We consider the equations

$$u_{tt} - \beta u_{xx} = \alpha (u_x)^2 \ , \tag{0.1}$$

for $t \geq 0$ and $x \in \mathbb{R}$. When $\beta \neq 0$ we can change variables to obtain a normalized equation

$$v_{tt} - v_{xx} = \frac{\alpha}{\beta}(v_x)^2 \ , \tag{0.2}$$

with $v(x,t) = u(x/\sqrt{\beta}, t)$. When $\beta = 0$, we consider the equation

$$w_{tt} = \alpha (w_x)^2 \ , \tag{0.3}$$

When $\beta \neq 0$ we are interested in the interplay of two types of divergence, *local* divergence, and *wave divergence*. In the local divergence, u will diverge near $x = 1$ like a special solution of (**??**). In the wave divergence, there will be a divergence near the front of the advancing wave.

We illustrate the phenomenology with two numerical simulations:

We expect that, for fixed β a transition between local divergence and wave divergence will appear. The mathematical situation for the wave divergence can be explained by an adaptation of the work of Rammaha **?**. He shows that (**??**) will diverge in finite time under quite weak conditions: We adapt the proof to the 1d case in in Theorem **??**, which will be given at the end.

Take as initial conditions smooth functions $u(x,0) = f(x)$ and $u_t(x,0) = g(x)$, both with compact support and assume the support is in $|x| < X$. Fix an $X_0 < X$ and define

$$\varepsilon = \tfrac{1}{2} \int_{x > X_0} (f(x) + (x - X_0) g(x)) \mathrm{d}x \ .$$

with

$$M(x) = \tfrac{1}{2} \left(f(x) + \int_x^X g(\xi) \mathrm{d}\xi \right) > 0$$

for all $x \in (X_0, X)$. We show that the solution must diverge before a time T_*. We do not aim to get the best possible bound on this time.

Rammaha's result can be paraphrased as follows. The point of the proofs is that because of the wave character of (**??**), the solution will grow at the advancing front of the wave. The qualitative behavior is clearly seen in Fig. **??**.

This discussion settles the question of what must happen at the advancing fronts. However, there can be a local divergence which can appear earlier in the "center" between the waves.

We will now argue that for $\beta = 0$ there are (admittedly unbounded) initial conditions for which local divergence will appear in arbitrarily short time. In particular, this time can be shorter than the wave divergence time.

This divergence is modeled most simply when $\beta = 0$. In this case, taking the (unbounded) initial condition $u(x,0) = Ax^2$, Eq.(**??**) has the explicit solution

$$u(x,t) = \frac{3}{2\alpha (t - t_0)^2} x^2 \ ,$$

when

$$u(x,0) = \frac{3}{2\alpha t_0^2} x^2 \ ,$$

peter.png

Figure 1: Local divergence:The parameters of (**??**) are $\alpha = 5$ and $\beta = 0.025$ with initial condition $u(x,0) = -\exp(-30(x-1)^2)$, $u_t(x,0) = 0$. (Numerically we take periodic boundary conditions.) Note that the second derivative diverges in finite time. The simulation corresponds to $\alpha/\beta = 200$.

$$u_t(x,0) = \frac{3}{\alpha t_0^3} x^2 \ .$$

Clearly, t_0 is then just given by (This result is nice and ok, but it does not work when we want to have $u(x,0) = 0$, which is the initial condition that Peter considered. There they showed that the curvature of the minimum blowsup.)JPE: need to discuss what to do here

JPE:probably omit what comes next

$$t_0 = 2\frac{u(x,0)}{u_t(x,0)} \ .$$

So we can make t_0 arbitrarily short as announced. Finally, in perturbation theory (up to order x^4 included) one can solve (**??**) explicitly (when $\alpha/\beta = 1$) in the form

$$u(x,t) = a(t)\left((x-t)^2 + (x+t)^2\right) + b(t)x + c(t) \ ,$$

divergence.png

Figure 2: Wave divergence:The nature of divergence "at infinity." As time goes on, the support of the function spreads, and the function at the edge gets steeper, until the derivative will diverge at some time T_*. The simulation is for the variant equation (**??**), with $\alpha = 0.01$ and $\beta = 0.025$ and initial condition $u(x,0) = -\exp(-30(x-1)^2)$, $u_x(x,0) = 0$. (Numerically we take periodic boundary conditions.) Now, $\alpha/\beta = 0.4$

with

$$
\begin{aligned}
a(t) &= \frac{3}{4A(t-t_0)^2}\,, \\
b(t) &= C_1(t-t_0)^3 + C_2\frac{1}{(t-t_0)^2}\,, \\
c(t) &= 3\frac{\log(t_0-t)}{A} - 3\frac{t_0}{A(t-t_0)} - 3\frac{t_0^2}{2A(t-t_0)^2} + C_3 t + C_4\,.
\end{aligned}
$$

