Trying wave at $x = 0$. Consider, locally

$$u(x,t) = a(t) + b(t)x^2 \ .$$

The equation

$$u_{tt}(x,t) = A(u_x(x,t))^2 + u_{xx}(x,t)$$

leads to $A(u_x(x,t))^2 = 4Ab(t)^2x^2$ and $u_{xx}(x,t) = 2b(t)$. Thus we get the coupled equations

$$\ddot{a}(t) = 2b(t) \ ,$$
$$\ddot{b}(t) = 4Ab(t)^2 \ .$$

The initial conditions are $u(x,0) = 0$, and $u_t(x,0) = \mu + \nu x^2$, locally near 0. This corresponds to

$$a(0) = b(0) = 0 \ , \quad \text{and} \quad \dot{a}(0) = \mu, \dot{b}(0) = \nu \ .$$

Maple says that the solutions are Weierstrass P functions, but the result is bizarre.

Anyway, I think that numerically one can easily integrate this problem, and, because $A > 0$ and $\dot{b}(0) > 0$ (positive curvature), I believe that, indeed the second derivative diverges in some finite time (at 0). This says nothing about any divergence of the adwancing front.

What do you think?