2 The equations

We consider the equations

$$u_{tt} - \beta u_{xx} = \alpha (u_x)^2 \,, \quad \alpha \neq 0 \,, \tag{2.1}$$

for $t \geq 0$ and $x \in \mathbb{R}$. When $\beta \neq 0$ we define $u(x,t) = \frac{\beta}{\alpha} v(\sqrt{|\alpha|} x/\beta, |\alpha| t/\beta)$ and obtain

$$v_{tt} - v_{xx} = (v_x)^2 \,. \tag{2.2}$$

Thus, it suffices to consider (2.2).
When $\beta = 0$, we consider the equation

$$w_{tt} = (w_x)^2 \,. \tag{2.3}$$

Equations like (2.1)-(2.3) have been studied in great detail, but mostly in dimensions 2 and higher. Here, we are interested in the questions of divergence of such equations (in finite time). Such questions are of interest in cosmological models. The behavior if the solutions can be quite different in the case $\beta = 0$ and $\beta \neq 0$. When $\beta = 0$, there is no wave equation and therefore we are confronted with what is essentially a set of local problems.

In the next section we discuss $\beta = 0$ and after that, we discuss $\beta \neq 0$.