Therefore, to this order, one can also produce arbitrary rapid local divergence, just slightly shifted (by t_0) from the origin. (I don't understand the previous argument. The solution does not give us a reasonable behaviour. Because at $t = t_0$ it seems that the logarithm also has problem.)

1 Wave divergence

Consider the equation

$$u_{tt} - u_{xx} = u_x^2 \ .$$

Assume the initial conditions are

$$u(x,0) = f(x) \ , \quad u_t(x,0) = g(x) \ ,$$

with f, g having support in $|x| < X$. Define, for $x > 0$,

$$M(x) = \tfrac{1}{2} f(x) + \tfrac{1}{2} \int_x^X g(\xi) \mathrm{d}\xi \ .$$

Theorem 1.1. *Assume there is an $X_0 \in (0, X)$ for which $M(x) \geq 0$ for all $x \in (X_0, X)$, and also*

$$\int_{X_0}^X M(\xi) \mathrm{d}\xi \equiv \varepsilon > 0 \ .$$

Then the local existence time is **finite***.*

Proof. It suffices to consider the situation where $u(x,0)$ and $u_t(x,0)$ have support in $|x| \leq X$, but we will consider only the side of positive x in the sequel. Clearly, $u(x,t) = 0$ for $x \geq t+X$. One estimates now the function

$$H(t) = \int_{X_1}^t (t - \tau) \int_{\tau + X_0}^{\tau + X} u(\xi, \tau) \mathrm{d}\xi \, \mathrm{d}\tau \ .$$

Here, $X_1 = (X - X_0)/2$. From the definition, we get

$$H''(t) = \int_{t+X_0}^{t+X} u(\xi, t) \mathrm{d}\xi \ . \tag{1.1}$$

One has the explicit formula

$$u(x,t) = u_0(x,t) + \tfrac{1}{2} \int_0^t \int_{x-t+\tau}^{x+t-\tau} u_x(\xi, \tau)^2 \mathrm{d}\tau \, \mathrm{d}\xi \ , \tag{1.2}$$

with the "free evolution"

$$u_0(x,t) = \tfrac{1}{2} \left(f(x-t) + f(x+t) \right) + \tfrac{1}{2} \int_{x-t}^{x+t} g(\xi) \mathrm{d}\xi \ .$$

When $x \geq t + X_0$ and $X \geq t \geq X_1$, then $x + t \geq X$ and therefore $f(x+t) = 0$ and therefore, in this region,

$$u_0(x,t) = \tfrac{1}{2} f(x-t) + \tfrac{1}{2} \int_{x-t}^{x+t} g(\xi) \mathrm{d}\xi \ .$$

We get from (**??**) and (**??**),

$$H''(t) = G_0(t) + G_1(t) \ ,$$

with

$$G_0(t) = \int_{t+X_0}^{t+X} u_0(x,t) \mathrm{d}x = \int_{t+X_0}^{t+X} M(x-t) \mathrm{d}x = \int_{X_0}^X M(x) \mathrm{d}x = \varepsilon \ ,$$

by the definition of ε in Theorem **??**. The nonlinearity leads to

$$G_1(t) = \int_{t+X_0}^{t+X} \mathrm{d}\xi \int_0^t \mathrm{d}\tau \int_{\xi-t+\tau}^{\xi+t-\tau} \mathrm{d}\eta \, u_x(\eta,\tau)^2 \, .$$

In Lemma **??** below, we show

$$G_1(t) \geq \frac{1}{t+X_0} \int_{X_1}^t \mathrm{d}\tau \int_{\tau+X_0}^{\tau+X} \mathrm{d}\xi \, (t-\tau) \, (\xi-\tau-X_0) \, u_x(\xi,\tau)^2 \, . \tag{1.3}$$

