Trying wave at $x = 0$. Consider, locally

$$u(x,t) = a(t) + b(t)x^2 \ .$$

The equation

$$u_{tt}(x,t) = A(u_x(x,t))^2 + u_{xx}(x,t)$$

leads to $A(u_x(x,t))^2 = 4Ab(t)^2x^2$ and $u_{xx}(x,t) = 2b(t)$. Thus we get the coupled equations

$$\begin{aligned} \ddot{a}(t) &= 2b(t) \ , \\ \ddot{b}(t) &= 4Ab(t)^2 \ . \end{aligned}$$

The initial conditions are $u(x,0) = 0$, and $u_t(x,0) = \mu + \nu x^2$, locally near 0. This corresponds to

$$a(0) = b(0) = 0 \ , \quad \text{and} \quad \dot{a}(0) = \mu, \dot{b}(0) = \nu \ ,$$

or

$$\mathrm{d}t = \frac{\mathrm{d}b}{\sqrt{\frac{8A}{3}b^3 + c}} \ .$$

Similar to the $\beta = 0$ case, we require here $\nu > 0$, qnd also $A < 0$. We find

$$\tfrac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(\dot{b})^2 = \tfrac{4}{3}A\frac{\mathrm{d}}{\mathrm{d}t}(b^3) \ .$$

Since $b(0) = 0$, we find from $\dot{b}(0) = \nu$ that $c = \nu^2$, so that

$$t_* = \int_0^\infty \frac{\mathrm{d}b}{\sqrt{\frac{8A}{3}b^3 + c}} \sim 2.022296539\, A^{-1/3}\, c^{-1/6} = 2.022296539\, A^{-1/3}\, \nu^{-1/3} \ .$$

This says nothing about any divergence of the advancing front.