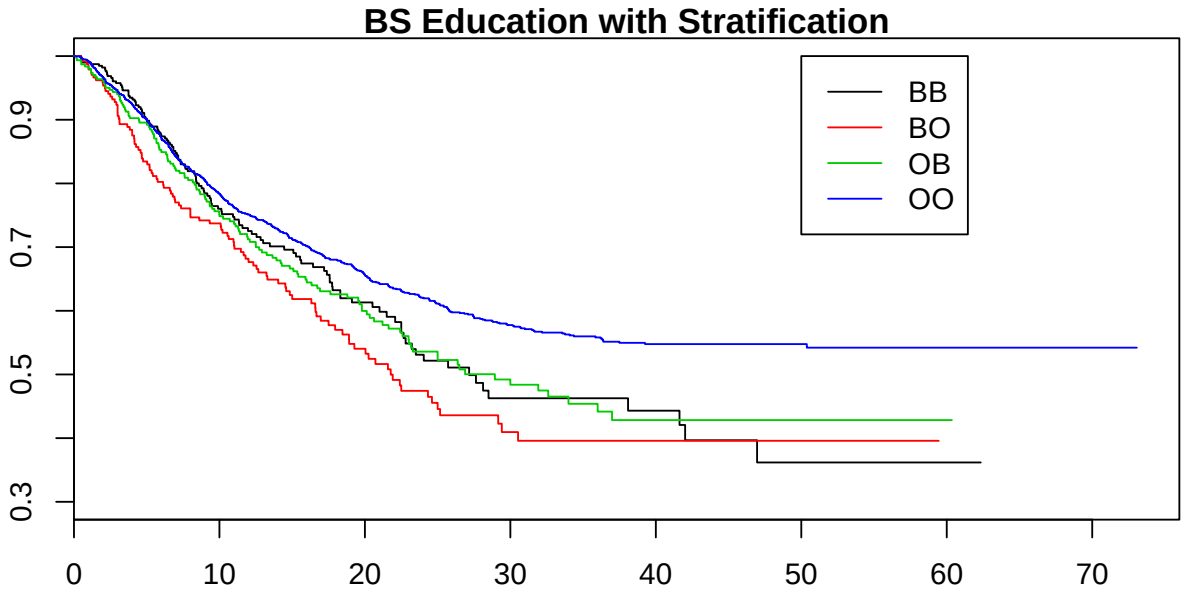
Stratification

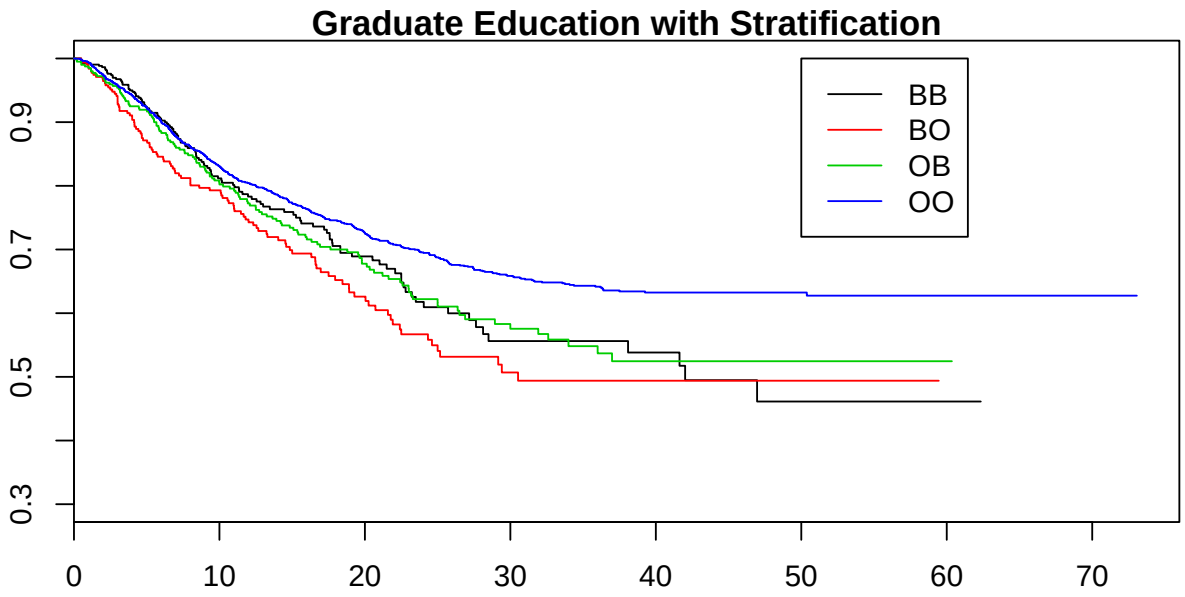
```
# Make Cox and KM models with stratified variable
edu_strata_coxph <- coxph(Surv(years, div) ~ edu + strata(couple), data = divorce)
edu_strata_zph <- cox.zph(edu_strata_coxph)
strata_BIC <- round(BIC(edu_strata_coxph), r)
```

```
##           rho chisq      p
## edu1   0.0332  1.16 0.2804
## edu2   0.0521  2.87 0.0904
## GLOBAL     NA  3.07 0.2151

## Base model BIC = 15685.14
## Strata model BIC = 13543
```