We use now the Schwarz inequality in the form

$$\int \varphi\psi = \int \varphi^{1/2}(\varphi^{1/2}\psi) \leq \left(\int \varphi\psi^2\right)^{1/2} \, (\int \varphi)^{1/2} \, ,$$

with $\varphi = (t-\tau)(\xi-\tau-X_0)$ and $\psi = u_x$. This leads to

$$G_1(t) \geq F^2(t)/J(t) \, ,$$

with

$$F(t) = \int_{X_1}^t \int_{\tau+X_0}^{\tau+X} (t-\tau)(\xi-\tau-X_0) u_x(\xi,\tau) \mathrm{d}\xi \, \mathrm{d}\tau$$

and

$$J(t) = \int_{X_1}^t \int_{\tau+X_0}^{\tau+X} (t-\tau)(\xi-\tau-X_0) \mathrm{d}\tau \, \mathrm{d}\xi = \frac{(X-X_0)^2(t-X_1)^2}{4} \, .$$

If we integrate the expression for F by parts (in ξ), we get

$$F(t) = -\int_{X_1}^t \int_{\tau+X_0}^{\tau+X} (t-\tau) u(\xi,\tau) \mathrm{d}\xi \, \mathrm{d}\tau = -H(t) \, .$$

Therefore, we find finally

$$H''(t) \geq G_0(t) + \frac{H(t)^2}{J(t)} \, . \tag{1.4}$$

Fix now T and we will show that the solution cannot exist for $T > T_*$, where T_* will be computed in the proof: We use here Lemma 1 from **?** adapted to the 1d case. The ingredients are that

$$H''(t) \geq G_0(t) = \varepsilon > 0 \, , \tag{1.5}$$

for all $t \geq 0$ and

$$H''(t) \geq G_1(t) \geq 4\frac{H(t)^2}{(X-X_0)^2(t-X_1)^2} \, , \tag{1.6}$$

for $t > X_1$ (as long as the solution exists). Furthermore, $H(X_1) = H'(X_1) = 0$.

Fix now $T_1 = 2(X_1+1)$. Then, for $t > T_1$, we have $t - X_1 > \frac{1}{2}(t+1)$, and we replace from now on (**??**) by the simpler

$$H''(t) \geq K_1 \frac{H(t)^2}{(t+1)^2} \, , \text{ for } t > T_1 \, , \tag{1.7}$$

with $K_1 = 16/(X-X_0)^2$.

The idea is now to deduce from (**??**) and (**??**) an inequality of the form

$$H'(t) \geq C H^{1+\delta}(t) \text{ for } t > T_* \text{ with } \delta > 0 \, . \tag{1.8}$$

This implies divergence in finite time, when $H(T_0) > 0$. Indeed, if $H(T_0) = c^{-1/\delta} > 0$, then

$$H(t) = \frac{1}{(c - C\delta(t-T_0))^{1/\delta}} \; . \tag{1.9}$$

One can reformulate this as follows:JPE:But we dont know $H(T_0)$... If $H(T_0) = A$ and $A \leq 1/e$, the optimizing δ in (**??**) is ≤ 1 and therefore we find that the divergence time is proportional to $-\log(A)$. Note that, if, for example, the leading edge of the support (at $x = 0$) is like $|x|^2$ for $x < 0$, then this will lead to earlier divergence compared to $|x|^3$.

We now begin the proof proper. If $B > 0$, we will use repeatedly the inequality

$$\frac{x}{x+B} \geq \tfrac{1}{2} \; , \text{ for all } x \geq B \; . \tag{1.10}$$

From (**??**) we find

$$H(t) \geq K_2 \varepsilon t^2 \; , \text{ for all } t > 0 \; , \tag{1.11}$$

with $K_2 = \frac{1}{2}$.

Substituting (**??**) into (**??**), we get

$$H''(t) \geq K_1 K_2 \varepsilon H(t) \frac{t^2}{(t+1)^2} \geq K_3 \varepsilon H(t) \; , \text{ when } t > T_2 \; , \tag{1.12}$$

for some large enough $T_2 = \text{const.}T_1$, not depending on ε. Since $H'(t) > 0$, we can multiply (**??**) by H' and write it as

$$\frac{\mathrm{d}}{\mathrm{d}t}(H'(t)^2) \geq K_3 \varepsilon \frac{\mathrm{d}}{\mathrm{d}t}(H(t)^2) \text{ when } t > T_2 \; .$$

We integrate from T_2 to t and obtain

$$H'(t)^2 \geq K_3 \varepsilon \left(H(t)^2 + H'(T_2)^2 - H(T_2)^2 \right) = K_3 \varepsilon H(t)^2 + K_4 \varepsilon \text{ when } t > T_2 \; ,$$

for some K_4. From (**??**), we conclude that for large enough T_3, one has

$$K_3 \varepsilon H(t)^2 + K_4 \varepsilon \geq K_5 \varepsilon H(t)^2 \; , \text{ when } t > T_3 \; .$$

Combining the last two equations we find

$$H'(t) \geq K_5^{1/2} \varepsilon^{1/2} H(t), \quad \text{when } t > T_3 \; .$$

Integrating from T_3 to t this leads to

$$H(t) \geq H(T_3) \exp\left(K_6 \varepsilon^{1/2}(t - T_3) \right) \geq H(T_3) \exp\left(\tfrac{1}{2} K_6 \varepsilon^{1/2} t \right) \; , \text{ when } t > 2T_3, \tag{1.13}$$

with $K_6 = K_5^{-1/2}$. Substituting again into (**??**), we get

$$H''(t) \geq K_7 H(t)^{1+\delta} \; , \text{ for any } \delta > 0 \; ,$$

since the exponential in (**??**) (to the power $\delta > 0$) will dominate the factor $(t+1)^{-2}$ of (**??**), if only t is sufficiently large. (Note that K_7 and this new minimal time T_4 will depend on δ.) We now multiply the last equation by H' and we get

$$\frac{\mathrm{d}}{\mathrm{d}t}(H'(t)^2) \geq \frac{2K_7}{2+\delta} \frac{\mathrm{d}}{\mathrm{d}t} \left(H(t)^{2+\delta} \right) \; , \text{ for } t > T_4 \; .$$

Integrating from T_4 to t we find

$$H'(t)^2 \geq \frac{2K_1}{2+\delta} \left(H(t)^{2+\delta} - H(T_4)^{2+\delta} \right) + H'(T_4)^2 \; .$$

Taking square roots on both sides and choosing T_* sufficiently larger than T_4, we finally arrive at (**??**) from which we see that there is divergence in finite time, as in (**??**). $\square$

We still need to show the inequality (**??**).

Lemma 1.2. *Let*

$$G_1(t) = \int_{t+X_0}^{t+X} \mathrm{d}x \int_0^t \mathrm{d}\tau \int_{x-t+\tau}^{x+t-\tau} \mathrm{d}\xi \, u_x(\xi,\tau)^2 \, .$$

Let $X > X_0 > 0$, and assume $u_x(x,t) = 0$ for all $|x| \geq t + X$, $0 \leq t \leq T$. Then one has for all $T \geq t \geq X_1 = (X - X_0)/2$, the inequality

$$G_1(t) \geq \frac{1}{t+X_0} \int_{X_1}^t \mathrm{d}\tau \int_{\tau+X_0}^{\tau+X} \mathrm{d}\xi \, (t-\tau)\,(\xi-\tau-X_0)\, u_x(\xi,\tau)^2 \, . \tag{1.14}$$

Proof.

$$\begin{aligned} G_1(t) &= \int_{t+X_0}^{t+X} \mathrm{d}x \int_0^t \mathrm{d}\tau \int_{x-t+\tau}^{x+t-\tau} \mathrm{d}\xi \, u_x(\xi,\tau)^2 \\ &= \int_0^t \mathrm{d}\tau \int_{t+X_0}^{t+X} \mathrm{d}x \int_{x-t+\tau}^{x+t-\tau} \mathrm{d}\xi \, u_x(\xi,\tau)^2 \, . \end{aligned}$$