<<<<<<< talk about zph output here >>>>>>>... The numerical tests show that using *couple* as a stratified variable improves on the previous model using *education* and *couple* (as a non-stratified variable).


HS Education with Stratification
BB
BO
OB
OO
0.3
0.5
0.7
0.9
0
10
20
30
40
50
60
70


BS Education with Stratification
BB
BO
OB
OO
0.3
0.5
0.7
0.9
0
10
20
30
40
50
60
70



Plot interpetation from top to bottom. Please note that the y-axis minimum is set to 0.30

Fig. 9

Fig. 10

Fig. 11

Time Transform

Observations split on time == 47, we will use this time transformation (instead of the more involved approach) because it does not introduce a new β.

```
# Split on t = exp(3.85) = 46.99
edu_split_df <- survSplit(Surv(years, div) ~ edu + couple, data = divorce, cut = 46.99, episode = "Ep")
edu_surv_split <- coxph(Surv(years, div) ~ edu:Ep, data = edu_split_df)
```

```
## Warning in fitter(X, Y, strats, offset, init, control, weights = weights, :
## Loglik converged before variable 1,2,3 ; beta may be infinite.
```

```
edu_couple_surv_split <- coxph(Surv(years, div) ~ edu + couple, data = edu_split_df)
edu_surv_split
```

```
## Call:
## coxph(formula = Surv(years, div) ~ edu:Ep, data = edu_split_df)
##
##              coef exp(coef)  se(coef)     z    p
## edu0:Ep -1.73e+01  3.14e-08  4.88e+02 -0.04 0.97
## edu1:Ep -1.70e+01  4.00e-08  4.88e+02 -0.03 0.97
## edu2:Ep -1.74e+01  2.91e-08  4.88e+02 -0.04 0.97
##
## Likelihood ratio test=251  on 3 df, p=0
## n= 3589, number of events= 1032
```

Final Summary

For this project we decided to explore if and how a husband's education affects divorce rates over time while taking into account the race of the couple. Our goal was to find the most significant model using various statistical methods that we learned in class as well as from our textbook. We determined that education definitely has an impact on divorce rate and then tested how other variables contributed to our model. As a final model we chose to include education and mixed race (stratified) as the variable of interest and control respectively. Our data tells us that when the husband has a bachelor's degree the marriage will fail 1.3 times faster than if he had a highschool degree. When having a Master's degree or above the marriage will fail .99 times faster in comparison to a highschool degree (Roughly the same rate). This tells us that getting a Master's degree or above will