Note that for $t \geq X_1$, the function $u_x(\xi,\tau)$ vanishes for $\xi \geq \tau + X$. Therefore, since also $2t - \tau \geq \tau$, we find

$$\begin{aligned} &\int_0^{t-X_1} \mathrm{d}\tau \int_{t+X_0}^{t+X} \mathrm{d}x \int_{x-t+\tau}^{x+t-\tau} \mathrm{d}\xi \, u_x(\xi,\tau)^2 \\ &= \int_0^{t-X_1} \mathrm{d}\tau \int_{\tau+X_0}^{2t+X-\tau} \mathrm{d}\xi \int_{t+X_0}^{\xi+t-\tau} \mathrm{d}x \, u_x(\xi,\tau)^2 \\ &= \int_0^{t-X_1} \mathrm{d}\tau \int_{\tau+X_0}^{\tau+X} \mathrm{d}\xi \, u_x(\xi,\tau)^2 \int_{t+X_0}^{\xi+t-\tau} 1 \, \mathrm{d}x \, . \end{aligned}$$

Similarly,

$$\begin{aligned} &\int_{t-X_1}^t \mathrm{d}\tau \int_{t+X_0}^{t+X} \mathrm{d}x \int_{x-t+\tau}^{x+t-\tau} \mathrm{d}\xi \, u_x(\xi,\tau)^2 \\ &= \int_{t-X_1}^t \mathrm{d}\tau \int_{\tau+X_0}^{2t-\tau+X_0} \mathrm{d}\xi \, u_x(\xi,\tau)^2 \int_{t+X_0}^{\xi+t-\tau} 1 \mathrm{d}x \\ &\quad + \int_{t-X_1}^t \mathrm{d}\tau \int_{2t-\tau+X_0}^{\tau+X} \mathrm{d}\xi \, u_x(\xi,\tau)^2 \int_{\xi-t+\tau}^{\xi+t-\tau} 1 \mathrm{d}x \, . \end{aligned}$$

Therefore, when $t \geq X_1$ the function G_1 can be decomposed as

$$\begin{aligned} G_1(t) &= \int_0^{t-X_1} \mathrm{d}\tau \int_{\tau+X_0}^{\tau+X} \mathrm{d}\xi \, u_x(\xi,\tau)^2 \int_{t+X_0}^{\xi+t-\tau} 1 \mathrm{d}x \\ &\quad + \int_{t-X_1}^t \mathrm{d}\tau \int_{\tau+X_0}^{2t-\tau+X_0} \mathrm{d}\xi \, u_x(\xi,\tau)^2 \int_{t+X_0}^{\xi+t-\tau} 1 \mathrm{d}x \\ &\quad + \int_{t-X_1}^t \mathrm{d}\tau \int_{2t-\tau+X_0}^{\tau+X} \mathrm{d}\xi \, u_x(\xi,\tau)^2 \int_{\xi-t+\tau}^{\xi+t-\tau} 1 \mathrm{d}x \, . \end{aligned}$$

The integrals over $\mathrm{d}x$ lead to

$$\begin{aligned}G_1(t) = &\int_0^{t-X_1} \mathrm{d}\tau \int_{\tau+X_0}^{\tau+X} \mathrm{d}\xi\ (\xi - \tau - X_0)\, u_x(\xi,\tau)^2 \\ &+ \int_{t-X_1}^{t} \mathrm{d}\tau \int_{\tau+X_0}^{2t-\tau+X_0} \mathrm{d}\xi\, (\xi - \tau - X_0)\, u_x(\xi,\tau)^2 \\ &+ \int_{t-X_1}^{t} \mathrm{d}\tau\, 2(t-\tau) \int_{2t-\tau+X_0}^{\tau+X} \mathrm{d}\xi\ u_x(\xi,\tau)^2 \,.\end{aligned}$$

Since

$$t - \tau \le t + X_0 \ , \ \text{and}\ \ \xi - \tau - X_0 \le t + X_0 \ ,$$

and both are positive, we find finally that

$$G_1(t) \ge \frac{1}{t+X_0} \int_{X_1}^{t} \mathrm{d}\tau \int_{\tau+X_0}^{\tau+X} \mathrm{d}\xi\, (t-\tau)\, (\xi - \tau - X_0)\, u_x(\xi,\tau)^2 \,.$$

□

2 Wittwer

Apart from the obviously diverging solution (9) (in PRL) there is a more general argument which shows that, locally, for any non-trivial initial condition (??) the solution of $u_{tt} = (u_x)^2$ will blow up in finite time. This is seen as follows: Differentiate the equation twice w.r.t. x, and this leads to

$$\begin{aligned}(u_{tt}) &= (u')^2 \ ,\\ (u_{tt})' &= 2u'u'',\\ (u_{tt})'' &= 2(u'')^2 + 2u'u''' \ .\end{aligned}$$

Define now $a(t) = u'(0,t)$ and $b(t) = u''(0,t)$. Then if $x = 0$ is an extremum of u at $t = 0$ we have $a(0) = \dot{a}(0) = 0$ and furthermore $\ddot{a} = 2a \cdot b$ (by the second equation above). Obviously, we have $a(t) = 0$ for all t for which the solution exists, and this means that the extremum does not move away from $x = 0$. But the equation for b is more interesting. Since $\ddot{b} = 2b^2 + 0$, we find

$$\ddot{b}\dot{b} = 2b^2\dot{b} \ ,$$

and therefore

$$\frac{1}{2}(\dot{b}) = \frac{2}{3}b^3 + c \ .$$

This diverges in finite time, as soon as $b(0)$ and $\dot{b}(0)$ do not vanish simultaneously. And the divergence time is an elliptic integral of the form ???

$$\int_{b(0)}^{\infty} \frac{\mathrm{d}b}{\sqrt{\frac{4}{3} + x}},$$

(I think $x = \dot{b}(0)$ or something like that...)

Let's discuss whether an extremum at initial time would remain an extremum at all times?

Let's try to show that if a point is an extremum it always remain an extremum! To do so we use the following equation:

$$a''(t) = 2a(t)b(t) ,$$

Where $a(t) = u_x(t)|_{x_0}$, $b(t) = u_{xx}(t)|_{x_0}$. At time $t = 0$ since the point is an extremum so $a_{tt}(t)|_{t=0} = a(t = 0) = 0$, if $a_t(t)|_{t=0}! = 0$ the extrema can move in space indeed, cant?

Now assume we have an extremum at point $x = x_0$ at all times, i.e. $(\frac{du}{dx})(t)_{x_0} = 0$, now lets write down all the relevant equations we obtain employing this assumption:

$$\begin{aligned}
(u_{tt})|_{x_0} &= (u')^2|_{x_0} = 0 , \\
(u_{tt})'|_{x_0} &= 2u'|_{x_0}u''|_{x_0} = 0 , \\
(u_{tt})''|_{x_0} &= 2(u'')^2|_{x_0} + 2u'u'''|_{x_0} = 2(u'')^2|_{x_0} , \\
(u_{tt})'''|_{x_0} &= 4u'''|_{x_0}u''|_{x_0} .
\end{aligned}$$

On the other hand we also have,

$$\begin{aligned}
(u_{tt})|_{x_0} &= (u')^2|_{x_0} = 0 , \\
(u_{ttt})|_{x_0} &= 2u'|_{x_0}u'_t|_{x_0} = 0 , \\
(u_{tttt})|_{x_0} &= 2u'_t|_{x_0}u'_t|_{x_0} + 2u'|_{x_0}u'_{tt}|_{x_0} = 2u'_t|_{x_0}u'_t|_{x_0} , \\
(u_{ttttt})|_{x_0} &= 4u'_t|_{x_0}u'_{tt}|_{x_0} + 2u'_t|_{x_0}u'_{tt}|_{x_0} + 2u'|_{x_0}u'_{ttt}|_{x_0} = 0 .
\end{aligned}$$

If we consider $\alpha(t) = u(t)|_{x_0}$, $a(t) = u_x(t)|_{x_0}$, $b(t) = u_{xx}(t)|_{x_0}$, $c(t) = u_{xxx}(t)|_{x_0}$ we have,

$$\begin{aligned}
\ddot{\alpha}(t) &= 0 , \\
\ddot{a}(t) &= 0 , \\
\ddot{b}(t) &= 2b(t)^2 , \\
\ddot{c}(t) &= 2b(t)c(t) .
\end{aligned}$$

and,

$$\begin{aligned}
\dddot{a}(t) &= 0 , \\
a^{(4)}(t) &= 2a(t)^2 , \\
a^{(5)}(t) &= 0 ,
\end{aligned}$$

$\ddot{\alpha}(t) = 0$ implies that $u(t)|_{x_0} = \frac{du}{dt}(t_0)|_{x_0}t + u(t_0)|_{x_0}$! If the initial condition for $\frac{du}{dt}(t_0)|_{x_0}$ is 0 then the value of the extrema do not change! Right? otherwise it should change!